

Chapter Two :Theory of Consumer Behaviors

Chapter objectives

- ✓ After successful completion of this chapter, you will be able to:
- ✓ Explain consumer preferences and utility
- ✓ Differentiate between cardinal and ordinal utility approach
- ✓ Define indifference curve and discuss its properties
- ✓ Derive and explain the budget line
- ✓ Describe the equilibrium condition of a consumer

2.1 Consumer preferences

- ✓ A consumer makes choices by comparing bundle of goods.
- *Strict preference*
 - Given any two consumption bundles, the consumer either decides that one of the consumption bundles is strictly better than the other, or decides that she is indifferent between the two bundles.
 - If she always chooses X when Y is available, then it is natural to say that this consumer prefers X to Y .
 - We use the symbol $>$ to mean that one bundle is strictly preferred to another, so that $X > Y$ should be interpreted as saying that the consumer strictly prefers X to Y .
 - Given any two consumption bundles (X_1, X_2) and (Y_1, Y_2) , if $(X_1, X_2) > (Y_1, Y_2)$ or if he chooses (X_1, X_2) when (Y_1, Y_2) is available, the consumer definitely wants the X bundle than Y .

2.1 Consumer Preference

- *Weak preference*

- Given any two consumption bundles (X_1, X_2) and (Y_1, Y_2) , if the consumer is indifferent between the two commodity bundles or if $(X_1, X_2) \succsim (Y_1, Y_2)$ or $(Y_1, Y_2) \succsim (X_1, X_2)$, the consumer would be equally satisfied if he consumes (X_1, X_2) or (Y_1, Y_2)

- *Completeness*

- For any two commodity bundles X and Y , a consumer will prefer X to Y , Y to X or will be indifferent between the two.

- *Transitivity*

- It means that if a consumer prefers basket A to basket B and to basket C , then the consumer also prefers A to C .

- *Continuity*

- if A is preferred to B , then situations suitably “close to” A must also be preferred to B
- It is used to analyze individuals’ responses to relatively small changes in income and prices

- **More is better than less**

- Consumers always prefer more of any good to less and they are never satisfied or satiated. However, bad goods are not desirable and consumers will always prefer less of them.

2.2 The concept of utility

- ✓ Economists use the term utility to describe the satisfaction or pleasure derived from the consumption of a good or service.
- ✓ In other words, utility is the power of the product to satisfy human wants.
- ✓ In defining strict preference, we said that given any two consumption bundles (X_1, X_2) and (Y_1, Y_2) the consumer definitely wants the X bundle than the Y bundle if $(X_1, X_2) \succ (Y_1, Y_2)$. This means the consumer preferred bundle (X_1, X_2) to (Y_1, Y_2) .

2.2 The concept of utility.....

- ✓ **Utility** and **Usefulness** are not synonymous. For example, paintings by Picasso may be useless functionally but offer great utility to art lovers. Hence, usefulness is product centric whereas utility is consumer centric.
- ✓ **Utility is subjective.** The utility of a product will vary from person to person
- ✓ That means, the utility that two individuals derive from consuming the same level of a product may not be the same.

- Utility is affected by the consumption of physical commodities, psychological attitudes, peer group pressures, personal experiences, and the general cultural environment
- Economists generally devote attention to quantifiable options while holding constant the other things that affect utility
 - *ceteris paribus* assumption

2.2 The concept of utility.....

- A Consumer considers the following points to get maximum utility or level of satisfaction:
 - How much satisfaction he gets from buying and then consuming an extra unit of a good or service.
 - The price he pays to get the good.
 - The satisfaction he gets from consuming alternative products.
 - The prices of alternative goods and services.

2.3 Approaches of measuring utility

- ✓ There are two major approaches to measure or compare consumer's utility: **cardinal** and **ordinal** approaches.
- ✓ The **cardinalist** school postulated that utility can be measured objectively.
- ✓ According to the **ordinalist** school, utility is not measurable in cardinal numbers rather the consumer can rank or order the utility he derives from different goods and services.

2.3.1 The cardinal utility theory

- ✓ According to the cardinal utility theory, utility is measurable by arbitrary unit of measurement called utils in the form of 1, 2, 3 etc.
- ✓ For example, we may say that consumption of an orange gives Bilen 10 utils and a banana gives her 8 utils, and so on.
- ✓ From this, we can assert that Bilen gets more satisfaction from orange than from banana.

2.3.1.1 Assumptions of cardinal utility theory

- ✓ **1. Rationality of consumers.** The main objective of the consumer is to maximize his/her satisfaction given his/her limited budget or income.
- ✓ Thus, in order to maximize his/her satisfaction, the consumer has to be rational.
- ✓ **2. Utility is cardinally measurable.** According to the cardinal approach, the utility or satisfaction of each commodity is measurable.
- ✓ Utility is measured in subjective units called utils.

2.3.1.1 Assumptions of cardinal utility theory.....'cont

- ✓ **3. Constant marginal utility of money.** A given unit of money deserves the same value at any time or place it is to be spent.
- ✓ A person at the start of the month where he has received monthly salary gives equal value to 1 birr with what he may give it after three weeks or so.
- ✓ **4. Limited Money Income.** The consumer has limited money income to spend on the goods and services he/she chooses to consume.

3.3.1.1 Assumptions of cardinal utility theory.....'cont.

- ✓ **5. The total utility of a basket of goods depends on the quantities of the individual commodities.**
- ✓ If there are n commodities in the bundle with quantities X_1 , X_2 , ..., X_n , the total utility is given by $TU = f(X_1, X_2, \dots, X_n)$.
- ✓ **Utility is also additive, i.e., $U(X_1) + U(X_2) + U(X_3) + \dots + U(X_n) = \text{Total Utility (TU)}$**

2.3.1.2 Total and marginal utility

- ✓ **Total Utility (TU)** is the total satisfaction a consumer gets from consuming some specific quantities of a commodity at a particular time.
- ✓ **if a consumer consumes 4 units of a commodity and derives U1, U2, U3 and U4 from the successive units consumed, then $TU = U1 + U2 + U3 + U4$.**
- ✓ **In case the number of commodities consumed is greater than one, then**
$$TU = TU_x + TU_y + TU_z + \dots + TU_n$$
- ✓ As the consumer consumes more of a good per time period,

2.3.1.2 Total and marginal utility....

- ✓ **Marginal Utility (MU)** is the extra satisfaction a consumer realizes from an additional unit of the product.
- ✓ In other words, marginal utility is the change in total utility that results from the consumption of one more unit of a product. Graphically, it is the slope of total utility.
- ✓ Mathematically, marginal utility is $MU = \frac{\Delta TU}{\Delta Q}$ where, ΔTU is the change in total utility, and ΔQ is the change in the amount of product consumed.
- ✓ To explain the relationship between TU and MU, let us

2.3.1.2 Total and marginal utility....

Table 3.1: Total and marginal utility

Quantity	Total utility (TU)	Marginal utility (MU)
0	0	-
1	10	10
2	18	8
3	24	6
4	28	4
5	30	2
6	30	0
7	28	-2

2.3.1.2 Total and marginal utility....

✓ Graphically, the above data can be depicted as follows.

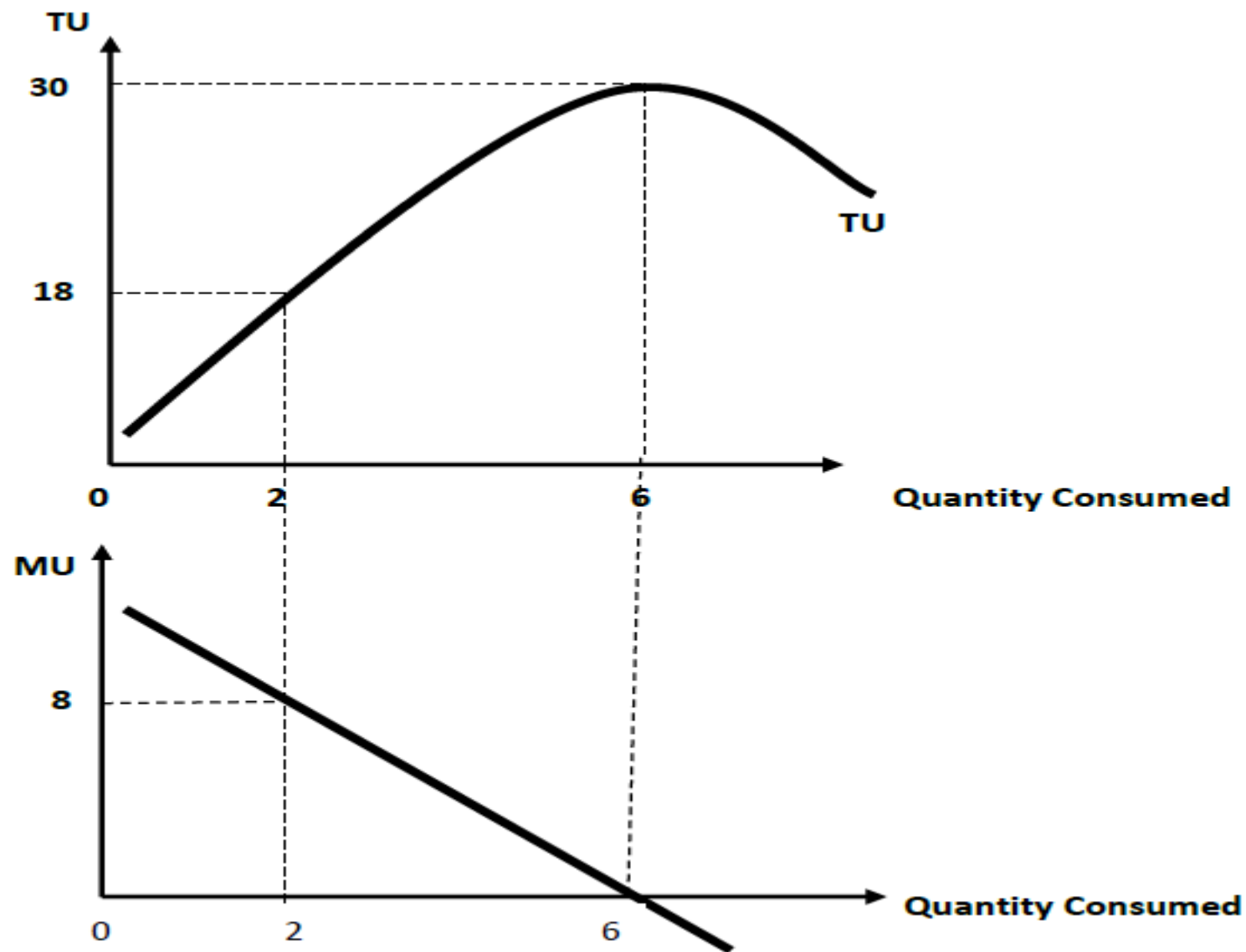


Figure 3.1: Total and marginal utility curves

2.3.1.2 Total and marginal utility.....

- ✓ As it can be observed from the above figure,
- ✓ □ When TU is increasing, MU is positive.
- ✓ □ When TU is maximized, MU is zero.
- ✓ □ When TU is decreasing, MU is negative.

2.3.1.3 Law of diminishing marginal utility (LDMU)

- ✓ The law of diminishing marginal utility states that as the quantity consumed of a commodity increases per unit of time, the utility derived from each successive unit decreases, consumption of all other commodities remaining constant.
- ✓ The law of diminishing marginal utility is based on **the following assumptions**.
- ✓ The consumer is rational
- ✓ The consumer consumes identical or homogenous product. The commodity to be consumed should have similar quality, color, design, etc.

• **Limitations or Exceptions**

- The law does not apply in the case of rare collections like stamps paintings and coins etc. In the case of rare collections, the larger the number the collects, the greater will be the pleasure. Hence, the law does not apply.
- When we discuss the law, we are applying it only to normal persons. But there are some abnormal persons too e.g., misers, drunkards etc. The more the money a miser has, the greater is the utility that he derives. The more the drunkard gets pleasure the more he drinks.

• **Importance**

- A. The law of diminishing marginal utility expresses us a basic principle of man's behaviour. It is of great practical value to human beings in every walk of life.
- B. The law is applied in the sphere of taxation. A rich man is taxed more, for the utility of money to a rich man is less than to a poor man. The principle of progressive taxation is based on this only.

- A. The law can also be applied in determining the prices of goods in the market. An increase in the stock of a commodity brings a person less satisfaction and therefore he can be induced to buy more only if the price is lowered. Hence, the greater the supply, the lower should be the price to clear it and vice-versa.
- B. The law regulates our daily expenditure pattern. We know that as we go on buying more of a commodity, its marginal utility falls. Having only a limited amount of money at our disposal, we do not want to waste it unnecessarily on the purchase of the same commodity in large quantity. We, therefore, stop purchasing it at a point where the utility of money spent is equal to the last unit of the good purchased. We spend the rest of our money on other goods.
- C. The marginal utility analysis of pricing and the diminishing marginal utility can quickly dispose of the diamond-water paradox with the aid of this analysis we can now explain that the relative scarcity of diamonds results in high price, while the relative abundance of water means that its marginal utility and consequently its price will be low despite its high total utility.

2.3.1.4 Equilibrium of a consumer

- ✓ Given his limited income and the price level of goods and services, what combination of goods and services should he consume so as to get the maximum total utility?

a) the case of one commodity

- **Suppose the consumer's utility function is given as**
 - $U = f(X)$
 - **his/her total income spent (expenditure) on commodity X**
 - **Total Expenditure would be: $TE = Q_x \cdot P_x$**
 - where Q_x is amount of commodity x and P_x is price of good X.

2.3.1.4 Equilibrium of a consumer

Suppose the consumer's utility function is given as $U = f(X)$

his/her total income spent (expenditure) on commodity X -Total Expenditure would be: $TE = Q_x P_x$

- where Q_x is amount of commodity x and**
- P_x is price of good X.**
- The consumer would like to maximize the difference between the utility (satisfaction) and expenditure (sacrifice).**
- The problem is a simple maximization of the function.**

2.3.1.4 Equilibrium of a consumer

- The equilibrium condition of a consumer that consumes a single good X occurs when the marginal utility of X is equal to its market price.

Proof

Given the utility function $U = f(X)$ $Q_X P_X$

- If the consumer buys commodity X, then his expenditure
- the consumer would like to maximizes the difference between his utility and expenditure.

- **Max U - TE or**
 $Max(U - Q_X P_X)$

- **The problem is a simple maximization of the function.**
- **Two conditions must be fulfilled**
 - **Necessary Condition (F.O.C)**
 - **Sufficient Condition (S.O.C)**
- **The necessary condition (First Order Condition) for maximum, require that the derivative of the function with respect to independent variable (Q_x) must be equal to zero.**

$$\frac{dU}{dQ_x} - \frac{d(Q_x P_x)}{dQ_x} = 0$$

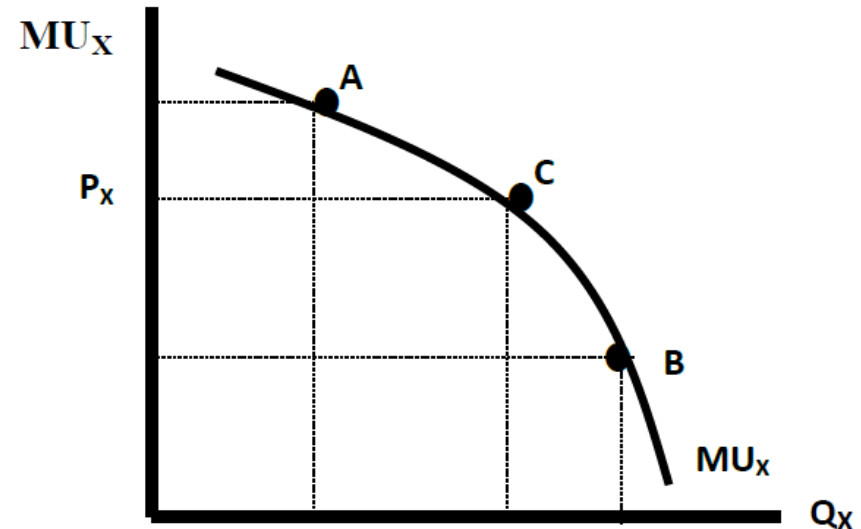
$$\square \quad MU_x - P_x = 0$$

$$\square \quad MU_x = P_x$$

$$\frac{MU_x}{P_x} = 1$$

2.3.1.4 Equilibrium of a consumer

- At any point above point C (like point A) where $MUX > PX$, it pays the consumer to consume more.
- When $MUX < PX$ (like point B), the consumer should consume less of X.
- At point C where $MUX = PX$ the consumer is in equilibrium.



1.

Consumers Equilibrium: The General Case: (The law of equi-marginal Utility)

- In reality, however, a consumer consumes a large number of goods.
- The MU schedules of different commodities may not be the same.
- A utility maximizing consumer consumes commodities in order of their utilities.
- He picks up the commodity, which yields the highest utility followed by the commodity yielding the second highest utility and so on.
- He switches his expenditure from one commodity to another in accordance, with their marginal utility.

Cardinal Utility (cont...)

- He continues to switch his expenditure from one commodity to the other till he reaches a stage where MU of each commodity is equal per unit of expenditure
- Therefore, the consumer optimum follows the law of equi-marginal utility.
- We can use a single commodity case to determine the general case.
- For commodity X, equilibrium occurs when $MU_x = P_x$
 - $MU_x = P_x$ and this can be written as
 - $MU_x / P_x = 1$

- if the consumer consumes a bundle of n commodities i.e X1, X2 X3..... Xn , he/she would be in equilibrium or utility is maximized if and only if:

i. when the marginal utility per money spent is equal for each good purchased

$$\frac{MU_{X_1}}{P_{X_1}} = \frac{MU_{X_2}}{P_{X_2}} = \dots\dots\dots = \frac{MU_{X_n}}{P_{X_n}}$$

ii. his money income available for the purchase of the goods is

e.g. $\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} = \dots = \frac{MU_N}{P_N}$ and $P_X Q_X + P_Y Q_Y + \dots + P_N Q_N = M$

In general

b) the case of two or more commodities

➤ For the case of two or more goods, the consumer's equilibrium is achieved when the marginal utility per money spent is equal for each good purchased and his money income available : $\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} = \dots = \frac{MU_N}{P_N}$ and $P_X Q_X + P_Y Q_Y + \dots + P_N Q_N = M$ l.

➤ Example: Suppose Saron has 7 Birr to be spent on two goods: banana and bread. The unit price of banana is 1 Birr and the unit price of a loaf of bread is 4 Birr.

➤ The total utility maximization problem for a consumer with a fixed income can be written as follows:

Cont.....

- **Exercise: Consider a consumer having only birr 7 in his pocket to buy bread and banana**

If the Price of banana is birr 4/kg and price of bread is birr one per unit determine.

- His marginal utility schedule for the two commodities**
- Determine his optimum consumption of these two goods**
- The total utility at optimum consumption**

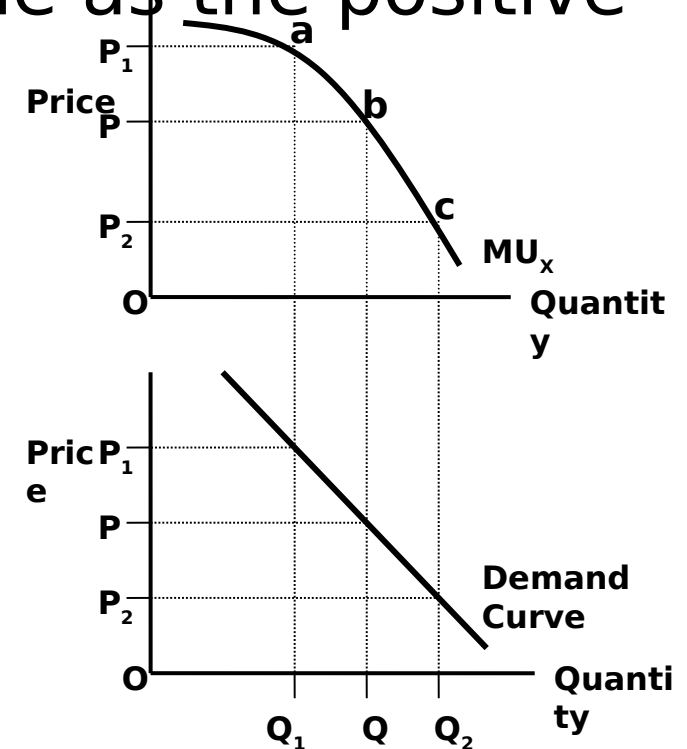
<i>Bread, Price=birr 1/unit</i>				<i>Banana, Price=4birr/kg</i>			
<i>Quantit y of</i>	<i>TU</i>	<i>MU</i>	<i>MU/P</i>	<i>Quantit y</i>	<i>TU</i>	<i>MU</i>	<i>MU/P</i>
<i>0</i>	<i>0</i>			<i>0</i>	<i>0</i>		
<i>1</i>	<i>6</i>			<i>1</i>	<i>12</i>		
<i>2</i>	<i>11</i>			<i>2</i>	<i>20</i>		
<i>3</i>	<i>14</i>			<i>3</i>	<i>26</i>		
<i>4</i>	<i>13</i>			<i>4</i>	<i>29</i>		
<i>5</i>	<i>13</i>			<i>5</i>	<i>32</i>		
<i>6</i>	<i>11</i>			<i>6</i>	<i>31</i>		

Solutions

<i>Bread , Price=birr 1/unit</i>				<i>Banana, Price=4birr/kg</i>			
<i>Quantity</i>	<i>TU</i>	<i>M</i>	<i>MU/P</i>	<i>Quantit y</i>	<i>TU</i>	<i>MU</i>	<i>MU/P</i>
0	0	-	-	0	0	-	-
1	6	6	6	1	12	12	3
2	11	5	5	2	20	8	2
3	14	3	3	3	26	6	1.5
4	13	1	1	4	29	3	0.75
5	13	0	0	5	32	2	.5
6	11	-2	-2	6	31	-1	-0.25

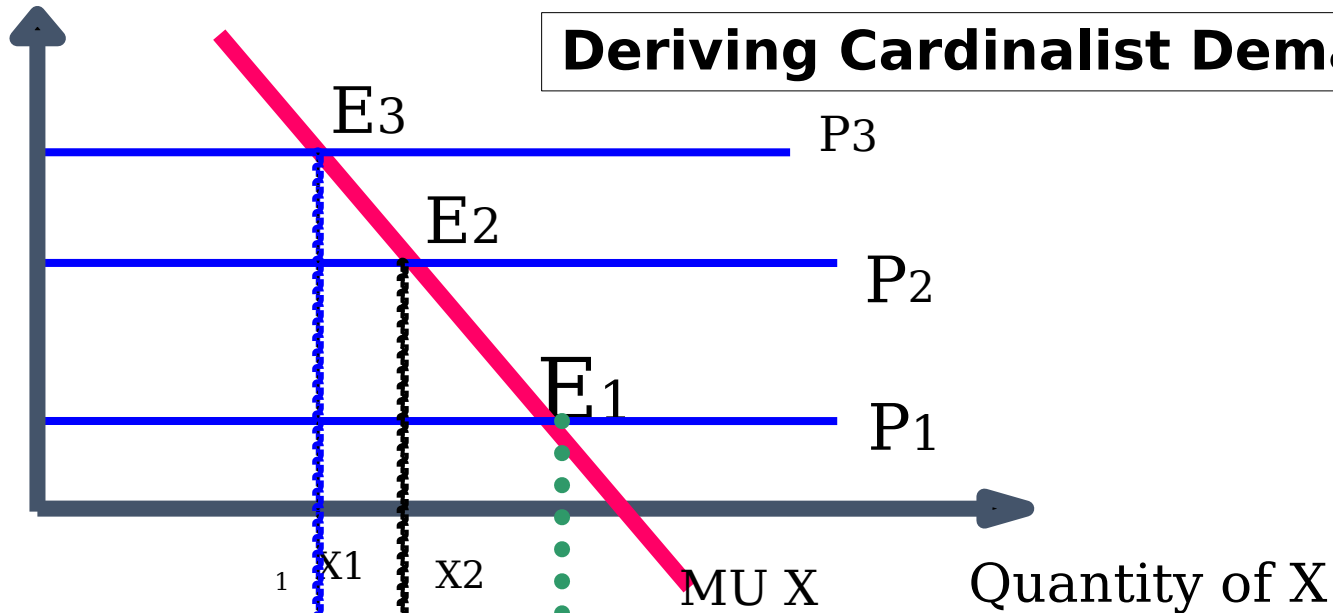
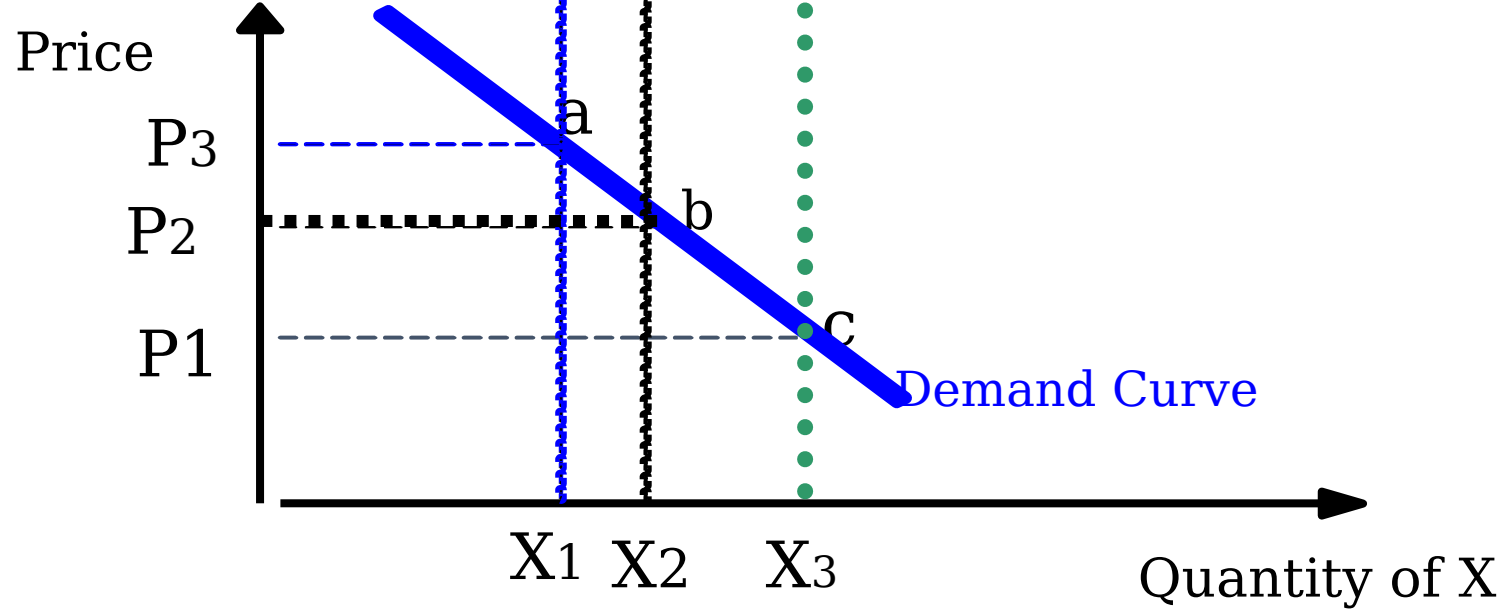
2.3.1.6. Deriving Cardinalist Demand

- The marginal utility is the slope of the total utility function.
- The derivation of demand curve is based on the concept of diminishing marginal utility.
- If the marginal utility is measured using monetary units the demand curve for a commodity is the same as the positive segment of the marginal utility curve.



Quantity of Y

Deriving Cardinalist Demand



2.3.1.5. Cardinal Utility (cont...)

Limitation of the Cardinal approach

- a) The assumption that utility is a cardinal concept (utility is objectively measurable) is doubtful.**
 - Utility is a subjective concept, which cannot be measured objectively.**
- b) The assumption of constant marginal utility of money is also unrealistic..**
- c) The psychological law of diminishing marginal utility has been established from introspection**
- d) The cardinal utility approach is on the basis of Ceteris Paribus assumption.**
 - As a result it ignores the substitution and income effect.**

2.3.2 The ordinal utility theory

- In the ordinal utility approach, it is not possible for consumers to express the utility of various commodities they consume in absolute terms, like 1 util, 2 utils, or 3 utils but it is possible to express the utility in relative terms.
- The consumers can rank commodities in the order of their preferences as 1st, 2nd, 3rd and so on.
- Therefore, the consumer need not know in specific units the utility of various commodities to make his choice.

3.3.2.1 Assumptions of ordinal utility theory

- **Consumers are rational** - they maximize their satisfaction or utility given their income and market prices.
- **Utility is ordinal** - utility is not absolutely (cardinally) measurable. Consumers are required only to order or rank their preference for various bundles of commodities.
- **Diminishing marginal rate of substitution:** The marginal rate of substitution is the rate at which a consumer is willing to substitute one commodity for another commodity so that his total satisfaction remains the same.

3.3.2.1 Assumptions of ordinal utility theory.....

- **Non-Satiation:** In any two consumption bundle A and B, A is preferred to B, if A contains, at least more of one commodity.

That is, more is preferred to less under normal condition.

- **Limited money income.**

Generally, ordinalist school simply argue that,

- individual tends to make consistent choice,
 - that the law of preference represents a good approximation of actual behavior of consumer
 - and thus, the law of preference are rules of rational choice.
-
- The **total utility** of a consumer is measured by the amount (quantities) of all items he/she consumes from his/her

- **Consumer's preferences are consistent:** For example, if there are three goods in a given consumer's basket, say, X, Y, Z and if he prefers X to Y and Y to Z, then the consumer is expected to prefer X to Z. This property is known as axioms of ***transitivity***.
- The ordinal utility approach is explained with the help of indifference curves.
- Therefore, the ordinal utility theory is also known as the indifference curve approach.

- Given these assumptions, it is possible to show that people are able to rank in order all possible situations from least desirable to most
- Economists call this ranking utility
 - if A is preferred to B, then the utility assigned to A exceeds the utility assigned to B

$$U(A) > U(B)$$

- Utility rankings are ordinal in nature
 - they record the relative desirability of commodity bundles
- Because utility measures are not unique, it makes no sense to consider how much more utility is gained from A than from B
- It is also impossible to compare utilities between people

- Assume that an individual must choose among consumption goods $x_1, x_2, \dots, x_n$
- The individual's rankings can be shown by a utility function of the form:

$$\text{utility} = U(x_1, x_2, \dots, x_n; \text{other things})$$

- this function is unique up to an order-preserving transformation

3.3.2.2 Indifference set, curve and map

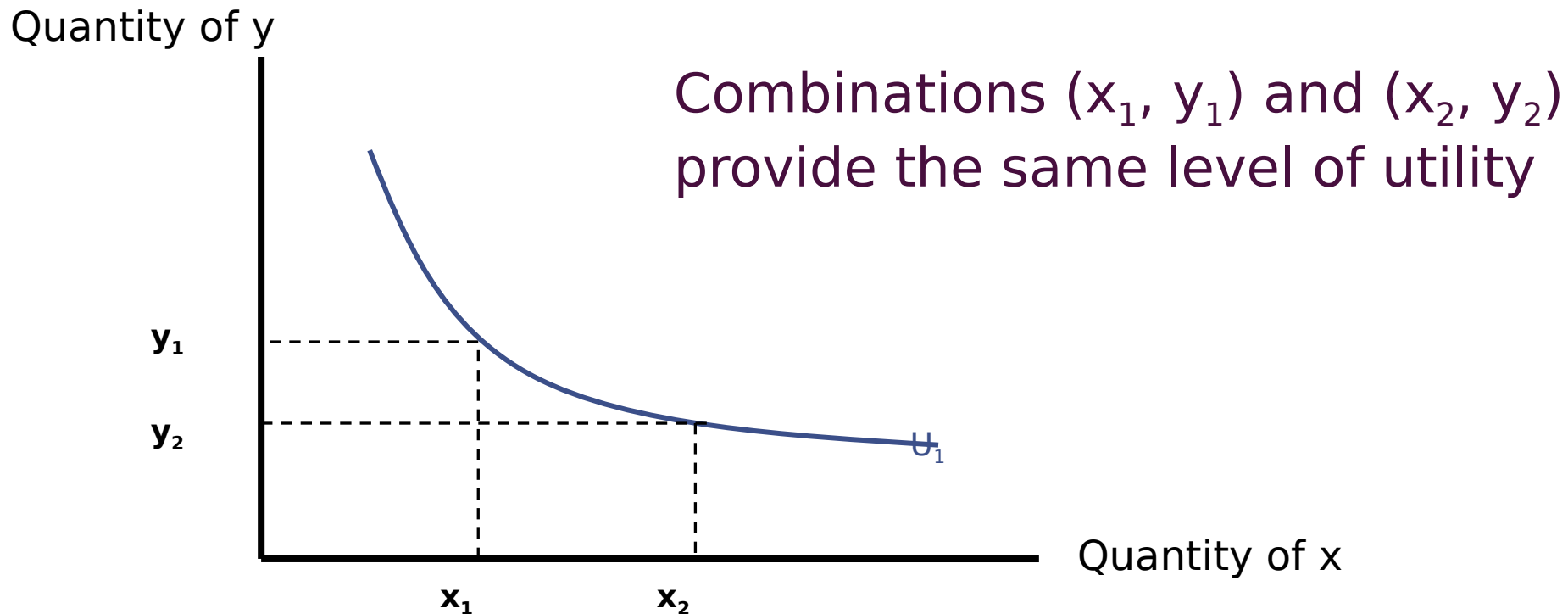
- ✓ ***Indifference set/ schedule*** is a combination of goods for which the consumer is indifferent.
- ✓ It shows the various combinations of goods from which the consumer derives the same level of satisfaction.
- ✓ Consider a consumer who consumes two goods X and Y (table 3.3).

Table 3.3: Indifference schedule

Bundle (Combination)	A	B	C	D
Orange	1	2	4	7
Banana	10	6	3	1

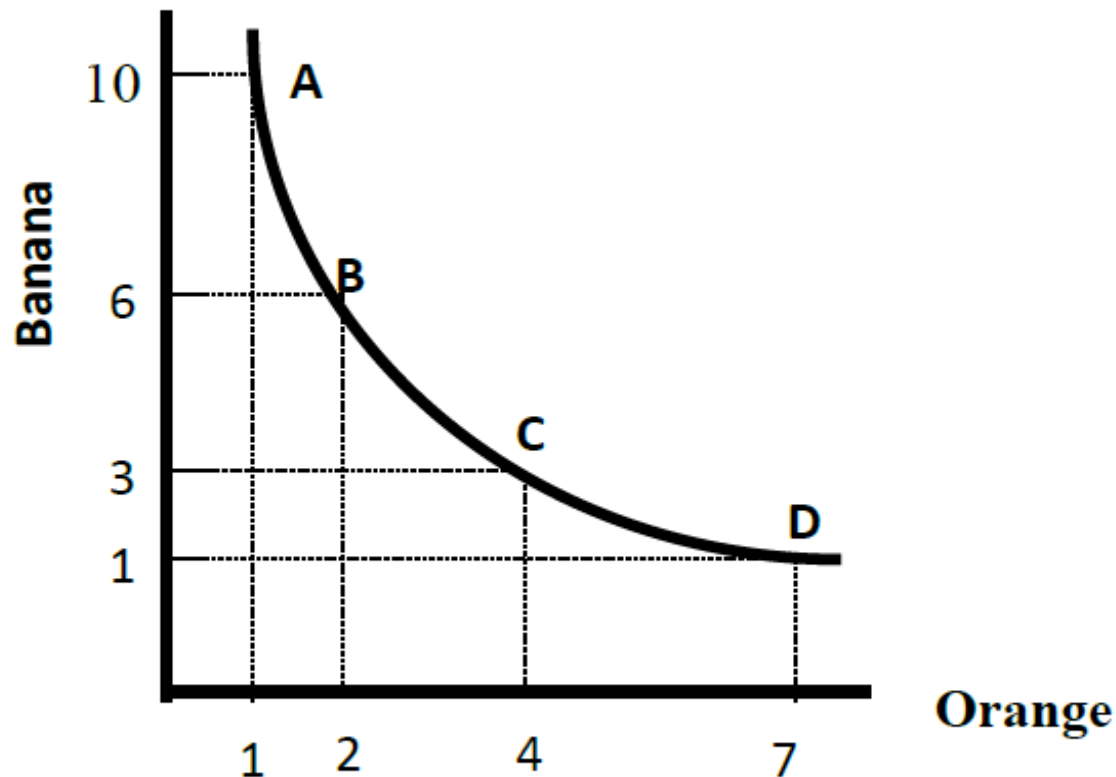
Indifference Curves

- An indifference curve shows a set of consumption bundles among which the individual is indifferent

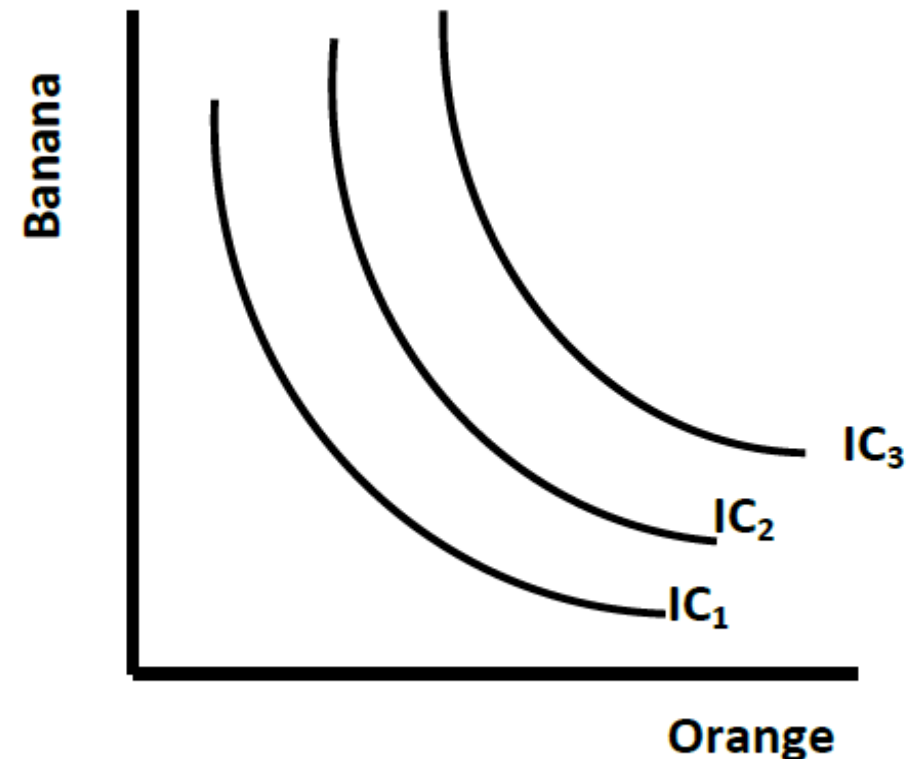


3.3.2.2 Indifference set, curve and map

- **Indifference curve:** An indifference curve shows different combinations of two goods which yield the same utility (level of satisfaction) to the consumer.



i) Indifference curve



ii) Indifference map

3.3.2.3 Properties of indifference curves

1. Indifference curves have negative slope (downward sloping to the right).

- Indifference curves are negatively sloped because the consumption level of one commodity can be increased only by reducing the consumption level of the other commodity.

2. Indifference curves are convex to the origin. The convexity of indifference curves is the reflection of the diminishing marginal rate of substitution.

3. A higher indifference curve is always preferred to a

3.3.2.3 Properties of indifference curves

- **4. Indifference curves never cross each other (cannot intersect).**
- The assumptions of consistency and transitivity will rule out the intersection of indifference curves. Figure 3.4
- From our transitivity if
 - The individual is indifferent between A and C. The individual is indifferent between B and C. Transitivity suggests that the individual should be indifferent between A and B.
 - the logic which says that if B is preferred to A and A is preferred to C, then B is preferred to C.

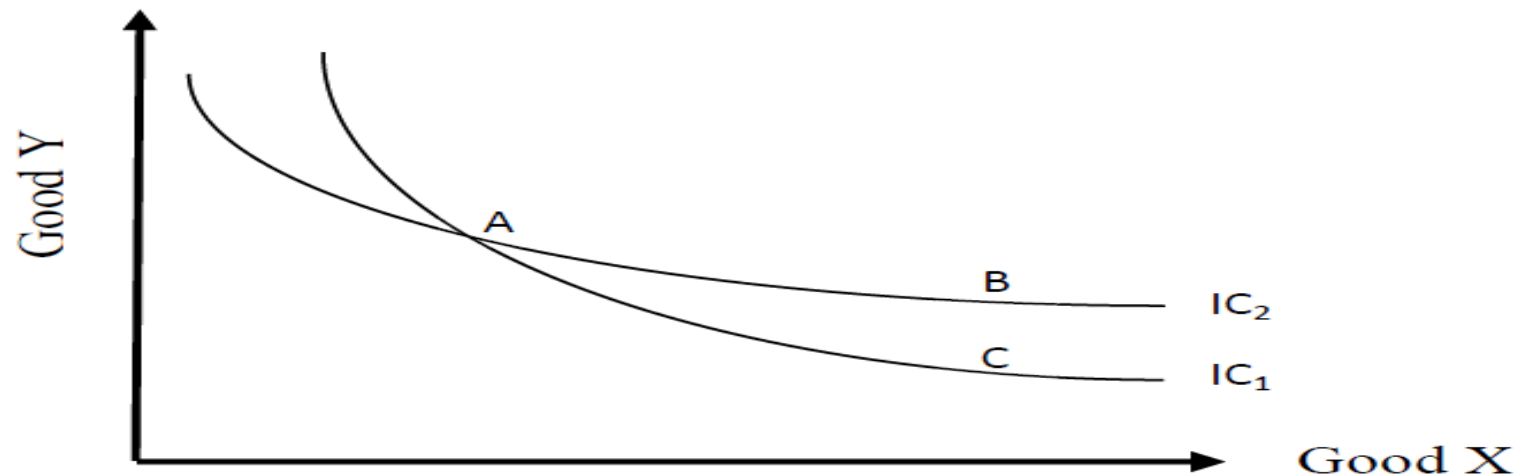
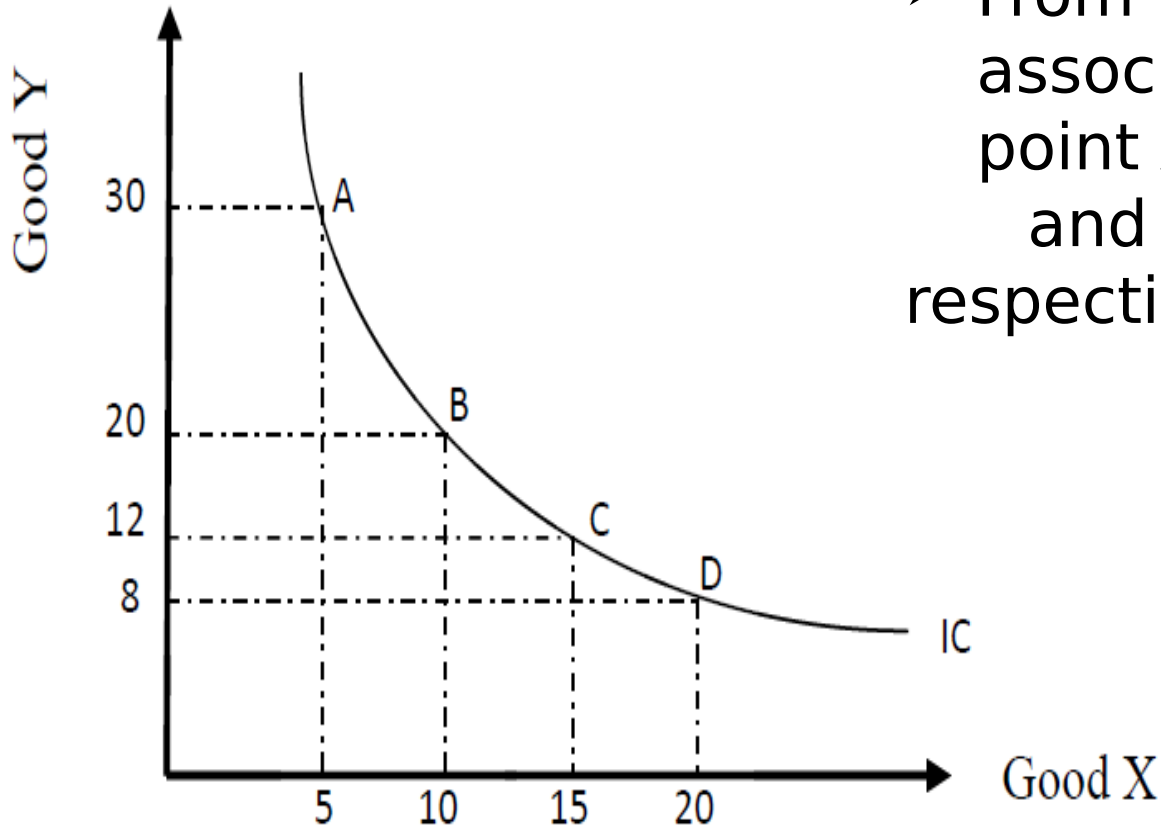


Figure 3.4: Intersection of indifference curves

3.3.2.4 Marginal rate of substitution (MRS)

- Marginal rate of substitution of X for Y is defined as the number of units of commodity Y that must be given up in exchange for an extra unit of commodity X so that the consumer maintains the same level of satisfaction.
- Since one of the goods is sacrificed to obtain more of the other good, the MRS is negative. Stated differently, the negative of the slope of $MRS_{X,Y} = \frac{\text{Number of units of Y given up}}{\text{Number of units of X gained}} = -\frac{\Delta Y}{\Delta X}$ called the marginal rate of substitution (MRS).

3.3.2.4 Marginal rate of substitution (MRS).....

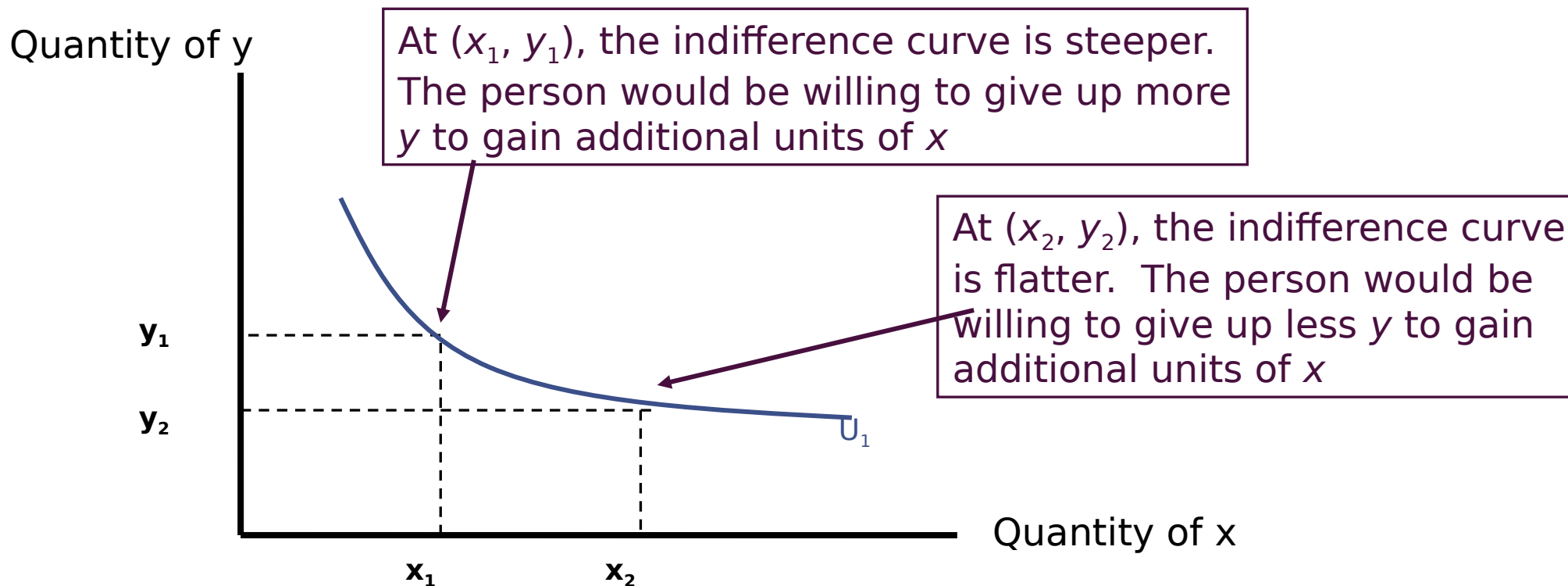


➤ From the above graph, $MRS_{X,Y}$ associated with the movement from point A to B, point B to C and point C to D is 2.0, 1.6, and 0.8 respectively.

Figure 3.5: Indifference curve for two products X and Y

- MRS changes as x and y change
 - reflects the individual's willingness to trade y for x

$$MRS = - \left. \frac{dy}{dx} \right|_{U=U_1}$$



3.3.2.4 Marginal rate of substitution (MRS)

- It is also possible to derive MRS using the concept of marginal utility. $MRS_{X,Y}$ is related to MU_X and MU_Y as follows.
- **Proof:** Suppose the utility function for two commodities X and Y is defined as: $U = f(X,Y)$
- Since utility is constant along an indifference curve, the total differential of the utility function will be zero.
-

$$dU = \frac{\partial U}{\partial X} dX + \frac{\partial U}{\partial Y} dY = 0$$

$$MU_X dX + MU_Y dY = 0$$

$$\frac{MU_X}{MU_Y} = -\frac{dY}{dX} = MRS_{X,Y}$$

$$\text{Similarly, } \frac{MU_Y}{MU_X} = -\frac{dX}{dY} = MRS_{Y,X}$$

- Example-1: Suppose an individual's preferences for hamburgers (y) and soft drinks (x) can be represented by

$$\text{utility} = 10 = \sqrt{x \cdot y}$$

- Solving for y , we get

$$y = 100/x$$

- Solving for $MRS = -dy/dx$:

- $MRS = -dy/dx = 100/x^2$

- Note that as x rises, MRS falls

- when $x = 5$, $MRS = 4$
 - when $x = 20$, $MRS = 0.25$

3.3.2.4 Marginal rate of substitution (MRS)...

- Cobb-Douglas Utility utility = $U(x,y) = x^\alpha y^\beta$ where α and β are positive constants
- The relative sizes of α and β indicate the relative importance of the goods

$$MRS = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = \frac{\alpha x^{\alpha-1} y^\beta}{\beta x^\alpha y^{\beta-1}} = \frac{\alpha}{\beta} \cdot \frac{y}{x}$$

Example-2. Suppose a consumer's utility function is given by 42

! Solution: $MRS_{x,y} = \frac{MU_x}{MU_y}$

$$MU_x = \frac{\partial U}{\partial X} = 4X^3Y^2 \quad \text{and} \quad MU_y = \frac{\partial U}{\partial Y} = 2X^4Y \quad \text{Hence, } MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{4X^3Y^2}{2X^4Y} = \frac{2Y}{X}$$

- Example -3: Suppose that the utility function is

$$\text{utility} = \sqrt{x \cdot y}$$

- We can simplify the algebra by taking the logarithm of this function
- $U^*(x,y) = \ln[U(x,y)] = 0.5 \ln x + 0.5 \ln y$

$$MRS = \frac{\frac{\partial U^*}{\partial x}}{\frac{\partial U^*}{\partial y}} = \frac{\frac{0.5}{x}}{\frac{0.5}{y}} = \frac{y}{x}$$

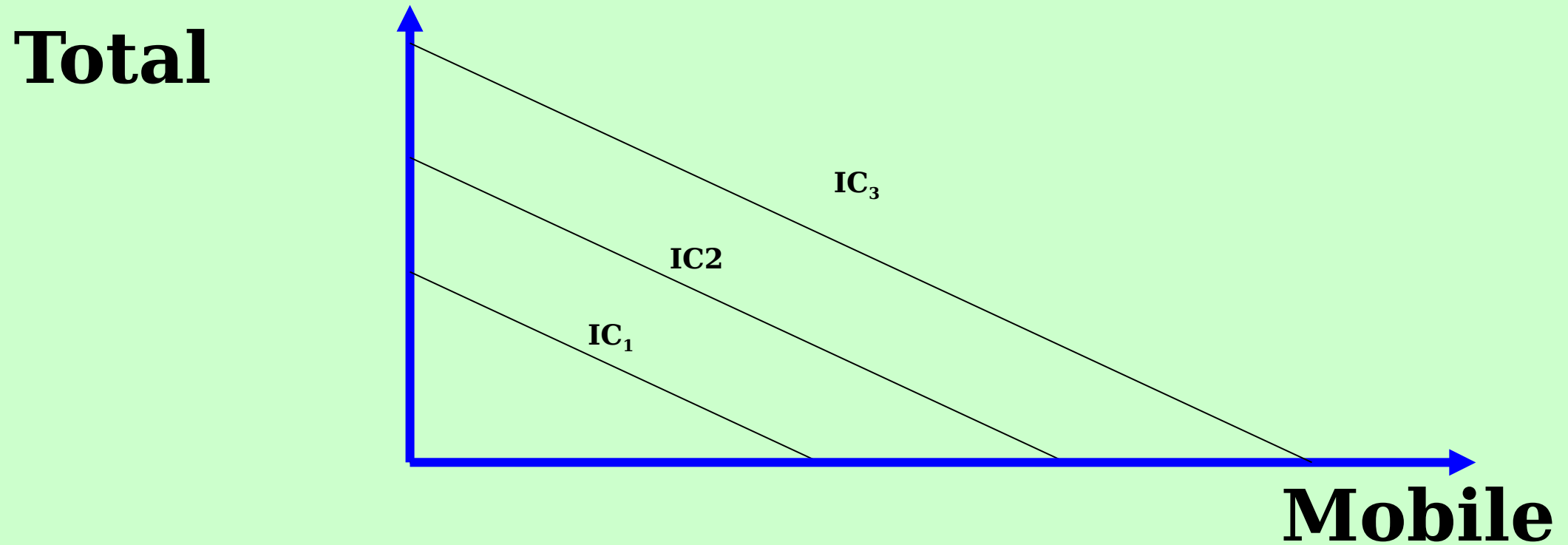
- Intuitively, it seems that the assumption of decreasing marginal utility is related to the concept of a diminishing *MRS*
 - diminishing *MRS* requires that the utility function be quasi-concave
 - this is independent of how utility is measured
 - diminishing marginal utility depends on how utility is measured
- Thus, these two concepts are different

Exceptional Indifference Curves

- **indifference curves are convex to the origin and downward sloping**
- However, the shape of the indifference curve reflects the degree of substitution between the two commodity
- The shape of an indifference curve might be different if the relationship between two commodities is unique
- **Perfect substitutes:** perfect substitutes are goods which can be replaced for one another at a constant rate.
 - $utility = U(x,y) = \alpha x + \beta y$
- The indifference curves will be linear. The MRS will be constant along the indifference curve.

Exceptional IC (Cont ...)

- **Perfect substitutes**



Exceptional IC (Cont ...)

- **Perfect complements:** perfect complements are goods which are to be consumed jointly at a constant rate
- If two commodities are perfect complements the indifference curve takes the shape of a right angle (L -shape)
- Graphically it is shown as follows.

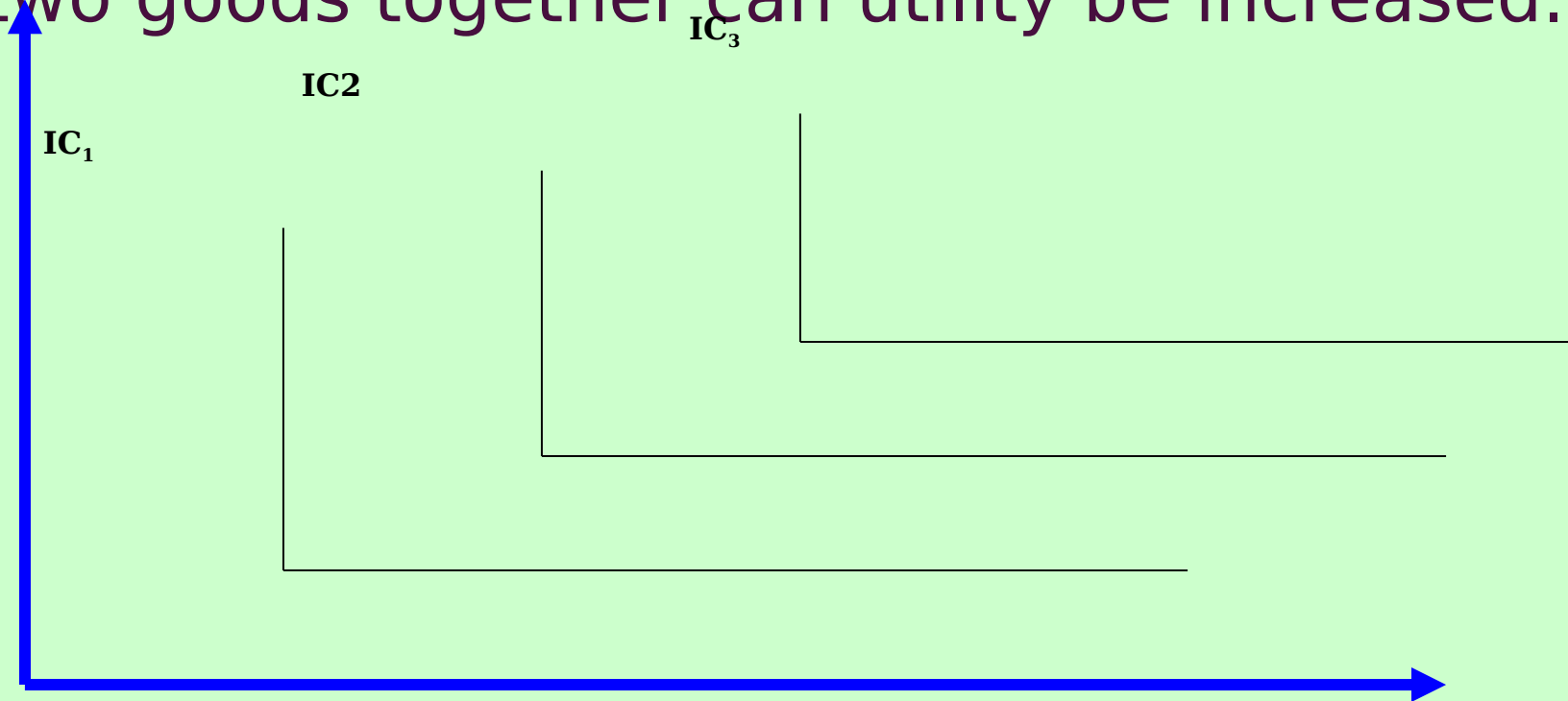
Exceptional IC (Cont ...)

- **Perfect complements**

- utility = $U(x,y) = \min(\alpha x, \beta y)$

- The indifference curves will be L-shaped. Only by choosing more of the two goods together can utility be increased.

Right shoe



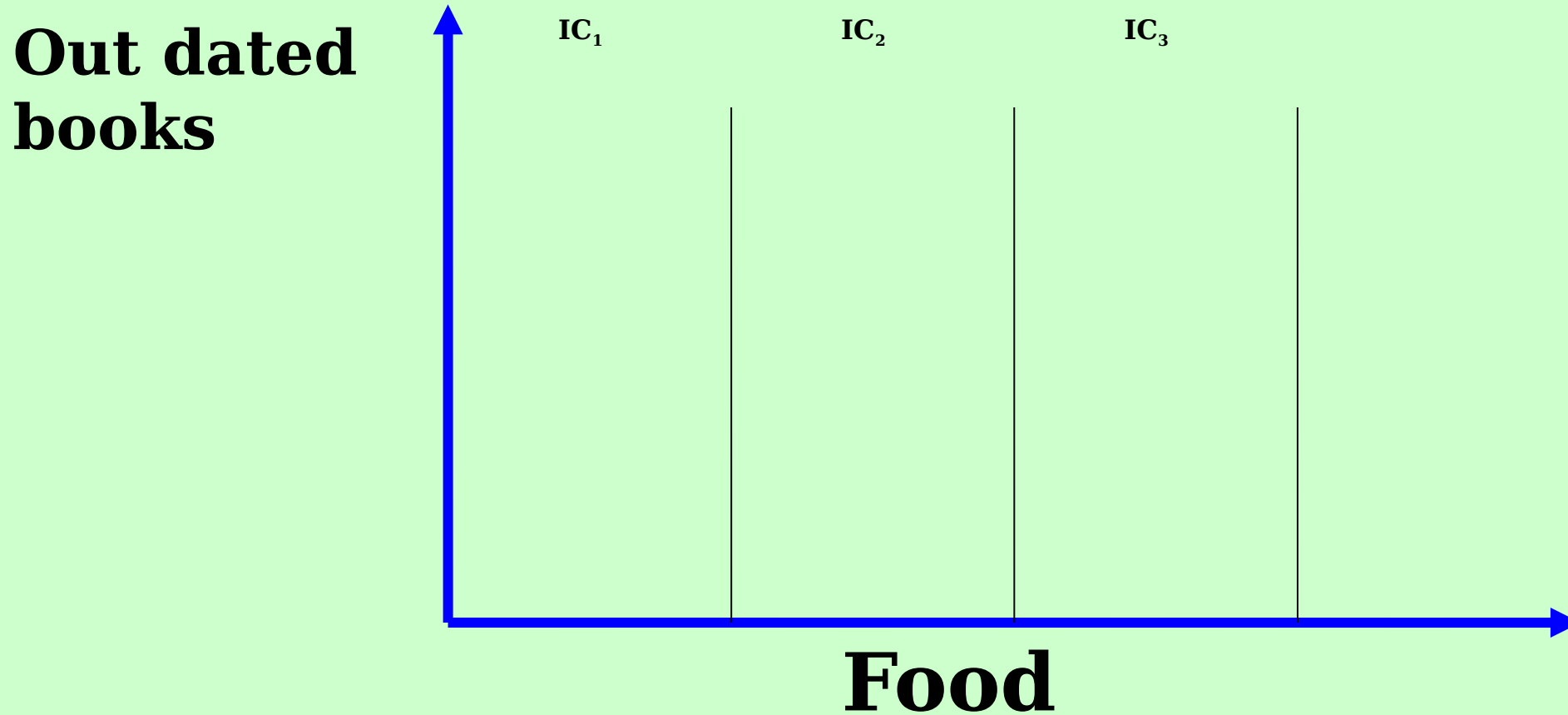
Left shoe

Exceptional IC (Cont ...)

- **A useless good:** This shows the relationship between useless good and another normal good.
- A good example is outdated book and food.
- since the outdated books are totally useless, increasing their purchases does not increase utility.
- The person enjoys a higher level of utility only by getting additional food consumption
- The indifference curve in this case will have a vertical one

Exceptional IC (Cont ...)

• Fig.



3.3.2.5 The budget line or the price line

- ✓ In reality, the consumer is constrained by his/her income and prices of the two commodities.
- ✓ This constraint is often presented with the help of the budget line.
- ✓ The budget line is a set of the commodity bundles that can be purchased if the entire income is spent.
- ✓ It is a graph which shows the various combinations of two goods that a consumer can purchase given his/her limited income and the prices of the two goods
- ✓ In order to draw a budget line facing a consumer, we consider the following assumptions.
 - There are only two goods bought in quantities, say, X and Y.
 - Each consumer is confronted with market determined

3.3.2.5 The budget line or the price line

- Assuming that the consumer spends all his/her income on the two goods (X and Y), we can express the budget constraint as:
- $M = P_x X + P_y Y$ By rearranging the above equation, we can derive the following general equation of a
$$Y = \frac{M}{P_y} - \frac{P_x}{P_y} X$$

- Graphically,

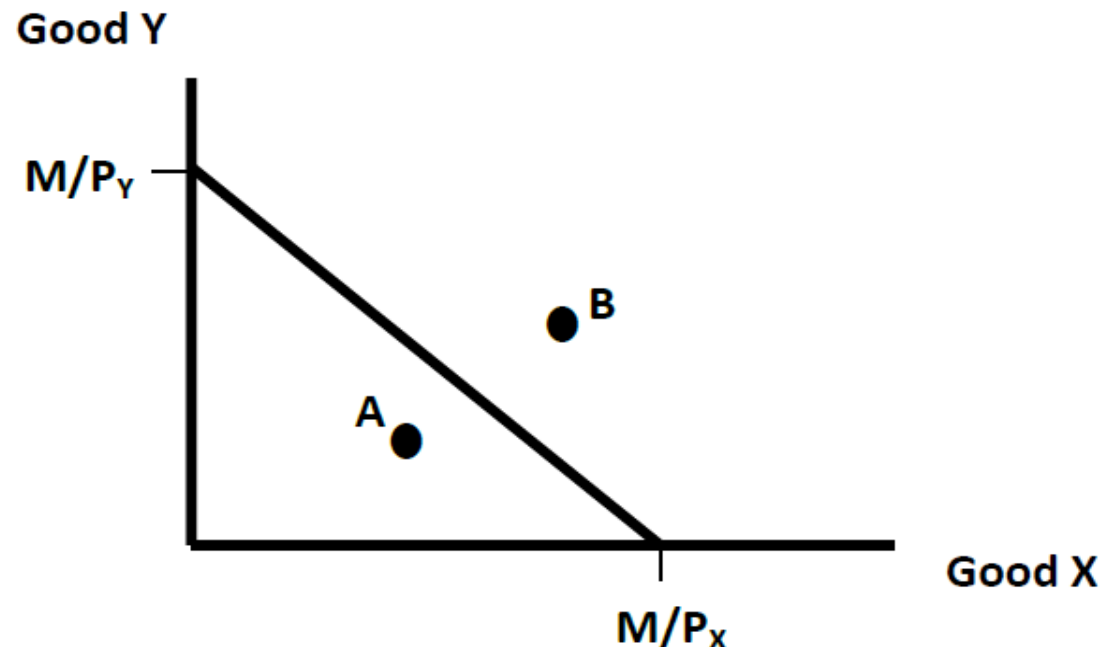


Figure 3.6: The budget line

3.3.2.5 The budget line or the price line

- Note that: The slope of the budget line is $-\frac{P_X}{P_Y}$ which is by (the ratio of the prices of the two goods).
- Any combination of the two goods within the budget line (such as point A) or along the budget line is attainable.
- Any combination of the two goods outside the budget line (such as point B) is unattainable (unaffordable).
- Example: A consumer has \$100 to spend on two goods X and Y with prices \$3 and \$5 respectively.
- Derive the equation of the budget line and sketch the graph.

3.3.2.5 The budget line or the price line

✓ **Solution:** The equation of the budget line can be derived as follows

$$P_X X + P_Y Y = M$$

$$3X + 5Y = 100$$

$$5Y = 100 - 3X$$

$$Y = \frac{100}{5} - \frac{3}{5}X$$

$$Y = 20 - \frac{3}{5}X$$



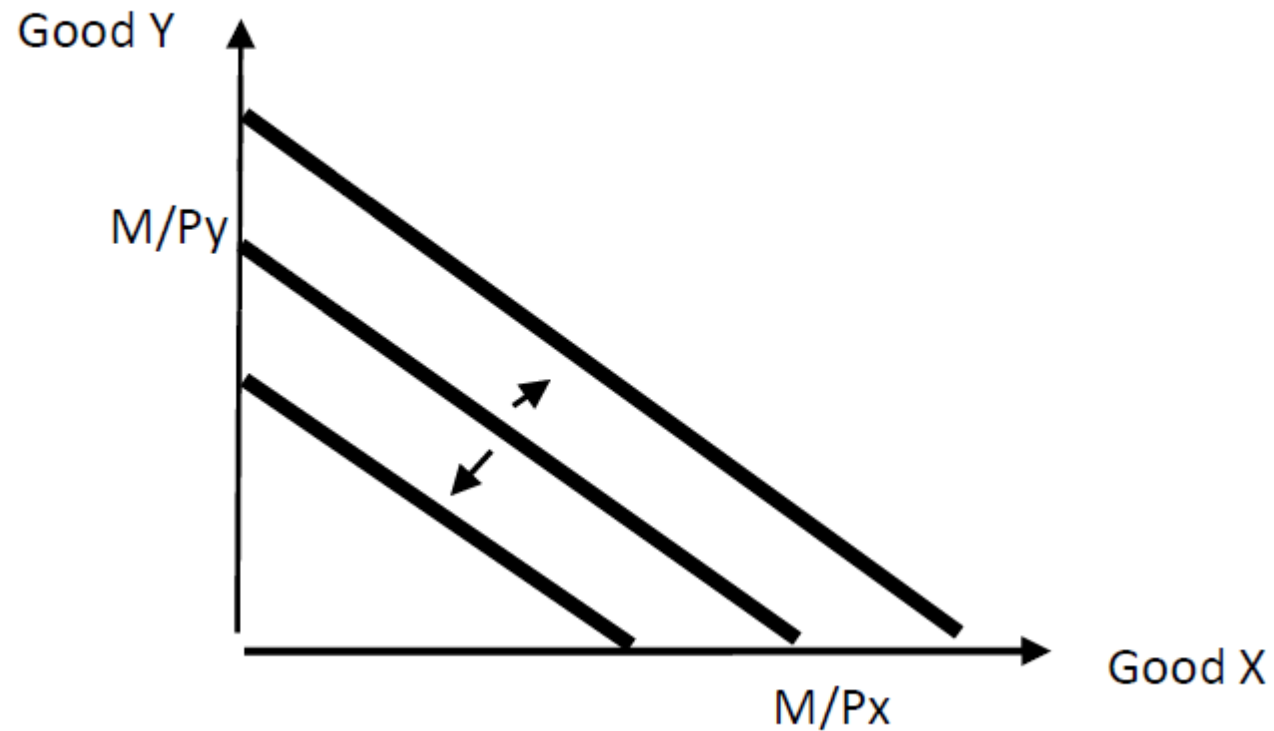
- ✓ When the consumer spends all of her income on good Y, we get the Y- intercept (0,20).
- ✓ Similarly, when the consumer spends all of her income on good X, we obtain the X- intercept (33.3,0). Using these two points we can sketch the graph of the budget line

Change in Income

- ✓ If the income of the consumer changes (keeping the prices of the commodities unchanged), the budget line also shifts (changes).
- ✓ Increase in income causes an upward/outward shift in the budget line that allows the consumer to buy more goods and services and decreases in income causes a downward/inward shift in the budget line that leads the consumer to buy less quantity of the two goods.
- ✓ It is important to note that the slope of the budget line (the

Change in Income

Figure 3.7: Effects of increase (right) and decrease (left) in income on the budget line

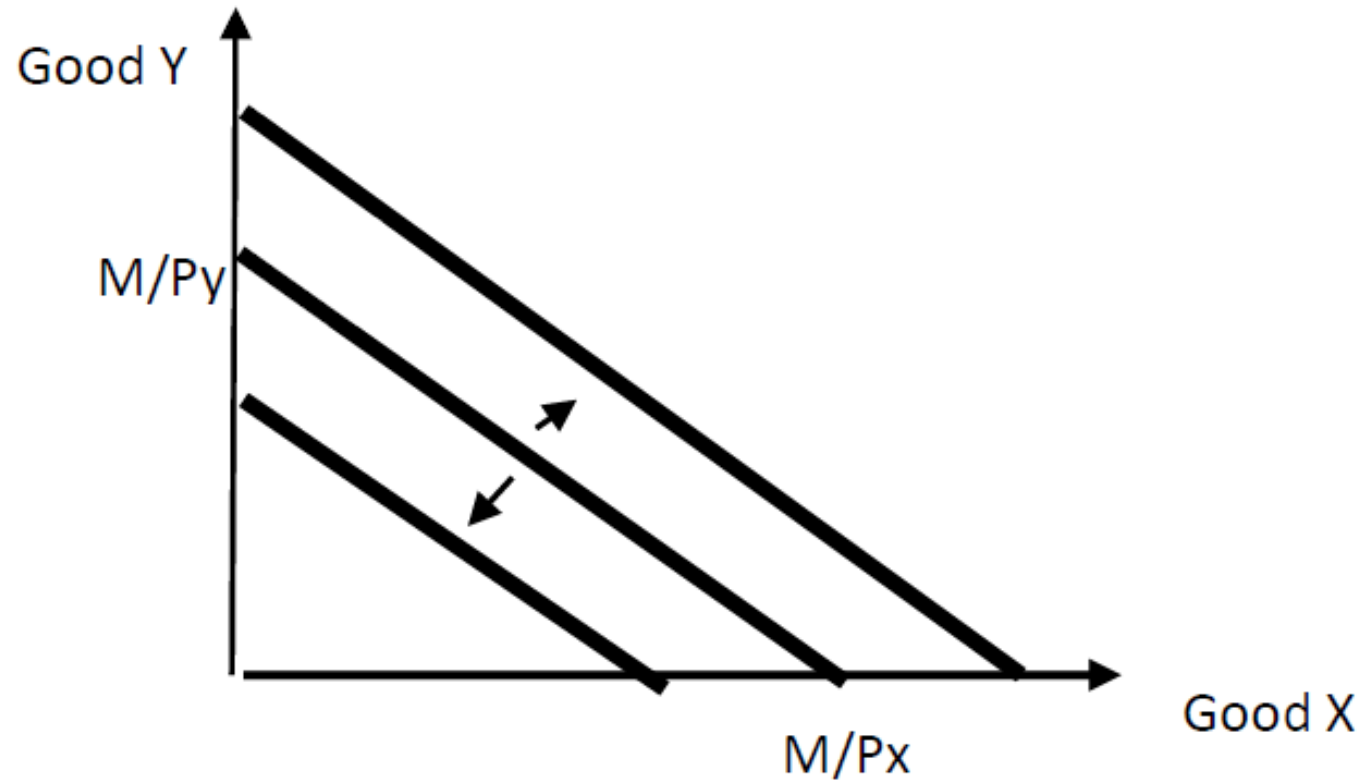


Change in prices:

- ✓ An equal increase in the prices of the two goods shifts the budget line inward.
- ✓ Since the two goods become expensive, the consumer can purchase the lesser amount of the two goods.
- ✓ An equal decrease in the prices of the two goods, on the other hand, shifts the budget line outward.
- ✓ Since the two goods become cheaper, the consumer can purchase the more amounts of the two goods.

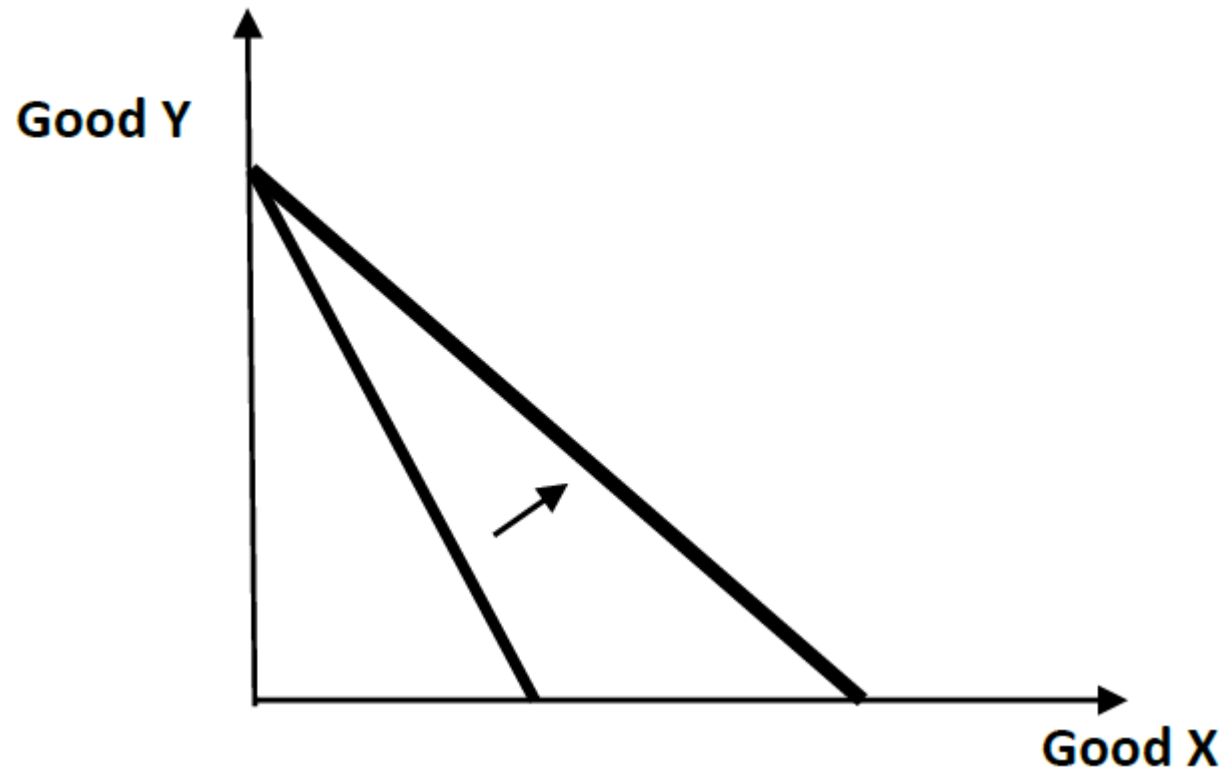
Change in prices:

- ✓ Figure 3.8: Effect of proportionate increase (inward) and decrease (outward) in the prices of both goods



Change in prices:

- ✓ Figure 3.9: Effect of decrease in the price of only good X on the budget line

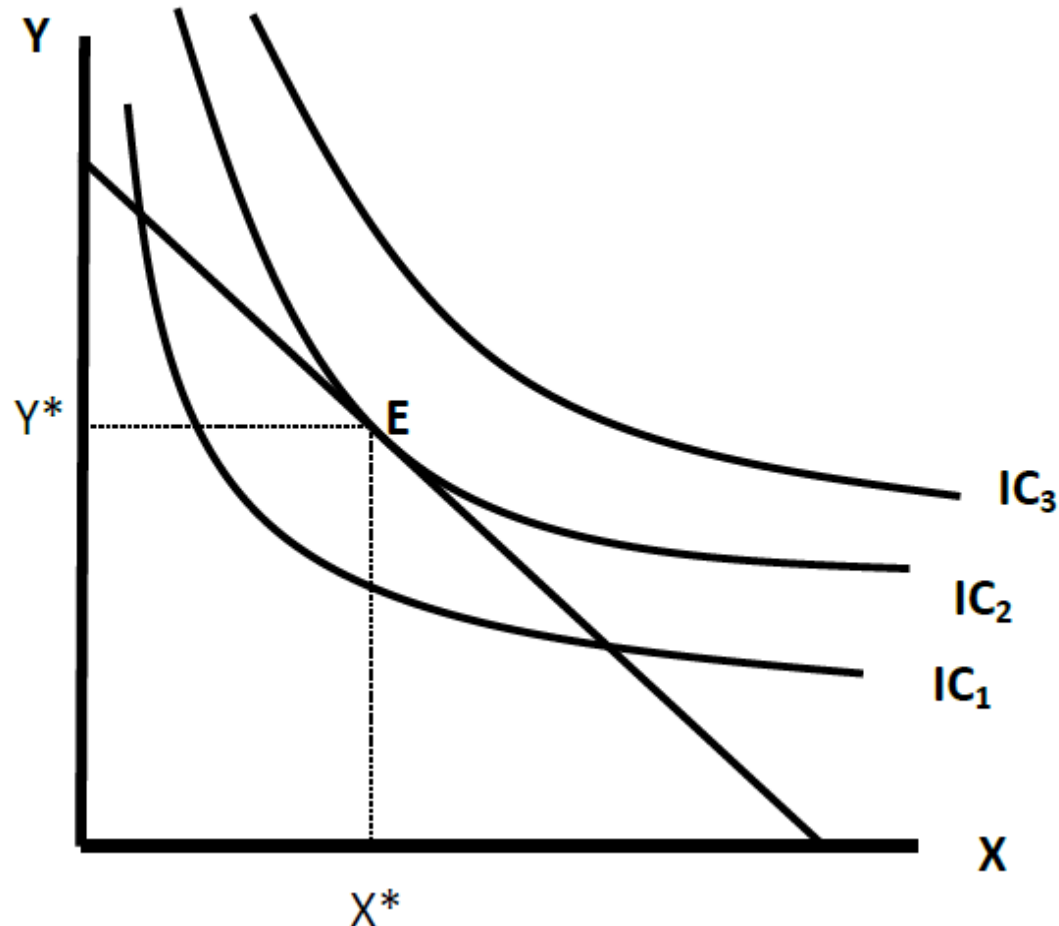


3.3.2.6 Equilibrium of the consumer

- ✓ The preferences of a consumer (what he/she wishes to purchase) are indicated by the indifference curve.
- ✓ The budget line specifies different combinations of two goods (say X and Y) the consumer can purchase with the limited income.
- ✓ Therefore, a rational consumer tries to attain the highest possible indifference curve, given the budget line.
- ✓ This occurs at the point where the indifference curve is tangent to the budget line so that the slope of the indifference curve

3.3.2.6 Equilibrium of the consumer.....

✓ Figure 3.10: Consumer equilibrium under indifference



- ✓ **Mathematically**, consumer optimum (equilibrium) is attained at the point where:
- ✓ Slope of indifference curve
= **MRS_{xy}** = **MU_x/MU_y**
- ✓ Slope of the budget line = **P_x/P_y**
- ✓ Slope of indifference curve = Slope of the budget line then

$$\Rightarrow \frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$

3.3.2.6 Equilibrium of the consumer

- ✓ The preferences of a consumer (what he/she wishes to purchase) are indicated by the indifference curve.
- ✓ The budget line specifies different combinations of two goods (say X and Y) the consumer can purchase with the limited income.
- ✓ Therefore, a rational consumer tries to attain the highest possible indifference curve, given the budget line.
- ✓ This occurs at the point where the indifference curve is tangent to the budget line so that the slope of the indifference curve

3.3.2.6 Equilibrium of the consumer

- ✓ Example: A consumer consuming two commodities X and Y has the utility function $U(X,Y) = XY + 2X$. The prices of the two commodities are 4 birr and 2 birr respectively. The consumer has a total income of 60 birr to be spent on the two goods.
- ✓ a) Find the utility maximizing quantities of good X and Y.
- ✓ b) Find the $MRS_{X,Y}$ at equilibrium.

3.3.2.6 Equilibrium of the consumer

- **Solution**

- **a)** The budget constraint of the consumer is given by:

- $P_X \cdot X + P_Y \cdot Y = M$

- $4X + 2Y = 60 \dots\dots\dots (i)$

Moreover, at equilibrium

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$

$$\frac{Y + 2}{X} = \frac{4}{2}$$

$$\frac{Y + 2}{X} = 2$$

$$Y = 2X - 2 \dots\dots\dots (ii)$$

- Substituting equation (ii) into (i), we obtain $Y = 14$ and $X = 8$.

3.3.2.6 Equilibrium of the consumer

- **Solution**

$$\text{b) } MRS_{X,Y} = \frac{MU_X}{MU_Y} = \frac{Y+2}{X} = \frac{14+2}{8} = 2$$

- **(At the equilibrium, MRS can also be calculated as the ratio of the prices of the two goods)**

- Suppose that the consumer consumes two commodities X and Y given their prices and level of money income M.
- Thus, the objective of the consumer is maximizing his utility function subject to his limited income and market prices.
- In utility maximization, the function that represents the objective that the consumer tries to achieve is called the objective function and the constraint that the consumer faces is called constraint function.
- The maximization problem may be formulated as follows:

$$\text{Maximize } U = f(X, Y)$$

$$\text{Subject to } P_X X + P_Y Y = M$$

- To solve the constrained problem; we use the Lagrange Multiplier Method. The steps involved are:
- A) Rewrite the constraint function as follows:

$$M - P_X X + P_Y Y = 0 \dots (\lambda \text{ will have positive value}) \text{ or}$$

$$P_X X + P_Y Y - M = 0, \dots \lambda \text{ will have negative value}$$

- B) Multiply the constraint by Lagrange multiplier

$$\lambda(M - P_X X + P_Y Y) = 0$$

- λ will have positive value

$$-\lambda(P_X X + P_Y Y - M) = 0$$

- λ will have positive value

- C) Form a composite function or the Lagrange function:

$$\ell = U(X, Y) + \lambda(M - P_X X + P_Y Y)$$

- Or

$$\ell = U(X, Y) - \lambda(P_X X + P_Y Y - M)$$

- D) The first order condition requires that the partial derivatives of the Lagrange function with respect to the two goods and the langrage multiplier be zero.

$$\frac{\partial \ell}{\partial X} = \frac{\partial U}{\partial X} - \lambda P_X = 0; \frac{\partial \ell}{\partial Y} = \frac{\partial U}{\partial Y} - \lambda P_Y = 0 \text{ and } \frac{\partial \ell}{\partial \lambda} = -(P_X X + P_Y Y - M) = 0$$

- From the above equations we obtain

$$\frac{\partial U}{\partial X} = \lambda P_X \quad \text{and} \quad \frac{\partial U}{\partial Y} = \lambda P_Y$$

- Or

$$\frac{\partial U}{\partial X} = MU_X \quad \text{and} \quad \frac{\partial U}{\partial Y} = MU_Y$$

- Therefore, substituting and solving for we get the equilibrium condition:

$$\lambda = \frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$$

- By rearranging we get:

- $$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$

- The second order condition for maximum requires that the second order partial derivatives of the Lagrange function with respect to the two goods must be negative.

$$\frac{\partial \ell^1}{\partial X^1} = \frac{\partial U^1}{\partial X^1} < 0 \quad \text{and} \quad \frac{\partial \ell^2}{\partial Y^1} = \frac{\partial U^2}{\partial Y^1} < 0$$

• A consumer consuming two commodities X and Y has the following utility function $U=XY+2X$. If the price of the two commodities are 4 and 2 respectively and his/her budget is birr 60.

A. form the budget equation

B. Find budget line

C. Find marginal rate of substitutions

D. Form lagrange equation

E. Find the quantities of good X and Y which will maximize utility.

F. Find the optimum level of utility .

A. $4X + 2Y = 60$

B. $Y = 30 - 2X$

C. $MRS_{xy} = M_{ux}/M_{uy} = 4/2 = 2$

D. The Lagrange equation will be written as follows:

$$\ell = XY + 2X + \lambda(60 - 4X - 2Y)$$

$$\frac{\partial \ell}{\partial X} = Y + 2 - \lambda 4 = 0$$

$$\frac{\partial \ell}{\partial Y} = X - \lambda 2 = 0$$

$$\frac{\partial \ell}{\partial \lambda} = 60 - 4X - 2Y = 0$$

From equation (1) we get $Y + 2 = 4\lambda$ and from equation (2) we get $X = 2\lambda$. Thus, we can get that

$$X = \frac{Y + 2}{2} \text{ and equation (2) gives as } \lambda = \frac{1}{2}X.$$

• Then By substituting $X = \frac{Y+2}{2}$ in to equation (2) we get $Y=14$ and $X=8$.

$$MRS_{X,Y} = \frac{MU_X}{MU_Y} = \frac{Y+2}{X}$$

After inserting the optimum value of $Y=14$ and $X=8$ we get 2 which equals to the price ratio of the two goods ($\frac{P_X}{P_Y} = \frac{4}{2} = 2$).

2.4 Effects of Changes in Income and Prices on Consumer's equilibrium

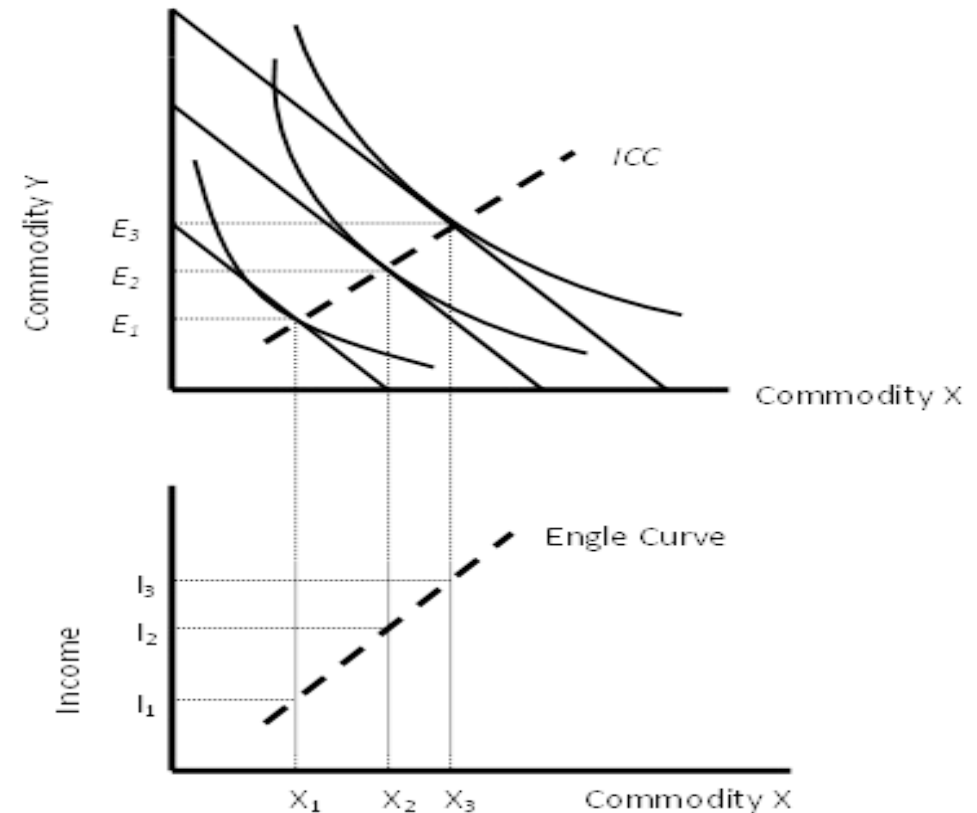
- let us now analyze the effect of changes in consumer's income and the price of the good on equilibrium .
- Let us first consider the effect of change in income on the equilibrium of the consumer all other things remaining constant.
- **Changes In Income: Income Consumption Curve and the Engel Curve**
 - **i. The case of normal goods**
 - In our previous discussion, we noted that an increase in the consumer's income (all other things held constant) results in an upward parallel shift of the budget line.
 - This allows the consumer to buy more of the two goods. And when the consumer's income falls, ceteris paribus, the budget line shifts downward, remaining parallel to the original one.

Effect of income cont

- If we connect all of the points representing equilibrium market baskets corresponding to all possible levels of money income, the resulting curve is called the ***Income consumption curve*** (ICC) or ***Income expansion curve*** (IEC).
- ICC is a locus of points representing various combinations of the two commodities purchased by the consumer at different levels of his income, all other things remaining the same
- The Income Consumption Curve is a curve joining the points of consumer optimum (equilibrium) as income changes (ceteris paribus).
- Or, it is the locus of consumer equilibrium points resulting when only the consumer's income varies.

Effect of income cont

- From the Income Consumption Curve, we can derive the Engle Curve.
- The Engle curve is named after Ernest Engel, the German Statistician who pioneered studies of family budgets and expenditure positions.



Effect of income cont

- **The Engle Curve** is the relationship between the equilibrium quantity purchased of a good and the level of income.
- It shows the equilibrium (utility maximizing) quantities of a commodity, which a consumer will purchase at various levels of income; (ceteris paribus) per unit of time.
- In relation to the shape of the income-consumption and Engle curves, goods can be categorized as normal (superior) and inferior goods.
- Thus, commodities are said to be normal, when the income consumption curve and its Engle curve are positively sloped; meaning that more of the goods are purchased at higher levels of income.
- On the other hand, commodities are said to be inferior when the income consumption curve and Engle curve is negatively sloped, i.e. their purchase decreases when income increases.

- **ii. The case of inferior goods**

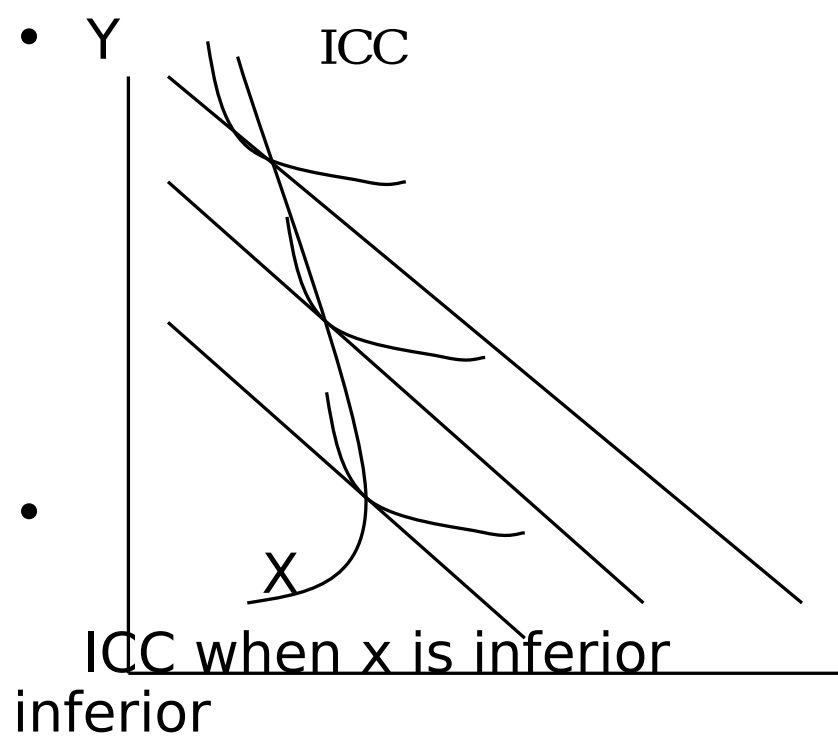
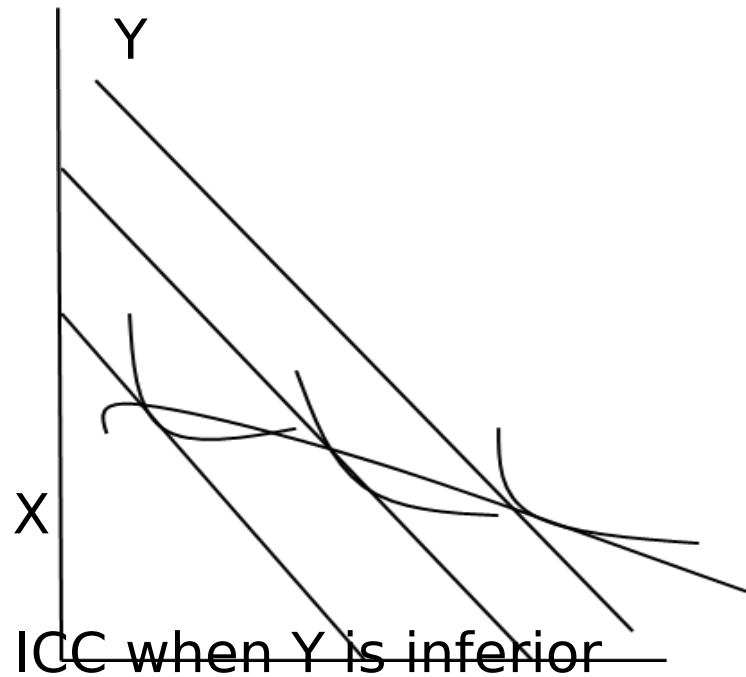
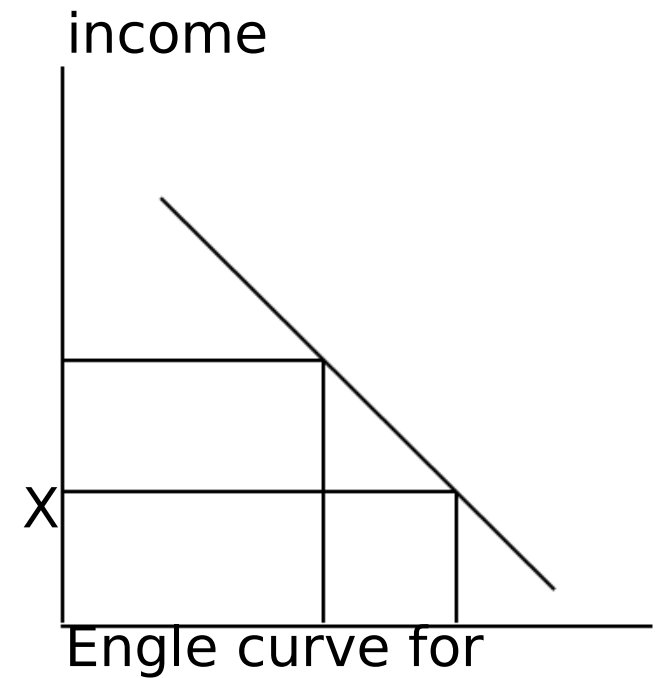
- An inferior commodity is a consumption good whose consumption decreases as income increases.

- ICC

- Y

- Y

-



- **Changes in Price: Price Consumption Curve (PCC) and Individual Demand Curve**
- the second factor that affects the equilibrium of the consumer is price of the goods. The effect of price on the consumption of good is even more important to economists than the effect of changes in income.
- Here, we hold money income constant and let price change to analyze the effect on consumer behavior.
- we have seen that an increase in the price of good X, for example, increases the absolute value of the slope of the budget line, but it does not affect the vertical (Y) intercept of the line.
- Thus, the change in the price of X will result in out ward shift of the budget line that makes the consumer to buy more of good X.
- If we connect all the points representing equilibrium market baskets corresponding to each price of good X we get a curve called price-consumption curve.

- **The price-consumption curve** is the locus of the utility-maximizing combinations of products that result from variations in the price of one commodity when other product prices, the money income and other factors are held constant.
- We can derive the demand curve of an individual for a commodity from the price consumption curve. Below is an illustration of deriving the demand curve when price of commodity X decreases from .

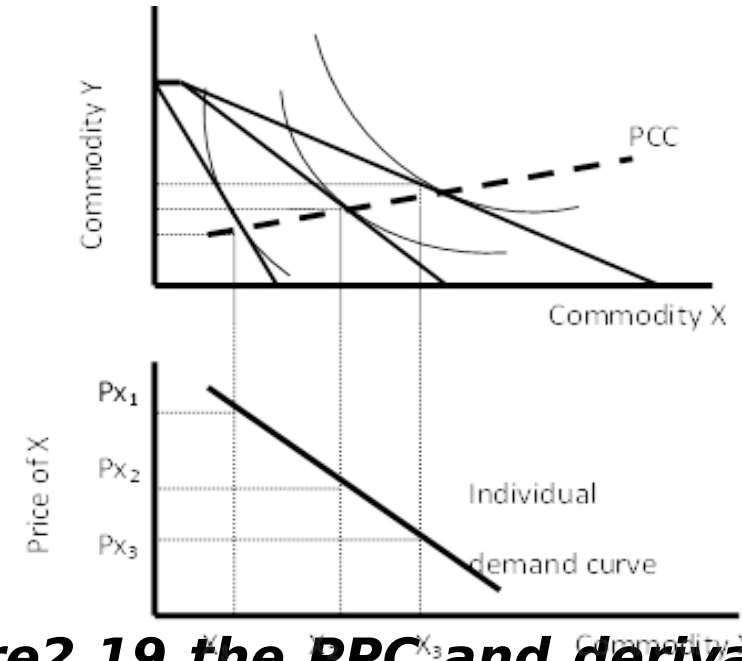


Figure 2.19 the PCC and derivation of the

- **demand curve**

2.5 Income and Substitution effects of a price change

- we now turn to a more complete analysis of why demand curves slope downward. In our previous discussion, we have noted that there are two effects of a price change.
- A fall in the price of a good has two effects:
 - *Substitution effect*
 - *Income effect*
- The combined effect of the two is known as ***the total effect*** (net effect).

A. Substitution Effect

- If price falls (rises), the good becomes cheaper (more expensive) relative to other goods; and consumers substitute toward (away from) the good. This is the substitution effect.
- a decrease in the price of a commodity induces some consumers (the consumer) to substitute it for others, which are now relatively expensive (higher price) commodities.
- Consumers will tend to buy more of the good that has become relatively cheaper, and less of the good that is now relatively more expensive.
- The *substitution effect* is the change in an item's consumption associated with a change in the price of the item, *with the level of utility held constant*.
- When the price of an item declines, the substitution effect always leads to an increase in the quantity of the item demanded.

B. Income Effect

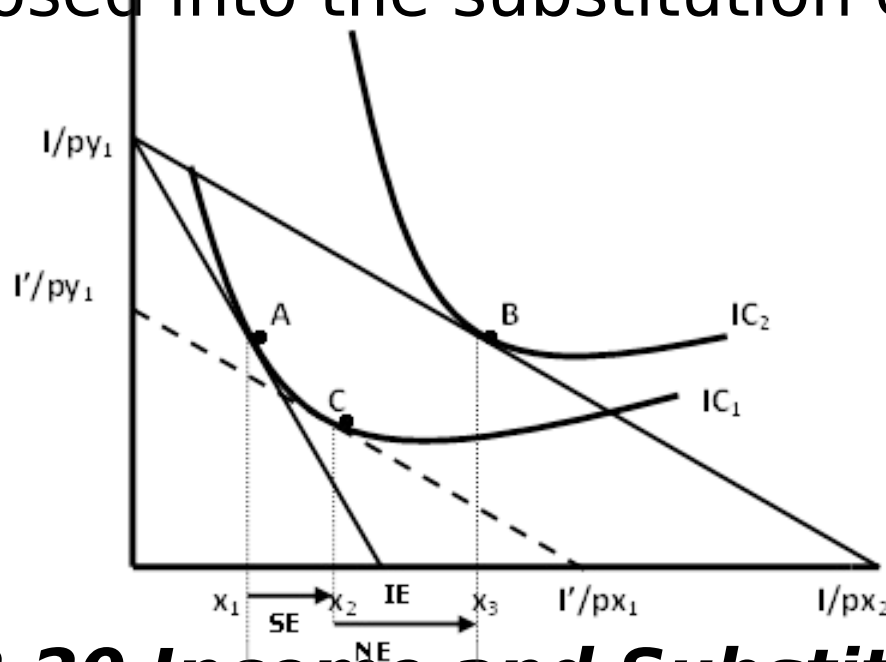
- A fall in the price of a good has *Income effect*
- Also, as price falls (rises), the consumer's purchasing power or real income increases (decreases). Since the set of consumption opportunities increases (decreases) as price changes, the consumer changes the mix of his or her consumption bundle. This effect is called the income effect.
- Consumers experience an increase in real purchasing power when the price of one good falls
- a decrease in price increases the consumer's real income (purchasing power), thus enhancing the ability to buy more goods and services to some extent.
- The *income effect* is the change in an item's consumption brought about by the increase in purchasing power, *with the price of the item held constant*.
- When a person's income increases, the quantity demanded for the product may increase or decrease.

B. The effects of a price-decline cont

- Let us analyze each effect in turn, and then combine the two in order to see why demand is assumed to be downward sloping.
- Suppose initially the income of the consumer is M , price of good Y is p_y , and Price of good X is P_x , we have the budget line with y-intercept I/p_y and X-intercept I/P_x .
- The consumer's equilibrium is point A that indicates the point of tangency between the budget line and indifference curve IC .
- As a result of a decrease in the price of X from P_{x1} to P_{x2} the budget line shifts outward with y-intercept remaining the same.
- Hence the new budget line passes through I/P_y & X-Intercept I/P_{x2} . The consumer's new equilibrium will be on point B .

B. The effects of a price-decline cont

- The total change in the quantity purchased of commodity X from the 1st equilibrium point at A to the second equilibrium point at B shows the Net effect or total effect of the price decline (change).
- The total effect of the price change can be conceptually decomposed into the substitution effect and income effect.



Note that:

$X_1X_3 = \mathbf{NE}$ = Total (net) effect

$X_1X_2 = \mathbf{SE}$ = Substitution effect

$X_2X_3 = \mathbf{IE}$ = Income effect

- **Figure 2.20 Income and Substitution effect for a normal good**

B. The effects of a price-decline cont

The Substitution Effect

- The substitution effect refers to the change in the quantity demanded of a Commodity resulting exclusively from a change in its price when the consumer's real income is held constant; thereby restricting the consumer's reaction to the price change to a movement along the original indifference curve.
- The decline in the price of X results in an increase in the consumer's real income, as evidenced by the movement to a higher indifference curve even though money income remains fixed.
- Now, imagine that we decrease the consumer's income by an amount just sufficient to return to the same level of satisfaction enjoyed before the price decline.
- Graphically, this is accomplished by drawing a fictitious (imaginary) line of attainable combinations with a slope corresponding to new ratio of the product price P_x/P_y so that it is just tangent to the original indifference curve .

B. The effects of a price-decline cont

- The point of tangency is the imaginary point **C** (imaginary equilibrium).
- The movement from point **A** to the imaginary intermediate equilibrium at point **C**, which shows increase in consumption of X from X_1 to X_2 is the substitution effect.
- In other words, the effect of a decrease in price encourages the consumer to increase consumption of X than Y .

B. The effects of a price-decline cont

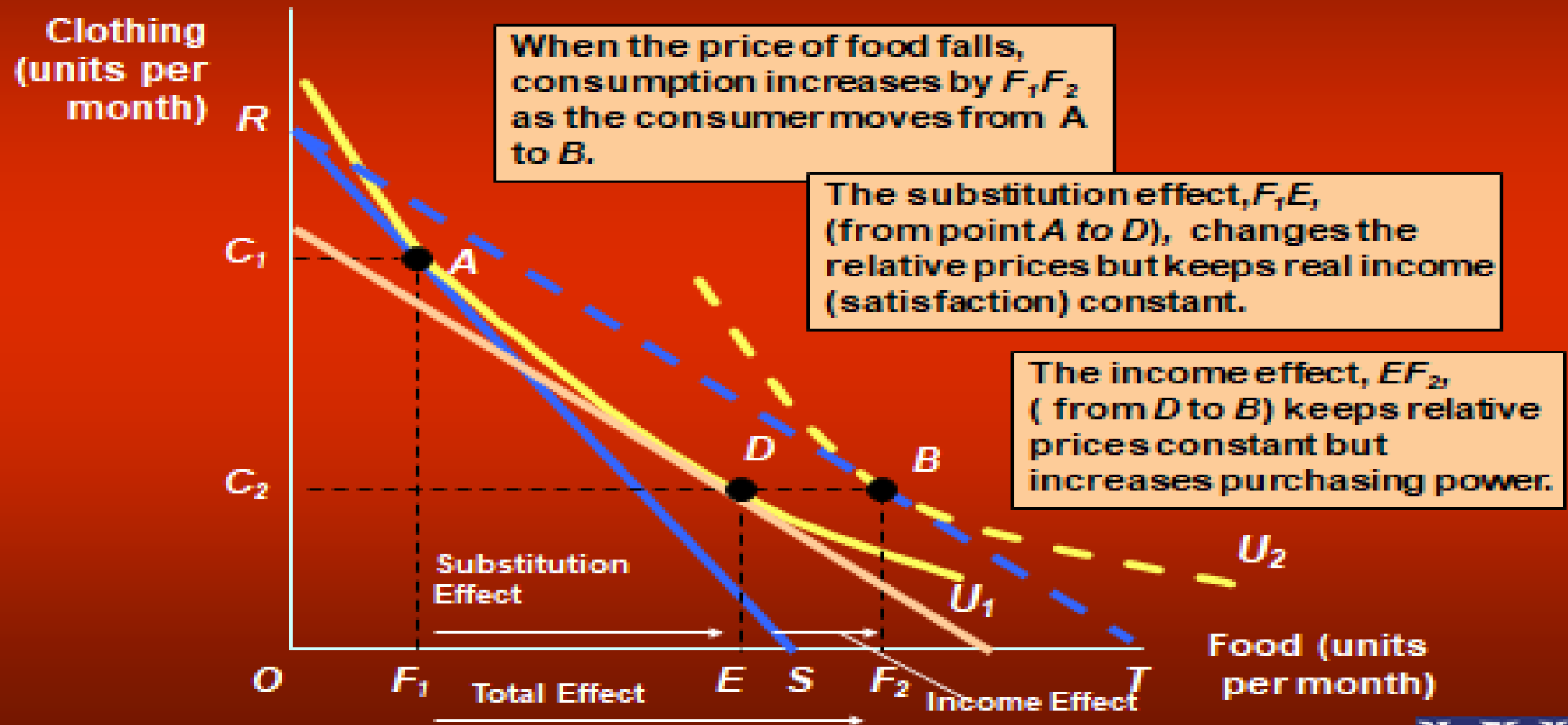
- **The Income Effect**

- The income effect may be defined as the change in the quantity demanded of a commodity exclusively associated with a change in real income.
- The income effect is determined by observing the change in the quantity demanded of a commodity that is associated solely with the change in the consumer's real income.
- In figure 2.20, letting the consumer's real income rise from its imaginary level (defined by the line of attainable combinations tangent to point **C**) back to its true level (defined by the line of attainable combinations tangent to point **B**) gives the income effect.
- Thus, the income effect is indicated by the movement from the imaginary equilibrium at point **C** to the actual new equilibrium at point **B**, the increase in the quantity of X purchased from X_2 to X_3 is the income effect.

- The income effect of a change in the price of good shows the change in quantity demanded via change in real income, while the relative price ratio remains constant.
- This movement does not involve any change in prices; the price ratio is the same in budget line 1 as in budget line 2.
- It is due to a change in total satisfaction and such a change is a movement from one indifference curve to another.
- When we look at both the substitution and income effects, the magnitude of the substitution effect is greater than that of the income effect.
- The reason is that:
 - Most goods have suitable substitutes and when the price of good falls, the quantity of the good purchased is likely to increase very much as consumers substitute the now cheaper good for others.
 - Spending only a small fraction of his /her income, i.e. with the consumers purchasing many goods and spending only a small fraction of their income on any one good, the income effect of a price change of any one good is likely to be small.

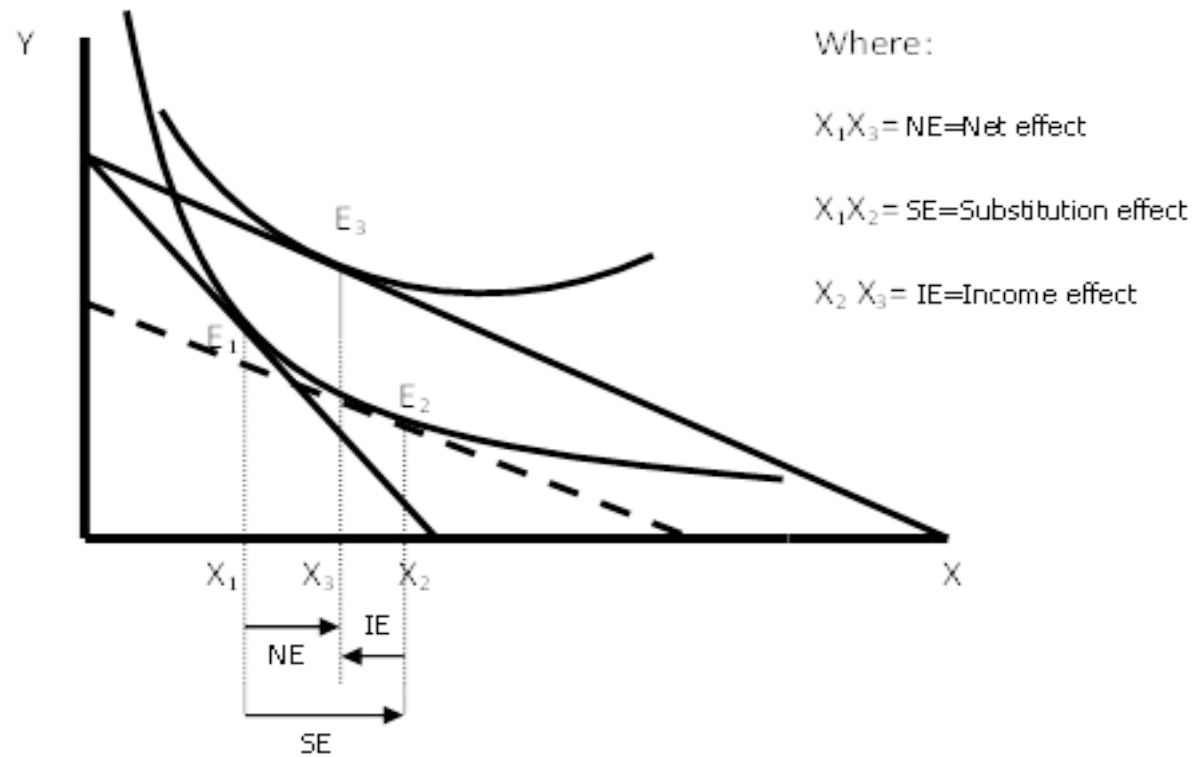
- Usually, the income and substitution effects reinforce one another i.e. they operate in the same direction.
- The substitution effect is always negative. i.e. if the price of a good X increases and real income is held constant, there will always be a decrease in the consumption of good X, and vice versa.
- This result follows from the fact that indifference curves have negative slopes. However, the income effect is not predictable from the theory alone.
- In most cases, one would expect that increases in real income would result in increases in consumption of a good. This is the case for so called Normal goods.
- In short, in the case of normal goods, the income effect and the substitution effects operate in the same direction –they reinforce each other. But, not all goods are normal. Some goods are called inferior goods because the income effect is the opposite (of that of a normal good) for them-they operate in opposite direction.

Income & Substitution Effects: Normal Good



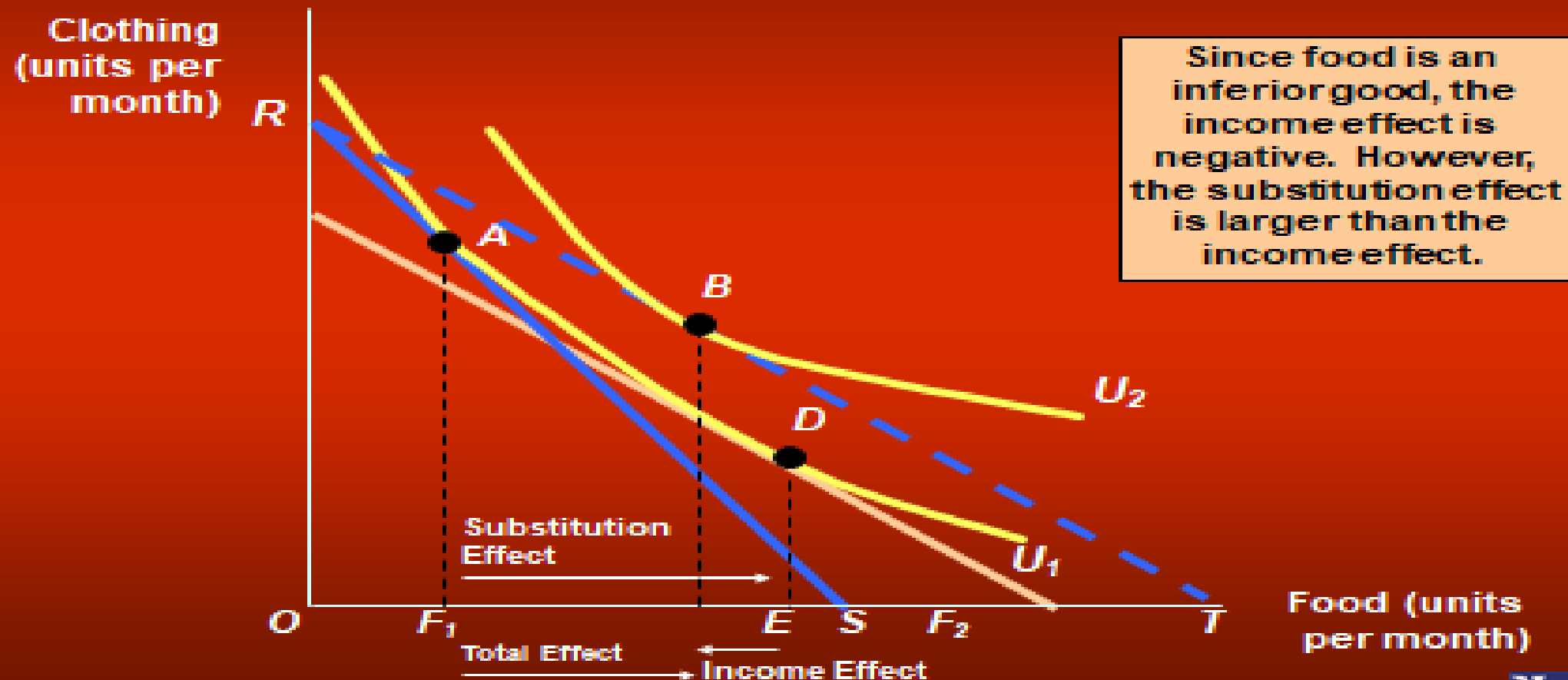
- **For an inferior good**, a decrease in the price of the commodity causes the consumer to buy more of it (the substitution effect), but at the same time the higher real income of the consumer tends to cause him to reduce consumption of the commodity (the income effect).
- We usually observe that the substitution effect still is the more powerful of the two; even though the income effect works counter to the substitution effect, it does not override it.
- Hence, the demand curve for inferior goods is still negatively sloped.

- Let us consider the following diagram that shows the income, substitution and net effect for an inferior commodity in the case of a decline in the price of good X.



- Figure 2.21 Income, Substitution, and Net effect for an inferior commodity**

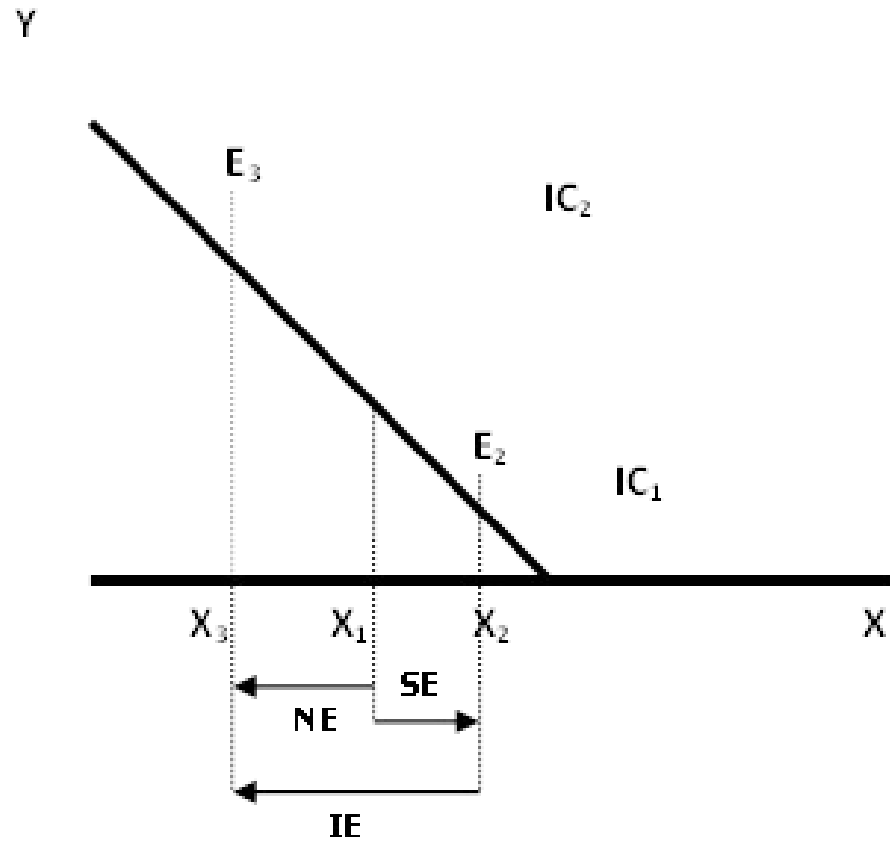
Income & Substitution Effects: Inferior Good



B. The effects of a price-decline cont

- In very rare occasions, a good may be so strongly inferior that the income effect actually overrides the substitute effect.
- Such an occurrence means that a decline in the price of a good would lead to a decline in the quantity demanded and that a rise in price will induce an increase in quantity demanded.
- In other words, price and quantity move in the same direction. The name given to such a unique situation is Giffen paradox; and it constitutes an exception to the Law of demand.
- That is for Giffen goods the income effect (which decreases the quantity demanded) is so strong that it offsets the substitution effect (which increases the quantity demanded), with the result that the quantity demanded is directly related to the price, at least over some range of variation of price.

- Mmm



- **Figure 2.22 Income, Substitution and net effects for a Giffen good,**
- **When there is a price decline.**
-

End of the Chapter

-
-

The End!!!
Thank you for
your
Attention

CHAPTER THREE

CHOICE UNDER UNCERTAINTY

• 3.0 Introduction

- In the previous chapter, you have examined the traditional consumer behavior and demand theory, which based its analysis on the assumption that consumers were making optimal consumption decisions as if they were facing riskless world.
- That is, consumers were assumed to face complete certainty as to the results of the choices they make.
- However, this is not the case in most instances.
- In contrary to our assumptions of price, income and other variables to be known with certainty, many of the choices that people make involve considerable uncertainty in real life decision phenomenon.
- In this chapter you will deal with how households are making consumption decisions when they face uncertainties in their day to day decisions.

3.2. Difference between the terms uncertainty and risk

- Although some people usually use these terms interchangeably, there is some difference between risk and uncertainty
- **Uncertainty:** refer to a situation when there is more than one possible outcome to a decision and where **the probability of each specific outcome is not known**. This may be due to insufficient past information or instability in the structure of the variables.
- **Risk:** refers to a situation when, there are more than one possible outcomes to a decision and the **probability of each specific outcome is known or can be estimated**.
- **Certainty:** refers to a situation when there is only one possible outcome to a decision and this **outcome is known precisely**. For example, investing on treasury bills leads to only one outcome (i.e. the amount of the interest yield), and this is known with certainty.

3.3. PROBABILITY

A. Definition of probability

- Probability is defined as a measure of the likelihood that a state of nature will occur. Probability is a number ranging from 0 to 1.
- It can also be expressed in terms of percentage or decimal.
- For example, a probability of 0 or 0% means that state is impossible; a probability of 1 or 100% means that it's certain.
- Example: Suppose that the Ethiopian government is currently undertaking oil exploration project in different part of the country.
- The probability of oil exploration project to be successful might be $\frac{1}{4}$, and the probability that it is unsuccessful $\frac{3}{4}$.
- This example implies that, the concept of probability depends on the nature of the uncertain event and on the beliefs of the people involved.
- Accordingly: probability can be objective or subjective.

3.3. PROBABILITY

I. Objective probability:

- the objective interpretation of probability relies on the ***frequency*** with which certain events tend to occur.
- The frequency of the occurrence of the state of nature depends on the past information under comparable conditions.
- From our example of oil exploration project, we know, in efforts of 100 oil explorations, 25 have succeeded and 75 have failed.
- Then the probability of success of $\frac{1}{4}$ is objective because it is based directly on the frequency of similar experiences.

3.3. PROBABILITY

- But if we do not have such similar experiences to help us measure probability, we use the concept of subjective probability.

II. Subjective probability:

- such probability is entirely dependent on personal perceptions and judgment to attach the chance of occurrence of an event.
- This shows that, different people may attach different probabilities to different outcomes.
- Accordingly, the choice of the outcomes differs. This is either due to lack of access to same information, or individual's ability to process the information at hand.
- From this you can see that subjective probability is determined independent of frequency of occurrences of an

3.3. PROBABILITY

- **B. Probability and its use**

- Whatever, the interpretation of probability, it is used in calculating two important measures that help us describe and compare risky choices: expected value and the variability of the possible outcomes.
- **3.3.1 EXPECTED VALUE/EXPECTED PAYOFF/**
- Each probabilistic situation has payoffs as it involves risk and uncertainties.
- It is not difficult to realize sometimes, risky choices have financial consequences, which is, known as ***expected values or payoffs***.
- We can use the concept of simple probability to calculate the expected returns from the outcome of risky choices.
- Payoffs can be either positive (gains) or negative (losses).
- To evaluate a choice, we need to know the probability distribution of the payoffs- that is, the likelihood of each possible payoff occurring.
- This means payoffs depend on understanding of events. As long as we know the probability of each possible state of nature, we can determine the probability of each possible payoff.

3.3. PROBABILITY

- Expected value associated with an uncertain situation measures the central tendency that is a weighted average of the payoffs or values associated with all possible outcomes, with the probabilities of each outcome used as weight.
- For example, consider the following economic gambles involving the toss of fair coin:

- We can calculate the expected payoffs of an outcome of an uncertain events, if the possible outcomes are given by $X_1, X_2, \dots, X_n$ and the probabilities denoted by $Pr_1, Pr_2, \dots, Pr_n$

$$EV = P_{r_1} X_1 + P_{r_2} X_2 + \dots + P_{r_n} X_n$$

$$EV = \sum_{i=1}^n P_{r_i} X_i$$

- Where, EV= expected value; Pr probabilities of the occurrence of events and X is the outcome and $i = 1, 2, 3, \dots, n$

3.3. PROBABILITY

- Example 1
- Consider the following economic gambles involving the toss of fair coin:
 - If one tosses a fair coin and the coin comes up heads, you win \$100; if tail, you lose \$0.50. This gamble implies, the winning outcome is 200 times larger than the losing outcome and two outcomes are equally likely.
 - If heads, you win \$200; if tails, you lose \$100. With winning outcome only twice as large as losing outcome, the bet is obviously less attractive than the first, but it too would be accepted by many people.
 - If heads, you win \$20,000; if tails, you lose \$10,000; losers are allowed to pay off their loss in small monthly payment spread out over 30 years.
- Which gamble is attractive to you?
 - The attractiveness of either of the gambles depends on the expected payoffs of each gamble. If a fair coin were tossed a very large number of times, it would come up heads half the time, tails the other half of the time. The probability that a fair coin comes up heads when tossed thus is $\frac{1}{2}$.

3.3. PROBABILITY

- **Solution:** to decide the attractiveness of the gamble we should compare their respective expected payoffs. Hence let us determine the expected values of each game
- Let EV_1 , EV_2 and EV_3 be the expected values of the three games respectively.
 - $EV_1 = \text{Pr}(\text{value of wine}) + \text{pr}(\text{value of lose}) = (1/2) \$100 + (1/2) (-\$0.50) = \$49.75;$
 - $EV_2 = \text{Pr}(\text{value of wine}) + \text{pr}(\text{value of lose}) = (1/2) \$200 + (1/2) (-\$100) = \50
 - $EV_3 = \text{Pr}(\text{value of wine}) + \text{pr}(\text{value of lose}) = (1/2) \$20,000 + (1/2)(-\$10,000) = \5000
- You can easily observe from our solution provided that the expected value of each gamble is positive.
- However, having positive expected value is by no means sufficient to make a gamble attractive.

3.3. PROBABILITY

- Why having positive expected value is not sufficient condition to make the game attractive?
 - The reason is that, in addition to the expected value of a gamble, the perception and attitude of most people about each possible outcome also matters.
 - Let us see the expected values of this particular game by eliminating method.
 - The third gamble is unattractive to most people since it involves a 50-50 chance of the extremely unpleasant outcome of losing \$10,000.
 - In the same way the second gamble also contains an unpleasant 50-50 possibility of losing the initial wealth of \$100.
 - Even though, the second games outcome is an outcome that many people feel they could live with, it is still unattractive to some people.
 - Therefore, the first gamble is by far the easiest choice for most people because its positive outcome is large enough to make a difference, while its negative outcome is too small to really matter.

3.3.2 VARIABILITY

- In previous topic, in section 3.3.1, we attempted to determine the average return of a risky state of nature, but we said nothing about the degree of riskiness of the outcome of that risky decision.
- However, is common that economists always gauge financial risk by measuring the variability of gains and losses from the event.
- Variability is the inconsistent or uneven probability distribution of the outcomes of the states of nature. Variability refers to the event to which the individual outcomes are likely to vary from their expected value.
 - *Variability is an indication of the riskiness of an event in such a way that: low variability shows a narrow range of likely payoffs.*
 - *On the other hand, high variability shows wide range of likely payoff.*
 - *Generally, an outcome with little variability indicates that the actual payoff is almost always close to the expected payoff; and with substantial variability, the two amounts often differ significantly.*
 - *The difference between the actual payoff and the expected payoff is called a **deviation**. Normally, greater variability is associated with larger deviations and vice versa.*

3.3.2 VARIABILITY

- As stated above economists usually measure the ***variability*** of a risky financial payoff or any other risky outcomes of economic decisions by calculating either variance or standard deviation of the distribution of outcomes.
- How to determine the variance of a distribution?
- The ***variance*** is the expected value of a squared deviation.

$$\text{Variance} = [X_1 - E(X_1)]^2 + [X_2 - E(X_2)]^2 + \dots + [X_n - E(X_n)]^2$$

- Its ***calculation involves three steps:***
 - Step1. Compute the deviations of all possible outcomes from the expected value;
 - Step2. Square each deviation;
 - Step3. Find the weighted average of all the possible squared deviations, using the probability of each squared deviation as its weight.

3.3.2 VARIABILITY

- The ***standard deviation*** is the square root of the variance. Note however, that both the variance and the standard deviation tell us the same thing about the size of the typical deviation.
- Example 1: Suppose you are choosing between two part-time sales jobs that have the same expected income of (birr 1500). The first job is based entirely on commission –the income earned depends on how much you sell (your performance). The second job is salaried. There are two equally likely incomes under the first job birr 2000 for good sales effort and birr 1000 for one that is only modestly successful. The second job pays birr 1510 most of the time, but you would earn birr 510 in unfavorable outcome moment pay if the company goes out of business.

3.3.2 VARIABILITY

- Which job is less risky and which is not?
- It may not be as such easy to judge the degree of riskiness of the outcome of these jobs. Thus we need to find the expected values and variability of the payment schemes.
- ***Solution:***
- Let us organized the jobs and their respective probabilities as follows

	probabil ity	Income(\$)	probability	Income (\$)
Job1:commission based	0.5	2000	0.5	1000
Job2: salary based	0.99	1510	0.01	510

3.3.2 VARIABILITY

- The expected payoff of each job is
- $EV_1 = \text{Pr}(\text{good sales}) + 1 - \text{Pr}(\text{modest sale})$; but we are told that these activity are equally likely, that is, they have the same chance to occur. That is, $\text{Pr} = 1 - \text{Pr} = 0.5$

$$\begin{aligned} EV_1 &= \text{Pr}(\text{pay from high performance}) + 1 - \text{Pr}(\text{pay from low performance}) \\ &= 0.5 \times 2000 + 0.5 \times 1000 = \text{birr}1500 \end{aligned}$$

$$\begin{aligned} EV_2 &= \text{pr}(\text{most frequent pay}) + 1 - \text{Pr}(\text{less frequent occurrences pay}) \\ &= 0.99(1510) + 0.01(510) = \text{birr}1500 \end{aligned}$$

3.3.2 VARIABILITY

- The expected values show that both jobs on average pay the same.
- But it does not say anything about which job is safe or more risky to the employee.
- The relative riskiness of these jobs can be determined by calculating the variability of the expected return from each employment.
- Remember that we have discussed that variability is measured by variance or deviation of the actual outcome from average value of the state of nature.
- Thus, since we have already calculated the expected value of the events, variance can be determined as follows:
- Variance of return from job one = $\text{Pr}(\text{pay}_1 - \text{EV}_1)^2 + 1 - \text{Pr}(\text{pay}_2 - \text{EV}_2)^2$
=> the square deviation of the actual values from the average values
- $\text{Variance}_1 = 0.5(2000 - 1500)^2 + 0.5(100 - 1500)^2$
 - $= 0.5 \cdot (500)^2 + 0.5 \cdot (-1400)^2 = 0.5 \cdot 250,000 + 0.5 \cdot 1,960,000 = \$1,105,000$

3.3.2 VARIABILITY

- Variance of return from job2 = $\text{Pr}(\text{pay1} - \text{EV1})^2 + 1 - \text{Pr}(\text{pay2} - \text{EV1})^2$ since expected values are the same for both jobs using EV1 or EV2 there is no difference.
 - $= 0.99(1510 - 1500)^2 + 0.01(510 - 1500)^2$
 - $= 0.99*(10)^2 + 0.01*(-990)^2 = 99 + 0.01*(980,100) = \$ \underline{9,900}$
- Now compare the magnitude of both variances calculated above.
- The outcome of the first business deviate from their expected value more than the second business and so is more risky.
- The preference of an individual will depend on his risk behavior. As our result shows the variance of the second job is lower than the variance of the first job.
- Therefore, the conclusion is that the job based on commission is more risky than job based on salary. Or
- Standard deviation (δ) of commission based job = = \$500
- Standard deviation of salary based job = = \$99.5
- The result indicates that whether we use variance or standard deviation to measure risk- both provide us with the same ranking of risky choices.
- Accordingly, the second job is substantially less risky than the second job. Because both the variance and the standard deviation of the income earned are lower.

3.4. *Utility Function under Uncertainty*

- The very ultimate measure of consumer's preference is the utility derived from consumption of riskless consumption decision as you have dealt with in the second chapter.
- However, under conditions of uncertainty some additional structure, called probability, needs to be added to the choice problem, if the consumer has reasonable preferences about consumption in different circumstances, a utility function can be used to describe these preferences.
- Thus the utility function is defined based on the consumption levels as well as the probabilities of the state of nature. Let C_1 & C_2 represent consumptions in two states and let π_1 & π_2 be their respective probabilities, then we can write our utility function for the different consumption states as $U(C_1, C_2, \pi_1, \pi_2)$.

A. EXPECTED UTILITY

- But at the beginning of this chapter we noted that consumption is always constrained by risky decisions.
- This formal economic theory of choice between uncertain alternatives was advanced by John Von Neumann, a distinguished mathematician at the Institute for Advanced Study, and Oskar Morgenstern, a Princeton economist.
- Its central argument is that, people choose the alternative that has the highest expected utility. Besides, their theory assumes that utility is cardinally measurable.
 - Expected utility is as usual the sum of the utilities associated with all possible outcomes, weighted by the probability that each outcome will occur.
 - Or expected Utility is outcome expected on average. It is the weighted average of all possible outcomes of an event (the weights being probabilities).
 - Eg. Take an event that can result in two possible outcomes: $X_1 = \text{Outcome 1}$ and $X_2 = \text{Outcome 2}$. $E(X) = \text{Expected Outcome}$

3.4. Utility Function under Uncertainty

- Let Π_1 is the probability of outcome 1 and Π_2 is the probability of outcome 2, then

- $$E(X) = \Pi_1 X_1 + \Pi_2 X_2.$$

- The formalization of consumer choice theory under uncertainty was initially by Neuman and Morgenton.
- The central premise of the theory is that consumer's choose the alternative with the highest expected utility rather than the highest value.
- Like other utility functions, the expected utility function can be subject to monotonic transformation.
- The specific form of the utility function depends on the nature of individual consumer preferences. In consumption possibilities state 1 & 2 are perfect substitute $U(C_1, C_2, \&) = \Pi_1 X_1 + \Pi_2 X_2$.
- The Cobb Douglas form is also possible. $U(C_1, C_2, \&) = C_1^{\Pi_1} C_2^{\Pi_2}$
- By taking the monotonic transformation we can re-write the utility function as

$\Pi_1 \ln C_1 + \Pi_2 \ln C_2$. But, the former representation doesn't have the expected utility property, while the latter does.

3.4. *Utility Function under Uncertainty*

- On the other hand, the expected utility function can be subject to some kinds of monotonic transformation and still have expected utility property.
- This special kind of monotonic transformation is called **positive affine transformation** which has the form of $V(U) = au + b$, where $a > 0$.
- Positive affine transformation means multiplying the utility function by some positive number and adding a constant.
- Economists argue that we can apply an affine transformation to expected utility and get another expected utility function that represents the same preferences. However, if any other kind of transformation is applied, it will destroy the expected utility property.
- There are compelling reasons why expected utility is a reasonable for choice problems in the face of uncertainty. The fact that outcomes of the random choice are consumption goods that will be consumed in different circumstances means that ultimately only one of those outcomes is actually going to occur.
- The consumption that will be available to the consumer under the two states of nature can be taken as independent with what the consumer will have if it didn't occur. And such additive form of expected utility function is acceptable for mutually exclusive uncertain outcomes.

3.4. Utility Function under Uncertainty

- Example1:
- Consider the gamble we have seen under section 3.1.2 above, if a consumer with initial wealth level of birr 1000 accepted the gamble and if she won the gamble, her total wealth will increase by the return from win.
- That is, birr 1000 + birr 100 = birr 1100, consequently, the consumer's wealth utility would be $U(1100)$ rather than being $U(1000)$.
- If she loses, her total wealth would fall to birr $(1000 - 0.50) =$ birr 999.5, thus, her utility of wealth after losing the game would be $U(999.5)$.
- More generally, if M_0 is the consumer's initial wealth level, and the new wealth after the gamble is undertaken be, M' . Then, the expected utility of accepting the gamble would be
- Expected utility (EU) = pr of win X (value of win) + pr of lose X (value of lose)
 - $\Rightarrow EU1 = \frac{1}{2} (U(M_0 + M')) + \frac{1}{2} (U(M_0 - M'))$
 - $= 0.5(1100) + 0.5(999.5) = 550 + 499.75 = 1049.75$

3.4. *Utility Function under Uncertainty*

- **Analyzing risky Consumption bundles**

- Consider a consumer who is uncertain about the supply of teff in winter and summer, the expectation is that supply of teff will be low in summer, thus affects the consumers expenditure.
- The possible outcomes are winter enjera consumption and summer enjera consumption.
- Further, assume that our consumer is familiar with the possible outcomes from her past experience.
- Besides she is assumed to consume only those items made of teff, which is measured in kilograms.
- If she is assumed to smooth her consumption, the amount of teff purchased in winter and summer must be the same.
- So that the supply cut in summer will drive the price up and distort her consumption pattern.
- That is, high price forces her to consume less in summer. Therefore, her consumption bundle must list both the amount of food she eats when it is winter and summer.

3.5. Preferences towards Risk:

- So far discussion we have seen that, the prevalence of risk in almost all decisions the consumers expected returns from consumption, investment or any other activities outcome is always less than the actual payoffs of the events.
- Under this part you will study the various attitudes towards risk and how individuals react to the uncertain outcomes to reduce the risky outcomes influences.
- Due to individual differences and preferences are just subjective, utility functions assumes different functional forms, though one can expect that utility should increase with the decision- makers' income, the shape of the utility function can vary greatly depending on the person's attitude towards risk.
- There are three common types of utility functions in relation to preference to wards risky events:

3.5. Preferences Toward Risk

- Choosing Among Risky Alternatives
 - Assume
 - Consumption of a single commodity
 - The consumer knows all probabilities
 - Payoffs measured in terms of utility
 - Utility function given
- Example
 - A person is earning \$15,000 and receiving 13 units of utility from the job.
 - She is considering a new, but risky job.

3.5. Preferences Toward Risk

- She has a .50 chance of increasing her income to \$30,000 and a .50 chance of decreasing her income to \$10,000.
- She will evaluate the position by calculating the expected value (utility) of the resulting income.
- The expected utility of the new position is the sum of the utilities associated with all her possible incomes weighted by the probability that each income will occur.
- The expected utility can be written:
 - $$\begin{aligned} E(u) &= (1/2)u(\$10,000) + (1/2)u(\$30,000) \\ &= 0.5(10) + 0.5(18) \\ &= 14 \end{aligned}$$
 - $E(u)$ of new job is 14 which is greater than the current utility of 13 and therefore preferred.

3.5. Preferences Toward Risk

- people Preferences Toward can be categorized in to three:
 - *risk averse*
 - *risk neutral* or
 - *risk loving*
- **3.5.1 Risk Aversion**
 - Risk aversion is a tendency to prefer a given amount of income with certainty than taking a gamble with high possible return and possible loss.
 - *Risk Averse:* A person who prefers a certain given income to a risky income with the same expected value. The term risk averse refers to the attempt of an individual to avoid uncertain or risky events in his possible outcomes.

3.5. Preferences Toward Risk

- For risk averse person the utility from a given amount of income is greater than the expected utility from different levels of income with similar expected value.
- The risk averse consumer has a concave utility curve. This implies that the slope of a utility curve gets flatter as wealth increases.
- A person is considered risk averse if they have a diminishing marginal utility of income. The reason is that risk averse consumer has a diminishing marginal utility of income.
- The marginal utility of income decreases with increasing level of wealth. This implies that the gain in utility from a given amount of additional wealth is smaller than the loss in utility from a comparable loss in wealth. That is why such a person always refuses to accept a gamble whose expected value is less than expected utility.

3.5. Preferences Toward Risk

- The use of insurance demonstrates risk averse behavior.
- The utility function of such individuals is concave, implying that his utility function increases with the level of his income at a decreasing rate. That is, the marginal utility of 1 birr increase in income generates lesser satisfaction.
- When he is confronted with uncertain situations that offer equal expected monetary values, he always prefers situations in which the outcome is more certain. Meaning he would always refuse a gamble whose expected value of zero. I.e. Gambles with an expected value of zero are called fair gambles.

3.5. Preferences Toward Risk

- What are the characteristics of risk averse consumer?
- Take a person whose utility depends on the level of income as follows. Suppose the person currently has 20,000 she is considering a business which will increase her wealth to 30,000 if the business succeeds but reduce her wealth to 10,000 if it fails. Each success and failure has equal probability of 0.5. A utility for each state is given as follows.

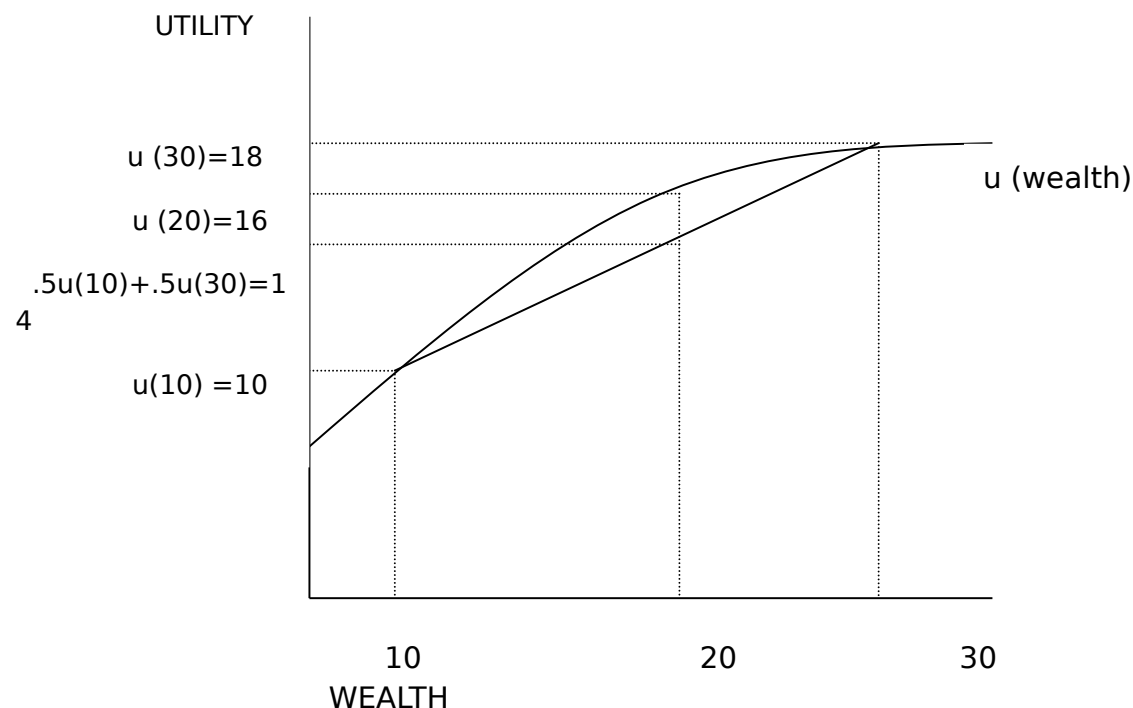
$$U(20,000) = 16, U(10,000) = 10, U(30,000) = 18$$

$$\begin{aligned}\text{Expected utility of wealth} &= 0.5U(30) + 0.5U(10) \\ &= 0.5(18) + 0.5(10) \\ &= 14\end{aligned}$$

- $U(20) > E(U)$, for $U(20)$ is 16 and $E(U)$ is 14. Because of this the risk averse consumer accepts this gamble.
- *Note that: Risk aversion* **prefers a certain income to a risky income with the same expected value.**

3.5. Preferences Toward Risk

- This is depicted in the following figure.



Risk Aversion. For a risk-averse consumer the utility of the expected value of wealth, $u(20)$, is greater than the expected utility of wealth, $.5u(10) + .5u(30)$.

3.5. Preferences Toward Risk

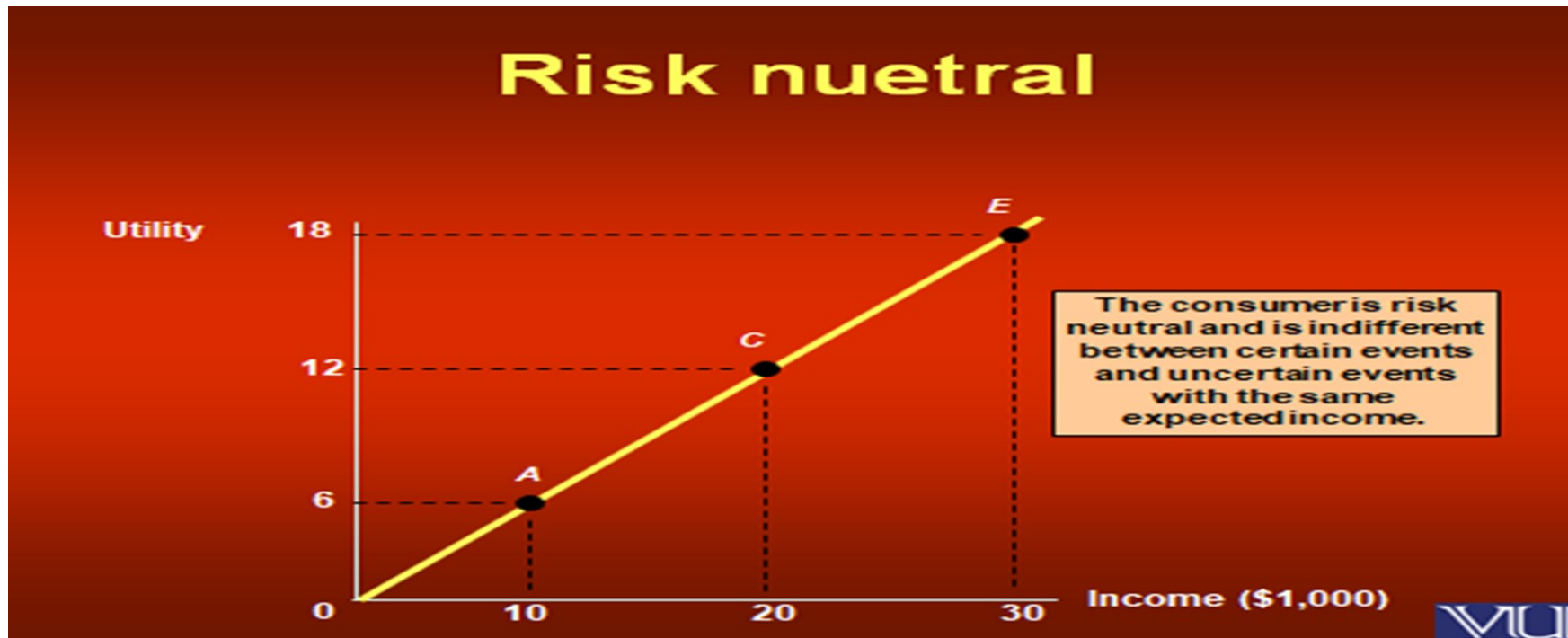
- Risk Averse: A Scenario
 - A person can have a \$20,000 job with 100% probability and receive a utility level of 16.
 - The person could have a job with a .5 chance of earning \$30,000 and a .5 chance of earning \$10,000.
- Expected Income = $(0.5)(\$30,000) + (0.5)(\$10,000) = \$20,000$
- Expected income from both jobs is the same -- risk averse may choose current job

3.5. Preferences Toward Risk

- The expected utility from the new job is found:
 - $E(u) = (1/2)u(\$10,000) + (1/2)u(\$30,000)$
 - $E(u) = (0.5)(10) + (0.5)(18) = 14$
 - $E(u)$ of Job 1 is 16 which is greater than the $E(u)$ of Job 2 which is 14.
- This individual would keep their present job since it provides them with more utility than the risky job.
- They are said to be *risk averse*.

3.5. Preferences Toward Risk

- Risk Neutral
 - A person is said to be *risk neutral* if they show no preference between a certain income, and an uncertain one with the same expected value.



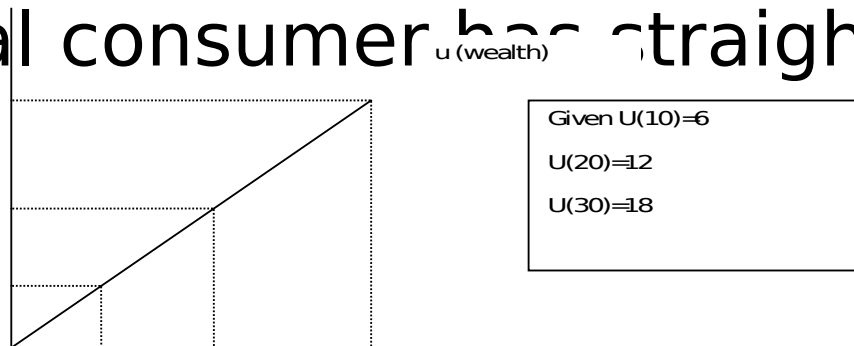
3.5. Preferences Toward Risk

- **Risk-Neutral:**

- If a person is indifferent between a given amount of wealth with certainty and expected value the person is said to be risk neutral.
- The utility function of such people is upward sloping straight line with constant slope. It shows utility increases with income at a constant rate. That is, there is a linear relationship between income and utility.
- These people always act to maximize expected monetary value regardless of the risk level. That means, he/she is indifferent between accepting or refusing a fair gamble.

3.5. Preferences Toward Risk

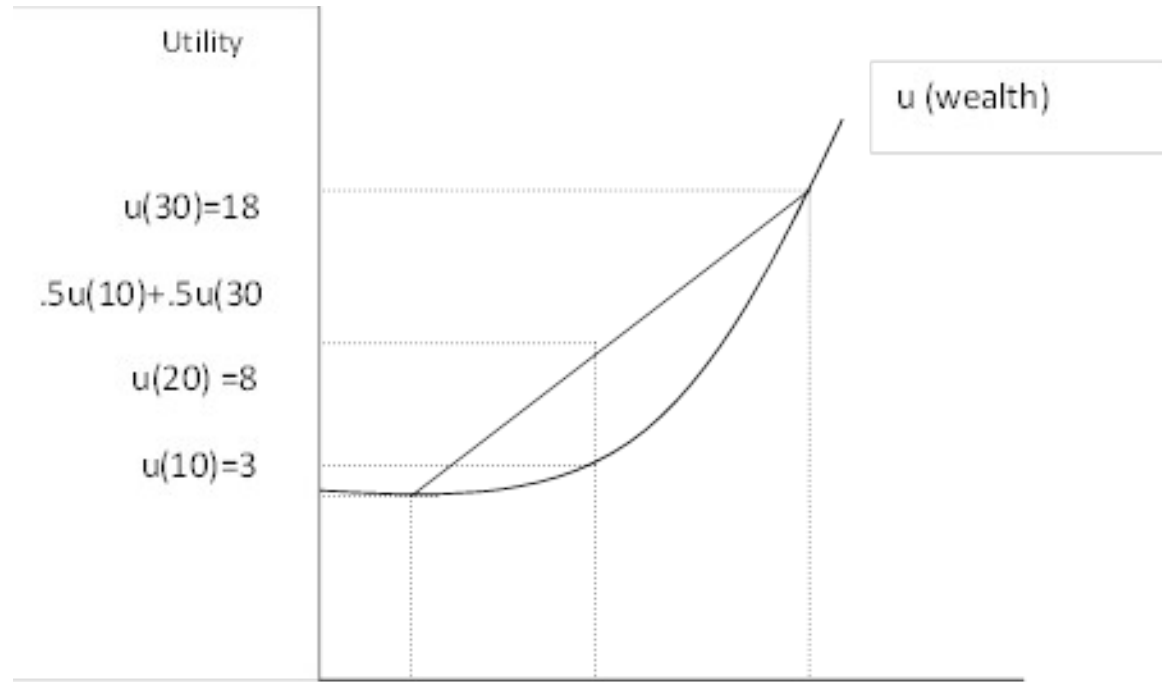
- We have noted above that there were some circumstances under which it would be rational to choose the action or gamble that has the largest expected monetary value.
- That is, making decision on the basis of expected monetary value not expected utility.
- The decision maker must be risk neutral here, because maximizing expected monetary value is exactly equivalent to maximizing expected utility.
- A risk neutral consumer has a straight line utility curve.



3.5. Preferences Toward Risk

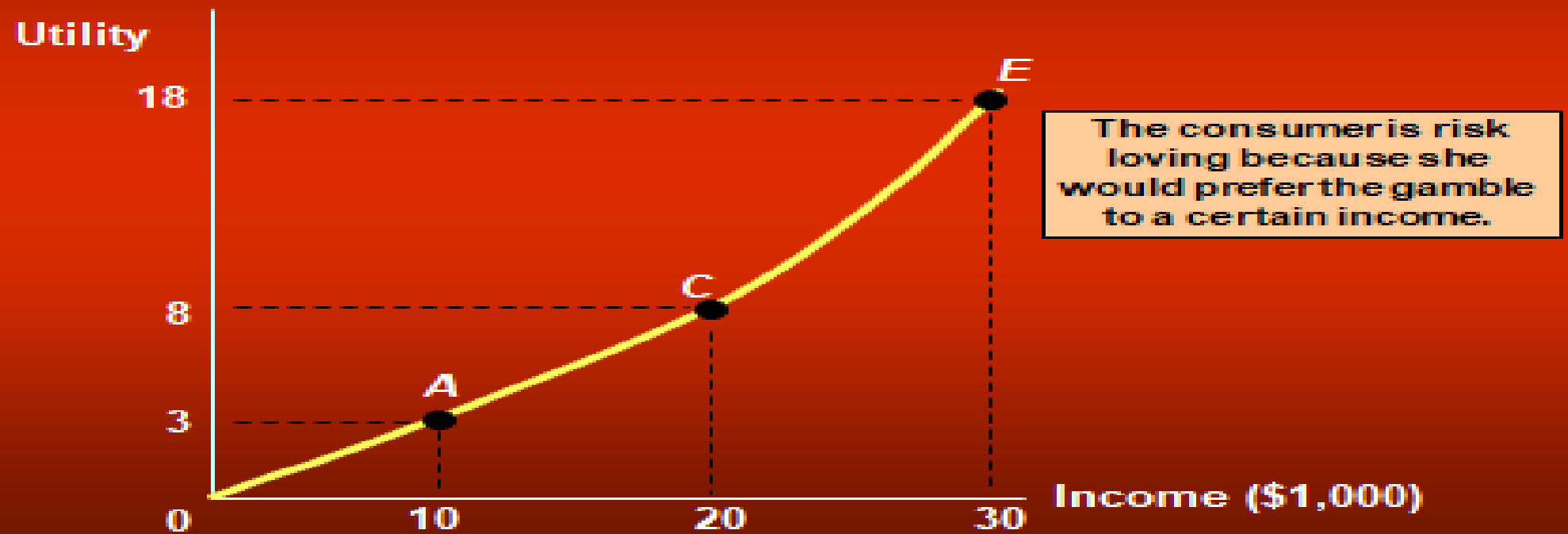
- Risk Loving
 - This is a situation where the utility from a given amount of wealth (income) is less than the expected utility.
 - **Risk lover:** the utility functions of risk lovers increases at an increasing rate with the increase in income.
 - In other words, an increase in income of \$1 is associated with larger and larger increase in utility as income increases in size.
 - When risk lover people are confronted with uncertain situations with equal expected monetary values, they prefer situations in which the outcome is less certain.
 - This implies, their utility function is convex in total wealth, which implies that the expected utility of accepting a fair gamble will be larger than the utility of refusing it.
 - A person is said to be *risk loving* if they show a preference toward an uncertain income over a certain income with the same expected value.
 - Examples: Gambling, some criminal activity

- Geometrically, a convex utility function is one whose slope increases with total wealth.



- The risk lover consumer has a convex utility function.
- The slope of the utility function gets steeper and steeper as wealth increases. For such consumer, marginal utility increases with increasing level of wealth.
- This implies that the gain in utility from a given amount of additional wealth is greater than the loss in utility from a comparable loss in wealth.
- That is when such a person always accepts a fair gamble.
- The marginal utility of income decreases with increasing level of wealth.
- This implies that the gain in utility from a given amount of additional wealth is smaller than the loss in utility from a comparable loss in wealth.
- That is why such a person always refuses to accept a gamble whose expected value is less than expected utility.
- The curvature of the utility function measures the consumer's attitude towards risk. The more concave the utility curve, the more risk averse the consumer is. In contrast, the more convex the utility curve, the more risk lover the consumer is.

Risk loving



- Risk-averse individuals do not avoid risk at all costs. Starting from a riskless position, they are usually willing to accept some risk, provided that they receive adequate compensation in the form of higher expected consumption.
- For example reconsider our consumer in summer and winter season, if she prefers bundle B to bundle A, even though bundle B involves more variability, and in the event of a summer, lower consumption than bundle A.
- **3.4.3. Certainty Equivalents and Risk Premiums:**
- In the previous sections of our discussion we have seen that a risk averse consumer attempts to avoid all risk outcomes.
- This is so because she or he knows that any exposure to risk reduces their well-being. But it is not possible to avoid risky outcomes freely.
-

- *The certainty equivalent of a risky bundle is the amount of consumption which, if provided with certainty, would make the consumer equally well off.*
- A risky bundle's certainty equivalent tells us exactly what it is worth to the consumer.
- When faced with a choice between two risky bundles, a risk averse consumer always chooses the one with the higher certainty equivalent. Note that for a risk-averse individual, the certainty equivalent of a risky bundle is always less than expected consumption. Why?

- Risk Premium

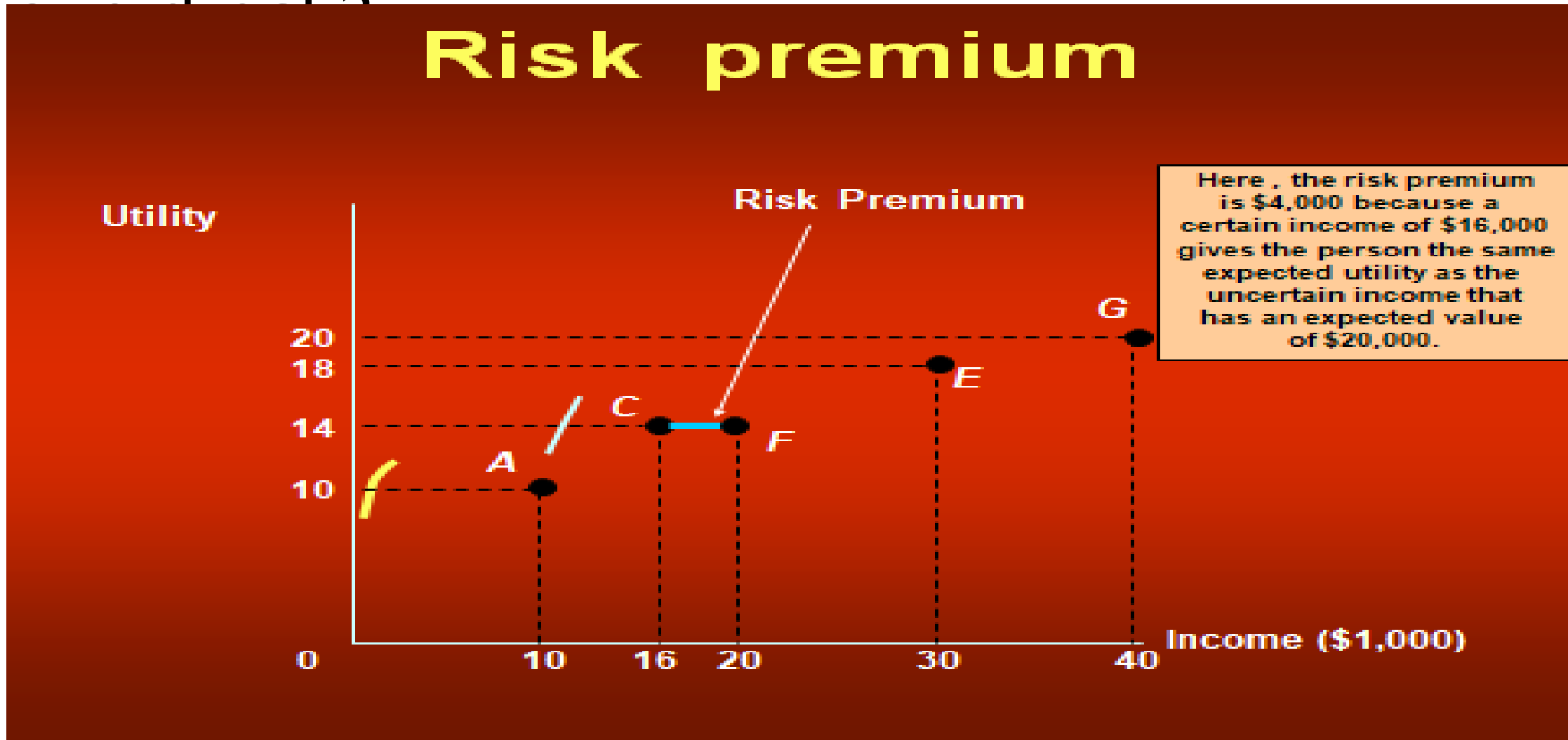
- The *risk premium* is the amount of money that a risk-averse person would pay to avoid taking a risk.
- **Risk Premium:** it's the amount by which the consumer is willing to reduce expected consumption to eliminate all risk. It is measured by the difference between its expected consumption and the consumer's certainty equivalent.

- Risk Premium: A Scenario

- The person has a .5 probability of earning \$30,000 and a .5 probability of earning \$10,000 (expected income = \$20,000).
- The expected utility of these two outcomes can be found:
 - $E(u) = .5(18) + .5(10) = 14$

Preferences Toward Risk

- Question: How much would the person pay to



Preferences Toward Risk

- Risk Aversion and Income
 - Variability in potential payoffs increase the risk premium.
 - Example:
 - A job has a .5 probability of paying \$40,000 (utility of 20) and a .5 chance of paying 0 (utility of 0).
- Solution
 - The expected income is still \$20,000, but the expected utility falls to 10.
 - Expected utility = $.5u(\$) + .5u(\$40,000)$
 $= 0 + .5(20) = 10$
 - The certain income of \$20,000 has a utility of 16.
 - If the person is required to take the new position, their utility will fall by 6.

Preferences Toward Risk

- The risk premium is \$10,000 (i.e. they would be willing to give up \$10,000 of the \$20,000 and have the same $E(u)$ as the risky job.
- Therefore, it can be said that *the greater the variability, the greater the risk premium.*

Business Executives and the Choice of Risk

- Example: Study of 464 executives found that:
 - 20% were risk neutral
 - 40% were risk takers
 - 20% were risk averse
 - 20% did not respond
- Those who liked risky situations did so when losses were involved.
- When risks involved gains the same, executives opted for less risky situations.
- The executives made substantial efforts to reduce or eliminate risk by delaying decisions and collecting more information.

3.6. Approaches of Reducing Risk

- so far discussions we tried to develop some models of consumption or other risk involving decisions that reduces or improves the welfare of an economic agent depending on his or her preference and attitude towards risk.
- However, Sometimes consumers choose risky alternatives that suggest risk-loving rather than risk-averse behavior. Nonetheless, in the face of a broad variety of risky situations, consumers are generally risk averse.
- There are usually four ways to reduce the level of risk:
 - 1) Diversification
 - 2) Insurance
 - 3) spreading
 - 4) Obtaining more information about choices and payoffs of an events outcome.

A. Reducing Risk: Diversification

- Suppose a firm has a choice of selling air conditioners, heaters, or both.
- The probability of it being hot or cold is 0.5.
- The firm would probably be better off by diversification.

A. Reducing Risk: Diversification

Income from Sales of Appliances

	hot weather	Cold weather
Air conditioner sales	\$30,000	\$12,000
Heater sales	12,000	30,000

* 0.5 probability of hot or cold weather

- If the firm sells only heaters or air conditioners their income will be either \$12,000 or \$30,000.
- Their expected income would be:
 - $1/2(\$12,000) + 1/2(\$30,000) = \$21,000$

A. Reducing Risk: Diversification

- If the firm divides their time evenly between appliances their air conditioning and heating sales would be half their original values.
- If it were hot, their expected income would be \$15,000 from air conditioners and \$6,000 from heaters, or \$21,000.
- If it were cold, their expected income would be \$6,000 from air conditioners and \$15,000 from heaters, or \$21,000.
- With diversification, expected income is \$21,000 with no risk

A. Reducing Risk: Diversification

- Firms can reduce risk by diversifying among a variety of activities that are not closely related.
- Stock Market
 - How can diversification reduce the risk of investing in the stock market?
 - Can diversification eliminate the risk of investing in the stock market?

B. Reducing Risk: Insurance

- Risk averse are willing to pay to avoid risk.
- If the cost of insurance equals the expected loss, risk averse people will buy enough insurance to recover fully from a potential financial loss.

B. Reducing Risk: Insurance

The Decision to Insure

Insurance	Burglary (Pr = .1)	No Burglary (Pr = .9)	Expected Wealth	Standard Deviation
No \$9,055	\$40,000		\$50,000	\$49,000
Yes	49,000	49,000	49,000	0

B. Reducing Risk: Insurance

- While the expected wealth is the same, the expected utility with insurance is greater because the marginal utility in the event of the loss is greater than if no loss occurs.
- Purchases of insurance transfers wealth and increases expected utility.

B. Reducing Risk: Insurance

- The Law of Large Numbers
 - Although single events are random and largely unpredictable, the average outcome of many similar events can be predicted.
- Examples
 - A single coin toss vs. large number of coins
 - Whom will have a car wreck vs. the number of wrecks for a large group of drivers

B. Reducing Risk: Insurance

- Assume:

- 10% chance of a \$10,000 loss from a home burglary
- Expected loss = $.10 \times \$10,000 = \$1,000$ with a high risk (10% chance of a \$10,000 loss)
- 100 people face the same risk

- Then:

- \$1,000 premium generates a \$100,000 fund to cover losses
- Actuarial Fairness
 - When the insurance premium = expected payout

B. Reducing Risk: Insurance

- The Value of Title Insurance When Buying a House
- A Scenario:
 - Price of a house is \$200,000
 - 5% chance that the seller does not own the house
- Risk neutral buyer would pay:
$$(.95\$200000 + .05\$0) = \$190000$$
- Risk averse buyer would pay much less
- By reducing risk, title insurance increases the value of the house by an amount far greater than the premium.

C. Reducing Risk: Value of Complete Information

- The difference between the expected value of a choice with complete information and the expected value when information is incomplete.
- Suppose a store manager must determine how many fall suits to order:
 - 100 suits cost \$180/suit
 - 50 suits cost \$200/suit
 - The price of the suits is \$300

C. Reducing Risk: Value of Complete Information

- Suppose a store manager must determine how many fall suits to order:
 - Unsold suits can be returned for half cost.
 - The probability of selling each quantity is .50.

C. Reducing Risk: Value of Complete Information

The Decision to Insure

	Sale of 50	Sale of 100	Expected Profit
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1. Buy 50 suits	\$5,000	\$5,000	\$5,000
2. Buy 100 suits	1,500	12,000	6,750

C. Reducing Risk: Value of Complete Information

- With incomplete information:
 - Risk Neutral: Buy 100 suits
 - Risk Averse: Buy 50 suits
- The expected value with complete information is \$8,500.
 - $8,500 = .5(5,000) + .5(12,000)$
- The expected value with uncertainty (buy 100 suits) is \$6,750.

C. Reducing Risk: Value of Complete Information

- The value of complete information is \$1,750, or the difference between the two (the amount the store owner would be willing to pay for a marketing study).
- An Example
 - Per capita packed milk consumption has fallen over the years
 - The milk producers engaged in market research to develop new sales strategies to encourage the consumption of packed milk.

The Demand for Risky Assets

- Assets

- Something that provides a flow of money or services to its owner.
- The flow of money or services can be explicit (dividends) or implicit (capital gain).

- Capital Gain

- An increase in the value of an asset, while a decrease is a capital loss.

The Demand for Risky Assets

- Risky & Riskless Assets
 - Risky Asset
 - Provides an uncertain flow of money or services to its owner.
 - Examples
 - apartment rent, capital gains, corporate bonds, stock prices
 - Riskless Asset
 - Provides a flow of money or services that is known with certainty.
 - Examples
 - short-term government bonds, short-term certificates of deposit

The Demand for Risky Assets

- Asset Returns
 - Return on an Asset
 - The total monetary flow of an asset as a fraction of its price.
 - Real Return of an Asset
 - The simple (or nominal) return *less* the rate of inflation.

The Demand for Risky Assets

- Asset Returns

$$\text{Asset Return} = \frac{\text{Monetary Flow}}{\text{Purchase Price}}$$

$$\text{Asset Return} = \frac{\text{Flow}}{\text{Bond Price}} = \frac{\$100/\text{yr.}}{\$1,000} = 10\%$$

The Demand for Risky Assets

- Expected vs. Actual Returns
 - Expected Return
 - Return that an asset should earn on average
- Actual Return
 - Return that an asset earns

The Demand for Risky Assets

- Expected vs. Actual Returns
 - Higher returns are associated with greater risk.
 - The risk-averse investor must balance risk relative to return

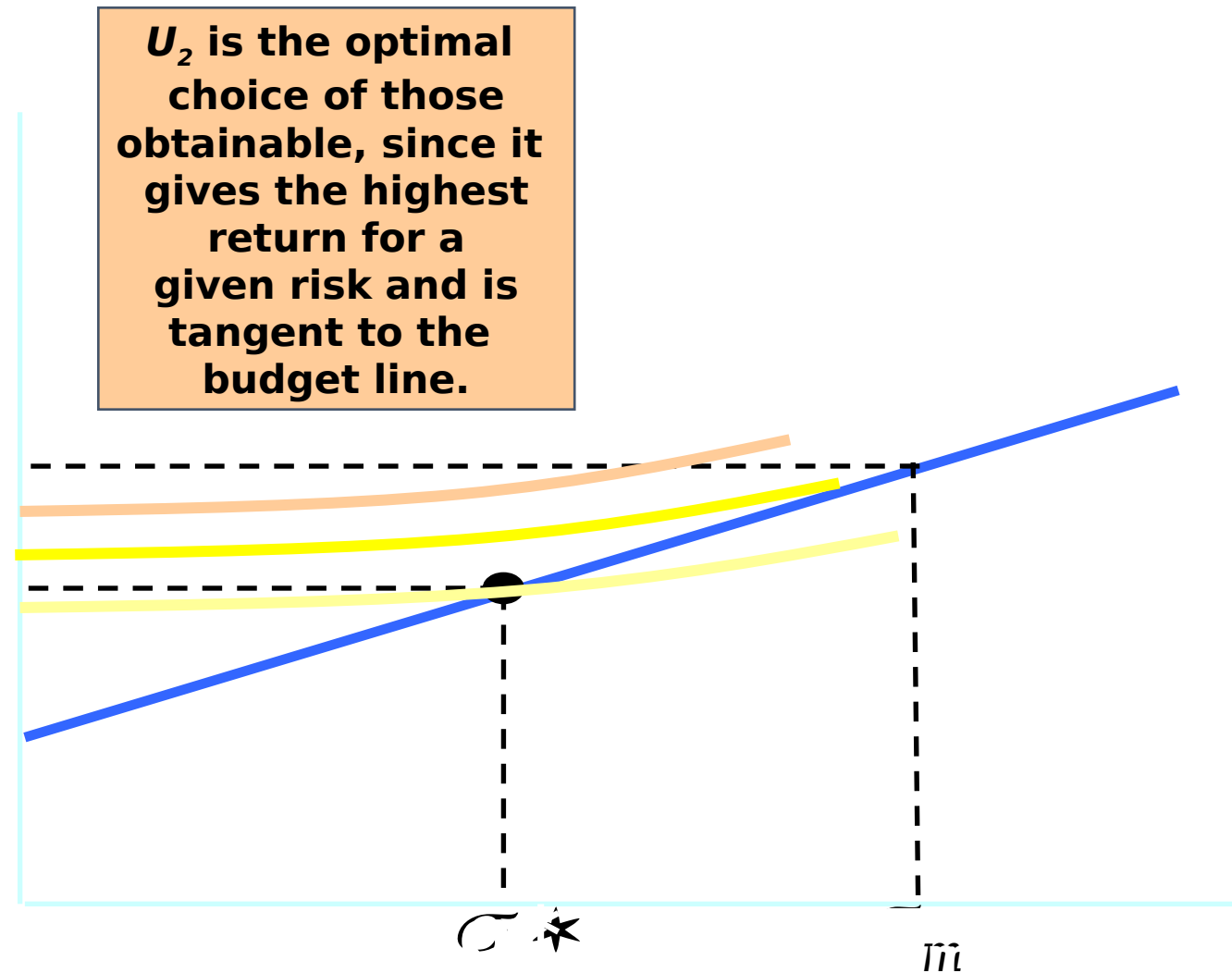
The Demand for Risky Assets

- Risk and Budget Line

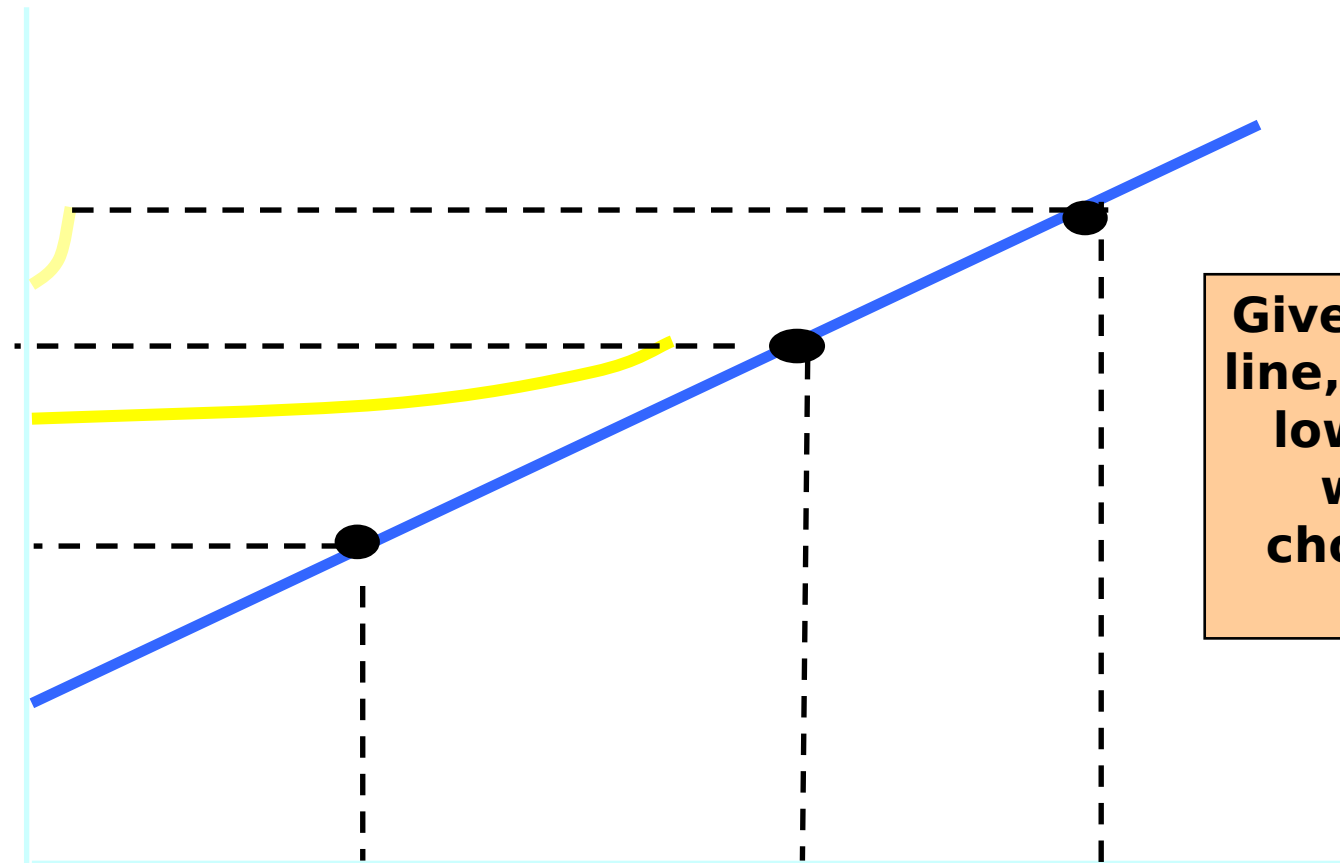
Expected return, R_p , increases as risk increases.

The slope is the price of risk or the risk-return trade-off.

Choosing Between Risk and Return



The Choices of Two Different Investors



Given the same budget line, investor A chooses low return-low risk, while investor B chooses high return-high risk.

Summary

- Reducing risk
 - Diversification
 - Insurance
 - Obtaining More Information
- Demand for Risky Assets

Upcoming Topics

- Production Function
- Isoquants
- Production With One Variable Input
 - Law of Diminishing Marginal Returns
 - Labor Productivity