3.Soil permeability and seepage analysis

- A material is permeable if it contains continuous voids. All materials such as rocks, soils etc are permeable materials since they allow flow of water through them.
- The flow of water through all of them obeys approximately the same laws.
- permeability of soils has effect on the stability of foundations, seepage loss through embankments of reservoirs, drainage of sub-grades.

3.1 Head

- before we look into in to Darcy's law which is governing law for flow of water through the soil mass, we will discuss an important principle in fluid mechanics – Bernoulli's principle – which is essential in understanding flow through soils.

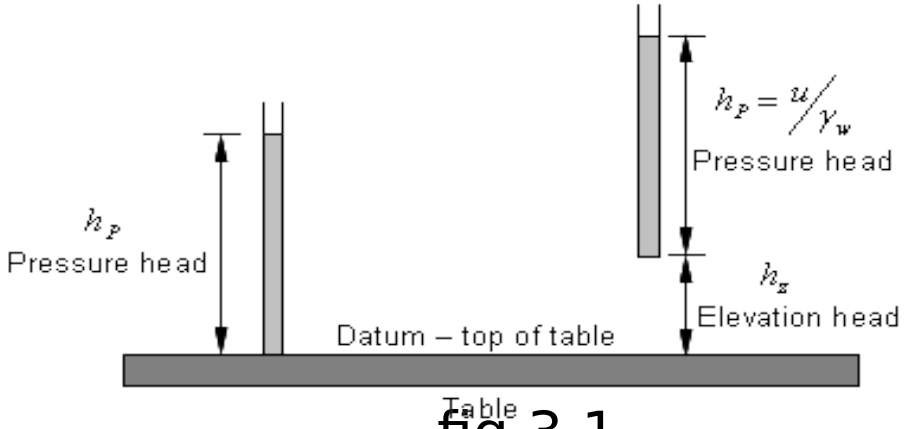


fig 3.1

- If you cap one end of a tube, fill the tube with water, and then rest it on your table (Fig. 3.1), the height of water with reference to your table is called the **pressure head (h_p).**

Cont…

- If you raise the tube above the table, the mechanical energy or total head increases. You now have two components of total head **– the pressure head (h_p) and the elevation head (h_z).**
- If water were to flow through the tube with a **velocity v** , under steady state condition, then we have an additional head due to the velocity given as .
- As per *Bernoulli's equation*, the total head at any point in water under steady flow condition may be expressed as:

- Total head = pressure head + velocity head + elevation head
  - H= hp +hz + hv
  - **H = hz (3.1)**

cont…

- The velocity of flow through soils is generally small (< 1 cm/s) and so,we usually neglect the velocity head. The total head in soils is then:

  - **$H = h_z + h_p = \frac{u}{\gamma_w}$ (3.2)**
  - **U=pore water pressure, γ_w =unit weight of the water**

- Now , Consider a cylinder containing a soil mass with water flowing through it at a constant rate as depicted in Fig. 3.2. If we connect two tubes, A and B, called piezometers, at a distance *L* apart, the water will rise to different heights in each of the tubes.

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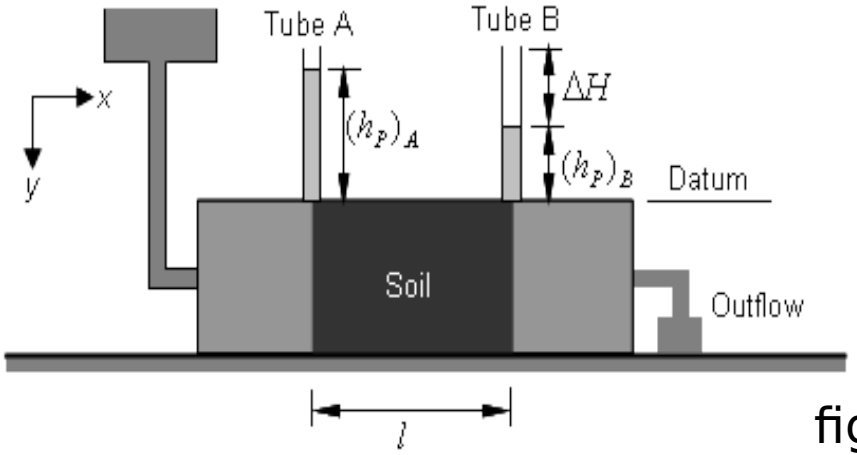


fig 3.2

- As the water flows through the soil, energy is dissipated through friction with the soil particles, resulting in a loss of head.
- The head loss between A and B assuming datum selected at the top of cylinder is :

$$\Delta H=(h_P)_A-(h_P)_B$$

$\Delta H/L=i$,called hydraulic gradient

3.2 Darcy's law

- Darcy (1856) proposed that average flow velocity through soils is proportional to the gradient of the total head. The flow in any direction, *j*, is:

$$v_j = k_j \frac{dH}{dx_j} \qquad \textbf{(3.3)}$$

where *v* is the average flow velocity, *k* is a coefficient of proportionality called the coefficient of permeability and ΔH is the change in total head over a distance Δx. The unit of measurement for *k* is length/time, that is, cm/s.

With reference to Fig. 3.2, Darcy's law becomes:

$$v_x = k_x \frac{\Delta H}{l} = k_x i \qquad (3.4)$$

where i= *ΔH/l is called hydraulic gradient*

Cont...

- Flow through soils, however, happens only through the interconnected voids. The velocity through the void spaces is called the ***seepage velocity*** (v_s) and is obtained by dividing the average velocity by the porosity of the soil:

$$v_s = \frac{k_j}{n} i \qquad (3.5)$$

The volume rate of flow, q_v, or, simply, flow rate is the product of the average velocity and the cross-sectional area:

$$q_v = v_j A = A k_j i \qquad (3.6)$$

Qv =measured in m3/s or cm3/s

The conservation of flow (law of continuity) stipulates that the volume rate of inflow $(q_v)_{in}$ into a soil element must equal the volume rate of outflow, $(q_v)_{out}$, or, simply, inflow must equal outflow:

$$(q_v)_{in} = (q_v)_{out}.$$

Table 3.1: Coefficient of permeability for common soil types

soil tyes	Kz (cm/s
Clean gravel	> 1.0
Clean sands, clean sand & gravel mix	1.0 to 10^{-3}
Fine sands, silts, mixtures comprising sands, silts, and clays	10^{-3} to 10^{-7}
Homogeneous clays	$< 10^{-7}$

but ,Homogeneous clays are practically impervious and used construction earth dam core.

For a homogeneous soil, the coefficient of permeability depends predominantly on its void ratio.

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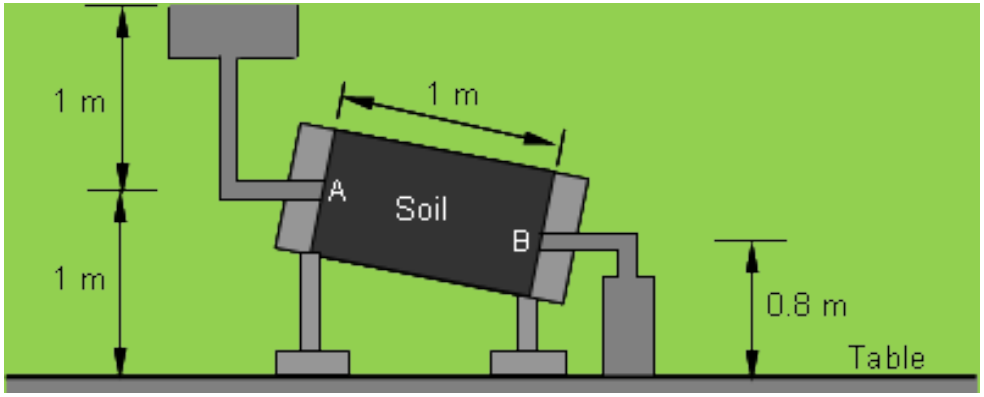
- A number of empirical relationships have been proposed linking *k* to void ratio and grain size for coarse-grained soils.
- Hazen (1930) proposed one of the early relationships as:

$$k_z = C^* \text{ in cm/s}$$

- where *C* is a constant varying between 0.4 and 1.2 if the unit of measurement of D_{10} (effective diameter) is mm.
- Typically, C = 1.0.

EXAMPLE 3.1

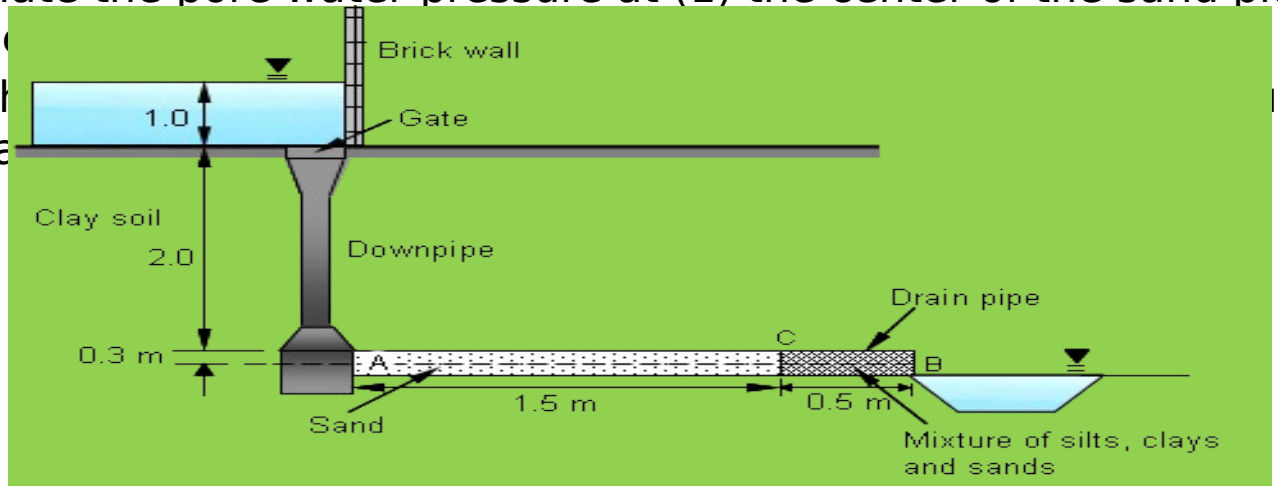
- A soil sample 10 cm in diameter is placed in a tube 1 m long. A constant supply of water is allowed to flow into one end of the soil at *A* and the outflow at *B* is collected by a beaker (Fig. E3.1). The average amount of water collected is 1 cm^3 for every 10 seconds. The tube is inclined as shown in Fig. 3.1. Determine the (a) hydraulic gradient, (b) flow rate, (c) average velocity, (d) seepage velocity, if e = 0.6, and (e) coefficient of permeability.



- ***Strategy***: For flow problems, you must define a datum position. So your first task is to define the datum position and then find the difference in total head between A and B. Use the head difference to calculate the hydraulic gradient and use Eqs. (3.4 to 3.6) to solve the problem.

Example 3.2

- A drainage pipe (Fig. E3.2) became completely blocked during a storm by a plug of sand, 1.5 m long, followed by another plug of a mixture of clays, silts, and sands, 0.5 m long. When the storm was over, the water level above ground was 1 m. The coefficient of permeability of the sand is 2 times that of the mixture of clays, silts, and sands.
  - A) Plot the variation of pressure, elevation, and total head over the length of the pipe.
  - B)Calculate the pore water pressure at (1) the center of the sand plug and (2) the c
  - C)Find th ure of clays, silts a



3.3 Flow Parallel to Soil Layers and normal to soil layer (reading assignment)

- When the flow is parallel to the soil layer (Fig. 3.2), the hydraulic gradient is the same at all points.
- The flow through the soil mass as a whole is equal to the sum of the flow through each of the layers.
- If we consider a unit width (in the y direction) of flow and use Eq. (3.6), we obtain.

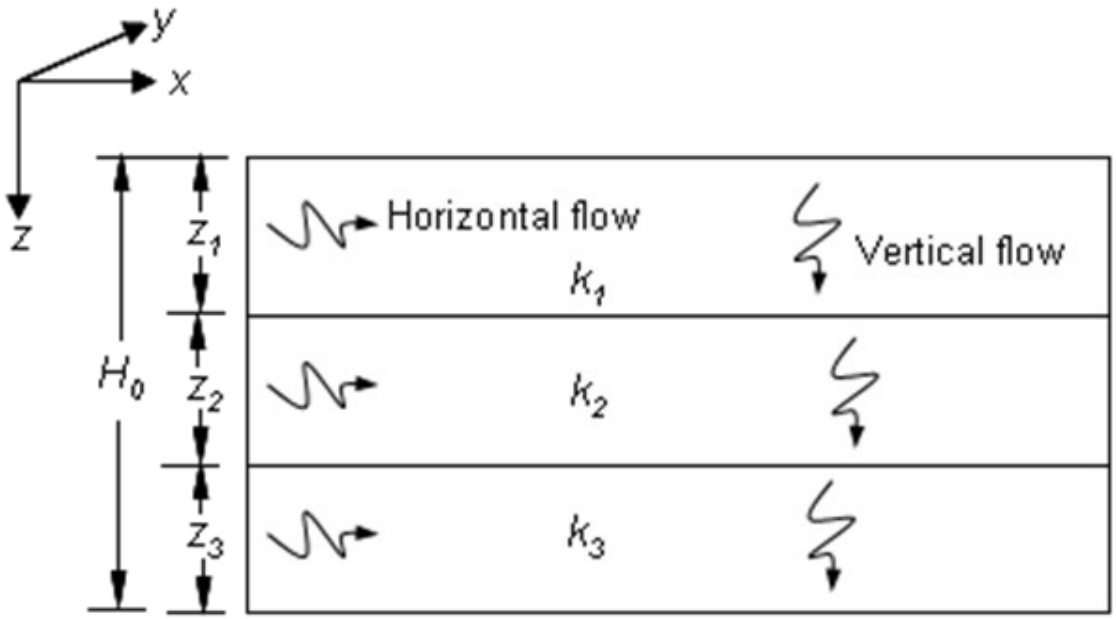
$$q_v = Av = (1 \times H_0)k_{x(eq)}i = (1 \times z_1)k_{x1}i + (1 \times z_2)k_{x2}i + \cdots + (1 \times z_n)k_{xn}i$$

- where H_0 is the total thickness of the soil mass, $k_{x(eq)}$ is the equivalent permeability in the horizontal (x) direction, z_1 to z_n are the thickness of the first to the n^{th} layers, and k_{x1} to k_{xn} are the horizontal permeability of first to the n^{th} layer.

Cont....

- Solving Eq. (3.6) for $k_{x(eq)}$,

$$k_{x(eq)} = \frac{1}{H_0}(z_1 k_{x1} + z_2 k_{x2} + \cdots + z_n k_{xn})$$



- For flow normal to the soil layers, the head loss in the soil mass is the sum of the head losses in each layer:

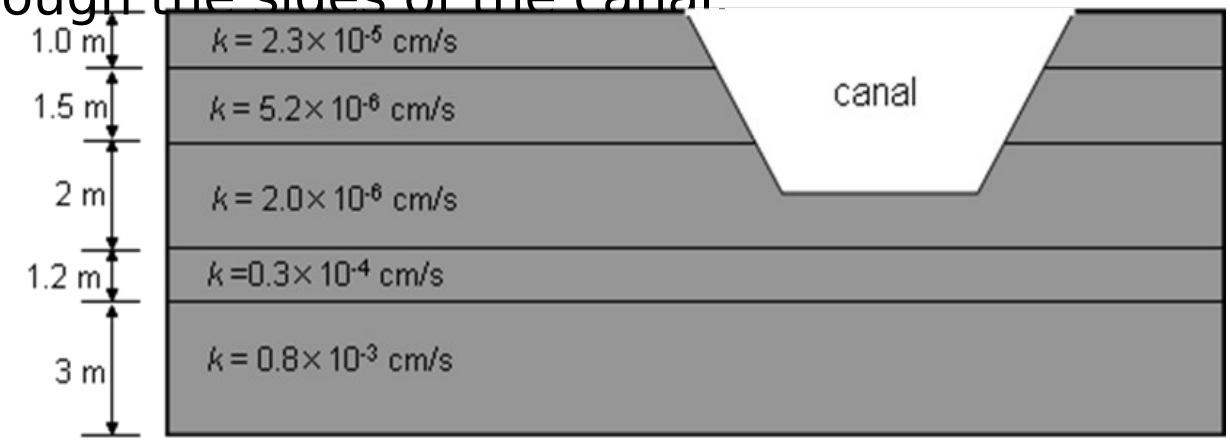
$$\Delta H = \Delta h_1 + \Delta h_2 + \cdots + \Delta h_n$$

- where ΔH is the total head loss, and Δh_1 to Δh_n are the head losses in each of the n layers. The velocity in each layer is the same.

$$k_{z(eq)} = \frac{H_0}{\frac{z_1}{k_{z1}} + \frac{z_2}{k_{z2}} + \cdots + \frac{z_n}{k_{zn}}}$$

Example 3.3

- A canal is cut into a soil with a stratigraphy shown in Fig. E3.3. Assuming flow takes place laterally and vertically through the sides of the canal and vertically below the canal, determines the equivalent permeability in the horizontal and vertical directions. Calculate the ratio of the equivalent horizontal permeability to the equivalent vertical permeability for flow through the sides of the canal.



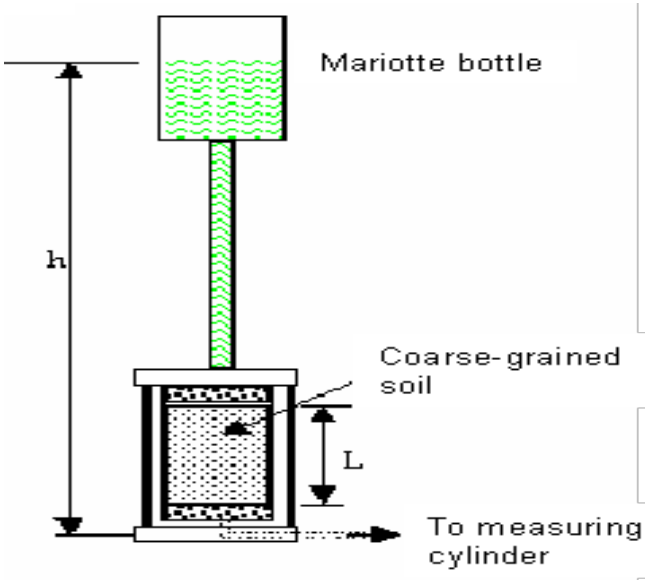
- ***Strategy***: find the equivalent horizontal and vertical permeability's over the depth of the canal (3 m) and then find the equivalent vertical permeability below the canal. To make the calculations easier, convert all exponential quantities to a single exponent.

3.4 Determination of the Coefficient of Permeability

- **Constant-Head Test:**

The constant-head test is used to determine the coefficient of permeability of coarse-grained soils.

- Water is allowed to flow through a cylindrical sample of soil under a constant head (*h*). The outflow (Q) is collected in a graduated cylinder at a convenient duration (*t*).

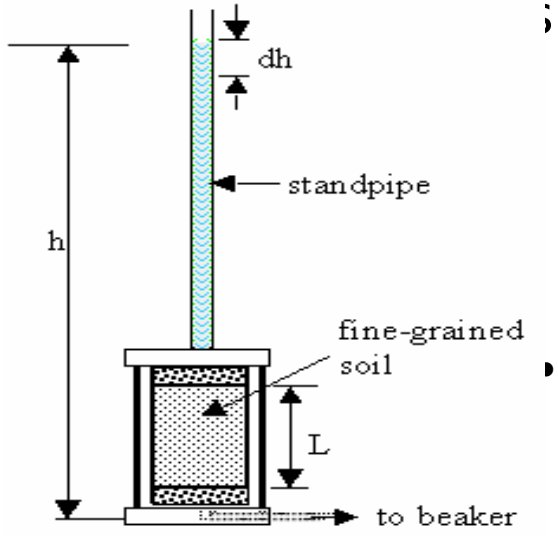


- the flow rate through the soil is $q_v = Q/t$, where Q is the total quantity of water collected in the measuring cylinder over time t.
- But qv=kz*A*i, i=h/l and kz=qv/Ai ,substituting for qv and I

$$k_z = \frac{q_v}{Ai} = \frac{QL}{tAh} \qquad (3.11)$$

Falling head test

- The falling-head test is used for fine-grained soils because the flow of water through these soils is too slow to get reasonable measurements from the constant-head test.
- A compacted soil sample or a sample extracted from the field is placed in a metal or acrylic cylinder (Fig. 3.4).
- Porous stones are positioned at the top and bottom faces of the sample to prevent its disintegration and to allow water to percolate through it. Water flows through the sample from a standpipe attached to the top of the cylinder. The head of water (h) changes with time as flow occurs through the soil. At different times, [illegible] recorded.
- [illegible]
- [illegible]
- [illegible]



$$k = k_z = \frac{aL}{A(t_2 - t_1)} \ln\left(\frac{h_1}{h_2}\right)$$

- a=x-sectional area of tube,A=X-sectional area of sample

3.5 seepage through the soil

In many practical situation flow around sheet pile wall and earth dam is not one dimensional but two dimensional which means velocity component in horizontal and vertical directions vary from point to point within the cross section of the soil mass.

The flow of water through soils is described by Laplace's equation. The popular form of Laplace's equation for two-dimensional flow of water through soils is:

$$k_x \frac{\partial^2 H}{\partial x^2} + k_z \frac{\partial^2 H}{\partial^2 z} = 0 \qquad (3.13)$$

where H is the total head and k_x and k_z are the coefficients of permeability in the X and Z directions.

Laplace's equation expresses the condition that the changes of hydraulic gradient in one direction are balanced by the changes in the other directions.

Cont…

- The assumptions in Laplace's equation are:
  - ✓ Darcy's law is valid.
  - ✓ The soil is homogeneous and saturated.
  - ✓ The soil and water are incompressible.
  - ✓ No volume change occurs.
- If the soil were an isotropic material then $k_x = k_z$ and Laplace's equation becomes:

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial^2 z} = 0 \qquad (3.14)$$

The velocities of flow in the X and Z directions are:

$$v_x = k_x \frac{\partial H}{\partial x} = \frac{\partial \xi}{\partial x} \qquad v_z = k_z \frac{\partial H}{\partial x} = \frac{\partial \xi}{\partial z} \qquad (3.15)$$

- The inference from Eqs. (5.4) and (5.5) is that the velocity of flow (v) is normal to lines of constant total head (also called constant piezometric head or ***equipotential lines***) as illustrated in Fig. 5.1.

Cont...

- The direction of *v* is in the direction of decreasing total head. The head difference between two equipotential lines is called a potential drop or head loss.

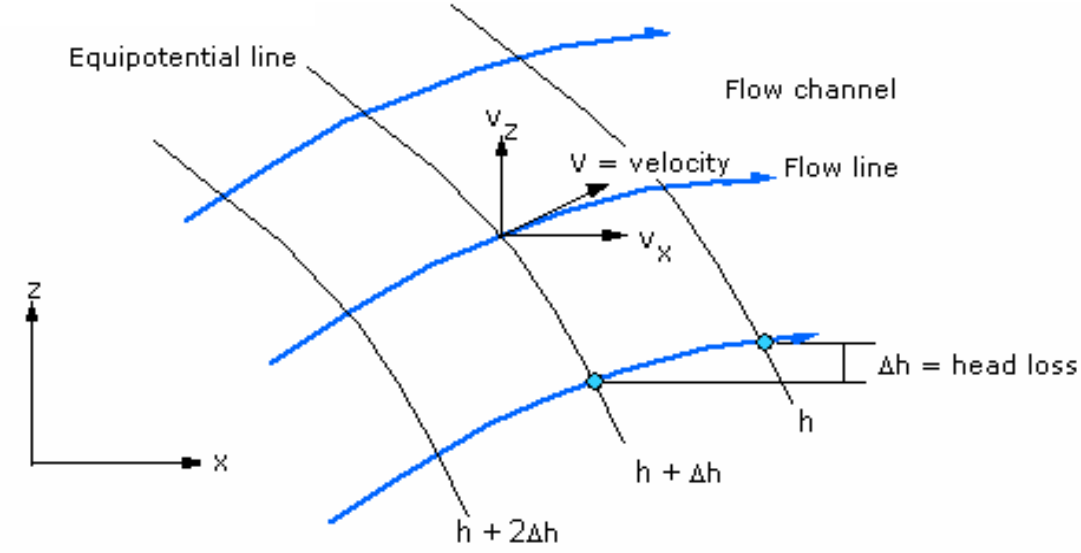


fig 3.4. illustration of flow terms

Cont...

- If lines are drawn that are tangent to the velocity of flow at every point in the flow field in the *XZ* plane, we will get a series of lines that are normal to the equipotential lines.

- These tangential lines are called ***streamlines or flow lines*** (Fig. 5.1). A flow line represents the flow path that a particle of water is expected to take in steady state flow.

- Since flow lines are normal to equipotential lines, there can be no flow across flow lines. The rate of flow between any two flow lines is constant. The area between two flow lines is called a flow channel (fig. 5.1). Therefore, the rate of flow is constant in a flow channel.

Flow net sketching

- **Criteria for Sketching Flow Nets**

- A flow net is a graphical representation of a flow field that satisfies Laplace's equation and comprises a family of flow lines and equipotential lines.
- A flow net must meet the following criteria:
  1. The boundary conditions must be satisfied.
  2. Flow lines must intersect equipotential lines at right angles.
  3. The area between flow lines and equipotential lines must be curvilinear squares. A curvilinear square has the property that an inscribed circle can be drawn to touch each side of the square.
  4. The quantity of flow through each flow channel is constant.
  5. The head loss between each consecutive equipotential line is constant
  6. A flow line cannot intersect another flow line.
  7. An equipotential line cannot intersect another equipotential line

Procedure for Sketching Flow Nets

1. Draw the structure and soil mass to a suitable scale.
2. Identify impermeable and permeable boundaries. The soil-impermeable boundary interfaces are flow lines because water can flow along these interfaces. The soil-permeable boundary interfaces are equipotential lines because the total head is constant along these interfaces.
3. Sketch a series of flow lines (four or five) and then sketch an appropriate number of equipotential lines such that the area between a pair of flow lines and a pair of equipotential lines (cell) is approximately a curvilinear square.

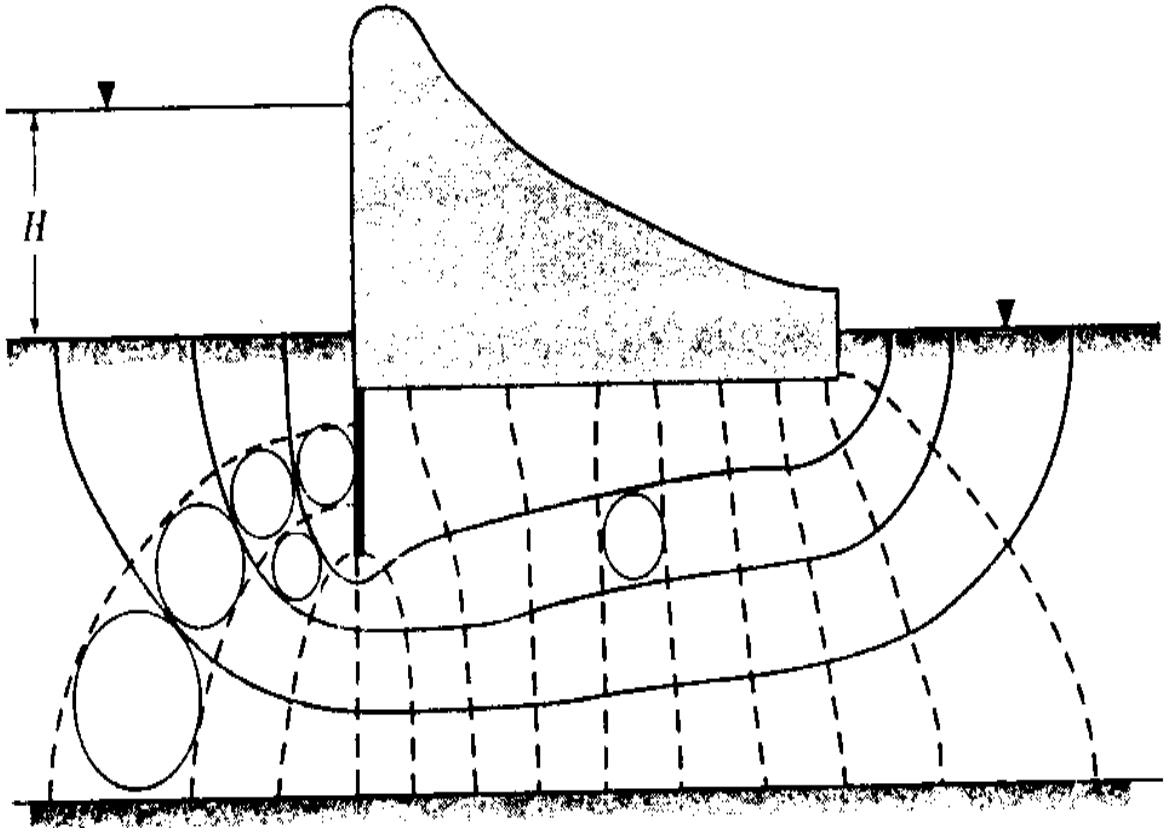


Figure : Flow net under a dam with a cutoff curtain (sheet pile) on the upstream

Flow Through Earth Dams

- Flow through earth dams is an important design consideration.
- We need to ensure that the pore water pressure at the downstream end of the dam will not lead to instability and the exit hydraulic gradient does not lead to piping.
- So, the major exercise is to find the top flow line called the phreatic surface (Fig . below) because,the pressure head on the phreatic surface is zero.
- Casagrande (1937) showed that the phreatic surface can be approximated by a parabola with corrections at the points of entry and exit. The focus of the parabola is at the toe of the dam, point F.

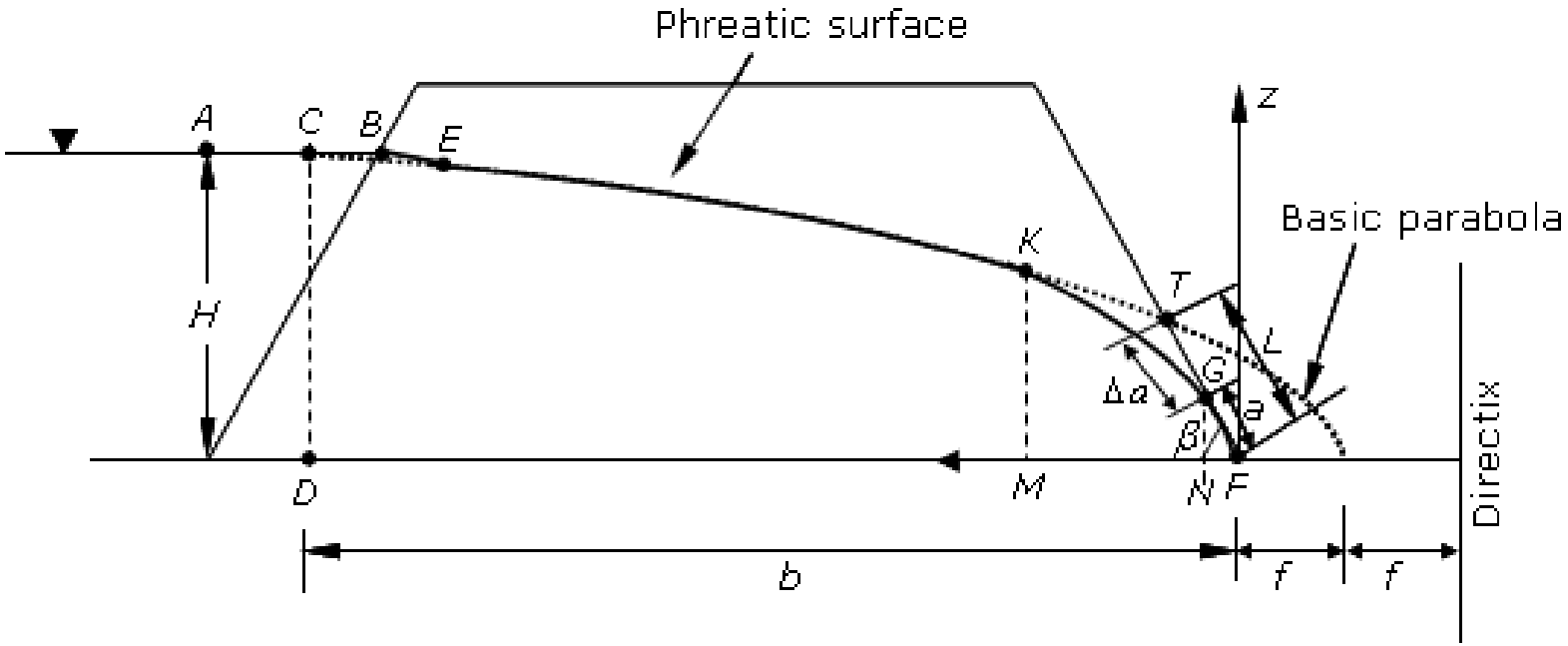


Figure : Phreatic surface within an earth dam

- The procedure to draw a phreatic surface within an earth dam, with reference to Fig. above, is as follows.
    1. Draw the structure to scale.
    2. Locate a point *A* at the intersection of a vertical line from the bottom of the upstream face and the water surface, and a point *B* where the waterline intersects the upstream face.
    3. Locate point *C*, such that $BC=0.3AB$.
    4. Project a vertical line from *C* to intersect the base of the dam at *D*.
    5. Locate the focus of the basic parabola. The focus is located at the toe of the dam
    6. Calculate the focal distance, $f=(\sqrt{b^2+H^2}-b)/2$
       where b is the distance FD and *H* is the height of water at the upstream face.

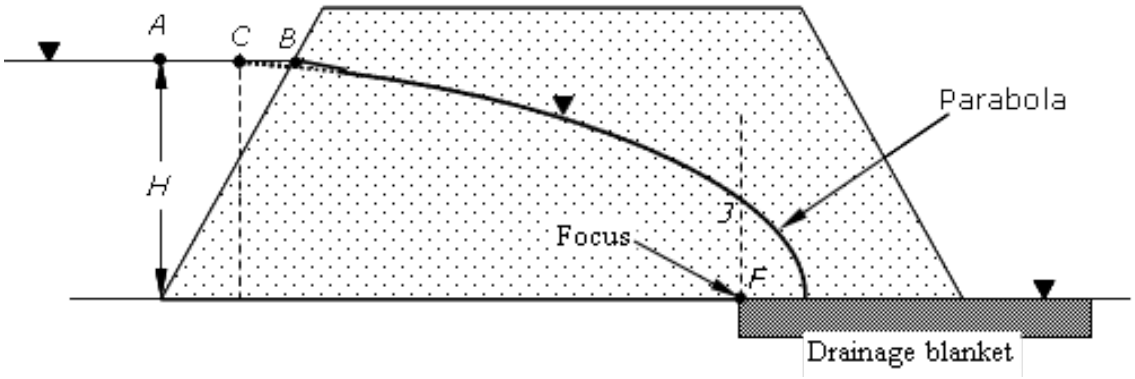
- 7. Construct the basic parabola from $x = \frac{y^2}{4(f+x)}$
- 8. Sketch in a transition section BE.
- 9. Calculate the length of the discharge face, a, using

$$a = \frac{b}{\cos\beta} - \cos\beta\sqrt{b^2 - H^2\cot^2\beta}; \quad \beta \le 30^0.$$

For ß>30, use Fig.5.7 and (a) measure the distance TF, where T is the intersection of the basic parabola with the downstream face; (b) for the known angle ß , read the corresponding factor Δa/L from the chart; and (c) find the distance $a = TF - \Delta a$.

- 10.Measure the distance a from the toe of the dam along the downstream face to point G.
- 11.Sketch in a transition section, *GK*
- 12.Calculate the flow using $q = gk \sin\beta \tan\beta$, where k is the coefficient of permeability.
- If the downstream slope has a horizontal drainage blanket as shown in Fig. 5.3, the flow is calculated using

$$q = 2fk$$



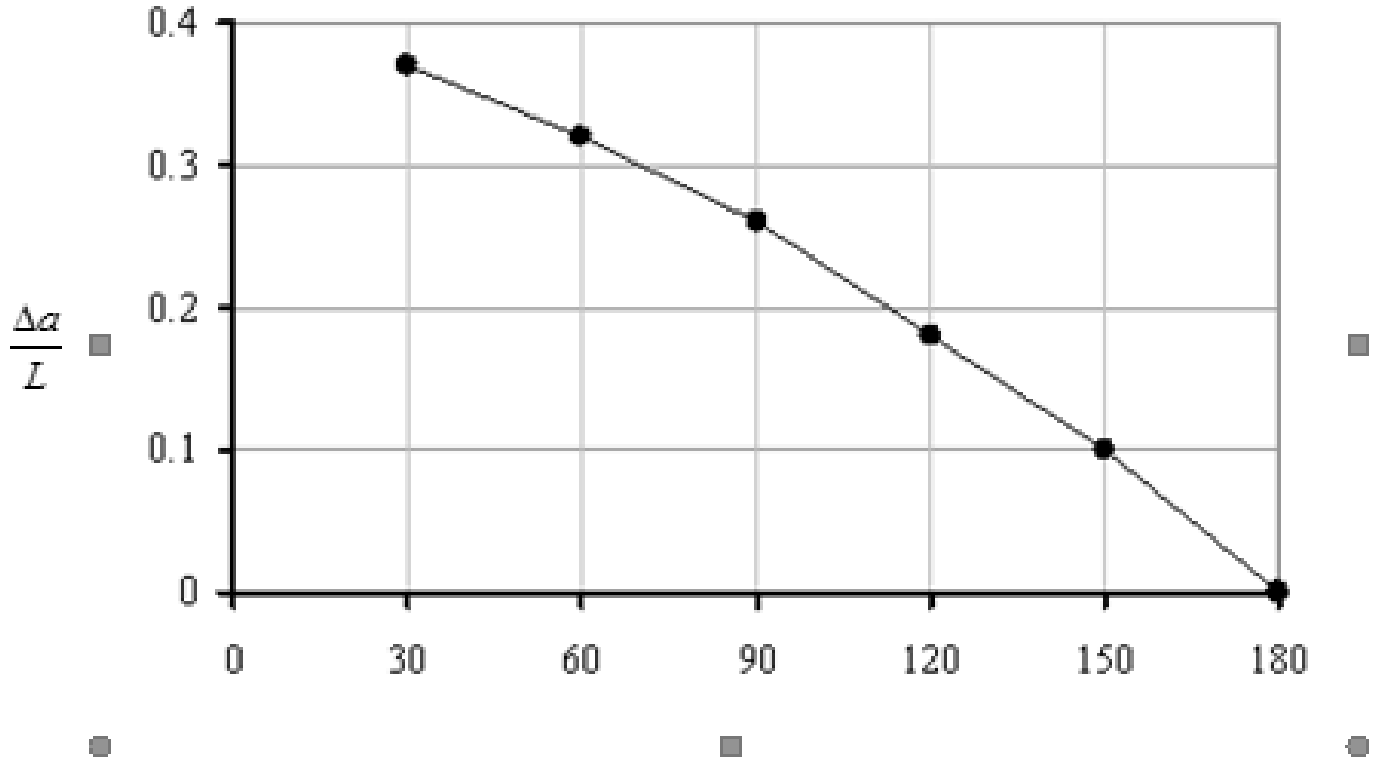


Figure 5.7: Correction : β(degrees) wnstream curve.