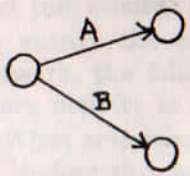
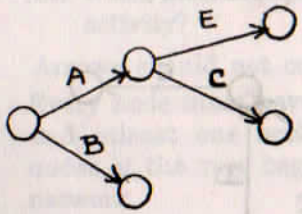
Example 2: Draw a network for the simple project of erection of steelworks for a shed. The various activities of the project are as follows.

activity	Job description	preceded by
A	Erect site workshop	–
B	Fence site	–
C	Bend Reinforcement	*A*
D	Dig foundation	*B*
E	Fabricate steelwork	*A*
F	Install concrete pillars	*B*
G	Place Reinforcement	*C, D*
H	Concrete foundation	*G, F*
I	Erect steelwork	*E*
J	Paint steel work	*H, I*
K	Give finishing touch	*J*

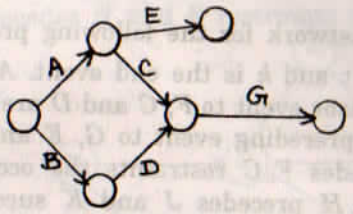
Solution: *A* and *B* have no preceding activities. ∴ They can commence concurrently.



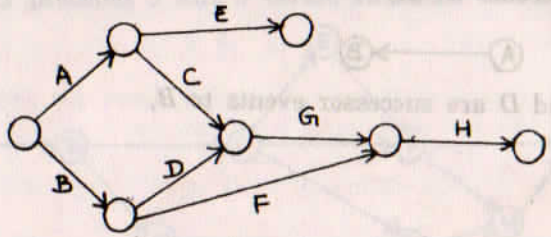
Activities *C* and *E* preceded by *A*.
(i.e.) *C* and *E* can start once *A* has finished.



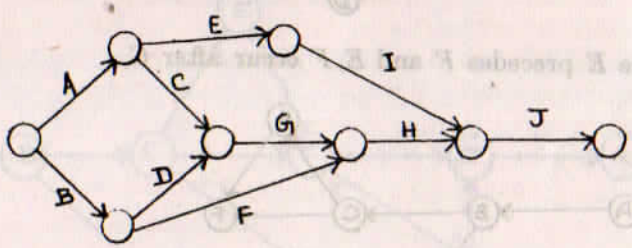
D and *F* can start once *B* has finished. But *C, D* have the same head event, since *G* is preceded by *C* and *D*.



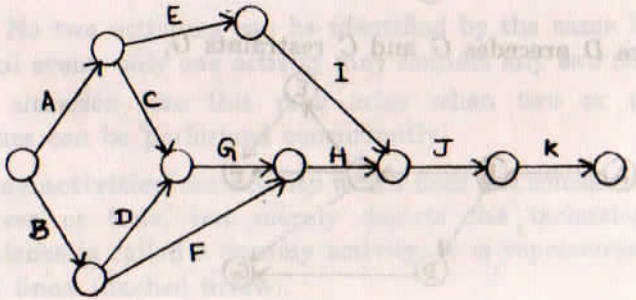
H is preceded by *G, F* (i.e.) *H* can start only both *G* and *F* have finished.



I is preceded by *E* but *H* and *I* have the same head event, since *J* is preceded by *H* and *I*.



Since *K* is preceded by *J*, we have the following network.

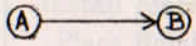


Example 3: Draw a network for the following project.

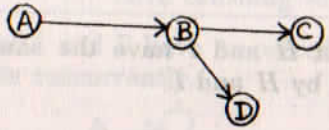
A is the start event and k is the end event. A precedes event B, J is the successor event to F, C and D are successor events to B, D is the preceding event to G, E and F occur after event C, E precedes F, C restraints the occurance of G and G precedes H, H precedes J and K succeeds J, F restraints the occurence of H.

Solution: Since A is the start event, we have (A)

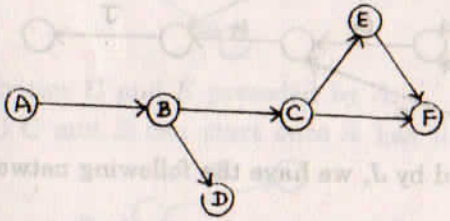
Since A precedes B, $(A) \longrightarrow (B)$



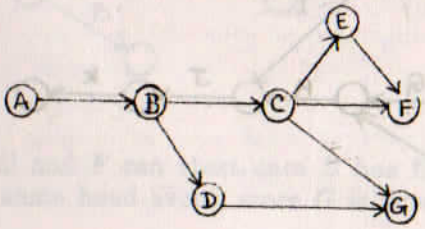
Since C and D are successor events to B,



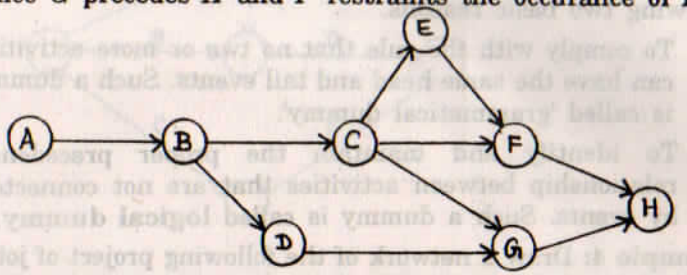
Since E precedes F and E, F occur after C,



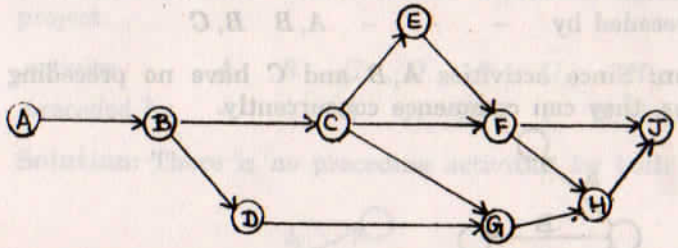
Since D precedes G and C restraints G,



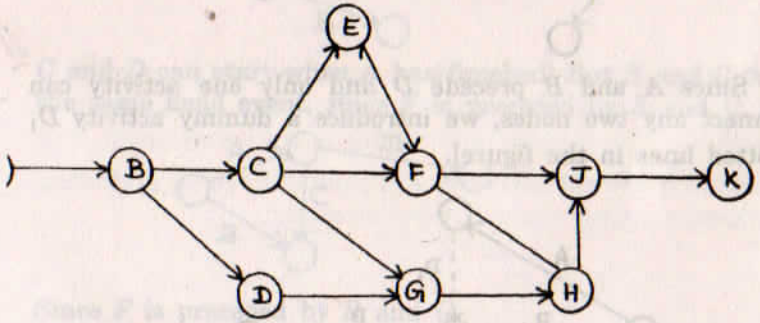
Since G precedes H and F restraints the occurance of H,



Since H precedes J and J is the successor to F,



Since K succeeds J and K is the last event,



Note: No two activities can be identified by the same head and tail event. only one activity may connect any two nodes.

A situation like this may arise when two or more activities can be performed concurrently.

Dummy activities: An activity which does not consume any resources or time, but merely depicts the technological dependence is called a dummy activity. It is represented by dotted lines. (dashed arrow).

A dummy activity is introduced in the network for the following two basic reasons.

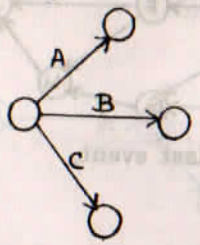
(i) To comply with the rule that no two or more activities can have the same head and tail events. Such a dummy is called 'grammatical dummy'.

(ii) To identify and maintain the proper precedence relationship between activities that are not connected by events. Such a dummy is called **logical dummy**.

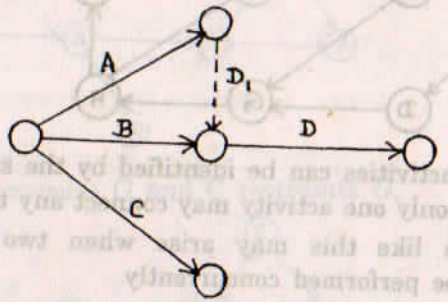
Example 4: Draw a network of the following project of jobs.

Jobs	A	B	C	D	E
preceded by	–	–	–	A, B	B, C

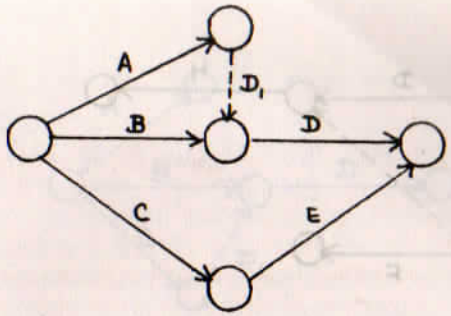
Solution: Since activities A, B and C have no preceding activities, they can commence concurrently.



Since A and B precede D and only one activity can connect any two nodes, we introduce a dummy activity D_1 [dotted lines in the figure].



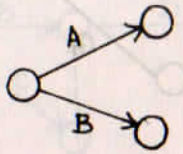
E can start only when Both B and C have finished and since any two nodes can be connected by only one node, introduce a dummy activity D_2.



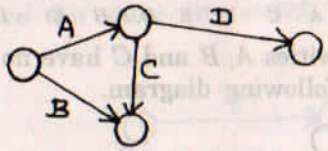
Example 5: Draw a network to represent the following project.

activity	A	B	C	D	F	G	H	I
preceded by	–	–	A	A	B, C	F	B, C, D	H, G

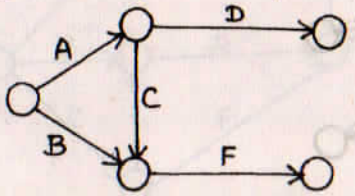
Solution: There is no preceding activities for both A and B.



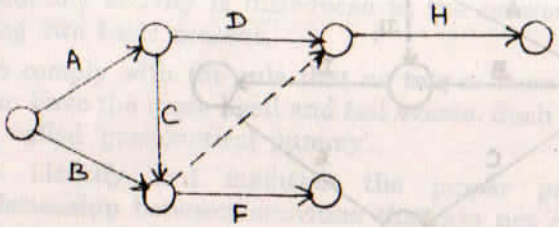
C and D can start when A has finished. But B and C have the same head event, since F is preceded by B and C.



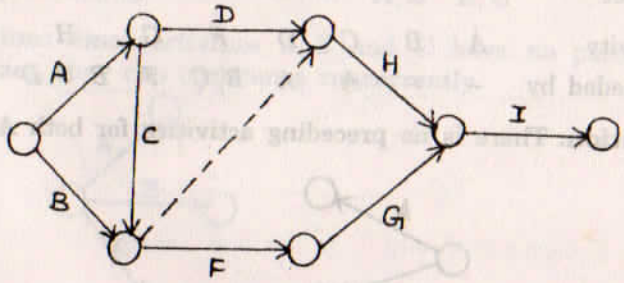
Since F is preceded by B and C,



Since H is preceded by B, C and D, we introduce a dummy activity D, (dotted lines in the figure).



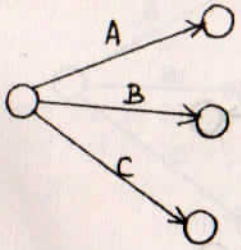
G is preceded by F and H, G have the common end event, since I is preceded by H and G.



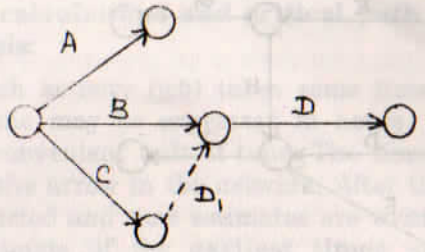
Example 6: Draw a network of a project of 12 activities.

activity	A	B	C	D	E	F	G	H	I	J	K	L
Dependence	–	–	–	B, C	A	C	E	E	D, F, H	E	I, J	G

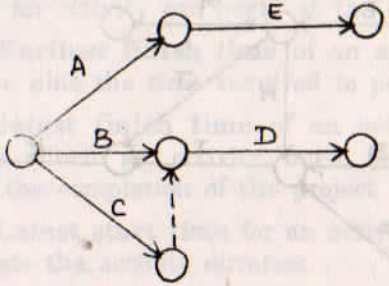
Solution: Since the activities A, B and C have no preceding activities, we have the following diagram.



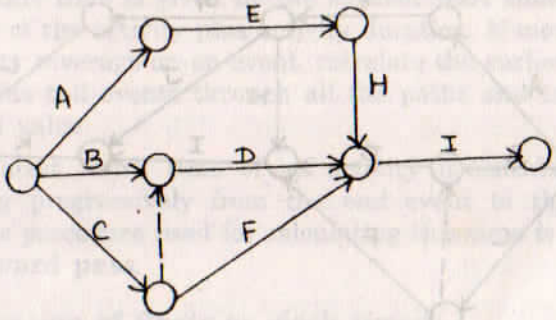
Since D is preceded by B and C, we introduce a dummy activity D_1.



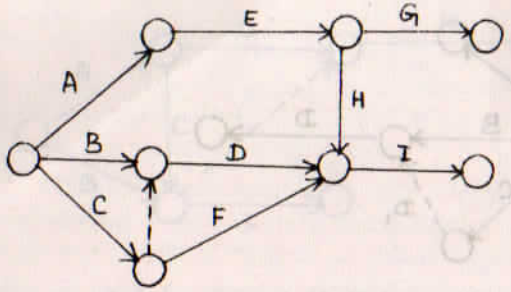
E is preceded by A.



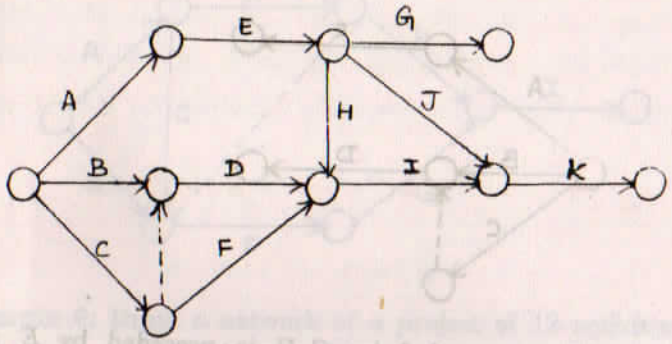
Since F is preceded by C, H is preceded by E and D, F, H have the same end event, since I is preceded by D, F, H



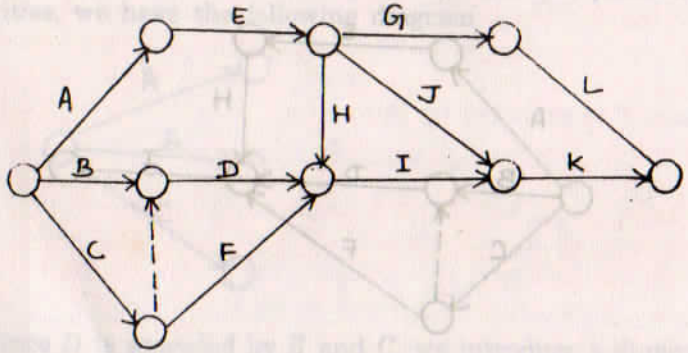
G is preceded by E



J is preceded by E. But J, I have the same end event since K is preceded by I, J



Since L is preceded by G, we have the following network.



Time calculations and critical path in Network analysis:

Each activity (job) takes some time for its completion. The time may be expressed in hours, days, weeks or any other convenient unit of time. The time estimate is written above the arrow in the network. After the network has been constructed and time estimates are written above the arrow, an estimate of the **earliest times** and **latest times** is obtained for each activity.

Earliest start time of an event represents the time at which an event is expected to occur at the earliest.

(i.e.) **Earliest start time** for an activity represents the time at which an activity can begin at the earliest.

The **Earliest finish time** of an activity is its earliest start time plus the time required to perform the activity.

The **latest finish time** of an activity represents the latest by which an activity must be completed without delaying the completion of the project.

The **Latest start time** for an activity is its latest finish time minus the activity duration.

Note: The earliest start time of an event can be determined by proceeding progressively from left to right (i.e) **forward pass method** according to the following rule.

If only one activity converges at a node (event), then its earliest start time is given by the earliest start time of the tail event of the activity plus activity duration. If more than one activity coverage on an event, calculate the earliest start times of the tail events through all the paths and take the maximum value.

The latest finish time of an activity is calculated by proceeding progressively from the end event to the start event. The procedure used for calculating this time is known as **Backward pass**.

Determination of floats or slack times

Total float: Total float is the amount of time within which an activity can be delayed without affecting the project

completion time. It is the difference between the late start and early start times of the same activity.

Mathematically, the total float of an activity $i-j$ is given by

$TF(i-j) = (LST - EST)$ for activity $(i-j)$

Free Float: It is the amount of time an activity can be delayed without affecting the early start time of any other activity. Free float of an activity is the difference between its early finish time and the earliest of the early start times of all its immediate successors.

(i.e) $FF[i-j] = EST[j-k] - EFT[i-j]$

Independent float: The amount of time by which the start of an activity can be delayed without affecting the earliest start time of any immediately following activities, assuming that the preceding activity has finished at its earliest finish time.

Mathematically,

$IF[i-j] = E_j - L_i - t_{ij}$

$E_j \rightarrow$ Early start time of an event j

$L_i \rightarrow$ Late finish time of an event i

$t_{ij} \rightarrow$ time duration of an activity $(i-j)$

If the result is negative, it is taken as zero.

Note: Independent float ≤ Free float ≤ Total float.

Critical Path Method: After drawing the network of all the activities of a project, we have to find the minimum time required for completion of the entire project. For this we must find the longest path with sequence of connected activities through the network. The longest path is called the **critical path**.

A **path** is defined as a set of nodes connected by lines which begin at the initial node of a network and end at the terminal node.

A **critical path** defines a chain of critical activities that connects the start and end nodes (events) of the network. An activity is said to be **critical** if a delay in its start will

cause a delay in the completion date of the entire project.

Critical date of the network determines the time for completion of the project. The minimum time to complete the project is the time taken for the longest path.

Procedure of determining the critical path

Method 1

* List all the activities [Jobs] and draw a network diagram.

* Consider the activity times to be deterministic. Indicate them above the arrow representing them.

* Calculate the earliest start time and earliest finish time for each activity. Similarly calculate the latest start time and the latest finish time for each activity.

* Determine the **'Total slcak'** for each activity.

* Connect the activities having no slacks, we get the critical path.

[Activity having no slack is called critical activity.]

Method 2: If an activity satisfies all the three conditions given below, it is critical.

(i) $EST = LFT$ for tail event

(ii) $EST = LFT$ for head event

(iii) EST of an event $j - EST$ of an event $i = LFT$ of an event $j - LFT$ of an event $i = t_{ij}$ [$t_{ij} \rightarrow$ duration of an activity $(i-j)$].

Method 3: Find the various paths and their durations. The path which is the longest in time is called the critical path.

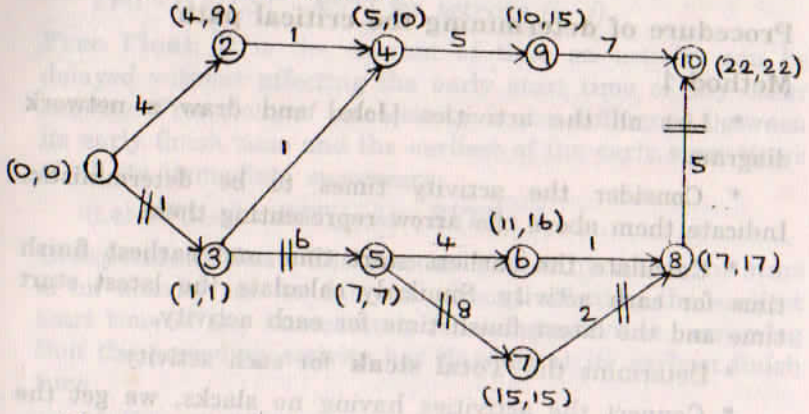
Example: 7 A project scheduling has the following characteristics.

activity:	1–2	1–3	2–4	3–4	3–5	4–9	5–6	5–7	6–8	7–8	8–10	9–10
Time (days):	4	1	1	1	6	5	4	8	1	2	5	7

Summarize the CPM calculations in a Tabular form and determine the critical path. (BE, MU Ap' 90)

Solution:

The 1st number in the braces represents the Early start time of the event (node). While the 2nd number represents Late finish time of the event.



First we calculate the Early start time of each activity [calculations begin from the start node and move to the end node].

Since activities $1-2$ and $1-3$ have no preceding activities, they can start immediately. (i.e.). They can start at time $t = 0$.

$\therefore E_1 = 0$

$E_1 \rightarrow$ Early start time of the event 1.

The activity $2-4$ is preceded by an activity $1-2$. $\therefore$ Early start time of the activity $2-4$ is Early start time of activity $1-2$ (or) event 1, plus time required for the completion of $1-2$

$\therefore$ activity $2-4$ can start at $t = 0+4 = 4$.

i.e. $E_2 = E_1 + t_{12} = 0+4 = 4$

$E_2 \rightarrow$ early start time of the event 2

$t_{12} \rightarrow$ duration for the activity $1-2$

Since the activity $3-4$ is preceded by an activity $1-3$, Early start time of $3-4$ is Early start time of $1-3$ plus duration for the activity $1-3$.

$\therefore$ activity $3-4$ can start at $t = 0+1 = 1$

i.e. $E_3 = E_1 + t_{13} = 0+1 = 1$

Similarly the early start time of $3-5$ is 1, since it is preceded by $1-3$

(i.e.) $E_3 = E_1 + t_{13} = 0+1 = 1$

Activity $(4-9)$ can start if both the activities $2-4$ and $(3-4)$ have finished.

Early start time of $(4-9)$ = Early start time of $(2-4)$ + duration of $(2-4) = 4+1 = 5$

Also early start time of $(4-9)$ = Early start time of $(3-4)$ + duration of $(3-4) = 1+1 = 2$

i.e. $E_4 = 5$

$E_4 = 2$

$\therefore E_4 = \text{Max}\,[5, 2] = 5$, sine at node 4 we have two incoming activities.

Similarly $E_5 = E_3 + t_{35} = 1+6 = 7$

$E_6 = E_5 + t_{56} = 7+4 = 11$

$E_7 = E_5 + t_{57} = 7+8 = 15$

$E_8 = \text{Max}\,[E_6 + t_{68}, E_7 + t_{78}]$

$= \text{Max}\,[11+1, 15+2] = 17$

$E_9 = E_4 + t_{49} = 5+5 = 10$

$E_{10} = \text{Max}\,[E_8 + t_{8-10}, E_9 + t_{9-10}]$

$= \text{Max}\,[17+5, 10+7] = 22$

Next we calculate Late Finish time of each activity. Here the calculations begin at the end of the network and work backwards. We begin with the node 10. The activity leading to node 10 are $(8-10)$, $(9-10)$.

They must be completed by day 22 so as not to delay the project.

$\therefore$ Late finish of activities $(8-10)$ and $(9-10)$ is 22 days.

i.e. $L_{10} = 22$

L_{10} = late finish of the event 10.

The activity (8 – 10) has two predecessors (7 – 8), (6 – 8).

∴ Late finish time for both the activities

= late finish of (8 – 10) – $t_{(}8 - 10)$ = 22 – 5 = 17

i.e. $L_8 = L_{10} - t_{8-10} = 22 - 5 = 17$

The activity (9 – 10) has one predecessor (4 – 9)

∴ Late finish time of (4 – 9)

= late finish of (9 – 10) – $t_{9,10}$ = 22 – 7 = 15

i.e. $L_9 = L_{10} - t_{9-10} = 22 - 7 = 15$

Similarly late finish of (5 – 7)

= late finish of (7 – 8) – t_{78} = 17 – 2 = 15

$L_7 = L_8 - t_{78} = 17 - 2 = 15$

Late finish of (5 – 6) = Late finish of (6 – 8) – t_{68}

= 17 – 1 = 16

i.e. $L_6 = L_8 - t_{68} = 17 - 1 = 16$

Late finish time of (3 – 5)

= Late finish time of (5 – 6) – t_{56}

= 16 – 4 = 12

Also Late finish time of (3 – 5)

= Late finish time of (5 – 7) – t_{57}

= 15 – 8 = 7

∴ Late finish time of (5 – 6) = Min $\{12, 7\} = 7$, since at node 5 we have two outgoing activities.

i.e. $E_5 = \text{Min}\{L_6 - t_{56}, L_7 - t_{57}\}$

$= \text{Min}\{16 - 4, 15 - 8\} = 7$

Late finish time of (3 – 4) = Late finish time of (4 – 9) – t_{49}, since (3 – 4) precedes (4 – 9)

∴ $L_4 = L_9 - t_{49} = 15 - 5 = 10$

Late finish time of (2 – 4) = Late finish of (4 – 9) – t_{49}, since (2 – 4) precedes (4 – 9)

∴ Late finish time of (2 – 4) = 15 – 5 = 10

(i.e.) $L_4 = 10$

Late finish of (1 – 3) = Min [late finish time of (3 – 5) – t_{35}; late finish time of (3 – 4) – t_{34}]

i.e. $L_3 = \text{Min}\{L_5 - t_{35}, L_4 - t_{34}\}$

$= \text{Min}\{7 - 6, 10 - 1\} = 1$

Late finish time of (1 – 2) = late finish time of (2 – 4) – t_{24}, since 1-2 precedes 2-4.

(i.e.) $L_2 = L_4 - t_{24} = 10 - 1 = 9$

Finally $L_1 = \text{Min}\{L_3 - t_{13}, L_2 - t_{12}\}$

$= \text{Min}\{1 - 1, 9 - 4\} = 0$

Now we have the following tabular form.

Activity	Duration	*EST*	*EFT*	*LST*	*LFT*	SLACK *LST – EST*
1 – 2	4	0	4(0 + 4)	5(9 – 4)	9	5
1 – 3	1	0	1(0 + 1)	0(1 – 1)	1	**0**
2 – 4	1	4	5(4 + 1)	9(10 – 1)	10	5
3 – 4	1	1	2(1 + 1)	9(10 – 1)	10	8
3 – 5	6	1	7(1 + 6)	1(7 – 6)	7	**0**
4 – 9	5	5	10(5 + 5)	10(15 – 5)	15	5
5 – 6	4	7	11(7 + 4)	12(16 – 4)	16	5
5 – 7	8	7	15(7 + 8)	7(15 – 8)	15	**0**
6 – 8	1	11	12(11 + 1)	16(17 – 1)	17	5
7 – 8	2	15	17(15 + 2)	15(17 – 2)	17	**0**
8 – 10	5	17	22 (17 + 5)	17(22 – 5)	22	**0**
9 – 10	7	10	17 (10 + 7)	15(22 – 7)	22	5

Total slack is zero for the activities 1 – 3, 3 – 5, 5 – 7, 7 – 8 and 8 – 10. Hence they are critical activities.

Critical path is a path connected by the critical activities.

$\therefore$ Critical Path $= 1-3-5-7-8-10$

Project duration = sum of the duration of the critical activities.

$\therefore$ Project duration = 22 days.

Note: From the network diagram, we have the following

event	EST and LFT	
1	(0, 0)	The 1st two conditions of method 2 are satisfied
3	(1, 1)	
5	(7, 7)	
7	(15, 15)	
8	(17, 17)	
10	(22, 22)	

EST of j – EST of i: and LFT of $j - LFT$ of i

EST of $3 - EST$ of $1 = 1 - 0 = LFT$ of $3 - LFT$ of $1 = t_{13}$

EST of $5 - EST$ of $3 = 7 - 1 = 6 = LFT$ of $5 - LFT$ of $3 = t_{35}$

EST of $7 - EST$ of $5 = 15 - 7 = 8 = LFT$ of $7 - LFT$ of $5 = t_{57}$

EST of $8 - EST$ of $7 = 17 - 15 = 2 = LFT$ of $8 - LFT$ of $7 = t_{78}$

EST of $10 - EST$ of $8 = 22 - 17 = 5 = LFT$ of $10 - LFT$ of $8 = t_{8-10}$

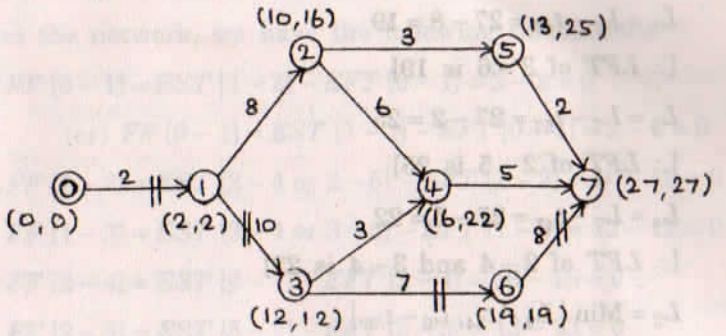
$\therefore$ 3rd condition of method 2 is satisfied. 1 – 3, 3 – 5, 5 – 7, 7 – 8 and 8 – 10 are critical activities and hence the critical path is 1 – 3 – 5 – 7 – 8 – 10

Example: 8 The utility data for a network is given below. Determine the total, free and independent floats and identify the critical path.

activity	0 – 1	1 – 2	1 – 3	2 – 4	2 – 5	3 – 4	3 – 6	4 – 7	5 – 7	6 – 7
duration (days)	2	8	10	6	3	3	7	5	2	8

(BE, MU Ap' 93)

Solution:



From the network given above, we have the following Early start time and Late finish time.

$E_0 = 0$ (i.e.) Early start time of an activity 0 – 1 is 0.

$E_1 = E_0 + t_{01} = 0 + 2 = 2$

[Early start time of 1 – 2 and 1 – 3 is 2]

$E_2 = E_1 + t_{12} = 2 + 8 = 10$

[$\therefore$ Early start time of 2 – 4 and 2 – 5 is 10, since both the activities have the same head event. (i.e.) beginning node].

$E_3 = E_1 + t_{13} = 2 + 10 = 12$

[$\therefore$ Early start time of 3 – 4 and 3 - 6 is 12].

$E_4 = \text{Max}\{E_2 + t_{24}, E_3 + t_{34}\} = \text{Max}\{10 + 6, 12 + 3\} = 16$

[$\because$ at the node E_4, we have two incoming activities. Early start time of 4 – 7 is 16].

$E_5 = E_2 + t_{25} = 10 + 3 = 13$

[$\therefore$ Early start time of 5 – 7 is 13]

$E_6 = E_3 + t_{36} = 12 + 7 = 19$

[$\therefore$ Early start time of 6 – 7 is 19]

$E_7 = \text{Max}\{E_4 + t_{47}, E_5 + t_{57}, E_6 + t_{67}\}$

$= \text{Max}\{16 + 5, 13 + 2, 19 + 8\} = 27$

Now we calculate LFT of each activity.

$L_7 = E_7 = 27$ [$\therefore$ LFT of 4 – 7, 5 – 7, 6 – 7 is 27.]

$L_6 = L_7 - t_{67} = 27 - 8 = 19$

[$\therefore$ *LFT* of 3 – 6 is 19]

$L_5 = L_7 - t_{57} = 27 - 2 = 25$

[$\therefore$ *LFT* of 2 – 5 is 25]

$L_4 = L_7 - t_{47} = 27 - 5 = 22$

[$\therefore$ *LFT* of 2 – 4 and 3 – 4 is 22]

$L_3 = \text{Min}\{L_4 - t_{34}, L_6 - t_{36}\}$

$= \text{Min}\{22 - 3, 19 - 7\} = 12$

[$\therefore$ *LFT* of 1 – 3 is 12]

$L_2 = \text{Min}\{L_4 - t_{24}, L_5 - t_{25}\}$

$= \text{Min}\{22 - 6, 25 - 3\} = 16$

[$\therefore$ *LFT* of 1 – 2 is 16]

$L_1 = \text{Min}\{L_2 - t_{12}, L_3 - t_{13}\}$

$= \text{Min}\{16 - 8, 12 - 10\} = 2$

[$\therefore$ *LFT* of 0 – 1 is 2]

$L_0 = L_1 - t_{01} = 2 - 2 = 0$

Thus we have the following Tabular form.

Activity	*EST*	*EFT*	*LST*	*LFT*	Total Float	Free Float	Indept. Float
0 – 1	0	2 (0 + 2)	0 (2 – 2)	2	0	0	0
1 – 2	2	10 (2 + 8)	8 (16 – 8)	16	6	0	0
1 – 3	2	12 (2 + 10)	2 (12 – 10)	12	0	0	0
2 – 4	10	16 (10 + 6)	16 (22 – 6)	22	6	0	6
2 – 5	10	13 (10 + 3)	22 (25 – 3)	25	12	0	– 6
3 – 4	12	15 (12 + 3)	19 (22 – 3)	22	7	1	1
3 – 6	12	19 (12 + 7)	12 (19 – 7)	19	0	0	0
4 – 7	16	21 (16 + 5)	22 (27 – 5)	27	6	16	0
5 – 7	13	15 (13 + 2)	25 (27 – 2)	27	12	12	0
6 – 7	19	27 (19 + 8)	19 (27 – 8)	27	0	0	0

$FF\,[i - j] = EST\,[J - K] - EFT\,[I - J]$

From the network, we have the following calculations

$FF\,[0 - 1] = EST\,[1 - 2] - EFT\,[0 - 1] = 2 - 2 = 0$

(or) $FF\,[0 - 1] = EST\,[1 - 3] - EFT\,[0 - 1] = 2 - 2 = 0$

$FF\,[1 - 2] = EST\,[2 - 4 \text{ or } 2 - 5] - EFT\,[1 - 2] = 10 - 10 = 0$

$FF\,[1 - 3] = EST\,[3 - 4 \text{ or } 3 - 6] - EFT\,[1 - 3] = 12 - 12 = 0$

$FF\,[2 - 4] = EST\,[4 - 7] - EFT\,[2 - 4] = 16 - 16 = 0$

$FF\,[2 - 5] = EST\,[5 - 7] - EFT\,[2 - 5] = 13 - 13 = 0$

$FF\,[3 - 4] = EST\,[4 - 7] - EFT\,[3 - 4] = 16 - 15 = 1$

$FF\,[3 - 6] = EST\,[6 - 7] - EFT\,[3 - 6] = 19 - 19 = 0$

$FF\,[4 - 7] = 27 - EFT\,[4 - 7] = 27 - 21 = 16$

$FF\,[5 - 7] = 27 - EFT\,[5 - 7] = 27 - 15 = 12$

$FF\,[6 - 7] = 27 - EFT\,[6 - 7] = 27 - 27 = 0$

Independent Floats: $IF\,[i - j] = E_j - L_i - t_{ij}$

$IF\,[0 - 1] = E_1 - L_0 - t_{01} = 2 - 0 - 2 = 0$

$IF\,[1 - 2] = E_2 - L_1 - t_{12} = 10 - 2 - 8 = 0$

$IF\,[1 - 3] = E_3 - L_1 - t_{13} = 12 - 2 - 10 = 0$

$IF\,[2 - 4] = E_4 - L_2 - t_{24} = 16 - 16 - 6 = -6$

$IF\,[2 - 5] = E_5 - L_2 - t_{25} = 13 - 16 - 3 = -6$

$IF\,[3 - 4] = E_4 - L_3 - t_{34} = 16 - 12 - 3 = 1$

$IF\,[3 - 6] = E_6 - L_3 - t_{36} = 19 - 12 - 7 = 0$

$IF\,[4 - 7] = E_7 - L_4 - t_{47} = 27 - 22 - 5 = 0$

$IF\,[5 - 7] = E_7 - L_5 - t_{57} = 27 - 25 - 2 = 0$

$IF\,[6 - 7] = E_7 - L_6 - t_{67} = 27 - 19 - 8 = 0$

Critical path is a path connected by the activities 0 – 1, 1 – 3, 3 – 6, 6 – 7

i.e. Critical Path: $0-1-3-6-7$

project duration = 27 days.

Example: 9 A project consists of a series of tasks labelled $A, B, C, D, E, F, G, H, I$ with the following relationships.

$W < X, Y$ means X and Y can not start until W is completed. $X, Y < W$ means W cannot start until both X and Y are completed.

With this notation construct the network diagram having the following constraints.

$$A < D, E;\ B, D < F;\ C < G;\ B, D < H;\ F, G < I$$

Find also the minimum time of completion of the project when the time (in days) of completion of each task is as follows

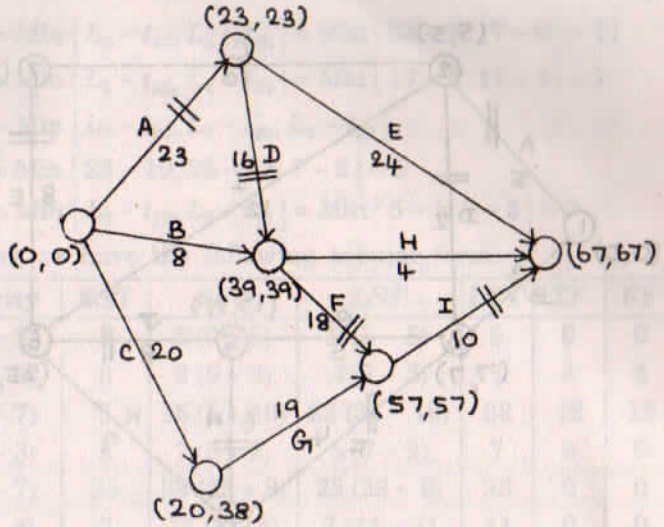
Task	A	B	C	D	E	F	G	H	I
Time	23	8	20	16	24	18	19	4	10

(BE, MU Nov' 92)

Solution:

From the network given above, we have the following. Tabular form.

activity	*EST*	*EFT*	*LST*	*LFT*	*TF*
A	0	23	0	23	0
B	0	8	31	39	31
C	0	20	18	38	18
D	23	39	23	39	0
E	23	47	43	67	20
F	39	57	39	57	0
G	20	39	38	57	18
H	39	41	63	67	24
I	57	67	57	67	0



Critical path: $A - D - F - I$

Project duration = 67 days

Example: 10 A small project consists of 11 activities $A, B, C, D, E, F, G, H, I, J$ and K. The precedence relationship are: A, B can start simultaneously; $A < C, D, I$; $B < G, F$; $D < G, F$; $F < H, K$; $G, H < J$; $I, J, K < E$. The duration of the activities are as follows.

activity	A	B	C	D	E	F	G	H	I	J	K
duration (days)	5	3	10	2	8	4	5	6	12	8	9

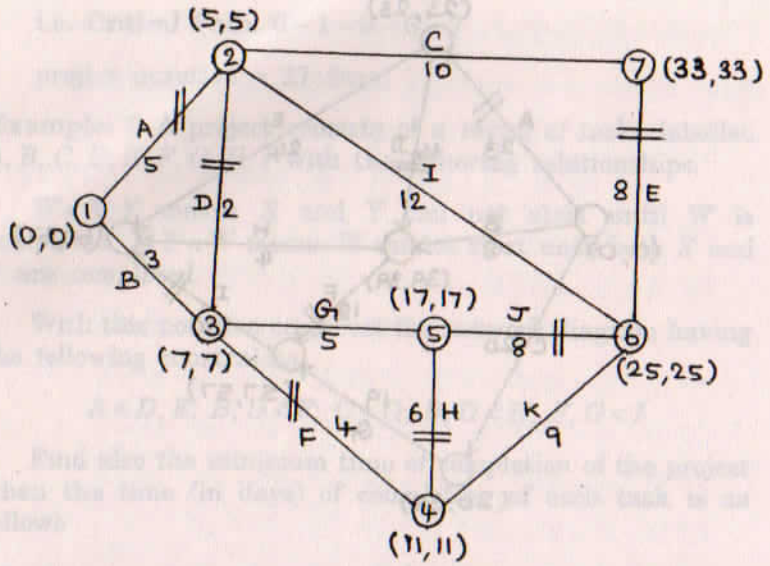
Draw the network of the project. Summarize the CPM calculations in a tabular form, computing total and free floats of activities and hence determine the critical path.

(BE, MU Ap' 96)

Solution:

For conveniance, take the nodes as 1, 2, 3, 4, 5, 6, 7 as shown in the figure.

$1-2$ corresponds to an activity A. $1-3$ corresponds to



an activity B and so on. Now we calculate the **Early start time of each activity.**

$E_1 = 0$

$E_2 = E_1 + t_{12} = 0 + 5 = 5$

$E_3 = \text{Max}\{E_2 + t_{23}, E_1 + t_{13}\} = \text{Max}\{5 + 2, 0 + 3\} = 7$

$E_4 = E_3 + t_{34} = 7 + 4 = 11$

$E_5 = \text{Max}\{E_3 + t_{35}; E_4 + t_{45}\} = \text{Max}\{7 + 5, 11 + 6\} = 17$

$E_6 = \text{Max}\{E_2 + t_{26}, E_5 + t_{56}, E_4 + t_{46}\}$

$= \text{Max}\{5 + 12, 17 + 8, 11 + 9\} = 25$

$E_7 = \text{Max}\{E_2 + t_{27}, E_6 + t_{67}\} = \text{Max}\{5 + 10, 25 + 8\} = 33$

Latest Finish time

$L_7 = E_7 = 33$

$L_6 = L_7 - t_{67} = 33 - 8 = 25$

$L_5 = L_6 - t_{56} = 25 - 8 = 17$

$L_4 = \text{Min}\{L_6 - t_{46}, L_5 - t_{45}\} = \text{Min}\{25 - 9, 17 - 6\} = 11$

$L_3 = \text{Min}\{L_5 - t_{35}, L_4 - t_{34}\} = \text{Min}\{17 - 5, 11 - 4\} = 7$

$L_2 = \text{Min}\{L_7 - t_{27}, L_6 - t_{26}, L_3 - t_{23}\}$

$= \text{Min}\{33 - 10, 25 - 12, 7 - 2\} = 5$

$L_1 = \text{Min}\{L_2 - t_{12}, L_3 - t_{13}\} = \text{Min}\{5 - 5, 7 - 3\} = 0$

Thus we have the following tabular form.

Activity	*EST*	*EFT*	*LST*	*LFT*	*TF*	*FF*
A (1 – 2)	0	5 (0 + 5)	0 (5 – 5)	5	0	0
B (1 – 3)	0	3 (0 + 3)	4 (7 – 3)	7	4	4
C (2 – 7)	5	15 (5 + 10)	23 (33 – 10)	33	18	18
D (2 – 3)	5	7 (5 + 2)	5 (7 – 2)	7	0	0
E (6 – 7)	25	33 (25 + 8)	25 (33 – 8)	33	0	0
F (3 – 4)	7	11 (7 + 4)	7 (11 – 4)	11	0	0
G (3 – 5)	7	12 (7 + 5)	12 (17 – 5)	17	5	5
H (4 – 5)	11	17 (11 + 6)	11 (17 – 6)	17	0	0
I (2 – 6)	5	17 (12 + 5)	20 (25 – 5)	25	15	8
J (5 – 6)	17	25 (17 + 8)	17 (25 – 8)	25	0	0
K (4 – 6)	11	20 (11 + 9)	16 (25 – 9)	25	5	5

Critical path is a path connected by the activities $A - D - F - H - J - E$

project duration = 33 days.

Free Float

$FF\,[i - j] = EST\,[j - k] - EFT\,[i - j]$

From the network diagram, we have

$FF\,[A] = FF\,[1 - 2] = EST\,[2 - 3 \text{ (or) } 2 - 7]$

$- EFT\,[1 - 2] = 5 - 5 = 0$

$FF\,[B] = FF\,[1 - 3] = EST\,[3 - 4 \text{ (or) } 3 - 5]$

$- EFT\,[1 - 3] = 7 - 3 = 4$

$FF\,(C) = FF\,[2 - 7] = 33 - EFT\,[2 - 7] = 33 - 15 = 18$

$FF\,[D] = FF\,[2 - 3] = EST\,[3 - 4 \text{ (or) } 3 - 5]$

$- EFT\,[2 - 3] = 7 - 7 = 0$

$FF\,[E] = FF\,(6-7) = 33 - EFT\,[6-7] = 33 - 33 = 0$

$FF[F] = FF\,[3-4] = EST\,[4-5 \text{ (or) } 4-6]$

$\quad - EFT\,[3-4] = 11 - 11 = 0$

$FF\,[G] = FF\,[3-5] = EST\,[5-6]$

$\quad - EFT\,[3-5] = 17 - 12 = 5$

$FF\,[H] = FF\,[4-5] = EST\,[5-6]$

$\quad - EFT\,[4-5] = 17 - 17 = 0$

$FF\,[I] = FF\,[2-6] = EST\,[6-7] - EFT\,[2-6] = 25 - 17 = 8$

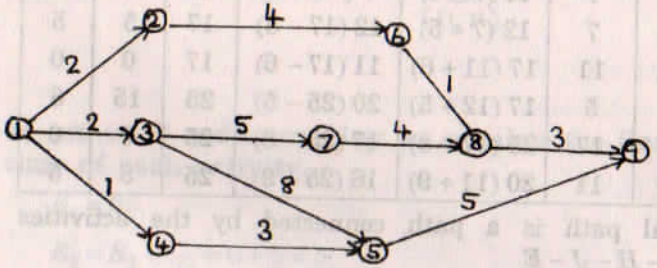
$FF\,[J] = FF\,[5-6] = EST\,[6-7]$

$\quad - EFT\,[5-6] = 25 - 25 = 0$

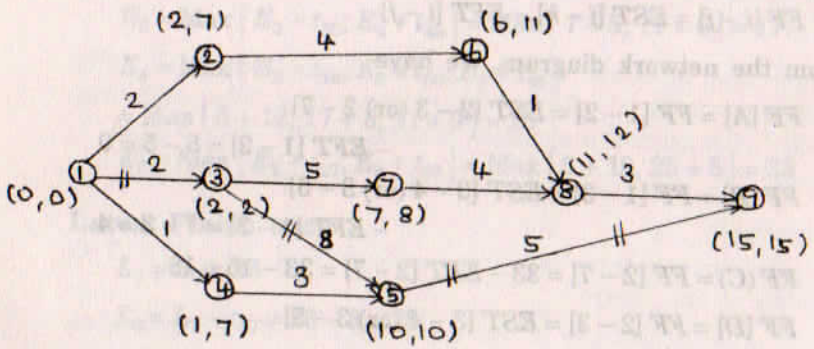
$FF\,[K] = FF\,[4-6] = EST\,[6-7]$

$\quad - EFT\,[4-6] = 25 - 20 = 5$

Example: 11 Find the critical path and calculate the slack time for each event for the following PERT diagram.



Solution:



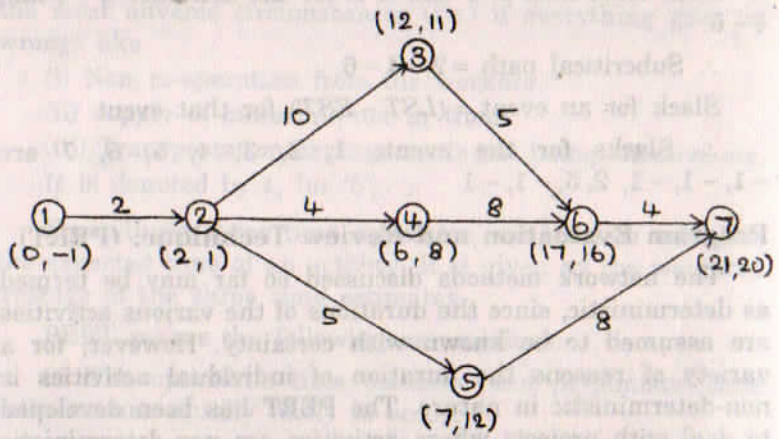
Activity	EST	EFT	LST	LFT	SLACK
1 – 2	0	2	5	7	5
1 – 3	0	2	0	2	**0**
1 – 4	0	1	6	7	6
2 – 6	2	6	7	11	5
3 – 5	2	10	2	10	**0**
3 – 7	2	7	3	8	1
4 – 5	1	4	7	10	6
5 – 9	10	15	10	15	**0**
6 – 8	6	7	11	12	5
7 – 8	7	11	8	12	1
8 – 9	11	14	10	15	– 1

Critical path = 1 – 3 – 5 – 9

Slack for an event = $(LST - EST)$ for that event

$\therefore$ Slack for the events $E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_9$ are 0, 5, 0, 6, 0, 5, 1, 1, 0.

Example: 12 For the network given below determine the float and find the critical path if the scheduled completion time for the project is 20 weeks. Also find the subcritical path and slacks for events.



Solution: The Scheduled project completion time is 20 weeks

i.e. LFT of the event 7 = 20

activity	EST	EFT	LST	LFT	Total float (Slack)
1 – 2	0	2	– 1	1	– 1
2 – 3	2	12	1	11	– 1
2 – 4	2	6	4	8	2
2 – 5	2	7	7	12	5
3 – 6	12	17	11	16	– 1
4 – 6	6	14	8	16	2
5 – 7	7	15	12	20	5
6 – 7	17	21	16	20	– 1

Here the least (most – *ve*) slack is – 1.

It is for the activities 1 – 2, 2 – 3, 3 – 6, 6 – 7 (i.e., for the events 1, 2, 3, 6, 7).

∴ Critical path is a path joining the events 1 – 2 – 3 – 6 – 7.

The path connecting the next least floats of the activity is called the **subcritical path**.

Next least float is 2 and it is for the activities 2 – 4 and 4 – 6.

∴ Subcritical path = 2 – 4 – 6

Slack for an event = (*LST* – *EST*) for that event

∴ Slacks for the events 1, 2, 3, 4, 5, 6, 7 are – 1, – 1, – 1, 2, 5, – 1, – 1

Program Evaluation and Review Technique: (PERT)

The network methods discussed so far may be termed as deterministic, since the durations of the various activities are assumed to be known with certainty. However, for a variety of reasons the duration of individual activities is non-deterministic in nature. The PERT has been developed to deal with projects whose activities are non-deterministic in nature. PERT was developed for the purpose of solving problems in aerospace industries, particularly in research and development programmes. The activities which are based on research and development are usually uncertain in

nature. For example, a part of the project may be dependent upon the successful development of a new electronic device. Accordingly the corresponding duration will be uncertain depending upon the likelihood of successful development of the device.

The PERT system is based on three time estimates of the performance time of an activity.

(i) optimistic time estimate

(ii) most likely time estimate

(iii) pessimistic time estimate

The **optimistic time estimate** is the smallest time to complete the activity if everything goes on exceptionally well or under best possible conditions. It may be less than the most likely time. Optimistic time is denoted by t_0 (or '*a*').

The **most likely time estimate** is the estimate of normal time an activity would take. It is the best possible time estimate that a given activity would take under normal conditions and is denoted by t_m (or '*m*').

The **pessimistic time estimate** is the best guess of the longest (maximum) time that an activity would take under the most adverse circumstances (i.e.) if everything goes on wrong) like

(i) Non co-operation from the workers

(ii) supply of materials not in time

(iii) Transportation arrangements not being effective etc.

It is denoted by t_p (or '*b*').

From these three time estimates, we have to calculate the expected time of an activity. It is given by the weighted average of the three time estimates.

PERT makes the following assumptions.

(i) The optimistic time estimate and pessimistic time estimate are equally likely to occur.

(ii) The most likely time estimate is four times more likely to occur than either of the other two.

The above assumptions are based on the properties of β distribution.

With this assumption, the expected time (t_e) [Mean or average] for each activity can be computed as follows

$$t_e = \frac{1}{6}[t_o + 4t_m + t_p]$$

[β distribution with weights of 1, 4, 1 for optimistic, most likely and pessimistic time estimates respectively.]

Variance (V_t) of the activity is given by

$$V_t = \left[\frac{t_p - t_o}{6}\right]^2$$

The expected length (duration), denoted by T_e, of the entire project is the length of the critical path. (i.e) the sum of the t_e's of all activities along the critical path.

Variance of T_e equals the sum of all the variances of the critical activities.

With PERT, it is possible to determine the probability of completing a project on schedule.

Example: 13 Draw the network of the following project of activities. Calculate the length and variance of the critical path. The three time estimates t_0, t_m and t_p are in days.

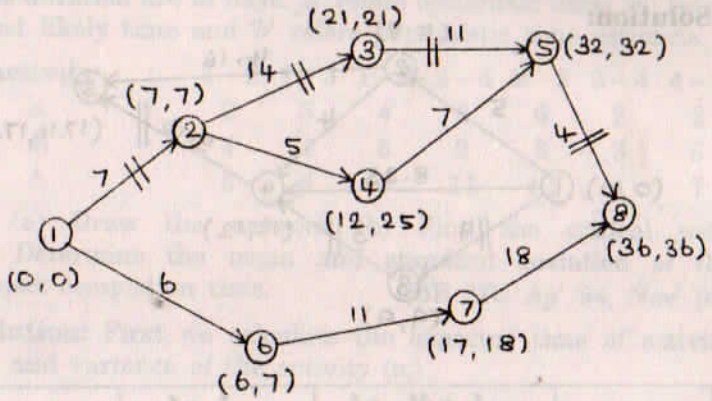
Activity	1 – 2	1 – 6	2 – 3	2 – 4	3 – 5	4 – 5	6 – 7	5 – 8	7 – 8
t_0	3	2	6	2	5	3	3	1	4
t_m	6	5	12	5	11	6	9	4	19
t_p	15	14	30	8	17	15	27	7	28

First we calculate the expected time and variance of each activity.

Solution:

activity	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$S_t = \frac{t_p - t_0}{6}$	v_t
1 – 2	7	2	4
1 – 6	6	2	4
2 – 3	14	4	16

activity	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$S_t = \frac{t_p - t_0}{6}$	v_t
2 – 4	5	1	1
3 – 5	11	2	4
4 – 5	7	2	4
6 – 7	11	4	16
5 – 8	4	1	1
7 – 8	18	4	16



Numbers on the arrow represents the expected time of the activities.

Critical path is a path connected by the activities 1 – 2, 2 – 3, 3 – 5, 5 – 8.

Critical path: 1 – 2 – 3 – 5 – 8

Length of the critical path = 7 + 14 + 11 + 4 = 36 (i.e.) Mean of the project completion time = 36 (days).

Variance of the project duration = sum of all the variances of critical activities.

= 4 + 16 + 4 + 1 = 25

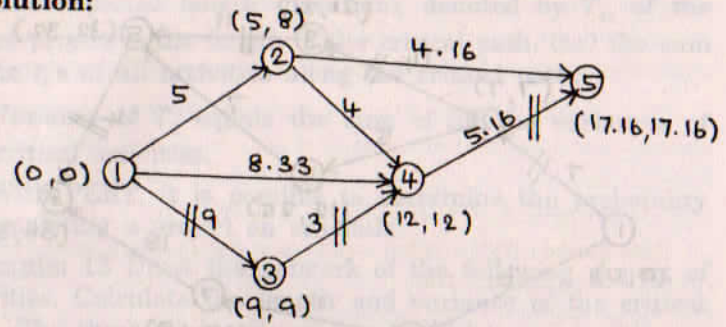
(i.e.) Variance of the critical path = 25 days.

Example: 14 The following indicates the details of the activities of a project. The duration are in days

activity	1 – 2	1 – 3	1 – 4	2 – 4	2 – 5	3 – 4	4 – 5
t_0	4	8	6	2	3	2	3
t_m	5	9	8	4	4	3	5
t_p	6	11	12	6	6	4	8

(a) Draw the network (b) Find the critical path (c) Find the mean and standard deviation of the project completion time. (BE MU Nov' 88)

Solution:



activity	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$s_t = \frac{t_p - t_0}{6}$	v_t
1 – 2	5	$\frac{1}{3}$	$\frac{1}{9}$
1 – 3	9	$\frac{1}{2}$	$\frac{1}{4}$
1 – 4	8.33	$\frac{1}{2}\frac{2}{3}$	$\frac{1}{4}\frac{1}{9}$
2 – 4	4	$\frac{1}{3}$	$\frac{1}{9}$
2 – 5	4.16	$\frac{1}{2}$	$\frac{1}{4}$
3 – 4	3	$\frac{1}{3}$	$\frac{1}{9}$
4 – 5	5.16	$\frac{5}{6}$	$\frac{25}{36}$

Critical path: 1 – 3 – 4 – 5

Mean of the project completion time

$= 9 + 3 + 5.16 = 17.16$

Standard deviation of the project completion time

$= \sqrt{\text{sum of the variances of critical activities}}$

$= \sqrt{\frac{1}{4} + \frac{1}{9} + \frac{25}{36}} = \sqrt{\frac{38}{36}}$

Example: The following indicates the details of a project. The duration are in days. 'a' refers optimistic time, 'm' refers most likely time and 'b' refers persimistic time estimate.

Activity	:	1 – 2	1 – 3	1 – 4	2 – 4	2 – 5	3 – 4	4 – 5
a	:	2	3	4	8	6	2	2
m	:	4	4	5	9	8	3	5
b	:	5	6	6	11	12	4	7

(a) Draw the network (b) Find the critical path (c) Determine the mean and standard deviation of the project completion time. [BE MU Ap' 94, Nov' 95]

Solution: First we calculate the expected time of activity (t_e) and variance of the activity (v_t)

Activity	:	1 – 2	1 – 3	1 – 4	2 – 4	2 – 5	3 – 4	4 – 5
$t_e = \frac{t_0 + 4t_m + t_p}{6}$	:	3.83	4.16	5	9.16	8.33	3	4.83
$v_t = \left(\frac{t_p - t_0}{6}\right)^2$	:	0.25	0.25	0.11	0.25	1	0.11	0.69

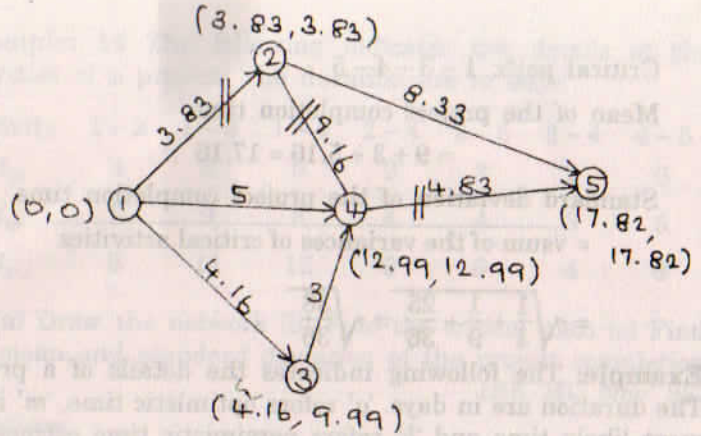
By making use of the expected time of activities we have the following network

There are four paths:

1 – 2 – 5	=	12.16 days
1 – 2 – 4 – 5	=	17.82 days
1 – 4 – 5	=	9.83 days
1 – 3 – 4 – 5	=	11.99 days

1 – 2 – 4 – 5 is the longest path and hence the critical path.

Mean of the project completion time = 17.82 days (Expected length of critical path.)



Variance of the project completion time
= sum of the variances of critical activities
$= 0.25 + 0.25 + 0.69 = 1.19$

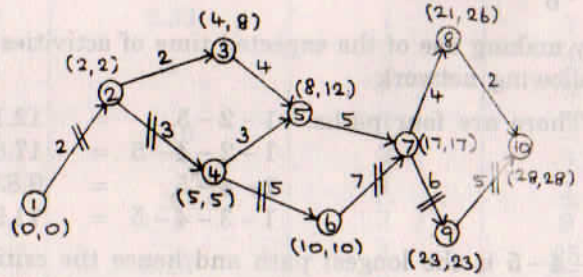
$\therefore$ S.D. of the project completion time $= \sqrt{1.19}$

Example: 16 A project schedule has the following characteristics.

activity:	1-2	2-3	2-4	3-5	4-5	4-6	5-7	6-7	7-8	7-9	8-10	9-10
t_0:	1	1	1	3	2	3	4	6	2	4	1	3
t_m:	2	2	3	4	3	5	5	7	4	6	2	5
t_p:	3	3	5	5	4	7	6	8	6	8	3	7

Construct a PERT network and find out (a) The earliest possible time T_E to complete the different stages of the project (*EFT*) (b) The latest allowable time T_L for them (*LST*) (c) The slack values (d) The critical path (e) The probability factor for completing the project in 30 weeks.

(BE, MU Nov' 92)



The numbers on the arrow diagram represents the expected time of the activities.

Activity	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	*EST*	*EFT*	*LST*	*LFT*	SLACK	V_t
1 – 2	2	0	2	0	2	0	0.11
2 – 3	2	2		6	8	4	
2 – 4	3	2	5	2	5	0	0.45
3 – 5	4	4	8	8	12	4	
4 – 5	3	5	8	9	12	4	
4 – 6	5	5	10	5	10	0	0.45
5 – 7	5	8	13	12	17	4	
6 – 7	7	10	17	10	17	0	0.11
7 – 8	4	17	21	22	26	5	
7 – 9	6	17	23	17	23	0	0.45
8 – 10	2	21	23	26	28	5	
9 – 10	5	23	28	23	28	0	0.45

Critical path 1 – 2 – 4 – 6 – 7 – 9 – 10

Expected project duration = 28 weeks

(i.e.) $T_e = 28$ weeks

Now we calculate the probability for completing the project in 30 weeks.

Here $T = 30$ Now $z = \dfrac{T - T_e}{S.D.}$

Variance of the project completion time

$= 0.11 + 0.45 + 0.45 + 0.11 + 0.45 + 0.45 = 2.02$

SD $\sqrt{2.02} = 1.42$ $\quad \therefore Z = \dfrac{30 - 28}{1.42} = 1.41$

$P[T = 30] = P[Z = 1.41] =$ 0.4207

Probability for completing the project in 30 weeks is 0.4207

Example: 17 The following table shows the jobs of a network along with their time estimates.

Job:	1-2	1-6	2-3	2-4	3-5	4-5	6-7	5-8	7-8
a (days):	1	2	2	2	7	5	5	3	8
m (days):	7	5	14	5	10	5	8	3	17
b (days):	13	14	26	8	19	17	29	9	32

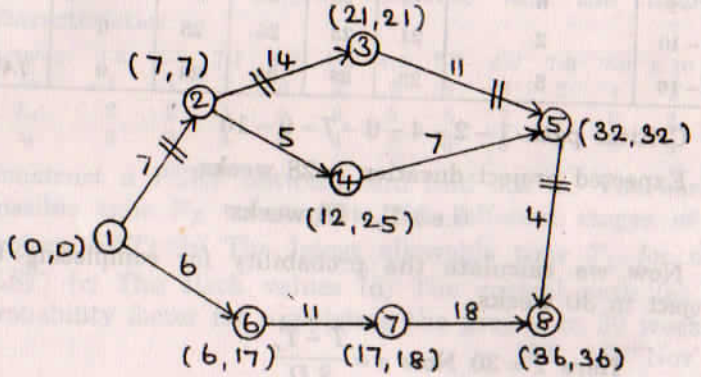
Draw the project network and find the probability that the project is completed in 40 days.

(BE Mech. MU Oct' 96)

Solution: First we calculate the expected time and S.D. of each activity.

Activity:	1-2	1-6	2-3	2-4	3-5	4-5	6-7	5-8	7-8
t_e:	7	6	14	5	11	7	11	4	18
s_t:	2	2	4	1	2	2	4	1	4

Making use of expected time of the activities, we can determine the project duration.



Expected project duration = 36 days.

Critical Path: 1 – 2 – 3 – 5 – 8

Now we calculate the probability that the project is completed in 40 days

Here $T_e = 36$, $T = 40$

$$SD = \sqrt{4 + 16 + 4 + 16} = \sqrt{40} = 6.3$$

$$z = \frac{T - T_e}{\sigma} = \frac{40 - 36}{6.3} = 0.6$$

$P[T = 40] = P[z = 0.6] = 0.2257$

∴ Probability for completing the project in 40 days is 0.2257

Example: 18 Data for a network is given below.

Activity:	1 – 2	1 – 3	1 – 4	2 – 5	3 – 5	4 – 6	5 – 6
t_0:	1	1	2	1	2	2	3
t_m:	1	4	2	1	5	5	6
t_p:	7	7	8	1	14	8	15

(a) Draw the project network (b) Find the probability that the project will be completed

(i) atleast 4 weeks earlier than expected time (ii) no more than 4 weeks later than expected time (c) If the project due date is 20 weeks, what is the probability of not meeting the due date. (d) if the probability for the completion of the project is 90%, find the scheduled completion time.

Solution:

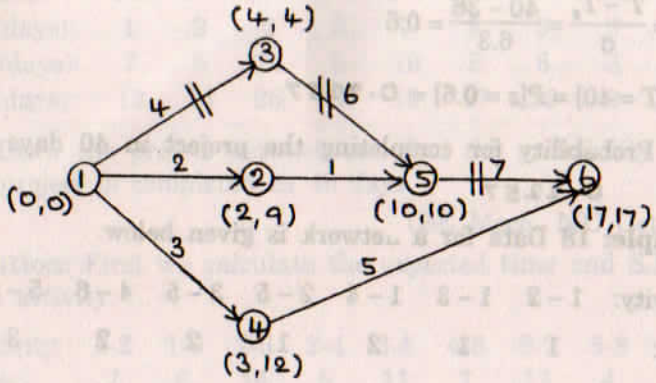
First we calculate the expected time and variance of each activity.

Activity:	1 – 2	1 – 3	1 – 4	2 – 5	3 – 5	4 – 6	5 – 6
t_e:	2	4	3	1	6	5	7
v_t:	1	1	1	0	4	1	4

Activity duration (expected time) are written along the activity arrows.

The earliest start and latest finish times are represented in the network around the respective nodes.

Critical path: 1 – 3 – 5 – 6



Length of the critical path $= 4 + 6 + 7 = 17$

(i.e.) The expected project length = 17 weeks

Variance of the project length

= sum of the variances of the activities on the critical path

$= 1 + 4 + 4 = 9$ weeks

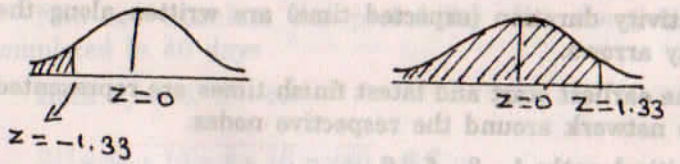
$\therefore S.D. = 3$

Thus $T_e = 17$ and $S.D. = 3$

(b) (i) P [the project will be completed atleast 4 weeks earlier than expected time]

$= P[T \le 13]$

$$z = \frac{T - T_e}{S.D.} = \frac{13 - 17}{3} = -1.33$$



$\therefore P[X \le 13] = P[Z \le -1.33] = .5 - P[0 \le z \le -1.33]$

$= 0.5 -$ Table value of $z = -1.33$ (or) 1.33

$= 0.5 - 0.4082 = 0.0918$

(ii) P [The project will be completed no more than 4 weeks later than expected time].

$= P[T \le 21] = P[z \le 1.33] = 0.5 + p\ [0 \le z \le 1.33]$

$= 0.5 +$ Table value of $z = 1.33 = 0.5 + 0.4082 = 0.9082$

(c) Project due date $T = 20$ weeks

P[Not meeting the due date] = 1 — P[Meeting the due date]

$= 1 - p[T = 20] = 1 - P[z = 1] = 1 - 0.8413 = 0.1587$

(d) The probability for the completion of the project is 90% (i.e) 0.90.

$\therefore$ Corresponding value of $z = 1.28$

$$z = \frac{T - T_e}{S.D.} \Rightarrow 1.28 = \frac{T - 17}{3} \therefore T = 20.84 \text{ weeks}$$

Cost analysis and crashing network:

Each activity of the project consumes some resources and hence cost has associated with it. The total cost of any project comprises direct and indirect costs.

Direct cost: The cost is directly dependent upon the amount of resources involved in the execution of individual activities. It includes the direct material cost and labour cost. The direct costs increases if the activity duration is to be reduced. (i.e) if the activity duration is to be reduced, it involves additional cost due to overtime expenses, cost of additional labour etc.

Indirect cost: Indirect costs are associated with the overhead expenses such as managerial services, indirect supplies, general administration, stores expense and cost of security personnel, etc. The indirect cost is computed on a per day, per week, or per month basis. The indirect costs increase if the activity duration is to be increased.

The network diagram can be used to identify the activities whose duration should be shortened so that the

completion time of the project can be shortened in the most economic manner. This procedure is known as **Network-crashing**. It is usually achieved by adding extra resources to an activity. The procedure is explained using the following definitions.

Normal cost: The direct cost associated with an activity of a project at normal level of resource is known as **Normal cost** of the activity. The corresponding time taken by the activity is known as the **normal time.**

We note that for technical reasons, the duration of an activity cannot be reduced indefinitely. The **crash time** represents the minimum activity duration time that is possible, and any attempts to further 'crash' or 'squeeze' would only raise the direct costs of the activities without reducing the time. The activity cost corresponding to the crash time is called the 'Crash cost'.

Cost slope: The cost slope indicating the increase in cost per unit reduction in time and is defined as

$$\text{Cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

(i.e) The cost slope represents the rate of increase in the cost of performing the activity per unit reduction in time and is called time-cost trade off.

The algorithm for project crashing:

Step 1: Using the Normal time of the given activities, find the critical path and identify the critical activities.

Step 2: Compute the cost slope for the different activities.

Step 3: Since the critical path controls the project duration, crash the various critical activities step by step. Activity having least cost slope would be crashed first to the maximum extent possible. Calculate the new total cost of the project.

Step 4: By crashing the activities on the critical path, other paths also become critical.

In such cases the project duration can be reduced by simultaneous crashing of activities in the parallel critical paths.

Step 5: Find the total cost of the project at each step.

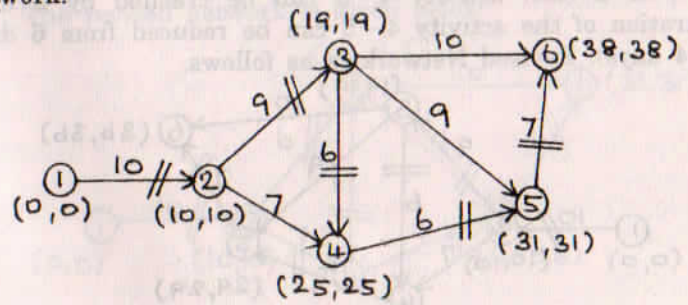
Step 6: Continue the process until all the critical activities are fully crashed or no further crashing is avaialable.

It should be noted that as the project duration is reduced, the total cost may also reduced, reaching the minimum value, beyond which it may increase. This minimum cost is called the **optimum project cost** and the corresponding time, **the optimum project time.**

Example: 1) The data for a network is given below. Crash the network to minimize the project duration and determine the project cost for that duration.

Activity	Normal		Crash	
	Time (days)	cost (Rs.)	Time (days)	cost (Rs.)
1 – 2	10	60	8	120
2 – 3	9	75	6	150
2 – 4	7	90	4	150
3 – 4	6	100	5	140
3 – 5	9	50	7	80
3 – 6	10	40	8	70
4 – 5	6	50	4	70
5 – 6	7	70	5	110

Solution: Making use of normal time we have the following network.



Critical path: 1 – 2 – 3 – 4 – 5 – 6

Project (Normal) duration = 38 days

Cost of the project = Sum of Normal cost of all activities

= 60 + 75 + 90 + 100 + 50 + 40 + 50 + 70

= Rs. 535/-

Now we require the minimum duration of the project.

Since the critical path controls the project duration, Crash atleast one of the activities on the critical path. For this first we compute incremental cost (or) cost slope of each activity. [cost of crashing the activity per day].

$$\text{cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

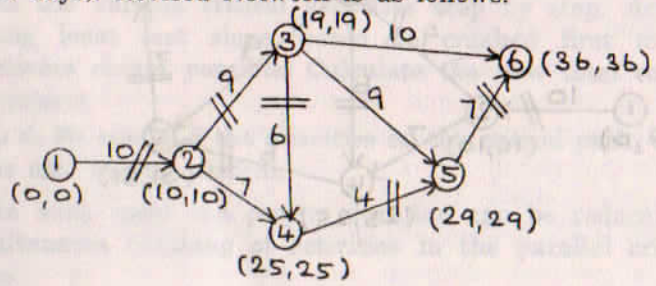
Activity	:	1 – 2	2 – 3	2 – 4	3 – 4	3 – 5	3 – 6	4 – 5	5 – 6
Cost slope	:	30	25	20	40	15	15	10	20

To reduce the project completion time, select the minimum cost slope on the critical path.

Here the minimum cost slope on the critical path is 10. Corresponding activity is 4 – 5. So we crash the activity 4 – 5 at an additional expense of Rs.10/- per day.

We have to see how many days are available for crashing and how much of it can be crashed before the other non-ctirical paths become ctirical. [Crashing of any activity should not affect the critical path].

Number of days available for crashing the activity 4 – 5 is 2 (i.e.) activity 4 – 5 can be crashed by 2 days (duration of the activity 4 – 5 can be reduced from 6 days to 4 days). Revised Network is as follows.



Critical path: 1 – 2 – 3 – 4 – 5 – 6

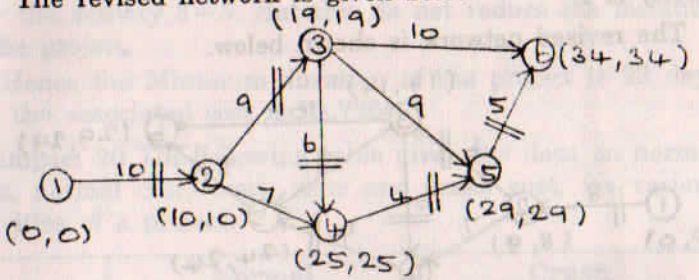
Project duration = 36 days

cost of the project = 38 days cost + cost of crashing

= 535 + 2(10) = 555/-

Minimum cost slope on the critical path is 20. So we can crash the activity 5 – 6 at an extra cost of Rs.20/day. Time available for crashing this activity is 2 days.

The revised network is given below



Again we have the same critical path.

Critical path: 1 – 2 – 3 – 4 – 5 – 6

Project duration = 34 days

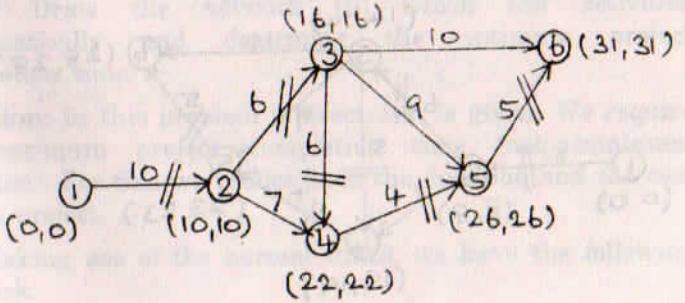
cost = 36 days cost + cost of crashing

= 555 + 2(20) = 595/- Rs.

Minimum cost slope on the critical path is 25.

∴ The activity 2 – 3 can be crashed at an extra cost of Rs.25/day. Time available for crashing this activity is 3 days. So we have to pay an additional amount of Rs.3(25) = 75

The revised network is given below.



Critical path: 1 – 2 – 3 – 4 – 5 – 6

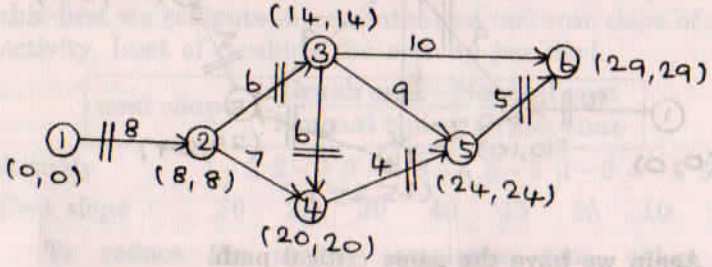
project duration = 31 days

cost of the project = 34 days cost + cost of crashing

= 595 + 3(25) = 670/- Rs.

Minimum cost slope on the critical path is 30. So we can crash the activity 1 – 2 at an additional expense of Rs.30/day. Number of days available for crashing this activity is 2.

The revised network is shown below.



Critical path: 1 – 2 – 3 – 4 – 5 – 6

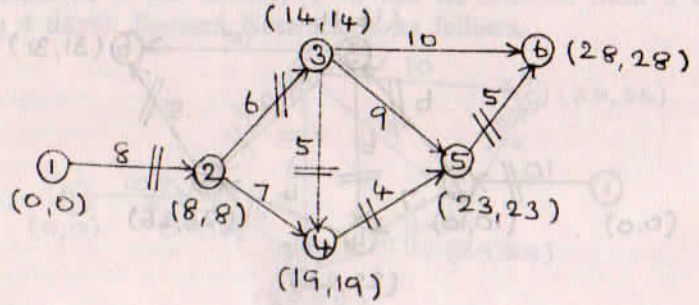
project duration = 29 days

cost of the project = 31 days cost + cost of crashing

= 670 + 2(30) = Rs.730/-

Minimum cost slope on the critical path is 40. So the activity 3 – 4 can be crashed by 1 day at an extra cost of Rs.40/day.

The revised network is depicted below.



Now we have two critical paths. One is 1 – 2 – 3 – 4 – 5 – 6 other one 1 – 2 – 3 – 5 – 6.

Project duration = 28 days

cost of the project = 29 days cost + cost of crashing

= 730 + 1(40) = Rs.770/-

All the activities on the critical path 1 – 2 – 3 – 4 – 5 – 6 are crashed already. So try to crash the activities on the critical path 1 – 2 – 3 – 5 – 6. We can crash only the activity 3 – 5. But this do not reduce the duration of the project.

Hence the Minimum duration of the project is 28 days and the associated cost is Rs.770/-.

Example: 20 The following table gives the data on normal time, normal cost, crash time and crash cost, for various activities of a project.

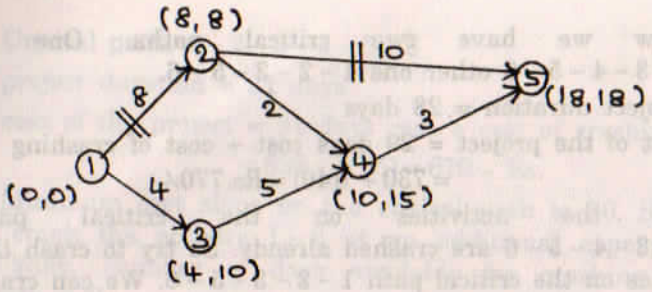
Activity	Normal		Crash	
	Time (days)	Cost (Rs.)	Time (days)	Cost (Rs.)
1 – 2	8	100	6	200
1 – 3	4	150	2	350
2 – 4	2	50	1	90
2 – 5	10	100	5	400
3 – 4	5	100	1	200
4 – 5	3	80	1	100

Indirect cost per day is Rs.70/-.

(i) Draw the network (ii) Crash the activities systematically and determine the optimum project completion time.

Solution: In this problem indirect cost is given. We require the optimum project completion time (not minimum duration). For this we reduce both the duration and the cost of the project.

Making use of the normal times, we have the following network.



Critical path: 1 – 2 – 5

Project (Normal) duration = 18 days

cost of the project = Normal cost of all activities + Indirect cost

$= 100 + 150 + 50 + 100 + 100 + 80 + 18\,(70)$

= 1840/- Rs.

Now we calculate the cost slope of each activity.

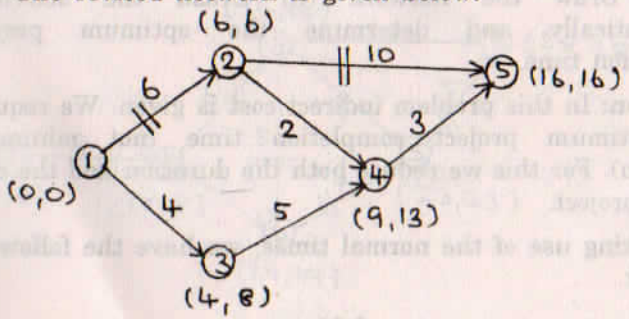
activity	:	1 – 2	1 – 3	2 – 4	2 – 5	3 – 4	4 – 5
cost slope	:	50	100	40	60	25	20

We can reduce the project duration from 18 days by crashing the time for activities on the critical path.

Minimum cost slope on the critical path is 50.

So we can crash the activity 1 – 2 at an extra cost of Rs.50/day. Since the indirect cost is Rs.70/day, there is a net saving of Rs.20 (70 – 50) if we reduce the activity duration by a day. Time available for crashing this activity is 2 days.

The revised network is given below.



Again we have the same critical path. 1 – 2 – 5

Project duration = 16 days.

Cost of the project = 18 days cost + cost of crashing — Indirect cost saving

$= 1840 + 2(50) - 2(70)$

= 1800 Rs.

Now the minimum cost slope on the critical path is 60.

∴ activity 2 – 5 can be crashed at an extra cost of Rs.60/day. There is still a net saving of Rs.70 – 60 = 10 if we reduce the time taken for activity by a single day. Number of days available for crashing this activity is 5. But the activity 2 – 5 can not be crashed by 5 days, since it will affect the original critical path.

[At the same time, if we crash the activity 2 – 5 by 5 days, then the project duration is 12 days. Duration of the project is reduced from 16 to 12. Cost of crashing is 5(60) = 300. Indirect cost saving = 4(70) = 280

∴ Cost of the project = 16 days cost + cost of Crashing — Indirect cost saving

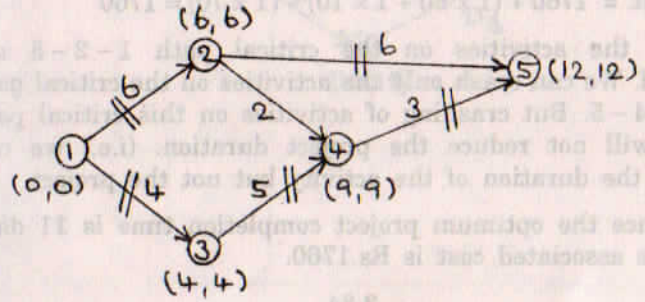
$= 1800 + 300 - 280 = 1820$

The cost of the project is not reduced.

∴ Cost of crashing should not exceed the indirect cost saving.]

We can crash the activity 2 – 5 only by 4 days (i.e. we can reduce the duration from 10 days to 6 days).

The revised network is as follows



Now we have two critical paths

$$1-2-5 \text{ and } 1-3-4-5$$

project duration = 12 days

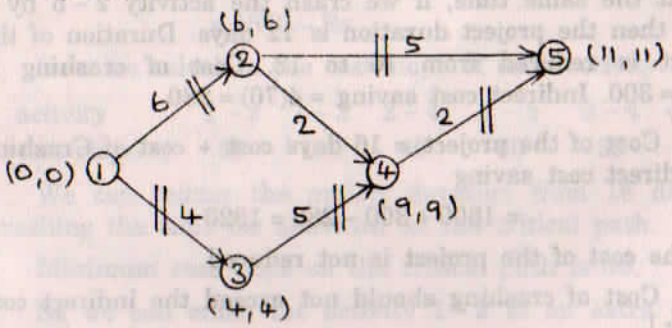
$$\text{cost} = 1800 + 4(60) - 4(70) = 1760$$

The minimum cost clope corresponds to an activity 4 – 5. But this cannot be crashed alone.

[Since crashing of 4 – 5 alone will not reduce the project duration.]

∴ Crash the activity 4 – 5 and 2 – 5 simultaneously [Not 4 – 5 and 1 – 2, since the minimum number of days required for 1 – 2 is 6.] Number of days available for crashing 4 – 5 is 2 and 2 – 5 is 1 (since the minimum number of days required for 2 – 5 is 5).

∴ Crash the activities 4 – 5 and 2 – 5 simultaneously by one day each.



Again we have the same critical paths 1 – 2 – 5 and 1 – 3 – 4 – 5

Project duration = 11 days

$$\text{Cost} = 1760 + (1 \times 60 + 1 \times 10) - (1 \times 70) = 1760$$

All the activities on the critical path 1 – 2 – 5 are crashed. We can crash only the activities on the critical path 1 – 3 – 4 – 5. But crashing of activities on this critical path alone will not reduce the project duration. (i.e.) we can reduce the duration of the activity but not the project.

Hence the optimum project completion time is 11 days and the associated cost is Rs.1760.

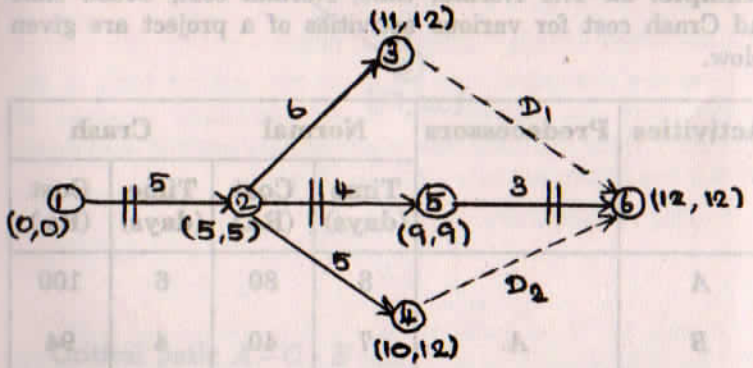
optimal time-cost trade off is time = 11 days and cost = Rs. 1760

Example: 21 For the network given below find the optimum cost schedule for the completion of the project.

activity	Normal		Crash	
	time (days)	cost (Rs.)	time (days)	cost (Rs.)
1 – 2	5	3000	4	4000
2 – 3	6	1200	2	2000
2 – 5	4	1000	3	1800
2 – 4	5	1200	3	2000
5 – 6	3	1600	3	1600

Indirect cost per day is Rs.300.

Solution: Making use of the normal time, we have the following network.



Critical path: 1 – 2 – 5 – 6

(Normal) Duration of the project = 12 days.

Cost of the project = Normal cost of all activities + Indirect cost

$= (3000 + 1200 + 1000 + 1200 + 1600) + 12(300)$

$= 11{,}600$ Rs.

Now we calculate the minimum cost slope of each activity.

activity	1 – 2	2 – 3	2 – 5	2 – 4	5 – 6
cost slope	1000	200	800	400	–

Minimum cost slope on the critical path is 800. So we crash the activity 2 – 5 at an extra cost of Rs.800/day. Number of days available for crashing this activity is 1.

Cost of crashing (800) exceeds the indirect cost savings (300). We can reduce only the duration but not the cost.

∴ optimum duration is 12 days.

optimum cost = Rs. 11,600.

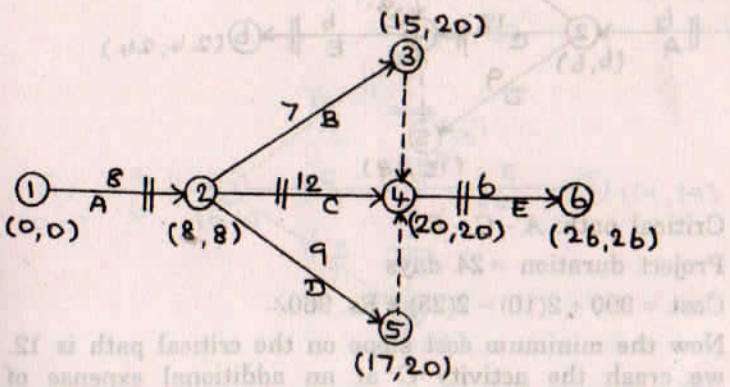
Example: 22 The Normal time, Normal cost, Crash time and Crash cost for various activities of a project are given below.

Activities	Predecessors	Normal		Crash	
		Time (days)	Cost (Rs.)	Time (days)	Cost (Rs.)
A	–	8	80	6	100
B	*A*	7	40	4	94
C	*A*	12	100	5	184
D	*A*	9	70	5	102
E	*B, C, D*	6	50	6	50

Indirect cost per day is Rs.25/-

(i) Draw the network (ii) Find (a) the normal duration of the project and associated cost (b) Minimum duration of the project and associated cost (c) Optimum duration and associated cost

Solution:



Critical path: $A - C - E$

Project (Normal) duration = 26 days.

Cost of the project

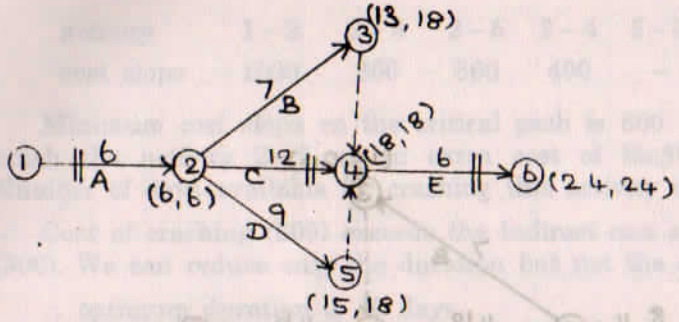
$= 80 + 40 + 100 + 70 + 50 + 26(25) =$ Rs. 990/-

Now we calculate the cost slope of each activity.

activity	A	B	C	D	E
C.S.	10	18	12	8	–

Minimum cost slope on the critical path is 10. So we crash the activity A at an extra cost of Rs.10/day. Number of days available for crashing this activity is 2 [8 – 6].

The Revised network is



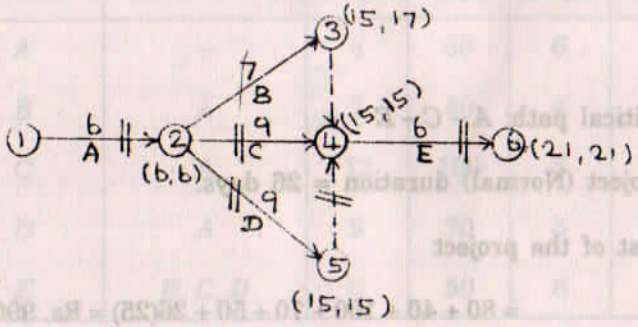
Critical path: $A - C - E$

Project duration = 24 days

Cost = 990 + 2(10) – 2(25) = Rs. 960/-

Now the minimum cost slope on the critical path is 12. So we crash the activity C at an additional expense of Rs.12/day. Time available for crashing this activity is 7 days. But we crash the activity only by 3 days.

∴ The revised network is



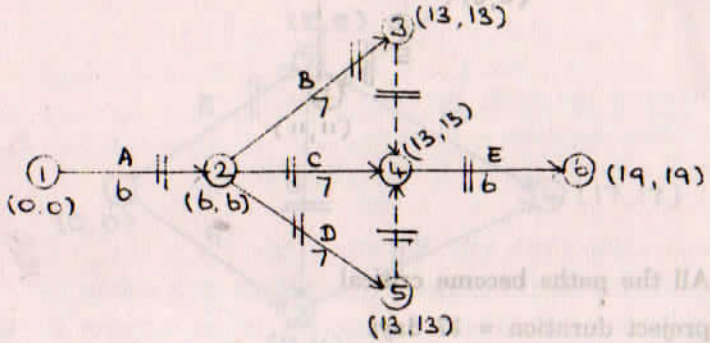
Critical paths: $A - C - E$ and $A - D - E$

project duration = 21 days

cost = 960 + 3(12) – 3(25) = Rs.921/-

The minimum cost slope on the critical path is 8. So we crash the activity D. But this can not be crashed alone. (since crashing of D will not reduce the project duration.) Next minimum cost slope corresponds to C. Number of days available for crashing D is 4, and C is 4. (Since C is crashed already by 3 days).

But the activities C and D can be crashed simultaneously by 2 days each.



Now all the paths become critical.

Project duration = 19 days

cost = 921 + [2(8) + 2(12)] – 2(25)

= Rs.911/-

The minimum cost slope corresponds to an activity D. This can not be crashed alone. So we crash the activities B, C, D simultaneously. Cost of crashing the same per day is 18 + 12 + 8 = 38 which exceeds the indirect cost per day. (i.e.) there is a net increase of Rs.13 if we reduce the duration of the activities B, C, D simultaneously by 1 day each.

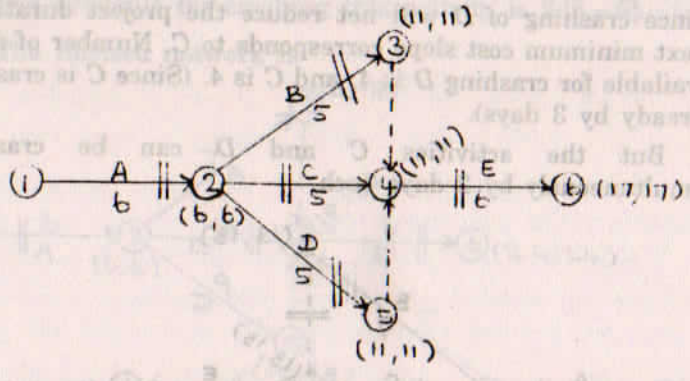
∴ optimum duration = 19 days

cost = Rs.911/-

If the cost is not in consideration, we proceed further to find the minimum duration.

Time available for crashing B, C, D simultaneously is 2 days.

The revised network is



All the paths become critical

project duration = 17 days

cost = 911 + 2 (18 + 12 + 8) – 2(25)

= Rs.937/-

Activities C and D have reached their minimum duration (though an activity B can be crashed by 1 more day).

Hence it is not possible to reduce further.

∴ Normal duration = 26 days; cost = Rs.990/-

Minimum duration = 17 days; cost = Rs.937/-

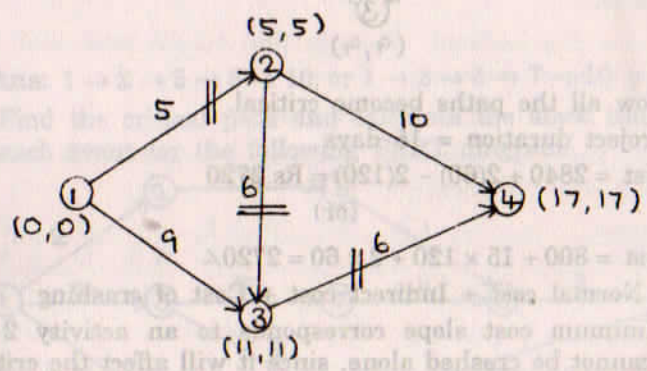
Optimum duration = 19 days; cost = Rs.911/-

Example: 23 For the data given below (i) draw the network of the project (ii) Find the normal duration and cost of the project (iii) Find the optimum duration and the corresponding minimum cost.

activity	Normal		Crash	
	Time (days)	Cost (Rs.)	Time (days)	Cost (Rs.)
1 – 2	5	170	4	240
1 – 3	9	310	6	550
2 – 3	6	80	4	200
2 – 4	10	130	8	230
3 – 4	6	110	4	290

Indirect cost of the project per day is Rs.120/-.

Solution: Making use of normal times, we have the following network.



Critical path: 1 – 2 – 3 – 4

Project (Normal) duration = 17 days

Cost of the project

= (170 + 310 + 80 + 130 + 110) + 17(120)

= Rs.2840/-

Slope of the activities are as follows.

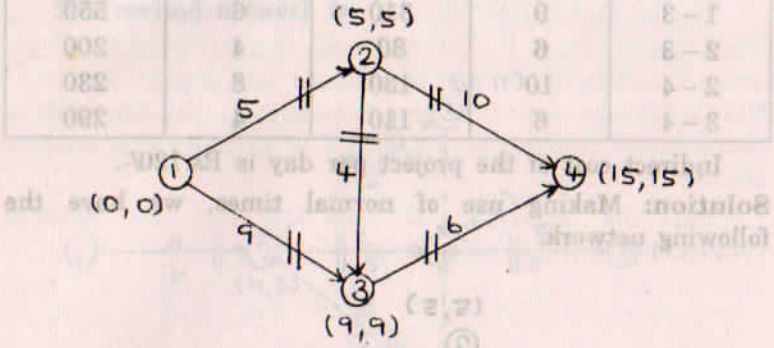
Activity	1 – 2	1 – 3	2 – 3	2 – 4	3 – 4
Slope	70	80	60	50	90

Minimum cost slope on the critical path corresponds to an activity 2 – 3.

So we can crash the activity 2 – 3 at an extra cost of

Rs.60/day. Number of days available for crashing this activity is 2.

The revised network is as follows.



Now all the paths become critical.

Project duration = 15 days

cost = 2840 + 2(60) − 2(120) = Rs.2720

(or)

cost = 800 + 15 × 120 + 2 × 60 = 2720/-

= Normal cost + Indirect cost + Cost of crashing

Minimum cost slope corresponds to an activity 2 – 4. This cannot be crashed alone, since it will affect the critical path. Next minimum cost slope corresponds to an activity 2 – 3. But this activity is already crashed.

So try to crash the activity 2 – 4 with 3 – 4 (or) 1 – 3.

Cost of crashing 2 – 4 with 3 – 4 per day = 50 + 90 = 140

Cost of crashing 2 – 4 with 1 – 3 per day = 50 + 80 = 130

least combination cost is 2 – 4 with 1 – 3

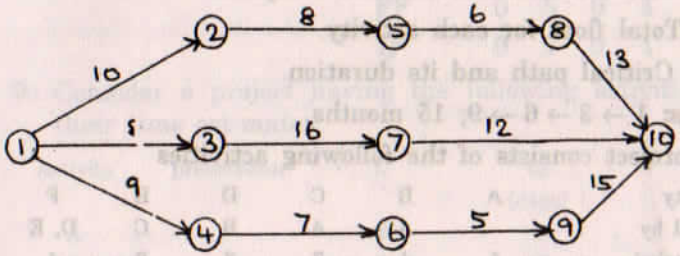
Indirect cost per day = 125

Cost of crashing 2 – 4 with 1 – 3 per day exceeds the indirect cost saving per day. ∴ We can reduce only the duration but not the cost of the project.

Hence the optimum duration is 15 days and the associated cost is Rs.2720/-.

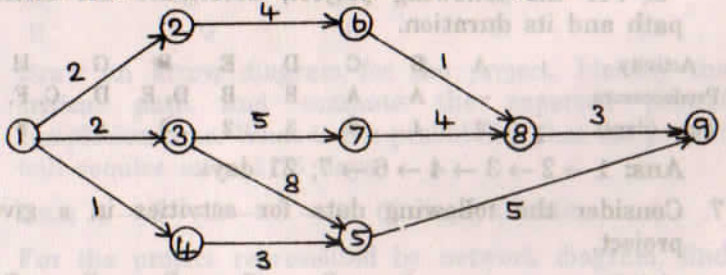
Exercise 3.2

1. Determine early start and late start in respect of all node points and identify the critical path in respect of the following network.



Ans: 1 → 2 → 5 → 8 → 10; or 1 → 2 → 5 → 7 → 10

2. Find the critical path and calculate the slack time for each event for the following PERT diagram.



Ans: 1 → 3 → 5 → 9

3. A project has the following time schedule:

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-9	8-10	9-10
Duration	4	1	1	1	6	5	4	8	1	2	1	8	7

Construct a PERT network and compute (i) T_E and T_L for each event and (ii) Float for each activity (iii) critical path and its duration.

Ans: 1 → 3 → 5 → 7 → 8 → 9 → 10; and 1 → 3 → 5 → 7 → 8 → 10

T_E :	0	0	4	1	1	5	7	7	11	15	17	17	18
T_L :	12	1	13	13	7	18	16	15	17	17	18	25	25

Duration = 25 weeks.

4. A project has the following time schedule:

Activity	1-2	1-3	1-4	2-5	3-6	3-7	4-6	5-8	6-9	7-8	8-9
duration (months)	2	2	1	4	8	5	3	1	5	4	3

Construct PERT network and compute:

(i) Total float for each activity

(ii) Critical path and its duration.

Ans: $1 \to 3 \to 6 \to 9$; 15 months.

5. A project consists of the following activities

Activity	A	B	C	D	E	F
Preceded by	—	A	A	B	C	D, E
Time (weeks)	5	4	7	8	9	4

Draw the network. Identify the critical path and find the project duration.

Ans: $A \to C \to E \to F$; 25 weeks

6. For the following project, determine the critical path and its duration.

Activity	A	B	C	D	E	F	G	H
Predecessors	—	A	A	B	B	D, E	D	C, F, G
Time (days)	2	4	8	3	2	3	4	8

Ans: $1 \to 2 \to 3 \to 4 \to 6 \to 7$; 21 days.

7. Consider the following data for actvities in a given project.

Activity	A	B	C	D	E	F
Predecessor	—	A	—	B, C	C	D, E
Time (days)	5	4	7	3	4	2

Draw the arrow diagram for the project. Compute the earliest and latest event times. What is the minimum project completion time? List the activities on the critical path.

Ans: $A \to B \to E \to$ dummy $\to F$; 15 days.

8. The following table gives the activities in a construction project and other relevant information:

Activity	1-2	1-3	2-3	2-4	3-4	4-5
Duration	20	25	10	12	6	10

(i) Draw the network for the project.

(ii) Find critical path.

(iii) Find free, total and independent floats for each activity.

Ans: $1 \to 2 \to 3 \to 4 \to 5$;

TF :	0	5	0	4	0	0
FF :	0	5	0	4	0	0
IF :	0	5	0	4	0	0

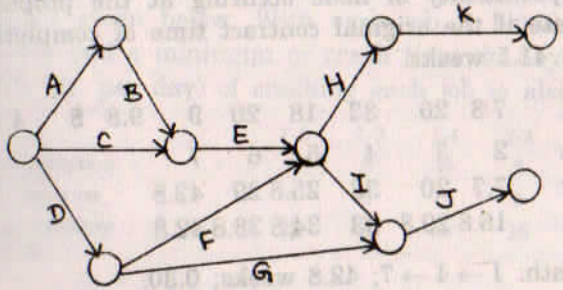
9. Consider a project having the following activities and their time estimates.

Activity	predecessor	t_o	t_m (days)	t_p
A	—	2	4	6
B	A	8	12	16
C	A	14	16	30
D	B	4	10	16
E	C, B	6	12	18
F	E	6	8	22
G	D	18	18	30
H	F, G	8	14	32

Draw an arrow diagram for the project. Identify the critical path and compute the expected project completion time. What is the probability that the project will require atleast 75 days.

Ans: $A \to B \to D \to G \to H$; 62 days; 0.9944

10. For the project represented by network diagram, find the earliest and start times to reach each node, given the following data.

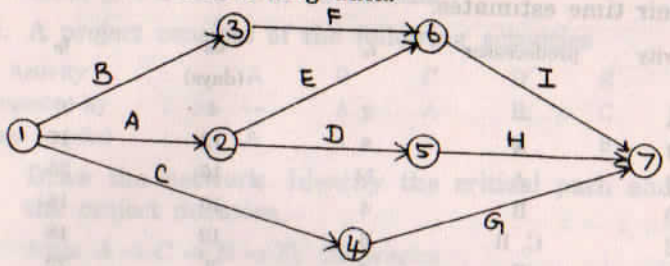


Job	:	A	B	C	D	E	F	G	H	I	J	K
Least time	:	4	5	8	2	4	6	8	5	3	5	6
Greatest time	:	8	10	12	7	10	15	16	9	7	11	13
Most likely time	:	5	7	11	3	7	9	12	6	5	8	9

Ans:

Node	:	2	3	4	5	6	7	8	9
E	:	5.3	3.5	12.5	19.5	25.8	24.5	34.9	32.5
L	:	5.3	10	12.5	19.5	25.8	24.5	34.9	32.5

11. A project is represented by the network shown below and has the following data:



Task	:	A	B	C	D	E	F	G	H	I
Least time	:	5	18	26	16	15	6	7	7	3
Greatest time	:	10	22	40	20	25	12	12	9	5
Most likely time	:	8	20	33	18	20	9	10	8	4

Determine the following:

(i) Expected task time and their variance.

(ii) The earliest and latest expected times to reach each node.

(iii) The critical path.

(iv) The probability of node occuring at the proposed completion date if the original contract time of completing the project is 41.5 weeks.

Ans:

t_e	:	7.8	20	33	18	20	9	9.8	8	4
Node	:	2	3	4	5	6	7			
E	:	7.7	20	33	25.8	29	42.8			
L	:	16.8	29.8	33	34.8	38.8	42.8			

Critical path: $1 \rightarrow 4 \rightarrow 7$; 42.8 weeks; 0.30.

12. Assuming that the expected times are normally distributed, find the probability of meeting the scheduled date as given for the network.

Activity	:	1-2	1-3	2-4	3-4	4-5	3-5
t_o	:	2	9	5	2	6	8
t_m	:	5	12	14	5	6	17
t_p	:	14	15	17	8	12	20

Scheduled project completion date is 30 days. Also find the date on which the project manager can complete the project with a probability of 0.90.

Ans: Critical paths: $1 \rightarrow 2 \rightarrow 4 \rightarrow 5$; $1 \rightarrow 3 \rightarrow 5$

(i) 0.71 (or) 0.81

13. The following cost table (time in weeks, cost in Rs.) applies to a project. Use it to arrive at the network associated with completing the project in minimum time at minimum cost.

Activity	Normal		Crash	
	Time	Cost	Time	Cost
1-2	2	800	1	1400
1-3	5	1000	2	2000
1-4	5	1000	3	1800
2-4	1	500	1	500
2-5	5	1500	3	2100
3-4	4	2000	3	3000
3-5	6	1200	4	1600
4-5	3	900	2	1600

(MU Ap'94; BE)

14. A small marketing project consists of the jobs in the table given below. With each job is listed its normal time and a minimum or crash time (in days). The cost (in Rs. per day) of crashing each job is also given.

Job	:	1-2	1-3	1-4	2-4	3-4	4-5
Normal duration	:	9	8	15	5	10	2
Crash duration	:	6	5	10	3	6	1
Cost of crashing (per day)	:	20	25	30	10	15	40

(i) What is the normal project length and the minimum project length. (ii) Overhead costs Rs. 60/day. What is the optimal length schedule duration of each job for your solution.

Ans: Normal duration = 20 days; Minimum duration = 12 days

Optimum duration of the project = 15 days

Optimum duration of each job 9, 8, 14, 5, 6, 1.

15. The following table gives data on normal time and cost and crash time and cost for a project.

Activity	NC (Rs.)	NT (weeks)	CT (weeks)	C.C Rs
1-2	300	3	2	400
2-3	30	3	3	30
2-4	420	7	5	580
2-5	720	9	7	810
3-5	250	5	4	300
4-5	0	0	0	0
5-6	320	6	4	410
6-7	400	4	3	470
6-8	780	13	10	900
7-8	1000	10	9	1200

(i) Draw the network and identify the critical path (ii) what are the normal project duration and the associated cost (iii) find the total float associated with each activity (iv) crash the relevant activities systematically and determine the optimal project completion time and cost.

Ans: (i) $1 \rightarrow 2 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8$; (ii) 32 weeks; 5820; (iii) TF: 0, 1, 2, 0, 1, 2, 0, 0, 1, 0; (iv) 29 weeks, Rs. 5805

UNIT IV

INVENTORY CONTROL

An inventory may be defined as any idle resource of an enterprise. It is a physical stock of items held in any business for the purpose of future affairs. In a factory the inventory may be in the form of raw materials, parts, semi finished goods and finished goods. In a hospital it is used to indicate number of beds, specialized personnel, stocks of drugs etc., In airline company the inventories could be in the form of parts of engine repairs, stewardness, mechanics etc.,

Reasons for maintaining inventories

Though inventory of materials is an idle resource (not to be used immediately) it is almost essential to maintain certain inventories for smooth functioning of an organisation. For example, In a service oriented organisation such as transport corportation, absence of spare part when it is required will affect the maintenance of vehicles as a result of which the quality of service will deteriorate. Consider an enterprise which has no inventory of materials. On receiving a sales order, the enterprise will have to order out for purchase of raw materials required to complete the order, wait till there arrive and then start production. This would keep the customers to wait for a long time for the delivery of goods ordered.

Some of the important reasons for carrying inventories are listed below.

To ensure smooth and efficient running of business.

To ensure an adequate and prompt supply of items to the customer to avoid the reputation for constantly being out of stock and avoid the shortage at a minimum cost.

To provide service to customers with varying demand and is various locations by maintaining a adequate supply to meet their immediate and seasonal needs.