

ADDIS ABABA SCIENCE AND TECHNOLOGY UNIVERSITY
COLLEGE OF NATURAL AND SOCIAL SCIENCES

APPLIED MATHEMATICS I**WORKSHEET 4 (Practice Problems on Derivatives)**

- Find the equations of the tangent (and normal) line(s) to the following curves at the indicated points.
 - $y = x^3$ at $x = 2$
 - $y = \sqrt{x}$ at $x = \sqrt{2}$
 - $y = \log_2 x$ at $x = 4$
 - $y = \cosh x$ at $x = 0$
 - $y = \sin x$ at $x = \frac{\pi}{2}$
 - $y = \sinh x$ at $x = 0$
- Using definition differentiate the following functions.
 - $y = \sqrt[3]{x}$
 - $y = \log_3 x$
 - $y = e^x$
 - $y = \sinh x$
 - $y = \cosh x$
 - $y = 2^x$
- If $f'(a) = 4$, then find
 - $\lim_{h \rightarrow 0} \frac{f(a+3h) - f(a)}{h}$
 - $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a-h)}{h}$
 - $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{\sqrt{x} - \sqrt{a}}$
 - $\lim_{h \rightarrow 0} \frac{f(a+6h) - f(a+3h)}{6h}$
- Find the positive integer n so that $\lim_{x \rightarrow 3} \frac{x^n - 3^n}{x - 3} = 108$.
- Find y' where
 - $y = \frac{1}{2-x}$
 - $y = \sin(2x^3)$
 - $y = \ln(\operatorname{sech}^2 x)$
 - $y = \sin(\ln(\cos(4x)))$
 - $y = e^{x^5}$
 - $y = 3^{\cos x}$
 - $y = x^2 \tanh^{-1}(2x)$
 - $y = \sin^{-1}(3x^2)$
 - $y = \tan^{-1}\left(\frac{1-x}{1+x}\right)$
 - $y = \cosh^{-1}((\cos x^2))$
 - $y = (\ln x)^{\ln x}$
 - $y = x^{x^x}$
- Find $\frac{d^2}{dx^2} f(x^3 - 2x^2 + 1)$ for differentiable function f satisfying $\frac{d}{dx}(f(x)) = 2x^2 + 3$.
- Given that $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$. Then find
 - $f(0)$
 - $f'(0)$
- Let f be a function for which $f(2) = -3$ and $f'(x) = \sqrt{x^2 + 5}$, if $g(x) = x^2 f\left(\frac{x}{x-1}\right)$. Find $g'(2)$.
- Determine a and b so that f is differentiable at $x = c$.
 - $f(x) = \begin{cases} \cos x, & x \geq 0 \\ ax + b, & x < 0 \end{cases}; c = 0$
 - $f(x) = \begin{cases} \frac{1}{|x|}, & x > 1 \\ ax^2 + b, & x \leq 1 \end{cases}; c = 1$
- Find $\frac{dy}{dt}$, if
 - $y = u^2 + 1, u = 2x, x = \sin t$
 - $y = e^x + x^e, x = \cosh u, u = \sqrt{t}$
 - $y = \frac{1+u}{1-u}, u = s - \frac{1}{s}, s = \sqrt{t}$
 - $y = \sinh^{-1} x, x = \sin x, x = 2t$
- Find $\frac{dy}{dx}$, if
 - $x^3 y + y^2 x = 2$
 - $\sin(x^2 + y) = y^2(3x + 1)$
 - $\ln(\cos(2y)) = x + y$
 - $x^2 + 3x \cos y = \frac{x}{y}$
 - $\ln(xy) + \sin(xy) = 0$
 - $\sinh(2y) + \cosh(2y) = 2x$
- Find all points on the graph of the given equation at which the tangent line is horizontal.
 - $x^4 + y^4 + 2 = 4xy^3$
 - $(x^2 + y^2)^2 = x^2 - y^2$
- Find all points on the graph of the given equation at which the tangent line is vertical.

