

EXERCISE # (S-1)

HCV Worked out Examples(Chapter No. 8 - 1,4,6,7)

1. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:
- work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.
 - work done by gravitational force in the above case,
 - work done by friction on a body sliding down an inclined plane,
 - work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,
 - work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

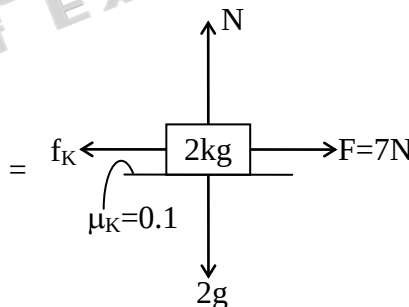
Ans. (a) +ve (b) -ve (c) -ve (d) +ve (e) -ve

Sol. If force and displacement are in same direction then work done is positive, otherwise it is negative.

2. A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the
- work done by the applied force in 10 s,
 - work done by friction in 10 s,
 - work done by the net force on the body in 10 s,
 - change in kinetic energy of the body in 10 s, and interpret your results.

Ans. (a) 882 J; (b) -247 J; (c) 635 J ; (d) 635 J;

Sol. $f_k = \mu N = \frac{1}{10} \times 2g = 2 \text{ N}$
 $a = \frac{F - f_k}{m} = \frac{7 - 2}{2} = \frac{5}{2} = 2.5 \text{ m/s}^2$
 In 10 sec block moves S
 $s = \frac{1}{2}at^2 = \frac{1}{2} \times 2.5 \times 100 = 125 \text{ m}$
 (a) $W_F = F \cos \theta S = 7 \cdot \cos 0 \cdot 125$
 $= 7 \times 125 = 875 \text{ J.}$
 (b) $W_{f_k} = F \cos 180^\circ S = 2 \times (-1) \cdot 125 = -250 \text{ J.}$
 (c) $\Sigma W_{\text{all}} = 875 - 250 = 625.$
 (d) $\Delta K = K_f - K_i$
 $= \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times 625 = 625 \text{ J}$
 $\therefore V = at = 2.5 \times 10 = 25$



3. A body constrained to move along the z-axis of a coordinate system is subject to a constant force F given by $F = -\hat{i} + 2\hat{j} + 3\hat{k} \text{ N}$ where $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors along the x-, y- and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis

Ans. 12 J

Sol. $W = F_Z \cdot \Delta Z = 3 \times 4 = 12 \text{ J}$

4. A particle of mass 2kg moves on a smooth horizontal plane under the action of a single force $\vec{F} = (3\hat{i} + 4\hat{j})$ Newton. Under this force it is displaced from (0,0) to (-1m, 1m). Find the work done by F (in Joule)

Ans. 1

Sol. $F_x = 3\hat{i}$ $F_y = 4\hat{j}$

$$\begin{aligned} W_F &= \int_0^{-1} F_x dx + \int_0^1 F_y dy \\ &= 3[x]_0^{-1} + 4[y]_0^1 \\ &= -3 + 4 = 1 \text{ J.} \end{aligned}$$

5. A point mass of 0.5 kg is moving along x-axis as $x = t^2 + 2t$, where, x is in meters and t is in seconds. Find the work done (in J) by all the forces acting on the body during the time interval [0, 2s]

Ans. 8

Sol. Method I: $W_{\text{all}} = \Delta KE = \frac{1}{2}m(v_2^2 - v_0^2)$

$$v = \frac{dx}{dt} = 2t + 2 = 2(t+1)$$

$$v_2 = 6 \text{ m/s}, v_0 = 2 \text{ m/s}$$

$$W_{\text{all}} = \frac{1}{2} \left(\frac{1}{2} \right) (6^2 - 2^2) = 8 \text{ J}$$

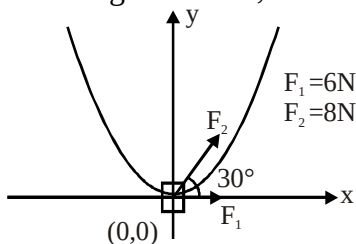
$$\text{Method II : } x = t^2 + 2t$$

$$a = \frac{d^2x}{dt^2} = 2 \text{ m/s}^2 = \text{constant}$$

$$\text{so } F = m(A) = 1 \text{ N} = \text{constant}$$

$$\text{so, } W = F.S = (1) (8-0) = 8 \text{ J}$$

6. A sleeve of mass 2 kg at origin can move on wire of parabolic shape $x^2 = 4y$. Two forces F_1 and F_2 are applied on the sleeve. F_1 is constant and is in x-direction. F_2 is constant in direction and magnitude. Body is displaced from origin to $x = 4$, then net work done by F_1 and F_2 is



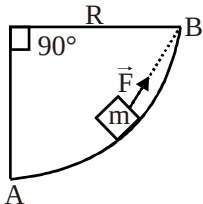
Ans. 67.7 J

Sol. $W_1 = \vec{F}_1 \cdot \vec{d}_1 = (6)(4) = 24 \text{ J}$

$W_2 = \vec{F}_2 \cdot \vec{d}_2$ where $\vec{F}_2 = 8\cos 30^\circ \hat{i} + 8\sin 30^\circ \hat{j} = (4\sqrt{3}\hat{i} + 4\hat{j})$, $\vec{s} = (4\hat{i} + 4\hat{j})$

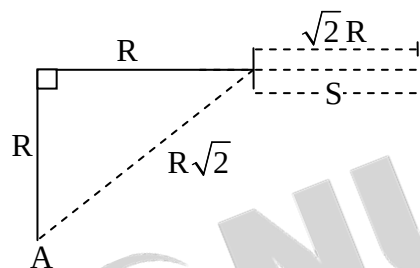
$\Rightarrow W_2 = 43.7 \text{ J}$ therefore $W_{\text{net}} = 67.7 \text{ J}$

7. A particle of mass m is taken from position A to position B along the circular path by a force $\vec{F}$, where magnitude of $\vec{F}$ is constant ($|\vec{F}|$) & is always directed towards point B. What will be the work done by the force $\vec{F}$? Here radius of track is R .



Ans. $FR\sqrt{2}$

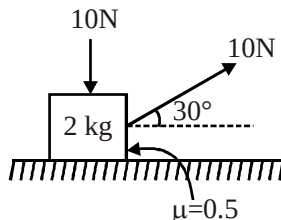
Sol.



$W_F = \vec{F} \cdot \vec{d}$
 $= F \cdot \sqrt{2} R$

$W_F = FR\sqrt{2}$

8. A body of mass 2 kg is kept on a rough horizontal surface as shown in the figure. Find the work done by frictional force in the time interval $t = 0$ to $t = 5 \text{ sec}$.



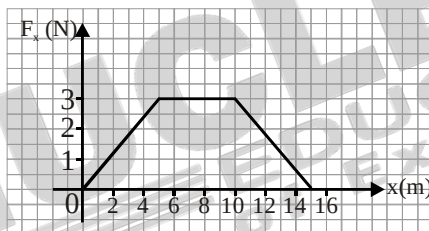
Ans. zero

Sol.

$N = 10 + 2g = 30$
 $f = \mu N$
 $= \frac{1}{2} \times 30 = 15$
 $f > F$

So in this case limiting friction is greater than applied force so block does not move
So, $W_f = 0$

9. A particle is subject to a force F_x that varies with position as in figure. Find the work done by the force on the body as it moves



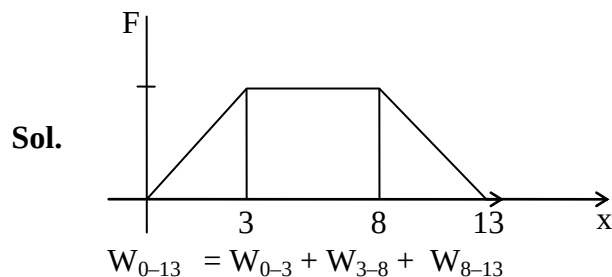
- (a) from $x = 0$ to $x = 5.00$ m,
 (b) from $x = 5.00$ m to $x = 10.0$ m, and
 (c) from $x = 10.0$ m to $x = 15.0$ m.
 (d) What is the total work done by the force over the distance $x = 0$ to $x = 15.0$ m?

Ans. (a) 7.5 J (b) 15 J (c) 7.5 J (d) 30 J

Sol. $W = \text{area under the graph.}$

10. The net force exerted on a particle acts in the $+x$ direction. Its magnitude increases linearly from zero at $x = 0$, to 24.0 N at $x = 3.0$ m. It remains constant at 24.0 N from $x = 3.0$ m to $x = 8.0$ m and then decreases linearly to zero at $x = 13.0$ m. Determine the work done to move the particle from $x = 0$ to $x = 13.0$.

Ans. 216 J



$$= \frac{1}{2} \times 3 \times 24 + 5 \times 24 + \frac{1}{2} \times 5 \times 24$$

$$= 36 + 120 + 60$$

$$W_{0-13} = 216 \text{ J}$$

HCV Exercises (Chapter No. 8 - 4,6,8,10)

HCV Worked out Examples (Chapter No. 8 - 2,3,5)

11. A force given by the relation $F = 8t$ (N/s) acts on a body of mass 2 kg initially at rest on a smooth surface where t is in seconds. Find the work done by this force on the body during first 2 seconds of its motion. Assume direction of force remain constant and body moves in the direction of force.

Ans. 64 J

Sol.

$$F = 8t$$

$$m = 3 \text{ kg}$$

$$a = \frac{F}{m} = \frac{8t}{2} = 4t$$

$$a = 4t$$

$$\frac{dv}{dt} = 4t$$

$$\int_0^v dv = 4 \int_0^2 t dt$$

$$v = 4 \left[\frac{t^2}{2} \right]_0^2$$

$$v = 4 [2 - 0]$$

$$v = 8 \text{ m/s}$$

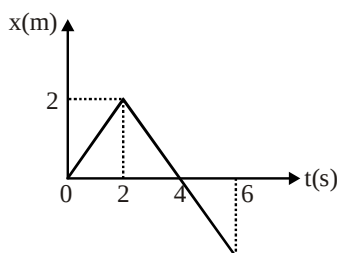
$$W_{\text{all}} = \Delta K$$

$$W_{\text{all}} = K_f - K_i$$

$$= \frac{1}{2} mv^2 = \frac{1}{2} \times 2 \times (8)^2 = 64$$

$$W = 64 \text{ J}$$

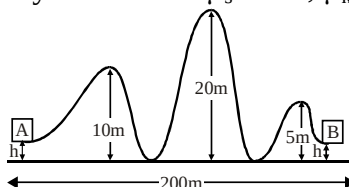
12. Position-time graph of a particle of mass 2 kg is shown in figure. Total work done on the particle from $t = 0$ to $t = 4$ s is



Ans. 0

Sol. Total work done = change in kinetic energy = $\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}(2) \times (1)^2 - \frac{1}{2} \times 2 \times (1)^2 = 0$

- 13.** A point object of mass 2 kg is moved from point A to point B very slowly on a curved path by applying a tangential force on a curved path as shown in figure. Then find the work done by external force in moving the body. Given that $\mu_s = 0.3$, $\mu_k = 0.1$. [$g = 10 \text{ m/s}^2$]



Ans. 400 J

Sol. $\Delta E = W_{fr} + W_g + W_{ext}$
 $= -(0.1 \times 20 \times 200) + 0 + W_{ext} = 400 \text{ J}$

- 14.** A 4 kg particle moves along the X-axis. Its position x varies with time according to $x(t) = t + 2t^3$, where x is in m and t is in seconds. Compute:

- (i) The kinetic energy at time t.
 (ii) The force acting on the particle at time t.
 (iii) The power delivered to the particle at time t.
 (iv) The work done on the particle from $t = 0$ to $t = 2$ seconds.

Ans. (i) $2 + 24t^2 + 72t^4 \text{ J}$, (ii) $48t \text{ N}$, (iii) $48t + 288t^3 \text{ W}$, (iv) 1248 J

Sol. $v = 1 + 6t^2$

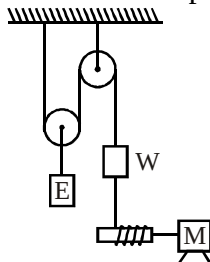
& $a = 12t$

$$KE = \frac{1}{2}m(1 + 36t^4 + 12t^2)$$

$$F = 4(12t) = 48t$$

$$P = F \cdot v$$

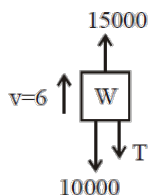
- 15.** The elevator E has a mass of 3000 kg when fully loaded and is connected as shown to a counterweight W of mass 1000 kg. Determine the power in kilowatts delivered by the motor



- (a) when the elevator is moving down at a constant speed of 3 m/s,
 (b) when it has an upward velocity of 3 m/s and a deceleration of 0.5 m/s^2 .

Ans. (a) -30 kW , 19.5 kW

Sol.



T = force applied by motor

$$\begin{aligned} \text{(a) } P &= F \cdot v = -T \times 6 \\ &= -5000 \times 6 \\ &= -30 \text{ kW} \end{aligned}$$

16. A block is released from rest from top of a rough curved track as shown in figure. It comes to rest at some point on the horizontal part. If its mass is 200 gm, calculate negative of work by friction is joules.



Ans. 4

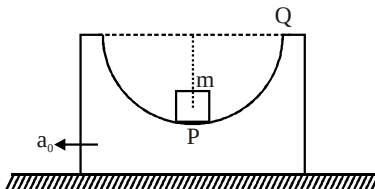
Sol. $W_f + W_g = 0 \Rightarrow W_f = -W_g = -mgh = -0.2 \times 10 \times 2 = -4 \text{ J}$

17. Power applied to a particle varies with time as $P = (3t^2 - 2t + 1)$ watt, where t is in second. Find the change in its kinetic energy between time $t = 2 \text{ s}$ and $t = 4 \text{ s}$.

Ans. 46 J

Sol. $P = F \cdot v = mv \cdot \frac{dv}{dt} \Rightarrow \frac{1}{2} m(v^2 - u^2) = \int P dt$
 $= \left[t^3 - t^2 + t \right]_2^4$

18. A small block of mass m is lying at rest at point P of a wedge having a smooth semicircular track of radius R . The minimum value of horizontal acceleration a_0 of wedge so that mass can just reach the point Q, is



Ans. g

Sol. By applying work energy theorem in wedge frame between point P and Q.

$$\Delta KE = W_g + W_{\text{pseudo force}} \Rightarrow 0 = -mgR + maR \Rightarrow a = g$$

19. A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the

work done by the gravitational force on the drop in the first and second half of its journey ?
What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 m s^{-1} ?

Ans. 0.082 J in each half : -0.163 J

Sol. Apply W.E.T between initial and final position.

$$W_g + W_R = K_f - K_i$$

$$W_R = K_f - W_g$$

$$= \frac{1}{2} \times 33.5 \times 10^{-6} \times 100 - 33.5 \times 10^{-6} \times 9.8 \times 500$$

$$= 0.00167 - 0.164$$

$$= -0.163 \text{ J.}$$

$$m = \rho V$$

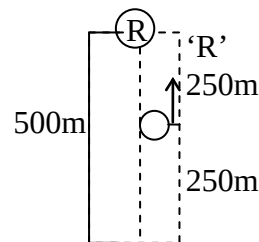
$$= 1000 \times \frac{4}{3} \times \frac{22}{7} \times 8 \times 10^{-9}$$

$$m = 33.49 \times 10^{-6}$$

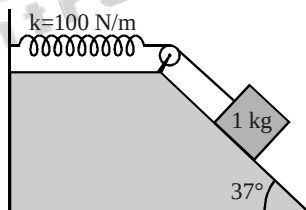
(ii) Work done by gravity in both case are same

$$W_g = mgh = 33.5 \times 10^{-6} \times 9.8 \times 250$$

$$= 0.082 \text{ J}$$



20. A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 N m^{-1} as shown in figure. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has a negligible mass and the pulley is frictionless.



Ans. 0.125

Sol. Let co-efficient of friction between block and incline plane
Apply W.E.T.

$$W_g + W_N + W_f + W_s = K_f - K_i$$

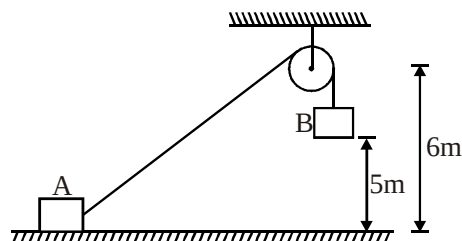
$$\sin 37 - \mu \cos 37 = \frac{100}{2} \left(\frac{100}{10000} \right)$$

$$\frac{3}{5} - \frac{1}{2} = \frac{\mu 4}{5}$$

$$\frac{1}{10} = \frac{\mu 4}{5} \quad \mu = \frac{1}{8}$$

$$\mu = 0.125$$

21. A block of mass m is held at rest on a smooth horizontal floor. A light frictionless, small pulley is fixed at a height of 6 m from the floor. A light inextensible string of length 16 m, connected with A passes over the pulley and another identical block B is hung from the string. Initial height of B is 5 m from the floor as shown in Fig. When the system is released from rest, B starts to move vertically downwards and A slides on the floor towards right.

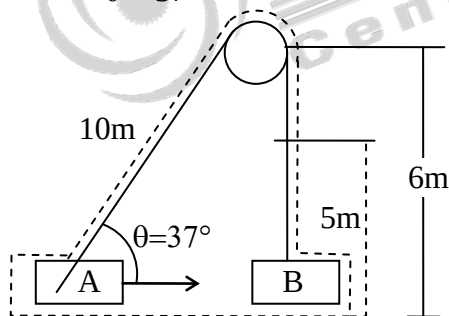


- (i) If at an instant string makes an angle θ with horizontal, calculate relation between velocity u of A and v of B.
(ii) Calculate v when B strikes the floor.

Ans. $u = v \sec \theta$, $v = \frac{40}{\sqrt{41}} \text{ m/s}$

Sol. (i) By string constraint
 $-TV + U \cos \theta T = 0$
 $v = U \cos \theta$
 or $U = v \sec \theta$

(ii) $\sin \theta = \frac{6}{10}$
 $\theta = 37^\circ$



Apply W.E.T on

A & B as a system

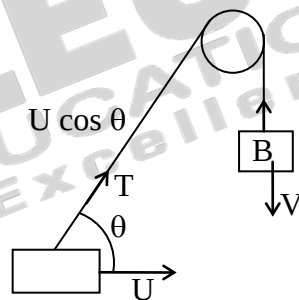
$$W_g + W_T = K_f - K_i$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mU^2$$

$$5g = \frac{1}{2} \left(v^2 + \frac{v^2}{\cos^2 \theta} \right)$$

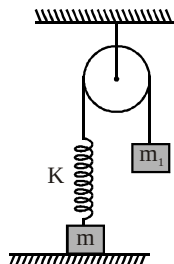
$$10g \times \frac{16}{25} = v^2 \left(\frac{41}{25} \right)$$

$$\therefore h = 5 \text{ m}$$



$$V = \frac{40}{\sqrt{41}}$$

22. For what minimum value of m_1 the block of mass m will just leave the contact with surface ?



Ans. $m_1 = m/2$

Sol. $ky = mg$

$$m_1gy = \frac{1}{2}ky^2$$

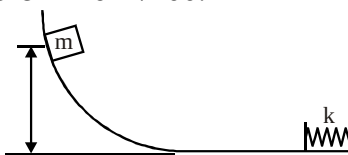
$$2m_1g = mg$$

$$m_1 = \frac{m}{2}$$

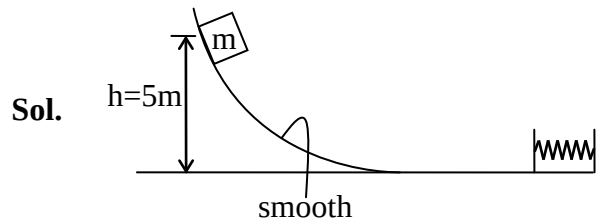
HCV Exercises (Chapter No. 8 - 15,17,19,21,23,26,31,35,37,40,43)

HCV Worked out Examples(Chapter No. 8 - 8,9,10,11,12)

23. A mass m slides from rest at height h down a smooth curved surface which becomes horizontal at zero height (see figure). A spring is fixed horizontally on the level part of the surface. The spring constant is k N/m. When the mass encounters the spring it compresses it by an amount $x = h/10$. If $m = 1$ kg, $h = 5$ m then find $k/100$.



Ans. 4



Apply W.E.T.

$$W_{all} = \Delta K$$

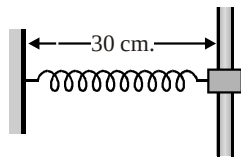
$$W_{g_1} + W_N + W_S + W_{g_2} = K_f - K_i$$

$$mgh = \frac{K}{2}(x_f^2 - x_i^2) \quad x_f = \frac{h}{10}$$

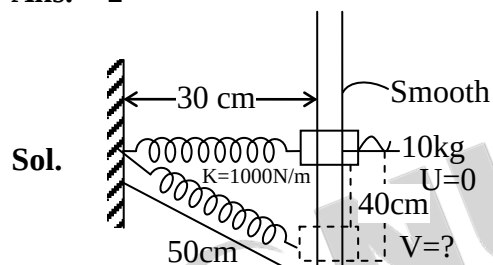
$$\frac{2mg}{h} = \frac{K}{100}$$

$$\frac{2 \times 1 \times 10}{5} \quad \therefore \quad \frac{K}{100} = 4$$

24. A sleeve of mass $m = 10 \text{ kg}$ can slide on a straight vertical smooth rod. The sleeve is connected to a spring of spring constant $k = 1000 \text{ N/m}$, whose other end is connected to a fixed support as shown in figure. The sleeve is released when the spring is horizontal and relaxed. Find the speed in SI units of the sleeve when it descends a distance of 40 cm . (Use $g = 10 \text{ m/s}^2$)



Ans. 2



$$x_f = 50 - 30 = 20 \text{ cm (total extension)}$$

$$x_i = 0 \text{ (Natural length)}$$

W.E.T

$$W_s + W_g = K_f - k_i$$

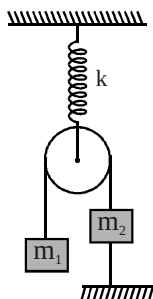
$$\frac{K}{2}(x_i^2 - x_f^2) + mg \times \frac{40}{100} = \frac{1}{2}mv^2$$

$$-20 + 40 = \frac{1}{2} \times 10V^2$$

$$\frac{20 \times 2}{10} = V^2$$

$$V = 2 \text{ m/s}$$

25. In the figure shown, pulley and spring are ideal. Find the potential energy stored in the spring ($m_1 > m_2$).



Ans. $\frac{2m_1^2 g^2}{k}$

Sol. $U = \frac{1}{2} kx^2$

$$= \frac{2m_1^2 g^2}{k}$$

in equilibrium,

$$T = m_1 g$$

$$kx = 2T = 2m_1 g$$

$$x = \frac{2m_1 g}{k}$$

26. The potential energy (in joules) function of a particle in a region of space is given as :

$$U = (2x^2 + 3y^3 + 2z)$$

Here x, y and z are in metres. Find the magnitude of x component of force (in Newton) acting on the particle at point P (1m, 2m, 3m).

Ans. 4

Sol. $F_x = -\frac{dU}{dx} = -4x$

27. The potential function for a conservative force is given by $U = k(x + y)$. Find the work done by the conservative force in moving a particle from the point A(1, 1) to point B (2, 3).

Ans. $-3k$

Sol. $U = K(x + y)$ (conservative)

$$F = -\frac{dU}{dr}$$

$$Fdr = -(d(x + y))$$

$$\int Fdr = -\int dU$$

$$W_{CF} = \left| U \right|_{1,4}^{2,3}$$

$$= -[K(2-1) + K(3-1)]$$

$$= -[K + 2K]$$

$$W_{CF} = -3K$$

28. The P.E. of a particle oscillating on x-axis is given as $U = 20 + (x - 2)^2$ here U is in Joules & x is in meters. Total mechanical energy of particle is 36 J

- (i) Find the mean position
(ii) Find the max. K.E. of the particle

Ans. (i) $x = 2$, (ii) 16 J

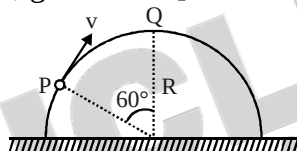
Sol. At mean position, U = minimum

$$U_{\min} = 20 \text{ J, at } x = 2\text{m, } KE_{\max} = 36 - 20 = 16 \text{ J}$$

HCV Exercises (Chapter No. 8 - 29,36,45,46,47,48,50,51)

HCV Worked out Examples(Chapter No. 8 - 13,14)

29. A particle is given a certain velocity v at point P as shown on a hemispherical smooth surface. The value of v in m/s, such that particle when reaches Q the normal reaction of surface is equal to particle's weight, is $[R = 1.6 \text{ m, } g = 10 \text{ m/s}^2]$

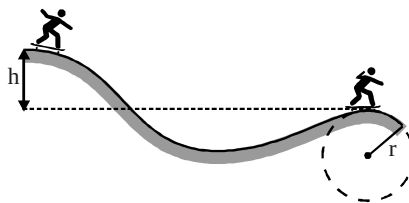


Ans. 4

Sol. $N = mg \Rightarrow v_f = 0$

$$mg \frac{R}{2} = \frac{1}{2} mv_1^2 \Rightarrow v = \sqrt{gR} = 4 \text{ m/s}$$

30. A skier starts from rest at the top of a hill. The skier coasts down the hill and up a second hill, as the drawing illustrates. The crest of the second hill is circular, with a radius of $r = 36 \text{ m}$. Neglect friction and air resistance. What must be the height h(in m) of the first hill so that the skier just loses contact with the snow at the crest of the second hill?



Ans. 018

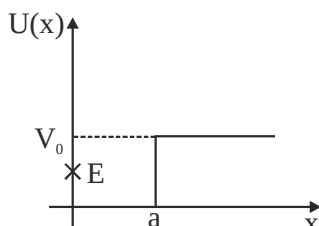
Sol. $mg = \frac{mv^2}{r} \quad \dots(i)$

$$v^2 = rg = 360$$

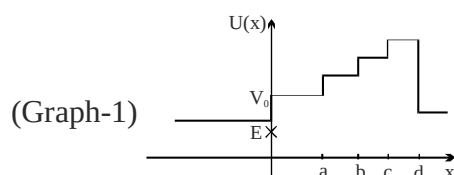
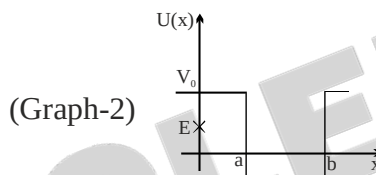
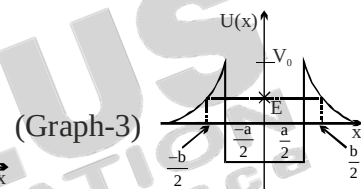
$$\text{from energy conservation } mgh = \frac{1}{2} mv^2$$

$$v^2 = 2gh \Rightarrow 360 = 2gh \Rightarrow h = 18 \text{ m}$$

31. The given graph is a potential energy function in one dimension. The total energy of particle is indicated by cross on the ordinate axis. The graph of figure-1 is given as an example. From the figure-1, it can be interpreted that for the given total energy indicated by cross on the ordinate axis the particle cannot be found in the Region : $x > a$. Now, for the following potential functions in one dimensions, specify the regions, in which the particle cannot be found for the energy marked as E on graphs.



(Figure-1)



Ans. Graph - 1 : For all x, Graph-2 : $x < a$ & $x > b$, Graph-3 : $kx - \frac{b}{2} < x < -\frac{a}{2}$ & $\frac{a}{2} < x < \frac{b}{2}$

Sol. PE must be smaller than or equal to total energy PE can not be more than total energy.

32. A ball is attached to a horizontal cord of length L whose other end is fixed, (a) If the ball is released, what will be its speed at the lowest point of its path ? (b) A peg is located a distance h directly below the point of attachment of the cord. If $h = 0.75 L$, what will be the speed of the ball when it reaches the top of its circular path about the peg ?

Ans. $\sqrt{2gL}$, $\sqrt{gL}$

Sol. Let mass of ball 'm'

(a) W.E.T.

$$W_g = K_f - K_i$$

$$mgL = \frac{1}{2}mv^2$$

$$V = \sqrt{2gL}$$

(b) About the peg length of cord is $\frac{L}{4}$

W.E.T.

$$W_g = K_f - K_i$$

$$-mgR \frac{L}{4} = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2$$

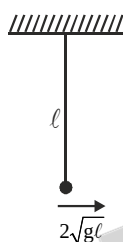
$$-\frac{mgL}{2} = \frac{1}{2}m(v'^2 - v^2)$$

$$-gL = v'^2 - 2gL$$

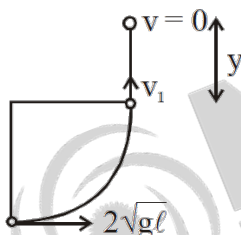
$$-v'^2 = -gL + 2gL$$

$$v' = \sqrt{gL}$$

33. One end of a string of length $\ell = \frac{14}{9}\text{m}$ is fixed and a mass of 1 kg is tied to the other end. The ball is given a velocity $2\sqrt{g\ell}$ at the bottom most point as shown in figure. The string is cut when the ball becomes horizontal. Find the distance (in m) travelled till it stop for the 1st time (Take $\pi = \frac{22}{7}$).



Ans. 4



Sol.

By energy conservation

$$v_1 = \sqrt{2g\ell} \text{ and } y = \frac{(2\sqrt{g\ell})^2}{2g} = \ell$$

$$\text{distance} = \frac{\pi\ell}{2} + \ell = \frac{14}{9} \left[\frac{11}{7} + 1 \right] = 4\text{m}$$

HCV Exercises (Chapter No. 8 - 49,52,53,54,55,56,58,60,61,62,64xx)

EXERCISE # (S-2)

1. Water is pumped from a depth of 10 m and delivered through a pipe of cross section 10^{-2} m^2 upto a height of 10 m. If it is needed to deliver a volume 0.2 m^3 per second, find the power required. [Use $g = 10 \text{ m/s}^2$]

Ans. 80 kW

Sol. $\therefore$ mass of deliver water = $1000 \frac{\text{kg}}{\text{m}^3} \times \frac{2 \text{ m}^3}{10 \text{ sec}}$
 $= 200 \frac{\text{kg}}{\text{sec}}$

$\therefore$ Required power = $\frac{W_{\text{required}}}{t}$

$W_g + W_{\text{req}} = K_f - K_i$

$-mgH + W_{\text{req}} = K_f$

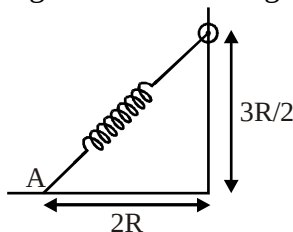
$W_{\text{req}} = \frac{1}{2}mv^2 + mgH$

$P_{\text{requ}} = \left(\frac{m}{t} \right) \left(\frac{1}{2} \times 400 + 10 \times 20 \right)$

$= 200 (400) = 80000$

$P_{\text{requ}} = 80 \text{ kW}$

2. A ring of mass m can slide over a smooth vertical rod. The ring is connected to a spring of force constant $K = \frac{4mg}{R}$ where $2R$ is the natural length of the spring. The other end of the spring is fixed to the ground at a horizontal distance $2R$ from the base of the rod. The mass is released at a height of $1.5R$ from ground
 (i) calculate the work done by the spring.
 (ii) calculate the velocity of the ring as it reaches the ground.



Ans. (i) $mgR/2$, (ii) $2\sqrt{gR}$

Sol. (i) $W_{\text{spring}} = -\Delta U$

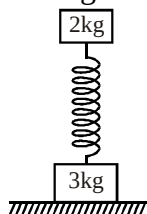
$= -\left(0 - \frac{1}{2}kx_0^2 \right)$

$x_0 = \sqrt{4R^2 + \frac{9R^2}{4}} - 2R = \frac{5R}{2} - 2R = \frac{R}{2}$

$$W_{\text{spring}} = \frac{1}{2} \times \frac{4mg}{R} \times \frac{R^2}{4} = \frac{1}{2} mgR$$

$$(ii) W_s + W_{mg} = \frac{1}{2} mv^2 \Rightarrow \frac{1}{2} mgR + \frac{3}{2} mgR = \frac{1}{2} mv^2 \Rightarrow v = 2\sqrt{gR}$$

3. The ends of spring are attached to blocks of mass 3kg and 2kg. The 3kg block rests on a horizontal surface and the 2kg block which is vertically above it is in equilibrium producing a compression of 1cm of the spring. The 2kg mass must be compressed further by at least _____, so that when it is released, the 3 kg block may be lifted off the ground.



Ans. 2.5 cm

Sol. $kx_0 = mg \Rightarrow k = 2000 \text{ N/m}$ [$x_0 = 1 \text{ cm}$ in inequilibrium]

to lift 3 kg mass, extension in the spring must be, $y = \frac{30}{2000} = \frac{3}{200} \text{ m}$

Let further compression in spring is x .

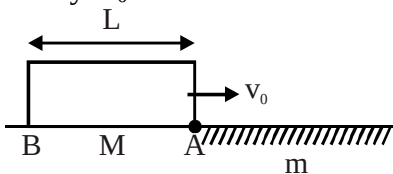
$$W_s + W_{mg} = \Delta KE \Rightarrow -\frac{1}{2} k \left[y^2 - \left(x + \frac{1}{100} \right)^2 \right] - mg \left[x + \frac{1}{100} + y \right] = 0$$

$$\Rightarrow -1000 \left[\frac{9}{40000} - \left(x + \frac{1}{100} \right)^2 \right] = 20 \left[\frac{3}{200} + \left(x + \frac{1}{100} \right) \right]$$

$$\Rightarrow -\frac{9}{800} + 50 \left(x + \frac{1}{100} \right)^2 = \frac{3}{200} + \left(x + \frac{1}{100} \right)$$

$$\Rightarrow 50 \left(x + \frac{1}{100} \right)^2 - \left(x + \frac{1}{100} \right) - \frac{21}{800} = 0 \Rightarrow x = 2.5 \text{ cm}$$

4. A uniform rod of mass m length L is sliding along its length on a horizontal table whose top is partly smooth & rest rough with friction coefficient μ . If the rod after moving through smooth part, enters the rough with velocity v_0 .

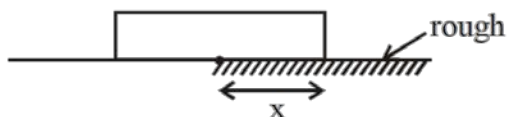


- (i) What will be the magnitude of the friction force when its x length ($< L$) lies in the rough part during sliding.

- (ii) Determine the minimum velocity v_0 with which it must enter so that it lies completely in rough region before coming to rest.
(iii) If the velocity is double the minimum velocity as calculated in part (a) then what distance does its front end A would have travelled in rough region before rod comes to rest.

Ans. (i) $f = -\frac{\mu mgx}{\ell}$; (ii) $\sqrt{\mu g \ell}$; (iii) $\frac{5\ell}{2}$

Sol. (a)



$$\Rightarrow N \text{ for rough part} = \frac{mgx}{\ell} \Rightarrow f_k = -\frac{\mu mgx}{\ell}$$

(b) By work-energy theorem $\Rightarrow -\frac{1}{2}\mu mg\ell = -\frac{1}{2}mv_0^2 \Rightarrow v_0 = \sqrt{\mu g \ell}$

(c) If rod travels a further distance s after it is completely inside the rough region

$$\Rightarrow -\frac{1}{2}\mu mg\ell - \mu mgs = -\frac{1}{2}m(4\mu g \ell) \Rightarrow s = \frac{3}{2}\ell$$

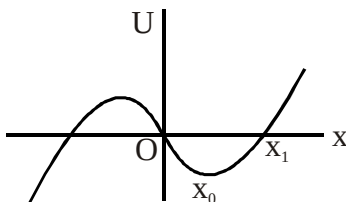
So distance travelled by A is $\ell + \frac{3\ell}{2} = \frac{5\ell}{2}$

5. A particle is confined to move along the $+x$ axis under the action of a force $F(x)$ that is derivable from the potential $U(x) = ax^3 - bx$.

(i) Find the expression for $F(x)$

(ii) When the total energy of the particle is zero, the particle can be trapped with in the interval $x = 0$ to $x = x_1$. For this case find the values of x_1 .

(iii) Determine the maximum kinetic energy that the trapped particle has in its motion. Express all answers in terms a and b . At what value of x will the kinetic energy be maximum ?



Ans. (i) $F = -3ax^2 + b$, (ii) $x = \sqrt{\frac{b}{a}}$, (iii) $KE_{\max} = \frac{2b}{3\sqrt{3}}\sqrt{\frac{b}{a}}$, $x = \sqrt{\frac{b}{3a}}$

Sol. (i) $F = -\frac{dU}{dx} = b - 3ax^2$

(ii) $U = 0$

$$x = \sqrt{\frac{b}{a}} = x_1$$

(iii) for KE = max., PE = min

$$\frac{dU}{dx} = 0 \Rightarrow x = \sqrt{\frac{b}{3a}}$$

6. Two trains of equal masses are drawn along smooth level lines by engines; one of them X exerts a constant force while the other Y works at a constant rate. Both start from rest & after a time t both again have the same velocity v. Find the ratio of travelled distance during the interval.

Ans. $\frac{3}{4}$

Sol. (i) $a = \frac{F}{m}$

$$s_x = \frac{1}{2} \left(\frac{F}{m} \right) t^2 \quad \dots(i)$$

$$v_x = \frac{F}{m} t$$

(ii) $P = mv_y \frac{dv}{dt}$

$$\frac{2Pt}{m} = v_y^2$$

$$v_y = \sqrt{\frac{2Pt}{m}}$$

$$\frac{ds_y}{dt} = \sqrt{\frac{2Pt}{m}}$$

$$s_y = \sqrt{\frac{2P}{m}} t^{3/2} \cdot \frac{2}{3}$$

$$s_y = \sqrt{\frac{2F^2 t}{2m^2}} t^{3/2} \cdot \frac{2}{3}$$

$$s_y = \frac{2}{3} \frac{F}{m} t^2 \quad \dots(ii)$$

$$\frac{s_x}{s_y} = \frac{3}{4}$$

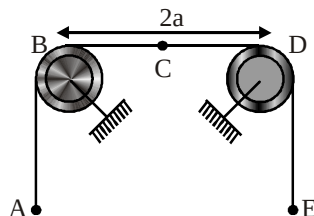
$$\therefore v_x = v_y$$

$$\frac{F}{m} t = \sqrt{\frac{2Pt}{m}}$$

$$\frac{F^2}{m^2} t^2 = \frac{2Pt}{m}$$

$$P = \frac{F^2 t}{2m}$$

7. A light string ABCDE whose mid point is C passes through smooth rings B and D, which are fixed in a horizontal plane distance 2a apart. To each of the points A, C and E is attached a mass m. Initially C is held at rest at O (mid point BD) and is then set free. What is the distance OC when C comes to instantaneous rest?



Ans. $4a/3$

Sol. Initially rest after release

Let the $OC = x$

then $BC = \sqrt{a^2 + x^2}$

Netw $AB = ED = \sqrt{a^2 + x^2} - a$

W.E.T.

$$W_g = K_f - K_i$$

$$-2mg\sqrt{a^2 + x^2} - a + mgx = 0$$

$$mgx = 2mg\sqrt{a^2 + x^2} - a$$

$$x = 2\sqrt{a^2 + x^2} - a$$

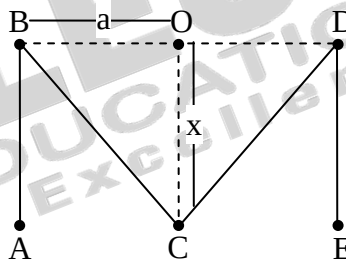
$$x + 2a = 2\sqrt{a^2 + x^2}$$

Squaring both side

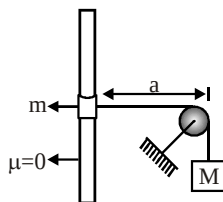
$$x^2 + 4a^2 + 4ax = 4a^2 + 4x^2$$

$$3x^2 = 4ax$$

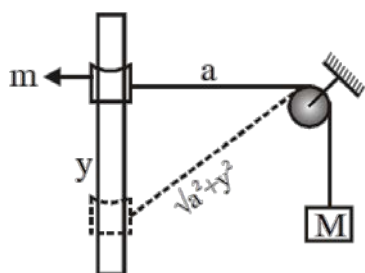
$$x = \frac{4a}{3}$$



8. A ring of mass m slides on a smooth vertical rod. A light string is attached to the ring and is passing over a smooth peg distant a from the rod, and at the other end of the string is a mass M ($M > m$). The ring is held on a level with the peg and released. Show that it first comes to rest after falling a distance $\frac{2mMa}{M^2 - m^2}$



Sol.



$$mgy = Mg \left[\sqrt{a^2 + y^2} - a \right]$$

$$\Rightarrow m^2 y^2 + m^2 a^2 + 2Mmay$$

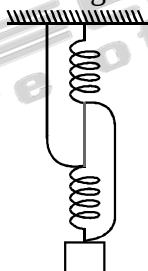
$$= M^2 a^2 + M^2 y^2$$

$$\Rightarrow y = \frac{2Mma}{M^2 - m^2}$$

9. In figure two identical springs, each with a relaxed length of 50 cm and a spring constant of 500 N/m, are connected by a short cord of length 10 cm. The upper spring is attached to the ceiling, a box that weighs 100N hangs from the lower spring. Two additional cords, each 85 cm long, are also tied to the assembly; they are limp (i.e. slack).

(i) If the short cord is cut, so that the box then hangs from the springs and the two longer cords, does the box move up or down?

(ii) How far does the box move before coming to rest again?



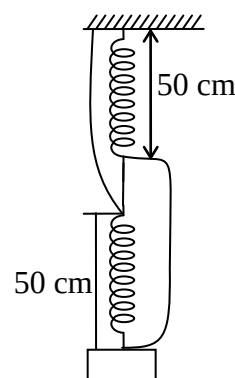
Ans. up, 10 cm

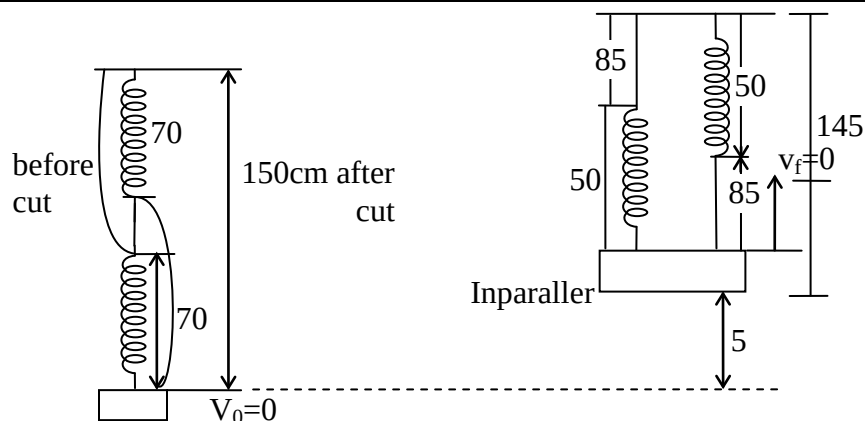
Sol. When string cut extension in spring in series combination

$$\frac{kx}{2} = 100 \text{ N}$$

$$x = \frac{2000}{500}$$

$$x = 0.4 \text{ m} \quad \Rightarrow \quad x = 40 \text{ cm}$$





$$2kx = 100$$

$$x = \frac{1}{10}$$

$$x = 10 \text{ cm}$$

10. A 650-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 1.75 m/s.

(i) What is the average power of the elevator motor during this period?

(ii) How does this power compare with its power when it moves at its cruising speed?

Ans. (i) $6 \times 10^3 \text{ W}$ (ii) $1.1 \times 10^4 \text{ W}$

Sol. $a = 1.75/3 = 0.58 \text{ m/s}^2 \Rightarrow s = \frac{1}{2}at^2 = 2.61 \text{ m}$

$$T = m(9 + a) = 6877 \text{ N} \Rightarrow P_{\text{avg}} = \frac{T \cdot s}{t} \approx 6 \times 10^3 \text{ W}$$

$$(ii) P = F \cdot v = 6500 \times 1.75 = 10^4 \text{ Watt}$$

11. A car's engine can deliver 90kW of power. The car's mass is 1000kg. Assume the total resistive force is proportional to the velocity: $|F_{\text{friction}}| = \alpha v$. The drag coefficient α is $\alpha = 100 \text{ N/s}$. Car can maintain on a level road a maximum speed of _____ m/s?



Ans. 30

Sol. $\therefore P = F V_{\text{max}} \quad \therefore F = \alpha V$

$$\frac{90 \times 1000}{\alpha} = V_{\text{max}}^2$$

$$\alpha = 100$$

$$\frac{90 \times 1000}{100}$$

$$V_{\max} = 30 \text{ m/s}$$



EXERCISE # (O-1)

1. A force of magnitude of 30 N acting along $\hat{i} + \hat{j} + \hat{k}$, displaces a particle from point (2, 4, 1) to (3, 5, 2). The work done during this displacement is

(A) 90 J (B) 30 J (C) $30\sqrt{3}$ J (D) $30/\sqrt{3}$ J

Ans. (C)

Sol. $\vec{F} = \frac{30}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$, $\Delta\vec{r} = \hat{i} + \hat{j} + \hat{k} \Rightarrow W = 30\sqrt{3}J$

2. The work done in joules in increasing the extension of a spring of stiffness 10 N/cm from 4 cm to 6 cm is:

(A) 1 (B) 10 (C) 50 (D) 100

Ans. (A)

Sol. $\therefore W_{\text{spring}} = \frac{K}{2}(x_i^2 - x_f^2)$
 $= \frac{1000}{2} \left(\frac{16}{10000} - \frac{36}{1000} \right)$
 $|W| = 1$

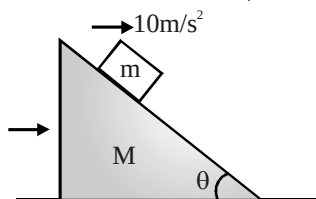
3. A rope is used to lower vertically a block of mass M by a distance x with a constant downward acceleration g/2. The work done by the rope on the block is :

(A) Mg x (B) $\frac{1}{2} Mg x^2$ (C) $-\frac{1}{2} Mg x$ (D) $Mg x^2$

Ans. (C)

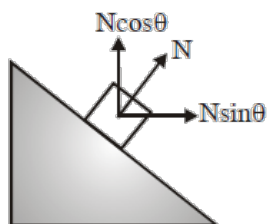
Sol. $T - mg = -\frac{mg}{2} \Rightarrow T = \frac{mg}{2} \Rightarrow W = -T.x = -\frac{1}{2}mgx$

4. In the figure shown all the surfaces are frictionless, and mass of the block, $m = 1$ kg. The block and wedge are held initially at rest. Now wedge is given a horizontal acceleration of 10 m/s^2 by applying a force on the wedge, so that the block does not slip on the wedge. Then work done by the normal force in ground frame on the block in $\sqrt{3}$ seconds is :



(A) 30 J (B) 60 J (C) 150 J (D) $100\sqrt{3}$ J

Ans. (C)



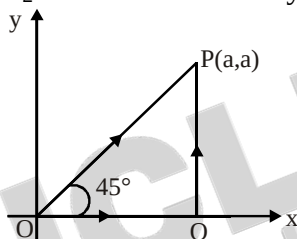
Sol.

$$N \sin \theta = ma = 10$$

$$\Delta x = \frac{1}{2} at^2 = 15$$

$$W = N \sin \theta \cdot \Delta x = 150 \text{ J}$$

5. A particle is moved from (0, 0) to (a, a) under a force $\vec{F} = (3\hat{i} + 4\hat{j})$ from two paths. Path 1 is OP and path 2 is OQP. Let W_1 and W_2 be the work done by this force in these two paths. Then :



- (A) $W_1 = W_2$ (B) $W_1 = 2W_2$ (C) $W_2 = 2W_1$ (D) $W_2 = 4W_1$

Ans.

(A)

Sol.

F is constant so it is conservative.

$$W_1 = W_2$$

6. A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is

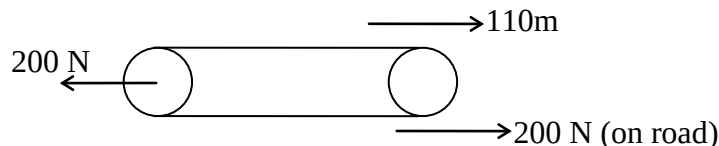
- (A) + 2000J (B) - 200J (C) zero (D) - 20,000J

Ans.

(C)

Sol.

Work done by cycle on the road or work done by the road on the cycle is zero. because there is no relative motion between tyre and road the point of contact motion zero.



7. A body of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5 \text{ m}^{-1/2} \text{ s}^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2 \text{ m}$ is

- (A) 1.5 J (B) 50 J (C) 10 J (D) 100 J

Ans.

(B)

Sol. $u = 0, v = 5(2)^{3/2} \Rightarrow W = \frac{1}{2}m(v^2 - u^2) = 50 \text{ J}$

8. A man who is running has half the kinetic energy of the boy of half his mass. The man speeds up by 1 m/s and then has the same kinetic energy as the boy. The original speed of the man was
(A) $\sqrt{2}$ m/s (B) $(\sqrt{2} - 1)$ m/s (C) 2 m/s (D) $(\sqrt{2} + 1)$ m/s

Ans. (D)

Sol. Mass of man is M and his speed is u. speed of boy is v

$$\Rightarrow \frac{1}{2}Mu^2 = \frac{1}{2} \times \left(\frac{1}{2} \times \frac{M}{2} v^2 \right) \quad \dots(i)$$

$$\& \frac{1}{2}M(u+1)^2 = \frac{1}{2} \times \left(\frac{M}{2} v^2 \right) \quad \dots(ii)$$

On solving equation (i) & (ii) we get

$$u = (\sqrt{2} + 1) \text{ m/sec}$$

9. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 revolutions per minute, its kinetic energy would be
(A) $250\pi^2$ (B) $100\pi^2$ (C) $5\pi^2$ (D) 0

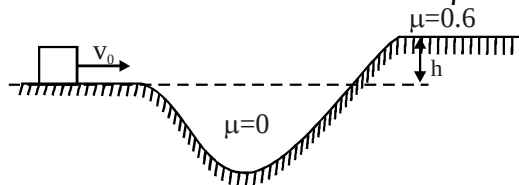
Ans. (A)

Sol. $\omega = 300 \times \frac{2\pi}{60} = 10\pi \text{ rad/sec}$

$$V = R\omega = 10\pi \times 1 = 10\pi \text{ rad/sec}$$

$$KE = \frac{1}{2} \times 5 \times 100\pi^2 = 250\pi^2 \text{ Joule}$$

10. In the figure, a block slides along a track from one level to a higher level, by moving through an intermediate valley. The track is frictionless until the block reaches the higher level. There a frictional force stops the block in a distance d. The block's initial speed v_0 is 6 m/s, the height difference h is 1.1 m and the coefficient of kinetic friction μ is 0.6. The value of d is



- (A) 1.17 m (B) 1.71 m (C) 7.11 m (D) 11.7 m

Ans. (A)

Sol. $\frac{1}{2}mv_0^2 = mgh + \mu mgd$

$$18 = 11 + 6d$$

$$d = \frac{7}{6}$$

11. A particle moves on a rough horizontal ground with some initial velocity say v_0 . If $3/4$ of its kinetic energy is lost in friction in time t_0 then coefficient of friction between the particle and the ground is

(A) $\frac{v_0}{2gt_0}$ (B) $\frac{v_0}{4gt_0}$ (C) $\frac{3v_0}{4gt_0}$ (D) $\frac{3v_0}{4gt_0}$

Ans. (A)

Sol. $\frac{1}{2}m(v^2 - u^2) = -\frac{3}{4} \times \frac{1}{2}mu^2$ & $a = -\mu g$

$$v = u + at \Rightarrow \frac{v_0}{2} = v_0 - \mu gt_0$$

12. A block of mass m is hung vertically from an elastic thread of force constant mg/a . Initially the thread was at its natural length and the block is allowed to fall freely. The kinetic energy of the block when it passes through the equilibrium position will be :

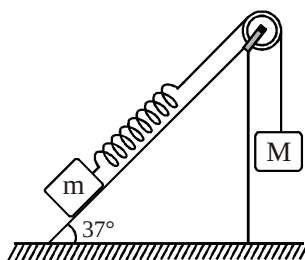
(A) mga (B) $mga/2$ (C) zero (D) $2mga$

Ans. (B)

Sol. $ky_0 = mg \Rightarrow y_0 = a$

$$KE = mga - \frac{1}{2}ka^2 = \frac{1}{2}mga$$

13. A block of mass m is attached with a massless spring of force constant k . The block is placed over a rough inclined surface for which the coefficient of friction is $\mu = \frac{3}{4}$. The minimum value of M required to move the block up the plane is : (Neglect mass of string and pulley and friction in pulley)



(A) $\frac{3}{5}m$ (B) $\frac{4}{5}m$ (C) $2m$ (D) $\frac{3}{2}m$

Ans. (A)

Sol. For mass m :

$$mg \sin \theta + f_{s, \max} = kx$$

$$mg \sin \theta + \mu mg \cos \theta = kx$$

for mass M :

$$Mgx = \frac{1}{2}kx^2 \Rightarrow kx = 2mg$$

$$\Rightarrow M = \frac{m(\sin + \mu \cos \theta)}{2}$$

14. A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to

(A) $t^{1/2}$ (B) t (C) $t^{3/2}$ (D) t^2

Ans. (B)

Sol. $S = \frac{1}{2}at^2$

$$P = \frac{dw}{dt}$$

$$P = \frac{f \cdot \frac{1}{2}at^2}{t}$$

$$P \propto t$$

15. A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to

(A) $t^{1/2}$ (B) t (C) $t^{3/2}$ (D) t^2

Ans. (C)

Sol. Power const.

$$S = Vt$$

$$P = F \left(\frac{ds}{dt} \right) = FV$$

$$FV = ma \frac{S}{t}$$

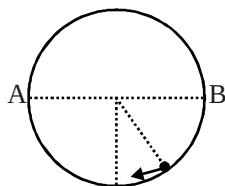
$$P = \frac{ms}{t^2} \times \frac{S}{t}$$

$$P = \frac{ms^2}{t^3}$$

$$S = \sqrt{\frac{Pt^3}{m}}$$

$$S \propto t^{3/2}$$

16. A particle of mass m moves inside a smooth spherical shell of radius R in a gravity free space. At any instant the kinetic energy of the particle is K . The force applied by particle on the shell at this instant is



(A) $\frac{K}{R}$

(B) $\frac{2K}{R}$

(C) $\frac{3K}{R}$

(D) $\frac{K}{2R}$

Ans. (B)

Sol. Gravity free space.

$$K.E = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}mv^2$$

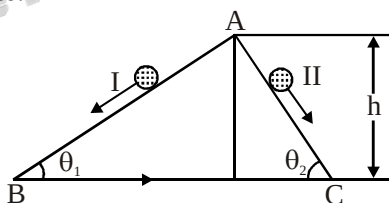
$$v^2 = \frac{2K}{m}$$

$$F = \frac{mv^2}{R}$$

$$= \frac{m2k}{Rm}$$

$$F = \frac{2k}{R}$$

17. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure. Which of the following statement is correct?



- (A) Both the stones reach the bottom at the same time but not with the same speed.
(B) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II.
(C) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I.
(D) Both the stones reach the bottom at different times and with different speeds.

Ans. (C)

Sol. $\frac{1}{2}mv^2 = mgh$

$$\Rightarrow v = \sqrt{2gh}$$

$$a = g \sin\theta$$

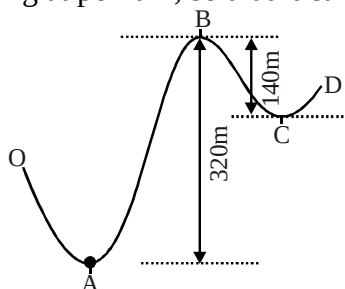
$$\theta_2 > \theta_1$$

$$\sin \theta_2 > \sin \theta_1$$

$$a_2 > a_1$$

$$t_2 < t_1$$

18. Track OABCD (as shown in figure) is smooth and fixed in vertical plane. What minimum speed has to be given to a particle lying at point A, so that it can reach point C?



(A) 60 m/s

(B) 100 m/s

(C) 70 m/s

(D) 80 m/s

Ans. (D)

Sol. To reach at C, particle must reach at B first.

$$\frac{1}{2}mu^2 = mgh$$

$$u^2 = 2 \times 10 \times 320$$

$$u = 80 \text{ m/sec}$$

19. In a shotput event an athlete throws the shotput of mass 10 kg with an initial speed of 1m/s at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 m/s^2 , the kinetic energy of the shotput when it just reaches the ground will be

(A) 2.5 J

(B) 5.0 J

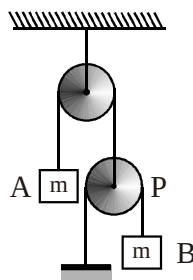
(C) 52.5 J

(D) 155.0 J

Ans. (D)

Sol. $\frac{1}{2}mv^2 = mgh + \frac{1}{2}mu^2 = 150 + 5 = 155 \text{ J}$

20. In the figure shown, the system is released from rest. Find the velocity of block A when block B has fallen a distance ' ℓ '. Assume all pulleys to be massless and frictionless.



- (A) $\sqrt{\frac{g\ell}{5}}$ (B) $\sqrt{g\ell}$ (C) $\sqrt{5g\ell}$ (D) None of these

Ans. (A)

Sol. $y_B = 2y_A \Rightarrow y_A = \frac{1}{2}\ell$ & $u_B = 2u_A$

$$mg\ell - \frac{1}{2}mg\ell = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 \Rightarrow \frac{1}{2}mg\ell = \frac{5}{2}mv_A^2$$

21. When a conservative force does positive work on a body
(A) the potential energy increases (B) the potential energy decreases
(C) total energy increases (D) total energy decreases

Ans. (B)

Sol. $-(\text{Work done by conservative force}) = \text{change in potential energy}$

22. A particle is released from rest at origin. It moves under influence of potential field $U = x^2 - 3x$, kinetic energy at $x = 2$ is

- (A) 2 J (B) 1 J (C) 1.5 J (D) 0 J

Ans. (A)

Sol. K.E at $x = 2$

$$K \cdot E = W_{\text{all}}$$

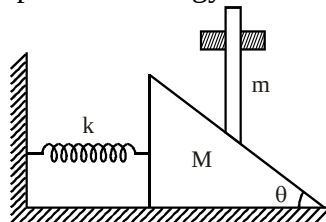
$$K \cdot E = -\Delta U$$

$$= -(x^2 - 3x)$$

$$K \cdot E = (3x - x^2)$$

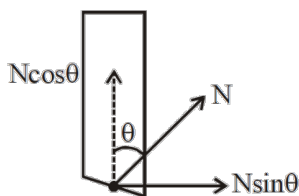
$$K \cdot E = 2 \text{ J}$$

23. A wedge of mass M fitted with a spring of stiffness ' k ' is kept on a smooth horizontal surface. A rod of mass m is kept on the wedge as shown in the figure. System is in equilibrium. Assuming that all surfaces are smooth, the potential energy stored in the spring is:



- (A) $\frac{mg^2 \tan^2 \theta}{2k}$ (B) $\frac{m^2 g \tan^2 \theta}{2k}$ (C) $\frac{m^2 g^2 \tan^2 \theta}{2k}$ (D) $\frac{m^2 g^2 \tan^2 \theta}{k}$

Ans. (C)



Sol.

$$N \cos \theta = mg \dots (i)$$

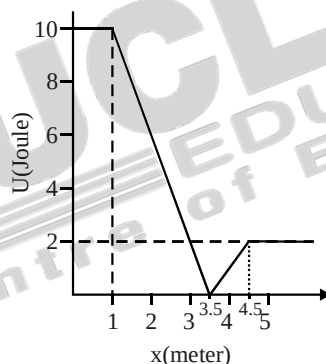
$$N \sin \theta = kx \dots (ii)$$

From equation (i) & (ii)

$$\frac{kx}{mg} = \tan \theta$$

$$\text{So potential energy} = \frac{1}{2} kx^2 = \frac{1}{2} k \left(\frac{mg \tan \theta}{k} \right)^2 = \frac{m^2 g^2 \tan^2 \theta}{2k}$$

24. A body with mass 2 kg moves in one direction in the presence of a force which is described by the potential energy graph. If the body is released from rest at $x = 2\text{m}$, then its speed when it crosses $x = 5\text{ m}$ is



(A) zero

(B) 1 ms^{-1}

(C) 2 ms^{-1}

(D) 3 ms^{-1}

Ans. (C)

Sol. $U(\text{at } x = 2) = 10 \left(\frac{3.5 - 2}{3.5 - 1} \right) = 6\text{J}$

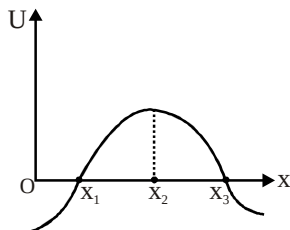
Applying COE

$$6 + 0 = 2 + \frac{1}{2}mv^2$$

$$v^2 = 4$$

$$\Rightarrow v = 2\text{m/sec}$$

25. In the figure shown the potential energy (U) of a particle is plotted against its position ' x ' from origin. Then which of the following statement is correct. A particle at :



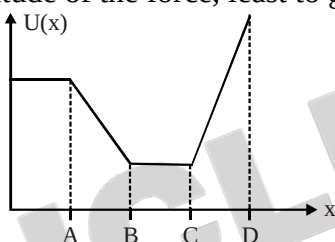
- (A) x_1 is in stable equilibrium
(B) x_2 is in stable equilibrium
(C) x_3 is in stable equilibrium
(D) None of these

Ans. (D)

Sol. Maxima of potential energy curve gives position of unstable equilibrium

So, at x_2 , there is unstable equilibrium.

26. As a particle moves along the x -axis it is acted upon by a conservative force. The potential energy is shown below as a function of the coordinate x of the particle. Rank the labelled regions according to the magnitude of the force, least to greatest.



- (A) AB, BC, CD (B) AB, CD, BC (C) BC, CD, AB (D) BC, AB, CD

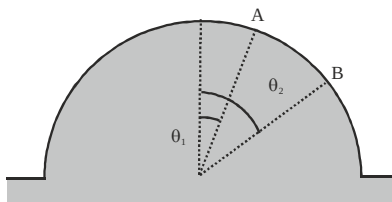
Ans. (D)

Sol. $\frac{dU}{dx}$ is zero for BC

$$F = -\frac{dU}{dx}$$

$\frac{dU}{dx}$ is maximum for CD.

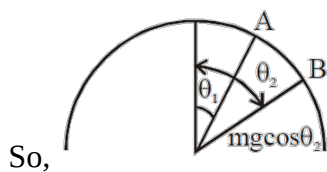
27. A small block slides down from rest at point A on the surface of a smooth circular cylinder, as shown. At point B, the block falls off (leaves) the cylinder. The equation relating the angles θ_1 and θ_2 is given by



- (A) $\sin \theta_2 = \frac{2}{3} \sin \theta_1$ (B) $\sin \theta_2 = \frac{3}{2} \sin \theta_1$ (C) $\cos \theta_2 = \frac{2}{3} \cos \theta_1$ (D) $\cos \theta_2 = \frac{3}{2} \cos \theta_1$

Ans. (C)

Sol. At B, normal becomes zero.



So,

$$mg \cos \theta_2 = \frac{mv_B^2}{R}$$

$$V_B = \sqrt{gR \cos \theta} \quad \dots(i)$$

$$\text{By COE } V_A = \sqrt{2gR(1 - \cos \theta)}$$

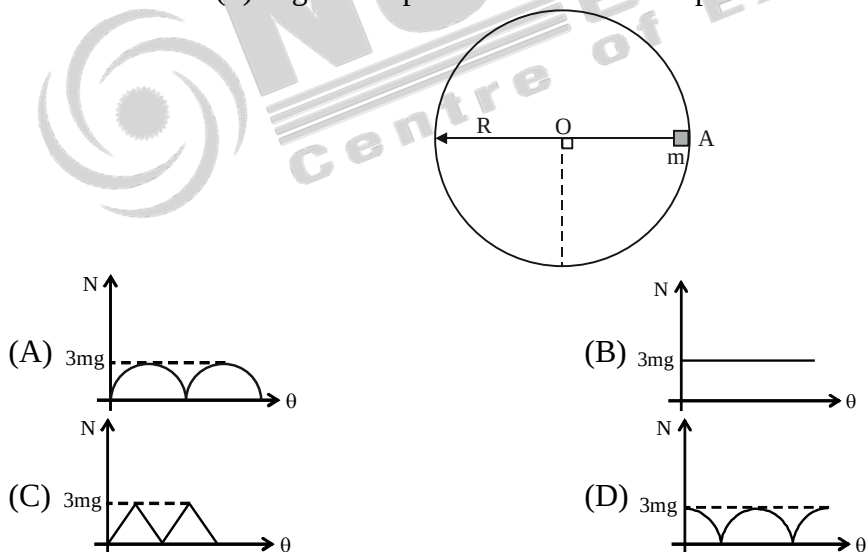
Applying COE at A & B

$$-mgR(1 - \cos \theta_1) = -mgR(1 - \cos \theta_2) + \frac{1}{2}mv_B^2$$

$$mgR \cos \theta_1 = mgR \cos \theta_2 + \frac{1}{2}mgR \cos \theta_2$$

$$mgR \cos \theta_1 = \frac{3mgR \cos \theta_2}{2} \Rightarrow \cos \theta_2 = \frac{2}{3} \cos \theta_1$$

28. A particle of mass m is released from point A on smooth fixed circular track in vertical plane as shown. If the particle is released from rest at $t = 0$, then variation of normal reaction N with (θ) angular displacement from initial position is –



Ans. (A)

Sol. For small

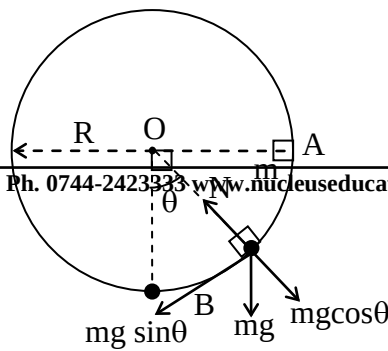
$$V = rw$$

$$= \frac{rd\theta}{dt}$$

Start from rest

$$U = 0$$

& $N = 0$ at point 'A'



at point 'B'

$N_{\max}$

$\theta = 90^\circ$

$$V = \sqrt{gR}$$

$$\frac{mv^2}{2} = mgR$$

$$V_B = \sqrt{2gR}$$

$$N - mg \cos \theta = \frac{mv^2}{R}$$

$$N = \frac{mv^2}{R} + mg \cos \theta$$

$$N = \frac{mv^2}{r}$$

$$= \frac{m2gR}{R}$$

$$= 2mg + mg (\cos \theta)$$

$$N = \frac{2}{3} mg$$

$$N \propto V^2$$

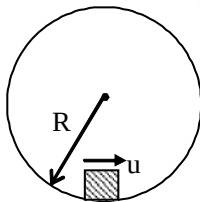
Parabola

$$N \propto (d\theta)^2$$

$$V = RW$$

$$= \frac{Rd\theta}{dr}$$

29. A particle is given an initial speed u inside a smooth spherical shell of radius $R = 1$ m that it is just able to complete the circle. Acceleration of the particle when its velocity is vertical is -



(A) $g\sqrt{10}$

(B) g

(C) $g\sqrt{2}$

(D) $3g$

Ans. (A)

Sol.

$$a_t = g$$

$$a_c = \frac{V_A^2}{R}$$

$$a_c = \frac{3gR}{R}$$

$$a_c = 3g$$

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{g^2 + 9g^2}$$

$$a = g\sqrt{10}$$

30. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position, and has a speed u . The magnitude of the change in its velocity as it reaches a position where the string is horizontal is -

(A) $\sqrt{u^2 - 2gL}$ (B) $\sqrt{2gL}$ (C) $\sqrt{u^2 - 2gL}$ (D) $\sqrt{2(u^2 - gL)}$

Ans. (D)

Sol. $|\vec{V}| = |\vec{V}_f - \vec{V}_i|$

Magnitude

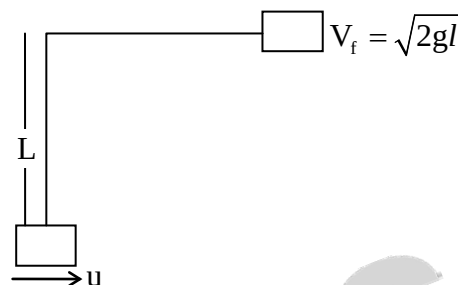
Apply W.E.T.

$$mgL = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$mgL = \frac{1}{2}mv_f^2 - \frac{1}{2}mu^2$$

$$V_f = \sqrt{u^2 - 2gL}$$

$$\begin{aligned} \text{change in velocity} &= \sqrt{u^2 + v_f^2} \\ &= \sqrt{2(u^2 - 2gL)} \end{aligned}$$



MULTIPLE CORRECT TYPE QUESTIONS

31. Which of the following statements is TRUE for a system comprising of two bodies in contact exerting frictional force on each other :

- (A) total work done by static friction on whole system is always zero.
(B) work done by static friction on a body is always zero
(C) work done by kinetic friction on a body is always negative
(D) total work done by internal kinetic friction on whole system is always negative

Ans. (AD)

Sol. In static friction, there is no relative motion between surfaces.

32. A particle of mass m is at rest in a train moving with constant velocity with respect to ground. Now the particle is accelerated by a constant force F_0 acting along the direction of motion of train for time t_0 . A girl in the train and a boy on the ground measure the work done by this force. Which of the following are **INCORRECT**?

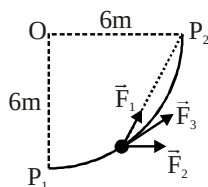
- (A) Both will measure the same work
(B) Boy will measure higher value than the girl
(C) Girl will measure higher value than the boy
(D) Data are insufficient for the measurement of work done by the force F_0

Ans. (AC)

Sol. displacement would be different for both observers.

33. A smooth track in the form of a quarter circle of radius 6 m lies in the vertical plane. A particle moves from P_1 to P_2 under the action of forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$. Force $\vec{F}_1$ is always toward P_2 and

is always 20N in magnitude. Force $\vec{F}_2$ always acts horizontally and is always 30 N in magnitude. Force $\vec{F}_3$ always acts tangentially to the track and is of magnitude 15 N. Select the correct alternative(s)



(A) work done by $\vec{F}_1$ is 120 J

(B) work done by $\vec{F}_2$ is 180 J

(C) work done by $\vec{F}_2$ is 45π

(D) $\vec{F}_1$ is conservative in nature

Ans. (BCD)

Sol. F_1 is a central force

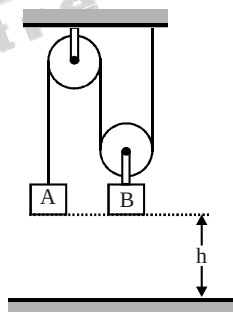
So it is conservative

$$W \text{ by } F_1 = 20 \times 6\sqrt{2}$$

$$= 120\sqrt{2}\text{J}$$

MATRIX MATCH TYPE QUESTIONS

34. In the figure shown are two blocks A and B of same mass connected with pulley and string to each other. Initially both of them are at a height of $h = 0.5$ m from ground. After they are released they move in either direction and one of them strike the ground. For, the interval from releasing to when one of them strike, some physical quantities are in column I and their modulus values in SI units are in column II.



Column I

- (A) Velocity of A immediately before any one of them strike ground.
(B) Velocity of B immediately before any one of them strike ground.
(C) Ratio of work done by gravity on A to work done by gravity on B.
(D) Acceleration of block A before any one of them strike ground.

Column II

- (P) 1
(Q) 2
(R) 3
(S) 4
(T) 5

Ans. (A)→Q; (B)→P; (C)→Q; (D)→S

Sol. $y_A = 2y_B \Rightarrow v_A = 2v_B$

A moves ↓ and

B moves ↑ as

tension on B is more than A.

$$mgh - \frac{mgh}{2} = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2$$

$$\frac{gh}{2} = \frac{5}{2}v_B^2$$

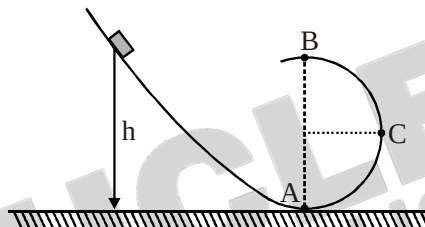
$$\Rightarrow v_B = \sqrt{\frac{5}{5}} = 1, v_A = 2$$

$$mg - T = ma_A$$

$$2T - mg = ma_B$$

$$a_A = 2a_B$$

35. A block of mass m is released from top of a smooth track as shown in the figure. The end part of the track is a circle in vertical plane of radius R . N is normal reaction of the track at any point of the track. Match the entries of column-I with entries of column-II.



Column I

- (A) $h = 5/2 R$
(B) $h = 9/2 R$
(C) $h = R$
(D) $h = 2R$

Column II

- (P) Net force on the block at C is mg
(Q) $N_A - N_B = 6mg$
(R) Block leaves contact before B
(S) Block will keep contact with the track in region between A & B.
(T) $N_C > mg$

Ans. (A)→QST; (B)→QST; (C)→P; (D)→RT

Sol. For (A) $v_A = \sqrt{2gh} = \sqrt{5gR}$ circle will be completed and $N_A - N_B = 6mg$

$$N_C = \frac{mv_C^2}{R} = 3mg$$

$$\text{For (B) } v_A = \sqrt{9gR} \quad N_C = \frac{mv_C^2}{R} = 7mg$$

$$\text{For (C) } v_A = \sqrt{2gR} \quad v_C = 0, N_C = 0, F_{\text{net}} \text{ at C} = mg$$

$$\text{For (D) } v_A = \sqrt{4gR} \quad \text{leaves the circle in region B-C}$$

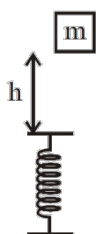
$$N_C = \frac{mv_C^2}{R} = 2mg$$

EXERCISE # (O-2)

1. A light spring of length 20 cm and force constant 2 N/cm is placed vertically on a table. A small block of mass 1 kg falls on it. The length h from the surface of the table at which the block will have the maximum velocity is :

(A) 20 cm (B) 15 cm (C) 10 cm (D) 5cm

Ans. (B)



Sol.

$$mgx_0 = \frac{1}{2} k$$

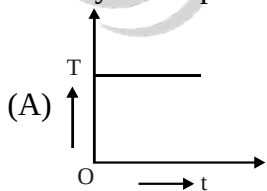
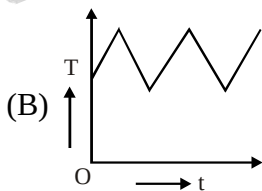
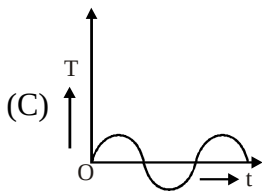
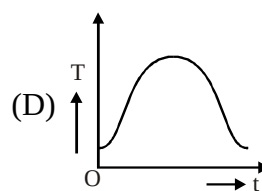
Velocity is maximum at equilibrium position

$$kx_0 = mg$$

$$x_0 = \frac{mg}{k} = 5\text{cm}$$

$$\text{height from surface of table} = 20 - 5 = 15\text{ cm}$$

2. A particle of mass m is suspended from a fixed point O by a string of length ℓ . At $t = 9$ s, it is displaced from equilibrium position and released. The graph, which shows the variation of the tension T in the string with time ' t ', may be at the equilibrium, it changes with change in velocity of the particle.



Ans. (D)

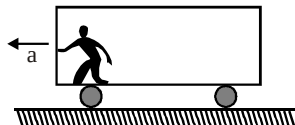
Sol. Initial $t = 9$ sec

$$T - mg \cos\theta = \frac{mv^2}{l}$$

$$T = mg \cos\theta + \frac{mv^2}{l}$$

$$T \propto V^2 \quad \text{Parabola}$$

3. As shown in the figure, a man pushes forward on the compartment which is accelerating uniformly to the left. If the man stays at rest relatively to the compartment, which of the following is correct?



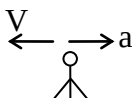
- (A) The man does positive work on the compartment
(B) The man does negative work on the compartment
(C) The man does zero work on the compartment
(D) It cannot be determined.

Ans.

(B)

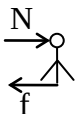
Sol.

Relative to compartment



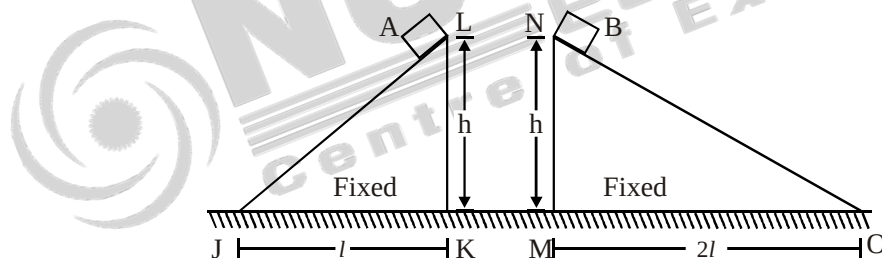
$$W = -ve$$

In general frame



$$f > N$$

4. Two identical blocks A and B are placed on two inclined planes as shown in diagram. Neglect air resistance and other friction. Choose the correct statement :



Statement I : Kinetic energy of 'A' on sliding to J will be greater than the kinetic energy of B on falling to M.

Statement II : Acceleration of 'A' will be greater than acceleration of 'B' when both are released to slide on inclined plane.

Statement III : Work done by external agent to move block slowly from position B to O is negative

(A) statement I is true

(B) statement II is true

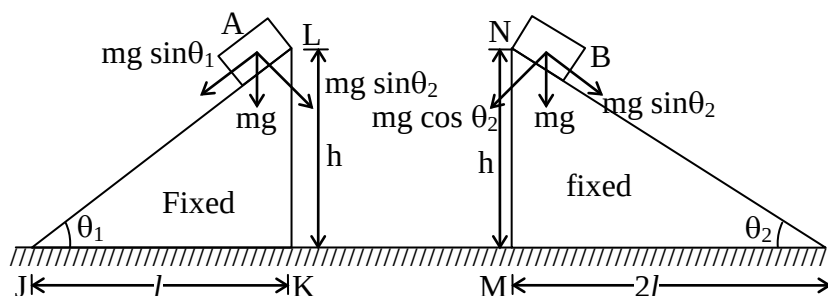
(C) statement I and III are true

(D) statement II and III are true

Ans.

(D)

Sol.

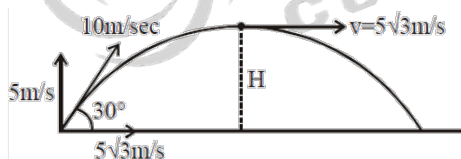


- (I) Both velocity same. So K.E same
 (II) $\theta_1 > \theta_2 \quad \therefore \quad a_A > a_B$
 (III) $W_{\text{ext}} + W_g = K_f - K_i$
 $W_{\text{ext}} = -W_g$

MULTIPLE CORRECT TYPE QUESTIONS

5. A particle of mass 2 kg is projected with an initial speed $u = 10 \text{ m/sec}$ at an angle $\theta = 30^\circ$ with the horizontal
- (A) The magnitude of work done on the particle during the first half of the total time of flight of the particle is 25J.
- (B) The magnitude of work done on the particle during the total time of flight of the particle is 0 J.
- (C) The magnitude of average power delivered to the particle during the first half of the flight is 50 watt.
- (D) The radius of curvature of the trajectory of the particle at the highest point of the projectile is 7.5m.

Ans. (ABCD)



Sol.

$$(A) W = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$W = \frac{1}{2} \times 2 \times (75 - 100) = -25 \text{ J}$$

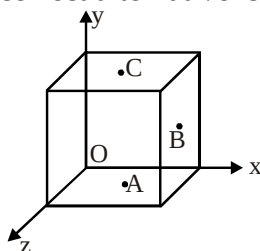
$$(B) \text{ Change in KE} = \text{work done} = 0$$

$$(C) t = \frac{u \sin \theta}{g} = \frac{5}{10} = \frac{1}{2} \text{ sec}$$

$$P_{\text{avg}} = \frac{W}{\text{time}} = \frac{-25}{\frac{1}{2}} = -50 \text{ watt}$$

$$(D) R_C = \frac{v_T^2}{a_e} = \frac{(5\sqrt{3})^2}{10} = 7.5\text{m}$$

6. A particle is shifted from A to B and then from B to C where A, B and C are the midpoints of the corresponding faces of a cube of side 2m. If a force $\vec{F} = (3\hat{i} + 4\hat{j} - 5\hat{k})\text{N}$ is continuously acting on the particle, then select correct alternative is



- (A) work done from A to B is 7 J
(B) work done from B to C is 1 J
(C) work done A to C is 8 J
(D) force $\vec{F}$, is conservative force

Ans. (ABCD)

Sol. $\vec{OA} = \hat{i} + \hat{k}$, $\vec{OB} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{OC} = \hat{i} + 2\hat{j} + \hat{k}$

$$\Rightarrow \vec{AB} = \vec{OB} - \vec{OA} = \hat{i} + \hat{j}, \vec{BC} = \vec{OC} - \vec{OB} = \hat{j} - \hat{i}, \vec{AC} = \vec{OC} - \vec{OA} = 2\hat{j}$$

$$W_{AB} = \vec{F} \cdot \vec{AB} = 7, W_{BC} = \vec{F} \cdot \vec{BC} = 1, W_{AC} = \vec{F} \cdot \vec{AC} = 8$$

A constant force is always a conservative force.

7. Which of the following is/are conservative force(s)?

- (A) $\vec{F} = 2r^3 \hat{r}$ (B) $\vec{F} = -\frac{5}{r} \hat{r}$ (C) $\vec{F} = \frac{x^2}{y} \hat{i} + \frac{y^2}{x} \hat{j}$ (D) $\vec{F} = y\hat{i} + x\hat{j}$

Ans. (ABD)

Sol. $\vec{F} = \frac{3\vec{r}}{r^3} = \frac{3\hat{r}}{r^2}$

$\vec{F}$ is a constant force and constant force is always conservative.

8. If one of the forces acting on a particle is conservative then :

- (A) Its work is zero when the particle moves exactly once around any closed path.
(B) Its work equals the change in the kinetic energy of the particle.
(C) It obeys Newton's second law.
(D) Its work depends on the end points of the motion, not on the path between.

Ans. (ACD)

Sol. Change in KE is due to work done by all forces and not only by a single conservative force

9. A particle of mass $m = 1\text{ kg}$ lying on x-axis experiences a force given by law $F = x(3x - 2)$ Newton, where x is the x-coordinate of the particle in meters. The points on x-axis where the particle is in equilibrium are :

(A) $x = 0$

(B) $x = 1/3$

(C) $x = 2/3$

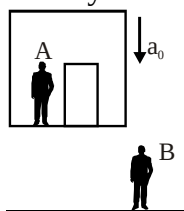
(D) $x = 1$

Ans. (AC)

Sol. For equilibrium, $F = 0 \Rightarrow x = 0, \frac{2}{3}$

Paragraph for Question 10 & 14

A block of mass m is kept in an elevator which starts moving downward with an acceleration a_0 as shown in figure. The block is observed by two observers A and B for a time interval t_0 .



10. The observer B finds that the work done by gravity is

(A) $\frac{1}{2}mg^2t_0^2$

(B) $-\frac{1}{2}mg^2t_0^2$

(C) $\frac{1}{2}mgat_0^2$

(D) $-\frac{1}{2}mgat_0^2$

Ans. (C)

Sol. $\Delta y_\ell = \frac{1}{2}a_0t_0^2$ in downward

$$W_g = +mg\Delta y$$

$$= \frac{1}{2}mga_0t_0^2$$

11. The observer B finds that work done by normal reaction N is

(A) zero

(B) $-Nat_0^2$

(C) $+\frac{Nat^2}{2}$

(D) None of these

Ans. (D)

Sol. $N = m(g - a_0)$; $W_N = -N\Delta y_\ell = -\frac{Na_0t_0^2}{2}$

12. The observer B finds that work done by pseudo force is

(A) zero

(B) $-ma^2t_0$

(C) $+ma^2t_0$

(D) $-mgat_0$

Ans. (A)

Sol. $\therefore$ Displacement zero.

$$\therefore W_{\text{pseudo}} = \text{zero}$$

13. According to observer B, the net work done on the block is

(A) $-\frac{1}{2}ma^2t_0^2$

(B) $\frac{1}{2}ma^2t_0^2$

(C) $\frac{1}{2}mgat_0^2$

(D) $-\frac{1}{2}mgat_0^2$

Ans. (B)

Sol. $W_{\text{net}} = W_g + W_N = \frac{1}{2} m g a_0 t_0^2 - \frac{m(g-a_0)a_0 t_0^2}{2}$

14. According to the observer A

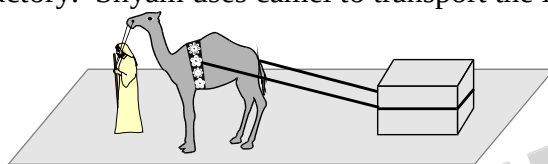
- (A) the work done by gravity is zero (B) the work done by normal reaction is zero
(C) the work done by pseudo force is zero (D) all the above

Ans. (D)

Sol. For observer A, displacement of mass is zero.

Paragraph for Question 15 & 19

Ram and Shyam are two fast friends since childhood. Shyam neglected studies and now has no means to earn money other than a camel whereas Ram becomes an engineer. Now both are working in the same factory. Shyam uses camel to transport the load within the factory.



Due to low salary & degradation in health of camel, Shyam becomes worried and meet his friend Ram and discusses his problem. Ram collected some data & with some assumptions concluded the following:

- (i) The load used in each trip is 1000 kg and has friction coefficient $\mu_k = 0.1$ and $\mu_s = 0.2$.
- (ii) Mass of camel is 500 kg.
- (iii) Load is accelerated for first 50 m with constant acceleration, then it is pulled at a constant speed of 5m/s for 2 km and at last stopped with constant retardation in 50 m.
- (iv) From biological data, the rate of consumption of energy of camel can be expressed as $P = 18 \times 10^3 v + 10^4$ J/s where P is the power and v is the velocity of the camel. After calculations on different issues Ram suggested proper food, speed of camel etc. to his friend. For the welfare of Shyam, Ram wrote a letter to the management to increase his salary.

(Assuming that the camel exerts a horizontal force on the load) :

15. Sign of work done by the camel on the load during parts of motion : accelerated motion, uniform motion and retarded motion respectively are :

- (A) +ve, +ve, +ve (B) +ve, +ve, -ve (C) +ve, zero, -ve (D) +ve, zero, +ve

Ans. (A)

Sol. (A) $W_{\text{CL}} + W_f = \Delta KE \therefore W_{\text{CL}} = \Delta KE - W_f$

(a) During accelerated motion negative work is done against friction and there is also change in kinetic energy. Hence net work needed is +ve.

(b) During uniform motion work is done against friction only and that is +ve.

(c) During retarded motion, the load has to be stopped in exactly 50 metres. If only friction is considered then the load stops in 12.5 metres which is less than where it has to stop.

Hence the camel has to apply some force so that the load stops in 50m (>12.5 m). Therefore the work done in this case is also +ve.

16. The ratio of magnitude of work done by camel on the load during accelerated motion to retarded motion is :

- (A) 3 : 5 (B) 2.2 : 1 (C) 1 : 1 (D) 5 : 3

Ans. (D)

Sol. $W_{CL}|_{\text{accelerated motion}} = \Delta KE - W_{\text{friction}}$ where W_{CL} is work done by camel on load.

$$= \left[\frac{1}{2}mv^2 - 0 \right] - [-\mu_k mg \cdot 50]$$

$$= \frac{1}{2} \times 1000 \times 5^2 + 0.1 \times 10 \times 1000 \times 50 = 1000 \left[\frac{125}{2} \right]$$

similarly, $W_{CL}|_{\text{retardation}} = \Delta KE - W_{\text{friction}}$

$$\left[0 - \frac{1}{2}mv^2 \right] - [-\mu_k mg \cdot 50] = 1000 \left[\frac{75}{2} \right]$$

$$\therefore \frac{W_{CL}|_{\text{accelerated motion}}}{W_{CL}|_{\text{retarded motion}}} = \frac{W_{CL}|_{\text{Rofjr xfr}}}{W_{CL}|_{\text{eafnr xfr}}}$$

$$= \frac{125}{75} = \frac{5}{3} \Rightarrow 5 : 3$$

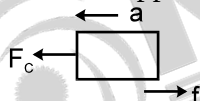
17. Maximum power transmitted by the camel to load is :

- (A) 6250 J/s (B) 5000 J/s (C) 10^5 J/s (D) 1250 J/s

Ans. (A)

Sol. Maximum power = $F_{\text{max}} \times V$

Maximum force applied by camel is during the accelerated motion.



We have $V^2 - U^2 = 2as$

$$25 = 0^2 + 2 \cdot a \cdot 50$$

$$a = 0.25 \text{ m/s}^2 \text{ ; for accelerated motion}$$

$$\therefore F_c - f = ma$$

$$\therefore F_c = \mu mg + ma = 0.1 \times 1000 \times 10 + 1000 \times 2.5$$

$$= 1000 + 250 = 1250 \text{ N}$$

This is the critical point just before the point where it attains maximum velocity of almost 5 m/s.

Hence maximum power at this point is = $1250 \times 5 = 6250 \text{ J/s}$.

18. The ratio of the energy consumed of the camel during uniform motion for the two cases when it moves with speed 5 m/s to the case when it moves with 10 m/s.

- (A) $\frac{19}{20}$ (B) $\frac{19}{10}$ (C) $\frac{10}{19}$ (D) $\frac{20}{19}$

Ans. (D)

Sol. We have

$$W = P\Delta T$$

$$P = 18 \times 10^3 \text{ W} + 10^4 \text{ J/s}$$

$$\therefore P_5 = 18 \times 10^3 \times 5 + 10^4 \text{ J/s} \quad \text{and} \quad \Delta T_5 = \frac{2000 \text{ m}}{5 \text{ m/s}} = 400 \text{ s}$$

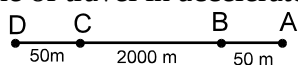
$$P_{10} = 18 \times 10^3 \times 10^4 \text{ J/s} \quad \text{and} \quad \Delta T_{10} = \frac{2000 \text{ m}}{10 \text{ m/s}} = 200 \text{ s}$$

$$\therefore W_5/W_{10} = \frac{10^4(9+1) \times 400}{10^4(18+1) \times 200} = \frac{20}{19}$$

19. The total energy consumed of the camel during the trip of 2100 m is
(A) $2.1 \times 10^6 \text{ J}$ (B) $4.22 \times 10^7 \text{ J}$ (C) $2.22 \times 10^4 \text{ J}$ (D) $4.22 \times 10^6 \text{ J}$

Ans. (B)

Sol. The time of travel in accelerated motion = time of travel in retarded motion.



$$T_{AB} = T_{CD} = \frac{V}{a} = \frac{5}{0.25} = 20 \text{ sec.}$$

$$\text{Now time for uniform motion} = T_{BC} = \frac{2000}{5} = 400 \text{ sec.}$$

$$\begin{aligned} \therefore \text{Total energy consumed} &= \int_0^{440} P dt \\ &= \int_0^{20} [18.10^3 V + 10^4] dt + \int_{20}^{420} [18.10^3 \cdot 5 + 10^4] dt + \int_{420}^{440} [18.10^3 V + 10^4] dt \\ &= \int_0^{20} 18.10^3 V dt + \int_0^{20} 10^4 dt + [10^5 t]_{20}^{420} + \int_{420}^{440} 18.10^3 V dt + \int_{420}^{440} 10^4 dt \end{aligned}$$

Putting $V dt = dx$ and changing limits appropriately.
 $V dt = dx$

$$\begin{aligned} \text{it becomes} & \int_0^{50} 18.10^3 dx + [10^4 t]_0^{20} + 10^5 [420 - 20] + \int_{2050}^{2100} 18.10^3 dx + [10^4]_{420}^{440} \\ & \int_0^{50} 18.10^3 dx + [10^4 t]_0^{20} + 10^5 [420 - 20] + \int_{2050}^{2100} 18.10^3 dx + [10^4]_{420}^{440} \\ &= 18.10^3 \cdot 50 + 10^4 [20] + 10^5 \cdot 400 + 18.10^3 [50] + 10^4 [20] \text{ Joules} \\ &= 90 \times 10^4 + 20 \times 10^4 + 400 \times 10^5 + 90 \times 10^4 + 20 \times 10^4 \text{ Joules} \\ &= 4.22 \times 10^7 \text{ Joules} \end{aligned}$$

Alternate :

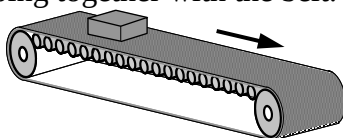
$$\begin{aligned} E &= \int P dt = \int (18 \times 10^3 V + 10^4) dt \\ &= 18 \times 10^3 \left[\int V dt \right] \rightarrow [\text{Total distance travelled}] \\ &+ 10^4 \int dt \\ E &= 18 \times 10^3 \times 2100 + 10^4 [T] \\ T &= \frac{50}{\left(0 + \frac{5}{2}\right)} \times 2 + \frac{2000}{5} \end{aligned}$$

$$T = \frac{440}{5}$$

$$E = 18 \times 10^3 \times 2100 + \frac{10^4 \times 440}{5}$$

Paragraph for Question 20 & 21

An object of mass M is gently placed on a horizontal conveyor belt, which is moving with uniform velocity v_0 as shown in the figure. The coefficient of static friction is μ_s , the coefficient of kinetic friction is μ_k , and the acceleration of gravity is g . Initially the object slips for a while but finally moves without slipping together with the belt.



20. How far the conveyor belt moves while the object is slipping?

(A) $\frac{v_0^2}{\mu_k g}$

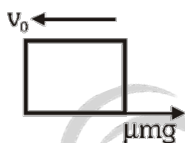
(B) $\frac{v_0^2}{2\mu_k g}$

(C) $\frac{v_0^2}{\mu_s g}$

(D) $\frac{v_0^2}{2\mu_s g}$

Ans. (A)

Sol. In the frame of belt $t = \frac{v_0}{\mu_k g}$



$$\text{displacement of belt} = v_0 \times \frac{v_0}{\mu_k g}$$

21. Work done on the object by friction force relative to the reference frame moving with the conveyor belt is

(A) $\frac{1}{2} M v_0^2$

(B) $-\frac{1}{2} M v_0^2$

(C) $M v_0^2$

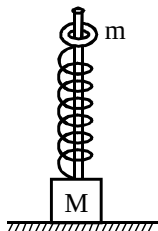
(D) zero

Ans. (B)

Sol. Work done by friction on block in belt frame $W = \Delta KE = \frac{-1}{2} m v_0^2$

Paragraph for Question 22 to 24

The vertical bar and its supporting base illustrated has a combined mass of M . At the lower end of the bar there is a spring of natural length $l_0 = 16$ cm attached to the base plate. The small bead of mass $m = 0.1$ kg can slide down on the bar without friction. When the bead is carefully placed onto the spring at equilibrium, the spring is compressed by $x_0 = 2$ cm. ($g = 10 \text{ m/s}^2$)



22. What is the spring constant?

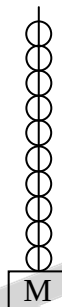
- (A) 40 N/m (B) 10 N/m (C) 50 N/m (D) 60 N/m

Ans. (C)

Sol. $Kx_0 = mg$

$$K = \frac{1}{10} \times \frac{10}{2} \times 100$$

$$K = 50 \text{ N/m}$$



23. At what height h from base should the bead be released, so that the spring should compressed by 8 cm?

- (A) 20 cm (B) 24 cm (C) 40 cm (D) 35 cm

Ans. (B)

Sol. $mgh = \frac{K}{2}(x_f^2 - x_i^2)$

$$h = \frac{510}{2} \times \frac{8}{100} \times \frac{10}{1 \times 10}$$

$$h = 20 \text{ cm}$$

24. Let us assume that in previous question during impact the bead gets glued (attached) to the free end of the spring. What is the minimum mass M needed to prevent the support from jumping up?

- (A) 0.2 kg (B) 0.15 kg (C) 1.5 kg (D) 0.3 kg

Ans. (A)

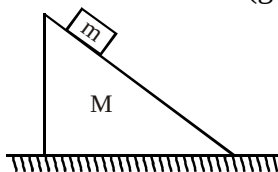
Sol. $Kx = Mg$

$$x = \frac{Mg}{K}$$

$$M = \frac{K(x)}{g}$$

MATRIX MATCH TYPE QUESTIONS

25. A block of mass m lies on wedge of mass M . The wedge in turn lies on smooth horizontal surface. Friction is absent everywhere. The wedge block system is released from rest. All situation given in column-I are to be estimated in duration the block undergoes a vertical displacement 'h' starting from rest (assume the block to be still on the wedge). Match the statement in column-I with the results in column-II. (g is acceleration due to gravity)



Column I

- (A) Work done by normal reaction acting on the block is
(B) Work done by normal reaction (exerted by block) acting on wedge is
(C) The sum of work done by normal reaction on block and work done by normal reaction (exerted by block) on wedge is
(D) Net work done by all forces on block is

Column II

- (P) positive
(Q) negative
(R) zero
(S) less than mgh in magnitude

Ans. (A) → Q,S; (B) → P,S; (C) → R,S; (D) → P,S;

Sol. for block, $W_N + W_g = \frac{1}{2}mv_b^2$

$$\Rightarrow W_N + mgh = \frac{1}{2}mv_b^2$$

$$\Rightarrow W_N = \frac{1}{2}mv_b^2 - mgh$$

$$\text{for wedge, } W_N = \frac{1}{2}Mv_w^2$$

$$\text{By C.O.M.E, } mgh = \frac{1}{2}mv_b^2 + \frac{1}{2}Mv_w^2$$

$$\Rightarrow \left(\frac{1}{2}mv_b^2 - mgh \right) + \frac{1}{2}Mv_w^2 = 0$$

$$W_{N, \text{ block}} + W_{N, \text{ wedge}} = 0$$

EXERCISE # (JM)

1. If a body loses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest? [AIEEE 2002, 4/300]

(1) 1 cm (2) 2 cm (3) 3 cm (4) 4 cm

Ans. (1)

Sol. Ist condition

$$u = v_0 \quad V = \frac{v_0}{2}$$

$$a = \text{retard}$$

$$v^2 = u^2 + 2as$$

$$\frac{V_0^2}{4} = v_0^2 - 2a \times 3$$

$$\frac{-3v_0^2}{4} = -2a \times 3$$

$$a = \frac{v_0^2}{8} \quad \dots(i)$$

IInd condition

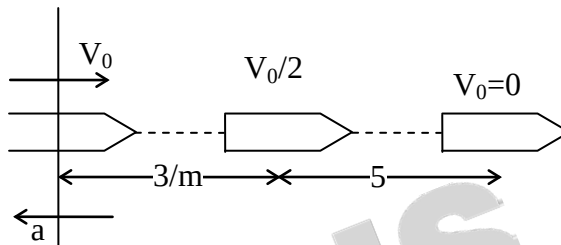
$$u = \frac{v_0}{2}$$

$$v = 0$$

$$v^2 = u^2 - 2as$$

$$u^2 = 2as \Rightarrow \frac{v_0^2}{4} = 2 \frac{v_0^2}{8} S$$

$$S = 1 \text{ cm}$$



2. A spring of force constant 800 N/m has an extension of 5cm. The work done in extending it from 5cm to 15cm is 800 N/m [AIEEE 2002, 4/300]

(1) 16 J (2) 8 J (3) 32 J (4) 24 J

Ans. (2)

Sol. $W_s = \frac{K}{2} (x_i^2 - x_f^2) = \frac{800}{2} (25 - 225)$

$$= \frac{800}{2} \left[\frac{-200}{10000} \right]$$

$$W_s = |-8J| = 8J$$

3. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is : [AIEEE 2003, 4/300]

(1) 12.50 N-m (2) 18.75 N-m (3) 25.00 N-m (4) 6.25 N-m

Ans. (2)

Sol. $W_{0-5} = \frac{K}{2} \left[\frac{25}{10000} \right]$

$$= \frac{5 \times 10}{2} \times \frac{25}{10000}$$

$$W_{0-s} = \frac{25}{4} = 6.25$$

4. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table? [AIEEE 2004, 4/300]

(1) 7.2 J (2) 3.6 J (3) 120 J (4) 1200 J

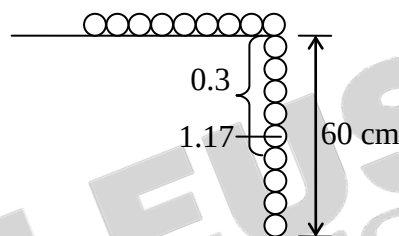
Ans. (2)

Sol. $m' = \frac{4}{2} \times \frac{60}{100}$

$$W_g = mgh$$

$$= \frac{6}{5} \times 10 \times 0.3$$

$$W_g = 3.6 \text{ J}$$



5. A force $\vec{F} = (5\hat{i} + 3\hat{j} + 2\hat{k})$ N is applied over a particle which displaces it from origin to the point $\vec{r} = (2\hat{i} - \hat{j})$ m. The work done on the particle in joules is : [AIEEE 2004, 4/300]

(1) -7 (2) +7 (3) +10 (4) +13

Ans. (2)

Sol. $F = 5\hat{i} + 3\hat{j} + 2\hat{k}$

$$W = (5\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j})$$

$$W = 10 - 3 = 7 \text{ J}$$

6. A body of mass m is accelerated uniformly from rest to a speed v in a time T . The instantaneous power delivered to the body as a function of time, is given by :

[AIEEE 2005, 4/300]

(1) $\frac{mv^2}{T^2} \cdot t$ (2) $\frac{mv^2}{T^2} \cdot t^2$ (3) $\frac{1}{2} \frac{mv^2}{T^2} \cdot t$ (4) $\frac{1}{2} \frac{mv^2}{T^2} \cdot t^2$

Ans. (1)

Sol. Uniformly acceleration

$$u = 0$$

$$V = u + at$$

$$t = T$$

$$V = aT$$

$$a = \frac{V}{t}$$

Instantaneous power

$$P = FV$$

$$= maV$$

$$= \frac{mV}{T} \times at$$

$$= \frac{mV}{T} \times \frac{V}{T} t = \frac{mv^2}{T^2} t$$

7. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. the work done by the force of gravity during the time the particle goes up is [AIEEE 2006, 1.5/180]
 (1) -0.5 J (2) -1.25 J (3) +1.25 J (4) 0.5 J

Ans. (2)

Sol. $W_g = -mgh$

$$= -\frac{100}{1000} \times 10 \times \frac{25}{20}$$

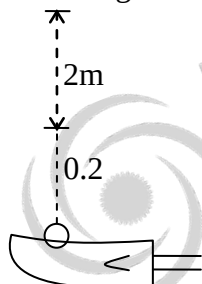
$$W_g = -1.25$$



8. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. Consider $g = 10 \text{ m/s}^2$ [AIEEE 2006, 3/180]
 (1) 22 N (2) 4 N (3) 16 N (4) 20 N

Ans. (1)

Sol. $m = 0.2 \text{ kg}$



$$W_{\text{hand}} + (-W_g) = K_f - K_i$$

$$W_{\text{hand}} = W_g$$

$$F_{\text{hand}} = 22 \text{ N}$$

9. A particle is projected at 60° to the horizontal with a kinetic energy K . The kinetic energy at the highest point is [AIEEE 2007, 3/120]
 (1) K (2) zero (3) $K/4$ (4) $K/2$

Ans. (3)

Sol. Initial K.E (K) = $\frac{1}{2}mv^2$... (i)

Find K.E

$$K_f = \frac{1}{2}mv^2 \cos^2 60 \quad \dots (ii)$$

from (i) & (ii)

$$K_f = \frac{K_i}{4}$$

10. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range [AIEEE 2008, 3/105]

- (1) $2 \times 10^5 \text{ J} - 3 \times 10^5 \text{ J}$ (2) 20,000 J – 50,000 J
(3) 2,000 J – 5,000 J (4) 200 J – 500 J

Ans. (3)

Sol.

$$\begin{aligned} K.E &= \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times m(100) \\ &= 50 \times 50 = 2500 \end{aligned}$$

11. At time $t = 0$ s a particle starts moving along the x-axis. If its kinetic energy increases uniformly with time 't', the net force acting on it must be proportional to : [AIEEE 2011]

- (1) constant (2) t (3) $\frac{1}{\sqrt{t}}$ (4) $\sqrt{t}$

Ans. (3)

Sol.

$$\begin{aligned} K &\propto t \\ V^2 &\propto t \\ V &\propto \sqrt{t} \quad \text{or} \quad V \propto t^{1/2} \\ \text{differential w.r.t 't'} \\ a &\propto \frac{dv}{dt} \propto t^{-1/2} \\ \Sigma F = ma &\propto \frac{1}{\sqrt{t}} \end{aligned}$$

12. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements. [AIEEE-2012]

If two springs S_1 and S_2 of force constants k_1 and k_2 , respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2 .

Statement-1: If stretched by the same amount, work done on S_1 , will be more than that on S_2

Statement-2 : $k_1 < k_2$

- (1) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of Statement-1.
(2) Statement-1 is false, Statement-2 is true
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of statement-1.

Ans. (2)

Sol.

$$W_f = \frac{1}{2} Kx^2$$

$$W_1 = \frac{1}{2} K_1 x_1^2 \quad W_2 = \frac{1}{2} K_2 x_2^2$$

$$\frac{W_1}{W_2} > 1 \quad (\text{where } w_1 \text{ has more work})$$

$$\frac{W_1}{W_2} > \frac{K_1 x_1^2}{K_2 x_2^2}$$

So $K_2 > K_1$ $x \uparrow K \downarrow$
Statement (2) true

$$x_1 = x_2 = x$$

$$\frac{W_1}{W_2} = \frac{K_1}{K_2} \quad \therefore K_2 > K_1$$

$$\frac{K_1}{K_2} < 1$$

$W_2 > W_1$ Statement (1) false.

13. When a rubber-band is stretched by a distance x , it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is:- **[JEE-Main-2014]**

(1) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (2) $\frac{1}{2} \left(\frac{aL^2}{2} + \frac{bL^3}{3} \right)$ (3) $aL^2 + bL^3$ (4) $\frac{1}{2} (aL^2 + bL^3)$

Ans. (1)

13.

$$F = ax + bx^2$$

$$w = \int F dx$$

$$= \int_0^L (ax + bx^2) dx$$

$$w = \frac{aL^2}{2} + \frac{bL^3}{3}$$

14. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8 \text{ ms}^{-2}$:- **[JEE-Main-2016]**

(1) $12.89 \times 10^{-3} \text{ kg}$ (2) $2.45 \times 10^{-3} \text{ kg}$ (3) $6.45 \times 10^{-3} \text{ kg}$ (4) $9.89 \times 10^{-3} \text{ kg}$

Ans. (1)

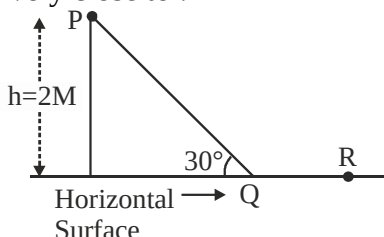
Sol.

$$W_g = 10 \times 1 \times 10 \times 1000$$

$$10 \times 10 \times 1000 = 3.8 \times 10^7 m_1 \times 0.2$$

$$m_1 = \frac{1 \times 10^{-2}}{3.8 \times 0.2} \\ = 12.89 \times 10^{-3}$$

15. A point particle of mass, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ . The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and PR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance $x(= QR)$ are, respectively close to :- **[JEE MAIN - 2016]**



- (1) 0.29 and 6.5 m (2) 0.2 and 6.5 m (3) 0.2 and 3.5 m (4) 0.29 and 3.5 m

Ans. (4)

Sol. Given,

lost energy PQ = lost QR

$$\mu mg l = \mu mg x$$

$$x = 2\sqrt{3}$$

$$\mu = \frac{20}{20\sqrt{3} + 10 \times 2\sqrt{3}} = \frac{20}{40\sqrt{3}} \times \frac{1}{2\sqrt{3}}$$

$$\mu = 0.29$$

16. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10 \text{ ms}^{-1}$. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be :-

- (1) $10^{-4} \text{ kg m}^{-1}$ (2) $10^{-1} \text{ kg m}^{-1} \text{ s}^{-1}$ (3) $10^{-3} \text{ kg m}^{-1}$

[JEE Main - 2017]

- (4) $10^{-3} \text{ kg s}^{-1}$

Ans. (1)

Sol. after 10sec

$$\frac{1}{8}mv_0^2 = \frac{1}{2}mv_f^2$$

$$v_f = \frac{V_0}{2}$$

$$F = -KV^2$$

$$m \frac{dv}{dt} = -KV^2$$

$$m \int_{10}^5 \frac{dv}{V^2} = -K \int_0^{10} dt$$

$$m \frac{dv}{V^2} = -Kdt \quad -m \left| \frac{1}{V} \right|_{10}^5 = K(10)$$

$$-\frac{1}{100} \left(\frac{2-1}{10} \right) = -K10$$

$$K10 = 10^{-3}$$

$$K = 10^{-4}$$

17. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be : [JEE Main - 2017]

(1) 9J (2) 18 J (3) 4.5 J (4) 22 J

Ans. (3)

Sol.

$$F = 6t$$

$$W_f = F ds$$

$$ma = 6t$$

$$m \frac{dV}{dt} = 6t \quad mv = \frac{6t^2}{2}$$

$$m \int dV = \int 6t dt \quad V = 3 \text{ m/s}$$

18. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R . If the period of rotation of the particle is T , then: [JEE Main - 2018]

(1) $T \propto R^{n/2}$ (2) $T \propto R^{3/2}$ for any n .

(3) $T \propto R^{\frac{n}{2}+1}$ (4) $T \propto R^{(n+1)/2}$

Ans. (4)

Sol.

$$F_c \propto \frac{1}{R^n}$$

$$F_c = \frac{k}{R^n} = mR\omega^2$$

$$\frac{K}{R^n} = \frac{mR4\pi^2}{t^2}$$

$$T^2 \propto R^{n+1}$$

$$T \propto R^{\frac{n+1}{2}}$$

19. A particle is moving in a circular path of radius a under the action of an attractive potential

$$U = -\frac{k}{2r^2}. \text{ Its total energy is:}$$

[JEE Main - 2018]

(1) $-\frac{3k}{2a^2}$ (2) $-\frac{k}{4a^2}$ (3) $\frac{k}{2a^2}$ (4) zero

Ans. (4)

Sol.

$$u = \frac{-K}{2r^2}$$

$$F = \frac{-du}{dr} =$$

$$\frac{-du}{dr^2} = \frac{K}{2} \frac{dr^{-2}}{dr} = \frac{K}{2} (-2) \frac{1}{r^3}$$

$$F = \frac{-K}{r^3}$$

$$ma_c = \frac{-K}{r^3}$$

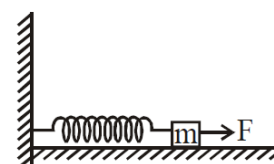
$$\frac{mv^2}{r} = \frac{-K}{r^3}$$

$$K.E = \frac{-K}{2r^2}$$

$$K.E = \frac{K}{2r}$$

$$T.E = U + K.E = \frac{-K}{2r^2} + \frac{K}{2r^2} = 0$$

20. A block of mass m , lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k . The other end of the spring is fixed, as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F , the maximum speed of the block is :



[JEE Main-2019]

(1) $\frac{\pi F}{\sqrt{mk}}$

(2) $\frac{2F}{\sqrt{mk}}$

(3) $\frac{F}{\sqrt{mk}}$

(4) $\frac{F}{\pi\sqrt{mk}}$

Ans. (3)

Sol. Maximum speed is at mean position (equilibrium). $F = kx$

$$x = \frac{F}{k}$$

$$W_F + W_{sp} = \Delta KE$$

$$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

$$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2 \Rightarrow v_{\max} = \frac{F}{\sqrt{mk}}$$

21. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds ? [JEE Main-2019]

(1) 950 J

(2) 875 J

(3) 900 J

(4) 850 J

Ans. (3)

Sol. $x = 3t^2 + 5$

$$v = \frac{dx}{dt} = 6t$$

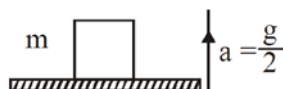
At $t = 5$ sec, $v = 6 \times 5 = 30$ m/s

$$W = \Delta KE = \frac{1}{2}mv^2 - 0$$

$$= \frac{1}{2} \times 2 \times 900 = 900 \text{ J}$$

22. A block of mass m is kept on a platform which starts from rest with constant acceleration $g/2$ upward, as shown in fig. Work done by normal reaction on block in time t is :

[JEE Main-2019]



[JEE Main-10-01-2019\Slot-1\WPE\M]

- (1) $\frac{mg^2t^2}{8}$ (2) $-\frac{mg^2t^2}{8}$ (3) $\frac{3mg^2t^2}{8}$ (4) 0

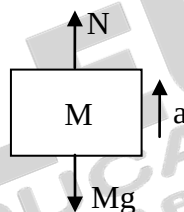
Ans. (3)

Sol. $N - mg = ma$

$$N - mg = \frac{mg}{2} \Rightarrow N = \frac{3mg}{2}$$

$$\text{Now, work done } W = \vec{N} \cdot \vec{S} = \left(\frac{3mg}{2} \right) \left(\frac{1}{2}gt^2 \right)$$

$$\Rightarrow W = \frac{3mg^2t^2}{4}$$



23. A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of $\vec{d} = 4\vec{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ?

[JEE Main-2019]

- (1) 9 J (2) 12 J (3) 10 J (4) 15 J

Ans. (4)

Sol. $W_{\text{all}} = \Delta KE$

$$W_F = K.E_f - K.E_i$$

$$K.E_f = 3 + 3 \times 4 = 15 \text{ J}$$

24. A body of mass 1 kg falls freely from a height of 100 m on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6 \text{ N/m}$. The body sticks to the platform and the spring's maximum compression is found to be x . Given that $g = 10 \text{ ms}^{-2}$, the value of x will be close to :

[JEE Main-2019]

- (1) 40 cm (2) 8 cm (3) 4 cm (4) 80 cm

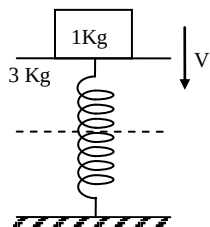
Ans. **BONUS [None of the above option matches]**

Sol. $V = \sqrt{2gh} = \sqrt{2000}$

Applying momentum conservation
for 1 kg and 3kg block.

$$1 \times 1 \times \sqrt{2gh} = 4(V)$$

$$\Rightarrow V = \frac{\sqrt{2000}}{4}$$



By energy conservation

$$\frac{1}{2} \times 4 \left(\frac{\sqrt{2000}}{4} \right)^2 = -40x + \frac{1}{2} \times 1.25 \times 10^6 \times X^2$$

$$250 = -40X + \frac{1.25 \times 10^6}{2} \times X^2$$

$$1.25 \times 10^6 X^2 - 40X - 500 = 0$$

$$\Rightarrow \frac{X \approx 2\text{cm}}{(\text{Bonus})}$$

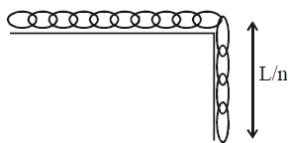
25. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $(1/n)^{\text{th}}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be:

[JEE Main-2019]

(1) $nMgL$ (2) $\frac{2MgL}{n^2}$ (3) $\frac{MgL}{2n^2}$ (4) $\frac{MgL}{n^2}$

Ans. (3)

Sol. Mass of the hanging part = $\frac{M}{n}$



$$h_{\text{COM}} = \frac{L}{2n}$$

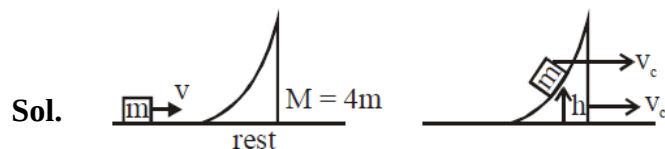
$$\text{work done } w = mgh_{\text{COM}} = \left(\frac{M}{n} \right) g \left(\frac{L}{2n} \right) = \frac{MgL}{2n^2}$$

26. A wedge of mass $M = 4m$ lies on a frictionless plane. A particle of mass m approaches the wedge with speed v . There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by:

[JEE Main-2019]

- (1) $\frac{v^2}{2g}$ (2) $\frac{2v^2}{5g}$ (3) $\frac{2v^2}{7g}$ (4) $\frac{v^2}{g}$

Ans. (2)



Applying Linear momentum conservation $mv = (m + M)v_c$

$$v_c = \frac{v}{5}$$

Applying work energy theorem

$$-mgh = \frac{1}{2}(m + M)v_c^2 - \frac{1}{2}mv^2$$

$$\text{solve, } h = \frac{2v^2}{5g}$$

27. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to:

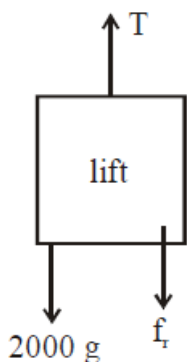
(1 HP = 746 W, $g = 10 \text{ ms}^{-2}$)

[JEE Main-2020]

- (1) 1.7 ms^{-1} (2) 1.9 ms^{-1} (3) 1.5 ms^{-1} (4) 2.0 ms^{-1}

Ans. (2)

Sol.



Let elevator is moving upward with constant speed V .

Tension in cable

$$T = 2000g + f_r = 2000 + 4000$$

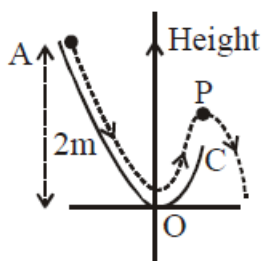
$$T = 24000 \text{ N}$$

$$\text{Power } P = TV$$

$$\Rightarrow 60 \times 746 = (24000) V$$

$$V = \frac{60 \times 746}{24000} = 1.865 \approx 1.9 \text{ m/s}$$

28. A particle ($m=1 \text{ kg}$) slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaches its highest point P (height 1 m), the kinetic energy of the particle (in J) is : (Figure drawn is schematic and not to scale; take $g = 10 \text{ ms}^{-2}$). [JEE Main-2020]



Ans. (10)

Sol. Mechanical energy conservation between A & P

$$U_1 + K_1 = K_2 + U_2$$

$$mg \times 2 = mg \times 1 + K_2$$

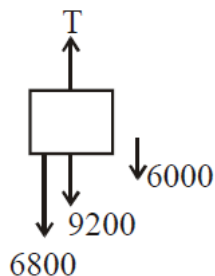
$$K_2 = mg \times 1 = 10 \text{ J.}$$

29. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator ($g = 10 \text{ m/s}^2$) must be at least : [JEE Main-2020]

- (1) 56300 W (2) 66000 W (3) 62360 W (4) 48000 W

Ans. (2)

Sol.



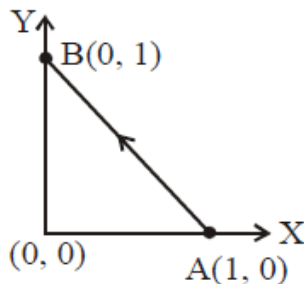
elevator moving with constant speed hence

$$T = 6800 + 9200 + 6000$$

$$T = 22000 \text{ N}$$

$$\text{power} = T \cdot v = 22000 \times 3 \\ = 66000 \text{ W}$$

30. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point A(1, 0) to B(0, 1) along the line segment is :
(all quantities are in SI units) [JEE Main-2020]



- (1) 2 (2) $\frac{1}{2}$ (3) 1 (4) $\frac{3}{2}$

Ans. (3)

Sol. $W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$

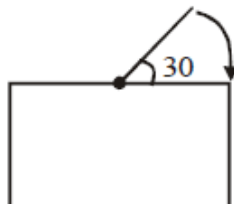
$$W = \int_1^0 -x dx + \int_0^1 y dy$$

$$W = \left[-\frac{x^2}{2} \right]_1^0 + \left[\frac{y^2}{2} \right]_0^1 \\ = -\left(\frac{0^2}{2} - \frac{1^2}{2} \right) + \left(\frac{1^2}{2} - \frac{0^2}{2} \right)$$

$$W = 1 \text{ J}$$

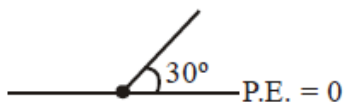
31. One end of a straight uniform 1 m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n} \text{ s}^{-1}$, where n is an integer. The value of n is _____.

[JEE Main-2020]



Ans. (15.00)

Sol.



From mechanical energy conservation.

$$U_i + K_i + U_f + K_f$$

$$\Rightarrow mg \frac{l}{2} \sin 30^\circ + 0 = 0 + \frac{1}{2} I \omega^2$$

$$\Rightarrow mg \times \frac{1}{2} \times \frac{1}{2} + 0 = 0 + \frac{1}{2} \times \frac{m(1)^2}{3} \omega^2$$

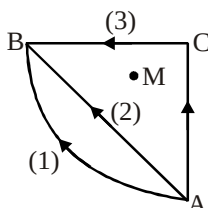
$$\Rightarrow \omega^2 = \frac{3g}{2} \Rightarrow \omega = \sqrt{15}$$

$$\therefore n = 15$$



EXERCISE # (JA)

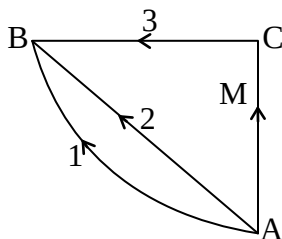
1. In a region of only gravitational field of mass 'M' a particle is shifted from A to B via three different paths in the figure. The work done in different paths are W_1, W_2, W_3 respectively then [IIT-JEE 2003]



- (A) $W_1 = W_2 = W_3$ (B) $W_1 = W_2 > W_3$ (C) $W_1 > W_2 > W_3$ (D) $W_1 < W_2 < W_3$

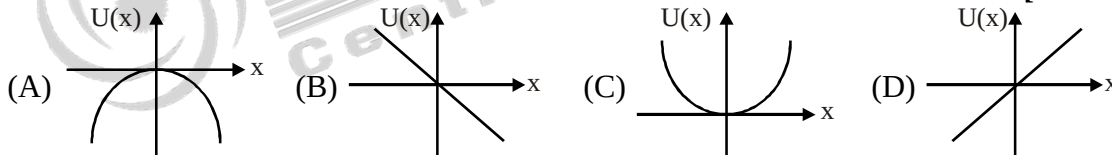
Ans. (A)

Sol. $F_{\text{gravitation}} = \text{constant}$ which is not depend on path
Same for all $W_1 = W_2 = W_3$



2. A particle is placed at the origin and a force $F = kx$ is acting on it (where k is positive constant). If $U(0) = 0$, the graph of $U(x)$ versus x will be (where U is the potential energy function) :

[IIT-JEE 2004]



Ans. (A)

Sol. $F = Kx,$ $F = -\frac{du}{dx}$

$$Fdx = -du$$

$$Kx dx = -du$$

$$\int du = Kx dx$$

$$du = (-)K\left(\frac{x^2}{2}\right)$$

3. **Statement-I** : A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is

made to go up on the surface with the same initial velocity v . The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

and

Statement-II : The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination. [IIT-JEE 2007]

- (A) statement-I is true, statement-II is true; statement-II is a correct explanation for statement-I
(B) statement-I is true, statement-II is true, statement-II is NOT a correct explanation for statement-I
(C) statement-I is true, statement-II is false
(D) statement-I is false, statement-II is true

Ans. (C)

Sol. Due to inclination, some K.E. is also converted into P.E. So, loss of energy is smaller. μ does not depend on inclination

4. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10 \text{ m/s}^2$, find the work done (in joules) by the string on the block of mass 0.36 kg during the first second after the system is released from rest. [IIT-JEE 2009]



Ans. 8

Sol.
$$a = \frac{(0.72 - 0.36) \times 10}{(0.72 + 0.36)}$$

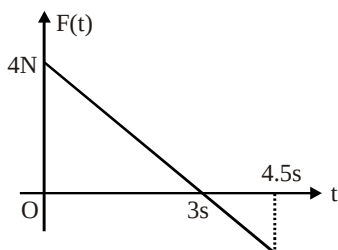
$$= \frac{10}{3} \text{ m/s}^2$$

$$T = 0.36 \left(10 + \frac{10}{3} \right) = 4.8 \text{ N}$$

$$\Delta y = \frac{1}{2} \times \frac{10}{3} \times 1^2$$

$$W_T = 4.8 \times \frac{5}{3} = 8 \text{ J}$$

5. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x-direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 second is [IIT-JEE-2010]



- (A) 4.50 J (B) 7.50 J (C) 5.06 J (D) 14.06 J

Ans.

(C)

Sol.

$$F = ma$$

$$F = m \frac{dv}{dt}$$

$$\int F dt = \int m dv$$

$$\text{Area} = mv$$

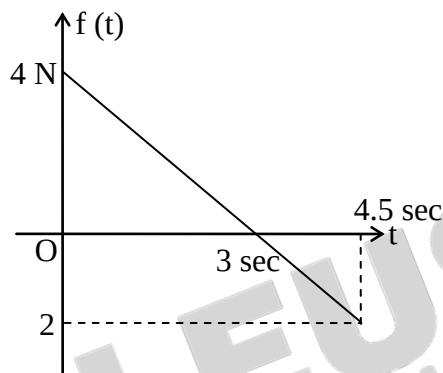
$$\frac{\text{Area}}{m} = v$$

$$v = \frac{6 - 1.5}{2}$$

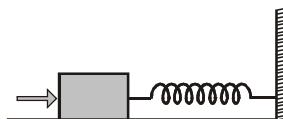
$$v = \frac{4.5}{2}$$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times \frac{(4.5)^2}{4}$$

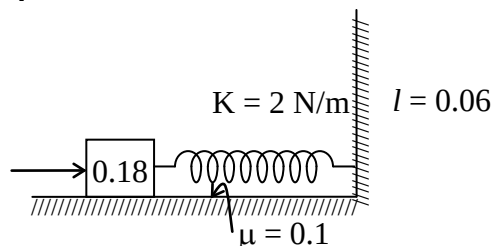
$$= 5.06 \text{ J}$$



6. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is unstretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is $V = N/10$. Then N is [IIT-JEE-2011]



Ans. 4



Sol.

W.E.T

$$W_f + W_s = K_f - K_i$$

$$\begin{aligned}
 -\mu mgl + \frac{K}{2}(x_i^2 - x_f^2) &= -\frac{1}{2}mv^2 \\
 \frac{1}{10} \times \frac{18}{100} \times 10 \times \frac{6}{100} + \frac{2}{2} \frac{36}{10000} &= \frac{1}{2} \times \frac{18}{100} \times v^2 \\
 \frac{108}{100} + \frac{36}{100} &= 9v^2 \\
 v^2 &= \sqrt{\frac{144}{100 \times 9}} \\
 v &= \frac{12}{10 \times 3} = 0.4 \\
 v &= \frac{N}{10} \Rightarrow N = 4
 \end{aligned}$$

7. The work done on a particle of mass m by a force, $K \left[\frac{x}{(x^2 + y^2)^{3/2}} \hat{i} + \frac{y}{(x^2 + y^2)^{3/2}} \hat{j} \right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius a about the origin in the x - y plane is : [IIT JEE-2013]

- (A) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$ (C) $\frac{K\pi}{2a}$ (D) 0

Ans. (D)

Sol. suppose $x = r \cos\theta$
 $y = r \sin\theta$

force on particle is $\frac{K}{r^3} (r \cos\theta \hat{i} + r \sin\theta \hat{j})$

force is in radial direction so work done by this force along given path (circle) is zero.

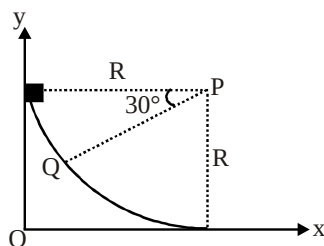
8. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5s is : [IIT JEE-2013]

Ans. 5

Sol. $E = P \cdot t = 0.5W \times 5s = 2.5 \text{ J} = \frac{1}{2} mv^2 \Rightarrow v = 5 \text{ m/s}$

Paragraph for Questions 9 and 10

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure below, is 150 J. (Take the acceleration due to gravity, $g = 10 \text{ m s}^{-2}$) [IIT JEE-2013]



Ans. (A)

9. The magnitude of the normal reaction that acts on the block at the point Q is
(A) 7.5 N (B) 8.6 N (C) 11.5 N (D) 22.5 N

Ans. (B)

10. The speed of the block when it reaches the point Q is
(A) 5 ms^{-1} (B) 10 ms^{-1} (C) $10\sqrt{3} \text{ ms}^{-1}$ (D) 20 ms^{-1}

Ans. (B)

Sol.9,10

$$R = 40 \text{ m}$$

$$m = 1 \text{ kg}$$

$$N - mg \cos 60^\circ = \frac{mv^2}{R}$$

$$N = \frac{mv^2}{R} + mg \cos 60^\circ \quad \dots(i)$$

W.E.T.

$$mgR(1 - \cos 60^\circ) - 150 = K_f - K_i$$

$$mgR\left(1 - \frac{1}{2}\right) - 150 = \frac{1}{2}mv^2$$

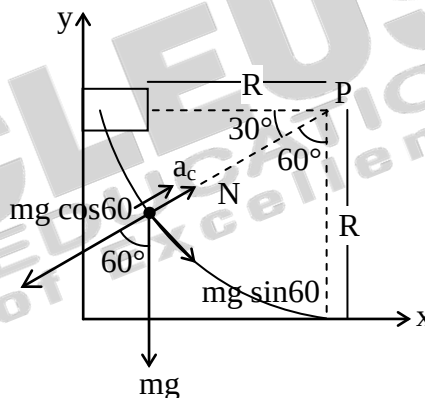
$$1 \times 10 \times 40\left(\frac{1}{2}\right) - 150 = \frac{1}{2} \times 1 \times v^2$$

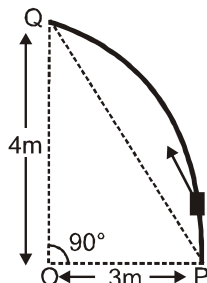
$$v = 10 \text{ m/s}$$

9. $N = 7.5 \text{ N}$

10. $v = 10 \text{ m/s}$

11. Consider an elliptically shaped rail PQ in the vertical plane with $OP = 3\text{m}$ and $OQ = 4\text{m}$. A block of mass 1kg is pulled along the rail from P to Q with a force of 18 N , Which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is $(n \times 10)$ joules. The value of n is (take acceleration due to gravity $= 10 \text{ ms}^{-2}$)
[IIT JEE Adv.-2014]

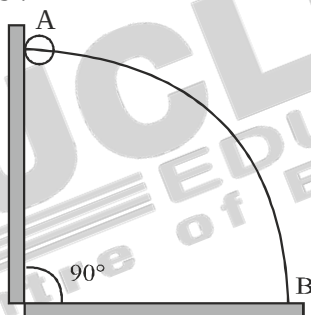




Ans. 5

Sol. $W_F + W_g = K_f - K_i$
 $18 \times 5 + 1g(-4) = K_f$
 $90 - 40 = K_f$
 $K_f = 50\text{J} = 5 \times 10\text{J}$

12. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is :- **[JEE Advanced-2014]**



- (A) Always radially outwards
 (B) Always radially inwards
 (C) Radially outwards initially and radially inwards later.
 (D) Radially inwards initially and radially outwards later.

Ans. (D)

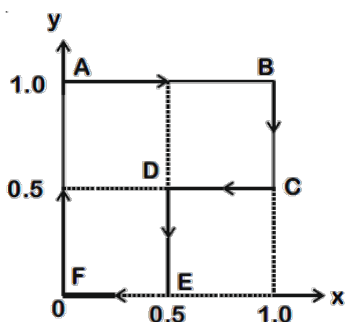
13. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x -axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true

[JEE Advanced-2018]

- (A) The force applied on the particle is constant
 (B) The speed of the particle is proportional to time
 (C) The distance of the particle from the origin increases linearly with time
 (D) The force is conservative

Ans. (ABD)

14. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})$ N, where x and y are in meter and $\alpha = -1 \text{ Nm}^{-1}$. The work done on the particle by this force $\vec{F}$ will be _____ Joule. [JEE Advanced-2019]



Ans. 0.75

Sol. Path Information

$$A \rightarrow B; Y = 1$$

$$B \rightarrow C; X = 1$$

$$C \rightarrow D \rightarrow Y = \frac{1}{2}$$

$$D \rightarrow E; X = \frac{1}{2}$$

$$E \rightarrow F \rightarrow Y = 0$$

$$F \rightarrow A \rightarrow X = 0$$

Putting path equation:

$$W_{A \rightarrow B} = \int_0^1 a(1) + 2ax \cdot \hat{j} \cdot dx \Rightarrow a$$

$$W_{B \rightarrow C} = 2a \left(-\frac{1}{2} \right) = -a$$

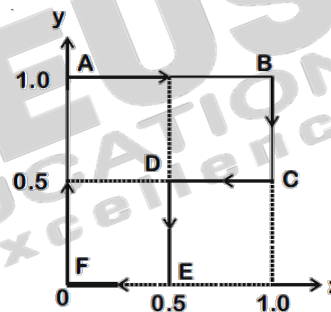
$$W_{C \rightarrow D} = -a \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) = -\frac{a}{4} \text{ Joule}$$

$$W_{D \rightarrow E} = -a \cdot \frac{1}{2} = -\frac{a}{2} \text{ Joule}$$

$$W_{E \rightarrow F} = 0$$

$$W_{F \rightarrow A} = 0$$

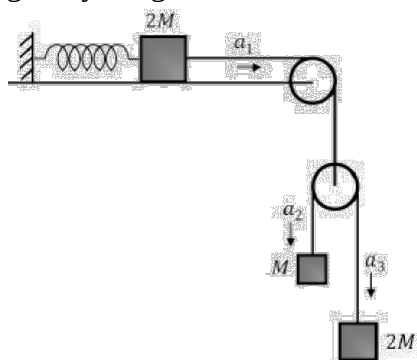
$$W_{\text{net}} = -\frac{3a}{4} = 0.75 \text{ Joule}$$



15. A block of mass $2M$ is attached to a massless spring with spring-constant k . This block is connected to two other blocks of masses M and $2M$ using two massless pulleys and strings. The accelerations of the blocks are a_1 , a_2 and a_3 as shown in the figure. The system is released from

rest with the spring in its unstretched state. The maximum extension of the spring is X_0 . Which of the following option(s) is are correct?

[g is the acceleration due to gravity. Neglect friction]



(1) $x_0 = \frac{4Mg}{k}$

[JEE Advanced-2019]

(2) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block

connected to the spring is $3g\sqrt{\frac{m}{5k}}$

(3) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected

to the spring is $\frac{3g}{10}$

(4) $a_2 - a_1 = a_1 - a_3$

Ans.

(4)

Sol.

$$a_1 = \frac{a_2 + a_3}{2}$$