

ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Applied Mathematics II (Math2041)
Exercise on chapter Four

1. Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or the divergence of the following series?

a) $\sum_{n=0}^{\infty} c_n$ b) $\sum_{n=0}^{\infty} c_n 7^n$ c) $\sum_{n=0}^{\infty} c_n 3^n$ d) $\sum_{n=0}^{\infty} c_n (-2)^{2n+1}$ e) $\sum_{n=0}^{\infty} c_n (-5)^n$

2. Suppose that the power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ has radius of convergence R_1 and $\sum_{n=0}^{\infty} d_n (x-a)^n$ has radius of convergence R_2 . What can you say about the radius of convergence of the given power series? Explain your reasoning.

a) $\sum_{n=0}^{\infty} p c_n (x-a)^n$ (p any non zero constant) b) $\sum_{n=0}^{\infty} (c_n + d_n)(x-a)^n$ c) $\sum_{n=0}^{\infty} c_n (x-a)^{2n}$

3. Find the domain of the function $f(x)$

a) $f(x) = 1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \dots$ b) $f(x) = 1 - \frac{x-4}{2} + \frac{(x-4)^2}{3^2} - \frac{(x-4)^3}{4^2} + \dots$

c) $f(x) = \sum_{k=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2k-1)}{(2k-2)!} x^k$ d) $f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (2k)!}{2^k} (x+3)^k$

4. Find the radius of convergence and the interval of convergence of the power series.

a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} x^n$ b) $\sum_{n=1}^{\infty} (n x^n)^2$ c) $\sum_{k=0}^{\infty} \frac{(-1)^k}{9^k} x^{2k}$
d) $\sum_{k=1}^{\infty} \frac{k}{6^k} (2x+1)^k$ e) $\sum_{k=0}^{\infty} \frac{(x-1)^k}{k+2}$ f) $\sum_{k=0}^{\infty} \frac{(\pi)^k (x-1)^{2k}}{(2k+1)!} x^{2k}$
g) $\sum_{n=0}^{\infty} \frac{3^n}{5^{2n}} x^{3n}$ h) $\sum_{k=0}^{\infty} (-1)^k \frac{(\ln k)^2}{n^2} x^n$ i) $\sum_{k=0}^{\infty} (-1)^k \frac{(x+1)^{2n+1}}{n^2 + 4}$
j) $\sum_{n=0}^{\infty} n^n x^{2n}$ k) $\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n)}$ l) $\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n)}{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)} x^n$

5. If k is a positive integer, find the radius of convergent of the series $\sum_{n=0}^{\infty} \frac{(n!)^k}{(kn)!} x^n$.

6. Find the Maclaurin series of $f(x)$.

a) $f(x) = \sin^2 x$ b) $f(x) = 5^x$ c) $f(x) = \frac{1}{\sqrt{4+x^2}}$
 d) $f(x) = \ln \frac{1-x}{1+x}$ e) $f(x) = \frac{1}{\sqrt[5]{32-x}}$ f) $f(x) = e^{-x^2} + \cos x$

7. Find the Taylor series for $f(x)$ about a.

a) $f(x) = 1 + x + x^2, a = 2$ b) $f(x) = \cos x, a = \pi$ c) $f(x) = \frac{1}{\sqrt{x}}, a = 9$

8. Find the 1st four non zero terms of the Maclaurin series for the function by multiplying or dividing the Maclaurin series of the factors.

a) $f(x) = e^{-x^2} \cos x$ b) $f(x) = \frac{x}{\sin x}$ c) $f(x) = (1+x^2)^{\frac{4}{3}}(1+x)^{\frac{1}{3}}$
 d) $f(x) = \sec x$ e) $f(x) = \frac{\tan^{-1} x}{1+x}$ f) $f(x) = \frac{\sin x}{e^x}$

9. Find the first for five nonzero terms of the Maclaurin series for the function by using partial fraction and a known Maclaurin series.

a) $f(x) = \frac{4x-2}{x^2-1}$ b) $f(x) = \frac{x^3+x^2+2x-2}{x^2-1}$

10. Show that $(1-x) \sum_{n=0}^{\infty} nx^n = \frac{x}{1-x}$ for $|x| < 1$.

11. Differentiate the Maclaurin series for $f(x) = xe^x$ and use the result to find the sum of $\sum_{n=0}^{\infty} \frac{n+1}{n!}$

12. Using the $\int \frac{1}{\sqrt{1+x^2}} dx = \sinh^{-1} x + c$, Find the Maclaurin series of $\sinh^{-1} x$ and give the 1st four nonzero terms of the series.

13. Using the $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$, Find the Maclaurin series of $\sin^{-1} x$ and give the 1st four nonzero terms of the series.

14. Using the Maclaurin series of $f(x) = x^2 \cos 2x$ find $f^{(50)}(0)$ and $f^{(99)}(0)$.

15. Show that

a) $\lim_{n \rightarrow \infty} \frac{x^{2n}}{n^n} = 0, \forall x$ b) $\lim_{n \rightarrow \infty} \frac{x^{3n}}{8^n} = 0, \text{ for } |x| < 2$ c) $\lim_{n \rightarrow \infty} \frac{(-1)^n x^{2n}}{2^{2n} (n!)^2} = 0, \forall x$

16. Evaluate the indefinite integral as an infinite series.

a) $\int xe^{-x^3} dx$ b) $\int \frac{\sin x}{x} dx$ c) $\int x\sqrt{1+x^4} dx$ d) $\int \frac{x^2}{1+x^4} dx$

17. Use series to evaluate the limit

a) $\lim_{x \rightarrow 0} \frac{x - \tan^{-1} x}{x^3}$ b) $\lim_{x \rightarrow 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$ c) $\lim_{x \rightarrow 0} \frac{\sin^2 x - x^2}{(e^{x^2} - 1)^2}$ d) $\lim_{x \rightarrow 0} \frac{(e^{x^2} - 1)^2}{\sin^3 x^2}$