

VECTORS

1. (a) The position vector of a point A is $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $AB = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$, find the coordinates of B.

- (b)(i) Find the column vector $\mathbf{m}$ such that $\begin{pmatrix} -5 \\ 2 \end{pmatrix} - \mathbf{m} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

(ii) Hence find $|\mathbf{m}|$

- (c). Given that $PQ = \begin{pmatrix} -1 \\ 9 \end{pmatrix}$, find QP in component form.

Solutions

1.(a) $AB = AO + OB$

$$\begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + OB$$

$$OB = \begin{pmatrix} -3 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$B(-1, 1)$$

(b)(i) $\begin{pmatrix} -5 \\ 2 \end{pmatrix} - \mathbf{m} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} -5 \\ 2 \end{pmatrix} - \begin{pmatrix} -8 \\ 6 \end{pmatrix} = \mathbf{m}$$

$$\mathbf{m} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

(ii) $|\mathbf{m}| = \sqrt{3^2 + 4^2}$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= 5$$

$$(c). PQ = \begin{pmatrix} -1 \\ 9 \end{pmatrix}$$

$$QP = \begin{pmatrix} 1 \\ -9 \end{pmatrix}$$

$$2. \text{ Given that } u = \begin{pmatrix} 6 \\ -8 \end{pmatrix}, v = \begin{pmatrix} -9 \\ 10 \end{pmatrix} \text{ and } w = \begin{pmatrix} 15 \\ p \end{pmatrix}, \text{ find}$$

$$(a) (i) |u|$$

$$(ii) 2u + v$$

$$(b). \text{ Given that vector } w \text{ is parallel to vector } u, \text{ calculate the value of } p.$$

Solution

$$2(a)(i) |u| = \sqrt{6^2 + 8^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100}$$

$$= 10 \text{ units}$$

$$(ii) 2u + v = 2 \begin{pmatrix} 6 \\ -8 \end{pmatrix} + \begin{pmatrix} -9 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} 12 \\ -16 \end{pmatrix} + \begin{pmatrix} -9 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ -6 \end{pmatrix}$$

$$(iii) u = kw \text{ where } k \text{ is a constant then}$$

$$\begin{pmatrix} 6 \\ -8 \end{pmatrix} = k \begin{pmatrix} 15 \\ p \end{pmatrix}$$

$$\begin{pmatrix} 6 \\ -8 \end{pmatrix} = \begin{pmatrix} 15k \\ kp \end{pmatrix}$$

$$6 = 15k$$

$$k = \frac{2}{5},$$

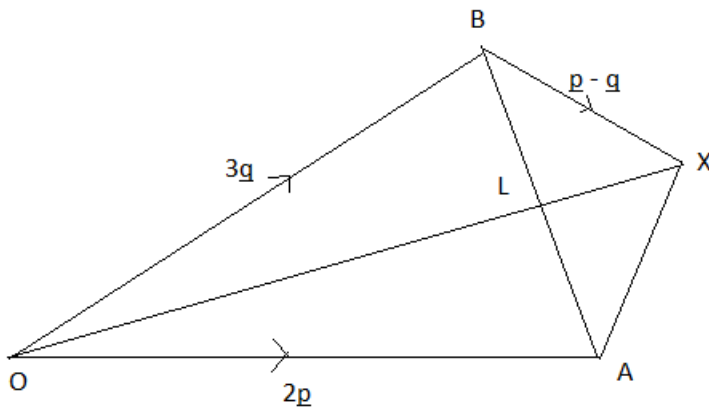
$$-8 = kp, \text{ but } k = \frac{2}{5}$$

$$-8 = \frac{2}{5}p$$

$$2p = -40$$

$$p = -20$$

3. In the diagram, $OA = 2p$, $OB = 3q$ and $BX = p - q$. The lines OX and AB intersect at L .



- (i) Express as simply as possible in terms of p and / or q
 - (a) OX
 - (b) AB
- (ii) Given that $AL = hAB$, express AL in terms of p , q and h .
- (iii) Hence show that $OL = (2 - 2h)p + 3hp$

Solution

$$\begin{aligned} 3. (i)(a) \quad OX &= OB + BX \\ &= 3q + p - q \\ &= 2q + p \end{aligned}$$

$$= 3q - q + p$$

$$(b) \quad AB = AO + OB$$

$$= -2p + 3q$$

$$= 3q - 2p$$

$$(ii). \quad AL = hAB$$

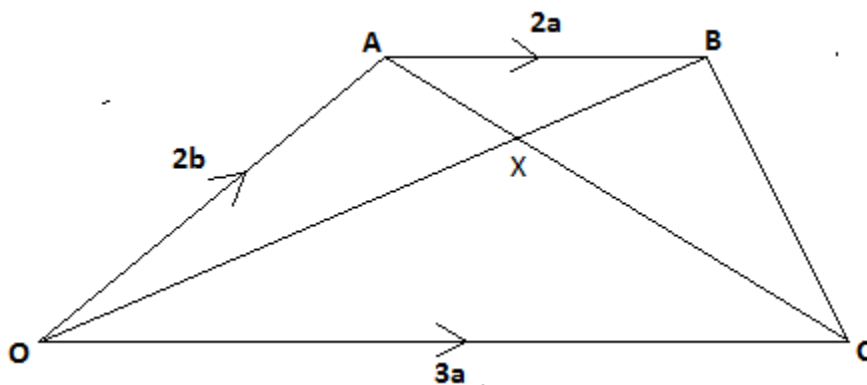
$$\begin{aligned}
 AL &= h(3q - 2p) \\
 &= 3hq - 2h \\
 \text{(iii) } OL &= OA + AL
 \end{aligned}$$

$$= 2p + 3hq - 2hp$$

$$= 2p - 2hp + 3hq$$

$$= (-2h)p + 3hq$$

4. In the diagram, $OA = 2b$, $OC = 3a$ and $AB = 2a$. The lines OB and AC intersect at X .



- (i) Express as simply as possible in terms of a and / or b .
 - (a) OB
 - (b) BC
- (ii) Given that $CX = hCA$, express CX in terms of a , b and h .
- (iii) Hence show that $OX = (3 - 3h)a + 2hb$

Solutions

$$4(i)(a) \quad OB = OA + AB$$

$$= 2b + 2a$$

$$(b) \quad BC = BO + OC$$

$$= -(2a + 2b) + 3a$$

$$= -2a - 2b + 3a$$

$$= 3a - 2a - 2b$$

$$= a - 2b$$

$$(ii) \quad \vec{CX} = h\vec{CA} \text{ but } \vec{CA} = \vec{CO} + \vec{OA}$$

$$= 2b - 3a$$

$$\vec{CX} = h(2b - 3a)$$

$$= 2hb - 3ha$$

$$(iii) \quad \vec{OX} = \vec{OC} + \vec{CX}$$

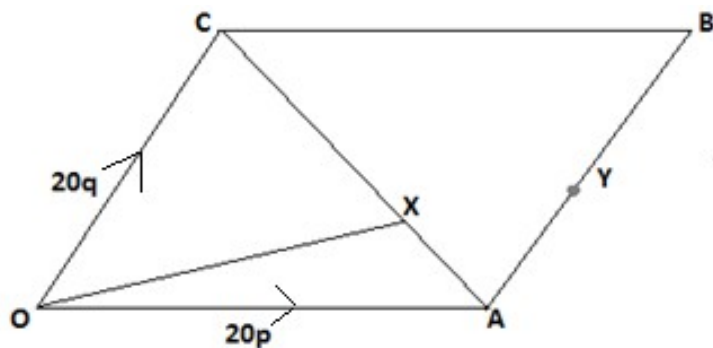
$$= 3a + 2hb - 3ha$$

$$= (3 - 3h)a + 2hb$$

$$= (3 - h)a + 2hb \text{ shown.}$$

PRACTICE QUESTIONS

(1)



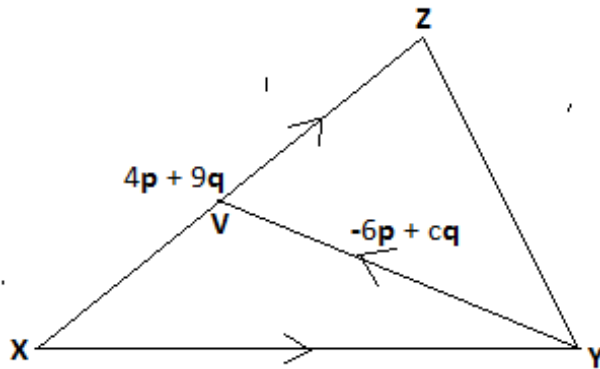
OABC is a parallelogram .

The point X on AC is such that $\vec{AX} = \frac{1}{5}\vec{AC}$. The point Y on AB is such that $\vec{AY} = \frac{1}{4}\vec{AB}$. Given that $\vec{OA} = 20\mathbf{p}$ and $\vec{OC} = 20\mathbf{q}$, express in terms of $\mathbf{p}$ and $\mathbf{q}$

- (i) $\vec{AC}$, (ii) $\vec{AX}$, (iii) $\vec{OX}$, (iv) $\vec{OY}$.

What do the results of (iii) and (iv) tell you about O, X and Y?

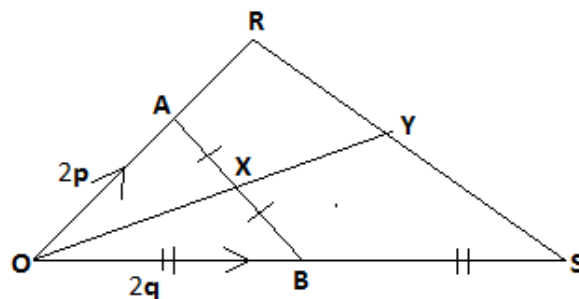
- (2) In the diagram XVZ is a straight line, $\vec{XY} = 8\mathbf{p}$, $\vec{XZ} = 4\mathbf{p} + 9\mathbf{q}$ and $\vec{YV} = -6\mathbf{p} + c\mathbf{q}$.



- (i) Express $\vec{XV}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and c .
- (ii) Given that $XV = h XZ$, form an equation involving $\mathbf{p}$, $\mathbf{q}$, h and c .
- (iii) The point K is outside triangle XYZ and is such that $\vec{XK} = -4\mathbf{p} + 3\mathbf{q}$.

Is $\mathbf{XK}$ parallel to $\mathbf{YV}$? Justify your answer.

- (3) In the triangle ORS , the point A on OR is such that $\vec{OA} = 2\mathbf{p}$. B is the midpoint of OS , X is the midpoint of AB and $\vec{OX}$ produced meets RS at Y . $\vec{OA} = 2\mathbf{p}$ and $\vec{OB} = 2\mathbf{q}$.



- (a) Express in terms of $\mathbf{p}$ and/or $\mathbf{q}$
- (i) $\vec{AB}$, (ii) $\vec{AX}$, (iii) $\vec{OX}$, (iii) $\vec{RS}$

(b) Given that $\vec{RY} = k \vec{RS}$, Express $\vec{RY}$ in terms of $\mathbf{p}, \mathbf{q}$ and h .

(c) Hence show that $\vec{OY} = 3(1-h)\mathbf{p} + 4h\mathbf{q}$.

(d) Given also that $\vec{OY} = k \vec{OX}$, Express $\vec{OY}$ in terms of $\mathbf{p}, \mathbf{q}$ and k .

(e) Using these two expressions for $\vec{OY}$, find the value of h and the value of k .

(f) Find the ratio $RY : YS$.

(g) Express $\vec{XY}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

(4) $\mathbf{p} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}, \quad \mathbf{q} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}, r = \begin{pmatrix} x \\ 7 \end{pmatrix} \text{ and } \mathbf{s} = \begin{pmatrix} 4 \\ y \end{pmatrix}.$

(a) Find (i) $2\mathbf{p} + 3\mathbf{q}$,

(ii) $\mathbf{p} - \mathbf{q}$.

(b) Given that $2\mathbf{p} = \mathbf{r} + \mathbf{s}$ calculate the value of x and the value of y .

(5) $\vec{OA} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $\vec{OB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

(a) Given that $\vec{OC} = \vec{OA} + \vec{OB}$, express $\vec{OC}$ as a column vector.

(b) Express $\vec{AB}$ as a column vector.

(c) If $\vec{LM} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$, what is the special name given to the quadrilateral ALMO?

(6) $\mathbf{p} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}, \quad \mathbf{q} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} m \\ 2 \end{pmatrix}.$

(a) Find $\mathbf{q}$.

(b) Express $2\mathbf{p} - \mathbf{q}$ as a column vector.

(c) Given that $\mathbf{p}$ is parallel to $\mathbf{r}$, find m.

ANSWERS:

(1) (i) $20(\mathbf{q}-\mathbf{p})$, (ii) $4(\mathbf{q}-\mathbf{p})$ (iii) $4(4\mathbf{p}+\mathbf{q})$ (iv) $5(4\mathbf{p}+\mathbf{q})$

The points O,X and Y lie on the same line.

(2) (i) $2\mathbf{p} + c\mathbf{q}$, (ii) $h = \frac{1}{2}$, $c = 4\frac{1}{2}$,

(ii) $\overrightarrow{YV} = \frac{3}{2}(-4\mathbf{p}+3\mathbf{q})$, $\overrightarrow{XK}$ is *parallel* to $\overrightarrow{YV}$.

(3) (a) (i) $2\mathbf{p} - 2\mathbf{q}$, (ii) $\mathbf{q} - \mathbf{p}$, (iii) $\mathbf{p} + \mathbf{q}$, (iv) $4\mathbf{q} - 3\mathbf{p}$.

(b) $h(4\mathbf{q} - 3\mathbf{p})$, (d) $k(\mathbf{p} + \mathbf{q})$, (e) $h = \frac{3}{7}$, $k = 1\frac{5}{7}$.

(f) $3 : 4$, (g) $\frac{5}{7}(\mathbf{p} + \mathbf{q})$

(4) (a) (i) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, (ii) $\begin{pmatrix} 8 \\ -3 \end{pmatrix}$, (b) $x = 6$, $y = -11$.

(5) (a) $\overrightarrow{OC} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$, (b) $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$, (c) Trapezium.

(6) (a) 5, (b) $\begin{pmatrix} 5 \\ -12 \end{pmatrix}$, (c) $-\frac{1}{2}$.