

Chapter Four

Transformations

Introduction

The term Transformation is generally referred to as converting a graphic into another graphic by applying some rules or algorithms. Sometimes an image or picture can be a combination of lines, rectangle, circle, and triangle. If we draw the basic and combination of pictures, then there should be a need to transform these images. Now we can perform the following actions to transform the images-

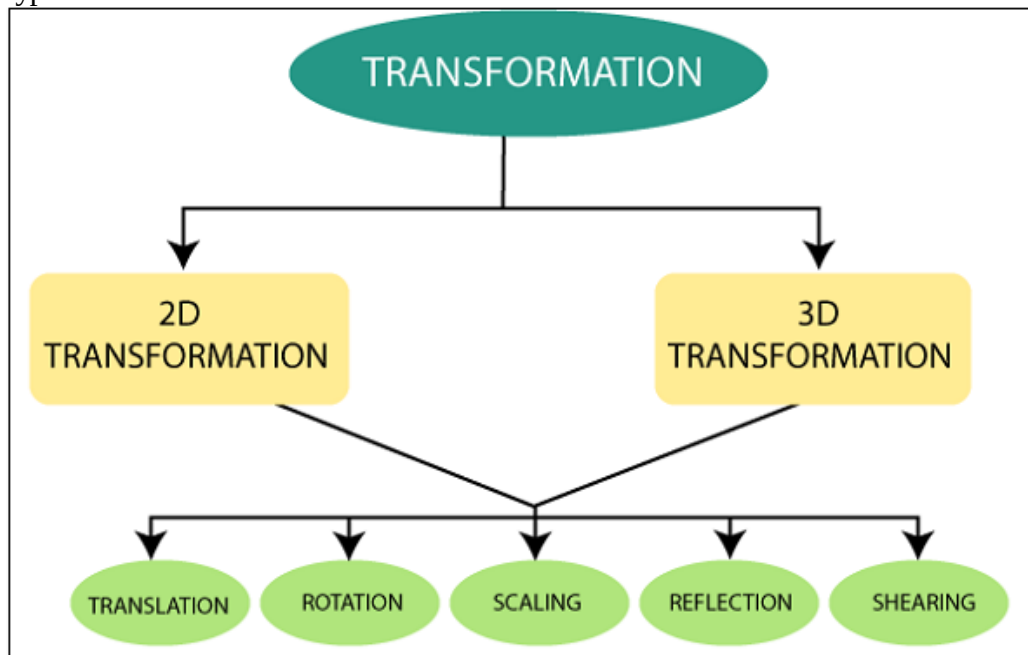
- We can change the position of an image.
- We can increase or decrease the size of an image.
- We can change the angle of the image.

By using the above actions, we will find a new image; this process is called Transformation. We can use some algorithms to produce new pictures.

The object transformation includes two important points-

- **Geometric Transformation:** When we are moving the picture, and the background is fixed, then it is a Geometric Transformation.
- **Coordinate Transformation:** When we are moving the background, and the picture is fixed, then it is Coordinate Transformation.

Types of Transformation



There are two basic kinds of Translation.

1. Two-Dimensional(2D) Transformation:

“When we translate, rotate, and scale object in the two-dimensional plane, then it is called a **Two-Dimensional Transformation**.” A two-dimensional plane consists of the x and y-axis.

The Two-Dimensional Transformation includes-

- **2D Translation:** “Translation is a mechanism used to move the object from one position to another position on the screen.”
- **2D Rotation:** “Rotation is a process used to rotate the object from origin to a particular angle.”
- **2D Scaling:** “Scaling is a process or technique used to resize the object in two-dimensional plane.”
- **2D Reflection:** “Reflection is a mechanism or process in which we can rotate the object at the angle of 180°”.
- **2D Shearing:** “Shearing is a process that is used to perform slanting on the object.” It is also called “**Skewing**.”

2. Three-Dimensional Transformation:

“When we translate, rotate, and scale object in the three-dimensional plane then, it is called **Three-Dimensional(3D) Transformation**”. A three-dimensional plane consists of x, y, and z-axis. The Three-Dimensional Transformation includes-

- **3D Translation:** “Translation is a mechanism used to move the object from one position to another position on the three-dimensional plane.”
- **3D Rotation:** “Rotation is a process used to rotate the object from origin to a particular angle in three-dimensional plane.”
- **3D Scaling:** “Scaling is a process or technique used to resize the object in three-dimensional plane”.
- **3D Reflection:** “Reflection is a mechanism or process in which we can rotate the object at the angle of 180° in three-dimensional plane.”
- **3D Shearing:** “Shearing is a process that is used to perform slanting on the object.” It is also called “**Skewing**”. It also includes z-axis.

1. 2D Translation

It is the straight-line movement of an object from one position to another is called Translation. Here the object is positioned from one coordinate location to another.

Translation of point:

To translate a point from coordinate position (x, y) to another (x_1, y_1) , we add algebraically the translation distances T_x and T_y to original coordinate.

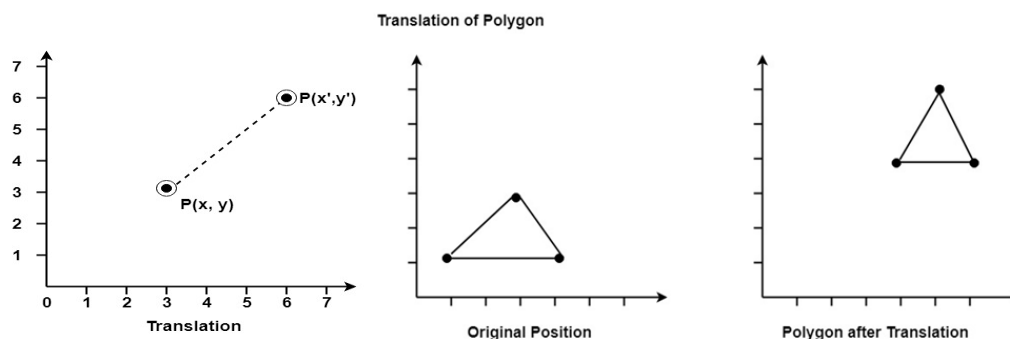
$$x_1 = x + T_x$$

$$y_1 = y + T_y$$

The translation pair (T_x, T_y) is called as **shift vector**. Translation is a movement of objects without deformation. Every position or point is translated by the same amount. When the straight line is translated, then it will be drawn using endpoints.

For translating polygon, each vertex of the polygon is converted to a new position. Similarly, curved objects are translated. To change the position of the circle or ellipse its center coordinates are transformed, then the object is drawn using new coordinates.

Let P is a point with coordinates (x, y) . It will be translated as (x^1, y^1) .



Consider a point object O has to be moved from one position to another in a 2D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- New coordinates of the object O after translation = (X_{new}, Y_{new})

- Translation vector or Shift vector = (T_x, T_y)

Given a Translation vector (T_x, T_y) -

- T_x defines the distance the X_{old} coordinate has to be moved.
- T_y defines the distance the Y_{old} coordinate has to be moved.

This translation is achieved by adding the translation coordinates to the old coordinates of the object as-

- $X_{new} = X_{old} + T_x$ (This denotes translation towards X axis)
- $Y_{new} = Y_{old} + T_y$ (This denotes translation towards Y axis)

In Matrix form, the above translation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

Translation Matrix

- The homogeneous coordinates representation of (X, Y) is $(X, Y, 1)$.
- Through this representation, all the transformations can be performed using matrix / vector multiplications.

The above translation matrix may be represented as a 3×3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Translation Matrix
(Homogeneous Coordinates Representation)

Example 1: An object is placed with respect to origin on (2,1). Now move the object away from the origin by 2 steps in x and 3 steps in y. What would be new coordinates to the moved object.

- I. The old coordinates of the object are $x=2$ and $y=1$. Translation coordinate in x direction $(tx) = 2$ and Translation coordinate in y direction $(ty) = 3$. Let's call the new coordinates as x' and y' .

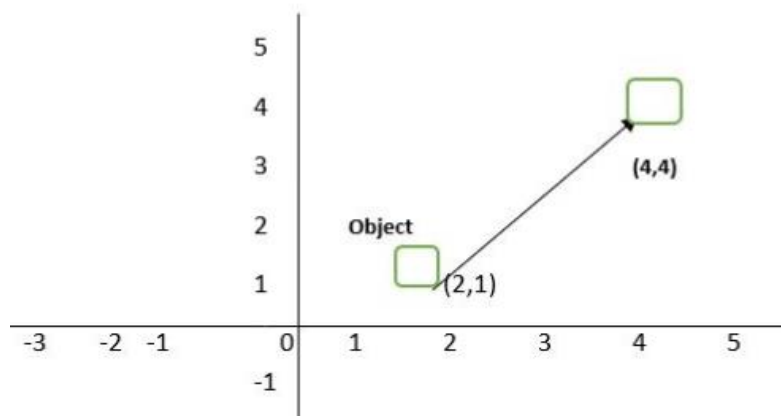
- II. Since the translated object is moving away from origin, hence it is a positive translation and new coordinates could be obtained in simple arithmetic as $x' = x + tx$ and $y' = y + ty$.
- III. Computer Graphics only deal with matrix calculations because all the transformation would not be as simple as in translation. Translation transformation of a single element at (x, y) coordinate is displayed as follows.
- IV. Just substitute the values of x, y as $(2, 1)$ with $tx = 2$ and $ty = 1$. Multiply all the matrices to get the result. This will give you new coordinates (x', y') of point (x, y) .

$$\begin{bmatrix} 1 \\ x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ x \\ y \end{bmatrix} \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix}$$

New Element Matrix Old Element Translation Matrix

$$\begin{bmatrix} 1 \\ x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

S-5: After Multiplication we would get the new element $x' = 2 + 2 = 4$ and $y' = 1 + 3 = 4$. The new element is also displayed in last figure.



Example 2: Given a square with coordinate points $A(0, 3)$, $B(3, 3)$, $C(3, 0)$, $D(0, 0)$. Apply the translation with distance 1 towards X axis and 1 towards Y axis. Obtain the new coordinates of the square.

Given-

- Old coordinates of the square = $A(0, 3)$, $B(3, 3)$, $C(3, 0)$, $D(0, 0)$
- Translation vector = $(T_x, T_y) = (1, 1)$

For Coordinates $A(0, 3)$: Let the new coordinates of corner $A = (X_{\text{new}}, Y_{\text{new}})$. Applying the translation equations, we have-

- $X_{\text{new}} = X_{\text{old}} + T_x = 0 + 1 = 1$
- $Y_{\text{new}} = Y_{\text{old}} + T_y = 3 + 1 = 4$:: Thus, New coordinates of corner A = (1, 4).

For Coordinates B(3, 3) : Let the new coordinates of corner B = $(X_{\text{new}}, Y_{\text{new}})$. Applying the translation equations, we have-

- $X_{\text{new}} = X_{\text{old}} + T_x = 3 + 1 = 4$
- $Y_{\text{new}} = Y_{\text{old}} + T_y = 3 + 1 = 4$:: Thus, New coordinates of corner B = (4, 4).

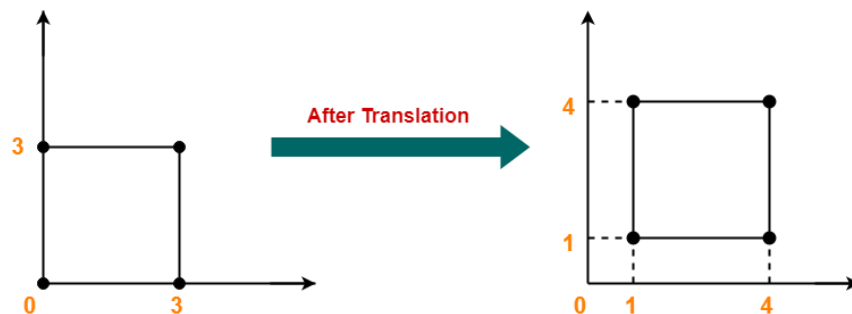
For Coordinates C(3, 0) : Let the new coordinates of corner C = $(X_{\text{new}}, Y_{\text{new}})$. Applying the translation equations, we have-

- $X_{\text{new}} = X_{\text{old}} + T_x = 3 + 1 = 4$
- $Y_{\text{new}} = Y_{\text{old}} + T_y = 0 + 1 = 1$:: Thus, New coordinates of corner C = (4, 1).

For Coordinates D(0, 0) : Let the new coordinates of corner D = $(X_{\text{new}}, Y_{\text{new}})$. Applying the translation equations, we have-

- $X_{\text{new}} = X_{\text{old}} + T_x = 0 + 1 = 1$
- $Y_{\text{new}} = Y_{\text{old}} + T_y = 0 + 1 = 1$:: Thus, New coordinates of corner D = (1, 1).

Thus, New coordinates of the square = A (1, 4), B(4, 4), C(4, 1), D(1, 1).



2. 2D Rotation

2D Rotation is a process of rotating an object with respect to an angle in a two-dimensional plane. Consider a point object O has to be rotated from one angle to another in a 2D plane. Let- Initial coordinates of the object O = $(X_{\text{old}}, Y_{\text{old}})$ Initial angle of the object O with respect to origin = Φ

The Rotation of any object depends upon the two points.

Rotation Point: It is also called **the Pivot point and the Rotation Angle:** It is denoted by **Theta** (θ).

We can rotate an object in two ways-

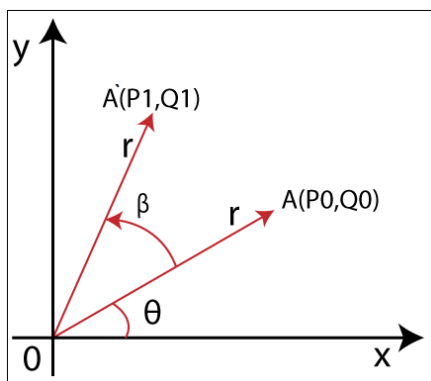
- **Clockwise:** An object rotates clockwise if the value of the Rotation angle is negative (-).
- **Anti-Clockwise:** An object rotates anti-clockwise if the value of the Rotation angle is positive (+).

We can apply Rotation on following objects-

- **Straight Lines**
- **Curved Lines**
- **Polygon**
- **Circle**

For Example-

Rotation of a Point: If we want to Rotate a point A (P_0, Q_0) that has a Rotation angle (θ) with distance r from origin to A' (P_1, Q_1) that has a Rotation angle β . Then, we can rotate by following Rotation equation-



$$P_1 = P_0 \times \cos\theta - Q_0 \times \sin\theta$$

$$Q_1 = P_0 \times \sin\theta + Q_0 \times \cos\theta$$

We can represent the coordinates of point A (P_0, Q_0) by using standard trigonometry-

$$P_0 = r \cos\theta \dots\dots\dots (1)$$

$$Q_0 = r \sin\theta \dots\dots\dots (2)$$

We can also define point A' (P_1, Q_1) in the same way-

$$P_1 = r \cos(\theta + \beta) = r \cos\theta \cos\beta - r \sin\theta \sin\beta \dots\dots\dots (3)$$

$$Q_1 = r \sin(\theta + \beta) = r \cos\theta \sin\beta + r \sin\theta \cos\beta \dots\dots\dots (4)$$

By using equation (1) (2) (3) (4), we will get-

$$P_1 = P_0 \cos\theta - P_0 \sin\theta$$

$$Q_1 = P_0 \sin\theta + P_0 \cos\theta$$

We can also represent the Rotation in the form of matrix-

$$\begin{bmatrix} P_1 \\ Q_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} P_0 \\ Q_0 \end{bmatrix}$$

Homogeneous Coordinates Representation: The Rotation can also be represented in the form of 3 x 3 Rotation matrix-

$$\begin{bmatrix} P_1 \\ Q_1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} P_0 \\ Q_0 \\ 1 \end{bmatrix}$$

Example- A line segment with the starting point (0, 0) and ending points (5, 5). Apply 30-degree rotation anticlockwise direction on the line. Find the new coordinates of the line?

Solution- We can rotate the straight line by its endpoints with the same angle.

$$P_1 = P_0 \times \cos\theta - Q_0 \times \sin\theta$$

$$= 5 \times \cos 30 - 5 \times \sin 30$$

$$= 5 \times (\sqrt{3}/2) - 5 \times (1/2)$$

$$= 4.33 - 2.5$$

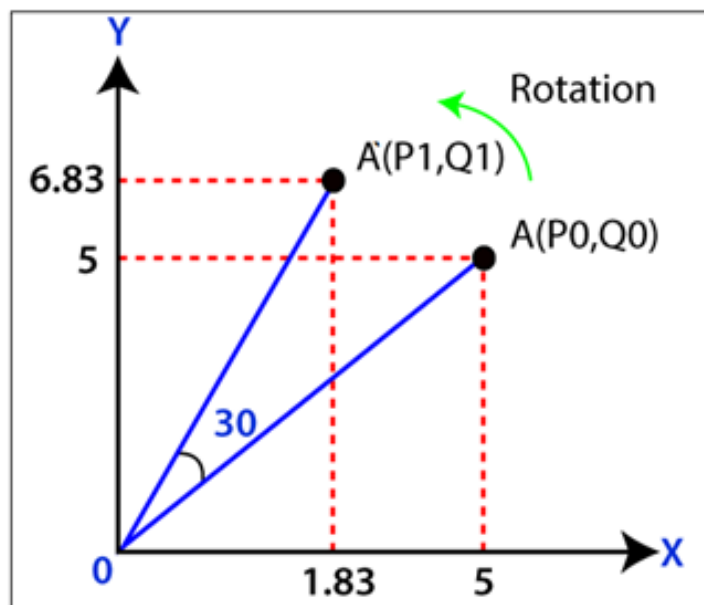
$$= 1.83$$

$$Q_1 = P_0 \times \sin\theta + Q_0 \times \cos\theta$$

$$= 5 \times \sin 30 + 5 \times \cos 30$$

$$= 5 \times (1/2) + 5 \times (\sqrt{3}/2)$$

$$= 2.5 + 4.33 = 6.83$$



We have, $(P_0, Q_0) = (0, 0)$

Rotation Angle $(\theta) = 30^\circ$

Let the new coordinates of line = (P_1, Q_1)

We can apply the rotation matrix, then, Thus, the new endpoint coordinates of the line are $= (P_1, Q_1) = (1.83, 6.83)$

Problem-01: Given a line segment with starting point as (0, 0) and ending point as (4, 4). Apply 30 degree rotation anticlockwise direction on the line segment and find out the new coordinates of the line.

Problem-02: Given a triangle with corner coordinates (0, 0), (1, 0) and (1, 1). Rotate the triangle by 90 degree anticlockwise direction and find out the new coordinates.

3. 2D Scaling

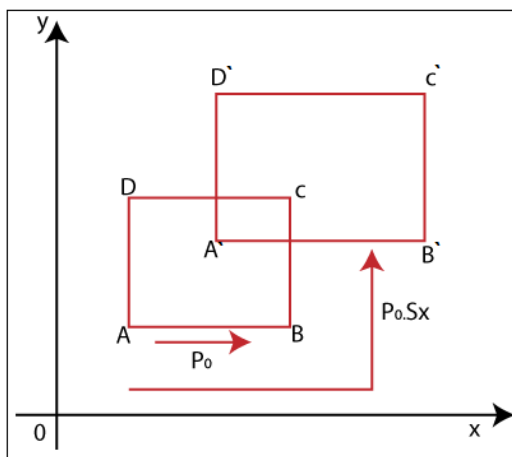
In scaling, we can expand or compress the size of any object. We can apply scaling on the object by multiplying the original coordinates with scaling factors. The term scaling factor is used to define whether the size of an object is increased or decreased.

- Scaling may be used to increase or reduce the size of object.
- Scaling subjects the coordinate points of the original object to change.
- Scaling factor determines whether the object size is to be increased or reduced.
- If scaling factor > 1 , then the object size is increased.
- If scaling factor < 1 , then the object size is reduced.

We can represent the scaling factor by 'S_x' for the x- axis and 'S_y' for the y-axis. For Example- If we want to scale an object that has R (P₀, Q₀) coordinate and the new coordinates of an object are R' (P₁, Q₁) then the equation will be-

$$P_1 = P_0 \cdot S_x$$

$$Q_1 = Q_0 \cdot S_y$$



$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Scaling Matrix

We can also represent Scaling in the form of Matrix-

Homogeneous Coordinates Representation: The scaling is also represented in the form of 3 x 3 matrix-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Scaling Matrix
(Homogeneous Coordinates Representation)

Example: A Square object with the coordinate points P (1, 4), Q (4, 4), R (4, 1), T (1,1). Apply the scaling factor 3 on the X-axis and 4 on the Y-axis. Find out the new coordinates of the square?

Solution: We have,

Coordinates Points for Square = P (1, 4), Q (4, 4), R (4, 1), S (1, 1)

Scaling factor along with X axis (S_x) = 3

Scaling factor along with Y axis (S_y) = 4

Applying the equation to find the new coordinates-

For Coordinate P (1, 4)- Let the new Coordinate for P = (P_1 , Q_1)

$$P_1 = P_0. S_x = 1 \times 3 = 3$$

$$Q_1 = Q_0. S_y = 4 \times 4 = 16 : \text{The new Coordinates} = (3, 16)$$

For Coordinate Q (4, 4)- Let the new Coordinate for Q = (P_1 , Q_1)

$$P_1 = P_0. S_x = 4 \times 3 = 12$$

$$Q_1 = Q_0. S_y = 4 \times 4 = 16 : \text{The new Coordinates} = (12, 16)$$

For Coordinate R (4, 1)- Let the new Coordinate for R = (P_1 , Q_1)

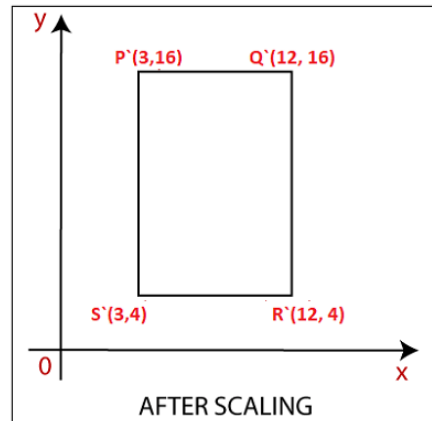
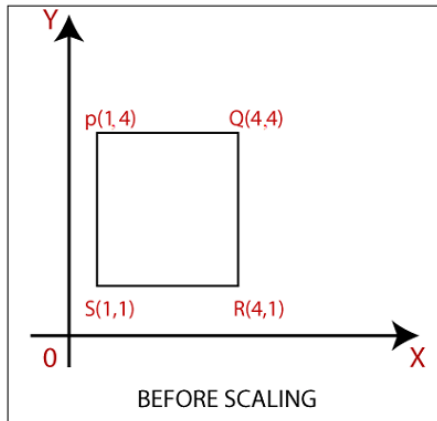
$$P_1 = P_0. S_x = 4 \times 3 = 12$$

$$Q_1 = Q_0. S_y = 1 \times 4 = 4 \text{ The new Coordinates} = (12, 4)$$

For Coordinate S (1, 1)- Let the new Coordinate for S = (P_1 , Q_1)

$$P_1 = P_0. S_x = 1 \times 3 = 3$$

$$Q_1 = Q_0. S_y = 1 \times 4 = 4 \text{ The new Coordinates} = (3, 4)$$



Thus, the new coordinates for square after scaling = P (3, 16), Q (12, 16), R (12, 4), S (3, 4).

Problem-01: Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.

4. 2D Reflection

The Reflection is a mirror image of the original object. In the Reflection process, the size of the object does not change.

We can represent Reflection by using four ways-

A. Reflection along X-axis: In this kind of Reflection, the value of X is positive, and the value of Y is negative. We can represent the Reflection along x-axis by following equation-

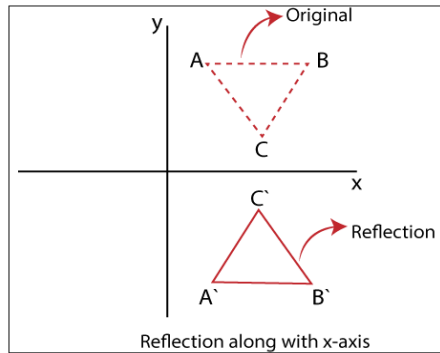
$$X_1 = X_0$$

$$Y_1 = -Y_0$$

We can also represent Reflection in the form of matrix-

$$\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}$$

Homogeneous Coordinate Representation: We can also represent the Reflection along x-axis in the form of 3 x 3 matrix-



$$\begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x} \begin{bmatrix} X_0 \\ Y_0 \\ 1 \end{bmatrix}$$

B. Reflection along Y-axis: In this kind of Reflection, the value of X is negative, and the value of Y is positive. We can represent the Reflection along y-axis by following equation-

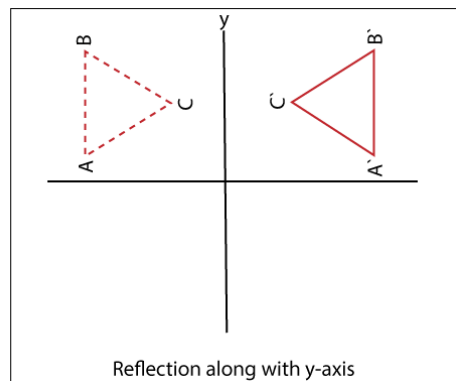
- $X_1 = -X_0$
- $Y_1 = Y_0$

We can also represent Reflection in the form of matrix-

$$\begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x} \begin{bmatrix} X_0 \\ Y_0 \\ 1 \end{bmatrix}$$

Homogeneous Coordinate Representation: We can also represent the Reflection along x-axis in the form of 3 x 3 matrix-

$$\begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x} \begin{bmatrix} X_0 \\ Y_0 \\ 1 \end{bmatrix}$$



C. Reflection perpendicular to XY plane: In this kind of Reflection, the value of both X and Y is negative. We can represent the Reflection along y-axis by following equation-

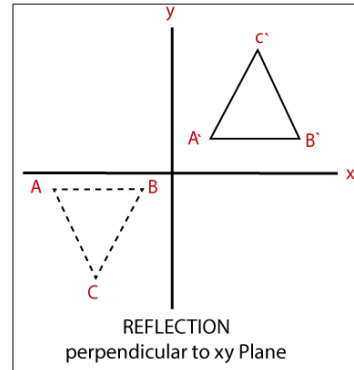
- $X_1 = -X_0$
- $Y_1 = -Y_0$

We can also represent Reflection in the form of matrix-

$$\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}$$

Homogeneous Coordinate Representation: We can also represent the Reflection along with x-axis in the form of 3 x 3 matrix-

$$\begin{pmatrix} X_1 \\ Y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} X_0 \\ Y_0 \\ 1 \end{pmatrix}$$



D. Reflection along with the line: In this kind of Reflection, the value of X is equal to the value of Y. We can represent the Reflection along y-axis by following equation-

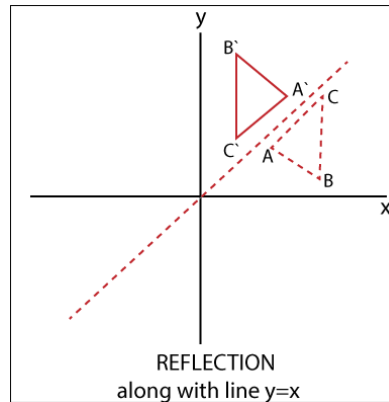
- $Y=X$, then the points are (Y, X)
- $Y= -X$, then the points are $(-Y, -X)$

We can also represent Reflection in the form of matrix-

$$\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}$$

Homogeneous Coordinate Representation: We can also represent the Reflection along with x-axis in the form of 3 x 3 matrix-

$$\begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_0 \\ Y_0 \\ 1 \end{bmatrix}$$



Example-01: Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the X axis and obtain the new coordinates of the object.

Given-

- Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)
- Reflection has to be taken on the X axis

For Coordinates A(3, 4) Let the new coordinates of corner A after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the reflection equations, we have-

- $X_{\text{new}} = X_{\text{old}} = 3$
- $Y_{\text{new}} = -Y_{\text{old}} = -4$: Thus, New coordinates of corner A after reflection = (3, -4).

For Coordinates B(6, 4) :Let the new coordinates of corner B after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the reflection equations, we have-

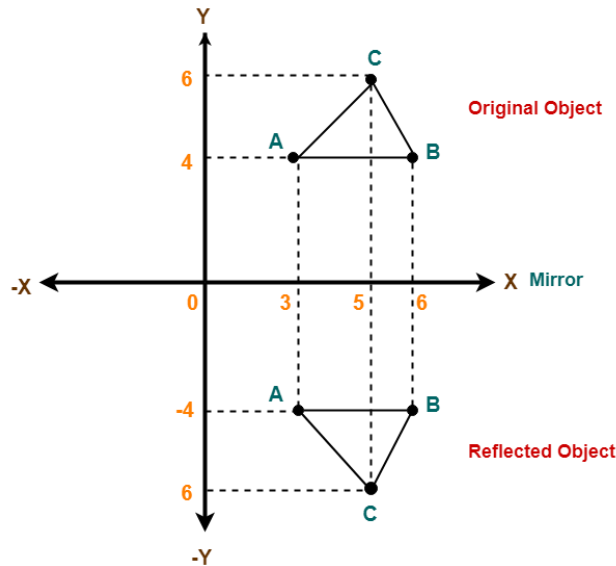
- $X_{\text{new}} = X_{\text{old}} = 6$
- $Y_{\text{new}} = -Y_{\text{old}} = -4$: Thus, New coordinates of corner B after reflection = (6, -4).

For Coordinates C(5, 6) :Let the new coordinates of corner C after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the reflection equations, we have-

- $X_{\text{new}} = X_{\text{old}} = 5$
- $Y_{\text{new}} = -Y_{\text{old}} = -6$: Thus, New coordinates of corner C after reflection = (5, -6).

Thus, New coordinates of the triangle after reflection = A (3, -4), B(6, -4), C(5, -6).



Problem-02: Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the Y axis and obtain the new coordinates of the object.

Given-

- Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)
- Reflection has to be taken on the Y axis

For Coordinates A(3, 4): Let the new coordinates of corner A after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the reflection equations, we have-

- $X_{\text{new}} = -X_{\text{old}} = -3$
- $Y_{\text{new}} = Y_{\text{old}} = 4$: Thus, New coordinates of corner A after reflection = (-3, 4).

For Coordinates B(6, 4): Let the new coordinates of corner B after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the reflection equations, we have-

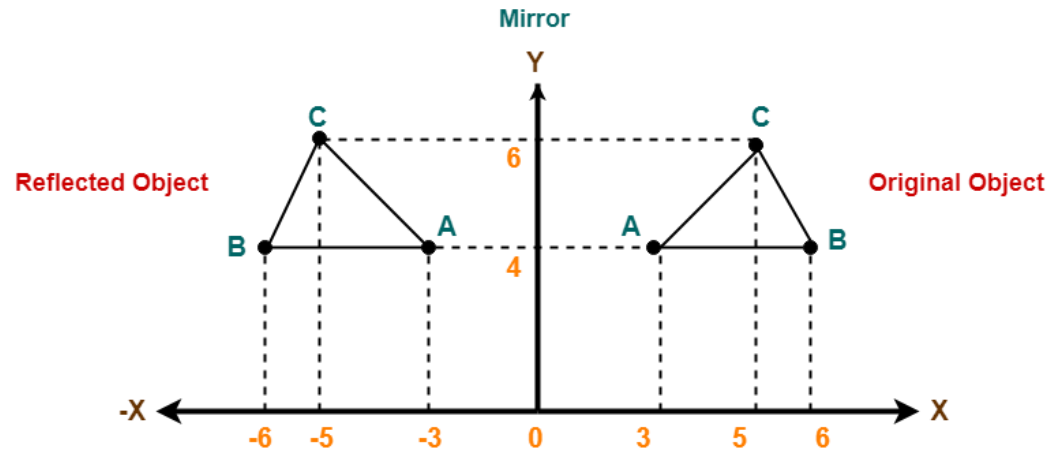
- $X_{\text{new}} = -X_{\text{old}} = -6$
- $Y_{\text{new}} = Y_{\text{old}} = 4$: Thus, New coordinates of corner B after reflection = (-6, 4).

For Coordinates C(5, 6): Let the new coordinates of corner C after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the reflection equations, we have-

- $X_{\text{new}} = -X_{\text{old}} = -5$
- $Y_{\text{new}} = Y_{\text{old}} = 6$: Thus, New coordinates of corner C after reflection = (-5, 6).

Thus, New coordinates of the triangle after reflection = A (-3, 4), B(-6, 4), C(-5, 6).



5. 2D Shearing

Shearing deals with changing the shape and size of the 2D object along x-axis and y-axis. It is similar to sliding the layers in one direction to change the shape of the 2D object. It is an ideal technique to change the shape of an existing object in a two dimensional plane.

We can denote shearing with ' SH_x ' and ' SH_y .' These ' SH_x ' and ' SH_y ' are called “**Shearing factor.**” We can perform shearing on the object in two ways-

- **Shearing along x-axis:** In this, we can store the y coordinate and only change the x coordinate. It is also called “**Horizontal Shearing.**”

We can represent Horizontal Shearing by the following equation-

$$X_1 = X_0 + SH_x \cdot Y_0$$

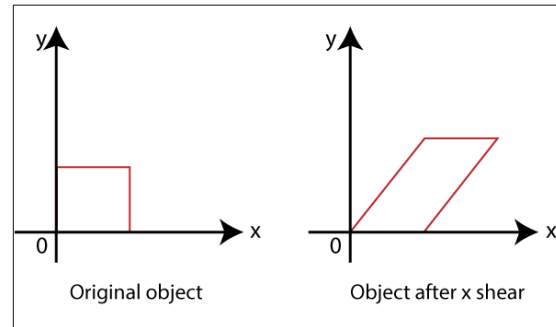
$$Y_1 = Y_0$$

We can represent Horizontal shearing in the form of matrix-

$$\begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = \begin{bmatrix} 1 & SH_x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix}$$

Homogeneous Coordinate Representation: The 3 x 3 matrix for Horizontal Shearing is given below-

$$\begin{pmatrix} X_1 \\ Y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ SH_x & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} X_0 \\ Y_0 \\ 1 \end{pmatrix}$$



- **Shearing along y-axis:** In this, we can store the x coordinate and only change the y coordinate. It is also called “Vertical Shearing.”

We can represent Vertical Shearing by the following equation-

$$X_1 = X_0$$

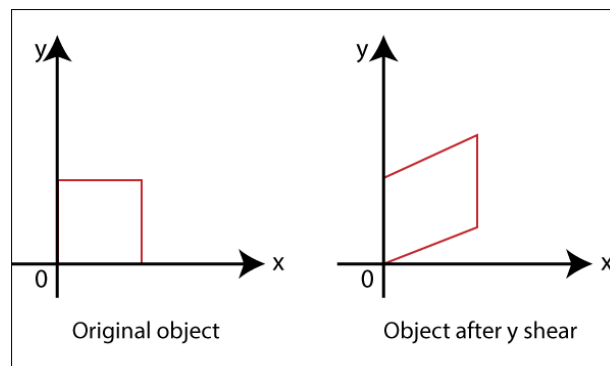
$$Y_1 = Y_0 + SH_y \cdot X_0$$

We can represent Vertical Shearing in the form of matrix-

$$\begin{pmatrix} X_1 \\ Y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ SH_y & 1 \end{pmatrix} \times \begin{pmatrix} X_0 \\ Y_0 \\ 1 \end{pmatrix}$$

Homogeneous Coordinate Representation: The 3x3 matrix for Vertical Shearing is given below-

$$\begin{pmatrix} X_1 \\ Y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & SH_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} X_0 \\ Y_0 \\ 1 \end{pmatrix}$$



Example: A Triangle with (2, 2), (0, 0) and (2, 0). Apply Shearing factor 2 on X-axis and 2 on Y-axis. Find out the new coordinates of the triangle?

Solution: We have, The coordinates of triangle = P (2, 2), Q (0, 0), R (2, 0)

- Shearing Factor for X-axis = 2
- Shearing Factor for Y-axis = 2 Now, apply the equation to find the new coordinates.

Shearing for X-axis: For Coordinate P (2, 2)- Let the new coordinate for P = (X₁, Y₁)

- $X_1 = X_0 + SH_x \cdot Y_0 = 2 + 2 \times 2 = 6$
- $Y_1 = Y_0 = 2$ **The New Coordinates = (6, 2)**

For Coordinate Q (0, 0)- Let the new coordinate for Q = (X₁, Y₁)

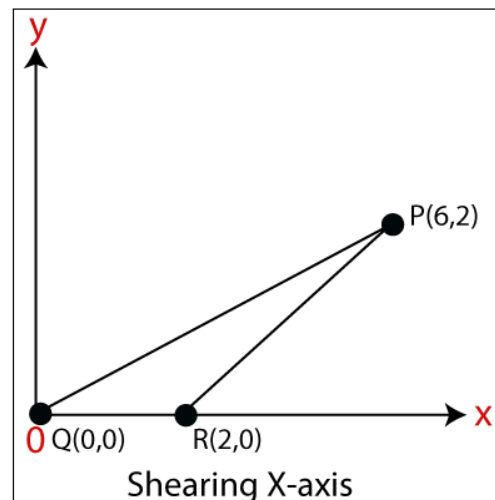
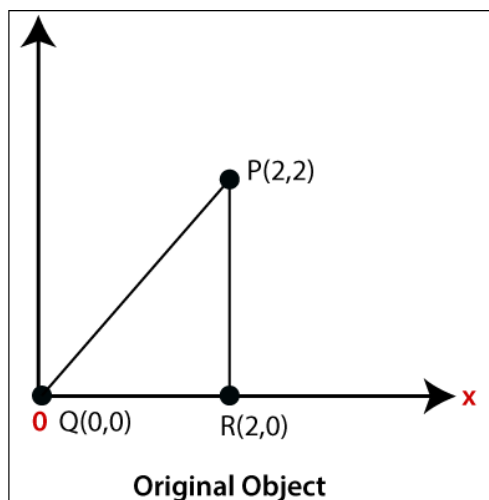
- $X_1 = X_0 + SH_x \cdot Y_0 = 0 + 2 \times 0 = 0$
- $Y_1 = Y_0 = 0$

The New Coordinates = (0, 0) For Coordinate R (2, 0)-

Let the new coordinate for R = (X₁, Y₁) ,

- $X_1 = X_0 + SH_x \cdot Y_0 = 2 + 2 \times 0 = 2$
- $Y_1 = Y_0 = 0$

The New Coordinates = (2, 0) , The New coordinates of triangle for x-axis = (6, 2), (0, 0), (2, 0)



Shearing for y-axis: For Coordinate P (2, 2)-

Let the new coordinate for P = (X₁, Y₁)

- $X_1 = X_0 = 2$
- $Y_1 = Y_0 + Sh_y \cdot X_0 = 2 + 2 \times 2 = 6$

The New Coordinates = (2, 6) , For Coordinate Q (0, 0)- Let the new coordinate for Q = (X_1 , Y_1)

- $X_1 = X_0 = 0$
- $Y_1 = Y_0 + Sh_y \cdot X_0 = 0 + 2 \times 0 = 0$

The New Coordinates = (0, 0), For Coordinate R (2, 0)- Let the new coordinate for R = (X_1 , Y_1)

- $X_1 = X_0 = 2$
- $Y_1 = Y_0 + Sh_y \cdot X_0 = 0 + 2 \times 2 = 4$

The New Coordinates = (2, 4) , The New coordinates of triangle for y-axis = (2, 6), (0, 0), (2, 4)

