

VARIATION

Direct variation: A quantity y is said to vary directly as another quantity x if the ratio y to $x = \frac{y}{x}$ is always constant. If this is the case then y is said to be directly proportional to x . The symbol $\propto$ denotes '**proportional to**'. Thus the statement y **is proportional to** x is expressed mathematically as $y \propto x$.

To find the equation connecting y and x , we introduce a **constant of proportionality** k , say. Hence the equation connecting y and x is

$$\frac{y}{x} = k \vee y = kx$$

Inverse variation: A quantity y varies **inversely** as another quantity x if the ratio $y \propto \frac{1}{x} = \frac{y}{\frac{1}{x}} = xy$ is constant for all pairs (x, y) . The statement y **varies**

inversely as x is written as $y \propto \frac{1}{x}$ and the formula linking y and x is

$y = \frac{k}{x}$ where k is a constant. If two quantities x and y vary in such a way that when one increases the other decreases, or vice versa, then the two are said to be showing inverse variation.

Joint variation: A variation in which one variable depends on two or more other variables.

Partial variation: Variation as the sum of parts.

Direct variation:

Question 1

It is given that y is directly proportional to the square of x and that $y = 1$ when $x = \frac{1}{2}$.

Find

- (a) the formula for y in terms of x ,
- (b) the values of x when $y = 9$.

Answer:

a) $y \propto x^2$

$$y = kx^2$$

$$1 = \frac{k}{4}$$

$$k = 4$$

$$\therefore y = 4x^2$$

$$\text{b) } 9 = 4x^2$$

$$x^2 = \frac{9}{4} \therefore x = \pm \frac{3}{2}$$

Question 2

y is directly proportional to the square root of x .
Given that $y = 12$ when $x = 36$,

find

(a) the formula for y in terms of x ,

(b) the value of x when $y = 10$.

Answer:

$$\text{a) } y = k\sqrt{x}$$

$$12 = k\sqrt{36}$$

$$12 = 6k$$

$$k = 2$$

$$y = 2\sqrt{x}$$

$$\text{b) } 10 = 2\sqrt{x}$$

$$\sqrt{x} = 5$$

$$x = 25$$

Inverse variation:

Question 1

A formula connecting x and y is $y = \frac{k}{x^3}$, where k is a constant.

Given that $y = -1$ when $x = 2$, calculate the value of

- (i) k ,
- (ii) x when $y = 64$.

Answer

$$(i) \quad -1 = \frac{k}{2^3} \therefore k = -8$$

$$(ii) \quad 64 = \frac{-8}{x^3}$$

$$64x^3 = -8$$

$$x^3 = \frac{-8}{64}$$

$$x^3 = \frac{-1}{8}$$

$$x = \frac{-1}{2}$$

Question 2

It is given that $p = \frac{12}{\sqrt{q}}$.

- (a) Describe the relationship between p and q in words by completing the sentence in the answer space.
- (b) Calculate q when $p = 4$.

Answer

a) p varies inversely as a square root of q .

$$b) \quad 4 = \frac{12}{\sqrt{q}}$$

$$4\sqrt{q} = 12$$

$$\sqrt{q} = 3$$

$$q = 3^2 \therefore q = 9$$

Joint variation:

Question 1

a. Given that y varies directly as x and z and that $y = 9$ when $x = 6$ and

$$z = \frac{1}{2}, \text{ find;}$$

- i. k (the constant of variation)
- ii. the value of y when $x=4$ and $z = 3$
- iii. the value of x when $y = 4\frac{1}{2}$ and $z = 5$

b. It is given that y varies inversely as x . Some corresponding values of x and y are given in the table below.

x	0.6	0.9	b
y	30	a	9

- i. Find the equation connecting x and y
- ii. Find the values of a and b

Solⁿ

a. .

$$\begin{aligned} \text{i. } y &\propto xz \\ y &= 3xz \\ y &= kxz \end{aligned}$$

$$4\frac{1}{2} = 3x(5)$$

$$9 = k(6)\left(\frac{1}{2}\right)$$

$$\frac{9}{2} = 15x$$

$$9 = 3k$$

$$30x = 9$$

$$3 = k$$

$$x = \frac{3}{10}$$

$$k = 3$$

$$\text{ii. } y = 3xz$$

$$y = 3(4)(3)$$

$$y = 36$$

iii.

b. .

$$\text{i. } y \propto \frac{1}{x}$$

$$y = \frac{18}{x}$$

$$\text{ii. } y = \frac{18}{x}$$

$$y = \frac{k}{x}$$

$$9 = \frac{18}{b}$$

$$30 = \frac{k}{0.6}$$

$$9b = 18$$

$$30 = \frac{k}{\frac{6}{10}}$$

$$b = 2$$

$$18 = k$$

$$y = \frac{18}{x}$$

$$a = \frac{18 \times 10}{0.9}$$

10 was multiplied to in order to do away of a decimal in 0.9

$$a = \frac{180}{9}$$

$$a = 20$$

Partial variation:

Question 1

The resistance (R Newtons) to the motion of the motor vehicle is partly constant and partly varies directly as the square of the velocity (v m/s). Write down a formula for r in terms of v.

Answer:

$$R = a + bv^2$$

Question 2

The velocity v m/s of a body moving with constant acceleration is given by $v = u + at$ where t seconds is the time the body is in motion and then u and a are constants. If $v = 30$ when $t = 10$ and $v = 40$ when $t = 15$, find the values of u and a.

Answer:

$$30 = u + 10a$$

.....
.(i)

$$40 = u + 15a$$

.....(ii)

Solving these equations simultaneously we get

a= 2 and u = 10.

PRACTICE QUESTIONS:

- (1) Given that y is proportional to x^3 and that $y = 250$ when $x=10$, find
- (a) the value of the constant k ,
 - (b) y when $x=4$,
 - (c) x when $y = 54$.
- (2) It is given that w varies directly as the square of x and inversely as y .
- (a) Write an expression for w , in terms of x, y and a constant k .
 - (b) If $x=-6, y=12$ and $w=15$, find k .
 - (c) Find the value of y when $x=8$ and $w=20$.
- (3) Two variables p and q have corresponding values as shown in the table below.

p	3	5	7	
q	$\frac{6}{5}$		$\frac{14}{5}$	6

Given that q varies directly as p , find

- (a) the constant of variation, k ,
- (b) the value of q when $p=5$,
- (c) the value of p when $q=6$.

- (4) Given that x varies as y and inversely as z^2 and that
 $x=12$ when $y=3$
and $z=2$, find
(a) the equation connecting x, y and z ,
(b) the value of x when $y=3$ and $z=4$,
(c) the values of z when $x=4$ and $y=25$.

ANSWERS:

- (1) (a) $k=\frac{1}{4}$, (b) $y=16$, (c) $x=6$.
- (2) (a) $w=k\frac{x^2}{y}$, (b) $k=5$, (c) $y=16$.
- (3) (a) $k=\frac{2}{5}$, (b) $q=2$, (c) $p=15$.
- (4) (a) $x=16\frac{y}{z^2}$, (b) $x=3$, (c) $z=10$.