

CHAPTER

6

VECTOR MECHANICS FOR ENGINEERS:
STATICS

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Analysis of Structures

Vector Mechanics for Engineers: Statics

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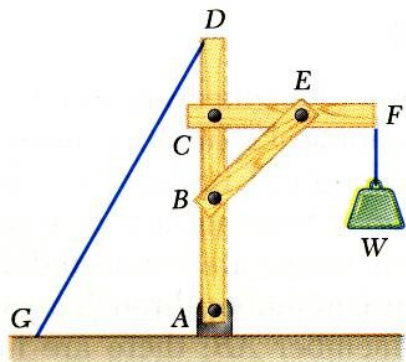
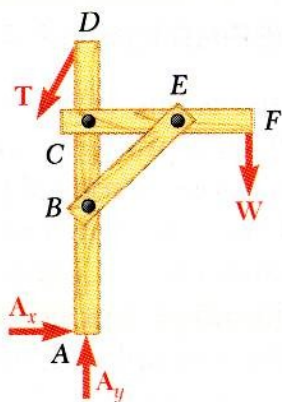
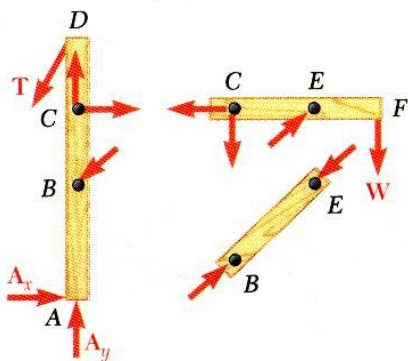
Sample Problem 6.4

Machines



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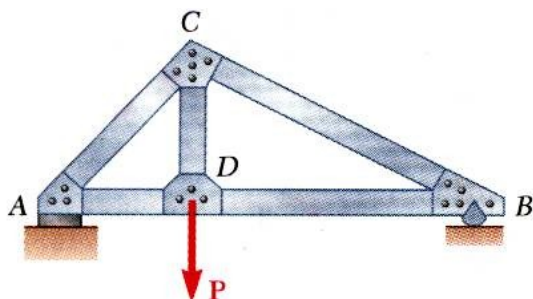
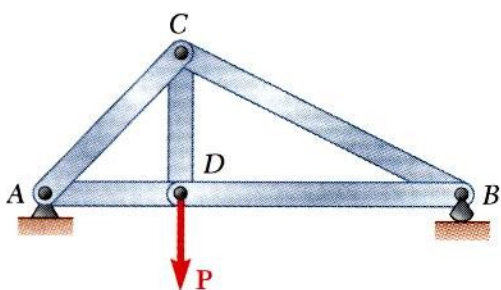
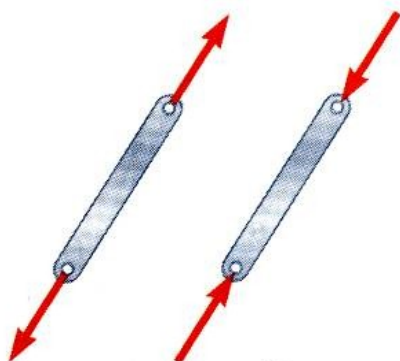
Introduction



- For the equilibrium of structures made of several connected parts, the *internal forces* as well the *external forces* are considered.
- In the interaction between connected parts, Newton's 3rd Law states that the *forces of action and reaction* between bodies in contact have the same magnitude, same line of action, and opposite sense.
- Three categories of engineering structures are considered:
 - a) *Frames*: contain at least one one multi-force member, i.e., member acted upon by 3 or more forces.
 - b) *Trusses*: formed from *two-force members*, i.e., straight members with end point connections
 - c) *Machines*: structures containing moving parts designed to transmit and modify forces.

Vector Mechanics for Engineers: Statics

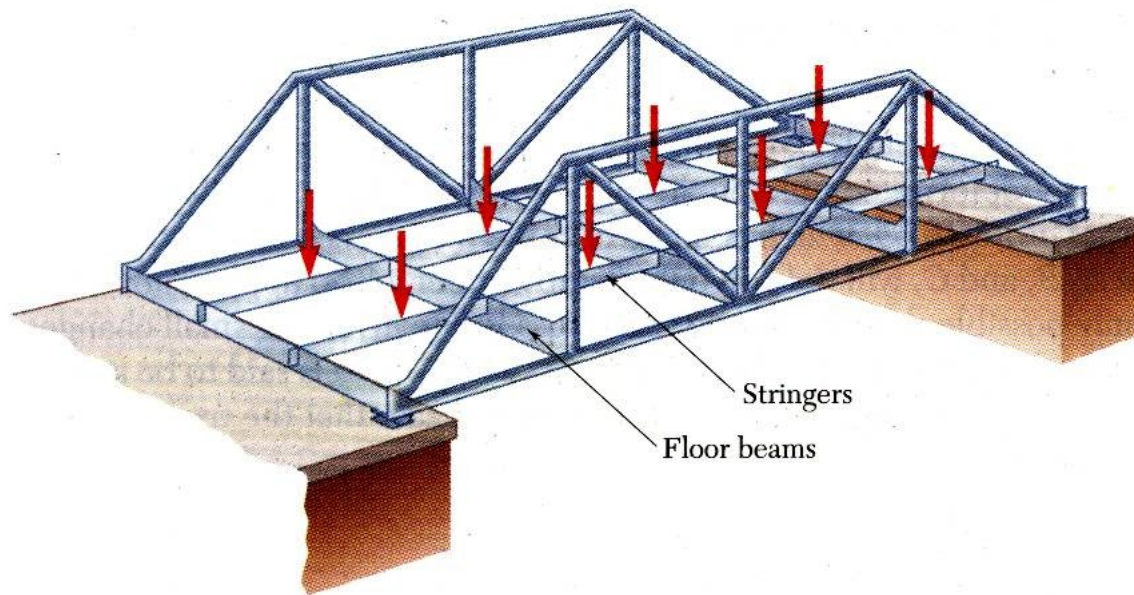
Definition of a Truss



- A truss consists of straight members connected at joints. No member is continuous through a joint.
- Most structures are made of several trusses joined together to form a space framework. Each truss carries those loads which act in its plane and may be treated as a two-dimensional structure.
- Bolted or welded connections are assumed to be pinned together. Forces acting at the member ends reduce to a single force and no couple. Only *two-force members* are considered.
- When forces tend to pull the member apart, it is in *tension*. When the forces tend to compress the member, it is in *compression*.

Vector Mechanics for Engineers: Statics

Definition of a Truss

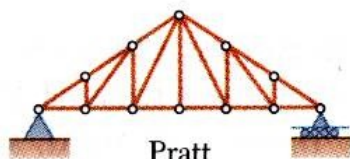


Members of a truss are slender and not capable of supporting large lateral loads. Loads must be applied at the joints.

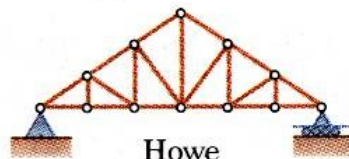


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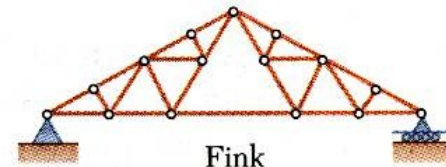
Definition of a Truss



Pratt

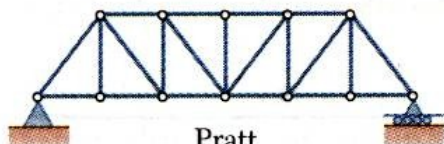


Howe

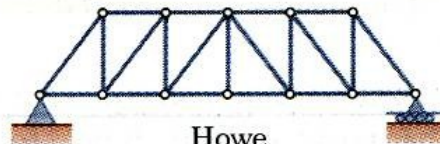


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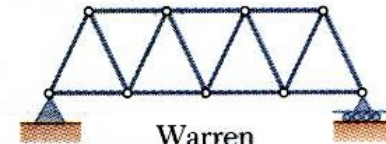
Typical Roof Trusses



Pratt



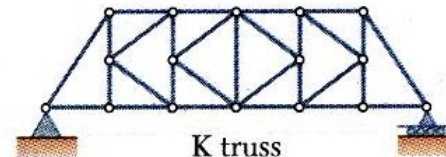
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Warren

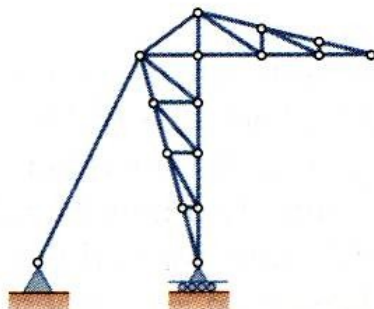


Baltimore

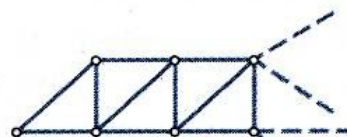


K truss

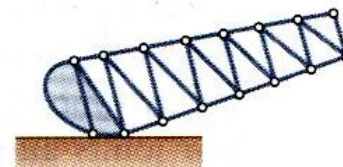
Typical Bridge Trusses



Stadium



Cantilever portion
of a truss

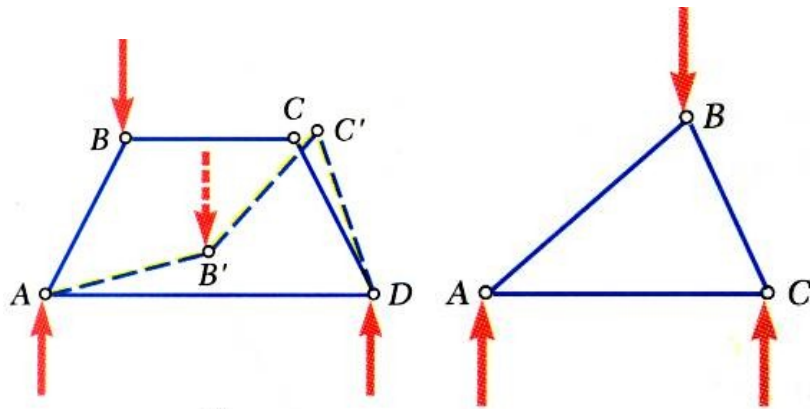
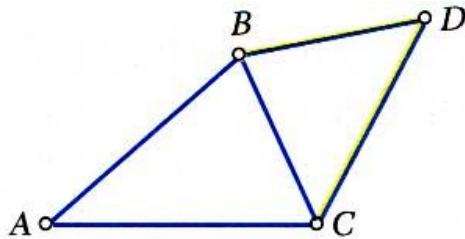
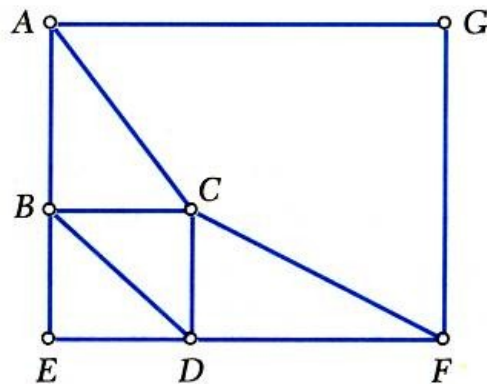


Bascule

Other Types of Trusses

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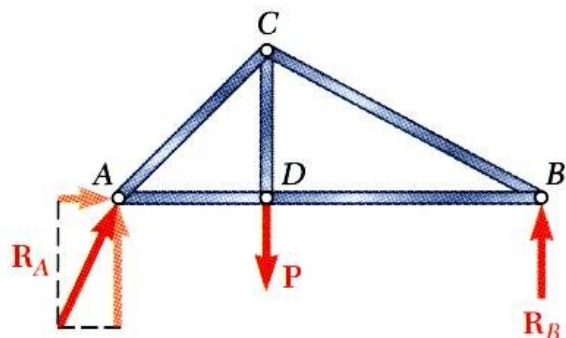
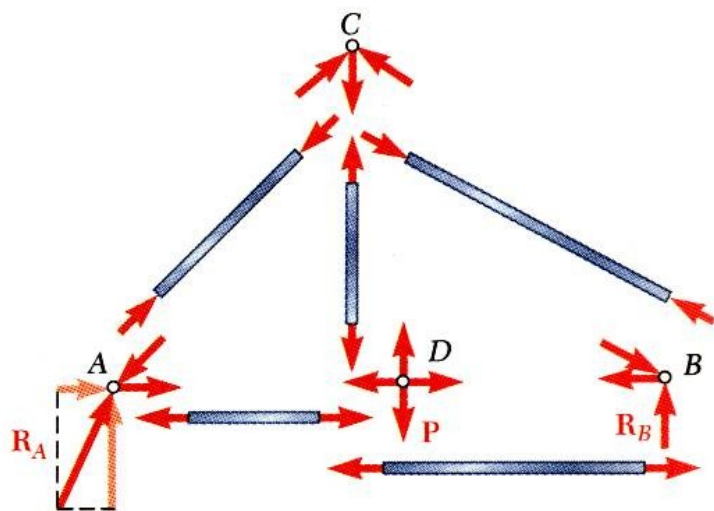
Simple Trusses



- A *rigid truss* will not collapse under the application of a load.
- A *simple truss* is constructed by successively adding two members and one connection to the basic triangular truss.
- In a simple truss, $m = 2n - 3$ where m is the total number of members and n is the number of joints.

Vector Mechanics for Engineers: Statics

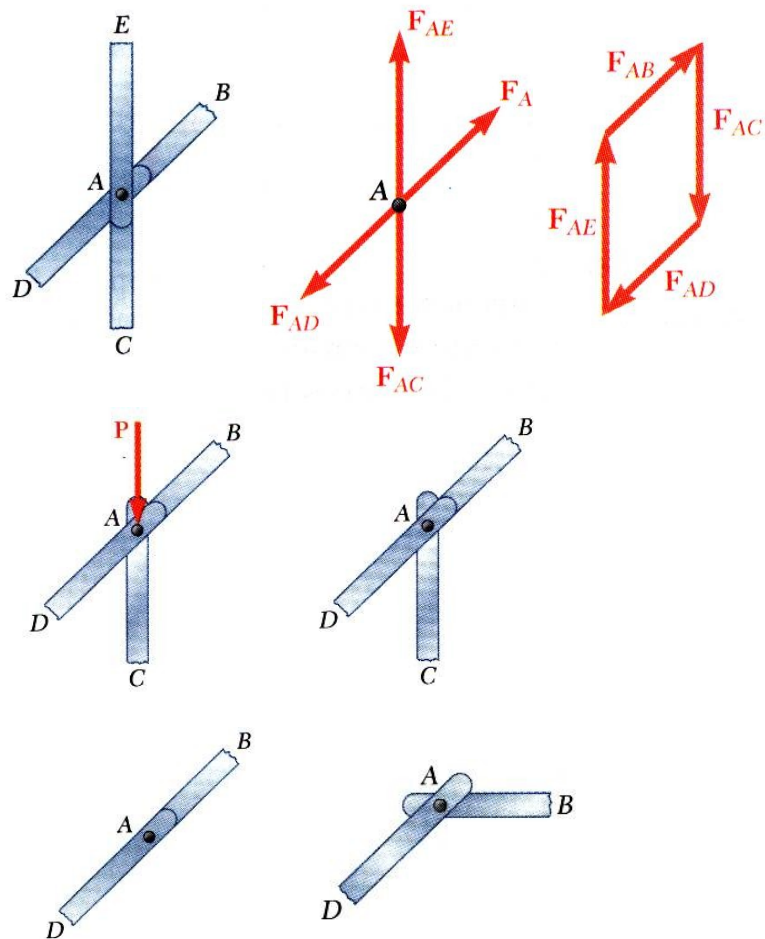
Analysis of Trusses by the Method of Joints



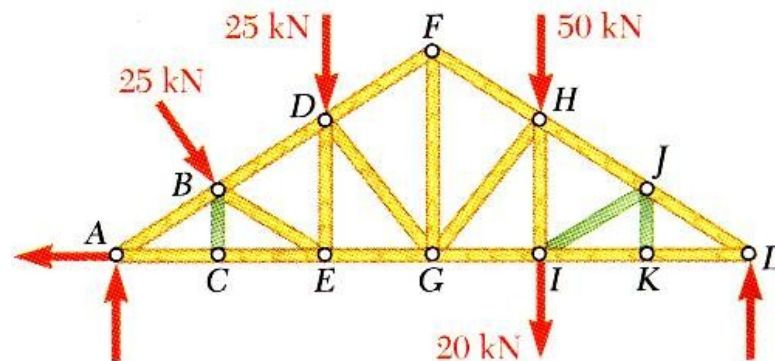
- Dismember the truss and create a freebody diagram for each member and pin.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium on the pins provide $2n$ equations for $2n$ unknowns. For a simple truss, $2n = m + 3$. May solve for m member forces and 3 reaction forces at the supports.
- Conditions for equilibrium for the entire truss provide 3 additional equations which are not independent of the pin equations.

Vector Mechanics for Engineers: Statics

Joints Under Special Loading Conditions

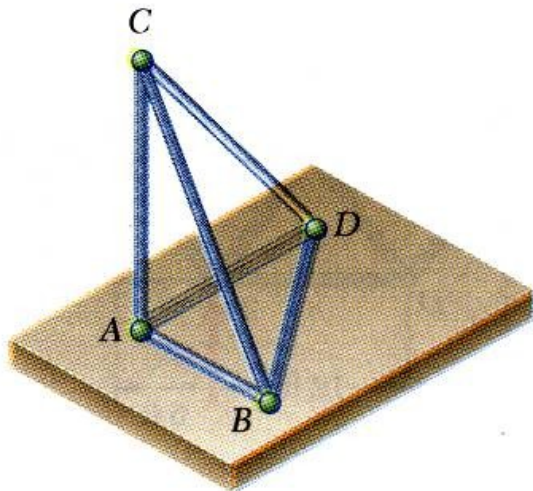
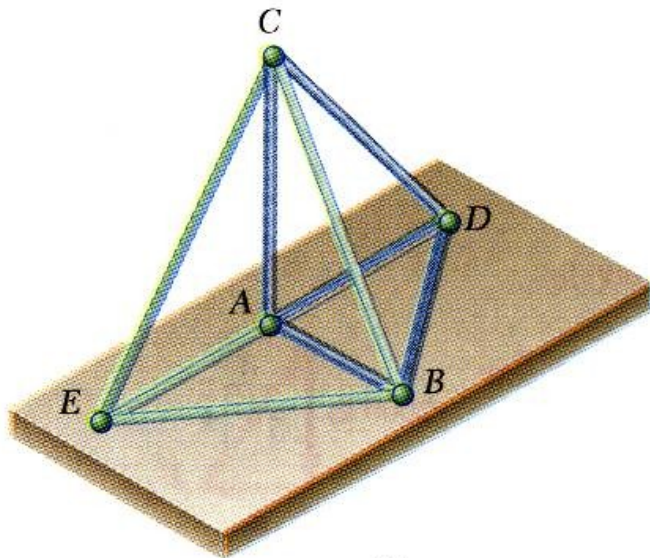


- Forces in opposite members intersecting in two straight lines at a joint are equal.
- The forces in two opposite members are equal when a load is aligned with a third member. The third member force is equal to the load (including zero load).
- The forces in two members connected at a joint are equal if the members are aligned and zero otherwise.
- Recognition of joints under special loading conditions simplifies a truss analysis.



Vector Mechanics for Engineers: Statics

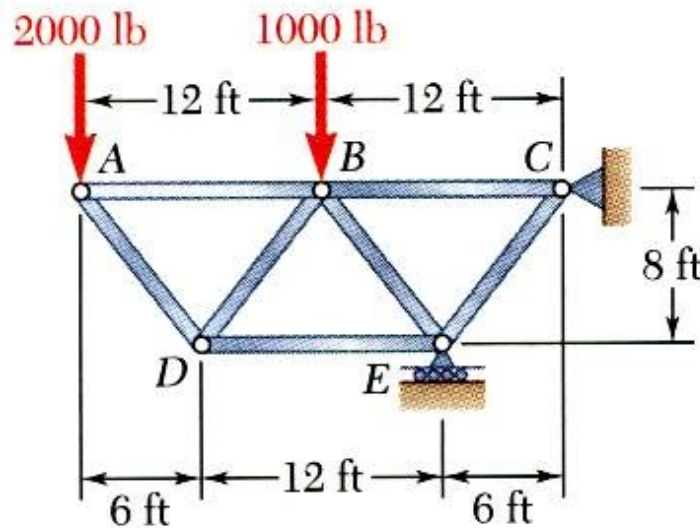
Space Trusses



- An *elementary space truss* consists of 6 members connected at 4 joints to form a tetrahedron.
- A *simple space truss* is formed and can be extended when 3 new members and 1 joint are added at the same time.
- In a simple space truss, $m = 3n - 6$ where m is the number of members and n is the number of joints.
- Conditions of equilibrium for the joints provide $3n$ equations. For a simple truss, $3n = m + 6$ and the equations can be solved for m member forces and 6 support reactions.
- Equilibrium for the entire truss provides 6 additional equations which are not independent of the joint equations.

Vector Mechanics for Engineers: Statics

Sample Problem 6.1



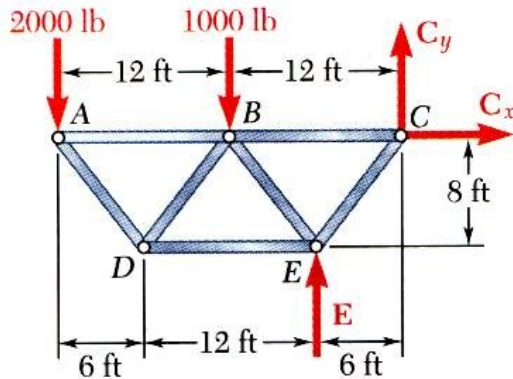
Using the method of joints, determine the force in each member of the truss.

SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *E* and *C*.
- Joint *A* is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.
- In succession, determine unknown member forces at joints *D*, *B*, and *E* from joint equilibrium requirements.
- All member forces and support reactions are known at joint *C*. However, the joint equilibrium requirements may be applied to check the results.

Vector Mechanics for Engineers: Statics

Sample Problem 6.1



SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at E and C .

$$\begin{aligned}\sum M_C &= 0 \\ &= (2000 \text{ lb})(24 \text{ ft}) + (1000 \text{ lb})(12 \text{ ft}) - E(6 \text{ ft})\end{aligned}$$

$$E = 10,000 \text{ lb} \uparrow$$

$$\sum F_x = 0 = C_x$$

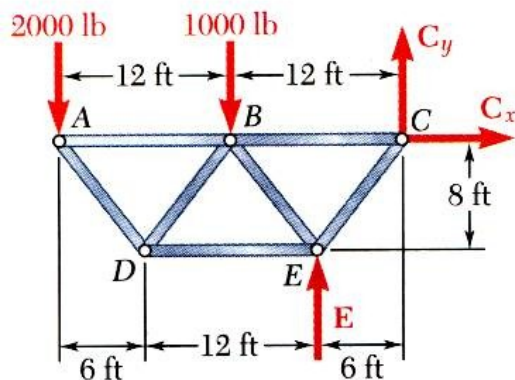
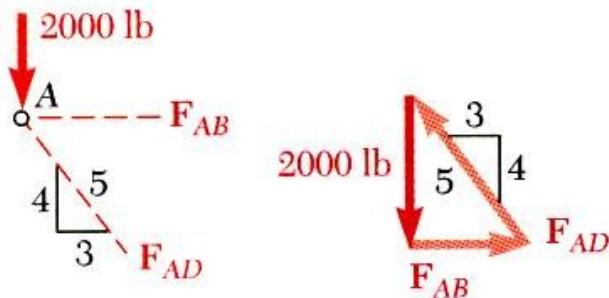
$$C_x = 0$$

$$\sum F_y = 0 = -2000 \text{ lb} - 1000 \text{ lb} + 10,000 \text{ lb} + C_y$$

$$C_y = 7000 \text{ lb} \downarrow$$

Vector Mechanics for Engineers: Statics

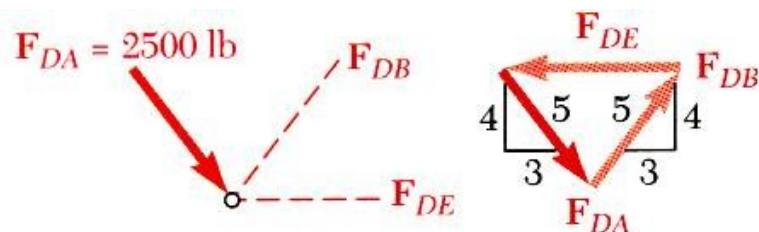
Sample Problem 6.1



- Joint A is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.

$$\frac{2000 \text{ lb}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$$\begin{aligned} F_{AB} &= 1500 \text{ lb } T \\ F_{AD} &= 2500 \text{ lb } C \end{aligned}$$



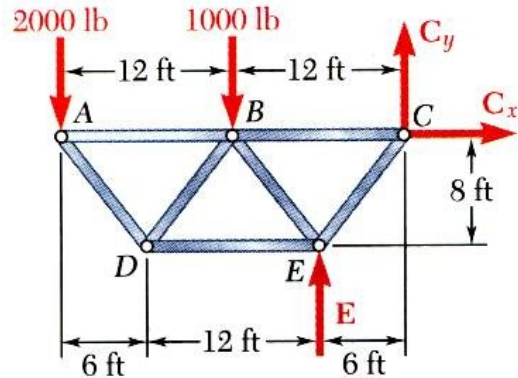
- There are now only two unknown member forces at joint D.

$$\begin{aligned} F_{DB} &= F_{DA} \\ F_{DE} &= 2\left(\frac{3}{5}\right)F_{DA} \end{aligned}$$

$$\begin{aligned} F_{DB} &= 2500 \text{ lb } T \\ F_{DE} &= 3000 \text{ lb } C \end{aligned}$$

Vector Mechanics for Engineers: Statics

Sample Problem 6.1



- There are now only two unknown member forces at joint B. Assume both are in tension.

$$\sum F_y = 0 = -1000 - \frac{4}{5}(2500) - \frac{4}{5}F_{BE}$$

$$F_{BE} = -3750 \text{ lb}$$

$$F_{BE} = 3750 \text{ lb C}$$

$$\sum F_x = 0 = F_{BC} - 1500 - \frac{3}{5}(2500) - \frac{3}{5}(3750)$$

$$F_{BC} = +5250 \text{ lb}$$

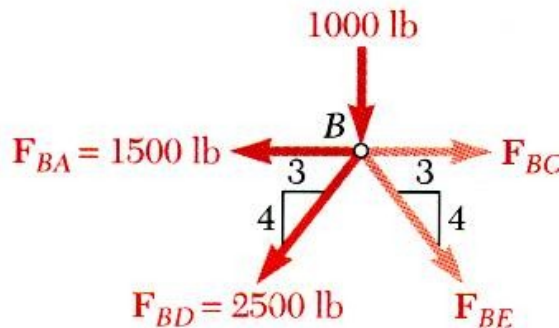
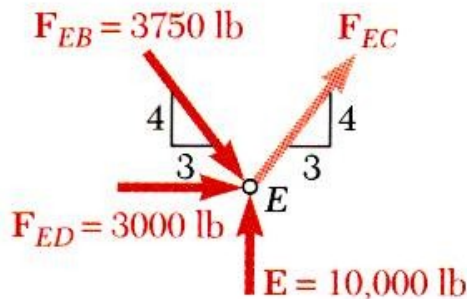
$$F_{BC} = 5250 \text{ lb T}$$

- There is one unknown member force at joint E. Assume the member is in tension.

$$\sum F_x = 0 = \frac{3}{5}F_{EC} + 3000 + \frac{3}{5}(3750)$$

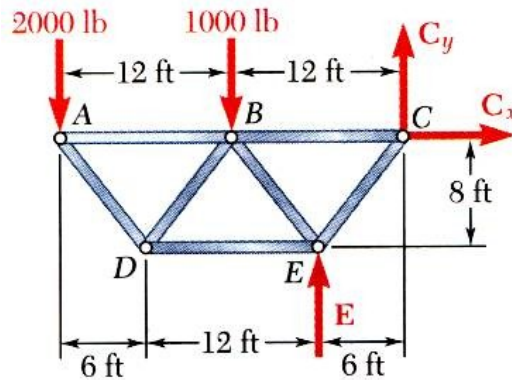
$$F_{EC} = -8750 \text{ lb}$$

$$F_{EC} = 8750 \text{ lb C}$$



Vector Mechanics for Engineers: Statics

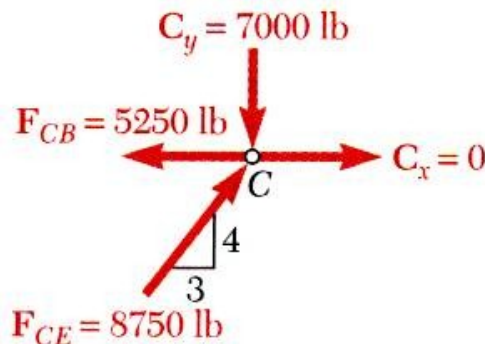
Sample Problem 6.1



- All member forces and support reactions are known at joint C. However, the joint equilibrium requirements may be applied to check the results.

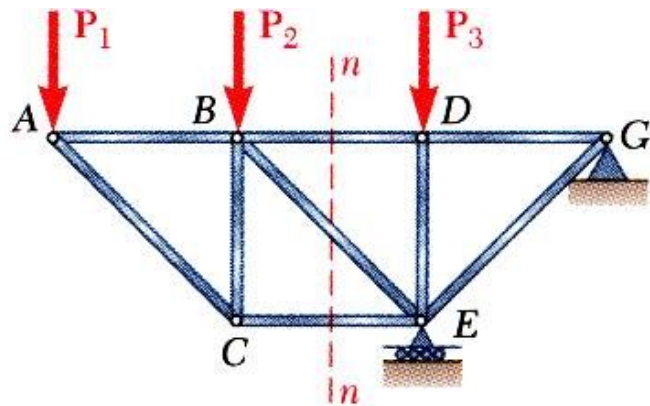
$$\sum F_x = -5250 + \frac{3}{5}(8750) = 0 \quad (\text{checks})$$

$$\sum F_y = -7000 + \frac{4}{5}(8750) = 0 \quad (\text{checks})$$

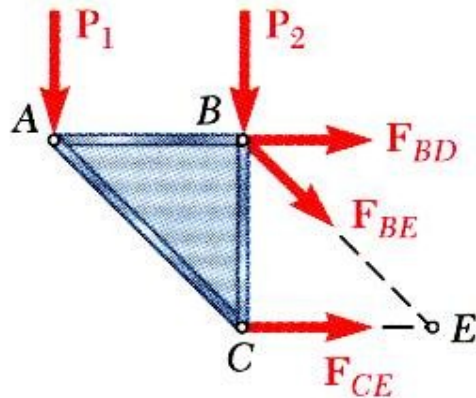


Vector Mechanics for Engineers: Statics

Analysis of Trusses by the Method of Sections

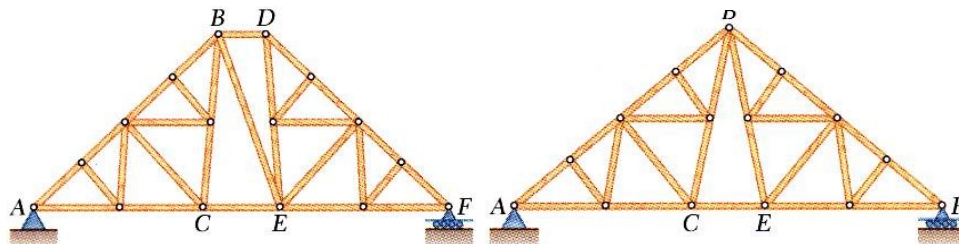


- When the force in only one member or the forces in a very few members are desired, the *method of sections* works well.
- To determine the force in member BD , *pass a section* through the truss as shown and create a free body diagram for the left side.
- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including F_{BD} .



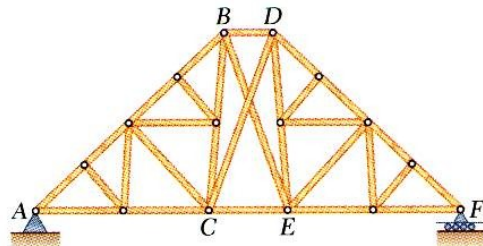
Vector Mechanics for Engineers: Statics

Trusses Made of Several Simple Trusses



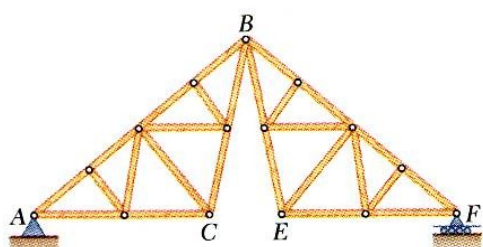
- Compound trusses are statically determinant, rigid, and completely constrained.

$$m = 2n - 3$$



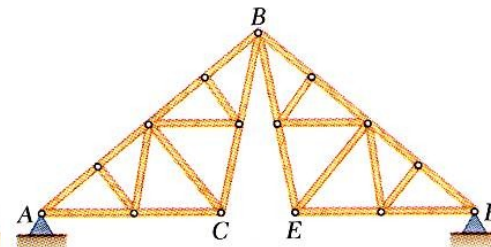
- Truss contains a *redundant member* and is *statically indeterminate*.

$$m > 2n - 3$$



non-rigid

$$m < 2n - 3$$



rigid

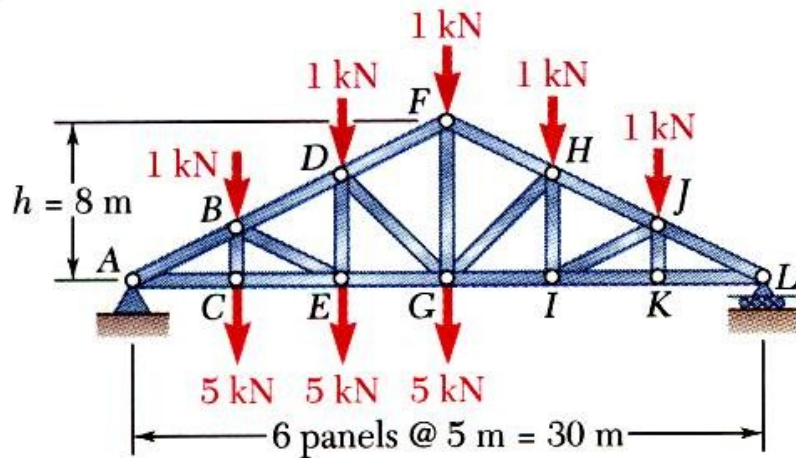
$$m < 2n - 4$$

- Additional reaction forces may be necessary for a rigid truss.
- Necessary but insufficient condition for a compound truss to be statically determinant, rigid, and completely constrained,

$$m + r = 2n$$

Vector Mechanics for Engineers: Statics

Sample Problem 6.3



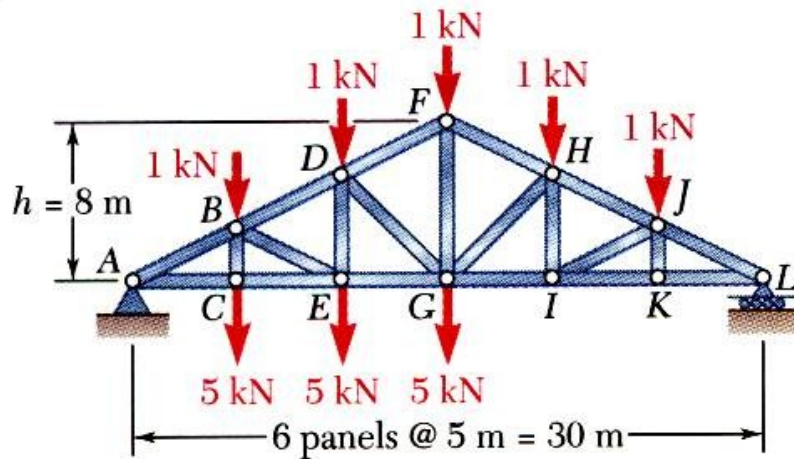
SOLUTION:

- Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at A and L .
- Pass a section through members FH , GH , and GI and take the right-hand section as a free body.
- Apply the conditions for static equilibrium to determine the desired member forces.

Determine the force in members FH , GH , and GI .

Vector Mechanics for Engineers: Statics

Sample Problem 6.3



SOLUTION:

- Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at A and L.

$$\sum M_A = 0 = -(5 \text{ m})(6 \text{ kN}) - (10 \text{ m})(6 \text{ kN}) - (15 \text{ m})(6 \text{ kN}) - (20 \text{ m})(1 \text{ kN}) - (25 \text{ m})(1 \text{ kN}) + (25 \text{ m})L$$

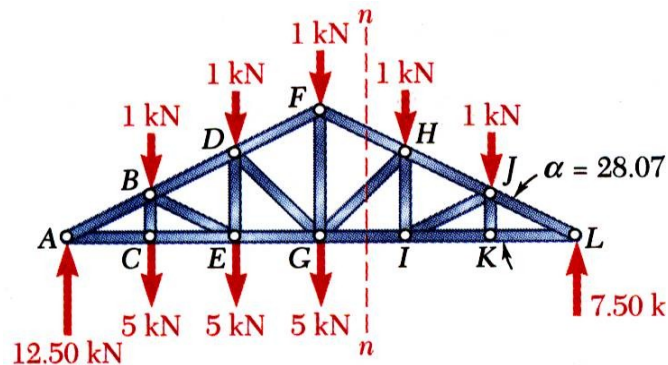
$$L = 7.5 \text{ kN} \uparrow$$

$$\sum F_y = 0 = -20 \text{ kN} + L + A$$

$$A = 12.5 \text{ kN} \uparrow$$

Vector Mechanics for Engineers: Statics

Sample Problem 6.3



- Pass a section through members FH , GH , and GI and take the right-hand section as a free body.

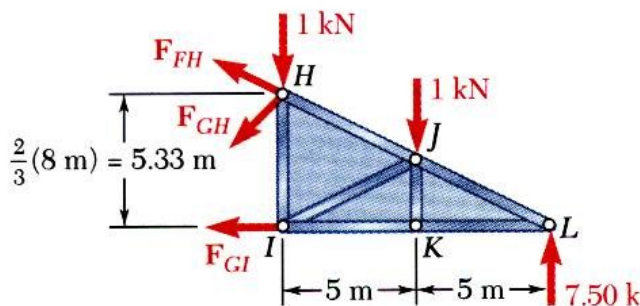
- Apply the conditions for static equilibrium to determine the desired member forces.

$$\sum M_H = 0$$

$$(7.50 \text{ kN})(10 \text{ m}) - (1 \text{ kN})(5 \text{ m}) - F_{GI}(5.33 \text{ m}) = 0$$

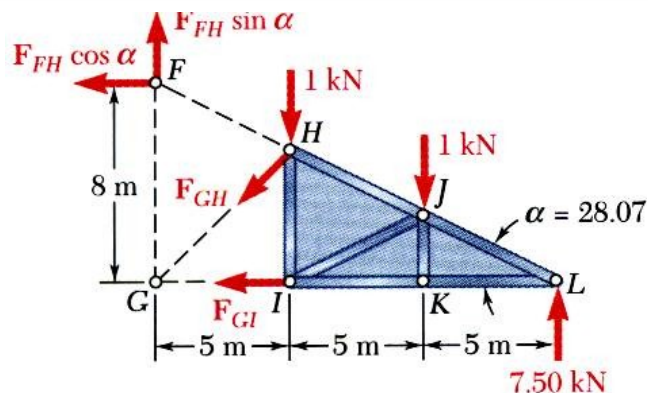
$$F_{GI} = +13.13 \text{ kN}$$

$$F_{GI} = 13.13 \text{ kN } T$$



Vector Mechanics for Engineers: Statics

Sample Problem 6.3



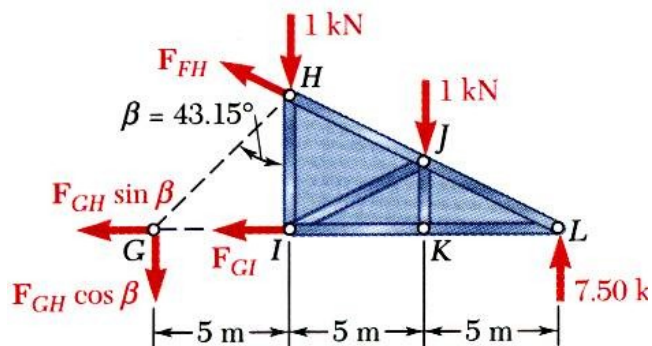
$$\tan \alpha = \frac{FG}{GL} = \frac{8 \text{ m}}{15 \text{ m}} = 0.5333 \quad \alpha = 28.07^\circ$$

$$\sum M_G = 0$$

$$(7.5 \text{ kN})(15 \text{ m}) - (1 \text{ kN})(10 \text{ m}) - (1 \text{ kN})(5 \text{ m}) + (F_{FH} \cos \alpha)(8 \text{ m}) = 0$$

$$F_{FH} = -13.82 \text{ kN}$$

$$F_{FH} = 13.82 \text{ kN C}$$



$$\tan \beta = \frac{GI}{HI} = \frac{5 \text{ m}}{\frac{2}{3}(8 \text{ m})} = 0.9375 \quad \beta = 43.15^\circ$$

$$\sum M_L = 0$$

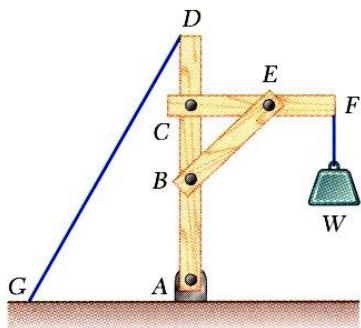
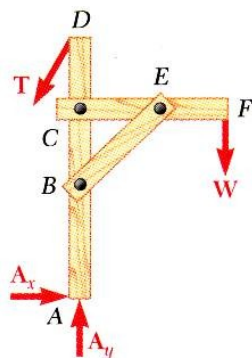
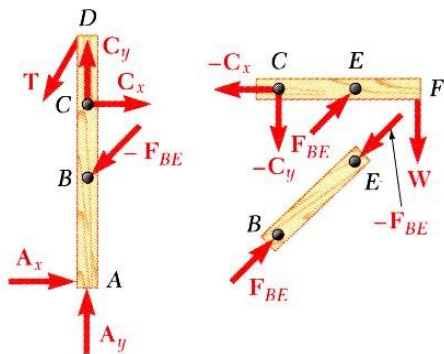
$$(1 \text{ kN})(10 \text{ m}) + (1 \text{ kN})(5 \text{ m}) + (F_{GH} \cos \beta)(10 \text{ m}) = 0$$

$$F_{GH} = -1.371 \text{ kN}$$

$$F_{GH} = 1.371 \text{ kN C}$$

Vector Mechanics for Engineers: Statics

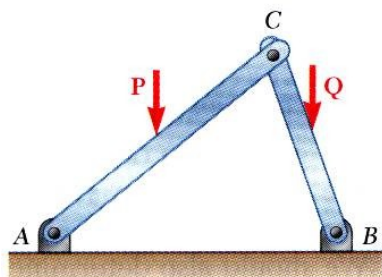
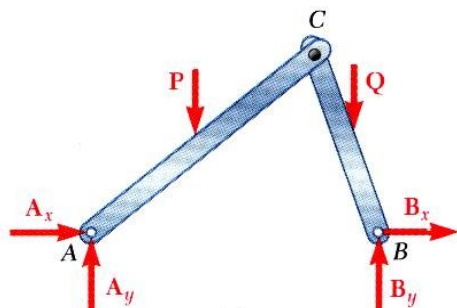
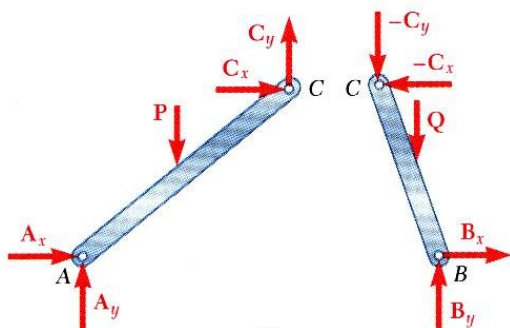
Analysis of Frames



- *Frames* and *machines* are structures with at least one *multiforce* member. Frames are designed to support loads and are usually stationary. Machines contain moving parts and are designed to transmit and modify forces.
- A free body diagram of the complete frame is used to determine the external forces acting on the frame.
- Internal forces are determined by dismembering the frame and creating free-body diagrams for each component.
- Forces on two force members have known lines of action but unknown magnitude and sense.
- Forces on multiforce members have unknown magnitude and line of action. They must be represented with two unknown components.
- Forces between connected components are equal, have the same line of action, and opposite sense.

Vector Mechanics for Engineers: Statics

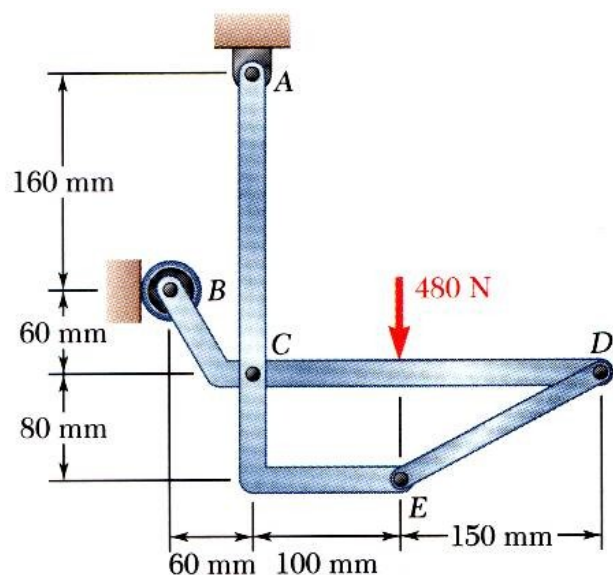
Frames Which Cease To Be Rigid When Detached From Their Supports



- Some frames may collapse if removed from their supports. Such frames can not be treated as rigid bodies.
- A free-body diagram of the complete frame indicates four unknown force components which can not be determined from the three equilibrium conditions.
- The frame must be considered as two distinct, but related, rigid bodies.
- With equal and opposite reactions at the contact point between members, the two free-body diagrams indicate 6 unknown force components.
- Equilibrium requirements for the two rigid bodies yield 6 independent equations.

Vector Mechanics for Engineers: Statics

Sample Problem 6.4



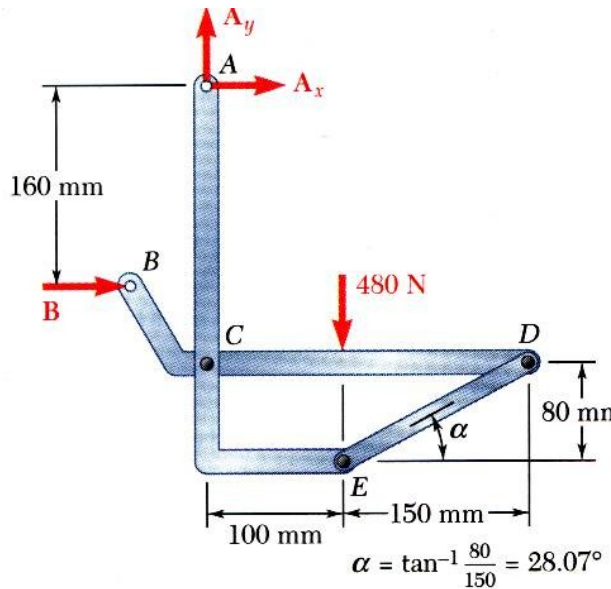
SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.
- Define a free-body diagram for member *BCD*. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing moments about *C*.
- With the force on the link *DE* known, the sum of forces in the *x* and *y* directions may be used to find the force components at *C*.
- With member *ACE* as a free-body, check the solution by summing moments about *A*.

Members *ACE* and *BCD* are connected by a pin at *C* and by the link *DE*. For the loading shown, determine the force in link *DE* and the components of the force exerted at *C* on member *BCD*.

Vector Mechanics for Engineers: Statics

Sample Problem 6.4



SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.

$$\sum F_y = 0 = A_y - 480 \text{ N}$$

$$A_y = 480 \text{ N } \uparrow$$

$$\sum M_A = 0 = - (480 \text{ N})(100 \text{ mm}) + B(160 \text{ mm})$$

$$B = 300 \text{ N } \rightarrow$$

$$\sum F_x = 0 = B + A_x$$

$$A_x = - 300 \text{ N } \leftarrow$$

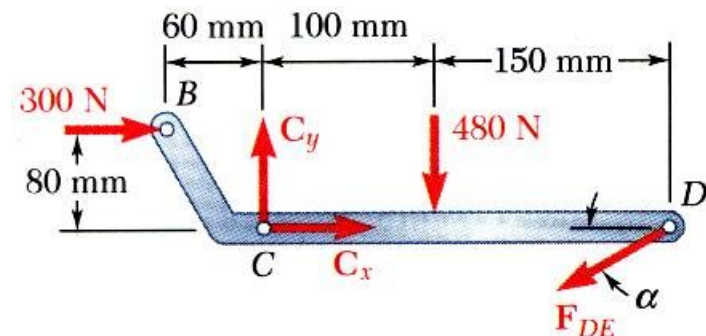
Note:

$$\alpha = \tan^{-1} \frac{80}{150} = 28.07^\circ$$

Vector Mechanics for Engineers: Statics

Sample Problem 6.4

- Define a free-body diagram for member *BCD*. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing moments about *C*.



$$\sum M_C = 0 = (F_{DE} \sin \alpha)(250 \text{ mm}) + (300 \text{ N})(60 \text{ mm}) + (480 \text{ N})(100 \text{ mm})$$

$$F_{DE} = -561 \text{ N}$$

$$F_{DE} = 561 \text{ N}$$

- Sum of forces in the *x* and *y* directions may be used to find the force components at *C*.

$$\sum F_x = 0 = C_x - F_{DE} \cos \alpha + 300 \text{ N}$$

$$0 = C_x - (-561 \text{ N}) \cos \alpha + 300 \text{ N}$$

$$C_x = -795 \text{ N}$$

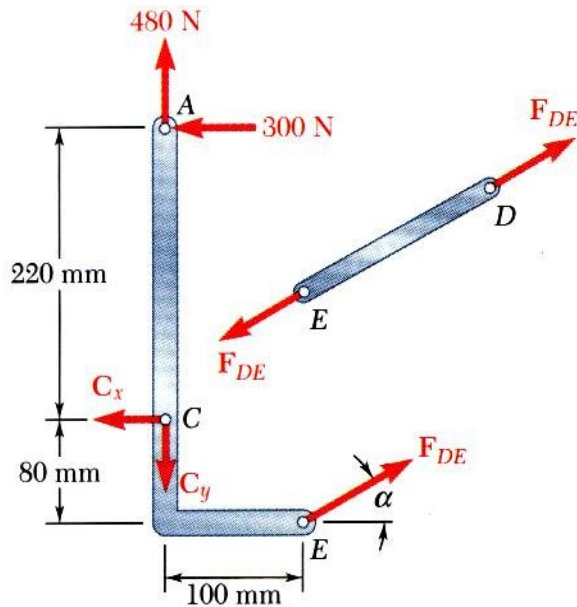
$$\sum F_y = 0 = C_y - F_{DE} \sin \alpha - 480 \text{ N}$$

$$0 = C_y - (-561 \text{ N}) \sin \alpha - 480 \text{ N}$$

$$C_y = 216 \text{ N}$$

Vector Mechanics for Engineers: Statics

Sample Problem 6.4



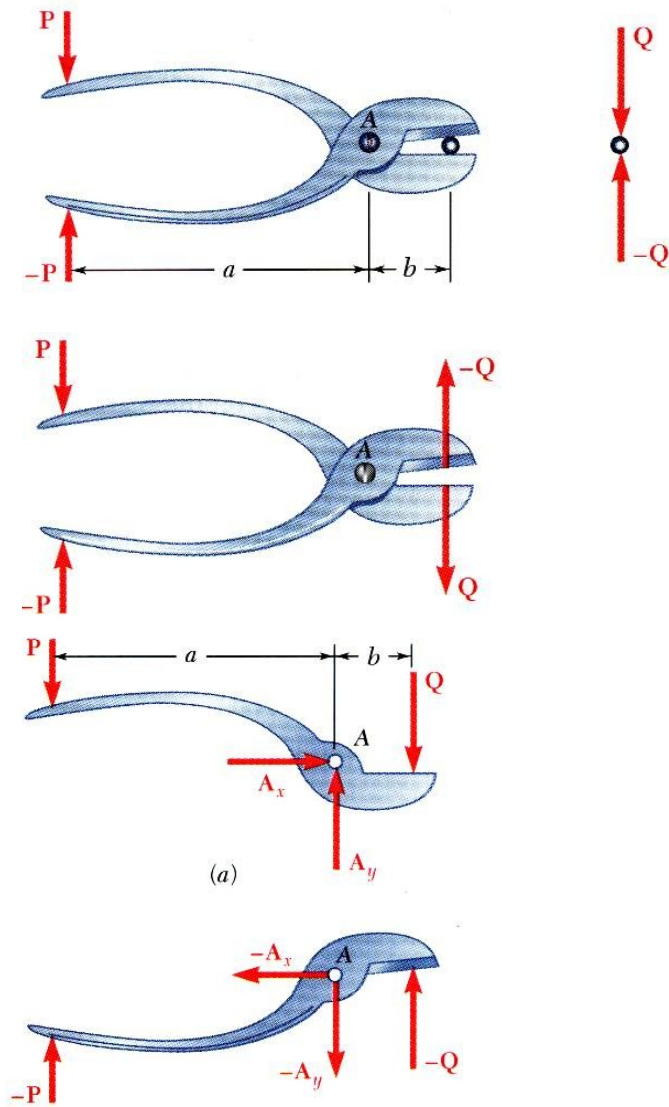
- With member *ACE* as a free-body, check the solution by summing moments about A.

$$\begin{aligned}\sum M_A &= (F_{DE} \cos \alpha)(300 \text{ mm}) + (F_{DE} \sin \alpha)(100 \text{ mm}) - C_x(220 \text{ mm}) \\ &= (-561 \cos \alpha)(300 \text{ mm}) + (-561 \sin \alpha)(100 \text{ mm}) - (-795)(220 \text{ mm}) = 0\end{aligned}$$

(checks)

Vector Mechanics for Engineers: Statics

Machines



- Machines are structures designed to transmit and modify forces. Their main purpose is to transform *input forces* into *output forces*.
- Given the magnitude of P , determine the magnitude of Q .
- Create a free-body diagram of the complete machine, including the reaction that the wire exerts.
- The machine is a nonrigid structure. Use one of the components as a free-body.
- Taking moments about A ,

$$\sum M_A = 0 = aP - bQ \quad Q = \frac{a}{b}P$$