

ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Applied Mathematics II (Math 2041)

Academic Year 2021/22, Semester I

Worksheet II

1. Evaluate the following integrals
 - a) $\int_0^1 \int_0^x \cos(x^2) dy dx$
 - b) $\int_0^1 \int_0^y x \sqrt{x^2 + y^2} dx dy$
2. Evaluate the double integrals on the region R bounded by the curves:
 - a) $\iint_R (x^3 + y^3) dA$, for $2y = x, y = x, x = 4$.
 - b) $\iint_R \cos(x^2) dA$, for $y = x, y = 0, x = 1$
 - b) $\iint_R \frac{y^3}{x^2} dA$, for $3y = x, y = \sqrt{x}$
 - c) $\iint_R (x^2 + 3y^2) dA$, for $x^2 + y^2 = 1$
3. Using double integrals compute the area of the region R which lies between the lines $2x - y = 0$, $y + x = 6$ and the x-axis.
4. By changing the order of the iterated integrals, when necessary, evaluate
 - a) $\int_0^1 \int_{y^2}^1 y^3 \sin(x^3) dx dy$
 - b) $\int_0^1 \int_x^1 x^2 e^{y^4} dy dx$
 - c) $\int_1^4 \int_{\sqrt{x}}^2 \cos(y^3 - 3y) dy dx$
 - d) $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \sin(5x^2) dx dy$
 - e) $\int_1^3 \int_0^{\ln y} e^{x+y} dx dy$
5. Evaluate the integrals
 - a. $\iint_R \sin(y^3) dA$, where R is the region bounded by $y = \sqrt{x}$, $y = 2$, and $x = 0$
 - b. $\iint_R xy dA$, where R is the region bounded by the circle with $r = 5$.
 - c. $\iint_R x^2 dA$, where R is the region bounded by the circle $r = 4 \sin \theta$
6. Change the integral to an iterated integral in polar coordinates and then evaluate
 - a) $\int_0^1 \int_0^{\sqrt{1-x^2}} dy dx$
 - b) $\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{1}{x^2 + y^2 + 1} dx dy$
 - c) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2} dy dx$
 - d) $\int_{-2}^2 \int_2^{\sqrt{8-x^2}} \frac{1}{\sqrt{x^2 + y^2}} dy dx$
7. Using double integrals, find the area of the region R bounded by
 - a) The curves $y = x^2$ and $y = \frac{2}{1+x^2}$
 - b) the cardioid $r = a(1 + \sin \theta), a > 0$
8. Find the surface area of:
 - a. the portion of the paraboloid $z = 9 - x^2 - y^2$ above the plane $z = 5$.
 - b. the surface $z = \sqrt{x^2 + y^2}$ below the plane $z = 9$.
 - c. the portion of the sphere $x^2 + y^2 + z^2 = 5$ that is inside the paraboloid $x^2 + y^2 = 4z$.
9. Find the volume of the solid D:
 - a. bounded by the paraboloids $z = 4 - x^2 - y^2$ and $z = 3x^2 + 3y^2$.

ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Applied Mathematics II (Math 2041)

Academic Year 2021/22, Semester I

- b. that lies inside the cylinder $x^2 + y^2 = 4x$ bounded from above by the upper nappe of the cone $z^2 = x^2 + y^2$ and from below by the xy -plane.

10. Evaluate

a) $\int_0^1 \int_0^{1-x} \int_0^{2-y} xyz \, dz dy dx$ b) $\int_0^{\pi/2} \int_0^1 \int_0^2 zr^2 \sin \theta \, dz dr d\theta$
c) $\int_0^2 \int_0^{\sqrt{2y-y^2}} \int_0^1 z\sqrt{x^2+y^2} \, dz dx dy$ d) $\int_0^\pi \int_0^{\pi/4} \int_0^{\sec \theta} \sin 2\theta \, d\rho d\phi d\theta$

11. Evaluate $\iiint_D e^y \, dV$, where D is the solid region bounded by the planes $y = 1$, $z = 0$, $y = x$, $y = -x$ and $z = y$.

12. Evaluate $\iiint_D x e^{x^2+y^2+z^2} \, dV$, where $D = \{(x,y,z): x^2 + y^2 + z^2 \leq 4\}$.

13. Evaluate using a suitable change of variables.

a) $\iint_R \left(\frac{x-y}{x+y} \right)^4 \, dA$, where R is the region bounded by the graphs of $x + y = 1$, $y = 0$ and $x = 0$
b) $\iint_R e^{\left(\frac{2x-y}{x+y} \right)} \, dA$, where R is bounded by $x = 2y$, $y = 2x$, $x + y = 1$ and $x + y = 2$
c) $\iint_R x^3 y \, dA$, where R is bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ for $x, y \geq 0$. (Hint: use $x = au \cos v$, $y = bu \sin v$)
d) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \int_0^a z\sqrt{x^2+y^2} \, dz dy dx$ (Use cylindrical coordinates)
e) $\iiint_D y \, dV$, where D is the ellipsoid $\frac{x^2}{9} + \frac{y^2}{4} + z^2 = 1$
(Hint: Let $x = 3u$, $y = 2v$, $w = z$)

14. Find the area of the region bounded by the curves $xy = 1$, $xy = 2$, $y = x$ and $y = 4x$ in the first quadrant (Hint: Let $x = \frac{u}{v}$, $y = v$)