

ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Applied Mathematics II (Math2041)

Exercise on chapter five

1. Graph the periodic extension of the following functions

a) $f(x) = \begin{cases} 8, & \text{if } 0 < x < 2 \\ -8, & \text{if } 2 < x < 4 \end{cases}$ Period 4

b) $f(x) = \begin{cases} -x, & \text{if } -4 \leq x \leq 0 \\ x, & \text{if } 0 \leq x \leq 4 \end{cases}$ Period 8

c) $f(x) = 4x, 0 < x < 4$ Period 10

d) $f(x) = \begin{cases} 0, & \text{if } -\pi < x < 0 \\ \sin x, & \text{if } 0 < x < \pi \end{cases}$ Period 2π

e) $f(x) = |x|, -1 < x < 1$ Period 2

f) $f(x) = \begin{cases} -1, & \text{if } -1 < x < -\frac{1}{2} \\ 0, & \text{if } -\frac{1}{2} < x < \frac{1}{2} \\ 1, & \text{if } \frac{1}{2} < x < 1 \end{cases}$ Period 2

2. Determine whether or not f satisfies Dirichlet conditions

a) $f(x) = \begin{cases} 0, & \text{if } -\pi < x < 0 \\ \sin x, & \text{if } 0 < x < \pi \end{cases}$ Period 2π

b) $f(x) = \begin{cases} -1, & \text{if } -1 < x < -\frac{1}{2} \\ 0, & \text{if } -\frac{1}{2} < x < \frac{1}{2} \\ 1, & \text{if } \frac{1}{2} < x < 1 \end{cases}$ Period 2

c) $f(x) = \sqrt{|x|}, -1 < x < 1$ Period 2

d) $f(x) = |x|^{\frac{3}{2}}, -1 < x < 1$ Period 2

3. Find the Fourier series of the following functions and tell to what value the series converges.

a) $f(x) = \begin{cases} 2x, & 0 < x < 3 \\ 0, & -3 < x < 0 \end{cases}$ Period 6

b) $f(x) = \begin{cases} 2-x, & 0 < x < 4 \\ x-6, & 4 < x < 8 \end{cases}$ Period 8

c) $f(x) = \frac{x^2}{4}, -\pi < x < \pi$ Period 2π

d) $f(x) = \begin{cases} -x^2, & -\pi < x < 0 \\ x^2, & 0 < x < \pi \end{cases}$ Period 2π

e) $f(x) = \begin{cases} 0, & \text{if } -\pi < x < 0 \\ \sin x, & \text{if } 0 < x < \pi \end{cases}$ Period 2π

f) $f(x) = 4x, 0 < x < 4$ Period 10

4. Represent the following functions by a Fourier sine series

a) $f(x) = x^2, 0 < x < 1$

b) $f(x) = x - x^2, 0 < x < 1$

c) $f(x) = \begin{cases} x, & 0 < x < 2 \\ 2, & 2 < x < 4 \end{cases}$

5. Represent the following functions by a Fourier Cosine series

a) $f(x) = \begin{cases} x, & 0 < x < 4 \\ 8-x, & 4 < x < 8 \end{cases}$

b) $f(x) = x - x^2, 0 < x < 1$

c) $f(x) = e^x, 0 < x < 1$