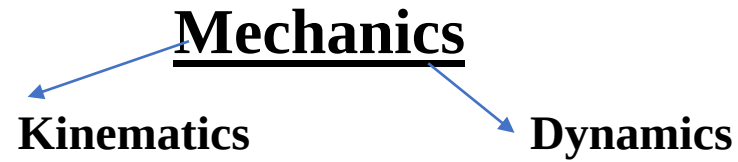


2. KINEMATICS AND DYNAMICS OF PARTICLES



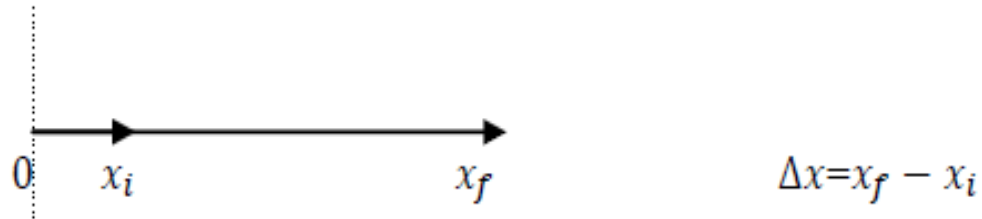
- ❖ **Mechanics** is the study of the physics of motions and how it relates to the physical factors that affect them, like force, mass, momentum and energy.
- ❖ **Dynamics**, which deals with the motion of objects with its cause – force;
- ❖ **kinematics** describes the possible motions of a body or system of bodies without considering the cause.
- ❖ Kinematics is concerned on analyzing kinematical quantities used to describe motion such as velocity, acceleration, displacement, time, and trajectory.
- ❖ Objects are in motion all around us.
- ❖ Planets moving around the sun, car moving along a road, blood flowing through veins, etc, are some examples of motion.

2.1. Kinematics in One and Two Dimensions

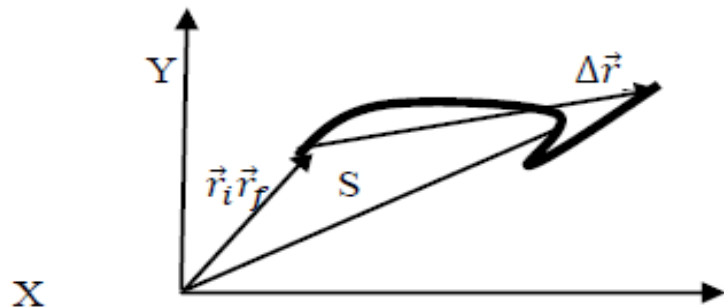
2.1.1. Displacement, velocity and Acceleration in 1D and 2D

Definition: **Kinematical Quantities**

- ❖ **Position:** -the location of an object with respect to a chosen reference point.
- ❖ **Displacement:** - The change in position of an object with respect to a given reference frame.
- ❖ For 1D (for one-dimensional motion)



- ❖ For 2D



$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i \text{-----} (2.1.1)$$

Cont.....

❖ **Distance (S):-** The length of the path followed by the object.

Average and Instantaneous Velocities:

❖ **Average Velocity ($\vec{v}_{av}$)** :-is the total displacement divided by the total time.

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} \dots\dots\dots (2.1.2)$$

❖ **Average Speed:** - is the total distance traveled by the object divided by the total elapsed time.

$$v_{av} = \frac{\text{total distance}(S)}{\text{total time interval}(\Delta t)} \dots\dots\dots (2.1.3)$$

- ❖ **Average speed** and **average velocity** of an object do not provide the detail information of the entire motion.
- ❖ We may need to know the velocity or speed of the particle at a **certain instant of time**.

Instantaneous Velocity ($\vec{v}$): - is the limiting value of the ratio $\frac{\Delta \vec{r}}{\Delta t}$ as Δt approaches zero.

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} \text{-----} (2.1.4)$$

- ❖ The **instantaneous speed**:-It is the magnitude of the instantaneous velocity.

Average and Instantaneous Accelerations:

- ❖ If the velocity of a particle changes with time, then the particle is said to be **accelerating**.

Cont.....

- ❖ Average acceleration: is the change in velocity ($\overrightarrow{\Delta v}$) of an object divided by the time interval during which that change occurs.

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} \text{-----} (2.1.5)$$

- ❖ **Instantaneous acceleration:** -The limit of average acceleration as Δt approaches zero.

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} \text{-----} (2.1.6)$$

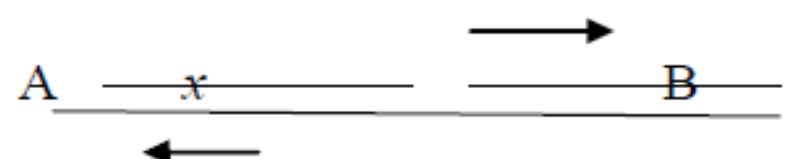
Cont.....example

Q₁ . A person walks first at a constant speed of 5m/s along the straight line from point A to point B, and then back along the same line from B to A at a constant speed of 3m/s.

- a) What is his average speed over the entire trip
- b) What is his average velocity over the entire trip

Solution:-

5m/s a) By definition, average speed is


$$v_{av} = \frac{\text{total distance}(S)}{\text{total time interval}(\Delta t)}$$

3m/s

The total distance covered in the entire trip is $S_{AB} + S_{BA} = 2x$

$$v_{av} = \frac{S_{tot}}{t_{tot}} = \frac{S_{AB} + S_{BA}}{t_{AB} + t_{BA}}$$

Cont....

$$\text{But, } t_{AB} = \frac{x}{v_{AB}} = \frac{x}{5\text{m/s}}$$

$$t_{BA} = \frac{x}{v_{BA}} = \frac{x}{3\text{m/s}}$$

$$\text{Therefore, } v_{av} = \frac{2x}{\left(\frac{x}{5\text{m/s}} + \frac{x}{3\text{m/s}}\right)} = 2x \left(\frac{15}{8x}\right) = \frac{15}{4} = 3.75\text{m/s}$$

b) Average velocity over the entire trip is zero, because for the entire trip

$$\vec{r}_f = \vec{r}_i \rightarrow \Delta\vec{r} = \vec{r}_f - \vec{r}_i = 0$$

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} = 0$$

2.1.2. Motion with Constant Acceleration

 For motion with constant acceleration,

- ❖ The velocity changes at the same rate throughout the motion.
- ❖ Average acceleration over any time interval is equal to the instantaneous acceleration at any instant of time.

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t}, \text{ assuming } t_i = 0.$$

Rearranging this equation gives

$$\vec{v}_f = \vec{v}_i + \vec{a}t \text{ ----- (2.1.7)}$$

- ❖ For motion with constant acceleration, average velocity can be written as:

$$\vec{v}_{av} = \frac{\vec{v}_f + \vec{v}_i}{2} \text{ ----- (2.1.8)}$$

Cont....

By definition $\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$ for $t_i=0$

$$\Delta \vec{r} = \vec{v}_{av} t$$

$$\vec{r}_f - \vec{r}_i = \frac{(\vec{v}_f + \vec{v}_i)t}{2} \text{ but } \vec{v}_f = \vec{v}_i + \vec{a}t$$

$$\vec{r}_f - \vec{r}_i = \vec{v}_i t + \frac{1}{2} \vec{a} t^2 \text{ ----- (2.1.9)}$$

Again, $\Delta \vec{r} = \vec{v}_{av} t$ but $\vec{v}_{av} = \frac{\vec{v}_f + \vec{v}_i}{2}$ and $t = \frac{\vec{v}_f - \vec{v}_i}{a}$

$$\Delta \vec{r} = \frac{(\vec{v}_f + \vec{v}_i)}{2} \frac{(\vec{v}_f - \vec{v}_i)}{a}$$

$$v_f^2 = v_i^2 + 2a\Delta r \text{ ----- (2.1.10)}$$

Cont....

For 2D motion $\vec{a} = a_x\hat{i} + a_y\hat{j}$, $\vec{v}_f = v_{xf}\hat{i} + v_{yf}\hat{j}$, $\vec{v}_i = v_{xi}\hat{i} + v_{yi}\hat{j}$

$$\vec{v}_f = \vec{v}_i + \vec{a}t \rightarrow \begin{cases} v_{xf} = v_{xi} + a_x t \\ v_{yf} = v_{yi} + a_y t \end{cases}$$

$$\vec{r}_f - \vec{r}_i = \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \rightarrow \begin{cases} \Delta x = v_{xi}t + \frac{1}{2}a_x t^2 \\ \Delta y = v_{yi}t + \frac{1}{2}a_y t^2 \end{cases}$$

Cont..

Q₂. A track covers 40m in 8.5s while smoothly slowing down to a final speed of 2.8m/s. Find

- a) Its original speed b) its acceleration

Solution;-

We are given that

$$a) \Delta \vec{r} = \vec{v}_{av} t$$

$S = 40m, \quad t = 8.5s$ and $v_f = 2.8m/s$ the magnitude of $\vec{v}_{av}$ is $v_{av} = \frac{v_f + v_i}{2}$

the magnitude of $\Delta \vec{r}$ is $S = 40m$ $S = \left(\frac{v_f + v_i}{2} \right) t \rightarrow v_i = \frac{2S}{t} - v_f$

We are asked to find a) v_i b) $\vec{a}$ $v_i = \frac{2(40m)}{8.5s} - 2.8m/s$

$$= 9.4 \frac{m}{s} - 2.8 \frac{m}{s}$$

$$= 6.6m/s$$

$$b) \quad \vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

The magnitude of the acceleration is

$$a = \frac{v_f - v_i}{t} = \frac{6.6m/s - 2.8m/s}{8.5s} = 0.447m/s^2$$

Cont.....

Q₃. A jet plane lands with a speed of 100m/s and slows down at a rate of 5m/s² as it comes to rest. **a)** What is the time interval needed by the jet to come to rest? **b)** Can this jet land on an airport where the runway is 0.8km long? *We are given that* *a) the magnitude of the acceleration is* $v_i = \frac{100\text{m}}{\text{s}}, a =$

Solution;- $-5 \frac{\text{m}}{\text{s}^2}, v_f = 0$

$$a = \frac{v_f - v_i}{t} \rightarrow t = \frac{v_f - v_i}{a}$$

We are asked to find a) t b) S

$$t = \frac{0\text{m/s} - 100\text{m/s}}{-5\text{m/s}^2} = 20\text{s}$$

b. To determine whether the jet can land on the 0.8km runway, we need to calculate the distance through which the jet moves as it comes to rest

$$\begin{aligned} S &= \left(\frac{v_f + v_i}{2} \right) t \\ &= \left(\frac{0\text{m/s} + 100\text{m/s}}{2} \right) 20\text{s} \\ &= 1000\text{m} = 1\text{km} \end{aligned}$$

Cont....

The jet can't land! because the runway(0.8km)is shorter than the distance the jet requires to come to rest(1km).

2.1.3. Free Fall Motion

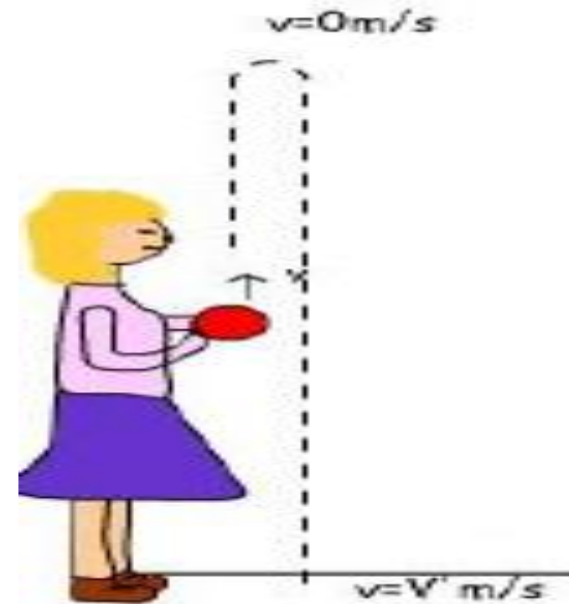
- ❖ The motion of an object near the surface of the Earth under the only control of the force of gravity is called free fall.
- ❖ In the absence of air resistance, all objects fall with constant acceleration, g , toward the surface of the Earth.
- ❖ On the surface of the Earth, the generally accepted value is $g = 9.8 \text{ m/s}^2$.
- ❖ The acceleration due to gravity varies with latitude, longitude and altitude on Earth's surface.
- ❖ And it is greater at the poles than at the equator and greater at sea level than a top mountain.

Cont....

- ❖ freely falling objects that we need to make in these equations is to note that the motion is in the vertical direction (the y direction) rather than in the horizontal direction (x) and that the acceleration is downward and has a magnitude of 9.80 m/s^2 . Therefore, we choose $a_y = -g = -9.80 \text{ m/s}^2$, where the negative sign means that the acceleration of a freely falling object is downward.

Q1. A girl throws a ball upwards, giving it an initial speed $u = 15 \text{ m/s}$. Neglect air resistance.

- (a) How long does the ball take to return to the boy's hand?
- (b) What will be its velocity then?



Cont..

Solution:

(a) We choose the positive y upward with its origin at the girl's hand, i.e. $y_i = 0$, see the Fig. above. Then, the ball's acceleration is negative (downward) during the ascending and descending motions, i.e. $a = -g = -9.8 \text{ m/s}^2$

When the ball returns to the girl's hand its position y is zero. Since

$u = 15 \text{ m/s}$, $y_i = 0$, $y = 0$, and $a = -g$, then we can find t from

$$y - y_i = ut - \frac{1}{2}gt^2 \quad \text{it follows:}$$

$$0 = (15 \text{ m/s}) t - \frac{1}{2}(9.8 \text{ m/s}^2) \times t^2$$

$$\Rightarrow t = 2 \times (15 \text{ m/s}) / 9.8 \text{ m/s}^2 = 3.1 \text{ s}$$

Cont..

(b) We are given $u = 15 \text{ m/s}$, $y_i = 0$, $y = 0$, and $a = -g = -9.8 \text{ m/s}^2$. To find v , we use

(c) $v^2 = u^2 - 2g(y - y_i)$ it follows that:

$$v^2 = u^2 - 0$$

$$\Rightarrow v = \pm u = \pm 15 \text{ m/s}$$

We should select the negative sign, because the ball is moving downward just before returning to the boy's hand, i.e. $v = -15 \text{ m/s}$.

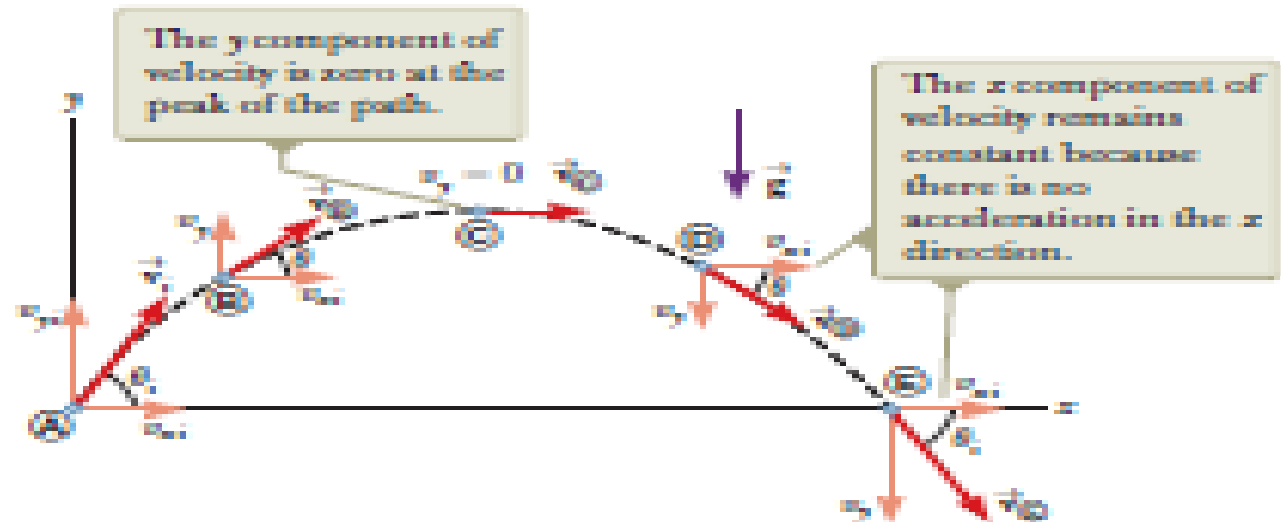
2.1.4. Projectile Motion

- ✓ Projectile is any object thrown obliquely into the space.
- ✓ Anyone who has observed a baseball in motion has observed projectile motion. The ball moves in a curved path and returns to the ground.
- ✓ A stone projected at an angle, a bomb released from an aero plane, a shot fired from a gun, a shot put or javelin thrown by the athlete are examples for the projectile.

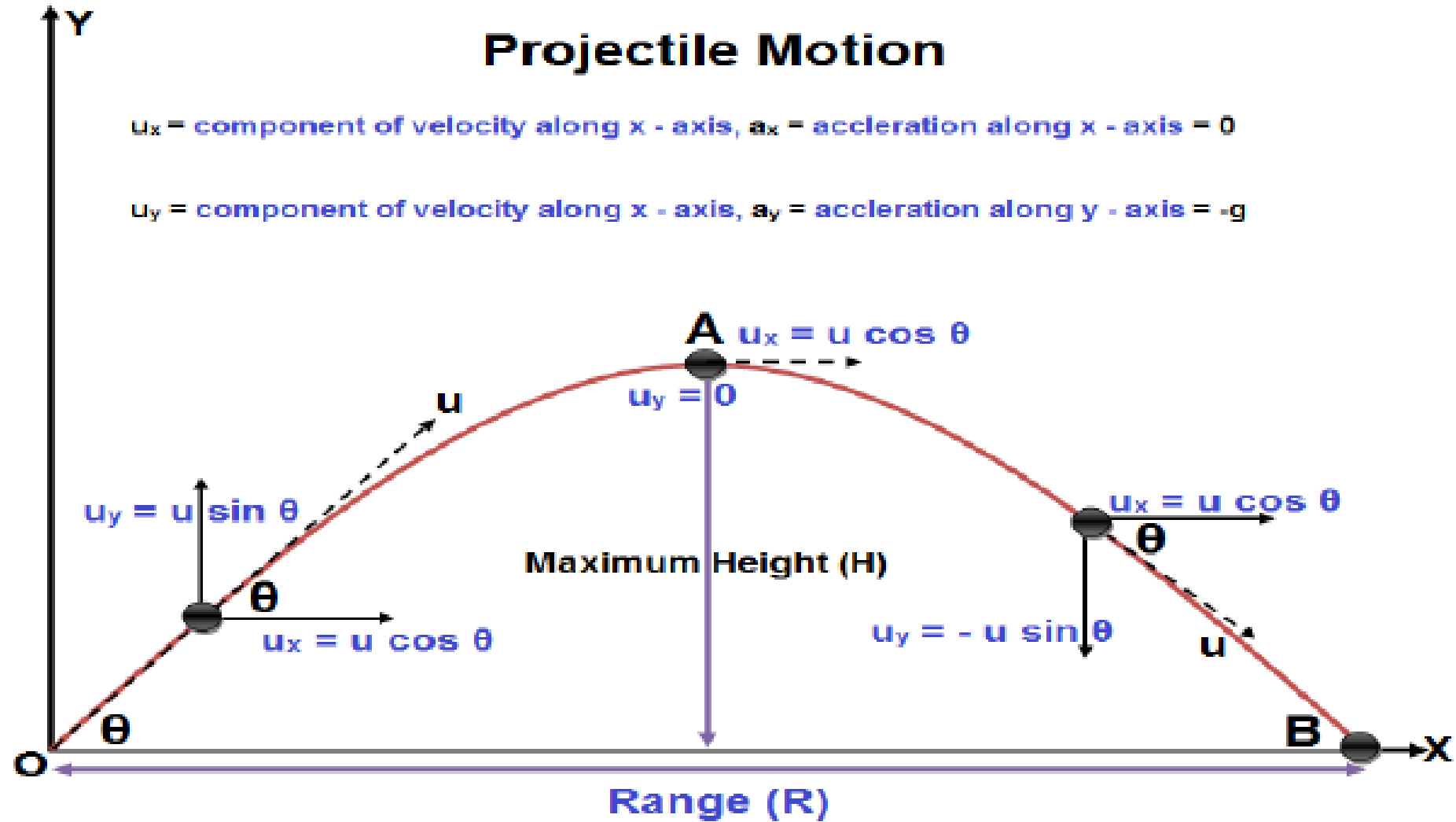
Cont..

➡ **Projectile motion of an object is simple to analyze if we make two assumptions:**

- ✓ (1) the free-fall acceleration is constant over the range of motion and is directed downward, and
 - ✓ (2) the effect of air resistance is negligible.
- ✓ With these assumptions, we find that the path of a projectile, which we call its **trajectory**, is always a **parabola** as shown in Figure below.



Cont....



Cont..

- Two-dimensional motion with constant acceleration can be analyzed as a combination of two independent motions in the x and y directions, with accelerations a_x and a_y .
- Projectile motion can also be handled in this way, with acceleration $a_x = 0$ in the x direction and a constant acceleration $a_y = -g$ in the y direction.
- Therefore, when solving projectile motion problems, use two analysis models:

(1) the particle under constant velocity in the horizontal direction

$a_x = 0$ (Because there is no force acting horizontally) The horizontal position of the projectile after some time t is:

$$\mathbf{x_f = x_i + v_{xi}t} \quad , \quad \Delta x = u_x t + \frac{1}{2} a_x t^2$$

Cont....

$(x_i, y_i) = (0, 0)$ if the projectile is initially at the origin

$$x_f = u \cos \theta t + \frac{1}{2} (0) t^2$$

$$x_f = u \cos \theta t$$

A

The vertical position of the projectile after some time t

$$\Delta y = u_y t + \frac{1}{2} a_y t^2$$

$$y_f = u \sin \theta t - \frac{1}{2} g t^2$$

B

The horizontal components of the velocity

$$v_x = u_x + a_x t \quad \text{But } a_x = 0$$

$$v_x = u_x = u \cos \theta$$

C

Cont....

The vertical components of the velocity

$$v_y = u_y + a_y t$$

$$v_y = u \sin \theta - gt$$

D

Horizontal Range and Maximum Height of a Projectile

- When the projectile reaches the maximum height (the peak), $v_y = 0$

$$0 = u \sin \theta - gt$$

$$t = \frac{u \sin \theta}{g} \text{ (time to reach maximum height)}$$

$$\text{At } y = h, t = \frac{u \sin \theta}{g} \rightarrow h = u \sin \theta \left(\frac{u \sin \theta}{g} \right) - \frac{1}{2} g \left(\frac{u \sin \theta}{g} \right)^2$$

$$h = \frac{u^2 \sin^2 \theta}{2g}$$

E

Cont....

The Range(R) is the horizontal displacement of the projectile covered in a total time of flight.

$$t_{tot} = 2t, \text{ Where } t = \frac{u \sin \theta}{g}$$

$$t_{tot} = \frac{2u \sin \theta}{g}$$

F

$$\text{When } x = R, t = t_{tot} = \frac{2u \sin \theta}{g}$$

$$R = (u \cos \theta) \left(\frac{2u \sin \theta}{g} \right) \quad \text{But, } 2 \sin \theta \cos \theta = \sin 2\theta$$

G

$$R = \frac{u^2 \sin 2\theta}{g}$$

The range (R) is maximum, when $\sin 2\theta = 1 \rightarrow 2\theta = 90^\circ \rightarrow \theta = 45^\circ$

$$R_{max} = \frac{u^2}{g}$$

Cont.....example

Q₁ . A rocket is fired with an initial velocity of 100m/s at an angle of 55° above the horizontal. It explodes on the mountain side 12s after its firing. What is the x-and y- coordinates of the rocket relative to its firing point?

Solution:- *We are given that*

$$u = 100\text{m/s}, \theta = 55^\circ, t = 12\text{s}$$

We are asked to find the horizontal position(x)

$$x(t) = u\cos\theta t$$

$$= (100\text{m/s})\cos 55^\circ (12\text{s})$$

$$x = 688.3\text{m}$$

and the vertical position (y) of the rocket

$$y(t) = u\sin\theta t - \frac{1}{2}gt^2$$

$$= (100\text{m/s})(\sin 55^\circ)(12\text{s}) - \frac{1}{2}(9.8\text{m/s}^2)(12\text{s})^2$$

$$= 983\text{m} - 706\text{m}$$

$$y = 277\text{m}$$

2.2. Particle Dynamics and Planetary Motion

- What do you think about the cause for the change in the state of motion of an object?
- What makes planets to revolve around the sun keeping their trajectory?

- ✓ **Force:** any interaction that changes the motion an object.
- ✓ A force moves or tends to move, stops or tends to stop the motion of the object.
- ✓ **Net force:** is defined as the vector sum of all the forces acting on the object. The object accelerates only if the net force (F_{net}) acting on it is not equal to zero.

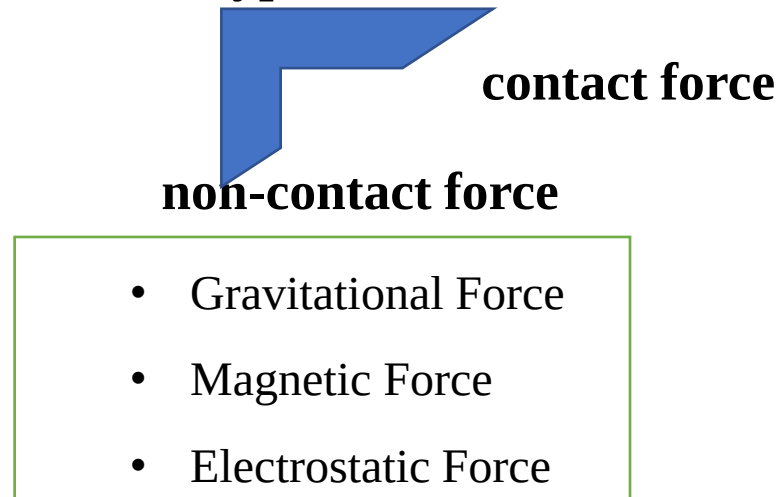
2.2.1. The Concept of Force as A Measure of Interaction

- ✓ **In physics**, any of the four basic forces gravitational, electromagnetic, strong nuclear and weak forces govern how particles interact.

Cont..

- ✓ The fundamental interactions are characterized on the basis of the following four criteria:
 - the types of particles that experience the force,
 - the relative strength of the force,
 - the range over which the force is effective, and
 - the nature of the particles that mediate the force.

2.2.2. Type of Forces



- Muscular Forces
- Frictional Forces
- Normal Force

2.2.3. Newton's Laws of Motion and Applications

✓ Newton's First law of Motion:

“Everybody continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by forces impressed upon it.”

This is sometimes called the **Law of Inertia**. Essentially, it makes the following two points:

- An object that is not in motion will not move until a force acting upon it.
- An object in constant motion will not change its velocity until a force acts upon it.

□ Another way of stating Newton's First Law of motion:

- A body that is acted on by **no net force** moves at a constant velocity (which may be zero) and **zero acceleration**.

Newton's Second law of Motion:

- The acceleration acquired by a point particle is directly proportional to the net force acting on the particle and inversely proportional to its mass and the acceleration is always in the direction of the net force.

$$\vec{a} \propto \frac{\sum \vec{F}}{m}$$

Mathematically, $\sum \vec{F} = m\vec{a}$

- Where $\sum \vec{F}$ is the net force acting on the particle, m is the mass of the particle and $\vec{a}$ is the acceleration of the particle.

Q1. A 3kg object undergoes an acceleration given by $\vec{a} = (2\hat{i} + 5\hat{j})\text{m/s}^2$. Find the magnitude of the resultant force.

We are given that

Solution:

$$m = 3\text{kg}, \vec{a} = (2\hat{i} + 5\hat{j})\frac{\text{m}}{\text{s}^2}$$

We are asked to find the magnitude of $\vec{F}$

$$\vec{F} = m\vec{a}$$

$$= (3\text{kg})(2\hat{i} + 5\hat{j})\text{m/s}^2$$

$$\vec{F} = (6\hat{i} + 15\hat{j})\text{N}$$

The magnitude of the force is

$$|\vec{F}| = \sqrt{6^2 + 15^2} = 16.15\text{N}$$

Newton's Third law of Motion

- States that “For every action there is always an equal and opposite reaction.”
- If two objects interact, the force $\mathbf{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\mathbf{F}_{21}$ exerted by object 2 on object 1:

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \quad ; \quad \mathbf{F}_{12} + (-\mathbf{F}_{21}) = 0$$

Note that:

- ✓ Action and reaction forces always exist in pair
- ✓ A single isolated force cannot exist
- ✓ Action and reaction forces act on different objects

Forces of Friction

- **Frictional force** refers to the force generated by two surfaces that are in contact and either at rest or slide against each other.
- The angle and position of the object affect the amount of frictional force.
 - If an object is placed on a horizontal surface against another object, then the frictional force will be equal to the weight of the object.
 - If an object is pushed against the surface, then the frictional force will be increased and becomes more than the weight of the object.
- Generally friction force is always proportional to the normal force between the two interacting surfaces.

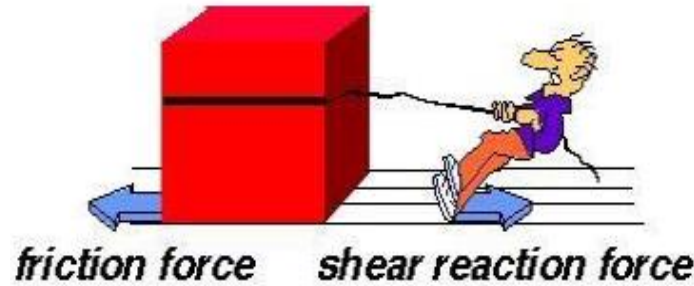
Mathematically

$$F_{frict} \propto F_{norm}$$

$$F_f = \mu F_N$$

Cont....

- Where the proportionality constant μ is the coefficient of friction.



- **Static friction:** exists between two stationary objects in contact to each other.

Mathematically static friction is written as; $f_s = f_{s,max} = \mu_s N$.

Where μ_s is the coefficient of static friction

- **Kinetic friction:** arises when the object is in motion on the surface. The magnitude of the force of kinetic friction acting between two surfaces is

$f_k = \mu_k N$ Where μ_k is called the coefficient of kinetic friction

Cont....example

Q1. A 25.0-kg block is initially at rest on a horizontal surface. A horizontal force of 75.0 N is required to set the block in motion. After it is in motion, a horizontal force of 60.0 N is required to keep the block moving with constant speed. Find the coefficients of static and kinetic friction from this information.

Solution: *We are given that*

$$m = 25\text{kg}, f_s = 75\text{N}, f_k = 60\text{N}$$

We are asked to find μ_s & μ_k

$$f_s = \mu_s N \quad \text{but } N = mg$$

$$\mu_s = \frac{f_s}{mg} = \frac{75\text{N}}{(25\text{kg})(9.8\text{m/s}^2)}$$

$$\mu_s = 0.31$$

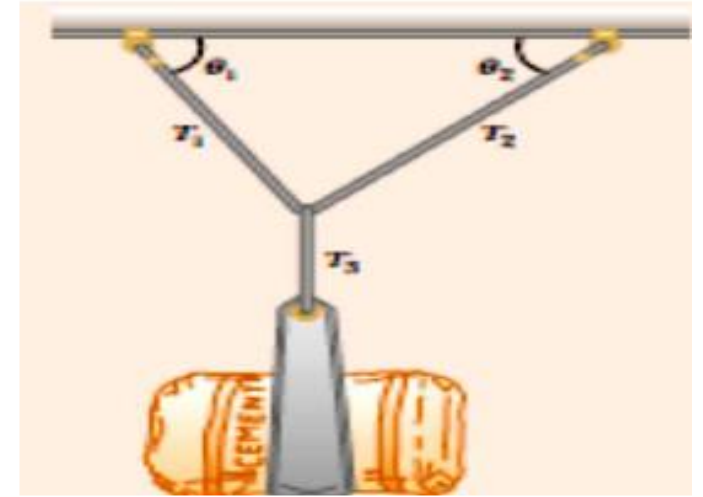
$$f_k = \mu_k N \quad \text{but } N = mg$$

$$\mu_k = \frac{f_k}{mg} = \frac{60\text{N}}{(25\text{kg})(9.8\text{m/s}^2)}$$

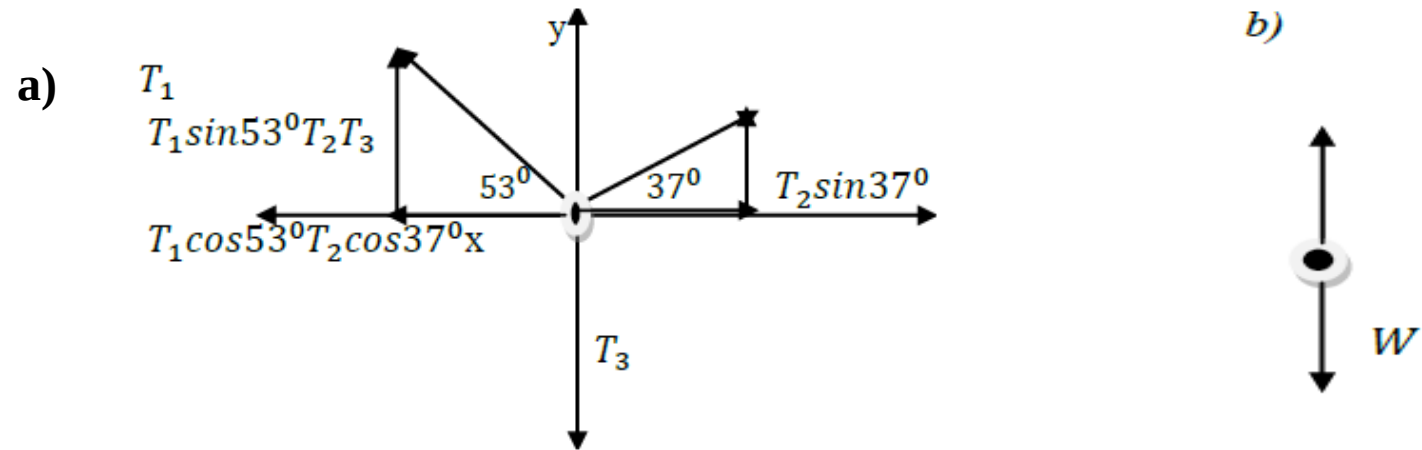
$$\mu_k = 0.245$$

Application of Newton's Laws of Motion

Q. A bag of cement of weight 300 N hangs from three ropes as shown in the figure below. Two of the ropes make angles of 53.0° and 37.0° with the horizontal. If the system is in equilibrium, find the tensions T_1 , T_2 and T_3 in the ropes.



Solution; We can draw two free body diagrams for the problem as follows



Cont..

Since the system is in equilibrium, $\sum \vec{F} = 0 \rightarrow \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases}$

From free body diagram (a) $\sum F_x = T_2 \cos 37^\circ - T_1 \cos 53^\circ = 0$

$$0.8T_2 = 0.6 T_1 \text{(a)}$$

$$\sum F_y = T_1 \sin 53^\circ + T_2 \sin 37^\circ - T_3 = 0$$

$$0.8 T_1 + 0.6 T_2 = T_3 \text{(b)}$$

From free body diagram (b) $\sum F_y = T_3 - W = 0 \rightarrow T_3 = W = 300N \text{(c)}$

Substituting(c) in (b) $0.8 T_1 + 0.6 T_2 = 300N \text{(d)}$

$$\text{But } 0.8T_2 = 0.6 T_1 \rightarrow T_2 = \frac{0.6 T_1}{0.8} = 0.75T_1$$

Substituting for T_2 in (d) Gives

$$0.8 T_1 + 0.6(0.75T_1) = 300N$$

$$T_1 = 240N$$

$$T_2 = 0.75T_1 = 0.75(240N)$$

$$T_2 = 180N$$

2.2.4. Uniform Circular Motion

❖ **Uniform Circular Motion** is motion of objects in a circular path with a **constant speed**. Objects moving in a circular path with a constant speed can have acceleration.

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

❖ There are two ways in which the acceleration can occur due to:

- change in magnitude of the velocity
- change in direction of the velocity

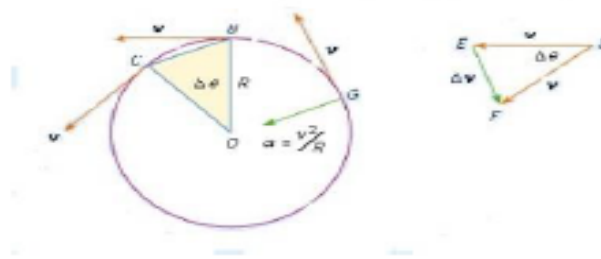
❖ For objects moving in a circular path with a constant speed, acceleration arises because of the change in direction of the velocity.

❖ Hence, in case of uniform circular motion:

- ✓ Velocity is always tangent to the circular path and perpendicular to the radius of the circular path.
- ✓ Acceleration is always perpendicular to the circular path, and points towards the center of the circle.

Such acceleration is called the **centripetal acceleration**.

Cont....



$$|\vec{r}_i| = |\vec{r}_f| = r$$

The angle $\Delta\theta$ in figure (a) and (b) are the same

By SAS similarity $\frac{|\Delta\vec{v}|}{|\Delta\vec{r}|} = \frac{v}{r} \rightarrow |\Delta\vec{v}| = \frac{(|\Delta\vec{r}|)v}{r}$

$$|\vec{a}| = \frac{|\Delta\vec{v}|}{\Delta t} \rightarrow \left(\frac{|\Delta\vec{r}|}{\Delta t}\right) \left(\frac{v}{r}\right) = \frac{v^2}{r}$$

$$a_c = \frac{v^2}{r} \text{ (Centripetal acceleration)}$$

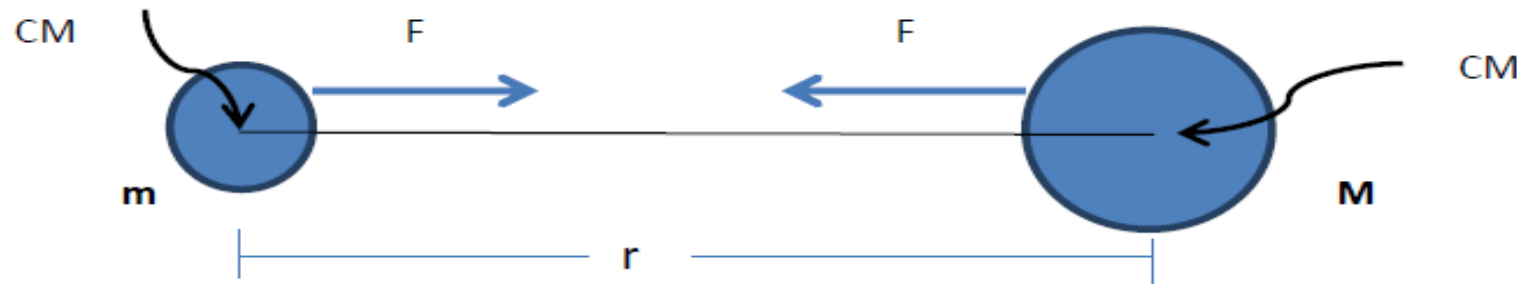
Period (T):- Time required for one complete revolution

For a particle moving in a circle of radius r with a constant speed

$$v = \frac{2\pi r}{T} \rightarrow T = \frac{2\pi r}{v}$$

2.2.5. Newton's Law of Universal Gravitation

- ✓ The gravitational force is always attractive, and it depends only on the masses involved and the distance between them.
- ✓ Newton's universal law of gravitation states that every particle in the universe attracts every other particle with a force along a line joining them.
- ✓ The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- ✓ Gravitational attraction is along a line joining the centers of mass of these two bodies. The magnitude of the force is the same on each, consistent with Newton's third law.



Cont....

- Interaction of two objects with gravitational force from the figure above. $\vec{F} = \frac{GmM}{r^2} \hat{r}$

Here, r is the distance between the centers of mass of the bodies, G is the gravitational constant, whose value found by experiment is $G = 6.674 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$ in SI units.

- Recall that the acceleration due to gravity g is about 9.8 m/s^2 on Earth and Earth's mass $6 \times 10^{24} \text{ kg}$.
- The weight of an object mg is the gravitational force between it and Earth. Substituting mg for F in Newton's universal law of gravitation gives $mg = \frac{GmM}{r^2}$

- Where, m is the mass of the object, M is the mass of Earth, and r is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). The mass m of the object cancels, leaving an equation for g : $g = \frac{GM}{r^2}$

Cont..

Substituting known values for Earth's mass and radius (to three significant figures),

$$g = (6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}) (\frac{5.98 \times 10^{24} \text{kg}}{6.38 \times 10^6 \text{m}})$$

and we obtain a value for the acceleration of a falling body:

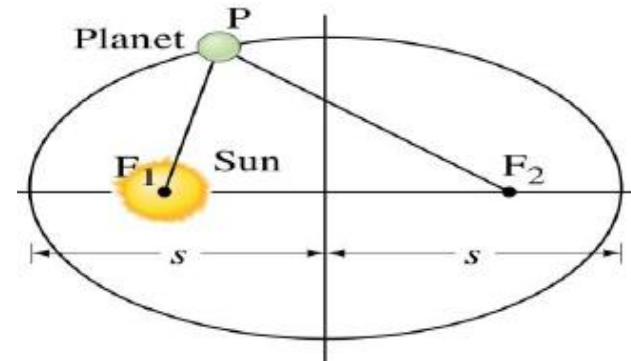
$$g = 9.8 \frac{\text{m}}{\text{s}^2}$$

This is the expected value and is independent of the body's mass.

2.2.6. Kepler's Laws, Satellites Motion and Weightlessness

- **Kepler's First Law (Law of Orbits)**

- ✓ States that the orbit of each planet in the solar system is an ellipse, the Sun will be on one focus. The points F_1 and F_2 represented in figure are known as the foci of the ellipse.
- ✓ Kepler's first law is rather simple - all planets orbit the sun in a path that resembles an ellipse, with the sun being located at one of the foci of that ellipse.

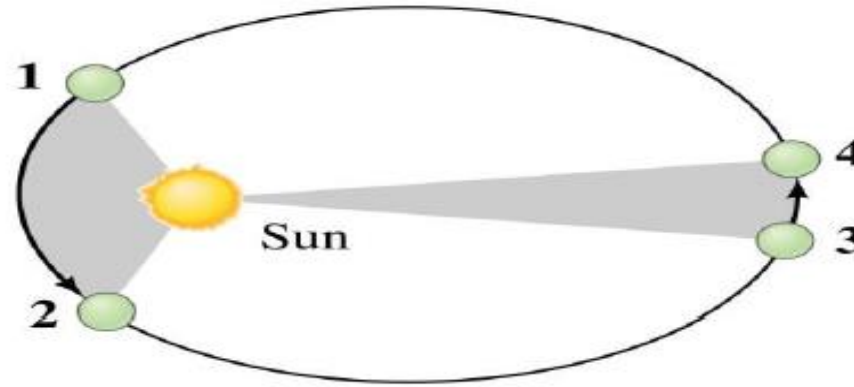


Kepler's Second Law (The Law of Areas)

States that —the radius vector connecting the centers of the Sun and the Planet sweeps out equal areas in equal intervals of time. The Figure below shows the two sectors of the ellipse having equal areas corresponding to the same time intervals.

Cont..

The second law describes the speed (which is constantly changing) at which any given planet will move while orbiting the sun. A planet moves fastest when it is closest to the sun and slowest when it is furthest from the sun. Yet, if an imaginary line were drawn from the center of the planet to the center of the sun, that line would sweep out the same area in equal periods of time.



Kepler's Third Law (The Law of Harmony)

- States that “the square of the orbital period of a planet is proportional to the cube of the average distance between the centers of the planet and the sun.”

Satellite motion and Weightlessness

- **Weightlessness** is simply a sensation experienced by an individual when there are no external objects touching one's body and exerting a push or pull upon it.
- **Weightless** sensations exist when all contact forces are removed, i.e. in a state of free fall.

Astronauts on the orbiting space station are **weightless** because:

- ✓ There is no gravity in space and they do not weigh anything.
- ✓ Space is a vacuum and there is no gravity in a vacuum.
- ✓ Space is a vacuum and there is no air resistance in a vacuum.
- ✓ The astronauts are far from Earth's surface at a location where gravitation has a minimal effect

2.3. Work, Energy and Linear Momentum

- **Work** to be done on an object, three essential conditions should be satisfied:-
 - ✓ Force must be exerted on the object
 - ✓ The force must cause a motion or displacement
 - ✓ The force should have a component along the line of displacement
- **The work (W)** done on a system by an agent exerting a constant force ($\vec{F}$) and moving the system through the displacement ($\Delta\vec{r}$) is
$$W = \vec{F} \cdot \Delta\vec{r} = |\vec{F}| |\Delta\vec{r}| \cos\theta$$
 Where θ is the angle between $\vec{F}$ and $\Delta\vec{r}$.

Work is a scalar quantity and its SI unit is **Joule (J)**. Where, $1J = 1N \cdot 1m$

The sign of work depends on the direction of $\vec{F}$ relative to $\Delta\vec{r}$. Hence, the work done by the applied force is positive when the projection of $\vec{F}$ onto $\Delta\vec{r}$ is in the same direction as $\Delta\vec{r}$.

Work Done by a Constant Force

- A particle moving in the xy plane undergoes a displacement given by $\vec{\Delta r} = (2.0\hat{i} + 3.0\hat{j}) \text{ m}$ as a constant force $\vec{F} = (5.0\hat{i} + 2.0\hat{j}) \text{ N}$ acts on the particle. Calculate the work done by $\vec{F}$ on the particle.

Solution

$$\begin{aligned} W &= \vec{F} \cdot \vec{\Delta r} = [(5.0\hat{i} + 2.0\hat{j}) \text{ N}] \cdot [(2.0\hat{i} + 3.0\hat{j}) \text{ m}] \\ &= (5.0\hat{i} \cdot 2.0\hat{i} + 5.0\hat{i} \cdot 3.0\hat{j} + 2.0\hat{j} \cdot 2.0\hat{i} + 2.0\hat{j} \cdot 3.0\hat{j}) \text{ N} \cdot \text{m} \\ &= [10 + 0 + 0 + 6] \text{ N} \cdot \text{m} = 16 \text{ J} \end{aligned}$$

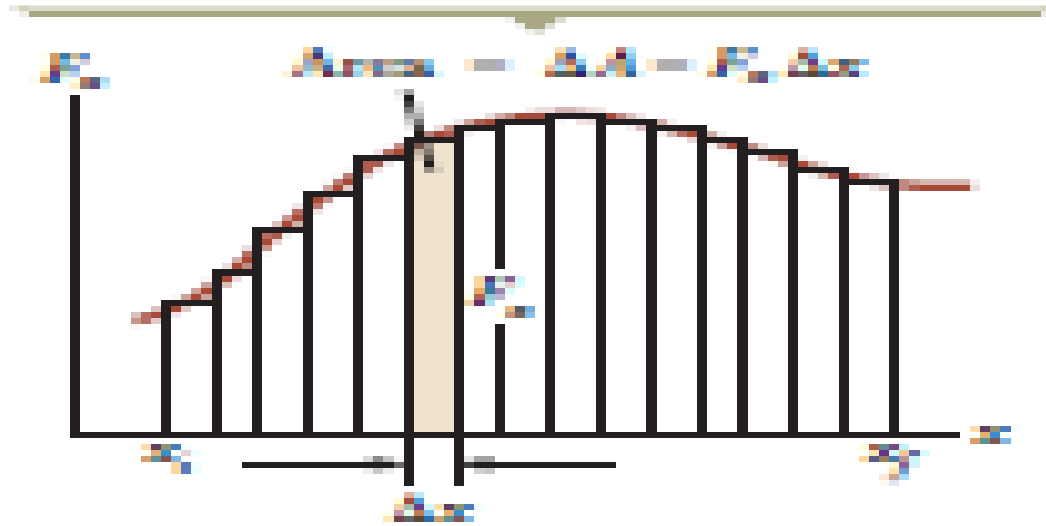
Work done by a varying force

- Consider a particle being displaced along the x axis under the action of a force that varies with position. The particle is displaced in the direction of increasing x from $x = x_i$ to $x = x_f$.
- We can approximate the work done on the particle by the force as $W = F_x \Delta x$

Cont.....

- The total work done for the displacement from $\mathbf{x}_i$ to $\mathbf{x}_f$ is approximately equal to the sum of a large number of such terms:

$$\sum_{xi}^{xf} F_x \Delta x$$

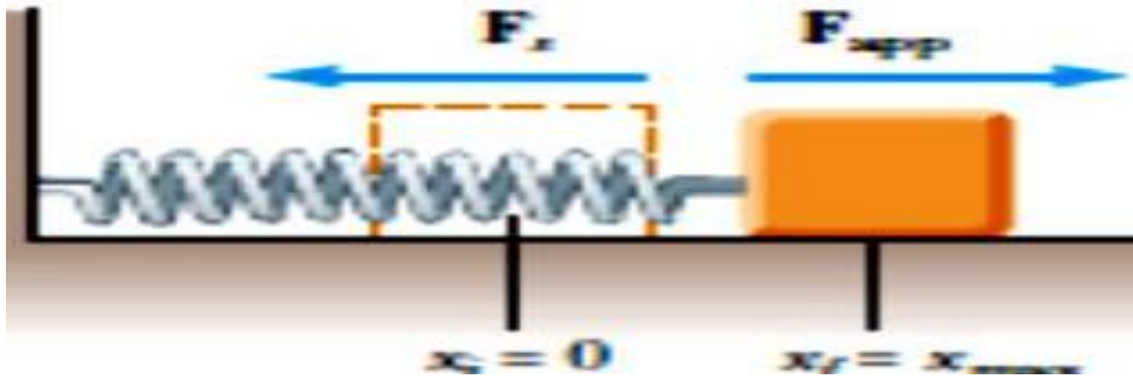


$$\lim_{\Delta x \rightarrow 0} \sum_{xi}^{xf} F_x \Delta x = \int_{xi}^{xf} F_x dx$$

The work done by $\mathbf{F}_x$ on the particle as it moves from $\mathbf{x}_i$ to $\mathbf{x}_f$ as

$$W = \int_{xi}^{xf} F_x dx, \text{ In general, } \mathbf{w} = \int_{ri}^{rf} \mathbf{F} \cdot d\mathbf{r}$$

Work done by spring



$$F_s = -kx \text{ (Hook's Law)}$$

Work done by the restoring force if the block undergoes an arbitrary displacement from x_i to x_f is the area enclosed by the above graph.

$$\text{Area} = W = \frac{1}{2}(F_s \times \Delta X), \quad \text{but} \quad F_s = -k\Delta X$$

$$W = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2. \quad \text{If the work is done by the applied force}$$

$$W_{F_{app}} = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2 \quad \text{because} \quad F_{app} = k\Delta X$$

Cont.. example

Q1. A particle is subject to a force that varies with position as in figure below. What is the total work done by the force over the distance, $x = 0$ to $x = 15.0$ m?



Solution

The total work done is equal to the area under the F_x versus x curve

$$\begin{aligned} W &= (F_x)(\Delta x) = \Delta A \\ &= \frac{1}{2}(5)(3) + (5)(3) + \frac{1}{2}(5)(3) \end{aligned}$$

$$W = 30J$$

Energy

- It is defined as the capacity of a physical system to perform work. And it exists in several forms such kinetic, potential, thermal, chemical and other forms. And its SI unit is joule (J).
- **Kinetic energy (KE)** is the capacity of an object to do work by virtue of its motion. For an object of mass m and moving with speed v , the kinetic energy is calculated as:

$$KE = \frac{1}{2}mv^2$$

- **Potential Energy (PE)** is the energy that is stored in an object due to its position relative to some zero position. $\Delta U_g = mg\Delta y$.

Work- Energy theorem

- **work** done on an object is the **change** in its **kinetic energy**.
- Let a force 'F' is applied on an object initially moving with velocity 'u'. If it is displaced to a displacement 's' and changes its velocity into 'v', then its motion will be expressed by :

$$v^2 - u^2 = 2as \quad \frac{mv^2}{2} - \frac{mu^2}{2} = mas;$$

$$\text{Hence, } \frac{mv^2}{2} - \frac{mu^2}{2} = Fs;$$

- Therefore, $\frac{mv^2}{2} - \frac{mu^2}{2} = W$; where $W = Fs$ is the work done by this force.

$$W_{net} = \Delta K.E$$

Cont..

Q1. A 0.600-kg particle has a speed of 2.00 m/s at point A and kinetic energy of 7.50 J at point B.

What is (a) its kinetic energy at A? (b) its speed at B? (c) the total work done on the particle as it moves from A to B?

We are given that

from description for the kinetic energy

Solution

$$m = 0.6\text{kg}, v_A = 2\text{m/s}, K_B = 7.5\text{J}$$

$$a) \quad K_A = \frac{1}{2}mv_A^2$$

We are asked to find

$$= \frac{1}{2}(0.6\text{kg})(2\text{m/s})^2$$

$$a) K_A \quad b) v_B \quad c) W_{AB} = 1.2\text{J}$$

$$b) \quad K_B = \frac{1}{2}mv_B^2$$

$$v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{2(7.5\text{J})}{0.6\text{kg}}} = \sqrt{25} = 5\text{m/s}$$

c) *From work-kinetic energy theorem*

$$W_{AB} = K_B - K_A = 7.5\text{J} - 1.2\text{J} = 6.3\text{J}$$

2.3.2. Power

➤ **Power** is defined as the time rate of energy transfer. $P_{av} = \frac{W}{\Delta t}$

➤ The instantaneous power P is defined as the limiting value of the average power as Δt approaches zero:

$$P = \lim_{\Delta t \rightarrow 0} \frac{W}{\Delta t} = \frac{W}{t} \quad \text{But, } W = \vec{F} \cdot \Delta \vec{r}$$
$$P = \frac{W}{\Delta t} = \vec{F} \cdot \frac{\Delta \vec{r}}{\Delta t} = \vec{F} \cdot \vec{v}$$

The SI unit of power is joules per second (J/s), also called the watt (W) (after James Watt):

$$1W = \frac{1J}{s} = 1kg \cdot \frac{m^2}{s^3}$$

$$1kWh = (10^3 W)(3600s) = 3.6 \times 10^6 J$$

2.3.3. Linear Momentum

➤ The **linear momentum** of a particle or an object that can be modeled as a particle of mass **m** moving with a velocity $\vec{v}$ is defined to be the product of its mass and velocity: $\vec{P} = m\vec{v}$

➤ **Momentum ($\vec{p}$)** is a vector quantity in the direction of the velocity with SI unit kgm/s.

➤ Using Newton's second law of motion, we can relate the linear momentum of a particle to the resultant force acting on the particle.

$$\vec{F} = m \frac{\Delta \vec{v}}{\Delta t} \Rightarrow \vec{F} = \frac{\Delta (m\vec{v})}{\Delta t} = \frac{\Delta \vec{P}}{\Delta t}$$

➤ Therefore, the time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.

$$\Delta \vec{P} = \vec{F} \Delta t$$

$$\vec{I}(\text{Impulse}) = \Delta \vec{P} = \vec{P}_f - \vec{p}_i = \vec{F} \Delta t$$

Conservation of Linear Momentum

➤ Recall Newton's third law:

- $\vec{F}_{21}$ = the force on m_1 from m_2

- $\vec{F}_{12}$ = the force on m_2 from m_1

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

$$m_1 \frac{\Delta \vec{v}_1}{\Delta t} + m_2 \frac{\Delta \vec{v}_2}{\Delta t} = 0$$

$$\Rightarrow m_1 \Delta \vec{v}_1 + m_2 \Delta \vec{v}_2 = 0$$

$$\Rightarrow m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$$

$$\vec{P}_{tot} = \vec{P}_1 + \vec{P}_2 = \text{constant}$$

Where $\vec{P}_{1i}$ and $\vec{P}_{2i}$ are the initial values and $\vec{P}_{1f}$ and $\vec{P}_{2f}$ the final values of the momenta for the two particles for the time interval during which the particles interact.

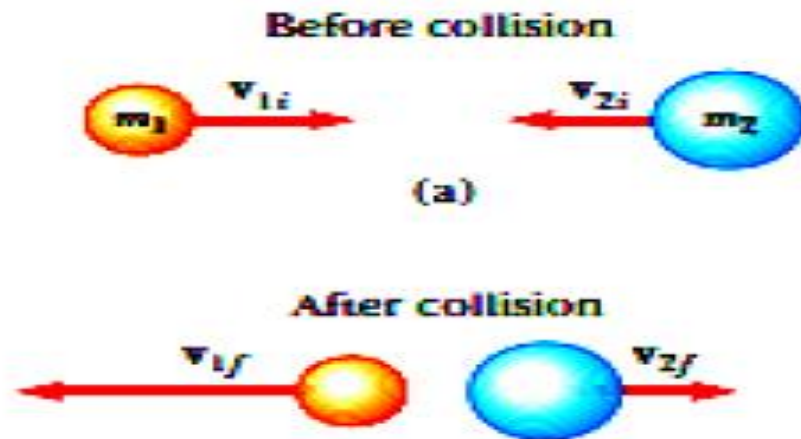
2.3.4. Collisions

- If the two particles form an isolated system, the momentum of the system must be conserved.
- Total momentum of an isolated system just before a collision equals the total momentum of the system just after the collision.
- But, the total kinetic energy of the system of particles may or may not be conserved, depending on the type of collision.

A. Elastic collision: - An elastic collision between two objects is one in which the total kinetic energy as well as total momentum of the system is conserved.

- Consider two particles of masses m_1 and m_2 moving with initial velocities $\mathbf{v}_{1i}$ and $\mathbf{v}_{2i}$ along the same straight line, as shown in the figure below.

Cont..



$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{-----}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

From conservation of kinetic energy,

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_1 (v_{1i}^2 - v_{1f}^2) = m_2 (v_{2f}^2 - v_{2i}^2)$$

Cont..

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) \quad (a) \quad m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \quad (b)$$

$$v_{1i} + v_{1f} = v_{2f} + v_{2i} \quad v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

B. Inelastic Collision: - An inelastic collision is one in which the total kinetic energy of the system is not conserved. But the momentum of the system is conserved.

Therefore, for inelastic collision of two particles:

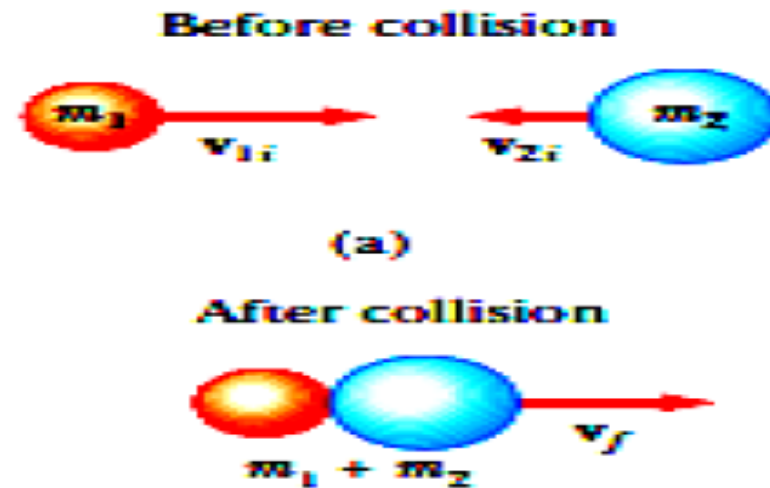
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 \neq \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Cont..

C. Perfectly Inelastic Collision: -When the colliding objects stick together after the collision, the collision is called perfectly inelastic. Consider two particles of m_1 and m_2 moving with initial velocities and along the same straight line, as shown in the figure below.

- The two particles collide head-on, stick together, and then move with some common velocity v_f after the collision.



Cont..

- The total momentum before the collision equals the total momentum of the composite system after the collision

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

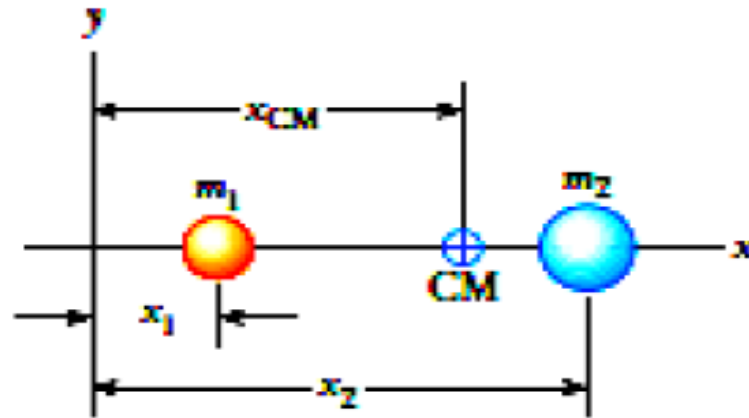
Q₁. An archer shoots an arrow toward a target that is sliding toward her with a speed of 2.50 m/s on a smooth surface. The 22.5-g arrow is shot with a speed of 35.0 m/s and passes through the 300-g target, which is stopped by the impact. What is the speed of the arrow after passing through the target?

Cont....

Q₂. A block of mass initially moving to the right with a speed of 4m/s on a horizontal frictionless track collides with a second block of mass initially moving to the left with a speed of 2.5m/s. If the collision is elastic, find the velocities of the two blocks after collision.

2.3.5. Center of Mass

- The center of mass is the point at which all the mass can be considered to be "concentrated".
- The center of mass of the pair of particles located on the x axis as shown in the figure below lies somewhere between the particles, it is given by:



$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Cont..

□ The x coordinate of the center of mass of n particles is defined to be

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 \dots + m_nx_n}{m_1 + m_2 + m_3 \dots + m_n} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{\sum_i m_i x_i}{M} \dots$$

□ Similarly, the y and z coordinates of the center of mass are

$$y_{CM} = \frac{\sum_i m_i y_i}{M} \quad \text{and} \quad z_{CM} = \frac{\sum_i m_i z_i}{M}$$

□ Therefore,

$$\vec{r}_{CM} = x_{CM}\hat{i} + y_{CM}\hat{j} + z_{CM}\hat{k} = \frac{\sum_i m_i x_i \hat{i} + \sum_i m_i y_i \hat{j} + \sum_i m_i z_i \hat{k}}{M} \qquad \vec{r}_{CM} = \frac{\sum_i m_i \vec{r}_i}{M}$$

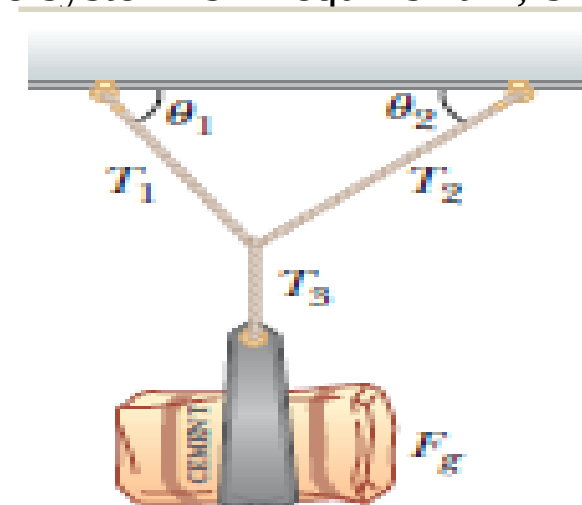
Cont.. Worksheet #2

Q₁. A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection?

Q₂. (A) A bag of cement weighing 325 N hangs in equilibrium from three wires as suggested in Figure below. Two of the wires make angles $\theta_1 = 60.0^\circ$ and $\theta_2 = 40.0^\circ$ with the horizontal. Assuming the system is in equilibrium, find the tensions T_1 , T_2 , and T_3 in the wires.

(B) A bag of cement whose weight is F_g hangs in equilibrium from three wires as shown in Figure below. Two of the wires make angles θ_1 and θ_2 with the horizontal. Assuming the system is in equilibrium, show that the tension in the left-hand wire is

$$T_1 = \frac{F_g \cos \theta_2}{\sin (\theta_1 + \theta_2)}$$



Cont..

Q₃. A block on a horizontal plane is given an initial speed v . Traveling in a straight line, it comes to rest after sliding a distance d . show that the coefficient of kinetic friction is $\mu_k = v^2 / 2gd$.

Q₄. A force has the dependence $F_x(x) = -kx^4$ on the displacement x , where the constant $k = 20.3 \text{ N/m}^4$.

How much work does it take to change the displacement from 0.73m to 1.35m?

Q₅. A particle of mass m moves from rest at $t=0$ under the influence of a single constant force F .

Show the power delivered by the force at time t is $P = F^2 t / m$.

Q₆. A ball of mass 0.200 kg has a velocity of 150m/s; a ball of mass 0.300 kg has a velocity of - 0.4m/s. They meet in a head-on elastic collision. Find their velocities after the collision.

Q₇. Four objects are situated along the y axis as follows: a 2.00 kg object is at +3.00 m, a 3.00-kg object is at +2.50 m, a 2.50-kg object is at the origin, and a 4.00-kg object is at -0.500 m .Where is the center of mass of these objects?