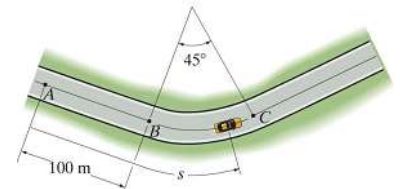


Worksheet 5

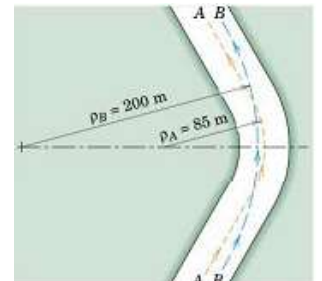
1. When designing a highway curve it is required that cars traveling at a constant speed of 25 m/s must not have an acceleration that exceeds 3 m/s^2 . Determine the minimum radius of curvature of the curve.
2. At a given instant, a car travels along a circular curve road with a speed of 20m/s while decreasing its speed at the rate of 3m/s^2 . If the magnitude of the car's acceleration is 5m/s^2 , determine the radius of curvature of the road.
3. A particle moves in a circular path of 0.3m radius. Calculate the magnitude of the acceleration of the particle (a) if its speed is constant at 0.6m/s and (b)if its speed is 0.6m/s but is increasing at a rate of 0.9m/s each seconds.
4. The car is traveling at a speed of 30 m/s. The driver applies the brakes at A and thereby reduces the speed at the rate of $a_t = (-\frac{1}{8}t)$ m/s^2 , where t is in seconds. Determine the acceleration of the car just before it reaches point C on the circular curve. It takes 15 s for the car to travel from A to C.



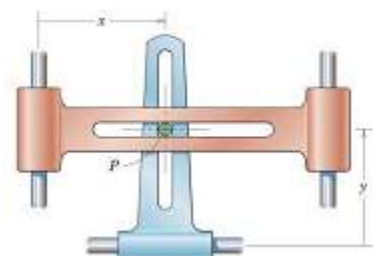
5. The car travels at a constant speed from the bottom A of the dip to the top B of the hump. If the radius of curvature of the road at A is $\rho_A = 120\text{m}$ and the car acceleration at A is $0.4g$, determine the car speed v . if the acceleration at B must be limited to $0.2g$. Determine the minimum radius of curvature ρ_B of the road at B.



6. The figure shows two possible paths for negotiating an unbanked turn on a horizontal portion of a race course. Path A-A follows the centerline of the road and has a radius of curvature $\rho_A = 85\text{m}$, while path B-B uses the width of the road to good advantage in increasing the radius of curvature to $\rho_B = 200\text{m}$. If the drivers limit their speeds in their curves so that the acceleration does not exceed $0.8g$, determine the maximum speed for each path.



7. During a short interval the slotted guides are designed to move according to $x = 16 - 12t + 4t^2$ and $y = 2 + 15t - 3t^2$, where x and y are in millimeters and t is in seconds. At the instant when $t=2\text{s}$, determine the radius curvature ρ of the path of the constrained pin P.



Exercise for students

1. Starting from rest the motorboat travels around the circular path, $p = 50\text{m}$, at a speed $v = (0.8t)$ m/s, where t is in seconds. Determine the magnitudes of the boat's velocity and acceleration when it has traveled 20m .

$$\text{ANS: } v = 6.66\text{m/s} \quad \|a\| = 1.02\text{ m/s}^2$$

2. A car moves along a circular track of radius 250ft , and its speed for a short period of time $0 \leq t \leq 2\text{s}$ is $v = 3(t+t^2)$ ft/s, where t is in seconds. Determine the magnitude of the car's acceleration when $t = 2\text{s}$. How far has it traveled in $t = 2\text{s}$?

$$\text{ANS: } \Delta s = 14\text{ft}$$

3. The train passes point B with a speed of 20m/s which is decreasing at $a_t = -0.5\text{ m/s}^2$. Determine the magnitude of acceleration of the train at this point. When the roller coaster is at B, it has a speed of 25m/s , which is increasing at $a_t = 3\text{ m/s}^2$. Determine the magnitude of the acceleration of the roller coaster at this instant and the direction angle it makes with the x-axis.

$$\text{ANS: } \|a\| = 0.511\text{ m/s}^2$$

4. A particle is traveling along a circular curve having a radius of 20 m . If it has an initial speed of 20 m/s and then begins to decrease its speed at the rate of $a_t = (-0.25\text{ s})$ m/s², determine the magnitude of the acceleration of the particle two seconds later.

$$\text{ANS: } \|a\| = 10.2\text{ m/s}^2$$

5. The truck travels along a circular road that has a radius of 50m at a speed of 4 m/s . For a short distance when $t = 0$, its speed is then increased by $a_t = (0.4t)$ m/s², where t is in seconds. Determine the speed and the magnitude of the truck's acceleration when $t = 4\text{s}$.

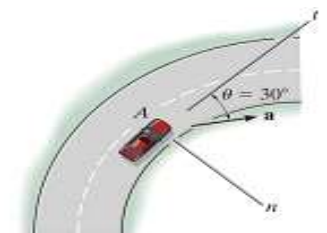
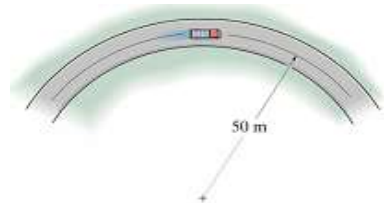
$$\text{ANS: } v = 7.20\text{m/s} \quad \|a\| = 1.91\text{ m/s}^2$$

6. The automobile has a speed of 80ft/s at point A and an acceleration a having a magnitude of 10ft/s^2 , acting in the direction shown. Determine the radius of curvature of the path at point A and the tangential component of acceleration.

$$\text{ANS: } \rho = 1280\text{ft} \quad a_t = 8.66\text{ft/s}^2$$

7. The position of a particle is defined by $r = \{2\sin(\frac{\pi}{4})ti + 2\cos(\frac{\pi}{4})tj + 3tk\}\text{m}$, where t is in seconds. Determine the magnitudes of the velocity and acceleration at any instant.

$$\text{ANS: } v = 3.39\text{m/s} \quad a = 1.23\text{ m/s}^2$$



8. An automobile is traveling on a horizontal circular curve having a radius of 800ft. If the acceleration of the automobile is 5 ft/s^2 , determine the constant speed at which the automobile is traveling.

ANS: $v = 63.2 \text{ ft/sec}$

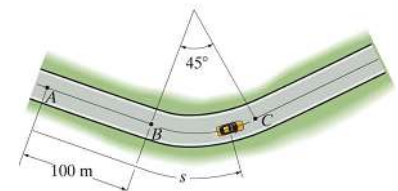
9. The car travels along the circular path such that its speed is increased by $a_t = (0.5et) \text{ m/s}^2$, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18 \text{ m}$ starting from rest. Neglect the size of the car.

ANS: $\|v\| = 19.9 \text{ m/s}$ $\|a\| = 24.2 \text{ m/s}^2$



10. The car is traveling at a speed of 30 m/s. The driver then applies the brakes at A and there by reduces the speed at the rate of $a_t = (-0.08) \text{ m/s}^2$, where v is in meter per seconds. Determine the acceleration of the car just before it reaches point C on the circular curve. It takes 15s for the car to travel from A to C.

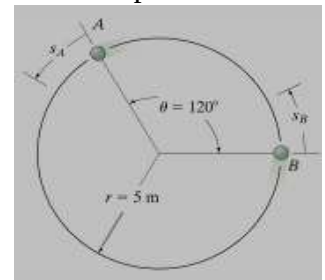
ANS: $\|a\| = 0.824 \text{ m/s}^2$



11. Particles A and B are traveling counter-clockwise around a circular track at a constant speed of 8m/s. If at the instant shown the speed of A begins to increase by $(at)A = (0.4s_A) \text{ m/s}^2$, where S_A is in meters, determine the distance measured counterclockwise along the track from B to A when $t=1\text{s}$. What is the magnitude of the acceleration of each particle at this instant?

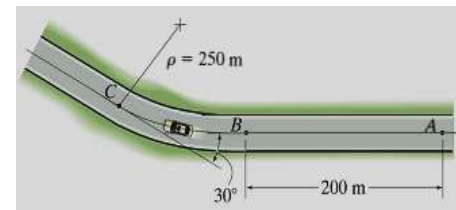
ANS: $s = 11.0 \text{ m}$ $a_A = 19.0 \text{ m/s}^2$

$a_B = 12.8 \text{ m/s}^2$



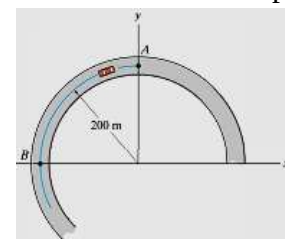
12. When the car reaches point A, it has a speed of 25 m/s. If the brakes are applied, its speed is reduced by $a_t = (0.001s - 1) \text{ m/s}^2$. Determine the magnitude of acceleration of the car just before it reaches point C.

ANS: $\|a\| = 0.730 \text{ m/s}^2$



13. The race car travels around the circular track with a speed of 16m/s. When it reaches point A it increases its speed at $a_t = (\frac{4}{3}v^{1/4}) \text{ m/s}^2$, where v is in m/s. Determine the magnitudes of the velocity and acceleration of the car when it reaches point B. Also, how much time is required for it to travel from A to B?

ANS: $t = 10.108 \text{ s}$ $v = 47.6 \text{ m/s}$
 $\|a\| = 11.8 \text{ m/s}^2$



14. The ball is kicked with an initial speed $V_A = 8 \text{ m/s}$ at an angle $\theta_A = 40^\circ$ with the horizontal. Find the equation of the path $y = f(x)$ and then determine the normal and tangential components of its acceleration when $t = 0.25 \text{ s}$.

ANS: $y = 0.839x - 0.131x^2$

$a_t = 3.94 \text{ m/s}^2$ $a_n = 8.98 \text{ m/s}^2$



15. The car passes through a dip in the road at A with a constant speed which gives its mass center G an acceleration equal to $0.5g$. If the radius of curvature of the road at A is 100 m , and if the distance from the road to the mass center G of the car is 0.6 m , determine the speed v of the car.



ANS: $v = 22.08 \text{ m/s}$

16. The driver of the truck has an acceleration of $0.4g$ as the truck passes over the top A of the hump in the road at constant speed. The radius of curvature of the road at the top of the hump is 98 m , and the center of mass G of the driver is 2 m above the road. Calculate the speed v of the truck.

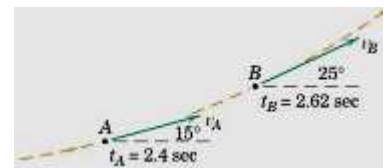


ANS: $v = 19.81 \text{ m/s}$

17. A train enters a curved horizontal section of track at a speed of 100 km/h and slows down with constant deceleration to 50 km/h in 12 seconds . An accelerometer mounted inside the train records a horizontal acceleration of 2 m/s^2 when the train is 6 seconds into the curve. Calculate the radius of curvature ρ of the track for this instant.

ANS: $\rho = 266.0 \text{ m}$

18. A particle moves along the curved path shown. The particle has a speed $v_A = 12 \text{ ft/sec}$ at a t_A and a speed $v_B = 14 \text{ ft/sec}$ at a time t_B . Determine the average values of the normal and tangential accelerations of the particles between point A and B.



ANS: $a_n = 10.31 \text{ ft/sec}^2$ $a_t = 9.09 \text{ ft/sec}^2$

19. A car travels along the level road with a speed which is decreasing at the constant rate of 0.6 m/s each second. The speed of the car as it passes point A is 16 m/s . Calculate the magnitude of the total acceleration of the car as it passes point B which is 120 m along the road from A. The radius of curvature of the road at B is 60 m .



ANS: $\|a\| = 1.961 \text{ m/s}^2$

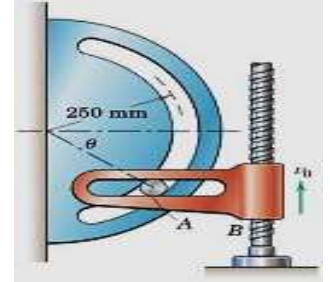
20. To simulate a condition of "weightlessness" in its cabin, a jet transport traveling at 800 km/h moves on a sustained vertical curve as shown. At what rate $\dot{\beta}$ in degree per second should the pilot drop his longitudinal line of sight to effect the desired condition? The maneuver takes place at a mean altitude of 8 km and the gravitational acceleration may be taken as 9.79 m/s^2 .



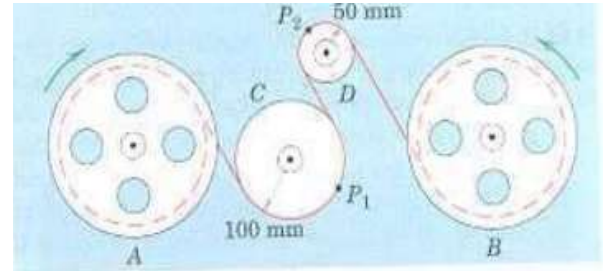
ANS: $\dot{\beta} = 2.52 \text{ deg/s}$

21. In the design of timing mechanism, the motion of the pin A in the fixed circular slot is controlled by the guide B, which is being elevated by its lead screw with a constant upward velocity $v_0 = 2 \text{ m/s}$ for an interval of its motion. Calculate both the normal and tangential components of pin A as it passes the position for which $\theta = 30^\circ$.

ANS: $a_n = 21.3 \text{ m/s}^2$ $a_t = 12.32 \text{ m/s}^2$



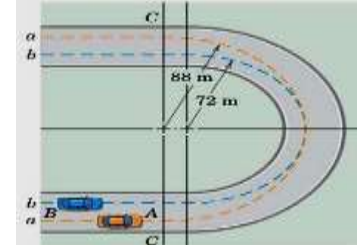
22. Magnetic tape is being transferred from reel A to reel B and passes around idler pulleys C and D. at a certain instant, point P_1 on the tape is in contact with pulley D. if the normal component of acceleration of P_1 is 40 m/s^2 and the tangential component of acceleration of P_2 is 30 m/s^2 at this instant, compute the corresponding speed v of the tape, magnitude of the total acceleration of P_1 and P_2 .



ANS: $v = 2 \text{ m/s}$, $a_1 = 50 \text{ m/s}^2$, $a_2 = 85.4 \text{ m/s}^2$

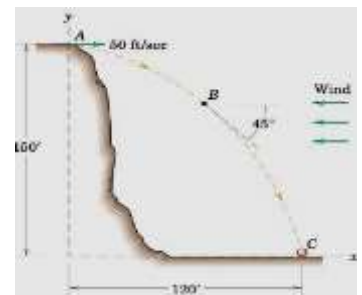
23. Race car A follows path $a-a$ while race car B follows path $b-b$ on the unbanked track. If each car has a constant speed limited to that corresponding to a lateral (normal) acceleration of $0.8g$, determine the times t_A and t_B for both cars to negotiate the turn as delimited by the line C-C.

ANS: $t_A = 10.52 \text{ s}$ $t_B = 10.86 \text{ s}$

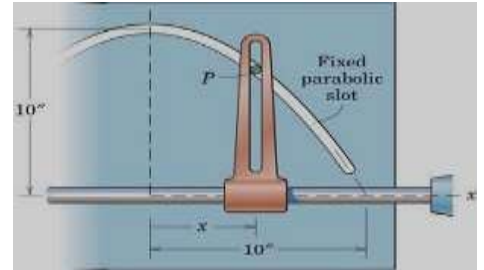


24. A ball is thrown horizontally from the top of a 150ft cliff at A with a speed of 50ft/sec and lands at point C. because of a strong horizontal wind, the ball has a constant acceleration in the negative x-direction. Determine the radius of curvature ρ of the path of the ball at B where its trajectory makes an angle of 45° with the horizontal. Neglect any effect of air resistance in the vertical direction.

ANS: $\rho = 121.7 \text{ ft}$



25. In the design of a control mechanism, the vertical slotted guide is moving with a constant velocity $\dot{x} = 15 \text{ in/sec}$ during the interval of motion from $x = -8 \text{ in}$ to $x = +8 \text{ in}$. For the instant when $x = 6 \text{ in}$, calculate the n- and t-components of acceleration of the pin P, which is confined to move in the parabolic slot. From these results, determine the radius of curvature ρ of the path at this position. (hint to calculate the radius of curvature for such parabolic



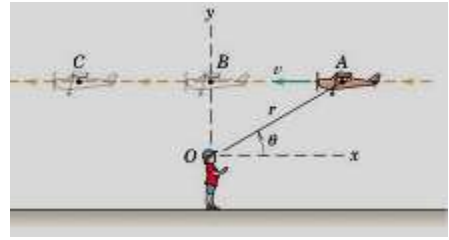
curve use $\rho_{xy} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} \bigg)$ ANS: $a_n =$

$28.8 \text{ in/s}^2 \quad a_t = 34.6 \text{ in/s}^2$

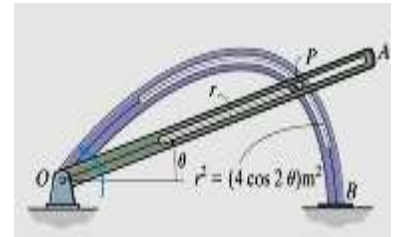
$\rho = -19.06 \text{ in}$

Worksheet 6 polar coordinate system

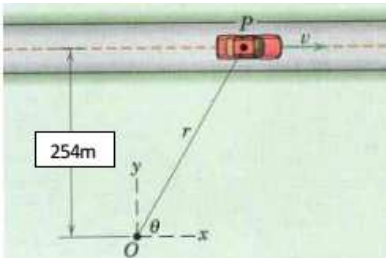
1. A model air plane flies over an observer O with constant speed in a straight line as shown. Determine the signs (plus, minus or zero) for $r, \dot{r}, \ddot{r}, \theta, \dot{\theta}$ and $\ddot{\theta}$ for each of the positions A, B and C.



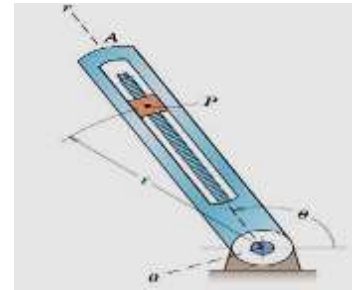
2. The position of a particle is described by $r = (t^3 + 4t - 4)m$ and $\theta = (t^{3/2})rad$, where t is in seconds. Determine the magnitudes of the particle's velocity and acceleration at the instant $t = 2s$.
3. The motion of peg P is constrained by the lemniscate curved slot in OB and by the slotted arm OA. If OA rotates counterclockwise with an angular velocity of $\dot{\theta} = (3t^{3/2})rad/s$, where t is in seconds, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30^\circ$. When $t = 0, \theta = 0^\circ$.



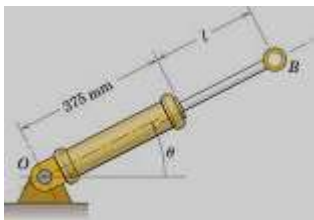
4. A car P travels along a straight road with a constant speed $v = 104.65 km/h$. At the instant when the angle $\theta = 60^\circ$, determine the values of $\dot{r}$ in m/sec and in $\dot{\theta}$ deg/sec.



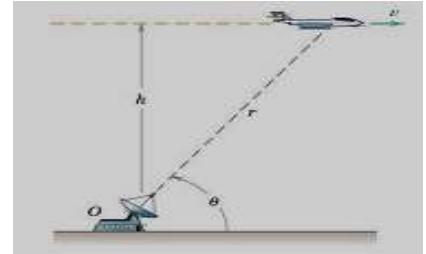
5. Motion of the sliding block P in the rotating radial slot is controlled by the power screw as shown. For the instant represented, $\dot{\theta} = 0.1 rad/sec$, $\ddot{\theta} = -0.4 rad/s^2$, $r = 300mm$. Also, the screw turns at a constant speed giving $\dot{r} = 40mm/sec$. For this instant, determine the magnitudes of the velocity v and acceleration a of P.



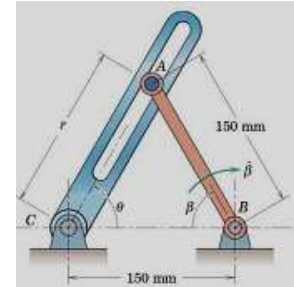
6. As the hydraulic cylinder rotates around O, the exposed length l of the piston rod P is controlled by the action of oil pressure in the cylinder. If the cylinder rotates at the constant rate $\dot{\theta} = 60 deg/s$ and l is decreasing at the constant rate of $150mm/s$, calculate the magnitudes of the velocity v and acceleration a of end B when $l = 125mm$.



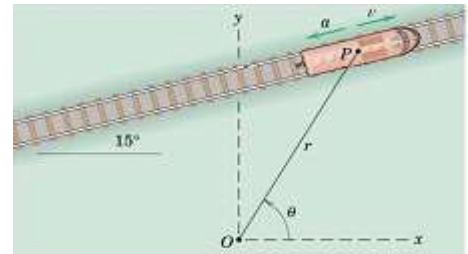
7. A jet plane flying at a constant speed at an altitude $h=10\text{km}$ is being tracked by radar located at O directly below the line of flight. If the angle θ is decreasing at a rate of 0.020rad/s when $\theta = 60^\circ$, determine the value of $\dot{r}$ at this instant and the magnitude of the velocity v of the plane.



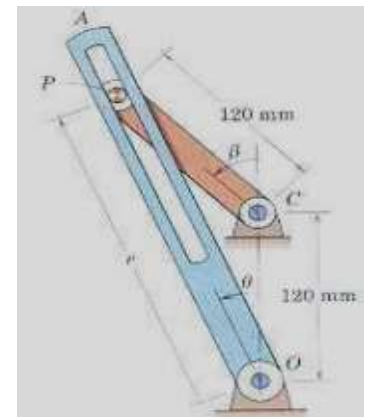
8. Link AB rotates through a limited range of the angle β , and its end A causes the slotted link AC to rotate also. For the instant represented where $\beta = 60^\circ$ and $\dot{\beta} = 0.6\text{rad/sec}$ constant, determine the corresponding values of $\dot{r}$, $\ddot{r}$, $\dot{\theta}$ and $\ddot{\theta}$. Use unit vector of that of the polar coordinate system.



9. A locomotive is traveling on the straight and level track with a speed $v = 90\text{km/h}$ and a deceleration $a = 0.5\text{m/s}^2$ as shown. Relative to the fixed observer at O, determine the quantities $\dot{r}$, $\ddot{r}$, $\dot{\theta}$ and $\ddot{\theta}$ at the instant when $\theta = 60^\circ$ and $r = 400\text{m}$.



10. For a limited range of motion, crank CP causes the slotted link OA to rotate. If β is increasing at the constant rate of 4rad/s when $\beta = 45^\circ$, determine the r - and θ -components of the acceleration of pin P for this position and specify the corresponding values of $\dot{r}$ and $\ddot{r}$.



11. Pin A moves in a circle of 90mm radius as crank AC revolves at the constant rate $\dot{\beta} = 60 \text{ rad/s}$. The slotted link rotates about point O as the rod attached to A moves in and out of the slot. For the position $\beta = 30^\circ$, determine $\dot{r}$, $\ddot{r}$, $\dot{\theta}$ and $\ddot{\theta}$.

