



Remedial Physics

Chapter 5: Work, Energy and Power

Gezahegn S.D.

`gezahegn.sufa@dbu.edu.et`

<https://sites.google.com/view/gezahegnsufa/>

May 26, 2023

☞ **Energy** is a system's ability to do work.

- It exists in different forms:
 - Mechanical energy
 - Chemical energy
 - Nuclear energy

☞ **Work** is the amount of energy transferred due to an object moving some distance because of an external force.

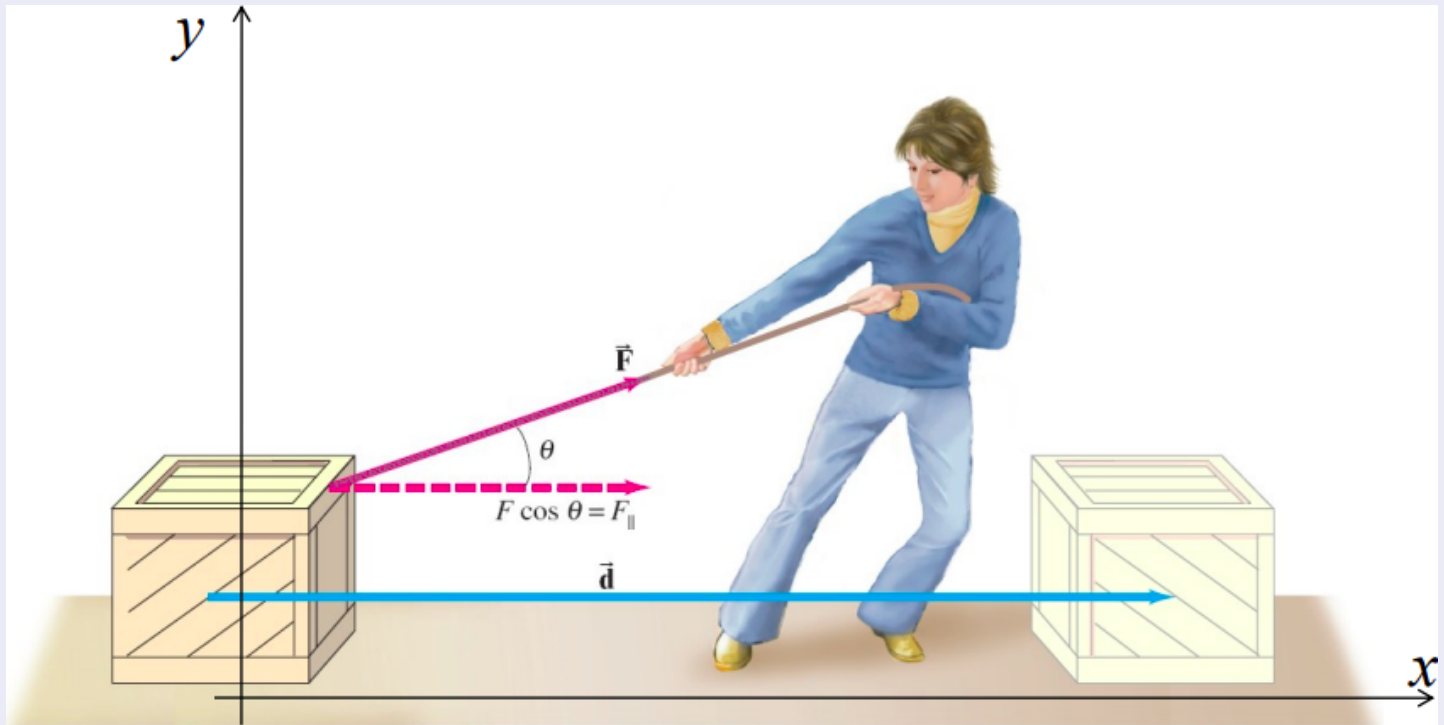
- For work to be done on an object
 - Force must be exerted on the object
 - The force must cause a motion or displacement
 - The force should have a component along the line of displacement

☞ Both Energy and Work have the same SI units, **Joules**

Work Done by a Constant Force, Cont'd



Suppose an object subjected to a constant force $\vec{F}$ undergoes a certain displacement, $\Delta\vec{r}$.



Work Done by a Constant Force, Cont'd



The work done W by the force is given by:

$$W = |\vec{F}| |\Delta\vec{r}| \cos \theta$$

☞ **this relationship applies only when F is constant in magnitude and direction.**

In other word, the work done by a force is the dot product between the force and the displacement:

$$W = \vec{F} \cdot \Delta\vec{r}$$

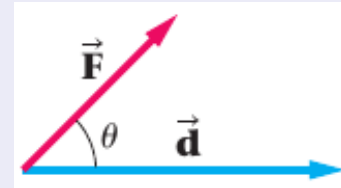
Component form (for 3D)

$$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

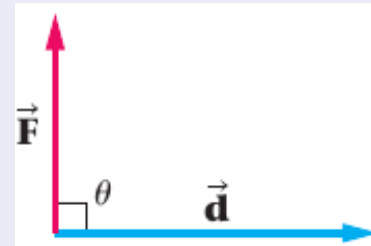
Work Done by a Constant Force, Cont'd



Work is positive if the force has a component in the direction of motion ($-90^\circ < \theta < 90^\circ$)



Work is zero if the force has no component in the direction of motion ($\theta \pm 90^\circ$)



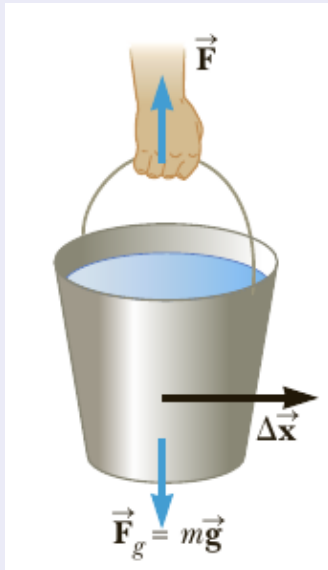
Work is negative if the force has a component opposite to the direction of motion ($90^\circ < \theta < 270^\circ$)



Work Done by a Constant Force, Cont'd

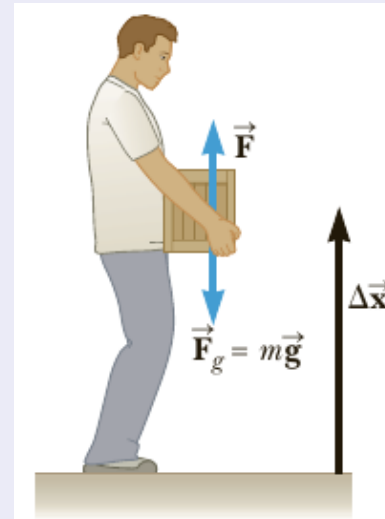


Example-1, Cont'd



No work is done on a bucket when it is moved horizontally because the applied force F is perpendicular to the displacement.

The student does positive work when he lifts the box from the floor, because the applied force F is in the same direction as the displacement. When he lowers the box to the floor, he does negative work.



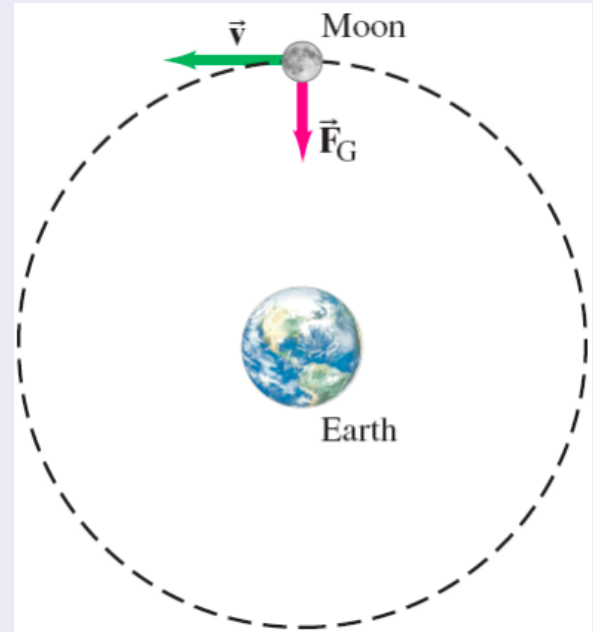
Work Done by a Constant Force, Cont'd



Example-2

The Moon revolves around the Earth in a nearly circular orbit, with approximately constant tangential speed, kept there by the gravitational force exerted by the Earth. Does gravity do

- (a) positive work,
- (b) negative work,
- (c) no work at all on the Moon?



ans: the Moon's displacement at any moment is tangent to the circle, perpendicular to the radius and the force of gravity - therefore the work done by gravity is zero

Work Done by a Constant Force, Cont'd



Example-3

A particle moving in the xy plane undergoes a displacement in $\Delta\vec{r} = (2\vec{i} + 3\vec{j})$ while a constant force $\vec{F} = (5\vec{i} + 2\vec{j})$ Nacts on the particle.

- (a) Calculate the magnitude of the displacemnet and the force.
- (b) Calculate the work done by the force $\vec{F}$.
- (c) Calculate the angle θ between $\vec{F}$ and $\vec{S}$.

Solution

- (a) The magnitude of the displacement

$$S = \sqrt{x^2 + y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}m$$

The magnitude of the force

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(5)^2 + (2)^2} = \sqrt{29} \text{ N}$$

Work Done by a Constant Force, Cont'd



Example-3, Cont'd

(b) Work done by the force

$$\mathbf{W} = \vec{F} \cdot \vec{S} = (5\vec{i} + 2\vec{j}) \cdot (2\vec{i} + 3\vec{j}) \text{ N} \cdot \text{m} = 16 \text{ N.m} = 16 \text{ J}$$

(c) The angle θ is given by the equation

$$\theta = \cos^{-1} \left[\frac{16}{\sqrt{377}} \right] = 34.5^\circ$$

Work Done by a Constant Force, Cont'd

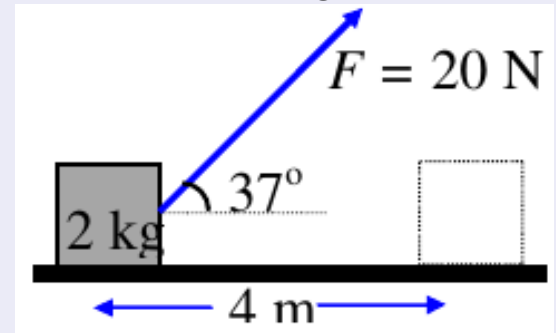


Example-4

A block of mass 2 kg moves under the influence of a force $F = 20\text{ N}$, which makes an angle of 37° above the horizontal. The block moved a distance $s = 4\text{ m}$, on a rough surface of $\mu = 0.2$, as shown in the figure below.

Calculate,

- the work done by the applied force
- the work done by friction
- the net work done on the block



Work Done by a Constant Force, Cont'd



Example-4, **Solution**

a) the work done by the applied force

$$W = |\vec{F}||\Delta\vec{r}| \cos \theta = (20)(4) \cos(37^\circ) = 63.9J$$

b) From y-component, we have $\sum F_y = 0$. Hence,

$$F_N + F \sin \theta - mg = 0$$

$$F_N = mg - F \sin \theta = 2 \times 9.8 - 20 \sin(37^\circ) = 7.56$$

Therefore, the work done by friction

$$W_f = \mu F_N \Delta r \cos \theta = 0.2 \times 7.56 \times 4 \cos(180^\circ) = -6.1J$$

c) the net work done on the block

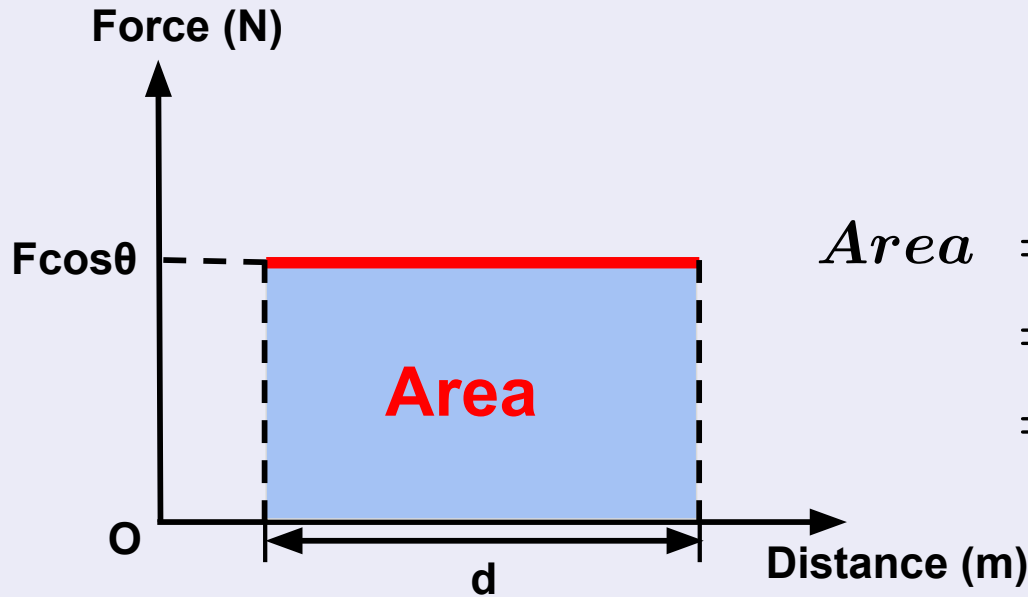
$$W_{net} = W_N + W_g + W_F + W_f = 63.9 + (-6.1) = 57.8J$$

Work Done by a Constant Force, Cont'd



Graphical Representation, Cont'd

Consider a constant force F applied on an object and displaces it through the distance d along the x-axis.



$$\begin{aligned} \text{Area} &= (F \cos \theta) d \\ &= F d \cos \theta \\ &= \text{Work} \end{aligned}$$

Therefore, the area under the force-displacement curve measures the work done

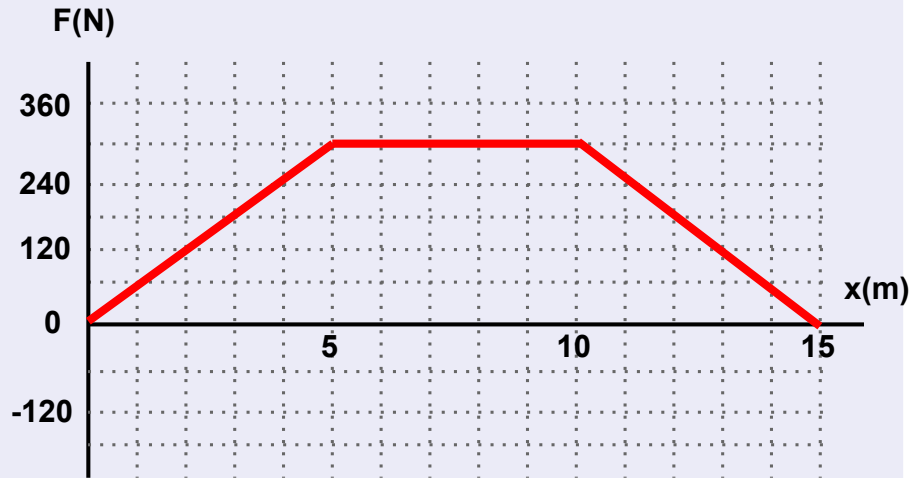
Work Done by a Varying Force, Cont'd



Graphical Representation, Example-5

A particle is acted by the force along x-axis which varies as given in below graph. Find the work done from

- a. $x=0$ to $x=5$ m
- b. $x=5$ to $x=10$ m
- c. $x=10$ to $x=15$ m
- d. $x=0$ to $x=15$ m



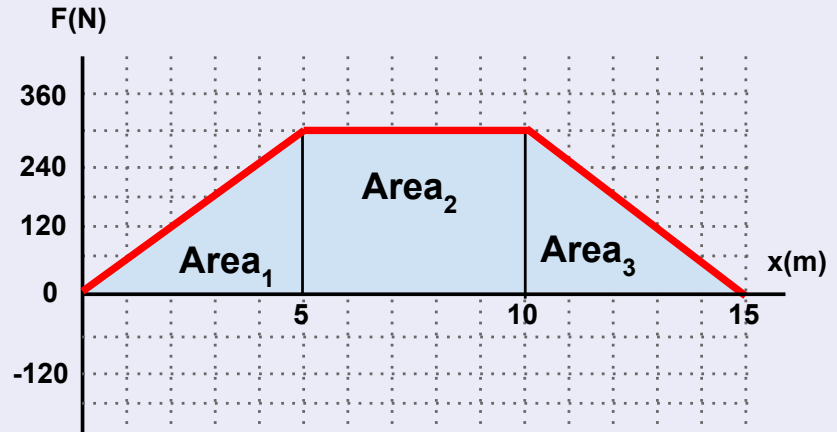
Work Done by a Varying Force, Cont'd



Graphical Representation, Example-5, Cont'd

- a. the work done from $x=0$ to $x=5$ m

$$\begin{aligned} W_1 &= Area_1 \\ &= \frac{1}{2} \times 5 \times 300 \\ &= 750J \end{aligned}$$



- b. the work done from $x=5$ to $x=10$ m

$$W_2 = Area_2 = 5 \times 300 = 1500J$$

- c. the work done from $x=10$ to $x=15$ m

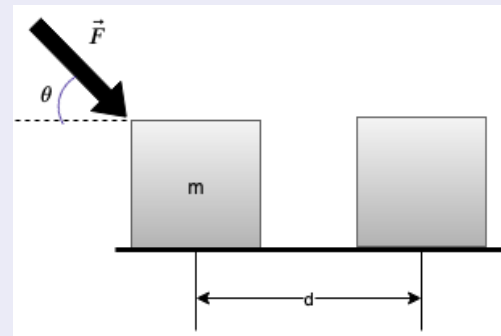
$$W_3 = Area_3 = \frac{1}{2} \times 5 \times 300 = 750J$$

Work Done by a Constant Force, Cont'd



Class Activity

1. The constant forces $\vec{F}_1 = (\hat{i} + 2\hat{j} + 3\hat{k})\text{N}$ and $\vec{F}_2 = (4\hat{i} - 5\hat{j} - 2\hat{k})\text{N}$ act together on a particle during a displacement from position $\vec{r}_2 = 7\hat{k}\text{cm}$ to position $\vec{r}_1 = (20\hat{i} + 15\hat{j})\text{cm}$. Determine the total work done on the particle. **ans: -0.48 J**
2. A block of mass $m = 2.50$ kg is pushed a distance $\Delta x = 2.20$ m along a frictionless horizontal table by a constant applied force of magnitude $F = 16.0$ N directed at an angle $\theta = 25.0^\circ$ below the horizontal as shown in the figure. Determine the work done by
 - (a) the applied force,
 - (b) the normal force exerted by the table,
 - (c) the force of gravity, and
 - (d) the net force on the block.



ans: a) 24.9J b) 0 c) 0 d) 24.9J

Work Done by a Constant Force, Cont'd

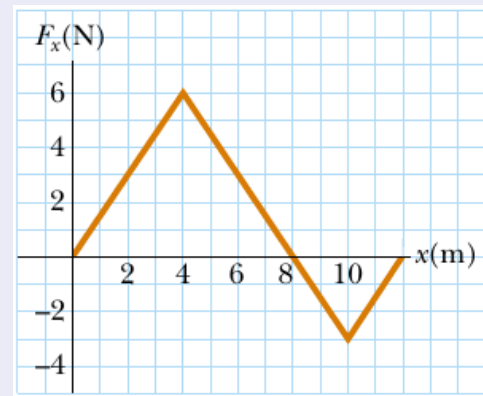


Class Activity, Cont'd

3. The force acting on a particle varies as in the figure. Find the work done by the force on the particle as it moves

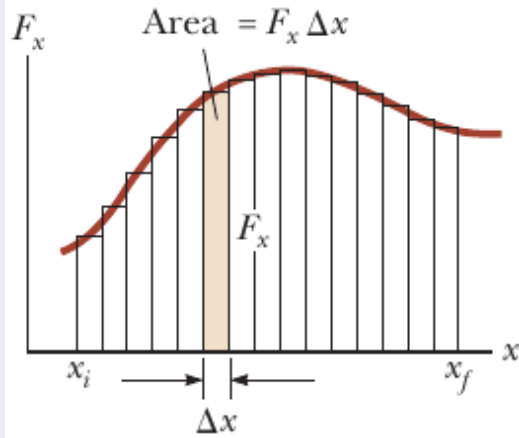
- (a) from $x = 0$ to $x = 8.00$ m,
- (b) from $x = 8.00$ m to $x = 10.0$ m,
- (c) from $x = 0$ to $x = 10.0$ m.

ans: a) 24.0J b) -3.00J c) 21.0J



Work Done by a Variable Force, Cont'd

Consider a particle being displaced along the x axis under the action of a force that varies with position.

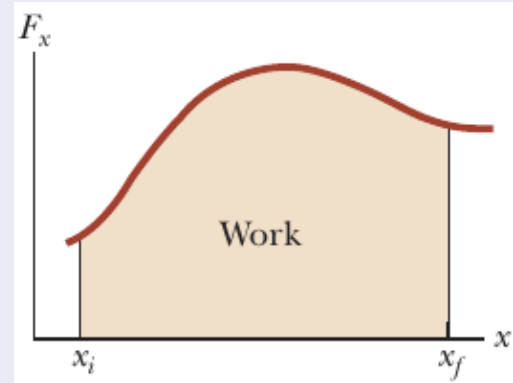


For a very small displacement Δx , F_x is approximately constant.

$$W \approx F_x \Delta x$$

The total work done

$$W \approx \sum^{x_f} F_x \Delta x$$



If the size of the small displacements is allowed to approach zero

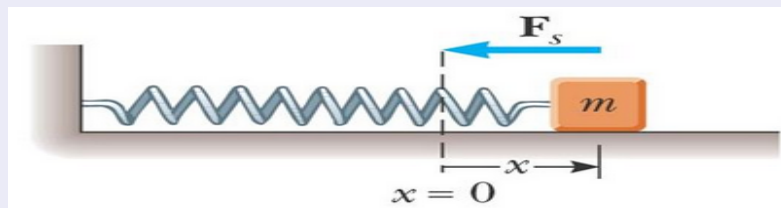
$$W = \lim_{\Delta x \rightarrow 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

Work Done by a Variable Force, Cont'd

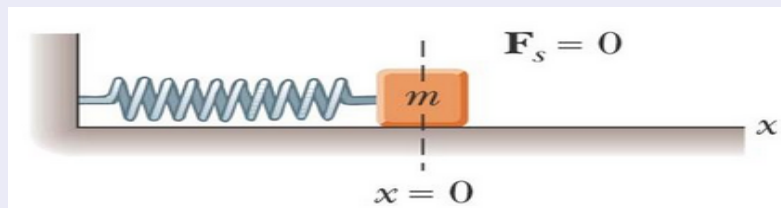


Example-1: Work Done By A Spring

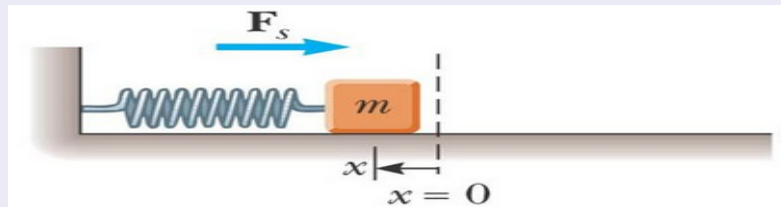
Consider the Mass-spring system where the force applied varies with position constantly. A block on a horizontal, frictionless surface is connected to a spring as shown in the figure below.



when $x > 0$ (stretched spring), the spring force is directed to the left, $F_s < 0, x > 0$



when $x = 0$ (natural length of the spring), the spring force is zero, $F_s = 0, x = 0$



when $x < 0$ (compressed spring), the spring force is directed to the right, $F_s > 0, x < 0$

Work Done by a Variable Force, Cont'd



Example-1: Work Done By A Spring, Cont'd

☞ Hooke's law states the restoring force exerted by the spring is

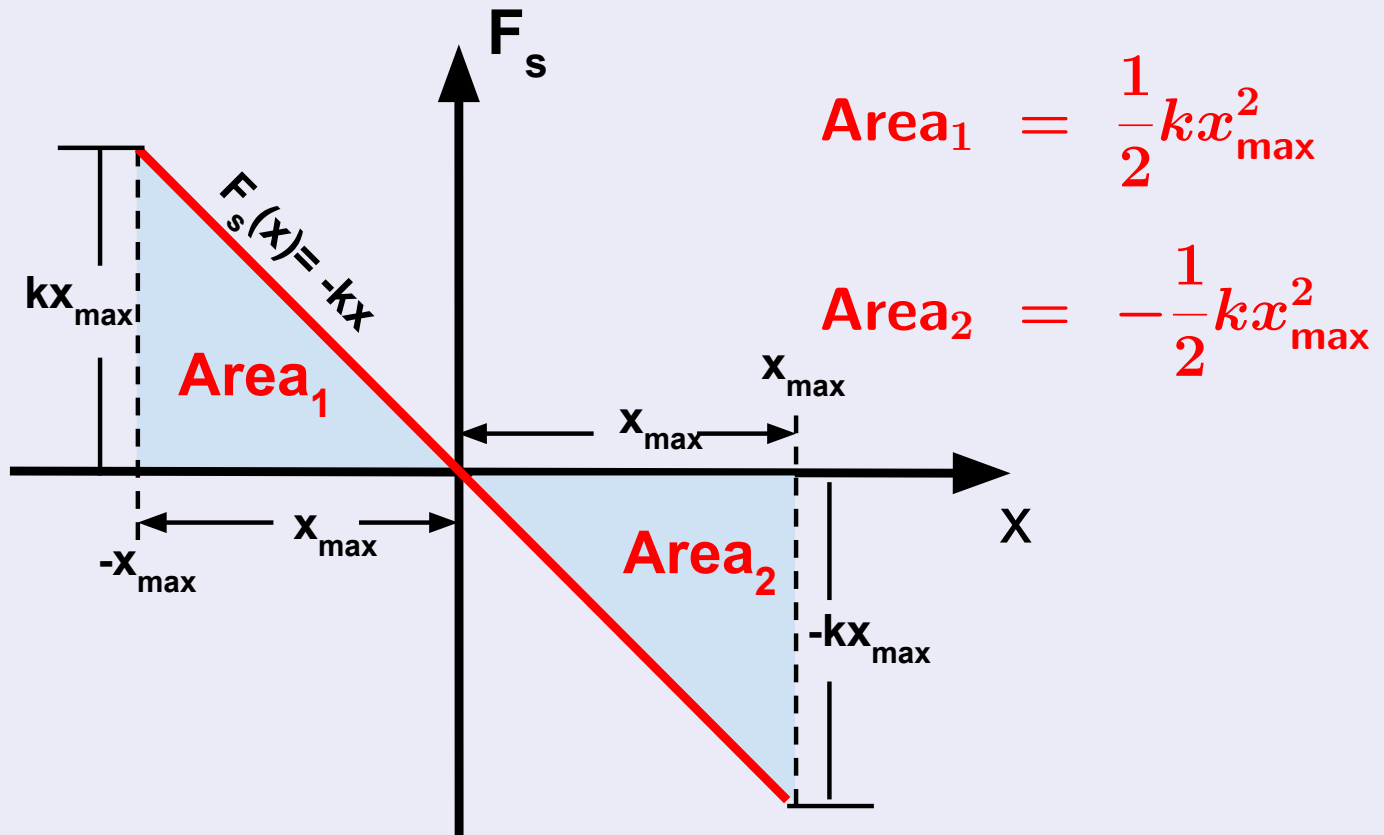
$$F_s = -kx$$

- x is the position of the block relative to its equilibrium
- k is the force constant or the spring constant of the spring. The value of k is a measure of the *stiffness* of the spring. Stiff springs have large k values, and soft springs have small k values.
- The negative sign implies the force exerted by the spring is always directed opposite to the displacement from equilibrium.

Work Done by a Variable Force, Cont'd



Example-1: Work Done By A Spring, Cont'd



Work Done by a Varying Force, Cont'd



Example-1: Work Done By A Spring, Cont'd

The work done by the spring force on the block as the block moves from $x_i = -x_{\max}$ to $x_f = 0$

$$W_s = \int_{x_i}^{x_f} F_s dx = \int_{-x_{\max}}^0 (-kx) dx = \frac{1}{2} kx_{\max}^2$$

☞ W_s is positive because the force is in the same direction as the displacement of the block (both are to the right).

As the block moves from $x_i = 0$ to $x_f = x_{\max}$, the work done by the spring force

$$W_s = -\frac{1}{2} kx_{\max}^2$$

☞ W_s is negative because the displacement is to the right and the spring force is to the left.

Work Done by a Varying Force, Cont'd



Example-1: Work Done By A Spring, Cont'd

Therefore, the net work done by the spring force as the block moves from $x_i = -x_{max}$ to $x_f = x_{max}$ is zero.

If the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$, the work done by the spring force on the block is

$$W_s = \int_{x_i}^{x_f} (-kx) dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Work Done by a Varying Force, Cont'd



Example-1: Work Done By A Spring, Cont'd

Consider the work done on the spring by an applied force that stretches the spring very slowly from $x_i = 0$ to $x_f = x_{max}$.

- the applied force F_{app} is equal in magnitude and opposite in direction to the spring force, $F_{app} = kx$
- the work done by this applied force on the mass-spring system is

$$W_{F_{app}} = \int_0^{x_{max}} F_{app} dx = \int_0^{x_{max}} kx dx = \frac{1}{2} kx_{max}^2$$

- The work done by an applied force on a mass-spring system between arbitrary positions of the block is

$$W_{F_{app}} = \int_{x_i}^{x_f} F_{app} dx = \int_{x_i}^{x_f} kx dx = \frac{1}{2} kx_f^2 - \frac{1}{2} kx_i^2$$

Work Done by a Variable Force, Cont'd



Example-2

- a) A person pulls on the spring in the above figure, stretching it 3.0 cm, which requires a maximum force of 75 N. How much work does the person do?
- b) If, instead, the person compresses the spring 3.0 cm, how much work does the person do?

Work Done by a Variable Force, Cont'd



Example-2: **Solution**

a) the spring constant

$$k = \frac{F_{max}}{x_{max}} = \frac{75N}{0.030m} = 2.5 \times 10^3 N/m$$

the work done by the person on the spring

$$W = \frac{1}{2} k x_{max}^2 = \frac{1}{2} (2.5 \times 10^3 N/m) (0.030m)^2 = 1.1J$$

b) The work done by the person is

$$W = \int_0^{-0.030m} F_x dx = \int_0^{-0.030m} kx dx = 1.1J$$

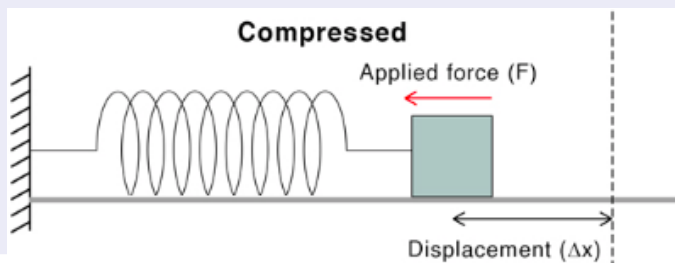
which is the same as for stretching it.

Work Done by a Variable Force, Cont'd



Example-3

2. An applied force $F_{app} = -5$ N is exerted on the block that is attached to the free end of the spring of the figure below. As a result of this force, the spring is compressed by 1 cm from its relaxed length.
- What is the spring constant of the spring?
 - What force does the spring exert on the block if the spring is compressed by 2.5 cm?
 - How much work does the spring force do on the block as the spring is compressed from the relaxed state by 2.5 cm?
 - How much work does the spring force do on the block during a total displacement starting from a compression of 2.5 cm, passing through the equilibrium, and then to a stretch of 2.0 cm?



Work Done by a Variable Force, Cont'd



Example-3: **Solution**

2. a) The applied force is equal in magnitude but opposite in direction.

$$k = -\frac{F_s}{x} = -\frac{5 \text{ N}}{(-1 \times 10^{-2} \text{ m})} = 500 \text{ N/m}$$

- b) Using Hooke's law,

$$F_s = -kx = -(500 \text{ N/m})(-2.5 \times 10^{-2} \text{ m}) = 12.5 \text{ N}$$

- c) the work done by the spring force on the block from $x_i = 0$ to $x_f = -2.5 \times 10^{-2} \text{ m}$ will be:

$$\begin{aligned} W_s &= \frac{1}{2}k x_i^2 - \frac{1}{2}k x_f^2 \\ &= 0 - \frac{1}{2}(500 \text{ N/m})(-2.5 \times 10^{-2} \text{ m})^2 \\ &= -0.156 \text{ J} \end{aligned}$$

Work Done by a Variable Force, Cont'd



Example-3: **Solution, Cont'd**

$$\begin{aligned} 2. \quad d) \quad W_s &= \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}k(x_i^2 - x_f^2) \\ &= \frac{1}{2}(500)[(-2.5 \times 10^{-2})^2 - (2.0 \times 10^{-2})^2] \\ &= 0.06 \text{ J} \end{aligned}$$

Work Done by a Variable Force, Cont'd



Example-4: **Solution, Cont'd**

2. A force $F = (2x + 5)\text{N}$ acts on a particle along the displacement. Find the work done by the force during the displacement of the particle from $x_0 = 0\text{m}$ to $x = 2\text{m}$.

$$\begin{aligned} W &= \int_0^2 F_x dx = \int_0^2 (2x + 5) dx \\ &= \left[\frac{2x^2}{2} + 5x \right]_0^2 \\ &= \left[\frac{2(2)^2}{2} + 5(2) \right] \\ &= 14J \end{aligned}$$

Work Done by a Variable Force, Cont'd



Class Activity

1. A perfectly elastic spring requires 0.54 J of work to stretch 6 cm from its equilibrium position, as in the above figure.

a) What is its spring constant k ?

ans: $k=3 \text{ N/cm}$

b) How much work is required to stretch it an additional 6 cm?

ans: 1.62 J

2. Calculate the work performed by each of the following forces in the specified domain.

(a) $F(x) = (6x^2 - 2x)N$, $x \in (0, 5m)$,

ans: 225 J

(b) $F = -\frac{1.3 \times 10^{22}}{x^2}$, $x \in (1.5 \times 10^{11}, 2.3 \times 10^{11})m$,

ans: $-3.0 \times 10^{10} J$

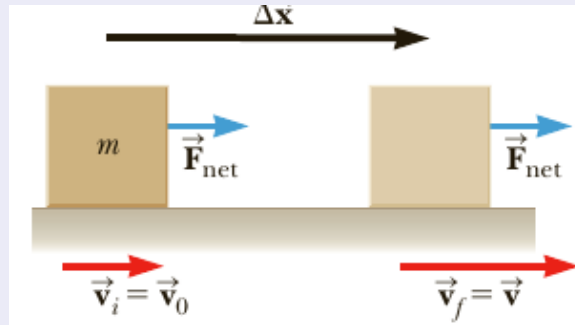
(c) $F(x) = F_0 \left(1 + \frac{1}{6} \frac{x^2}{x_0^2} \right)$, $F_0 = 2.0 \text{ N}$, $x_0 = 0.0070 \text{ m}$, and
 $x \in (0.01, 0.05)m$

ans: 0.36 J

Work-Energy theorem



Let a force F is applied on an object initially moving with velocity v_i .



If the object is displaced by Δx , the work done by $\vec{F}$ on the object is

$$W_{\text{net}} = \vec{F}_{\text{net}} \Delta x = (ma) \Delta x$$

an object undergoes constant acceleration:

$$v_f^2 = v_i^2 + 2a\Delta x \quad \text{or} \quad a\Delta x = \frac{v_f^2 - v_i^2}{2}$$

Work-Energy theorem, Cont'd



Substituting into the first equation

$$W_{\text{net}} = m \left(\frac{v_f^2 - v_i^2}{2} \right)$$

Therefore,

$$W_{\text{net}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The kinetic energy KE of an object of mass m moving with a speed v is

$$KE = \frac{1}{2}mv^2$$

Thus, the net work done on an object is:

$$W_{\text{net}} = KE_f - KE_i = \Delta KE$$

Work-kinetic energy theorem

“In the case in which work is done on a system and the only change in the system is in its speed, the work done by the net force equals the change in kinetic energy of the system.”

Work-Energy theorem, Cont'd



Example

A 6.0-kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3.0 m.

$$W = Fd = (12N)(3.0m) = 36J$$

$$W = \Delta KE$$

$$W = \frac{1}{2}mv_f^2 - 0$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(36)}{6}} = 3.5m/s$$

Work-Energy theorem, Cont'd



Example

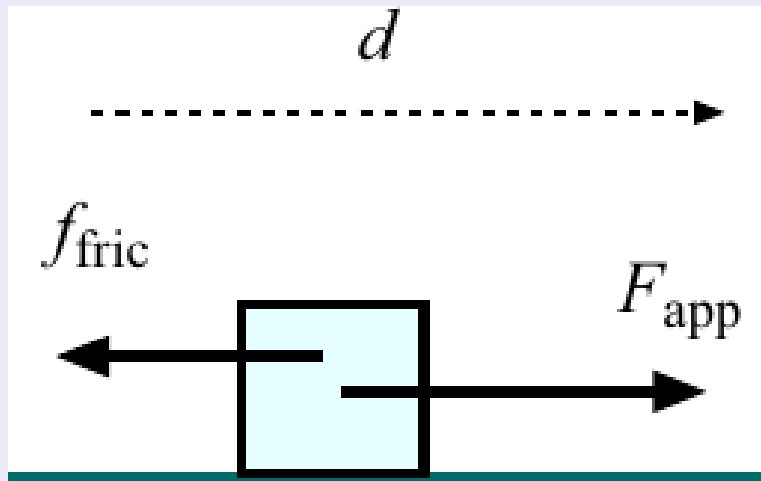
A 40 kg box initially at rest is pushed 5.0 m along a rough horizontal floor with a constant applied horizontal force of 130 N. If the coefficient of friction between the box and floor is 0.30, find

- a) the work done by the applied force,
- b) the energy lost due to friction,
- c) the change in kinetic energy of the box, and
- d) the final speed of the box.

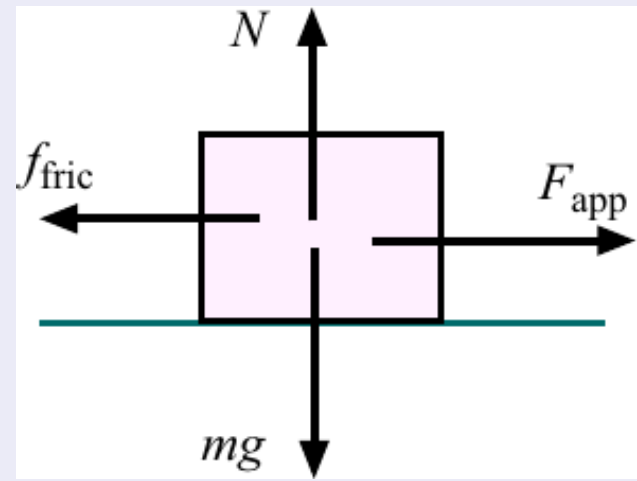
Work-Energy theorem, Cont'd



Example



Pictorial representation



Free-body diagram

Work-Energy theorem, Cont'd



Solution

a) the work done by the applied force,

$$W = Fd \cos \theta = (130N)(5.0m) \cos 0^\circ = 650J$$

b) the energy lost due to friction,

$$\begin{aligned} f_{fric} &= \mu mg = (0.30)(40kg)(9.80m/s^2) = 120N \\ W_{fric} &= Fd \cos \theta \\ &= f_{fric}d \cos(180^\circ) \\ &= (120N)(5.0m)(-1) \\ &= -590J \end{aligned}$$

590J of energy is lost due to friction.

Work-Energy theorem, Cont'd



Solution

- c) the change in kinetic energy of the box,
the net work done

$$W_{net} = W_c + W_{nc} = 650J - 590J = 60J$$

work-kinetic energy theorem

$$\Delta KE = W_{net} = 60J$$

- d) the final speed of the box.

$$\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2; \quad v_i = 0$$

$$v_f = \sqrt{\frac{2\Delta KE}{m}} = 1.8m/s$$

Work-Energy theorem, Cont'd



Example

A 5.00-kg package slides 1.5 m down a long ramp that is inclined at 12° below horizontal. The coefficient of kinetic friction between the package and the ramp is $\mu_k = 0.31$. Calculate

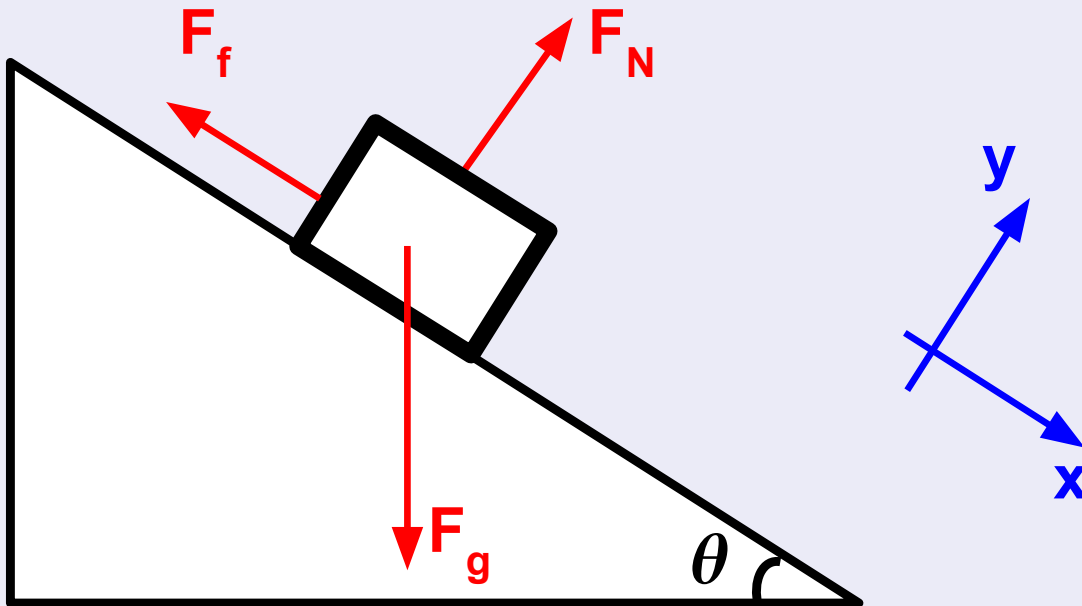
- the work done on the package by friction
- the work done on the package by the gravity
- the work done by the normal force
- the total work done on the package
- If the package had has a speed of 2.20 m/s at the top of the ramp, what is its speed after sliding 1.5 m down the ramp?

Work-Energy theorem, Cont'd



Example

Free-body diagram



Work-Energy theorem, Cont'd



Example

a) the work done on the package by friction

$$\begin{aligned}f_k &= \mu_k mg \cos \theta \\&= (0.31)(5.00)(9.8) \cos(12^\circ) \\&= 14.7N\end{aligned}$$

$$\begin{aligned}W_f &= f_k d \cos(180^\circ) \\&= (14.7)(1.50)(-1) \\&= -22.3J\end{aligned}$$

b) the work done on the package by the gravity

$$\begin{aligned}F_g &= mg = (5.00)(9.80) = 49.0N \\W_g &= Fd \cos \theta = F_g d \cos(78^\circ) = (49.0)(1.50) \cos(78^\circ) = 15.3J\end{aligned}$$

Work-Energy theorem, Cont'd



Example

c) the work done by the normal force

$$W_n = Fd \cos \theta = F_n d \cos(90^\circ) = 0J$$

d) the total work done on the package

$$\begin{aligned} W_{net} &= W_f + W_g + W_n \\ &= -22.3J + 15.3J + 0J \\ &= -7.0J \end{aligned}$$

e) If the package had has a speed of 2.20 m/s at the top of the ramp, what is its speed after sliding 1.5 m down the ramp?

From the work-energy theorem,

$$\Delta KE = W_{net} = -7.0J$$

Work-Energy theorem, Cont'd



Example

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_{net}$$

The speed at the end of the ramp

$$\begin{aligned} v_f &= \sqrt{v_i^2 + 2W/m} \\ &= \sqrt{2.2^2 + (2)(-7/5)} \\ &= 1.4m/s \end{aligned}$$

Work-Energy theorem, Cont'd



Class Activity

A horizontal spring has spring constant $k = 360 \text{ N/m}$.

- How much work is required to compress it from its uncompressed length ($x = 0$) to $x = 11.0 \text{ cm}$?
- If a 1.85-kg block is placed against the spring and the spring is released, what will be the speed of the block when it separates from the spring at $x = 0$? Ignore friction.
- Repeat part (b) but assume that the block is moving on a table and friction force of $F_f = 7.0 \text{ N}$ is acting to slow it down.

Class Activity, **Solution**

- a) The work needed to compress the spring a distance $x = 0.110$ m is

$$W = \frac{1}{2}(360 \text{ N/m})(0.110 \text{ m})^2 = 2.18 \text{ J},$$

- b) Using the work-kinetic energy theorem and noting that the initial kinetic energy is zero, we obtain

$$W = K_f - K_i = \frac{1}{2}mv_f^2 - 0$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(2.18)}{1.85}} = 1.54 \text{ m/s}$$

Work-Energy theorem, Cont'd



Class Activity, **Solution**

b) The work done by the friction

$$W_f = F_f x = -(7.0)(0.110) = -0.77J$$

This work is negative because the drag force acts in the direction opposite to the displacement x .

The net work done on the block is

$$W_{net} = W_a + W_f = 2.18 - 0.77 = 1.41J$$

Using, work-energy theorem

$$v_f = \sqrt{\frac{2W_{net}}{m}} = \sqrt{\frac{2(1.41)}{1.85}} = 1.23m/s$$

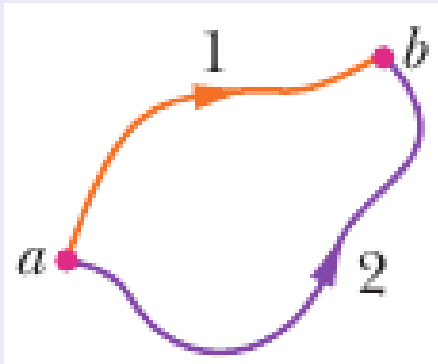
Conservative and Nonconservative Forces



Conservative Forces

a force is said to be **conservative force** if the work done by the force on an object moving from one point to another depends only on the initial and final positions of the object, and **is independent of the particular path taken**.

- ☞ The **work done** by a **conservative force** on a particle moving through any **closed path is zero**.



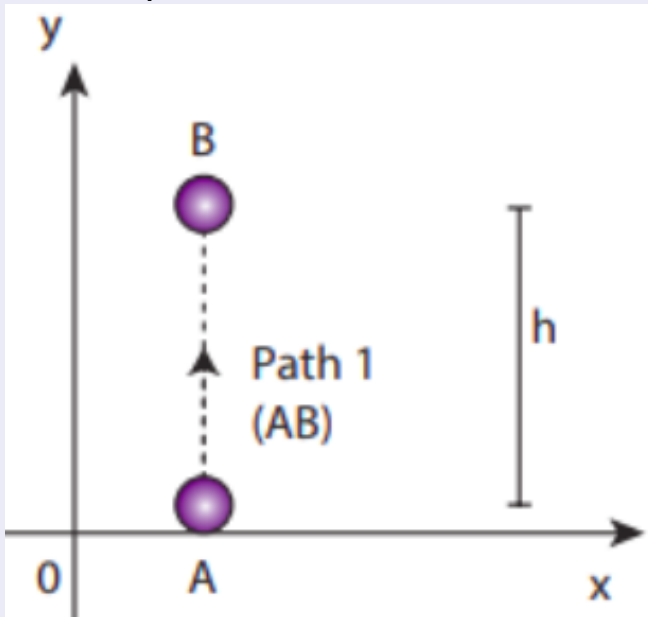
As the object does a work of W in moving from a to b , then it does a work of $-W$ from b to a . Hence, the total work done in going from a to b and back to a is $W + (-W) = 0$

Conservative and Nonconservative Forces, Cont'd

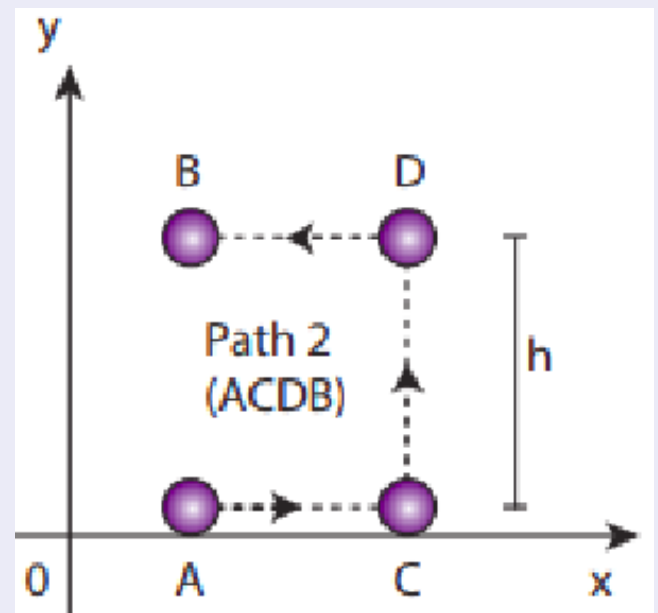


Conservative Forces, Example

Suppose an object of mass m is lifted upward to a height of h through two different paths as shown below.



Path-1



Path-2

Conservative and Nonconservative Forces, Cont'd



Conservative Forces, Example, Cont'd

The work done by the force along path 1.

$$W_{\text{path 1}} = \int_A^B \vec{F} \cdot d\vec{r} = \int_A^B (-mg\hat{j}) \cdot (dy\hat{j}) = -mg \int_0^h dy = -mgh$$

Total work done for path 2 is

$$\begin{aligned} W_{\text{path 2}} &= \int_A^B \vec{F} \cdot d\vec{r} \\ &= \int_A^C \vec{F} \cdot d\vec{r} + \int_C^D \vec{F} \cdot d\vec{r} + \int_D^B \vec{F} \cdot d\vec{r} \\ &= \int_A^B (-mg\hat{j}) \cdot (dx\hat{i}) + \int_C^D (-mg\hat{j}) \cdot (dy\hat{j}) + \int_A^B (-mg\hat{j}) \cdot (-dx\hat{i}) \\ &= 0 + -mg \int_0^h dy + 0 \\ &= -mgh \end{aligned}$$

Conservative Forces, Example, Cont'd

Therefore, the work done by the conservative force is independent of the path:

$$W_{\text{path 1}} = W_{\text{path 2}}$$

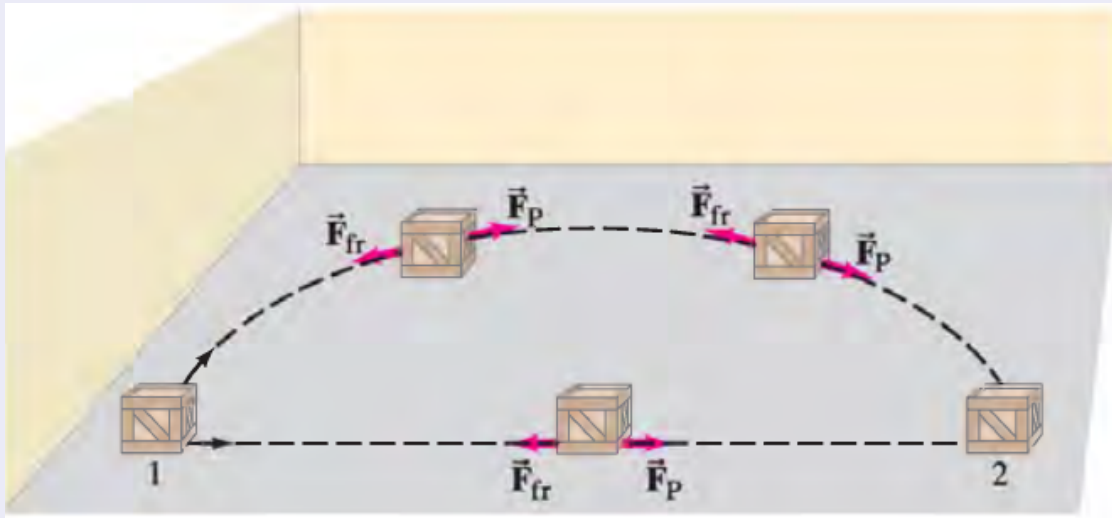
Therefore, the force of gravity is conservative!

- ☞ Other examples of conservative forces are: **Spring force, Electrostatic force, Magnetic force, etc**

Nonconservative Forces

Nonconservative forces do path dependent work and thus they cause a change in the mechanical energy and internal energy of the system.

- ☞ For example, if a crate is pushed from point 1 to point 2 along the longer semicircular path, you do more work against friction than if you push it along the straight path.

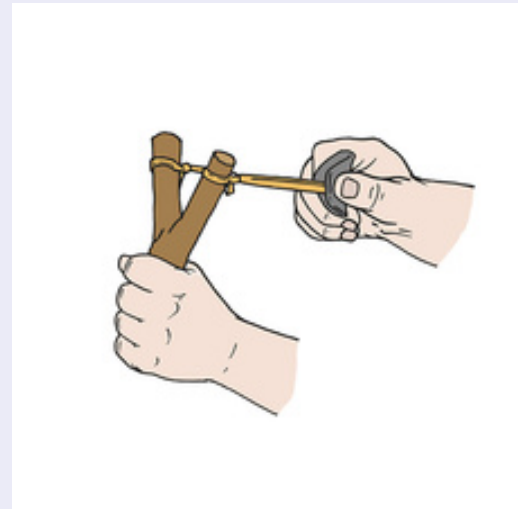
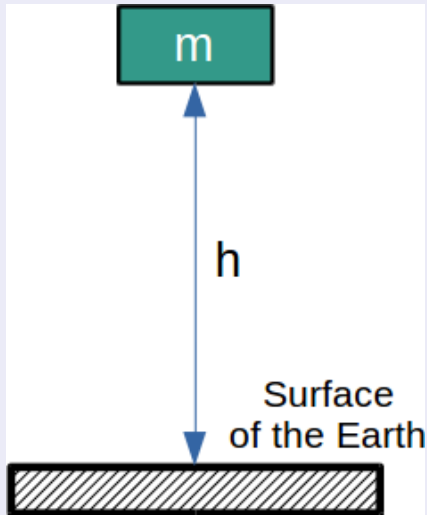


Examples:
Friction,
Air resistance,
rocket propul-
sion, etc

Potential Energy

The work done by conservative forces is stored as **potential energy** of the object.

- **Potential energy** refers to the energy possessed by an object due to its position, condition or configuration.
 - ☞ it is the amount of energy contained in an object at rest
- Eg: **gravitational potential energy** and **elastic potential energy**



work-potential energy theorem states that the work done by conservative forces on an object is equal to the change in its potential energy.

$$W_c = -(U_f - U_i) = -\Delta U$$

- ☞ any conservative field force performs a work at the expense of its potential energy.
- ☞ **Sign convention:** when work is done on an object by a conservative force, it is considered **negative** and it indicates energy is being transferred to the object or stored as potential energy.
- *For example, when an object falls, gravity does positive work on it and the gravitational potential energy decreases. Similarly, when an object is lifted, gravity does negative work and the gravitational potential energy increases.*

Potential Energy

Gravitational Potential Energy

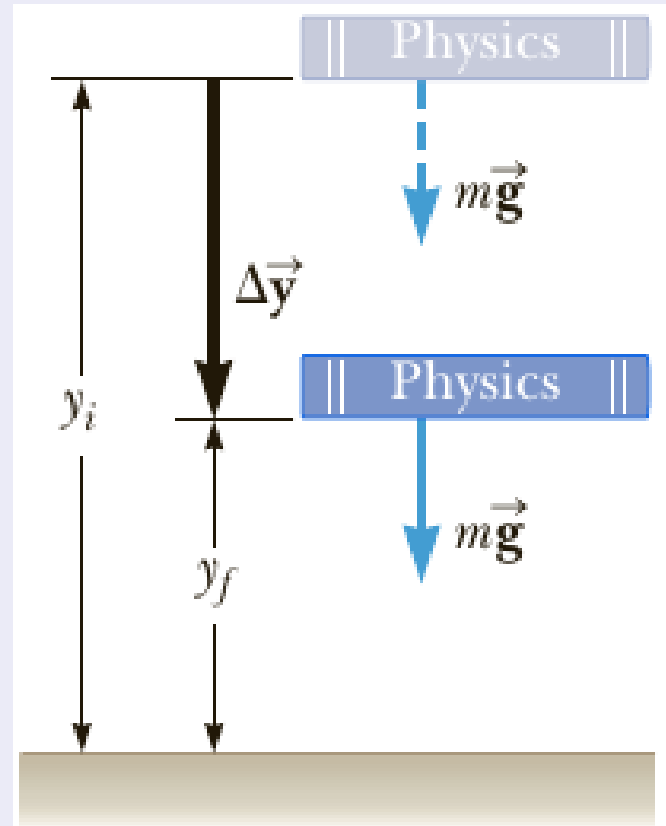
Suppose a book of mass m falls from a height y_i to a height y_f ,

$$\begin{aligned} W_g &= \vec{F} \cdot \Delta \vec{r} \\ &= (-mg\hat{j}) \cdot [-(y_i - y_f)\hat{j}] \\ &= -mg(y_f - y_i) \end{aligned}$$

The gravitational PE at any point near the surface of the Earth a vertical height y above some reference point:

$$U_g = mgy$$

Therefore, $W_g = -\Delta U_g$



Gravitational Potential Energy, Example

A book of mass 1.2 kg is on a horizontal table that is 1.5 m high. The book and the table are in a room of height 2.8 m.

- a) What is the gravitational PE of the book if the level of zero PE is taken to be
 - i) at the table's surface,
 - ii) at the floor, and
 - iii) at the ceiling?
- b) When the book drops to the floor, calculate ΔU for the three choices of the zero PE.

Gravitational Potential Energy, Example, Solution

a) Using Equation 7.4 we have

i) if the level of zero PE is the surface of the table,

$$y = 0, \quad U_g = 0$$

ii) if the level of zero PE is the floor,

$$y = 1.5m, \quad U_g = 17.67J$$

iii) if the level of zero PE is taken to be at the ceiling.

$$y = -(2.8 - 1.5) = -1.3m, \quad U_g = -15.29J$$

Gravitational Potential Energy, Example, Solution

b) We know that

$$\Delta U = mg(y_f - y_i) = mg\Delta y$$

Thus,

- i) $\Delta y = -1.5 - 0 = -1.5m$,
- ii) $\Delta y = 0 - 1.5 = -1.5m$, and
- iii) $\Delta y = -2.8 - (-1.3) = -1.5m$.

So for all the choices we obtain the same result

$$\Delta U = (1.2)(9.8)(-1.5) = -17.64J$$

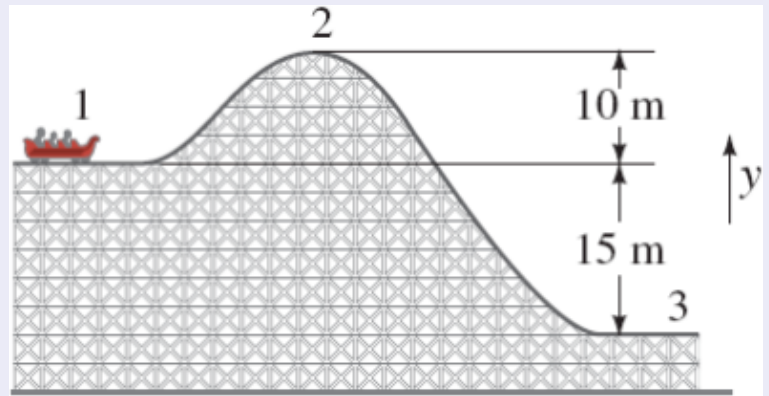
Potential Energy



Gravitational Potential Energy, Example-2

A 1000-kg roller-coaster car moves from point 1 to point 2 and then to point 3.

- (a) What is the gravitational PE at points 2 and 3 relative to point 1? That is, take $y = 0$ at point 1.
- (b) What is the change in PE when the car goes from point 2 to point 3?
- (c) Repeat (a) & (b), when the reference point ($y = 0$) is at point 3.



Potential Energy



Gravitational Potential Energy, Example-2, Solution

(a) We measure heights from point 1: $y_1 = 0$, $U_1 = 0$

At point 2,

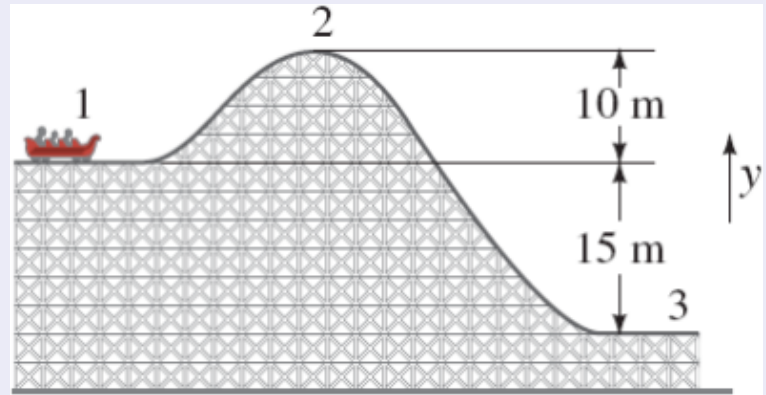
$$y_2 = 10m$$

$$\begin{aligned} U_2 &= mgy_2 \\ &= (1000)(9.8)(10) \\ &= 9.8 \times 10^4 J \end{aligned}$$

At point 3 ,

$$y_3 = -15m$$

$$U_3 = -1.5 \times 10^5 J$$



$$(b) U_3 - U_2 = -1.5 \times 10^5 J - 9.8 \times 10^4 J = -2.5 \times 10^5 J$$

Gravitational Potential Energy, Example-2, Solution

(c) We measure heights from point 3: $y_3 = 0$, $U_3 = 0$

At point 1,

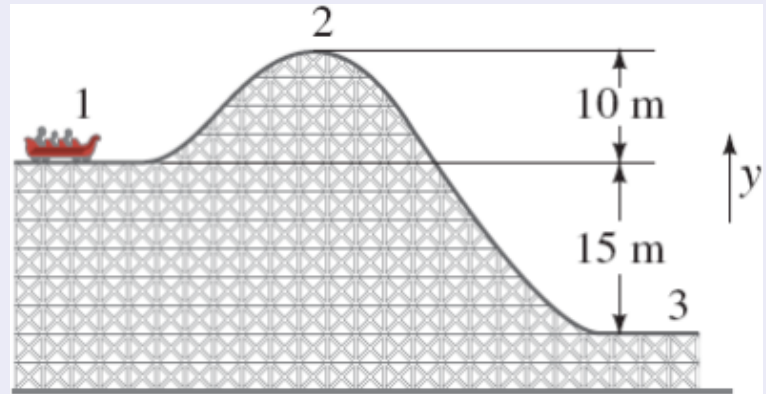
$$y_1 = +15m$$

$$U_1 = 1.5 \times 10^5 J$$

At point 2 ,

$$y_2 = +25m$$

$$U_2 = 2.5 \times 10^5 J$$



The change in potential energy going from point 2 to point 3 is

$$U_3 - U_2 = 0 - 2.5 \times 10^5 J = -2.5 \times 10^5 J$$

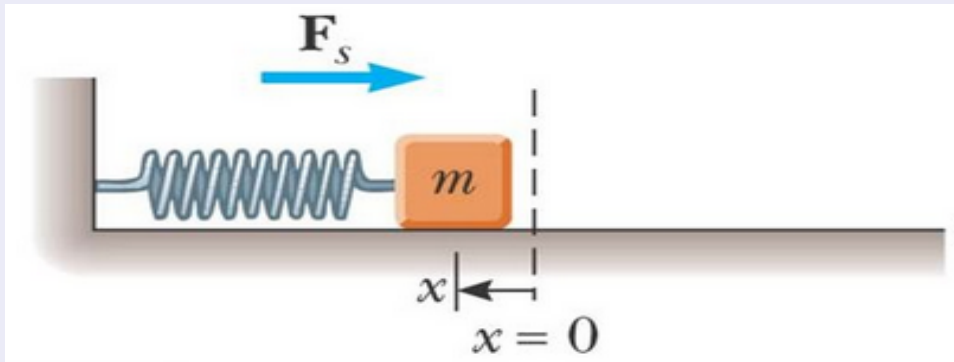
Changes in gravitational potential energy is path independent!

Potential Energy



Elastic Potential Energy

Consider a block attached to a spring with a spring constant k as in the figure below.



As the block moves from position x_i to position x_f the spring force does work on the block.

Potential Energy



Elastic Potential Energy

The corresponding change in elastic potential energy of the block-spring system,

$$\Delta U_s = - \int_{x_i}^{x_f} (-kx) dx = \frac{1}{2} kx_f^2 - \frac{1}{2} kx_i^2$$

It is usually convenient to choose the potential energy at $x = 0$ to be zero: $U(0) = 0$, so the potential energy of a spring compressed or stretched an amount x from equilibrium is

$$U_s = \frac{1}{2} kx^2$$

Elastic Potential Energy, Example

A system contains a perfectly elastic spring, with an unstretched length of 20 cm and a spring constant of 4 N/cm.

- a) How much elastic potential energy does the spring contribute when its length is 23 cm?
- b) How much more potential energy does it contribute if its length increases to 26 cm?

Elastic Potential Energy, Example, **Solution**

- a) The displacement of the spring is $x = 23 \text{ cm} - 20 \text{ cm} = 3 \text{ cm}$, so the contributed potential energy is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(4 \text{ N/cm})(3 \text{ cm})^2 = 0.18 \text{ J}.$$

- b) When the spring's displacement is $x = 26 \text{ cm} - 20 \text{ cm} = 6 \text{ cm}$, the potential energy is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(4 \text{ N/cm})(6 \text{ cm})^2 = 0.72 \text{ J}$$

which is a 0.54-J increase over the amount in part (a).

Conservation of Mechanical Energy



In the presence of both conservative and nonconservative forces,

$$W_{net} = W_c + W_{nc}$$

The work-energy theorem

$$W_c + W_{nc} = \Delta KE$$

The work done by gravity is the same as the negative of the change in gravitational potential energy.

$$W_g = -(U_f - U_i)$$

Thus, the work-energy theorem

$$W_{nc} = (K_f - K_i) + (U_f - U_i)$$

Conservation of Mechanical Energy



For an object falling only under the influence of gravity, nonconservative forces are absent thus, $W_{nc} = 0$. Thus,

$$K_i + U_i = K_f + U_f$$

In any isolated system of objects interacting only through conservative forces, the total mechanical energy,

$$E = K + U$$

and the total mechanical energy of the system is conserved

$$\Delta E = 0$$

Conservation of Mechanical Energy



For the object of mass m falling in a gravitational field

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

On the other hand, in the presence of friction, the change in mechanical energy is equal to the work done by friction. That is;

$$\Delta E = W_f$$

where W_f is work done by friction

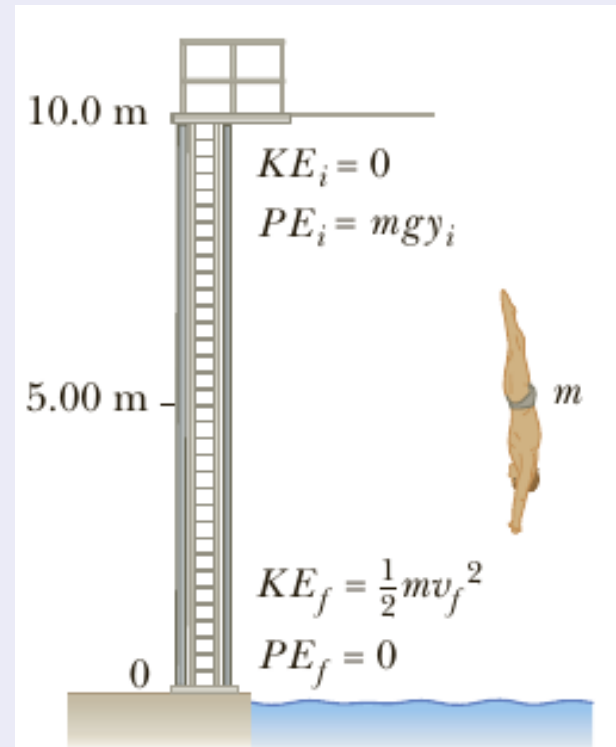
Conservation of Mechanical Energy



Example: **Falling diver**

A diver of mass m drops from a board 10.0 m above the water's surface, as in the Figure. Neglect air resistance.

- a) Use conservation of mechanical energy to find his speed 5.00 m above the water's surface.
- b) Find his speed as he hits the water.



Conservation of Mechanical Energy



Example: **Falling diver**

- a) Use conservation of mechanical energy to find his speed 5.00 m above the water's surface.

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

$$0 + gy_i = \frac{1}{2}v_f^2 + gy_f$$

$$\begin{aligned} v_f &= \sqrt{2g(y_i - y_f)} = \sqrt{2(9.80)(10.0 - 5.00)} \\ &= 9.90m/s \end{aligned}$$

- b) Find his speed as he hits the water.

$$0 + mgy_i = \frac{1}{2}mv_f^2 + 0$$

$$v_f = \sqrt{2gy_i} = \sqrt{2(9.80)(10.0)} = 14m/s$$

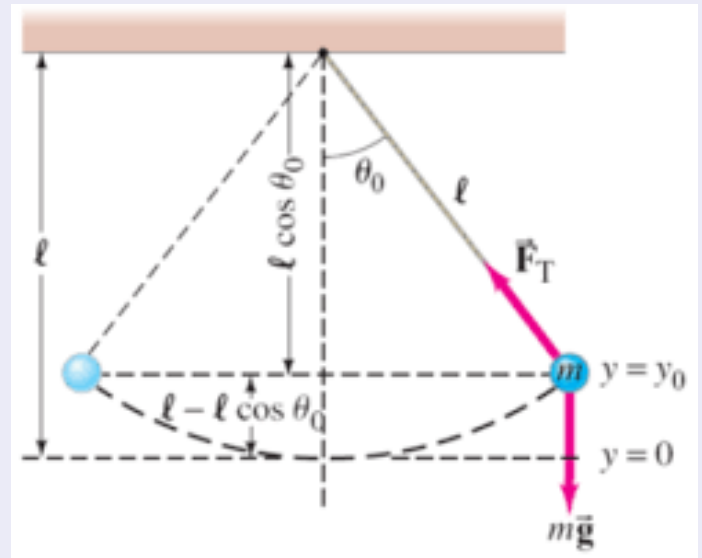
Conservation of Mechanical Energy



Example: **A swinging pendulum**

The simple pendulum shown in the figure consists of a small bob of mass m suspended by a massless cord of length ℓ . The bob is released (without a push) at $t = 0$, where the cord makes an angle $\theta = \theta_0$ to the vertical,

- Determine the speed of the bob at the lowest point of the swing.
- Find the tension in the cord, F_T . Ignore friction and air resistance.



Conservation of Mechanical Energy



Example: **A swinging pendulum**

$$\text{a) } \frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

$$0 + gy_0 = \frac{1}{2}v_f^2 + 0$$

$$g(\ell - \cos \theta_0) = \frac{1}{2}v_f^2$$

$$v_f = \sqrt{2g(\ell - \cos \theta_0)}$$

$$\text{b) } m\frac{v^2}{F} = F_T - mg \cos \theta$$

$$F_T = m\left(\frac{v^2}{\ell} + g \cos \theta\right)$$

$$= 2mg(\cos \theta - \cos \theta_0) + mg \cos \theta$$

$$= (3 \cos \theta - 2 \cos \theta_0)mg$$

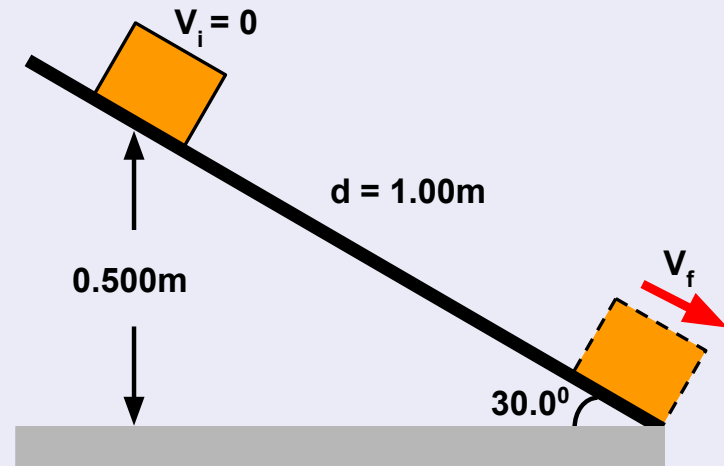
Conservation of Mechanical Energy



Example

A 3.00-kg crate slides down a ramp. The ramp is 1.00m in length and inclined at an angle of 30.0° , as shown in the figure below. The crate starts from rest at the top, experiences a constant friction force of magnitude 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp.

- Use energy methods to determine the speed of the crate at the bottom of the ramp.
- How far does the crate slide on the horizontal floor if it continues to experience a friction force of magnitude 2.00N ?



Conservation of Mechanical Energy



Example

- a) Use energy methods to determine the speed of the crate at the bottom of the ramp.

the mechanical energy of system when the crate is at the top

$$E_i = K_i + U_i = \frac{1}{2}mv_i^2 + mgy_i = 0 + mgy_i = mgy_i$$

the mechanical energy of system when the crate is at the bottom

$$E_f = K_f + U_f = \frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_f^2 + 0 = \frac{1}{2}mv_f^2$$

the change in mechanical energy of the system

$$\Delta E = E_f - E_i = \frac{1}{2}mv_f^2 - mgy_i$$

Conservation of Mechanical Energy



Example

in the presence of non-conservative force, i.e, friction force, the change in mechanical energy of the system

$$\Delta E = W_f$$

where W_f is the work done by the friction force which is given by

$$W_f = Fd \cos \theta = f_k d \cos(180) = -f_k d$$

Hence, equating the above equations, we get

$$\frac{1}{2}mv_f^2 - mgy_i = -f_k d$$

Therefore, the final speed becomes

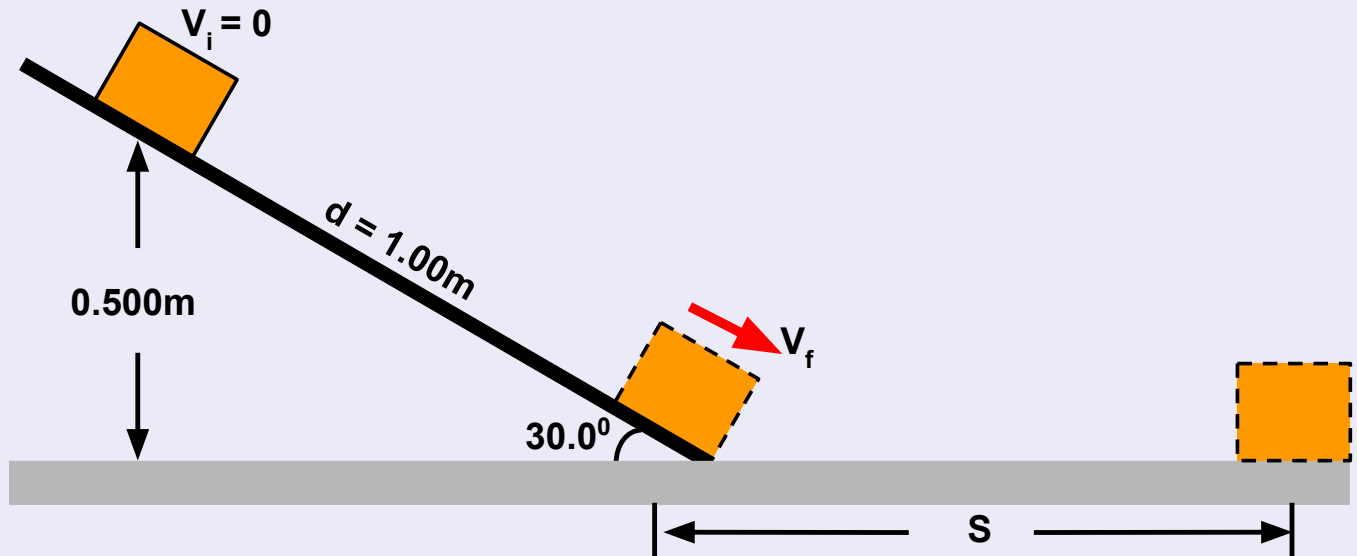
$$v_f = \sqrt{\frac{2}{m} (mgy_i - f_k d)} = 2.54 \text{ m/s}$$

Conservation of Mechanical Energy



Example

- b) How far does the crate slide on the horizontal floor if it continues to experience a friction force of magnitude 2.00N ?



Conservation of Mechanical Energy



Example

The mechanical energy of the system when the crate leaves the bottom of the ramp

$$E'_i = K_f + U_f = \frac{1}{2}mv_f^2 + 0 = \frac{1}{2}mv_f^2$$

The system comes to rest after sliding some distance, so that it experiences no mechanical energy

$$E'_f = 0$$

Hence, the change in mechanical energy becomes

$$\Delta E' = E'_f - E'_i = -\frac{1}{2}mv_f^2$$

Conservation of Mechanical Energy



Example

in the presence of non-conservative force

$$\Delta E' = -f_k d'$$

Equating the two equations

$$\begin{aligned} -\frac{1}{2}mv_f^2 &= -f_k d' \\ d' &= \frac{mv_f^2}{2f_k} \end{aligned}$$

When a work ΔW is done during a time interval Δt , average power P_{av}

$$P_{av} = \frac{\Delta W}{\Delta t}$$

the average power in terms of the average velocity

$$P_{av} = \frac{\Delta W}{\Delta t} = \frac{F \Delta x}{\Delta t} = F v_{av}$$

Instantaneous power P is the limit of this expression as $\Delta t \rightarrow 0$

$$P = Fv \quad v \text{ is instantaneous velocity}$$

We can also express in terms of the scalar product:

$$P = \vec{F} \cdot \vec{v}$$

The SI unit of power is the joule per second (J/s), also called the watt, named after James Watt:

$$1W = 1J/s = 1kg \cdot m^2/s^3$$

The unit of power in the U.S. customary system is the horsepower (hp), where

$$1hp = 746W$$

1kWh is the energy transferred in 1 h at the constant rate of $1kW = 1000J/s$.

$$1kWh = (10^3W)(3600s) = 3.6 \times 10^6J$$

Example

A 1500-kg car accelerates uniformly from rest to a speed of 10 m/s in 3 s. Find

- a) the work done on the car in this time,
- b) the average power delivered by the engine in the first 3 s,
- c) the instantaneous power delivered by the engine at $t=2$ s.

Example

Solution: After 3 s, $v_f = 10 \text{ m/s}$, $m = 1500 \text{ kg}$

a) the work done is given by

$$\begin{aligned} W &= \frac{1}{2}mv_f^2 - 0 \\ &= \frac{1}{2} \times 1500 \times (10)^2 \\ &= 7.50 \times 10^4 \text{ J} \end{aligned}$$

b) the average power delivered by the engine in the first 3 s,

$$\begin{aligned} \bar{P} &= \frac{7.5 \times 10^4}{3} \\ &= 2.5 \times 10^4 \text{ W} \end{aligned}$$

Example

c) let us find the acceleration a , from $v = v_o + at$, we have,

$$a = \frac{v - v_o}{t} = \frac{10 - 0}{3} = 3.33 \text{ m/s}^2.$$

The velocity at $t = 2$ s, is then

$$v = 0 + 3.33 \times 2 = 6.66 \text{ m/s}$$

and the force of the engine is calculated using Newton's second law

$$F = ma = 1500 \times 3.33 = 5000 \text{ N}.$$

Now, the instantaneous power is

$$P = Fv = 5000 \times 6.66 = 3.33 \times 10^4 \text{ W}.$$