Solid mechanics 1.1 – key points

- A **force** arises from the action (or reaction) of one body on another.
- Newton's third law tells us that the action and reaction forces in this situation (and generally) are equal and opposite.
- The SI unit of force is the newton (N).
- The **moment** of a force about a point is equal to the product of the magnitude of the force and the perpendicular distance from the point to the line of action of the force.

Solid mechanics 1.1 – key points

- The three conditions of equilibrium are:
  - (i) the sum of all the acting forces in the *x*-direction must be zero, i.e. $\sum Fx=0$.
  - (ii) The sum of all the acting forces in the *y*-direction must be zero, i.e. $\sum Fy=0$.
  - (iii) The sum of all the moments about *any* point must be zero.

Solid mechanics 1.1 – key points

- Key points about freebody diagrams:
  - (i) A freebody diagram, as the name implies, is a diagram of a freebody which shows *all* the external forces acting on the body.
  - (ii)Where several bodies (or subcomponents) interact as part of a more complex system, each body should be drawn separately and interacting bodies should be replaced at their contact points with suitable reaction forces and/or moments.

Solid mechanics 1.1 – key points

- The **two-force principle** states that for a two-force body (i.e. a body with forces applied at two points only) to be in equilibrium, both forces must act along the same line of action.

- The three-force principle states that for a three-force body (i.e. a body with forces applied at three points only) to be in equilibrium, the lines of action of these forces must pass through a common point.

Solid mechanics 1.1
Learning summary

By the end of section 1.1 you will have learnt:

- the basic definitions of forces, moments and couples;
- how to use ‘freebody diagrams’ and the basic ‘conditions of equilibrium’ to solve for unknown forces acting on or within a structure;
- how to analyse frictional forces at the contact between bodies or structures;
- general design principles including the two-force and three-force principles which can be used to simplify the solution of specific problems;
- analytical and geometrical methods for solving for forces acting on or within pin-jointed structures.

Solid mechanics 1.2 – key points

- The units of direct stress are the units of force divided by area, i.e. N/m^2, also known as pascal (Pa).
- Because strain is a ratio of lengths, it has no units
- Elastic materials are fully recoverable upon unloading.
- The ratio of σ/ε, i.e. the gradient of this line, is constant and called **Young's modulus** (after the scientist Thomas Young (1773–1829)).

Solid mechanics 1.2 – key points

- The equation $\boldsymbol{\sigma = E\varepsilon}$ is known as **Hooke's law** and, because it describes a straight line, materials obeying it are termed **linear** elastic (NB: there are some exceptions to this linear behaviour but most engineering solids, at least at small deformations, do obey the law).
- The ratio of lateral strain to longitudinal strain depends on the material and, like Young's modulus, is an important material property. It is called **Poisson's ratio**, denoted by the symbol $\boldsymbol{\nu}$ (nu), and named after the French mathematician Simeon Denis Poisson (1781–1840).

Solid mechanics 1.2 – key points

- The yield point is the point beyond which permanent deformations occur.
- **Shear stress** results when a force is applied *parallel* to an area.
- **Shear strain**, denoted by the symbol **γ** (gamma), is a measure of the angle of distortion resulting from the applied shear stress.
- Thus, shear stress is proportional to shear strain and the constant of proportionality, **G**, is termed the **shear modulus**.

Solid mechanics 1.2
Learning summary

By the end of this section you will have learnt:

- the definitions of *direct* stress and strain and *shear* stress and strain;
- the principles of linear elasticity and Hooke's law which relates stress and strain through the material property Young's modulus in direct stress conditions, and Shear modulus in shear conditions, both a measure of the stiffness of a material;
- how lateral deformations are quantified by the material property, Poisson's ratio;
- that there are generally two types of stress–strain problem, namely 'statically determinate' and 'statically indeterminate' problems. Statically determinate problems can be solved using the equations of statics alone (i.e. equilibrium and the stress-strain relationship), while statically indeterminate problems require further information such as the compatibility of strains or deformations;

(continued...)

Solid mechanics 1.2
Learning summary

By the end of this section you will have learnt:

- how temperature changes can give rise to stresses and strains in bodies or structures even in the absence of any applied loading. In such cases, an important material property, determining the magnitude of these ‘thermal’ stresses and strains, is the ‘coefficient of thermal expansion’ of the material;
- how to use the principle of superposition to solve problems involving both mechanical and thermal loading.

Solid mechanics 1.3 – key points

- The knife edge **simple support**, **Figure 1.41(a)**, is modelled with a vertical reaction force which constrains vertical movement but allows free rotation of the beam at the support.
- The **built-in support**, **Figure 1.41(b)**, is modelled with a vertical reaction force and a reaction moment which constrain vertical movement and rotation respectively at the support. This type of support is also referred to as **encastre**.

Solid mechanics 1.3 – key points

- A **point load** has units of newtons (N) and acts vertically at a point along the beam span. A **uniformly distributed load**, shortened to **UDL**, has units of Newtons per metre length of beam (Nm^{-1}), and acts over the full span or part of the span. Finally, a **point moment** has units of Nm and again acts at a point along the beam span.
- A *positive shear force* results in *anticlockwise rotation* of the face on which it acts.
- A *negative shear force* results in *clockwise rotation* of the face on which it acts.

Solid mechanics 1.3 – key points

- A *positive bending moment* results in *sagging* of the beam.
- A *negative bending moment* results in *hogging* of the beam.
- The **second moment of area** about the NA is a measure of the distribution of the area within the beam cross-section and is an important structural term in bending.

Solid mechanics 1.3
Learning summary

By the end of this section you will have learnt:

- various types of beam, their supports and loading conditions;
- how to calculate and draw shear force and bending moment distribution diagrams for beams under point loading, UDL and concentrated moments;
- beam bending theory and the derivation of the beam bending equation relating the bending moment to bending stress and radius of curvature;
- how to calculate the second moments of area of simple and more complex sections such as I-sections and T-sections;
- the general procedure for calculating stresses within beams.

Solid mechanics 1.4 – key points

- An *x*-plane is defined as a plane whose *normal* acts in the *x*-direction. Thus, the right hand edge and the left hand edge of the element are both *x*-planes.
- A *y*-plane is defined as a plane whose *normal* acts in the *y*-direction. Thus, the top edge and the bottom edge of the element are both y-planes.
- A *positive shear stress* is trying to rotate the element *anticlockwise.* τ_{xy} is therefore positive on both the right-hand and left-hand edges.

Solid mechanics 1.4 – key points

- A *negative shear stress* is trying to rotate the element *clockwise*. τ_{yx} is therefore negative on both the top and bottom edges.
- Mohr's circle is a graphical construction representing the above transformation equations.

Solid mechanics 1.4
Learning summary

By the end of this section you will have learnt:

- about stresses and strains acting in a single plane (two-dimensional) and how we define the general state of plane stress;
- the general equations for the angular transformation of stresses in two-dimensional and how these equations may be represented by a graphical construction called ‘Mohr’s circle for plane stress’;
- the use of Mohr’s circle to analyse stresses at a point in a material and determine the planes of maximum direct stress, i.e. the Principal Planes, on which the Principal Stresses act, and the planes of maximum shear stress;
- The derivation of ‘Generalised Hooke’s Law’ relating direct stresses to direct strains in three dimensions and its application in solving three-dimensional problems, including stresses in thin cylinders and spheres and hydrostatic stress/volumetric strain problems.

Solid mechanics 1.5 – key points

- When a moment is applied about the axis of a shaft or bar, as shown in **Figure 1.79**, it is termed a **torque** and the shaft is said to be under a state of **torsion.**
- We now define a new geometric quantity, **J, the polar second moment of area** given by

$$J = \int_A r^2 dA$$

Solid mechanics 1.5 – key points

- ***J*** is an important term in torsion and describes the distribution of the elements of area about the axis of the shaft. Its units are m^4.
- Note that for a given T and J, the shear stress, τ, is proportional to r and is a maximum at r = outer radius, i.e. on the surface of the shaft, and a minimum, $\tau = 0$, at the centre of the shaft. The shear stress varies linearly with radial position.[

Solid mechanics 1.5 – key points

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Solid mechanics 1.5
Learning summary

By the end of this section you will have learnt:

- the definition of torque and how it gives rise to twist and shear stresses within a shaft;
- the derivation of the ‘torsion equation’ and how it is used to calculate twist angle and shear stresses in a shaft subjected to a known torque;
- how to calculate the ‘polar second moment of area’ for solid and hollow circular cross-sections;
- how to solve torsion problems including ‘power transmission’, stepped and composite shafts.