UNIT EIGHT

Properties of bulk Matter

Introduction

The word "matter" refers to everything in the universe that has mass and takes up space. All matter is made up of atoms of elements. Sometimes, atoms bond together closely, while at other times they are scattered widely. There are four known states of matter: solids, liquids, gases, and plasmas. Plasmas are largely common in the Universe whereas in our environment on Earth, solids, liquids, and gases predominate.

States of matter are generally described on the basis of qualities that can be seen or felt. Matter that feels hard and maintains a fixed shape is called a solid; matter that feels wet and maintains its volume but not its shape is called a liquid. Matter that can change both shape and volume is called a gas.
They are refers to as **Bulk matter** which consists of large numbers of atoms, molecules, or ions.

Solids

A **solid** is a form of matter that adopts and maintains a shape that is independent of the container it occupies. It has a definite shape and volume. Its atoms held together by strong molecular bond.
A solid is nearly incompressible, meaning that the atoms are about as close together as they can get.

Liquids

A **liquid** has a definite volume but takes the shape of its container. Like a solid, a **liquid** is nearly incompressible too. A liquid flows and deforms to fit the shape of its container. The molecules are held together by weak molecular bonds and are therefore free to move around.

Gases

A gas has neither a definite volume nor a definite shape. Some gases can be seen and felt, while others are intangible for human beings.
Each molecule in a **gas** moves through space as a free, non-interacting particle until, on occasion, it collides with another molecule or with the wall of the container. A **gas**, like a liquid, is a fluid. It is also **highly compressible**, meaning that a lot of space exists between molecules.

Plasma

Plasma has neither a definite volume nor a definite shape. Plasma often is seen in ionized gases, but it is distinct from a gas because it possesses unique properties. Free electrical charges (not bound to atoms or ions) cause the plasma to be electrically conductive. The plasma may be formed by heating and ionizing a gas. Examples of plasma include stars, lightning, fluorescent lights, and neon signs.
Properties of bulk matter include key concepts such as **stress**, **strain**, **elasticity**, **Hook's law**, and other concepts related to fluids such as **Pascal's law**, Archimedes **viscosity, surface energy**, etc. In this section, we are going to discuss the elastic and plastic behaviors of bulk matter.

8.1 Elastic and plastic behavior

At the completion of this section, you should be able to:

• Define the terms elastic limit, **tensile stress and tensile strain in solid,** Young's modulus, shear modulus

• State Hooke's law and use it to solve relevant problems

• Carry out calculations involving stress, strain, Young's modulus and the energy stored in a stretched material.

• Describe the r/ship between tensile stress and strain (Young's modulus)

• Describe and calculate the following terms Shear stress, Shear strain, Shear modulus, Volume stress, Volume strain, Bulk modulus.

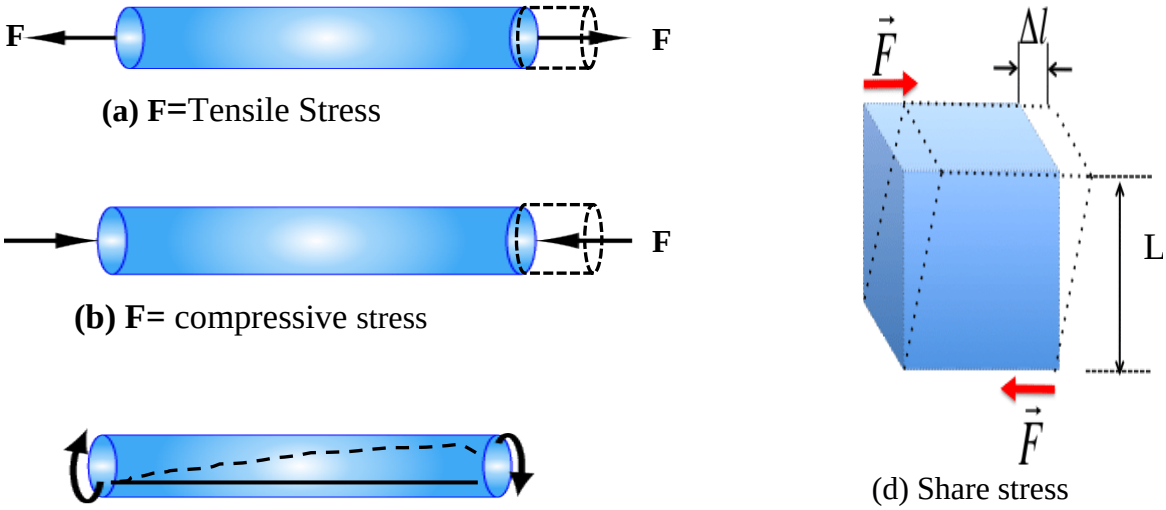
When a force is applied on a material it will cause the material to change in shape or size. A change in shape due to the application of force is known as **deformation.** Any object under the actions of external forces undergoes deformation**.** Any deformation can be of two types as follows:

1) **Elastic deformation**, it is reversible. It is a type of deformation that disappears after the removal of applied forces. It is performed within the **elastic limit**. The object returns to its original position. The force applied is directly proportional to the extension of the material. It is also known as **Temporary Deformation**. Here, Hooke's law is applicable or valid.
2) **Plastic deformation**, it is irreversible. It is a type of deformation that stays even after the removal of applied forces. In other words, it no longer returns to its original dimensions when the forces are removed. The force applied is not directly proportional to the extension of the material Deformation that does not returns to its original position. It is also known as **Permanent Deformation.** Here, Hooke's law is not applicable or it is invalid.

Deformation of Solids

A sufficiently large force will permanently deform or break an object, but otherwise, when the external forces are removed, the object tends to return to its original shape and size. Generally, there are mainly three ways to deform a solid depending on how the external forces are applied on it:

1. When a solid body is stretched by applying with two equal forces normal to cross-sectional area, it is known as **tensile stress** and the deformation is called **tensile deformation.**
2. When a solid body is compressed, restoring force is developed due to an externally applied force which is known as **compressive stress** and the deformation is called **compressive deformation.**
3. When a solid body is exposed to two equal and opposite forces parallel to the cross-sectional area, the tangential force that is developed is known as **tangential or shearing stress** and the deformation is called **shear deformation.**
4. When a torque is applied to the structure, it will twist along the long axis of the rod, and its cross section remains circular. The deformation is called **Torsional deformation.**



(a) F=Tensile Stress

(b) F= compressive stress

(c) τ =Torque

(d) Share stress

Figure 8.1.1 (a) Tensile, (b) Compressive, (c) Torsional and (d) Shear deformations

Elastic and inelastic behavior of materials

Elastic materials are materials that regain their original shape and size when the deforming force is removed. Eg. Rubber bands, Football, Billiard balls, Springs …etc

Inelastic material: Material that **does not return** to its original shape or position after being deformed. They do not regain their original shape and size when the deforming force is removed.

Elastic limit

The elasticity between metallic and non-metallic solids is different. For all materials there is an elastic limit.

Elastic limit is defined as the maximum capacity or stress that a material can withstand before the permanent deformation or the material can elastically stretched without deformed permanently.

It is the highest limit of the material before the plastic deformation of the material can occur. Once the stress or force is removed from the material, the material comes back to its original shape. Beyond this **limit**, a force applied on an object causes **permanent** and **irreversible** deformation (**plastic deformation)**.

The elastic limit depends on the internal structure of the material. For example, Steel (a metal) has a low elastic limit whereas rubber (non-metal) has a very much higher limit.

Hooke's Law

Hooke's law states that **the strain of the material is proportional to the applied stress within the elastic limit of that material**. When the elastic materials are stretched, the atoms and molecules deform until stress is applied, and when the stress is removed, they return to their initial state. Mathematically, Hooke's law is expressed as:

F = –kx ---------- (8.1.1)

In the equation, *F* is the force, *x* is the extension in length, k is the constant of proportionality known as the spring constant in N/m.

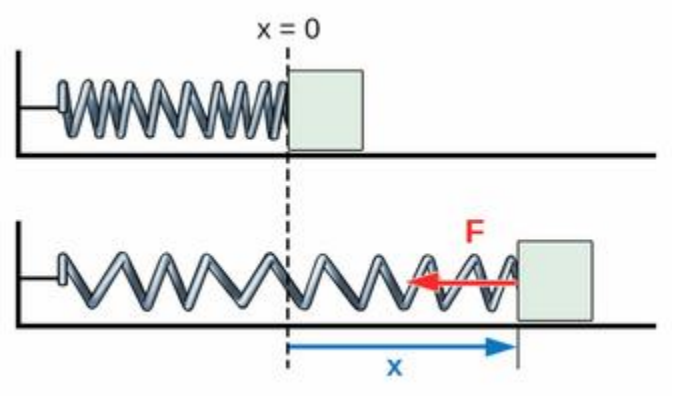


Figure 8.1.2: Hooke's Law

Depending on the material, different springs will have different spring constants, which can be calculated. The figure shows us two instances, the stable condition of the spring, the spring elongated to an amount x when it is pulled under a force equal to the restoring force. If the force is doubled, the elongation is also doubled.

Hooke's Law is linear. It states that the restoring force is directly proportional to the displacement. Hooke's law is applicable to a perfectly elastic material and does not apply beyond the elastic limit of any material. Hooke's Law is essential because it helps us to understand how a stretchy object will behave when stretched or compacted.

Elasticity

When an external force is applied on an object, its shape or size (or both) change, i.e. deformation takes place. The extent of deformation depends on the nature of the material and the external force applied on it. When the deforming forces are withdrawn, some objects try to regain their original shape and size whereas others do not regain their original shape and size. The property of matter to regain its original shape and size after removal of the deforming forces is known as **elasticity.**
The **elastic property** of materials is very important in our daily life. It is used to help us to determine the strength of cables to support the **weight of bodies** such as in **cable cars**, **cranes, lifts…** etc. We use this property to find the strength of beams for construction of buildings and bridges. The **elastic properties** of solid materials are described in terms of **stress** and **strain**.

Stress and Strain

When an external force or system of forces is applied on a body, it undergoes a change in the shape or size according to nature of the forces and it develops stress and strain. In the process of deformation, an internal restoring force is developed due to molecular displacements from their equilibrium positions. The **internal restoring force** opposes the **deforming force**.

Stress

The **stress** is the internal restoring force acting per unit area of the material, and the resulting strain is the deformation that occurs as a result of this stress. Strain and stress are strongly intertwined because strain occurs only as a result of stress. Stress arises from externally applied forces.
In equilibrium, the restoring force is equal in magnitude and opposite in direction to the **external deforming force**. Hence, **stress** is measured by the external force per unit area of cross-section when equilibrium is attained or **Stress** is the force per unit area that is causing some deformation on an object. If the magnitude of deforming force is **F** and it acts on area **A**, we can write.

$$\textbf{Stress} = \boldsymbol{\sigma} = \frac{\textbf{Restoring force}}{\textbf{Area(A)}} = \frac{\textbf{Deforming force}}{\textbf{Area(A)}} = \frac{\boldsymbol{F}}{\boldsymbol{A}}$$

$$\boldsymbol{\sigma} = \frac{\boldsymbol{F}}{\boldsymbol{A}} \text{ ---------- (8.1.2)}$$

Stress has SI unit N/m^2 or Pascal (Pa), the same as the unit of pressure.

b) Strain

Deforming forces produce changes in the dimensions of the body. In general, Strain is the amount of deformation (distortion) experienced by the body in the direction of force applied divided by the initial dimensions of the body. In other word, Strain is defined as the ratio of the change in dimension (e.g. length, shape or volume) to the original dimension of the body. The strain has no units. It is a dimensionless quantity.

$$\textbf{Strain} = \boldsymbol{\gamma} = \frac{\text{change in configuration}}{\text{initial configuration.}} = \frac{\Delta\ell}{\text{L}}$$

$$\boldsymbol{\gamma} = \frac{\Delta\ell}{\text{L}} \text{ ---------- (8.1.3)}$$

Generally, the main difference between stress and strain is that stress measures the deforming force per unit area of the object, whereas strain measures the relative change in length or in volume caused by a deforming force.

Types of Stress and Strain

Stress

The **stress** may also be categorized into **longitudinal**, **normal** or **shearing**. Let us see them one by one.

(i) Longitudinal Stress

If the deforming forces are along the length of the body, we call the stress produced as **longitudinal stress**. The Longitudinal Stress either stretches the object or compresses the object along its length. Thus, it can be further classified into two types based on the direction of deforming force- as shown in its two forms in Fig 8.1.3 (a) and Fig 8.1.3 (b). **Tensile stress** and **Compressive stress**

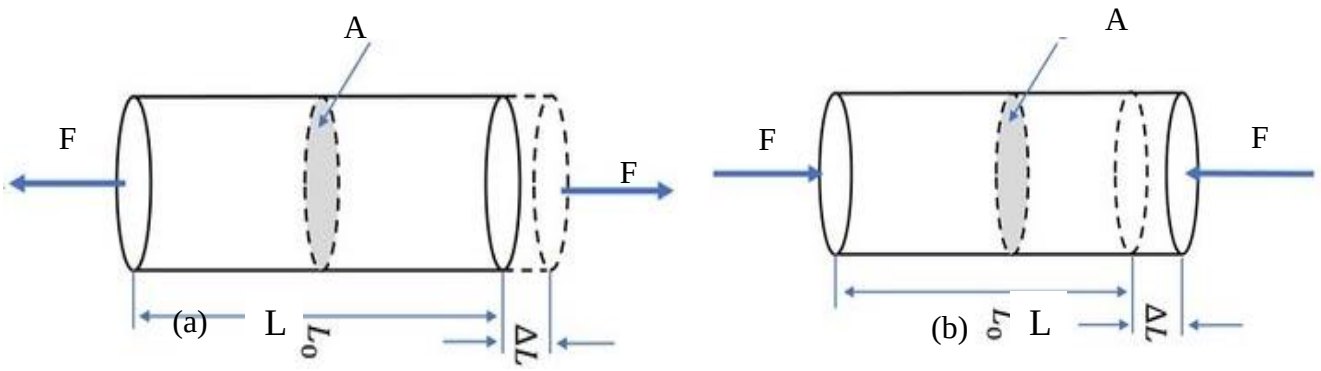


Figure 8.1.3: (a) Tensile stress, (b) Compressive stress

If the longitudinal deforming force or applied force results in the increase in the object's length (L) by ΔL, then the resulting stress is termed as **tensile stress**. For example: when the ends of a bar (rod or wire) of uniform cross-sectional area **A** are pulled with equal and opposite forces of magnitude **F**, the bar will undergo a stretch by the **tensile stress** as Fig.8.1.3 (a). It is defined as the ratio of the **force magnitude F** to **the cross-sectional area A**:

$$\text{Tensile stress} = \frac{F}{A} \quad (\text{in N/m}^2 \text{or Pascal})$$

The application of the forces to the object is:

- perpendicular to each surface
- balanced or no net force
- equal in magnitude but opposite in direction
- collinear forces

Compressive Stress

If the longitudinal deforming force or applied force results in the decrease in the object's length (L), then the resulting stress is termed as **compressive stress**. For example: When a rod or wire is compressed/squeezed by pushing it with equal and opposite forces (inwards) at both ends as Fig.8.1.2(b)

Compressive stress is a force that causes a material to deform and to occupy a smaller volume. It is can be determined in the same way **Tensile Stress**.

$$\textbf{Compressive stress} = \frac{F}{A} \quad (\text{in N/m}^2 \text{or Pascal})$$

(ii) Normal Stress

If the deforming forces are applied uniformly and normally all over the surface of the body so that the change in its volume occurs without change in shape, Fig. 8.1.3(b), then we call the stress produced as **normal stress**. You may produce normal stress by applying force uniformly over the entire surface of the body.

Deforming force per unit area normal to the surface is called **pressure** while restoring force developed inside the body per unit area normal to the surface is known as **stress**.

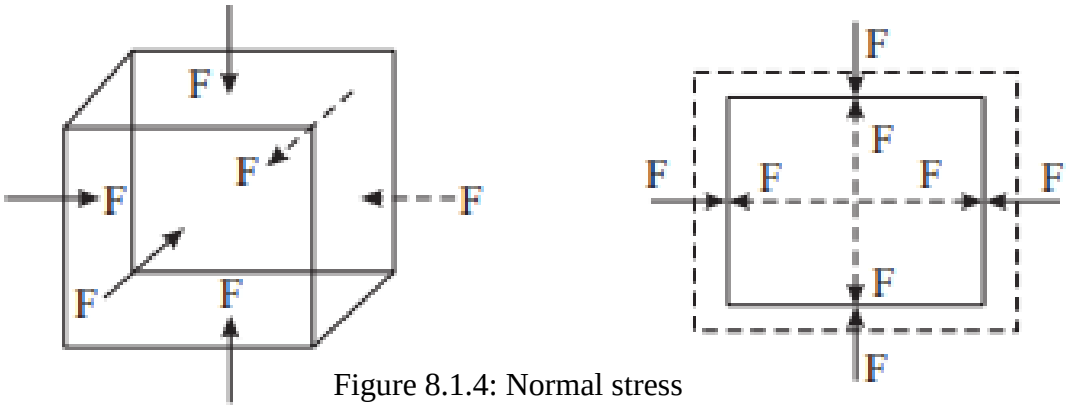


Figure 8.1.4: Normal stress

To calculate for normal stress, two essential parameters are needed and these parameters are **normal force (ΔN) and area (ΔA)**. Let's solve an example. Calculate the normal stress with a normal force of 12 and an area of 22. Therefore, the normal stress is 0.54 Pa.

$$\text{Normal stress} = \sigma = \frac{\textbf{Normal Force}}{\textbf{Area}} = \frac{F}{A}$$

(iii) Shear Stress

Consider a block of height h and area A, in which a tangential force **F** is applied to the upper surface. The lower surface is held fixed. The upper surface will shear by an angle γ corresponding to a horizontal displacementΔx. The geometry of the shearing action is shown in Fig.8.1.5.

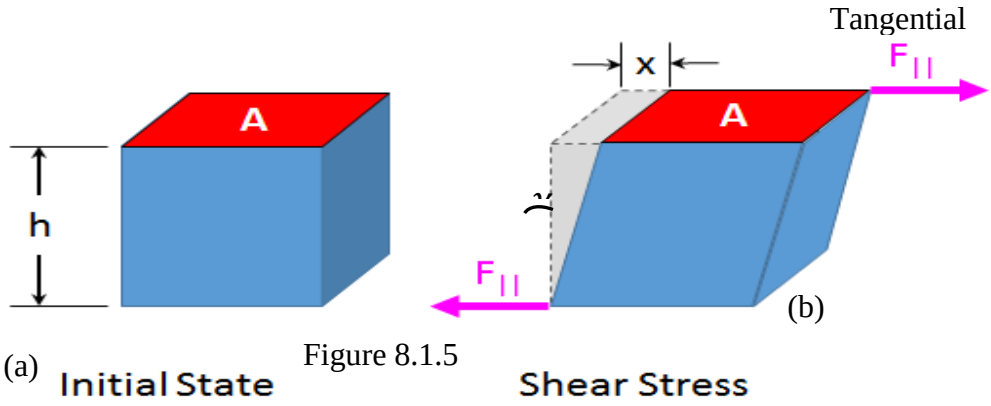


Figure 8.1.5

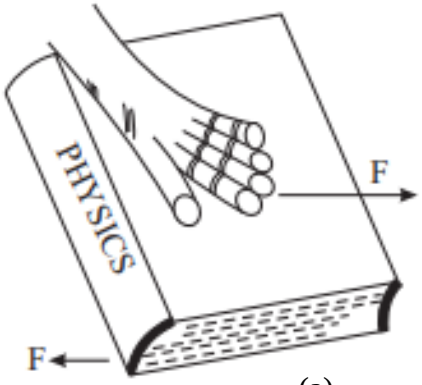
If the deforming forces act **tangentially** or parallel to the surface Fig 8.1.5(b) so that shape of the body changes without change in volume, then such stress is called **shear stress**.

"Shear stress is a type of stress that acts coplanar with cross section of material." Shear stress arises due to shear forces. They are the pair of forces acting on opposite sides of a body with the same magnitude and opposite direction as shown in Fig. 8.1.5(b).

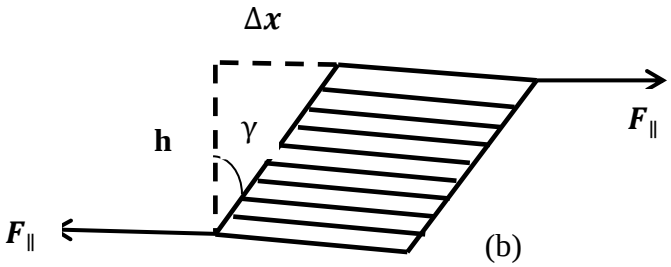
Mathematically, **shear stress** is defined as the ratio of the **tangential force** to the **area A** of the face being sheared.

$$\textbf{Shear stress} = \sigma = \frac{\mathbf{F}}{\mathbf{A}} \text{ ---------- (8.1.4)}$$

An example of shearing stress is shown in Fig 8.1.6 (a) in which a book is pushed sideways. Its opposite face is held fixed by the force of friction.



(a)



(b)

Figure 8.1.6: Pushing a book sideway

This deformation occurs when an object is subjected to a force parallel to one of its faces while the opposite face is **held fixed** by another force. A **shear stress** results in change in shape. Not change in volume of the object. The forces applied to an object are

- Non-collinear forces
- Parallel to the surfaces
- Applied to two separated surfaces
- Equal in magnitude but opposite in direction
- Twists the block and changes its shape
- Non-concurrent forces

(iv)Volumetric Stress

If the forces acting on an object deform it in such a way that there is a change to the volume of that object, then we are talking about volumetric stress.

In this case the force is applied to the body uniformly perpendicular to every surface. All forces have equal magnitude. They compress the body and reduce its volume.

Volumetric Stress is a stress which causes volume deformation on an object and defined as the ratio of the change in the applied force (ΔF) to the surface area (A**).**

$$\textbf{Volumetric Stress (pressure)} = \varepsilon = \frac{\Delta \mathbf{F}}{\mathbf{A}}$$

$$\varepsilon = \frac{\Delta \mathbf{F}}{\mathbf{A}} \text{ ---------- (8.1.5)}$$

Strain

Depending on the kind of **stress** applied, strains are of three types:

Longitudinal (linear) strain, shearing strain, volumetric (bulk) strain

(i) Longitudinal strain

When the deforming force produces a change in length under a **tensile stress**, then the strain is called **longitudinal strain or tensile strain.**

Longitudinal strain = Tensile strain

$$= \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta\ell}{\text{L}}$$

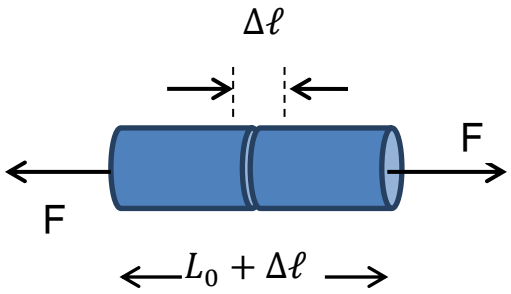


Figure 8.1.7: Tensile strain

(ii) Shear Strain

When the deforming force produces change in shape of the body, then the strain is called **shearing strain.** The **shear strain** is defined as the ratio Δx to h, where $\Delta x$the horizontal distance that the sheared face moves is and **h** is the height of the object. Strain is unit less quantity (Figure 8.1.6).

Shear strain = $\frac{\Delta \mathbf{x}}{\mathbf{h}} = \frac{\Delta \mathbf{l}}{\mathbf{L}} =$ tanγ and for very small angle, tan$\gamma \approx \gamma$

$$\gamma = \frac{\Delta \mathbf{x}}{\mathbf{h}} = \frac{\Delta \mathbf{l}}{\mathbf{L}} \quad \text{---------- (8.1.6)}$$

(iii) Volumetric Strain

When there is volumetric stress, volumetric deformation or volume strain changes the volume of the body. The change in volume is caused by the deforming force

Volume Strain is defined as the ratio of change in volume (ΔV) to the original volume (V_o)**)**

$$\textbf{Volumetric Strain} = \boldsymbol{\mu} = \frac{\text{change in volume}}{\text{Original volume}} = \frac{\Delta \boldsymbol{V}}{\boldsymbol{V}}$$

$$\boldsymbol{\mu} = \frac{\Delta \boldsymbol{V}}{\boldsymbol{V}} \quad \text{---------- (8.1.7)}$$

Similar to Tensile Strain, Volumetric Strain also has no units.

Worked Example 8.1.1

1. A load of 100 kg is suspended by a wire of length 1.0 m and cross sectional area 0.10 cm^2. The wire is stretched by 0.20 cm. Calculate the (i) tensile stress, and (ii) tensile strain in the wire. Given, g = 9.80 ms^{-2}.

Solution

(i) Tensile stress = $\frac{\text{F}}{A} = \frac{mg}{A} = \frac{100x9.8}{0.1x10^{-4}}$ N/m^2 = 9.8x10^7 N/m^2

(ii) Tensile strain = $\frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta\ell}{\text{L}}$

Tensile strain = $\frac{\Delta\ell}{\text{L}} = \frac{0.20\text{x}10^{-2}\text{m}}{1.0\text{m}} = 2.0\text{x}10^{-3}$

2. An object of mass 15kg is hanging from one end of a steel wire. The wire without the mass has an upstretched length of 0.50m and a change in length 1.125 x$10^{-6}m$. What are the resulting shear stress and shear strain of the wire? The cross-sectional area for steel wire is 1.4x$10^{-3}cm^2$.

Given

F= mg = 147N, L = 0.50m, A =1.4x$10^{-3}cm^2$ = 1.4x$10^{-6}m^2$ and Δl = 1.125 x$10^{-6}m$

Required: Shear stress and shear strain

(a) Shear stress = $\frac{\mathbf{F}}{\mathbf{A}} = \frac{\mathbf{147}}{\mathbf{1.4x10^{-6}}}$ N/m^2

= 1.05x10^4 N/m^2

(b) Shear strain = $\frac{\Delta \boldsymbol{l}}{\mathbf{L}}$

Shear strain = $\frac{\mathbf{1.125x10^{-6}m}}{\mathbf{0.50m}}$

= 2.25x10^{-6}

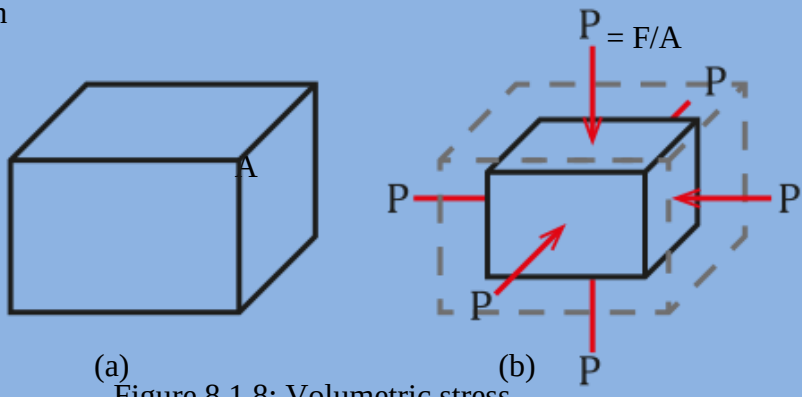


Figure 8.1.8: Volumetric stress

Stress-strain Curve for a Metallic Wire

Refer to Fig. 8.1.9, which shows variation of stress with strain when a metallic wire of uniform cross-section is subjected to an increasing load. Let us study the regions and points on this curve that are of particular importance.

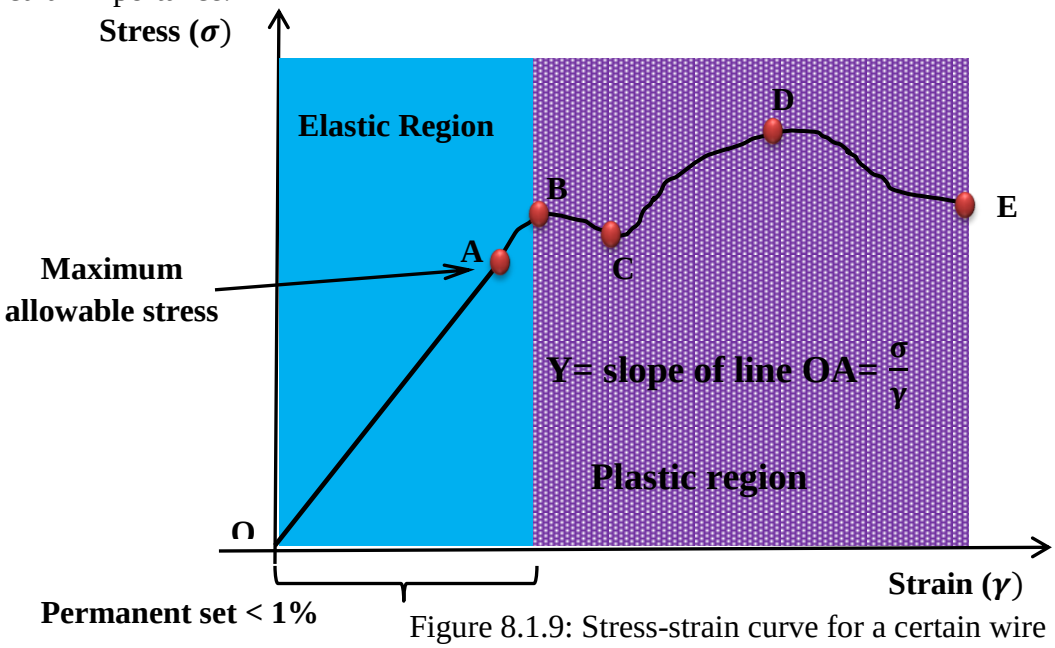


Figure 8.1.9: Stress-strain curve for a certain wire

Explanation

AS the load or the force applied increases, the line begins to curve. This point is marked by A on the graph and known as the limit of proportionality or **proportional limit**.

(i) Region of Proportionality

It is the region in the stress-strain curve that obeys Hooke's Law. In this limit, the stress-strain ratio gives us a proportionality constant known as **Young's modulus**. The point **A** in the graph represents the **proportional limit**. **OA** is a straight line which indicates that in this region, stress is linearly proportional to strain and the body behaves like a perfectly elastic body. Hooke's law is valid in this region.

(ii) Elastic Limit

It is the point, point B, in the graph up to which the material returns to its original position when the load acting on it is completely removed. Beyond this limit, the material doesn't return to its original position, and a **plastic deformation** starts to appear in it.

If we increase the strain a little beyond **A**, the stress is not linearly proportional to strain. The maximum value of strain for which a body (wire) shows elastic property is called **elastic limit**. Beyond the elastic limit, a body behaves like a plastic body.

(iii) Yield Point

When the wire is stretched beyond the **Elastic limit B**, the strain increases more rapidly and the body becomes plastic. It means that even if the deforming load is removed, the wire will not recover its original length.

The **yield point** is defined as a point at which the material starts to deform plastically. It is a point where there is a sudden increase in extension. After the yield point is passed, permanent plastic deformation occurs. There are two yield points (i) upper yield point, point **B** and (ii) lower yield point, point **C**. **The yield strength is the minimum stress which produces permanent plastic deformation**.

Permanent set is the deformation that stays in the material after the applied stress is removed. Permanent set refers to **plastic strain in a material beyond elastic limit**, since the elastic limit will be recovered. When a wire is stretched too much, then it has **permanent strain** even when the **stress is zero**. Then the wire is said to have **permanent set**.

(iv) Ultimate Stress Point (at point D)

It is a point that represents the maximum stress that a material can endure before failure. Beyond this point, failure occurs. After point **D** on the curve, no extension is recoverable

(iv) Fracture or Breaking point E

Beyond point **D**, strain increases very rapidly and near point E, the length of the wire increases continuously even without increasing of load. The wire breaks at point E. This is called the **breaking point** or **fracture point** and the corresponding stress is known as **breaking stress** or **tensile strength**.

Table 8.1: Ultimate Strength of Some Materials

Material	Tensile Strength (N/m^2)	Compressive Strength (N/m^2)
Iron	1.7×10^8	5.5×10^8
Steel	5.0×10^8	5.0×10^8
Aluminum	2.0×10^8	2.0×10^8
Bone	1.2×10^8	1.5×10^8
Marble	—	8.0×10^7
Brick	1×10^6	3.5×10^7
Concrete	2×10^6	2×10^7

Torsional shear stress and strain

Torsion is the twisting of an object caused by a moment or torque acting about the object's longitudinal axis. A common example of an object subjected to torsion is the transmission of shaft, which is used to transmit power by rotation. **Twisting** can be produced in the shaft when two equal and opposite **couples** acting in parallel planes.

A **couple** is two equal and opposite parallel forces acting upon a body with a different line of acting points said as a couple. Due to this torsion in the shaft, the stresses induced in the shaft are known as the **Torsional shear stress** or simply **Torsional stress**.

Consider a shaft is fixed at one end and another end is subjected to the torque as shown in figure 8.1.10. As a result, each and every cross section of the shaft is subjected to the **Torsional shear stress**. What happens to the shaft when we apply torque to a circular bar or shaft?

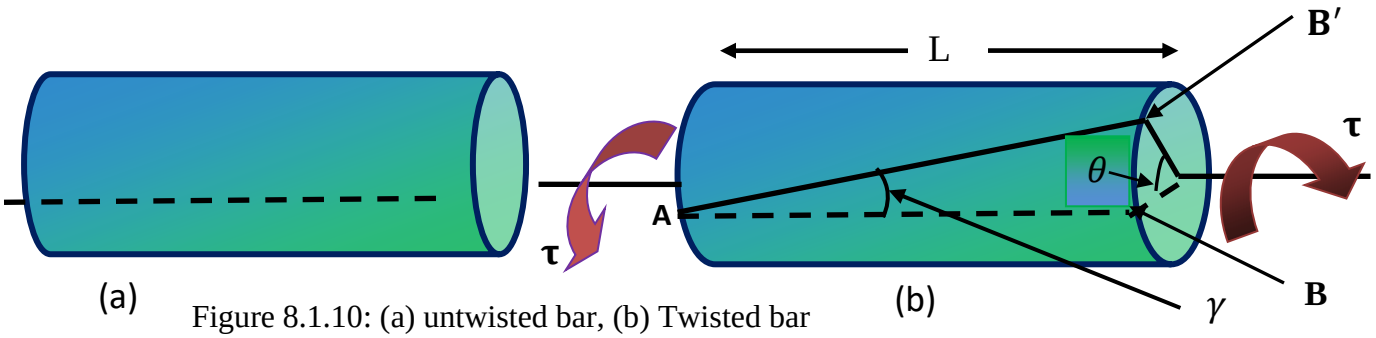


Figure 8.1.10: (a) untwisted bar, (b) Twisted bar

The applied torque cause the free end of the bar to rotate by an angle (θ). This angle is called the angle of twist. The angle of twist (θ) varies linearly from zero at the fixed end of the bar to Gamma (γ). We can calculate the angle of twist using the following equation.

$$\theta = \frac{\tau L}{GJ}$$ ---------- (8.1.8)

The angle of twist depends on the length (L) of the bar, the applied torque (τ), the shear modulus (G), which is a material property, and the polar moment of inertia (J).

Polar moment of inertia

Polar moment of inertia is defined as the resistance of a cross-section to torsional deformation when **torque** is applied in a plane that is parallel to the cross-section area or in a plane that is perpendicular
to the object's central axis, due only to the shape of the cross-section.

Solid Cylinder Shaft

To determine the polar moment of inertia for a solid cylindrical bar or shaft with an outer radius R is:

$$J = \frac{\pi R^4}{2} \text{ or } J = \frac{\pi D^4}{32}$$ ---------- (8.1.9)

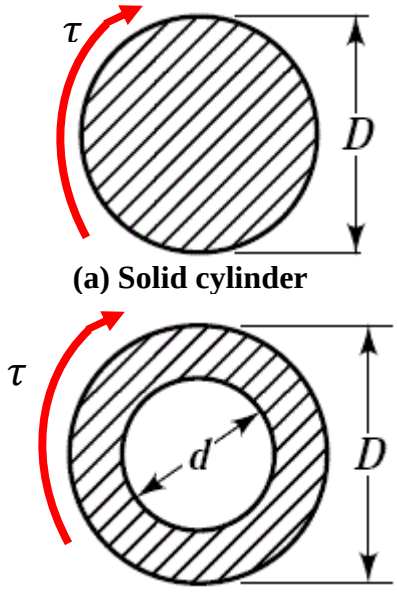
Hollow cylindrical bar or shaft

To determine the polar moment of inertia for a hollow cylindrical bar or shaft with an outer radius R and an inner radius r can be calculated by

$$J = \frac{\pi}{2}(R^4 - r^4)$$ ---------- (8.1.10)

In terms of diameters (R=D/2and r = d/2), it can be written as

$$J = \frac{\pi}{32}(D^4 - d^4)$$ ---------- (8.1.10*)



(a) Solid cylinder

(b) Solid cylinder

Figure 8.1.11: (a) Solid bar, (b) Hollow bar

Torsional shear strain

Torsion generates s**tress** and **strain** within the bar, which we can calculate if we are sure that our bar or shaft will not fail. To figure out how to calculate the stresses and strains, we can start by observing how the element or the shape of the rod in Fig. 8.1.9 (a) is initially cylindrical, but when the torque is applied, the shape of its elements get distorted and twisted by angle θ as shown in Fig.8.1.9 (b).

This gives rise to a **shear strain**, which corresponds to the angle you can see in Fig.8.1.9 (b) above. Considering the geometry of the bar and its deformation, we can calculate the shear strain. It corresponds to the angle between AB and AB^1 in Fig. 8.1.9(b). From trigonometry, for very small angle, Gamma (γ) will be approximately equal to the tangent of γ. That is, it is equal to the ratio of the arc length BB^1 to the length of the bar AB.

$\tan\gamma \approx \gamma = \frac{BB^1}{AB}$, but BB^1 = arc length = Rθ and AB = L

$\gamma = \frac{BB^1}{AB} = \frac{R\theta}{L}$, where R is the radius of the bar or the shaft

Therefore, the Torsional shear strain on the surface of the bar is given by

Torsional Shear strain = $\gamma_\tau = \frac{R\theta}{L}$

$$\gamma_\tau = \frac{R\theta}{L} \text{ ---------- (8.1.11)}$$

This is actually only an equation for **shear strain** on the surface of the bar or the shaft. What about inside it? The shear strains increases linearly with the distance from the center of the cross-section. So if we represent 'r' as the radial distance from the center of the cross-section, we can replace 'R' in equation (8.1.12) with 'r", to give us an equation we can use to calculate the **shear strain** due to torsion at any point within the bar or the shaft, for example, at point A.

Torsional shear strain within the bar is given by:

$$\gamma_{\tau r} = \frac{r\theta}{L} \text{ ---------- (8.1.12)}$$

Where θ is in radian, L and r are in meter.

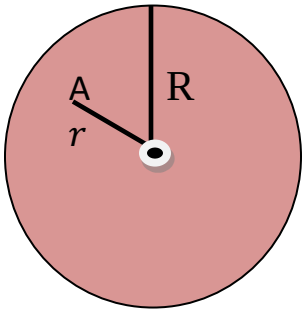
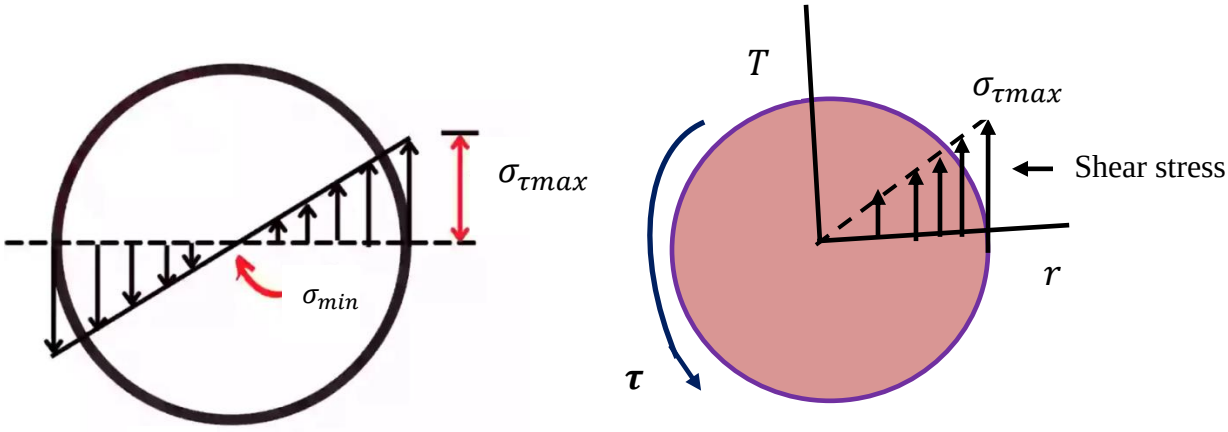


Figure 8.1.12: The free end of the bar

Like **shear strain**, shear stress increases linearly with the distance from the center as you can see in Fig.8.1.13.
This is true for both solid and hollow bar. It is maximum at the edge.



(a) (b)

Figure 8.1.13: Torsional shear stress increases radially outward.

The torsional **shear stress** at any point inside the cross-section is given by

Torsional shear stress = $\frac{\boldsymbol{Distance\ from\ center\ xTorque\ applied}}{\boldsymbol{Polar\ moment\ of\ inertia}}$

$$\boldsymbol{\sigma_{\tau}} = \frac{r\tau}{J} \text{ ---------- (8.1.13)}$$

The shear stress depends on the applied torque (τ), the distance (r) from center of the cross-section and the polar moment of inertia (J).

Due to the Circular section of the shaft, it has been considered that the shear stress at the center axis will be zero and it is maximum at the outer surface of the shaft. The maximum torsional shear stress can be obtained, when $\boldsymbol{r = R}$, at the surface of the bar or the shaft.

$$\boldsymbol{\sigma_{\tau max}} = \frac{R\tau}{J} \text{ ---------- (8.1.14)}$$

From Eq(8.1.8) and Eq(8.1.14), we have the following relationship. This equation is known as Torsional equation.

$$\frac{\tau}{J} = \frac{\sigma_{\tau max}}{R} = \frac{G\theta}{L} \text{ ---------- (8.1.15)}$$

$\boldsymbol{\sigma_{\tau max}}$ = Torsional stress induced at the outer surface of the shaft (Maximum Shear stress)

$\boldsymbol{R}$= Radius of the shaft, τ = Twisting Moment or Torque

J = Polar moment of inertia, **G** = Modulus of rigidity for the shaft material.

$\boldsymbol{L}$= Length of the shaft and **θ** = Angle of twist in radians

$$\boldsymbol{\sigma_{\tau max}} = \frac{G\theta R}{L} = \frac{R\tau}{J} \text{ ---------- (8.1.16)}$$

Torsion Rigidity

The ability to resist the torsion is known as the **torsional stiffness**. The measure of the torsional stiffness is known as the **polar moment of inertia**.

From this, the **torsional rigidity** can be defined as the product of the polar moment of inertia and the modulus of Rigidity of shaft material.

Torsion Rigidity = G x J ---------- (8.1.17)

Worked Example 8.1.2

1.What must be the length of a5mm diameter aluminum wire so that it can be twisted through 1complete revolution without exceeding a shear stress of 42N/mm^2? Take G=27GPa.

Given

D =5mm,R=2.5x10^{-3}m θ= 1rev =2πrad, σ = 42N/mm^{2}G = 27GPa

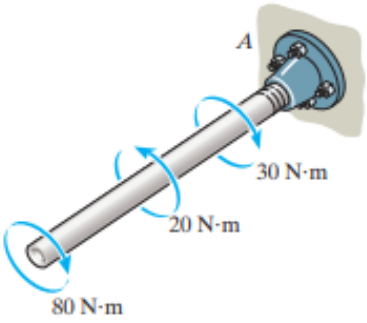
Solution

$\sigma = \frac{R\tau}{J}$ and $\frac{\sigma}{R} = \frac{\tau}{J}$, but $\frac{\tau}{J} = \frac{G\theta}{L}$

$$\frac{\sigma}{R} = \frac{G\theta}{L}$$

$$L = \frac{RG\theta}{\sigma} = \frac{0.0025mx27x10^9Pax6.28rad}{42x10^6Pa} = 10.09m = 10090\ mm$$

2. The copper pipe has an outer diameter of 40 mm and an inner diameter of 30 mm. If it is tightly secured to the wall at A and three torques are applied to it as shown, determine the torque at A and the absolute maximum shear stress developed in the pipe.



Given

R = 40mm/2=20mm, r = 30mm/2 =15mm

Solution

(a) $\sum \tau$= -80Nm + 20Nm -30Nm + τ_A= 0

-90Nm + τ_A = 0 and τ_A = 90Nm

(b) $\sigma_{max} = \frac{\tau_A R}{\frac{\pi}{2}(R^4 - r^4)}$

3. The aluminum shaft of length 1m, G=24GPa and having a diameter of 40mm is twisted by a torque of 100Nm. Find the (a) polar moment of inertia of the shaft. (b) angle of twist in the shaft.

Given:

G = 24 GPa= 24 × 10^3 N/mm²

d = 40 mm

τ = 100 N.m = 100 × 10^3 N.mm

L = 1 m = 1000 mm

Solution

(a) Polar moment of inertia

$J = \frac{\pi}{32} d^4 = \frac{\pi}{32}(40mm)^4$

$J = 251327\ mm^2$

b)The angle of twist is θ

$$\theta = \frac{TL}{GJ} = \frac{10^5x1000}{251327x24x10^3} = 0.016 \text{ radian}$$

Ductile and Brittle materials

The two terms ductile and brittle are used to describe two physical changes in substances.

Ductile materials

Ductile substances can be easily hammered or stretched into thin wires without breakage. This physical property is explained as **ductility**.

Ductility is a physical property of a material by which it can be stretched into wires without breakage. In other words, ductility is the ability to bend as external stress is applied. Ductile materials are those which can be easily drawn out into wires. Many of metals are best examples of ductile materials like copper, gold, silver, mild steel, and Rubber.

Most plastics Ductility of a material is dependent on temperature. Usually when metals are heated, their brittleness is reduced.

Brittle materials

The materials that are easily snapped or broken are known as brittle materials. These materials cannot withstand with continuous external loading. They instantly break as stress is applied, without any plastic deformation after producing a sound. Examples of brittle materials include glass, Cast iron, aluminum, ceramics, cement, Stone, concrete Ice … etc.

Brittle materials are liable to break easily. These materials are hard, and cannot be hammered or stretch like ductile materials; instead, they break. The main difference between ductile and brittle materials is that **ductile materials are can be drawn out into thin wires whereas brittle materials are hard but liable to break easily.**

Elastic Moduli of Materials

The elastic properties of solids are discussed in terms of stress and strain previously. Stress is the force per unit area causing a deformation; strain is a measure of the amount of the deformation. For sufficiently small stresses, stress is proportional to strain, with the constant of proportionality depending on the material being deformed and on the nature of the deformation. We call this proportionality constant as the **elastic modulus**:

$$\textbf{Stress} \propto \textbf{strain}$$

$$\frac{\textbf{Stress}}{\textbf{Strain}} = \text{constant proportionality}$$

Stress = elastic modulus x Strain---------- (8.1.18)

This constant proportionally is called the modulus of elasticity. This elastic modulus is analogous to a **spring constant**. It can be taken as the stiffness of a material that is, the **Elastic modulus** is a measure of the stiffness of a material. In other words, it is a measure of how easily any material can be bent or stretched. It is the slope of **stress** vs **strain** graph up to the limit of proportionality.

A material having a large elastic modulus is very stiff and difficult to deform. **Hooke's law**

This law states that, within the elastic limit, the stress is directly proportional to the strain. That is:

$$F = -k\Delta x$$

Where **F** is the applied force, k is the spring constant, and Δx is essentially the amount by which the spring is stretched or compressed.

Moduli of elasticity

There are three relationships having the form of Hooke's law, corresponding to **tensile**, **shear**, and **bulk** deformation, and all of them satisfy an equation similar to Hooke's law for springs:

These three types of moduli of elasticity are Young's Modulus or modulus of elasticity (Y), Shear (Rigidity) modulus and Bulk Modulus (B).

Young's Modulus

Consider a wire of length L and area of cross-section A fixed at one end to a rigid support. If a normal force F to the free end, the length of the wire will change. Suppose the length increases by $\Delta\ell$. Then the

Normal stress = $\frac{\mathbf{F}}{\mathbf{A}}$ and Longitudinal stress = $\frac{\Delta\ell}{\mathbf{L}}$

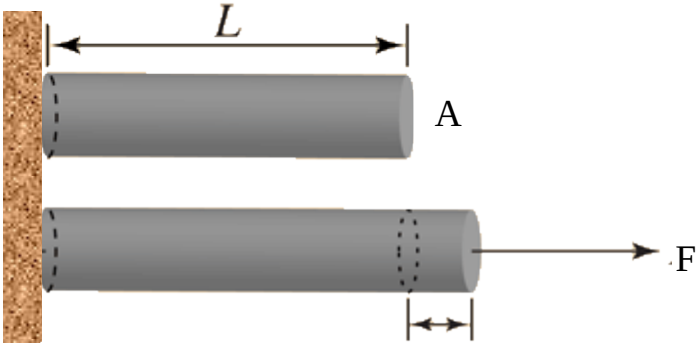


Figure 8.1.14: Stress

Young's Modulus is defined as the ratio of normal stress to the longitudinal strain within the elastic limit.

Young's Modulus =Y = $\frac{\textbf{Tensile stress}}{\textbf{Tensile Strain}}$ or

$= \frac{\textbf{Normal stress}}{\textbf{Longitudinal Strain}}$ **or**

$= \frac{\textbf{stress}}{\textbf{Strain}}$

$$Y = \frac{F/A}{\Delta\ell/L} \text{ ---------- (8.1.19)}$$

$$Y = \left(\frac{F}{A}\right)\left(\frac{L}{\Delta\ell}\right) \text{ ---------- (8.1.19*)}$$

Young's Modulus measured in N/m^2or Pascal-SI units

We can relate the **tensile stress** to **tensile strain** in this equation as follow

$$\frac{F}{A} = Y\frac{\Delta\ell}{L} \text{ ---------- (8.1.20)}$$

$$F = AY\frac{\Delta\ell}{L} = \frac{YA}{L}\Delta\ell \text{ ---------- (8.1.21)}$$

Notice that Equation (8.1.20) could be solved for F and put it as a form of: $F = k\Delta\ell$, where $k = \frac{YA}{L}$ making it looks just like Hooke's law, Equation (8.1.1) and with unit N/m.

A material having a large Young's modulus is difficult to stretch or compress. This quantity is typically used to characterize a rod or wire stressed under either **tension** or **compression.**
The value of **Young's modulus** for a given material depends on whether the material is stretched or compressed. It is constant only over the straight line part of the stress-strain graph or over the range for which Hooke's law applies.

Shear modulus

Consider a rectangular block whose lower face is fixed. Let's a tangential force F be applied on the upper force of the block tangentially. The force provide a shear stress which displaces the upper force through a very small distance Δx and γ. If BC = L and A is the area of the upper face, then the

Tangential stress = $\frac{\text{Normal Force}}{\text{Area}} = \frac{F}{A}$

Shear strain = $\tan\gamma = \gamma$

$$\gamma = \frac{BB'}{BC} = \frac{\Delta x}{L}$$

$$\gamma = \frac{\Delta x}{L}$$

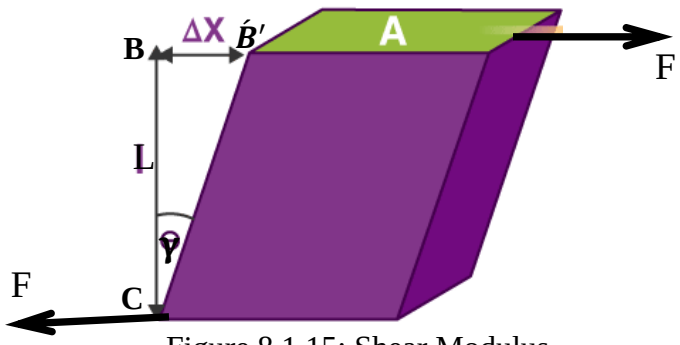


Figure 8.1.15: Shear Modulus

Shear modulus also known as **Modulus of rigidity** is the measure of the rigidity of the body.
It is defined as the ratio of **shear stress** to **shear strain** within the elastic limit or in the Hooke's law region. It denoted by symbol η.

$$\textbf{Shear modulus} = \eta = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$\textbf{Shear modulus} = \eta = \frac{F/A}{\Delta x/L} = \left(\frac{F}{A}\right)\left(\frac{L}{\Delta x}\right) \text{ ---------- (8.1.22)}$$

$$\frac{F}{A} = \eta\frac{\Delta x}{L}$$

$$F = \frac{\eta A}{L}\Delta x = k\,\Delta x$$

$$k = \frac{\eta A}{L} \text{ ---------- (8.1.23)}$$

(Where k is spring constant)

Bulk modulus (B): Volume elasticity

The bulk modulus characterizes the response of a substance to uniform squeezing or compressing. Suppose compressive forces or the external forces acting on a spherical shape are all perpendicular to the surface or normal everywhere on the surface of the sphere. All are distributed uniformly over

the surface of the object (Fig.8.1.21). This occurs when an object is immersed in a fluid. An object subject to this type of deformation undergoes a change in volume but no change in shape. Bulk modulus measures the resistance of solids or liquids to changes in their volume.
In compressing an object, an increase in presure(positive(ΔP)) causes a decrease in volume (negative(ΔV)) and vice versal.

The volumetric stress Δp is defined as the ratio of the magnitude of the change in the applied force ΔF to the surface area A. It is equivalent to Normal stress

Volumetric stress $= \Delta p = \frac{\Delta F}{A}$

The volumetric strain is equal to the change in volume ΔV divided by the original volume V.

Volumetric strain = $\frac{-\Delta V}{V_o}$

Again using Equation 8.1.23 and 8.1.24, we can relate a volume stress to a volume strain by the formula **Bulk modulus** is defined as the ratio of **volumetric stress** to **volumetric strain** within the elastic limit. It is denoted by symbol B.

Bulk modulus =B = $\frac{Volumetric\ stress}{Volumetric\ strain}$ ---------- (8.1.24)

$$B = \frac{Volumetric\ stress}{Volumetric\ strain} = \frac{\Delta F/A}{\frac{-\Delta V}{V}} = \frac{-\Delta P}{\frac{\Delta V}{V}}$$

$$B = -V\frac{\Delta P}{\Delta V} \text{ ---------- (8.1.25)}$$

measured in N/m^2

or

$$B = \frac{Change\ in\ Pressure}{Volumetric\ strain}$$

$$B = \frac{-\Delta P}{\frac{\Delta V}{V}} \text{ ---------- (8.1.26)}$$

ΔP = Change of the pressure that applied on the material

From eq.(8.1.26), we have

$$\Delta P = -B\frac{\Delta V}{V} \text{ ---------- (8.1.27)}$$

$$\Delta p = \frac{F}{A} = -B\frac{\Delta V}{V}$$

$$F = -B\frac{A\Delta V}{V} \text{ ---------- (8.1.28)}$$

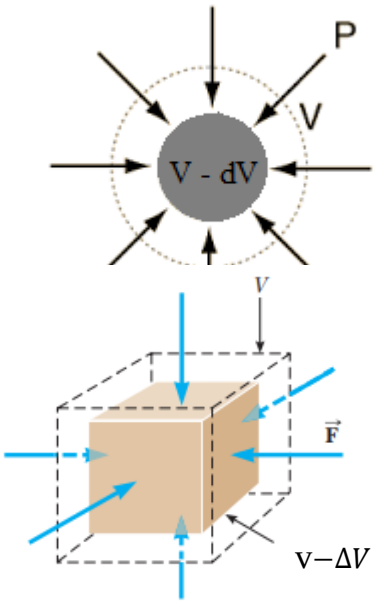


Figure 8.1.16

Notice: 1) B is positive number

2) Both Solids and liquids have a bulk modulus.

3) Liquids have no Shear modulus and Young's modulus because a liquid does not sustain a shearing stress or a tensile stress.

Compressibility

The reciprocal of bulk modulus of a material (1/ B) is called Compressibility
Compressibility = 1/B. Its S.I unit is the m^2/N or Pa^{-1}.

Worked Example 8.1.3

1. A vertical steel beam in a building supports a load of $6x10^4$ N. (a) If the length of the beam is 4 m and its cross-sectional area is $8x\ 10^{-3}\ m^2$, find the distance the beam is compressed along its length. (b) What maximum load(F) that the steel beam support before failing if its ultimate compressive strength is $5x10^8 N/m^2$?
$Y= 2x10^{11} N/m^2$, $A= 8x\ 10^{-3}\ m^2$

Solution

From $\frac{F}{A} = Y\frac{\Delta \ell}{L}$

(a) $\Delta \ell = \frac{FL}{YA} = 1.5 \times 10^{-4}$ m

(b) To find the maximum load that the beam can support, set the compressive stress equal to the ultimate compressive strength from Table 8.1, and solve for F.

(b) $\frac{F}{A} = \frac{F}{8x10^{-4}m^2}$ = Compressive stress

F = Compressive stress x A
$= 5x10^8 N/m^2 x 8 \times 10^{-4}\ m^2 = 4x10^4 N$

2. What is the bulk modulus of a body that experienced a change of pressure of $5x10^4 N/m^2$ and its volume goes from 4 cm^3 to 3.9 cm^3?

Solution

$\Delta P = 5x10^4 N/m^2$, $\Delta P = V - V_0 = 3.9cm^3 - 4cm^3 = -0.1cm^3$

$B = \frac{\Delta P}{\frac{-\Delta V}{V}} = (5x10^4 N/m^2 / -(-0.1cm^3/4cm^3) = 2x10^6 N/m^2$

Poisson's ratio (δ)

Poisson's ratio or is a mechanical property of a material that is the measure of its deformation perpendicular to the direction of loading. In other words, it is the negative value of the ratio of transverse or lateral strain (x-direction) to axial or longitudinal strain (y-direction).It measures the **Poisson effect**, which is the tendency of a material to compress in the direction perpendicular to

the applied force. For example, a rubber band becomes thinner when you stretch it or thicker when you compress it. This effect can be measured by **Poisson's ratio.**

In simple terms, Poisson's ratio measures a materials ability to thicken when compressed or thin when stretched.

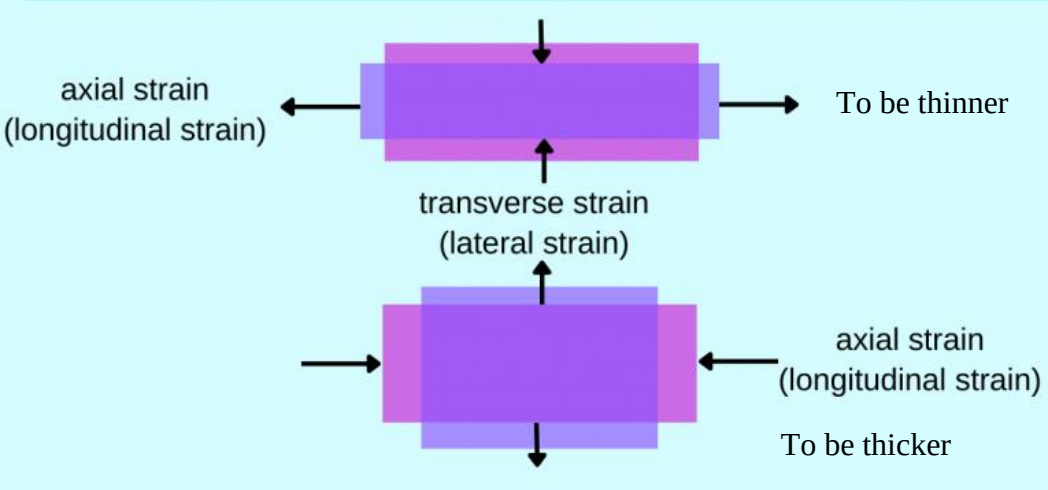


Figure 8.1.17: Poisson's ratio is the deformation of a material perpendicular to the loading direction. Tensile deformation is considered positive and compressive deformation is considered negative. Therefore, the definition of Poisson's ratio contains a **minus sign** so that normal materials have a positive ratio. This ratio is unitless.

Let's L and D are original length and diameter of the wire whereas ℓ is slightly increase in length and d is a corresponding slightly decrease in the diameter of the wire.

Longitudinal strain = $\frac{\ell}{L}$ and Lateral strain = $\frac{-d}{D}$

The negative sign shows that as the length of the wire increases, then at the same time its diameter decreases.

Poisson's ratio = $\delta = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

$$\delta = \frac{\frac{-d}{D}}{\ell/_L} = (\frac{-d}{D})(\frac{L}{\ell})$$

$$\delta = (\frac{-d}{D})(\frac{L}{\ell}) \text{ ---------- (8.1.29)}$$

Since 'd' is negative, then the value of the ratio (δ) is positive.

Poisson's ratio is never more than 0.5, most materials have Poisson's ratio values ranging between 0.0 and 0.5. Soft materials, like rubber, have Poisson's ratio values near 0.5. Steel and rigid polymers typically have values around 0.3. Porous materials typically have Poisson's ratio values close to zero. This is because they collapse when compressed.

Some materials have negative Poisson's ratio values. These materials are called **Auxetic materials**. They thicken when they are stretched and become thin when they are compressed. Origami-folded materials and certain crystals are auxetic materials.

Elastic strain energy in a stretching wire

Consider a wire of original length L and a cross-section area A stretched by a load or a force "F" acting along its length. As the wire is stretched, the stress and strain both increase gradually and proportionally until the elastic limit. When the wire is in equilibrium, the **Young's Modulus** of the material of the wire is:

$$Y = \frac{FL}{A\Delta\ell} \quad .$$

$$F = \frac{YA\Delta\ell}{L}$$

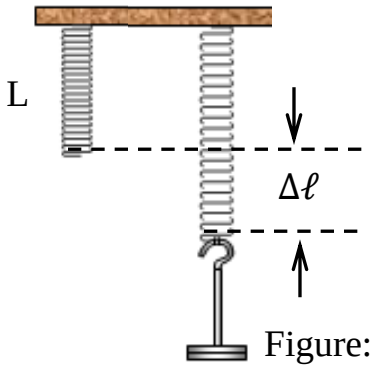


Figure: 8.1. 18

F is not constant, it increases gradually from zero to F and take the average

$$F_{av} = (0 + F)/2 = \frac{YA\Delta\ell}{2L}$$

Where '$\Delta\ell$' is the increases in length of the wire

The work done to extend the wire by an additional amount $\Delta\ell$ is given by

$$W = F_{av}\Delta\ell = \frac{1}{2}F\Delta\ell$$

$$W = \frac{YA\Delta\ell}{2L}\Delta\ell = \frac{YA\Delta\ell^2}{2L} \quad \text{---------- (8.1.30)}$$

The total work done in extending the wire by an amount "ℓ" is by an integral

$$W = \frac{YA\ell^2}{2L} \quad \text{---------- (8.1.31)}$$

We know that from Hooke's law

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta\ell/L} = \left(\frac{F}{A}\right)\left(\frac{L}{\Delta\ell}\right) \quad \text{---------- (8.1.32)}$$

But $\Delta\ell = \frac{FL}{YA}$

$$W = \frac{1}{2}F\Delta\ell = \frac{1}{2}F\left(\frac{FL}{YA}\right) = \frac{1}{2}\frac{F^2L}{YA}$$

$$W = \frac{1}{2}\frac{F^2L}{YA} \quad \text{---------- (8.1.33)}$$

Multiply the right side of this equation by $\frac{A}{A}$

$$W = \frac{1}{2}\frac{F^2 AL}{YA^2} = \frac{1}{2}\left(\frac{F}{A}\right)^2\left(\frac{LA}{Y}\right) = \frac{1}{2Y}(Stress)^2 \text{x volume}$$

$$W = \frac{1}{2Y}(Stress)^2 \text{x V} \text{ ---------- (8.1.34)}$$

But $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\gamma}$

$$W = \frac{1}{2Y}(\sigma)^2 \text{x V} \text{ ---------- (8.1.34*)}$$

$$W = \frac{1}{2}(Stress)^2\left(\frac{\text{Strain}}{\text{Stress}}\right) \text{xV}$$

$$W = \frac{1}{2}(\text{Stress})(\text{Strain})\text{xV} \text{ ---------- ----- (8.1.35)}$$

$$W = \frac{1}{2}(\sigma)(\gamma)\text{xV} \text{ ---------- ---------- --- (8.1.35*)}$$

The work done to deform the body is stored up in the body as the elastic potential energy and it is called the **Elastic Strain energy**.

Elastic Strain energy =U $= \frac{1}{2}(\sigma)(\gamma)\text{xV}$

The elastic Strain energy per unit volume refers to as Elastic strain energy density.

Strain energy density $(\text{U/v}) = \frac{1}{2}(\sigma)(\gamma)$ ---------- (8.1.36)

Worked Example 8.1.4

1. A wire of length 5m and its cross-sectional area $1mm^2$ is stretched or deformed in length by 1mm. $Y = 2x10^{11} N/m^2$

(a) Calculate the work done in stretching the wire

(b) Find the value of the spring constant

(c) Determine the stress and strain of the wire

(d) What are the elastic strain energy and elastic strain energy density?

Given

$L = 5m$, $A = 1mm^2 = 1x10^{-6}m^2$, $\Delta\ell = 1mm = 1x10^{-3}m$ and $Y = 2x10^{11}N/m^2$

Solution

(a) $W = \frac{YA\Delta\ell^2}{2L} = \frac{2x10^{11}x\ 10^{-6}x\ (10^{-3})^2}{2x5}J = \frac{2x10^{-1}}{10}J = 2x10^{-2}J$

(b) $K = \frac{YA}{L} = \frac{2x10^{11}x\ 10^{-6}}{5}$ N/m $= 4x10^{-4}$N/m

(c) Stress $= \frac{F}{A}$, but $F = \frac{YA\Delta\ell}{L}$

Stress = $\frac{F}{A} = \frac{Y\Delta\ell}{L} = \frac{2x10^{11}x\ 10^{-3}}{5}$ N/m = $4x10^{9}$ N/m

$Y = \frac{\text{Stress}}{\text{Strain}}$

Strain = $\frac{\text{Stress}}{Y} = \frac{4x10^{9}}{2x\ 10^{11}} = 2x10^{-2}$

(d) Elastic strain energy = Work done in a stretching wire = $2x10^{-2}$J

Elastic strain energy density = $\frac{W}{V}$, but V = AL = $(1x10^{-6}m^2)(5m) = 5x10^{-6}m^3$

Elastic strain energy density = $\frac{2x10^{-2}}{5x10^{-6}}$ J/m^3 = $4x10^{3}$ J/m^3

Summary

In this section you have learnt that:

- The elastic limit is the point beyond which all materials are permanently deformed.
- Tensile stress is tensile force divided by cross-sectional area *A* of the wire.
- Tensile strain is the extension *x* divided by the wire's original length, *L*.
- Shear stress is the force divided by the cross-sectional area being sheared.
- Shear strain = Δ*xL*
- Young's modulus is the ratio of tensile stress to tensile strain over the range for which Hooke's law applies.
- Hooke's law states that the extension is proportional to the applied force.
- Shear modulus is shear stress (*F/A*) divided by shear strain (Δ*x/L*).
- The energy stored in a stretched wire is $\frac{1}{2}$(*F)(x)* where *F* is the force applied and *x* is the extension.

Review questions

1. Define the terms a) elastic limit, b) stress, c) strain, d) Young's modulus, e) shear modulus.
2. A load of 2.0 kg is applied to the ends of a wire 4.0 m long, and produces an extension of 0.24 mm. If the diameter of the wire is 2.0 mm, find:
a) the stress on the wire
b) the strain it produces
c) the value of Young's modulus of the material from which it is made
d) the strain energy.
3. a) What load in kilograms must be hung from a steel wire 6.0 m long and radius 0.80 mm to produce an extension of 1.0 mm? Young's modulus for steel is 2.0×10^{11} Pa.
b) What is the strain energy?
4. Consider a long steel wire where a tension of 4.9 N causes an extension of 18 mm. Find the strain energy.
5. Repeat the previous problem but this time the shaft is hollow with inner diameter of 30mm.

6. A shaft 40mm diameter is made from steel and the maximum allowable shear stress for the material is 50Mpa. Calculate the maximum torque that can be safely transmitted. Take G= 90GPa. (628.3 Nm)

7. What are the assumptions made in theory of torsion. (i) The material of the shaft is uniform throughout.

(ii) The twist along the shaft is uniform.

(iii) The shaft is of uniform circular section throughout.

(iv) Cross-Sections of the shaft, which are plane before twist, remain plane after twist.

8. The solid shaft of radius r is subjected to a torque (τ). Determine the radius of the inner core of the shaft that resists one-half of the applied torque.

Solve the problem two ways:

(a) by using the torsion formula,

(b) by finding the resultant of the shear-stress distribution.

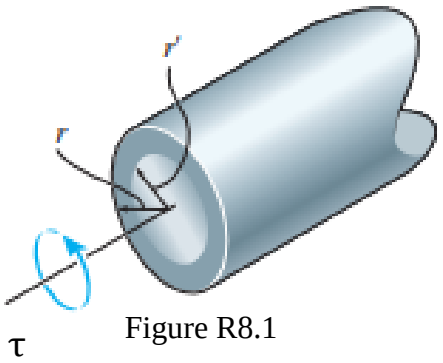


Figure R8.1

9. A shaft is made of a steel alloy having an allowable shear stress of σ_{max} =12kpa if the diameter of the shaft is 1.5cm, (a) determine the maximum torque that can be transmitted. (b) What would be the maximum torque if a 1cm.-diameter hole is bored through the shaft? Sketch the shear-stress distribution along a radial line in each case. (R=1.5cm/2, r =1cm/2)

10. For the aluminum shaft, the limiting value of angle of twist is 0.4 rad. If the shaft has a length of 1m with a diameter of 50mm, calculate a minimum permissible value of the torque. *(*Assume *G=25GPa)*

11. A shaft 50 mm diameters and 0.7m long is subjected to a torque of 1200Nm. Calculate the (a) polar moment of inertia (b) torsional shear stress (c) angle of twist of the shaft. Take G = 90Gpa.

8.2 Fluid Mechanics

Introduction

A substance that can flow is known as a fluid. All liquids and gaseous substances are considered to be fluids. Fluid mechanics deals with the study of all fluids under **static** and **dynamic** situations. Fluid mechanics is a branch of continuous mechanics which deals with a relationship between forces, motions, and statically conditions in a continuous material. This study area deals with many and diversified problems such as surface tension, fluid statics, flow in enclose bodies, or flow round bodies (solid or otherwise), flow stability, etc.

Fluid mechanics has a wide range of applications in Civil mechanical and chemical engineering, in biological systems, and in astrophysics.

There are two major branches in fluid mechanics, namely, **fluid statics** (hydrostatics) which is the study of fluid in a stationary state and **fluid dynamics** (hydrodynamics or aerodynamics) which is the study of fluid when it is flowing.

8.2.1 Fluid statics

- Define the terms density, atmospheric pressure, absolute pressure, pressure, volume.
- Describe the concepts related to hydraulic and pneumatic systems.
- State and apply Archimedes' principle.
- Define surface tension and surface energy.
- Define the angle of contact and account for the shapes of the surfaces of liquids.
- Determine the relationship for capillary rise and use it to solve problems.

Fluid statics is the part of fluid mechanics that deals with fluids when there is no relative motion between the fluid particles.
Fluid statics is an important field of study with numerous applications in various fields such as hydraulic systems, civil engineering, aerospace engineering, biomedical engineering, environmental engineering, and marine engineering. It is used to design and analyze systems that involve the transmission of force and energy using fluid power, as well as to study the movement of fluids in the environment and blood flow in the human body. Fluid statics principles are also used to calculate the buoyancy of objects in fluids and to analyze the stability of structures subjected to hydrostatic loads. Fluid statics is the study of fluids at rest. It involves the analysis of fluid pressures, forces, and moments acting on fluid systems in static equilibrium. Hydrostatic pressure, Pascal's principle, and the buoyancy force are all important concepts in fluid statics. Fluid statics is used in many practical applications, including designing hydraulic systems, analyzing fluid flow in pipelines, and predicting fluid behavior in tanks and reservoirs.

Density and Pressure in Static Fluids

Fluid density

Density of a fluid is defined as the ratio of mass of a fluid to the volume of the fluid.

$$\rho = \frac{m}{V} \text{ ---------- } (8.2.1)$$

Relative density is defined as the ratio of the density of a substance to the density of pure water. The density of pure water at $\mathbf{4^0}$ is used as standard ($1x10^3 kg/m^3$)

$$\textbf{Relative density (R.d)} = \frac{\text{Density of substance}}{\text{Density of water}} = \frac{\boldsymbol{\rho_s}}{\boldsymbol{\rho_w}}$$

Pressure is the ratio of the force acting perpendicular to s surface to the surface area (A) on which the force acts. 1atm equal to 101.3 kP_a,

$$P = \frac{F}{A} \text{ ---------- } (8.2.2)$$

Atmospheric pressure

An **atmosphere** is the layers or blanket of air surrounding the Earth or other planets. Earth's atmosphere is composed of gasses about 78% nitrogen, 21% oxygen, and one percent other gases. Atmospheric pressure depends on the weight or the density of the air. Earth's atmosphere is most

dense at the ground or at sea-level, estimated at about 1.3g/ liter extended to a height about 80 to several hundred kilometers. The higher one goes, the less dense is the air, since there is less weight of air to compress it.
Therefore, atmospheric pressure decreases as we go up from the Earth's surface because the density of the air decreases.

Measuring atmospheric pressure

There are several instruments to measure atmospheric pressure. The most common one is the Mercury Barometer. The principle of its operation is as shown in Fig.8.2.1. A glass tube closed at one end is filled with mercury. Cover the open end of the tube and then inverted in a dish of mercury leaving the open end under the surface of mercury. The level of mercury in the tube falls down, leaving an empty space (vacuum) at the top. Then the height of mercury column above the level of mercury in the dish indicates the atmospheric pressure.

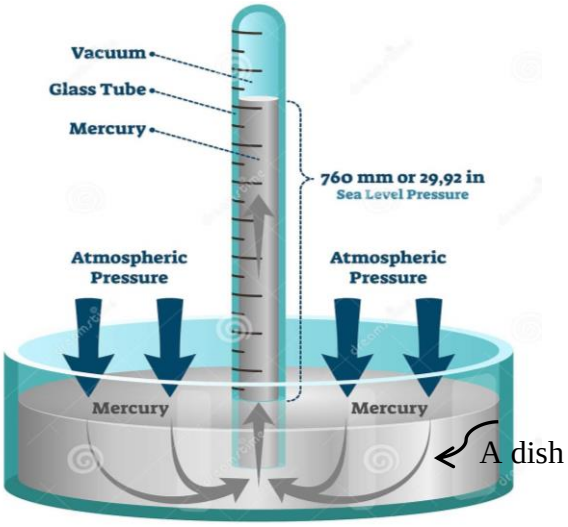


Figure 8.2.1: Mercury barometer

The column of mercury in the tube is supported by the atmospheric pressure (Pa) acting on the surface of mercury in the open dish.
The exact height of the mercury column inside the tube depends on the pressure of atmosphere acting on surface of mercury in the open dish. Therefore, it can be measured in terms of the height of the column of mercury. **Standard** or **normal** atmospheric pressure is taken to support a column of mercury 76cm height of mercury at sea-level.

1atm = 76cm-Hg = 76mm-Hg

To convert 76cm-Hg into N/m^2 or pascal (Pa),
we use that $Pa = \rho_{Hg}gh$, ρ_{Hg}=13600kg/m^3

$Pa = (13600kg/m^3)(9.8m/s^2)(0.76m)$

$= 1.013x10^5 N/m^2 = 1.013x10^5 Pa.$

$Pa = 1.013x10^5 Pa$

Variation of atmospheric pressure with altitude

Atmospheric pressure decreases with increase in altitude as a result of decreasing in the density of the air. **Density of air decreases as we rise up and air becomes rarer**. Therefore, as one goes higher and higher above the Earth's surface, the density of the air become smaller and smaller and the air pressure correspondingly deceases.

The atmospheric pressure varies with altitude as density of air decreases as we rise up and air becomes rarer. For example: Pressure at sea level is taken as 76 cm of Mercury Column, which is one atmosphere. A device that is used to measure altitude is called **altimeter**.

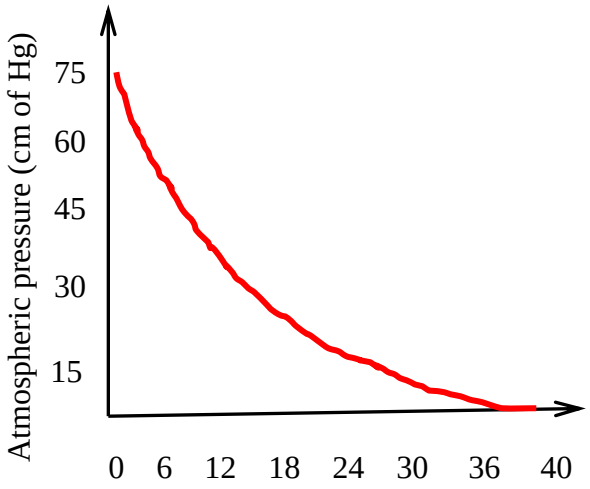


Figure 8.2.2: Height above sea level (km)

It is experimentally observed that nearly, for every 120m rise in altitude above sea level, the height of the column of mercury in the barometer deceases by 1cm of mercury. This implies that the atmospheric pressure decreases by 1 cm of mercury column in every 120m rise in altitude.

Worked example 8.2.1

1. The reading of a mercury barometer at the top of a certain mountain is found to be 60cm of mercury. Suppose for every 120m rise in altitude above the sea-level, the atmospheric pressure decreases by 1cm of mercury and the height of column of mercury at sea-level is 76cm-Hg. What is the altitude of the mountain from the sea-level?

Solution

The difference in height of column of Hg = the pressure difference b/n the top of the mountain and sea level

ΔP =76cm-Hg – 60cm-Hg = 16cm-Hg

For every rise in altitude by 120m, there is a corresponding 1cm-Hg decrease in column.

1cm-Hg = 120m

16cm-Hg = ? h

h= 16cm-Hgx120m/1cm-Hg = 1920m

Therefore, the height of the mountain is nearly 1920m.

Pressure Gauge

A pressure gauge is a device used to measure the pressure of a gas or liquid within a system. It monitors the fluid pressure in various applications such as automotive, aerospace, medical, and manufacturing industries. Pressure gauges measure relative to atmospheric pressure but absolute pressure is defined as force applied perpendicular to a particular area.

Absolute pressure/ System pressure (P_s)

The absolute pressure is the actual or the true pressure at a given point if all of the factors are taken into account.

$P_s = P_a + P_g$ ---------- (8.2.3)

Gauge pressure (P_g): is the difference in pressure between a system and the surrounding atmosphere.

Gauge pressure = System pressure − Atmospheric pressure

$P_g = P_s - P_a$ ---------- (8.2.4)



Figure 8.2.6: Pressure gauge

The reading on a pressure gauge is either positive or negative relative to the atmospheric pressure. If the reading on the pressure gauge is positive, it would mean that the system pressure is above the atmospheric pressure. On the other hand, if the reading on the pressure gauge is negative, it would mean that the system pressure is below the atmospheric pressure. If the gauge was disconnected, it would read zero Pa even though the absolute or system pressure is still 101.3kPa.

Pressure in a fluid

Fluid pressure is the pressure at a point within a fluid arising due to the weight of the fluid. There is a very significant difference between the way a force acts on a fluid and on a solid. Since a solid is a rigid body, it can withstand the application of a force without a significant change in shape. A liquid, on the other hand, can sustain a force only at an enclosed surface or boundary. If a fluid is not restrained, it will flow under a shearing stress instead of being deformed elastically. A solid object can exert only a downward force due to its weight whereas a liquid can exert force in all directions.

The force exerted by a fluid on the walls of its container must always act perpendicular to the wall. Fluids exert pressure in all direction.

Consider a liquid enclosed in a container is under pressure due to the force exerted by the face of the piston in Figure 8.2.4. The fluid exerts pressure in all directions perpendicularly to the wall of the container. At any particular point in a fluid, the pressure is the same in all directions. if this is not true , the fluid would flow under the influence of a resultant pressure until a new condition of equilibrium was reached.

The pressure in a fluid is different at different depths. In any fluid, fluid pressure increases as the depth of the liquid increases. The pressure in a liquid is due to the weight of the column of the liquid above its surface. Since the particles in a liquid are tightly packed, this pressure acts in all directions.

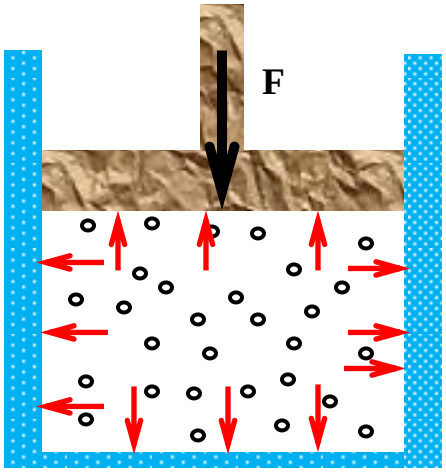


Figure 8.2.4: Fluids exert pressure in all directions

Consider water/fluid in a cylinder

The pressure produced by the column of fluid of height **h** and density **ρ** can be derived as:

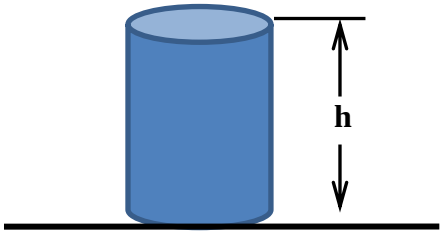


Figure 8.2.5: Water in a cylinder

Pressure = Weight of fluid in the cylinder / base area

= mg/A, but m = **ρv = ρAh**

= ρAhg/A

= ρgh

P = ρgh ---------- (8.2.5)

Note that:

1) All points at same level in a fluid have same pressure.
2) Fluid pressure increases with increase in the depth of the fluid.
3) Fluid pressure does not depend on the shape of the container.

The effect of atmospheric pressure in a fluid

Consider a person swimming under the water as shown below. Pressure is exerted perpendicular to all sides of this swimmer. The arrows represent the directions and magnitudes of the forces exerted at various points on the swimmer.

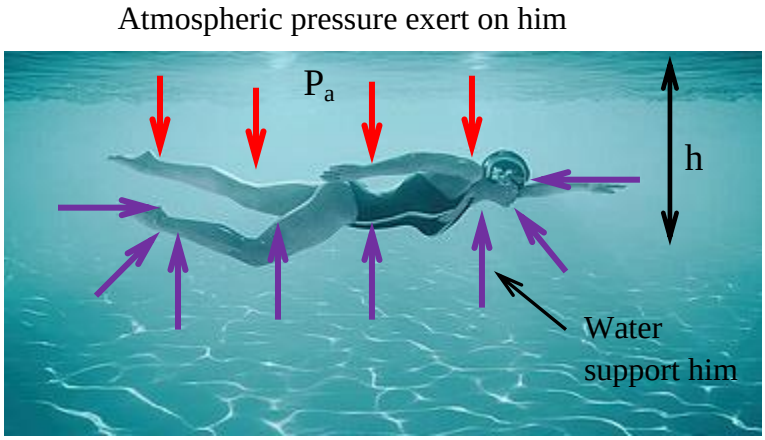


Figure 8.2.6: Effect of atmospheric pressure in a fluid

The pressure on the swimmer would be the sum of the pressure due to the fluid and the atmospheric pressure. The equation used to determine the total pressure exerted on the person is given by

P= Atmospheric + pressure due to
pressure the liquid

$P = Pa + \rho_\ell gh$

Where **ρ** the density of the liquid is, **h** is the depth of the liquid and **g** acceleration due to gravity.

Pascal's principle

This principle deals about the transmission of pressure in a fluid at rest. According to Pascal's principle,

"**The pressure applied to an enclosed fluid is transmitted to every part of the fluid, as well as to the walls of the container without reducing in value."** Any change in pressure is transmitted undiminished

The force in small cylinder must be exerted over a much large distance. Hydraulic systems are used to operate automotive brakes, hydraulic jacks, and numerous other mechanical systems.

When a force ($\mathbf{F_1}$) is applied on a small piston $\mathbf{A_1}$, pressure is transmitted equally through the fluid to the large piston ($\mathbf{A_2}$) and used to lift a large load ($\mathbf{F_2}$).

Pressure is defined as force per unit area.

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \text{---------- (8.2.6)}$$

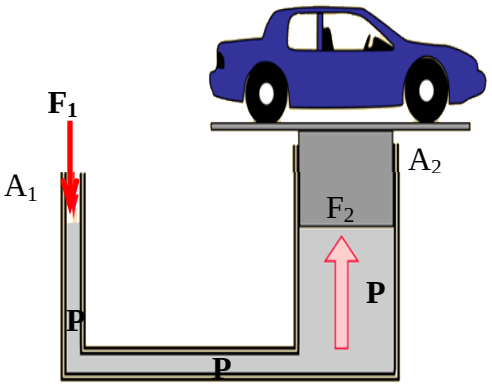


Figure 8.2.7: Hydraulic lift

Archimedes' principle

Archimedes' principle tells us about the relationship between the buoyant force exerted on a submerged by fluids and the weight of the liquid it displaced. It is stated as

A body partly or completely immersed in a fluid is buoyed up by a force equal to the weight of the displaced fluid.

Consider a block of mass "m" is partially or completely immersed into water as shown in Figure 8.2.7 below. Measure the weight of the block in air and in water by using spring balance.

Where W_1=Weight of a block in air

=Actual weight

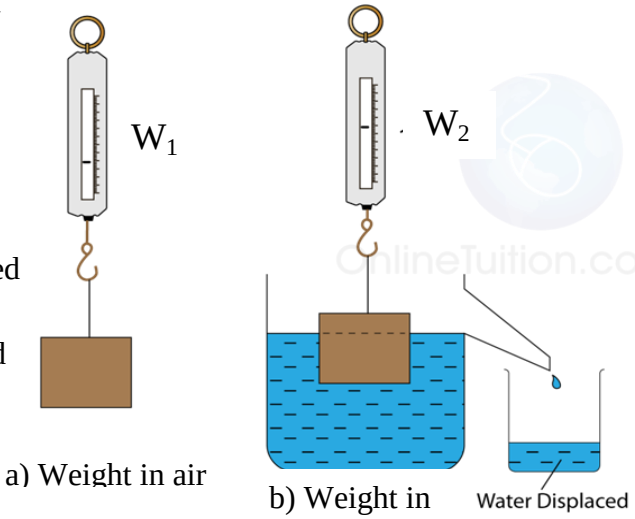
W_2 = Weight of the block in water when partly immersed

= Apparent weight

W_3 = Weight of the block in water when fully immersed

= Apparent weight

According to Archimedes' principle, the magnitude of the buoyant force is equal to the weight of the fluid displaced by the submerged object in all cases.



a) Weight in air b) Weight in water

Figure 8.2.8

Buoyant force (F_b) = Weight of liquid displaced

$F_b = W_{dil}$

$= M_{dil}\, g$

$F_b = \rho_l\, g V_{dil}$ ---------- (8.2.7)

(1) If the object is completely submerged in the fluid, the volume of displaced fluid (v_{dil}) is equal to the volume of the submerged object: $V_{dil} = V_{ob}$. Then

$F_b = \rho_l\, V_{ob}\, g$ ---------- (8.2.8)

The difference between the actual weight and the apparent weight is also equal to the buoyant force.

Buoyant force (F_b) = Weight in air –Weight in liquid

$F_b = W_1 - W_3$ ---------- (8.2.9)

From eq(2) and eq(3), we have

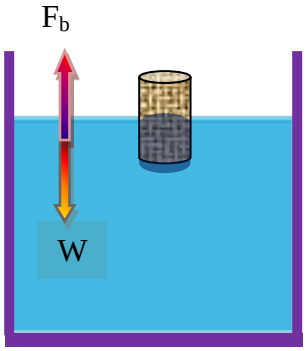
$\rho_l\, V_{ob}\, g = W_1 - W_3$ but $V_{ob} = m/\rho_{bo}$

$\rho_l\, m/\rho_{bo}\, g = W_1 - W_3$

$$\frac{\rho_l}{\rho_{bo}} = \frac{W_1 - W_3}{mg} = \frac{W_1 - W_3}{W_1}$$

$$\frac{\rho_{bo}}{\rho_l} = \frac{W_1}{W_1 - W_3}$$ ---------- ---- (8.2.10)

Condition for Flotation



An object will float if the buoyancy force exerted on it by the fluid balances its weight. This is often called the **"principle of flotation"** where a floating object displaces a weight of fluid equal to its own weight.

For a floating object, the weight of the displaced fluid is equal to the weight of the object

Weight of fluid displaced = Weight of the floating body

Buoyant force = Weight of liquid displaced

= Weight of the floating body

Consider a body of mass "m" and density of ρ_{bo} floating in a liquid of density ρ_l. From principle of floatation we have the following.

Weight of the body = Weight of liquid displaced

= Buoyant force

Mass of the body **x** gravity = mass of liquid displaced x gravity

From $m = \rho V$ and cancel gravity from both sides, then we have

Density of the body x volume of the body = Density of fluid displaced x volume of fluid displaced

$$\frac{\text{Density of a body}}{\text{Density of iquid}} = \frac{\text{Volume of fluid displaced}}{\text{Volume of a body}}$$

$$\rho_{bo}/\rho_l = V_{ld}/V_{bo} \quad \text{---------- (8.2.13)}$$

This equation is used to determine the fraction submerged or the part of the floating body under the fluid. The fraction submerged is the ratio of the volume submerged to the volume of the object, or the ratio of density of the object to the density of the fluid.

Worked Example 8.2.2

1. A floating wooden block has a volume of 600000cm^3 and displaces 240000cm^3 of water. Determine the density of the block. The density of water is 1000kg/m^3

Given: V_{ld}=240000cm^3, V_{bo}=600000cm^3 Required: ρ_{bo}=?

Solution:

$\rho_{bo}/\rho_w = V_{ld}/V_{bo}$

$\rho_{bo} = (V_{ld}/V_{bo})(\rho_w)$

$= (240000\ cm^3/600000\ cm^3) \times 1000kg/m^3$

$= 400kg/m^3$

2. A block of wood floats in fresh water with two-fifth of its volume (V_b) submerged and in oil with $0.75V_b$ submerged. Find the density of (a) the wood (b) the oil. (ρ_w=1000kg/m^3).

Given

$V_{wd} = \frac{2}{5}V_b$ and $V_{oid} = \frac{3}{4}V_b$

Solution

(a) $\rho_b/\rho_w = V_{wd}/V_b$

$\rho_b = (V_{wd}/V_b)(\rho_w)$

$= (\frac{2}{5}V_b/V_b)(1000kg/m^3)$

$= 400kg/m^3$

(b) $\rho_{oi}/\rho_w = V_{oid}/V_b$

$\rho_{oi} = (V_{oid}/V_b)(\rho_w)$

$= (\frac{3}{4}V_b/V_b)(1000kg/m^3)$

$=750kg/m^3$

Surface Tension and Surface Energy

All matter is made of molecules and all molecules have varying degrees of attraction to other molecules. This intermolecular attraction is why the molecules of your drink stay together in a glass or as a puddle on the table top rather than drift apart into the air. Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called **surface tension**. The surface tension of liquid results from an imbalance of intermolecular attractive forces or the cohesive forces between molecules.

Molecules inside a liquid are surrounded equally on all sides by other molecules. All of these experience cohesive forces with other molecules in all directions as shown in Figure 8.2.4.

Here, the attractive force on the molecule in one direction is counter balanced by the attractive force in another direction. They experience zero net force. But, molecules at the surface have molecules only below it and at the side but none above it. The downward pull does not balanced by the upward pull. Therefore, a molecule at the surface of the liquid experiences only net inward cohesive forces to reduce the surface area. Molecules at the surface are pulled into the liquid and the surface tends to become as small as possible and it seems to be an elastic membrane or a **thin elastic skin** on the surface.

(a) Attractive forces between molecules of the same type are called cohesive forces.

(b) Attractive forces between molecules of different type are called cohesive forces.

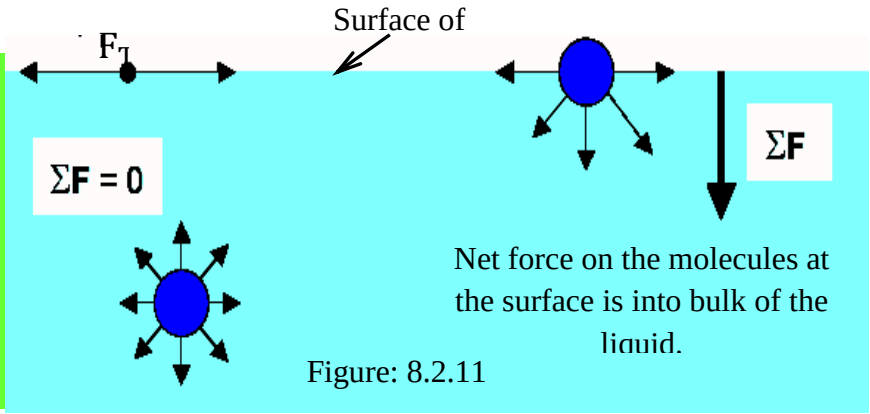
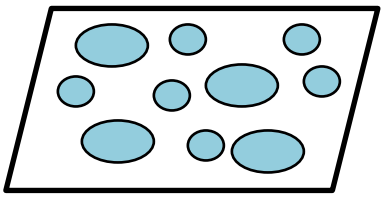


Figure: 8.2.11

The surface of a liquid behaves as though it is covered by a stretched membrane. The strength of the membrane depends on the liquid. It is, for example, much higher for pure water than for soapy water. Cohesive forces that act between the molecules of the liquid without chemical bonding give rise to the surface tension. Molecules in liquids experience strong intermolecular attractive forces. The attractive forces that act between like molecules are referred to as **cohesive forces**. The attractive forces that act between unlike molecules are referred to be **adhesive forces**.
The molecules at the surface do not have other neighbors of the same kind on all sides. They experience a net attractive force toward (inside) the bulk of the liquid. This force results in a surface "**film**" or "a **thin elastic skin** "which makes it more difficult to move an object through the surface than to move it, when it is completely submersed.
Depending on effects of surface tension,

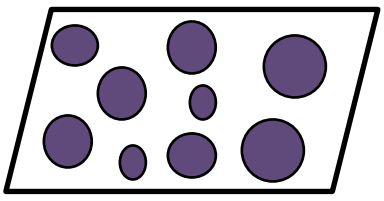
- a razor blade or a needle will float on the surface of water.
- drops of liquids try to take up a spherical shape, never spreads out completely on a flat surface.

The smaller the droplets of liquid, the more nearly spherical it is.



(a) Droplets of water on a glass plate.

Figure 8.2.12



(b) Droplets of mercury on a glass plate.

The mercury droplets tend to be more spherical than the water droplets since the surface tension of mercury is larger than that of water.

Consider a metal needle or paper clip which has higher density than that of water. So when it is dropped into water, it falls down into the bottom of water. However, if we put it gently horizontally on the surface of the water, then **it can float due to the surface tension of water or since it is supported by elastic skin on the water surface**. This is effect of surface tension.

Surface tension = Cohesive force/ length

$$T = F/L \quad \text{---------- (8.2.14)}$$

L= length of the needle

The surface tension T is defined as the force along a line of unit length. The force is parallel to the surface and perpendicular to the line of unit length. Surface tension is therefore measured in units of N/m in SI units.

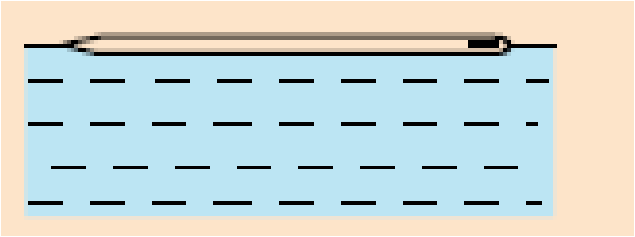


Figure 8.2.13: Steel needle floats on the surface of water

Surface energy is the equivalent attractive force present between the molecules at the surface of a solid substance. This attractive force is what holds a substance together into a cohesive form. We usually use surface tension when referring to liquids and surface energy when referring to solids, but in reality they are the same thing.

The main difference between surface energy and surface tension is that surface energy measures **how much work** is required per unit area to stretch, while surface tension measures the force per unit length**.** Surface tension and surface energy are important because they are measurable quantities that tell us how strong the attraction between molecules is. High surface energy (or tension) substances have high boiling points: they have to be heated up a lot in order to overcome the intermolecular attractions to form vapor or steam.

Class Activity 8.1: To demonstrate the action of surface tension on a thread using soap film.

Make a ring of thin thread. Tie the thread loosely across it(Fig.8.2.9(a). Dip the ring into soap solution or liquid detergent, so that a film forms across it (Fig.8.2.9 (b). Break the film on one side of the thread by touching it. What happens to the tread?

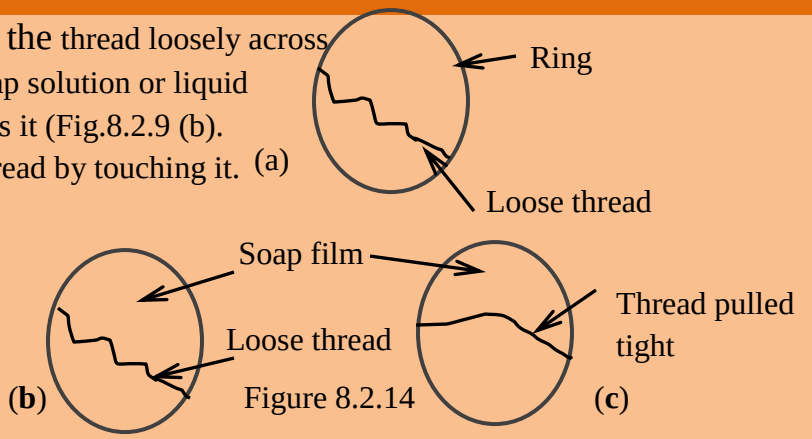


Figure 8.2.14

Note: While cutting the film you have to be careful. Use hot pin to do it. The thread is tightly drawn towards the side of the frame in which the film remains (Fig.8.2.9(c). What force is pulling the thread to the ring or to the right?

Some factors that affect surface tension

1. Temperature- the surface tension of a liquid decreases with increase in temperature. This is due to the fact that decreases the cohesive forces between the molecules as increased temperature.
2. **Soluble impurities/ Concentration:** The presence of soluble impurities may increase or decrease the surface tension. In case of solutions, the surface tension depends on the degree of concentration of the solution or solute.

If the solute is very soluble, then the surface tension increases, as for example dissolving salt in water. If the solute is less soluble, then the surface tension decreases, for example the addition of soap or phenol decreases surface tension of water.

(3) **Surroundings:** The surface tension of a liquid depends on the nature of the media in contact with its surface.

(4) **Contamination-** Contamination due to oil, grease, dust etc., reduces the surface tension of water considerably.

The decrease in surface tension can lead to larger contact angle and even make the Nano bubbles survivable on the highly hydrophilic surface.

Worked example 8.2.3

1. A needle 5 cm long can just rest on the surface of the water without wetting. What is its weight? Surface tension of water = 0.07 N/m.

Given: Length of needle = l = 5 cm = 5×10^{-2} m, Surface tension = T = 0.07 N/m

Solution

In case of a needle, the water wets it from two sides, Hence, the effective length = 2 L

T = F/2L = Weight of Needle/ L

Weight = 2TL =2x 0.07N/m x 5×10^{-2} m = $7x10^{-3}$ N

2. A light square wireframe each side of which is 10 cm long hangs vertically in the water with one side just touching the water surface. Find the additional force necessary to pull the frame clear of the water (T=0.074 N/m)

Given:

Length of frame in contact with water = L = 10 cm = 10×10^{-2} m, Surface tension = T = 0.074 N/m

Solution

In case of wire, the water wets it from two sides, hence, the effective length = 2 L

T= F/2L

F= Tx2L = 0.074N/mx2x10× 10^{-2} m = 0.0148N

The pull required is 0.0148N

Excess pressure inside a Liquid drop and Air bubble

Excess pressure inside a Liquid drop and Air bubble

When there is a curved surface of a liquid, every molecule on the surface will experience a force that is along the tangent to that surface. When these force vectors are added, the horizontal components cancel out each other and the vertical components add up. So, the resultant force is normal to the surface. The pressure on the concave side of the curve will be greater than the pressure on the convex side because the pressure on the concave side must balance out the pressure on the convex side and the net inward force generated due to surface tension as well.

Derivation

To derive an expression for excess pressure inside a liquid drop consider a spherical drop of liquid in shape as shown in the figure below. Let P be pressure inside the drop and P_0 be the outside pressure. The inside pressure P will be greater than the outside pressure P_0. So that, the excess pressure inside the liquid drop will be $P - P_0$.
Due to that pressure, let us assume the radius of the drop increases from r to r +Δr. Note that

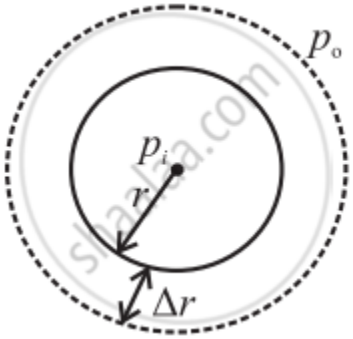


Figure 8.2.15

Δr <<< r so, the pressure inside the drop almost remains the same. Initial surface area of the drop $A_1 = 4\pi r^2$. Final surface area of the drop $A^2 = 4\pi(r+\Delta r)^2$.
Expanding, $A^2 = 4\pi(r^2+2r\Delta r + \Delta r^2)$. We can neglect Δr^2 as it is very small as compared to the rest.
So that, $A^2 = 4\pi r^2+8\pi r\Delta r$
Now, $\Delta A=A_2-A_1=8\pi r\Delta r$
Work done in increasing the surface area is the energy that will be converted to surface energy.
So, $\Delta W=T\Delta A=T(8\pi r\Delta r)$
$\Delta W=T\Delta A=T(8\pi r\Delta r)$ ---------- (8.2.15)
We know that work done also equals to the product of force F which caused an increase in surface area with the displacement Δr of the surface.
So, $\Delta W = F\Delta r$ ---------- (8.2.16)

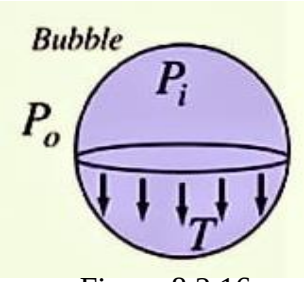
But, $F = (P'-P_0)4\pi r^2$ ---------- (8.2.17)

Equating (1) and (2),
$T(8\pi r\Delta r) = F\Delta r$
$T(8\pi r\Delta r) = (P'-P_0)4\pi r^2\Delta r$
$P = P_i - P_0 = 2T/r$
$P = 2T/r$ ---------- (8.2.18)

This is an expression to find the excess pressure inside a liquid drop and air bubble.

Excess pressure inside a soap bubble

Unlike a liquid drop, a soap bubble has two liquid surfaces in contact with air, one inside the bubble and the other outside the bubble. In balance, the air pressure surrounded by a soap bubble is enhanced than the atmospheric pressure. Consider a spherical soap bubble of radius R and surface tension T.



$$P_i - P_o = \frac{4T}{r} \quad \text{for a bubble}$$

Figure 8.2.16

This excess pressure is formed due to the surface tension of the soap solution. As there are two free surfaces of the soap (hemisphere) bubble surfaces, the excess pressure will also be doubled as the surface area is doubled. Therefore, there is more pressure inside than outside.
The net upward force on the top hemisphere of the bubble is just the pressure difference times the area of the equatorial circle:

$$F_{upward} = (P_i - P_o)\pi r^2 \quad \text{---------- (8.2.19)}$$

The force of the surface tension downward on the entire circumference of the circle is twice the surface tension times the circumference, since two surfaces contribute to the force:

$$F_{downward} = 2T(2\pi r) \quad \text{---------- (8.2.20)}$$

Force due to excess pressure (PA) = force produced due to surface tension.
Therefore the force due to surface tension = $2(2\pi r)T$
At equilibrium, force due to excess pressure (PA) = $(P_i - P_0)\,\pi r^2$
So, excess pressure inside the soap bubble is:

$P = (P_i - P_0)\,\pi r^2 = 2(2\pi r)T$

$P = P_i - P_0 = 2(2\pi r)T/\,\pi r^2$

$P = P_i - P_0 = 4T/r$

$P = 4T/r$ ---------- (8.2.21)

If you have noticed at home, it is difficult to blow small bubbles of soap or it is easier to inflate a balloon when it is already filled. This is because excess pressure is inversely proportional to the radius of the bubble. So, the lesser the radius, the more will be the surface pressure.
The surface tension T for different surfaces is listed in the table below.

Liquid	Surface Tension T (N/m)
water (20 ^{o}C)	0.073
water (100 ^{o}C)	0.059
soapy water (20 ^{o}C)	0.025
alcohol	0.022
glycerine	0.063
mercury	0.513

Worked example8.2.4

1. A rectangular film of liquid is formed in a frame of wire and a movable rod of length 4 cm. Let 2 .4×10^{-4} *J* of work is done to increase the area of a film of soap bubble from 50 cm^2 to 100 cm^2. What force must be applied to the rod to keep it in equilibrium if the surface tension of the liquid is
40 × 10^{-3} N/m?

Given:
Length of the rod = L = 4 × 10^{-2} m, Surface tension = T = 40 × 10^{-3} N/m, work done W = 2 .4×10^{-4} *J*

Solution:
In case of rod, the liquid wets it from two sides, first calculate the surface tension of soap solution.
A soap bubble has two free surfaces, therefore increase in surface area twice:
$\Delta A = A_2 - A_1$ = 2(100-50) × $10^{-4} m^2$ = 100 × $10^{-4} m^2$
Since, work done $W = T \times \Delta A \Rightarrow$
T =W/ ΔA = 2 .4×10^{-4} *J* /100x$10^{-4} m^2$
T =2.4x10^{-2} N/m
The force applied to the rod is F=2TL
F = 2x2.4x10^{-2} N/mx4 × 10^{-2} m = 1.92x10^{-5} N

2. If excess pressure is balanced by a column of oil (with specific gravity 0.8) 4 *mm* high, where r = 2.0 *cm*, find the surface tension of the soap bubble.
Given: ρ = 0.8x1000kg/m^3 = 800kg/m^3, h = 0.004m, r = 0.02m. T=?

Solution
The excess of pressure inside the soap bubble is
$\Delta P = P_i - P_o = \frac{4T}{r}$
But, $\Delta P = \rho gh$
$\frac{4T}{r} = \rho gh$
$T = \rho ghr/4$
= (800kg/m^3)(9.8m/s^2)(0.004m)(0.02m) = 0.1568N/m

3. Water rises in a capillary tube to a height of 2.0cm. How much will the water rise through another capillary tube whose radius is one-third of the first tube?

Solution
Consider two capillary tubes with radius r_1 and r_2 which are dipped in a liquid, and resulting capillary rises of height h_1 and h_2 respectively and $r_2 = \frac{1}{3} r_1$

From equation $h = \frac{4T}{r\rho g}$, we have

$h \propto 1/r \Rightarrow hr$ = constant, then

$h_2 r_2 = h_1 r_1$

$h_2 = h_1 r_1 / r_2 = 3h_1 = 3\text{x}2\text{cm} = 6\text{cm}$

4. Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 2 mm, made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside?. Surface tension of mercury T=0.456 Nm-1; Density of mercury ρ = 13.6 × 103 $kg\ m$-3

Solution

Capillarity depressed when the capillary immersed into mercury.

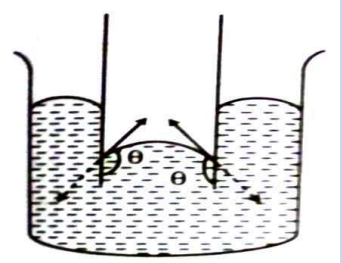


Figure 8.2.17

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2\times(0.465\,N\,m^{-1})(\cos 140^0)}{(2\times10^{-3}\,m)(13.6\times10^3)(9.8\,m\,s^{-2})}$$

$$\Rightarrow h = -6.89\times10^{-4}\,m$$

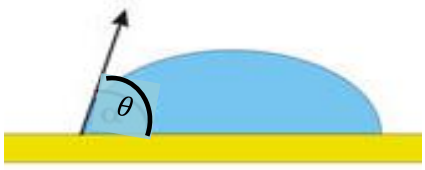
Where, negative sign here indicates that there is a fall of mercury (mercury is depressed) in glass tube.jh

Angle of contact and capillary action

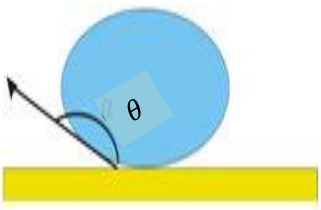
No liquid can exist in a perfect vacuum for very long, so the surface of any liquid is an interface between that liquid and some other medium. For example, the top surface of a puddle is an interface between the water in the puddle and the air. This means that surface tension is not just a property of the liquid, but a property of the liquid's interface with another medium. If a liquid is in a container, then besides the liquid/air interface at its top surface, there is also an interface between the liquid and the walls of the container (see Figure 8.16).

Usually, the surface tension between the liquid and air is greater than its surface tension with the walls of a container. Where the two surfaces meet, their geometry must be such that all forces balance. Where the two surfaces meet, they form a **contact angle**. **Angle** of contact is the angle which is tangent to a liquid surface at the point of contact makes with the solid surface inside the liquid.


(a) When $\theta = 0^0$
Complete wetting



(a) Acute Angle ($0<\theta<90^0$)
Good wetting



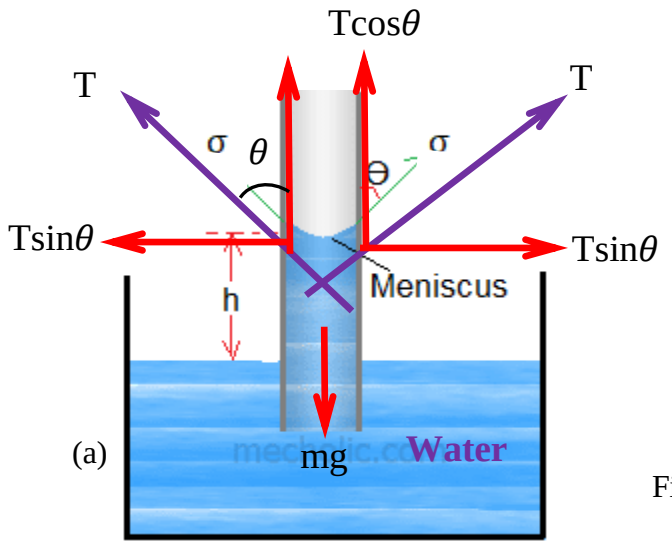
(b) Obtuse Angle
No wetting ($90^0 < \theta < 180^0$)

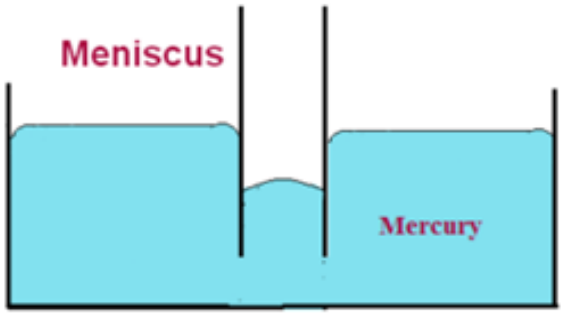
Figure 8.2.18: Droplets of liquids

Wetting is the ability of liquids to form interfaces with solid surfaces. The **contact angle** θ between the liquid and the solid surface is a measure of the degree of wetting. A smaller the contact angle and a smaller the surface tension result in a greater the degree of wetting. Depending on the nature of the droplet of liquids or liquid in the capillary tube, the angle of contact also will vary.
When adhesive forces are greater than cohesive forces, the contact angle θ lies between 0 and 90^o. When adhesive forces are smaller than cohesive forces, the contact angle θ lies between 90^o and 180^o. If no forces are dominating then the angle of contact is equal to 90° itself. This can be experienced with the pure silver and the pure glass.

Explaining capillary action in Capillary (thin) tube

Capillary tube is a fine tube whose length is very large when compared with its radius. Both of its ends always an open state and we can insert this in a liquid.





(b)

Figure 8.2.19

When a capillary tube is immersed in a liquid and depending on the angle of contact the liquid can either rises or falls inside the tube. This phenomenon is called **capillarity** or **capillary action**. The capillarity depends on the nature of the liquid in the capillary tube.
Capillary action occurs when the adhesion to the walls is stronger than the cohesive forces between the liquid molecules.
If the adhesive forces between the liquid and the vessels wall are dominating then there will be a capillary rise and the meniscus is **concave** (Figure 8.2.17(a)).
If the cohesive forces are dominating between the liquid and the vessel then there will be a capillary fall and the meniscus is **convex** (Figure 8.2.17(b)).
The surface tension keeps the surface intact, so the whole liquid surface is dragged upward.
Therefore, the relative rise or fall of a liquid in a narrow glass tube depends on whether the liquid is attracted or repelled by the glass. Water has a positive capillary rise and is drawn up a narrow glass tube. Mercury has a negative capillary rise and sinks down a narrow glass tube, relative to the

outside level. The horizontal component Tsinθ being active on the two ends of the tube in opposite direction, so they cancel out each other. So, due to vertical component Tcosθ, water rises through the capillary tube.

At equilibrium, the formula for capillary rise can be derived by balancing forces on the liquid column. The weight of the liquid ($\pi r^2 h\rho g$) is balanced by the upward force due to surface tension times with the circumference of the curve ($2\pi r T\cos\theta$).

Weight of the liquid = upward force due to surface tension

$$\pi r^2 h\rho g = 2\pi r T\cos\theta$$

$$h = 2\pi r T\cos\theta / \pi r^2 h\rho g$$

$$h = 2T\cos\theta / r\rho g \text{ ---------- (8.2.22)}$$

($2\pi r$ = the circumference of the meniscus)

This formula can also be derived using pressure balance.

(Pressure difference b/n inside and outside meniscus)(area of the capillary tube) = upward force due to surface tension

$$(P_i - P_o)A = 2\pi r T\cos\theta$$

$$(\rho gh)\,\pi r^2 = 2\pi r T\cos\theta$$

The maximum height to which the liquid will rise through capillary action is given by:

$$h = 2\pi r T\cos\theta / \rho gh\pi r^2$$

$$h = 2T\cos\theta / r\rho g$$

Where: T is the liquid–air surface tension, θ is the contact angle, ρ is the density of liquid g is acceleration due to gravity and r is radius of tube (length).

Independent of the shape of capillary

At equilibrium, the height (h) of column is independent of the shape of the capillary tube. If the radius of meniscus remains the same, then vertical height (h) of a liquid column in the capillaries of different size and shapes also remains same. The contact angle for various liquid/solid interfaces has been measured and Table 8.1 shows these results.

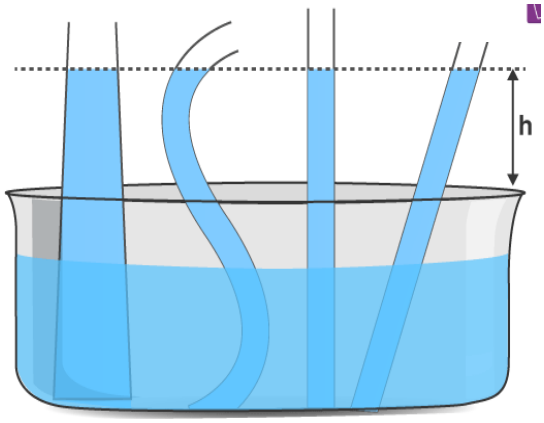


Figure 8.2.20: Capillary with respect to radius of meniscus

Table 8.1 Contact angles for different liquid/solid interfaces.

Liquid	Solid	Contact Angle
Water	**Soda-lime glass** **Lead glass** **Fused quartz**	**0**
Ethanol		
Diethyl ether		
Carbon Tetrachloride		
Glycerol		
Acetic Acid		
Water	**Paraffin wax**	**107^0**
	Silver, pure glass	**90^0**
Methyl Iodide	**Soda-lime glass**	**29^0**
	Lead glass	**30^0**
	Fused quartz	**33^0**
Mercury	**Soda-lime glass**	**140^0**

Applications of the capillarity

We can see the daily life applications of the capillarity it so many places.

1. Trees are able to pump the water even to the highest parts from the ground using the root system and this root system acts basing on the capillarity.
2. A kerosene lamp can pump kerosene through its wick basing on the capillarity and the same is the case with the paraffin candle.
3. The rise of ink in the nib of a fountain pen
4. Sucking of water by the towel from a bucket
5. Lubricating oil spread easily on all parts because of their low surface tension.
6. Cotton dresses are preferred in summer because cotton dresses have fine pores which act as capillaries for sweat.
7. Dirt gets removed when detergents are added while washing clothes because surface tension of water is reduced.
8. The absorption of ink by a blotting paper is due to capillary action, as the blotting paper is porous. When it is placed over the ink, the ink raises into the pores. Also rise of oil in the wick of a lamp is due to capillary action. etc.

Worked example 8.2.5

1. Water has a surface tension of 0.4N/m. If a 3mm diameter of vertical tube is dipped into the water, the water rises by 6mm above the liquid outside the tube. Calculate the contact angle.

From $h = \frac{4T\cos\theta}{\rho g d}$, then we have got: $\cos\theta = \frac{\rho g h d}{4T}$ and $\theta = 83^0$.

2. A soap bubble 50mm in diameter contains an excess of pressure of 2barr. Find the surface tension in the soap film.

ΔP= 2barr = $2x10^5 N/m^2$ and r = 0.025m

Solution

$\Delta P = \frac{4T}{r}$ and $T = \frac{\Delta Pr}{4} = 1250 N/m$

Summary

In this section you have learnt that:

• density defined as mass/volume.
• Pressure defined as Force per unit area
• Atmospheric pressure is the pressure excreted by air around us .
• Absolute pressure is the force applied perpendicular to a particular area.
• Hydraulic and pneumatic system rely on Pascal's principle:
• Liquid or air is used in a hydraulic or pneumatic system to transmit pressure from one place, such as brake pedal to another, such as brake drum.
• Arch mend's principle states that: Any object wholly or partly immersed into a fluid is buoyed up by a force equal to the weight of the fluid displaced by the object.
• Surface tension defined as a cohesive force effect at the surface of the liquid due to the force b/n the liquid molecules (T= Force/ Length).
• Surface energy measures the disruption to the forces and the molecules at the surface of the liquid, which seems to be tugged inward.
• The contact angle is the angle which is tangent to a line surface at the point of contact makes with the solid surface inside the liquid.
• The **contact angle** θ between the liquid and the solid surface is a measure of the degree of wetting.
•The height of column of liquids in a capillary tube is given by: $h = 2T\cos\theta/r\rho g$

Review questions

1. An object of density $7.8 g/cm^3$ appears 200 N lighter in water than in air.
(a) What is the volume of the object? (b) How much does it weigh in the air?
2. A wooden rectangular slab with surface area $5.7m^2$, volume V= $0.6m^3$ and density $600kg/m^3$ is placed slowly in freshwater. By what depth h is the slab submerged?
3. A spherical ball of density ρ = 0.70kgL has a radius of r=10cm. If the ball is placed on the surface of water and released, how much of the ball becomes submerged in the water?
4. How much of an iceberg is submerged below the water if the density of ice Is 917 kg/m^3 and the density of water is $1000kg/m^3$?
5. A barber raises his customer's chair by applying a force of 150N to a hydraulic piston of area 0.01 m^2. If the chair is attached to a piston of area 0.1 m^2, how massive a customer can the chair raise? Assume the chair itself has a mass of 5 kg.
6. A hydraulic system is used to lift a 2000-kg vehicle in an auto garage. If the vehicle sits on a piston of area 0.5 square meter, and a force is applied to a piston of area 0.03 square meters, what is the minimum force that must be applied to lift the vehicle?

7. The area of the piston in a force pump is $10m^2$. What force is required to raise water with the piston to a height of 100m?
8. The hydraulic lift has a large cross section and a small cross section. Large cross-sectional area is 20 times the small cross-sectional area. If on the small cross section is given an input force of 25 N, then determine the output force.
9. Two pistons of a hydraulic lift have diameters of 60 cm and 5 cm. What is the force exerted by the larger piston when 50 N is placed on the smaller piston?
10. Air is introduced through a nozzle into a tank of water to form a stream of bubbles. If the bubbles are intended to have a diameter of 2 mm, calculate how much the pressure of the air at the tip of the nozzle must exceed that of the surrounding water. Assume that the value of surface tension between air and water as 72.7×10^{-3} N/m. Density of the liquid is $1000kg/m^3$.
11. A capillary tube of a uniform bore is dipped vertically in a liquid which rises by 7 cm in the tube. Find the radius of the capillary if the surface tension is 70 dynes/cm g= 9.8 m/s^2. Assume $\theta = 0^0$
12. A liquid rises to a height of 4.5 cm in a glass capillary of radius 0.01 cm. What will be the height of the liquid column in a similar glass capillary of radius 0.02 cm?
13. When a capillary tube of radius 0.45 mm is dipped into water, the level inside the capillary tube rises to height 3 cm above the surface of the water. Calculate the surface tension of water, if its angle of contact is zero and its density is 1 g/cm^3, g = 9.8 m/s^2.
14 The height of a water column in a capillary tube is 9cm. If the diameter of the second capillary tube is three-fifth of that of the first, what is the height of the water column?
15. Two capillary tubes A and B of radii 0.2cm and 0.4cm respectively are dipped in the same liquid. What is the ratio of the height through which liquid rises in the tubes A and B?
16. A liquid of density 900 kg/m^3 rises to a height of 7 mm in a capillary tube of 2 mm internal diameter. If the angle of contact of the liquid in the glass is 25°, find the surface tension of the liquid.

8.2.2 Fluid dynamics

By the end of this section you should be able to:
- Define the terms laminar and turbulent flow, flow rate.
- Identify factors affecting and give examples of laminar flow.
- Identify factors that affect the streamlining of cars, boats and planes.
- Define the Reynolds number.
- State Bernoulli's principle.
- Explain applications of Bernoulli's principle.
- Use Bernoulli's equation to solve problems.
- State Stoke's law and use it to solve problems.
- Use equation of continuity to solve problems

Streamline and turbulent flow

Stream line flow

Stream line flow of a liquid is that flow in which each element of the liquid passing through a point travels along the same path and with the same velocity as the preceding element passes through that point as shown in Figure 8.21 a), the flow is known as streamline or laminar flow.



(a) Figure 8.2.1 (b)

a) Streamline flow (liquid in parallel layers) and b) turbulent flow (disruption between layers).

A streamline may be defined as the path, straight or curved, the tangent to which at any point gives the direction of the flow of liquid at that point. The two streamlines **cannot** cross each other and the greater is the crowding of streamlines at a place, the greater is the velocity of liquid particles at that place.

Laminar flow

If a liquid is flowing over a horizontal surface with a steady flow and moves in the form of layers of different velocities which do not mix with each other, then the flow of liquid is called laminar flow. In this flow the velocity of liquid flow is always less than the critical velocity of the liquid. The laminar flow is generally used synonymously with streamlined flow.

Turbulent flow

When a liquid moves with a velocity greater than its critical velocity, the motion of the particles of liquid becomes disordered or irregular as shown in Figure 8.21 b). Such a flow is called a turbulent

flow. In a turbulent flow, the path and the velocity of the particles of the liquid change continuously and haphazardly with time from point to point. In a turbulent flow, most of the external energy maintaining the flow is spent in producing eddies in the liquid and only a small fraction of energy is available for forward flow. For example, eddies are seen by the sides of the pillars of a river bridge.

Because the motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In our model of ideal fluid flow, we make the following four assumptions:

1. The fluid is **non-viscous**. In a non-viscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.
2. The flow is **steady**. In steady (laminar) flow, the velocity of the fluid at each point remains constant.
3. The fluid is **incompressible**. The density of an incompressible fluid is constant.
4. The flow is **irrotational**. In irrotational flow, the fluid has no angular momentum about any point. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, then the flow is **irrotational**

Bernoulli's principle, which states that a fluid (such as air) travelling over the surface of an object exerts less pressure than if the fluid were still, applies during laminar flow. Aeroplanes fly because of Bernoulli's principle. When an aeroplane takes off, air rushes over the top surface of its wing, reducing pressure on the upper surface of the wing. Normal pressure below the wing pushes the wing upward, carrying the airplane upward along with it.

Turbulent flow is chaotic and unpredictable. It consists of irregular eddies (circular currents) of air that push on a surface in unexpected ways. You may have experienced turbulence on a commercial aeroplane flight − your bumpy ride could have resulted from the development of turbulent flow over the aeroplane's wings as shown in the figure 8.22.

The flow rate is the volume of liquid flowing past a given point per unit time.

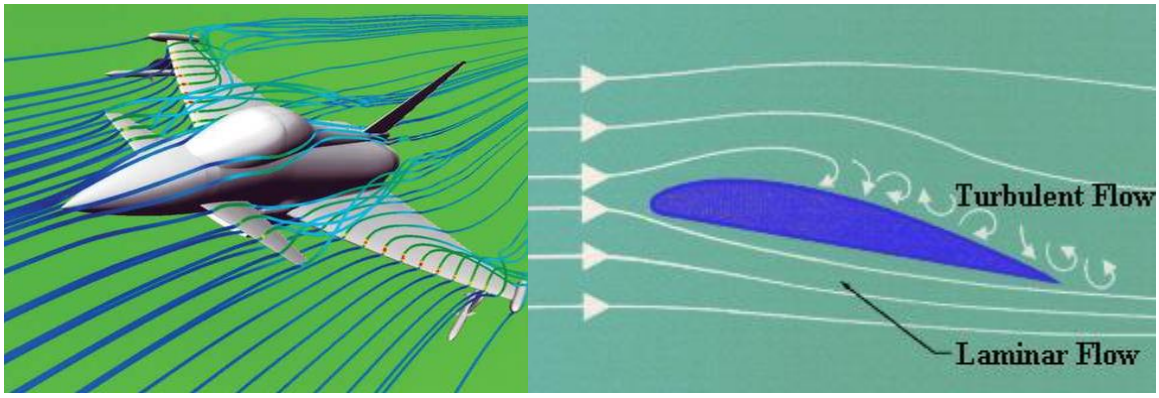


Figure 8.2.2 Turbulent flow over an aeroplane's wings

Factors affecting laminar flow

There are four properties of air that affect the way it flows past an object: density, compressibility (how much its volume can be reduced), temperature and viscosity.

The density and compressibility of air are important factors at high speeds. As an object travels rapidly through air, it causes air to become compressed and more dense. As a result, other properties of air then change.

The effects of temperature change on air flow also become important at high speeds. A regular commercial airplane, after landing, will feel cool to the touch.

Factors affecting streamlining of cars, boats and planes

Unless you keep pushing an object, as it moves through the air it will slow down because of air resistance acting to oppose the motion, as shown in Figure 8.23

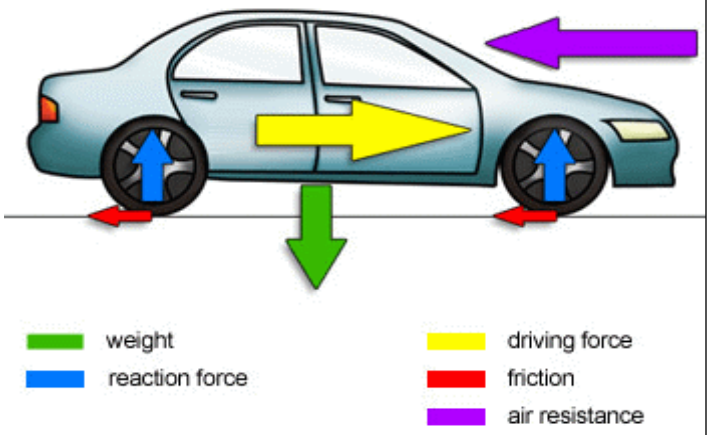


Figure 8.2.3 Air resistance opposes motion

Activity 8.2.1: Identifying streamlining features on motorsport vehicles

Compare the photographs of the ordinary car and Formula 1car here. What differences can you see?



Figure 8.2.4

Activity 8.2.2: Exploring streamlining

Obtain a long see-through tube which you can fill with a liquid such as water. In a small group, make various objects out of modelling clay, some of which should have features which make them more streamlined than others (e.g. lower surface area at front, etc.). Drop the objects into the top of the tube of liquid and time how long they take to reach the bottom. Do the ones that you thought you had designed to be more streamlined travel more quickly through the liquid?

KEY WORDS

Equation of continuity the mass flow rate of fluid flowing into a system is equal to the mass flow rate of fluid leaving the

In general, objects with larger surface areas will travel slower through the air (or any other fluid). There are certain shapes which help objects to speed up and travel faster through the air or other fluid than others – this is called streamlining. An application of streamlining is in motorsport, where the vehicles are designed so that their shape is streamlined to help their performance.

Total displacement = −50m + 40m = −10m

= 10m backward

Equation of continuity

The equation of continuity in fluid dynamics states that the volume flow rate of an incompressible ideal fluid flowing through a closed system is the same at every point.

In Figure 8.24, the system is closed and so V_{in}= V_{out}, where V_{in} is the volume flow rate of liquid in, Vout is the volume flow rate of liquid out.

$\mathbf{v_{in}} = \mathbf{v_1} = \mathbf{v_2} = \mathbf{v_{out}}$

Rate of flow = $\frac{\text{volume}}{time}$

Where v_1 is the volume flow rate of section 1,

v_2 is the volume flow rate of section 2.

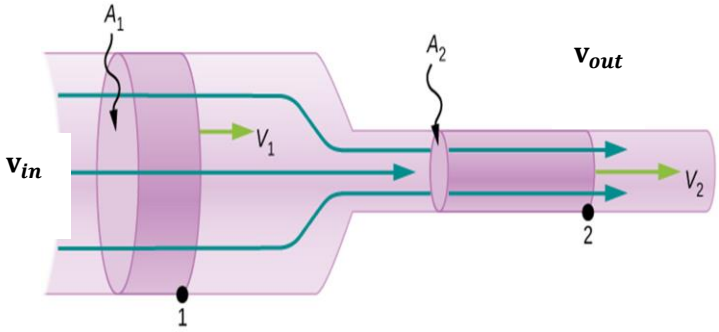


Figure 8.2.5

Volume flow in over $\mathbf{A_1}$=$\mathbf{A_1 v_1 \Delta t}$

Volume flow in over $\mathbf{A_2}$= $\mathbf{A_2 v_2 \Delta t}$

Rate of flow = $\frac{A_1 v_1 \Delta t}{\Delta t}$

Rate of flow = $\mathbf{A_1 v_1}$

= $\mathbf{A_2 v_2}$

= cross-sectional area x velocity of the fluid

Therefore, if the fluid is incompressible and we ignore the effect of internal friction, the rate of flow will remain constant. This means that a variation in the pipe cross section, as shown in Figure 8.24, will result in a change in speed of the fluid, but the product (vA) remains constant.

Mass in over A_1= $\rho A_1 v_1 \Delta t$ and Mass in over A_2 = $\rho A_2 v_2 \Delta t$, so that

$$A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

$\mathbf{A_1 v_1 = A_2 v_2}$ = $\mathbf{A_3 v_3}$ = **constant**

This is Equation of continuity

Generally, in streamline flow the product of the fluid velocity and the cross sectional area of the pipe is constant at any point.

Activity: 8.2.3

(a) Plot both the position-time and velocity-time graphs of the motion. Determine the displacement of the particle for the whole trip and compare your result with the previous answer.
(b) Write down the main difference between Average speed and Average velocity
(c) In uniform motion, describe the relationship between average velocity and instantaneous velocity in a given interval of time.

Worked example 8.15

An incompressible liquid flows through a horizontal tube as shown in the following figure 8.25. Then the velocity v of the fluid is

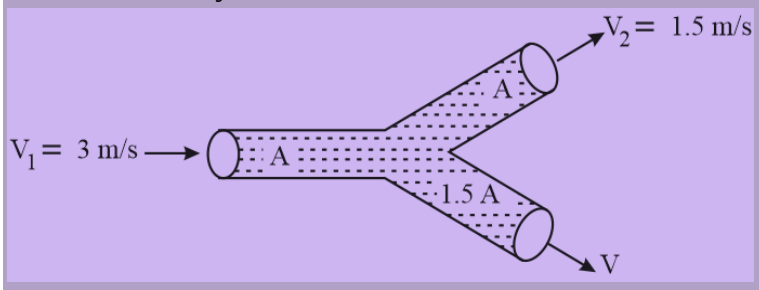


Figure 8.2.6

If the liquid is incompressible then mass of liquid entering through left end, should be equal to mass of liquid coming out from the right end

$M=\mathbf{m_1 + m_2}$

$\boldsymbol{Av_1 = Av_2 + 1.5Av}$

$\boldsymbol{A(3m/s) = A(1.5m/s) + 1.5Av}$

$\boldsymbol{v = \frac{3m/s - 1m/s}{1.5} = 1m/s}$

Worked example 8.15

Water flows at 10 m^3/h through a pipe connected to a tap. At what rate will the water leave the tap?

Use the equation of continuity.

The volume flow rate is the same at every point.

Volume flow rate = volume flow rate of tap

10 m3/h = 10 m3/h

Water will leave the tap at 10m^3/h

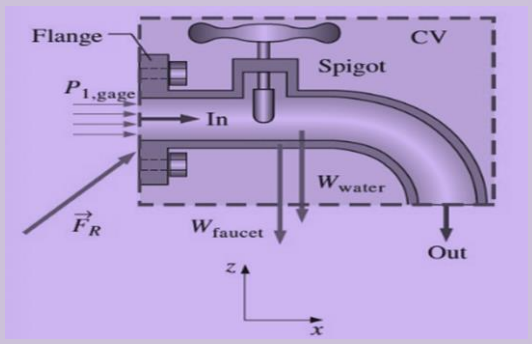


Figure 8.2.7

Worked example 8.16

Water flows at 10 m^3/h through a pipe with 100 mm inside diameter. The pipe is reduced to an inside dimension of 80 mm. Find the velocity of the water in each part of the pipe.

Using equation (2),

$\boldsymbol{q = v_{i1}A_{i1} + v_{i2}A_{i2} + \ldots + v_{in}A_{in}}$

$\boldsymbol{= v_{o1}A_{o1} + v_{o2}A_{i2} + \ldots + v_{on}A_{on}}$

the velocity in the 100 mm pipe can be calculated as

(10 m^3/h)(1/3600 h/s) = v_{100}(3.14 (0.1 m)^2/ 4)or

v_{100}= (10 m^3/h)(1/3600h/s)/(3.14 (0.1 m)^2/ 4) = 0.35 m/s
Using equation (2), the velocity in the 80 mm pipe can be calculated
(10 m^3/h)(1/3600 h/s) = v_{80}(3.14 (0.08 m)^2/ 4)or v_{80}= (10 m^3/h)(1/3600h/s)/(3.14 (0.08m)^2/4) = 0.55 m/s

Activity 8.2.4: Application of the equation of continuity

You can demonstrate the equation of continuity by taking a hose, watching the water flow out and then half covering the exit point with your thumb. What happens to the speed of the water coming out of the hose? Try to explain the observation using the equation of continuity before reading on

Common applications of the equation of continuity are pipes, tubes and ducts with flowing fluids or gases, rivers, overall processes as power plants, roads, computer networks and semiconductor technology.
Explaining the results of Activity 8.17
The rate of flow stays constant in a closed system. The decrease of the cross-section of the opening is balanced by the increased of the water. Note that you cannot apply the equation to the water once it has left the system as it is no longer in a closed system!

Activity 8.2.5:Demonstrating Bernoulli's principle 1

Take a ping-pong ball and a hairdryer. Use the flow of air from the hair-dryer to hold the ping-pong ball in the air.
How long can you support the ball with the air flow?
Try other light objects such as feathers and pieces of

Pressure and velocity

Consider a fluid moves through a narrow or constricted choke of a pipe as shown below. The fluid's speed increases when it flows through a constriction due to the presence of an accelerating force.
In order to accelerate the liquid as it enters the constriction, the pushing force from the large cross section must be greater than the resulting force from the constriction.
The increased velocity of a fluid flowing through a constriction indicates a drop in pressure.

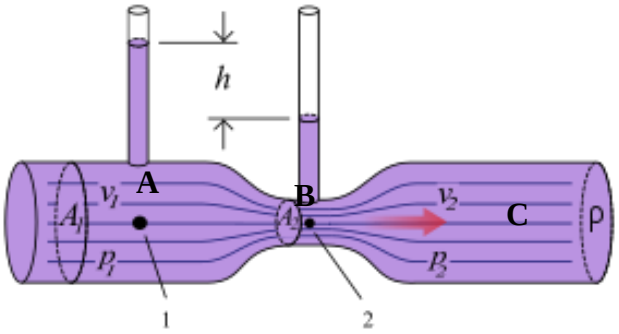


Figure 8.2.8: Venturi tube

In other words, the pressure at A and C in Figure 8.25, must be greater than the pressure at B.

The tubes inserted into the pipe above these points clearly indicate the difference in pressure. The fluid level in the tube above the restriction is lower than the level in the adjacent areas. If 'h' is the difference in height, the pressure differential is given by:

$$P_A - P_B = \rho gh$$

The above example illustrated in Figure 8.25, shows the principle of the ***Venturi meter.***

Venturi meter is a flow meter device or instrument that is used to measure the flow rate (discharge) in a pipe. The venturi meter is used in the water supply industry and it works on the basis of the Bernoulli theorem.

Bernoulli's equation

In most examples of liquid flow, we can consider that the fluid in the flow can be described as incompressible flow. Bernoulli performed his experiments on liquids and his equation in its original form is valid only for incompressible flow.

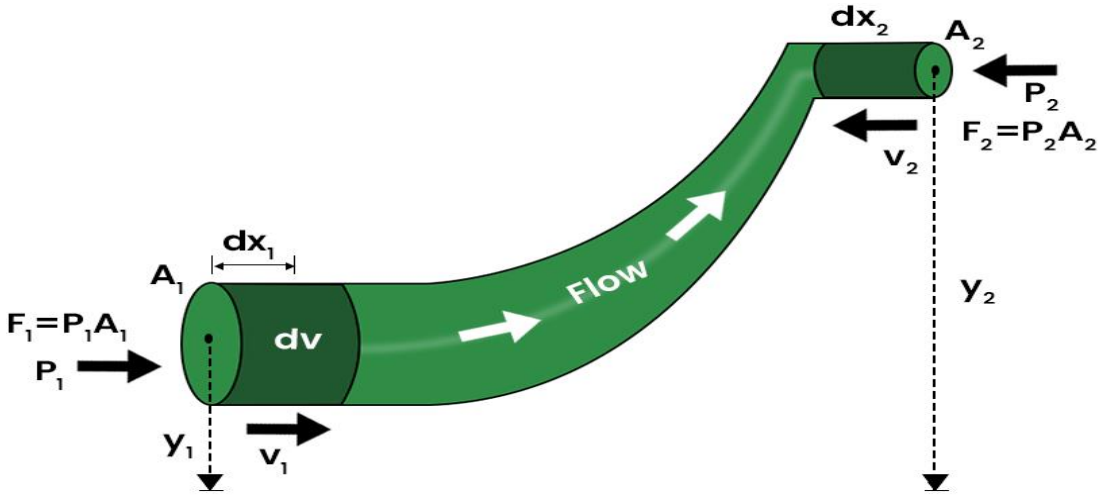


Figure 8.2.9 Bernoulli's principle

$W_{net} = \Delta KE + \Delta PE$ (the law of conservation of energy)

$F_1\Delta x_1 - F_2\Delta x_2 = \Delta KE + \Delta PE$

$P_1V_1 - P_2V_2 = \Delta KE + \Delta PE$

$m\frac{P_1}{\rho} + \frac{1}{2}mv^2{}_1 + mgh_1 = m\frac{P_2}{\rho} + \frac{1}{2}mv^2{}_2 + mgh_2$ = constant

Thus the above equation is the common form of Bernoulli's equation, which is valid at any arbitrary point along a streamline where gravity is constant, is given by:

$\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h$ = Pressure head+ kinetic head+ gravitational = constant

where: v is the fluid flow speed at a point on a streamline

g is the acceleration due to gravity

h is the elevation of the point above a reference plane, with the positive z-direction pointing upward — so in the direction opposite to the gravitational acceleration

p is the pressure at the point
ρ is the density of the fluid at all points in the fluid.

Worked example 8.2.6

Water is flowing through a horizontal pipe of varying cross section. At a certain point where the velocity is 0.12 m/s, the pressure of water is 1333Pa. What is the pressure at a point where the velocity is 0.24 m/s?

According to the Bernoulli's principle,

$m\frac{P_1}{\rho} + \frac{1}{2}mv^2{}_1 + mgh_1 = m\frac{P_2}{\rho} + \frac{1}{2}mv^2{}_2 + mgh_2 =$ constant

$P_2 = P_1 + \frac{1}{2}\rho_{water}(v_1{}^2 - v_2{}^2)$

$P_2 = (1{,}333\text{Pa}) + \frac{1}{2}\left(1000\ {}^{kg}/_{m^3}\right)((0.12m/s)^2 - (0.24m/s)^2)$

$= (1{,}333\text{Pa}) + 500(0.0144 - 0.0576)\text{pa}$

$= (1{,}333\text{Pa} - 21.6)\text{pa} = 1{,}311.4\text{pa}$

In many applications of Bernoulli's equation, we can use the following simplified form:

$p + q = p_0$ where p_0 is called total pressure, and q is dynamic pressure. The pressure p is often referred to as static pressure to distinguish it from total pressure p_0 and dynamic pressure q.

The simplified form of Bernoulli's equation can be summarized as:

static pressure + dynamic pressure = total pressure

Every point in a steadily flowing fluid, regardless of the fluid speed at that point, has its own unique static pressure p and dynamic pressure q. Their sum p+ q is defined to be the total pressure p0.

The significance of Bernoulli's principle can now be summarized as: Total pressure is constant along a streamline.

Worked Example: 8.2.7

Use the simplified version of Bernoulli's equation to find the total pressure when the static pressure is 5 Pa and the dynamic pressure is 7 Pa.

static pressure + dynamic pressure = total pressure

5Pa + 7 Pa = 12 Pa

Activity 8.2.6: Demonstrating Bernoulli's principle 2

Suspend two light bulbs from a table rod in the field of an overhead projector turned on its side so that you can see the magnified image on the ceiling. Blow sharply between the bulbs so that they clank together.

As an alternative, you could blow sharply between two pieces of paper or ping-pong balls.

Further applications of Bernoulli's principle

We have already learnt that Bernoulli's principle can be used to calculate the lift force on an aeroplane if you know the behaviour of the fluid flow in the vicinity of the wing in figure 5.28.

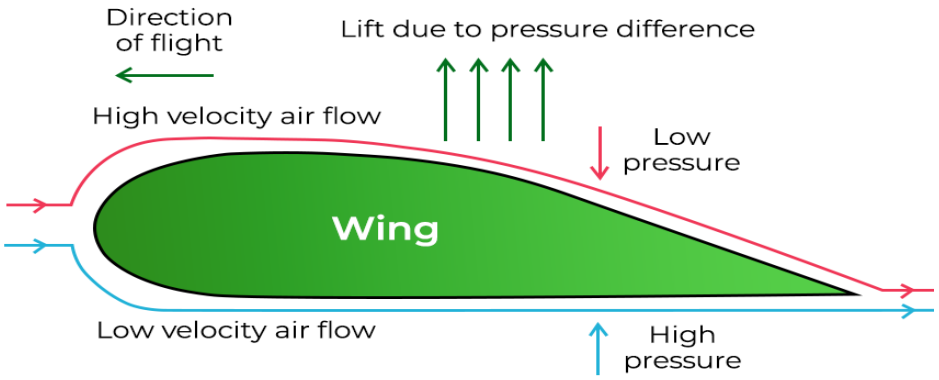


Figure 8.2.10 Lift of an air craft

Other applications of the principle include:

- The carburettor in engines contains a device to create a region of low pressure to draw fuel into the carburettor and mix it thoroughly with the incoming air. The low pressure in the device can be explained by Bernoulli's principle; in the narrow throat, the air is moving at its fastest speed and therefore it is at its lowest pressure.
- The pitot tube and static port on an aircraft are used to determine the airspeed of the aircraft. These two devices are connected to the airspeed indicator which determines the dynamic pressure of the airflow past the aircraft. Dynamic pressure is the difference between stagnation pressure and static pressure. Bernoulli's principle is used to calibrate the airspeed

indicator so that it displays the indicated airspeed appropriate to the dynamic pressure.

- The flow speed of a fluid can be measured using a device placed into a pipeline to reduce the diameter of the flow. For a horizontal device, the continuity equation shows that for an incompressible fluid, the reduction in diameter will cause an increase in the fluid flow speed. Subsequently Bernoulli's principle then shows that there must be a decrease in the pressure in the reduced diameter region.

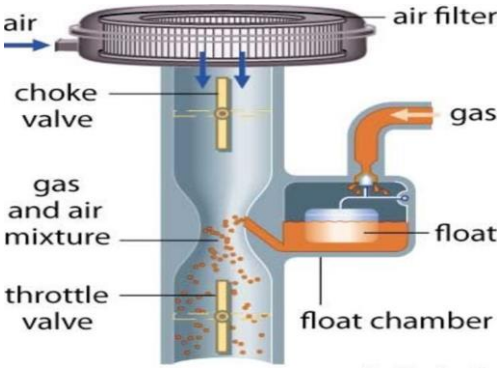


Figure 8.2.11. In the narrow throat, the air is moving at its fastest speed and is therefore at its lowest pressure



Figure 8.2.12 The wind passing in front of the sail is fast enough to experience a significant reduction in pressure and the sail is pulled forward

The maximum possible drain rate for a tank with a hole or tap at the base can be calculated directly from Bernoulli's equation, and is found to be proportional to the square root of the height of the fluid in the tank.

The principle also makes it possible for sail-powered craft to travel faster than the wind that propels them (if friction can be sufficiently reduced). If the wind passing in front of the sail is fast enough to experience a significant reduction in pressure, the sail is pulled forward, in addition to being pushed from behind.

Although boats in water must contend with the friction of the water along the hull, ice sailing and land sailing vehicles can travel faster than the wind.

Viscosity

The term **viscosity** is commonly used in the description of fluid flow to characterize the degree of internal friction in the fluid. This internal friction, or viscous force, is associated with the resistance that two adjacent layers of fluid have to moving relative to each other. Viscosity causes part of the kinetic energy of a fluid to be converted to internal energy. This mechanism is similar to the one by which an object sliding on a rough horizontal surface loses kinetic energy.

If water flows through a pipe, smooth streamline motion can ensue. The layer of water touching the walls is at rest, the water in the centre is moving fastest. The layers of water are travelling at an increasing speed as we go from the walls to the centre, and this means that each layer is being dragged forward by the faster layer one side but held back by the slower one on the other side. The overall outcome turns out to be a resultant drag force (Figure 8.31)

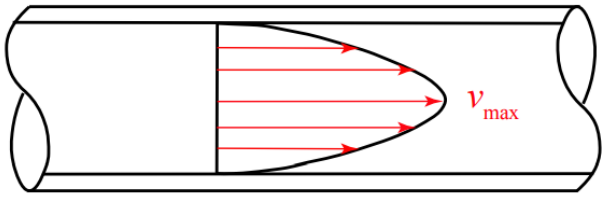


Figure 8.2.13 Velocity of layers of liquid in a Pipe

The force required to drive the layers

$F = \eta A\frac{\Delta v}{\Delta y}$

Where v = speed of flow

y = distance from container wall

A = cross-sectional area

η = viscosity

Some liquids flow move readily through pipes, or move out of the way more freely to allow a body to move through them. These are the liquids that have a low viscosity– to put it simply, they are very runny.

Activity 8.2.7: Comparing viscosity of various liquids

In a small group, devise an investigation to find out which of a selection of liquids is the most viscous.

You could compare water and cooking oil, for example.

Think carefully about your method and check it with your teacher before you begin. How will you record your observations so that you can draw conclusions?

The coefficient of viscosity

Thick oils and the like are quite the opposite. They are very viscous, and large drag forces appear when layers have to slide over one another. The viscosity of a liquid may be measured by a quantity called its coefficient of viscosity, given the Greek letter η ('eta').

Its units are kg/m/s, and a large number indicates a viscous liquid. It is not essential that you remember those units − so long as you put all the values into an equation in their basic SI units it is bound to work out. As a liquid warms up, its viscosity falls considerably.

The viscous force between layers is determined in part by how rapidly the velocity is changing as you go from layer to layer. This is specified by what is known as the velocity gradient. It is the change in velocity (in m/s) per meter as you move in from the wall of the pipe. It is measured in m/s per meter, and m/s/m reduces to just /s.

The second factor is just the area of contact A between the two layers. In a given liquid at a given temperature, the viscous drag force Facting between the layers is given by F= (a constant) × A× the velocity gradient.

The constant is defined to be the viscosity η of the liquid.

Poiseuille's Law

Poiseuille's law states that the rate of flow of (Q) of any liquid through a narrow tube is given by:

Volume flow rate= $\frac{\pi \Delta P r^4}{8\eta l} = \frac{\pi \rho r^2}{8\eta l}$

Where Δp is the difference in pressure between the ends of a pipe; R is the radius of the pipe, η is the coefficient of viscosity and L is the length of the pipe

Stokes's law and terminal velocity

There are three factors that affect the magnitude of the drag force F:

1. There is the size of the body. A reasonable guess might be that what matters is the area which the shape presents to the fluid, but this turns out not to be the case. The force is proportional to the radius r.
2. The force is also proportional to the velocity $\boldsymbol{v}$ at which the sphere moves through the air: double that velocity and the drag force doubles.
3. The third factor is the viscosity of the fluid, which is η.

Put them all together and we get

F= (some constant) × η× r× v.

A full theoretical treatment tells us that the constant is 6π, and the result is Stokes's law − the viscous drag force F that acts on a sphere of radius r travelling at a velocity v through a fluid of viscosity η is given by:

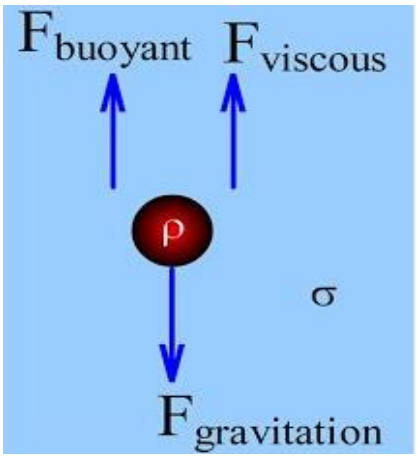
$$F = 6\pi\eta r v$$

If a ball is dropped through a fluid, the force of gravity causes it to go faster and faster. As its velocity increases, the drag force becomes bigger too. Eventually it is going so fast that the drag

force is as large as its weight and therefore cancels it out. The ball no longer accelerates. Instead it goes at a constant velocity – its terminal velocity through that fluid.

If you have spheres that are similar except that they are of, say, five different sizes, how could you improve the method? Could you plot a graph of your results and deduce the viscosity from that?

Consider a body moving through a fluid (or the fluid moving past the body, which is the same thing). The viscous drag force on the moving body (the 'head wind' effect) may be worked out. For simplicity we imagine the body is a sphere, of radius r.



$F_g = W = mg = V\rho g$

$W = \frac{4}{3}\pi r^3 \rho_{ob} g$

$F_v = 6\,\pi\eta r v$

$F_d = \frac{4}{3}\pi r^3 \rho_{flu} g$

When the ball attains terminal velocity, then

$F_d + F_v = W$

$6\,\pi\eta r v = \frac{4}{3}\pi r^3(\rho_{ob} - \rho_{flu})g$

$v = \frac{2r^2(\rho_{ob} - \rho_{flu})g}{9\eta}$

Worked Example: 8.2.8

Find the force on a ball bearing of radius 4×10^{-3}m falling through a liquid of viscosity 0.985 at a velocity of 0.5m/s.

$F = 6\pi\eta r v = 6\pi \times 0.985 \times 4\times10^{-3} \times 0.5$

$= 0.01182\pi$ N

Activity 8.2.8: Measuring

If you have available a long transparent tube, you may be able to fill it full of a liquid and measure terminal speeds through it to find the liquid's viscosity at room temperature. Steel ball bearings through an oil may be suitable, perhaps.

The details, and the theory behind it, are up to you. Start with just one ball. How will you measure its terminal velocity v? What other quantities will you have to measure, and how? How could you take a check reading?

Worked Example: 8.2.9

A metal plate of length 10 cm and breadth 5 cm is in contact with a layer of oil 1 mm thick. The horizontal force required to move it with a velocity of 4 cm/s along the surface of the oil is 0.32 N. Find the coefficient of viscosity of the oil. Also express it in poise.

Solution:

Data: A=10cm×5cm=50cm^2=5× 10^{-3}m^2

y=1$mm = 10^{-3}m$, $v_o = 4\,{}^{cm}/_{s} = 4 \times 10^{-2}\,{}^{m}/_{s}$

F=0.32N

$velocity\ gradient, \frac{v_o}{y} = \frac{4\times10^{-2}{}^{m}/_{s}}{10^{-3}m} = 40s^{-1}$

$viscous\ force, F = \eta A\frac{v_o}{y}$

∴ the coefficient of viscosity is

$$\eta = \frac{F}{A(\frac{v_o}{y})} = \frac{0.32N}{(5\times10^{-3}\mathrm{m}^2)(40s^{-1})}$$

$$= \frac{320}{200} = \mathbf{1.6Ns/m^2}$$

Since $1N\,{}^{S}/_{m^2} = 10$Poise

$\eta = 16\,poise$

Reynolds numbers

Reynold's number is a pure number which determines the nature of flow of liquid through a pipe, which does not have any units, is an important variable in equations that describe whether flow conditions lead to laminar or turbulent flow.

It is defined as the ratio of the inertial force per unit area to the viscous force per unit area for a flowing fluid.

Reynolds Number = Re

$$\mathrm{Re} = \frac{\text{Inertial Force}}{\text{Viscous Force}}$$

$$Re = \frac{\rho v\frac{dv}{dx}}{\mu\frac{d^2v}{dx^2}} = \frac{\rho v\frac{v}{L}}{\mu\frac{v}{L^2}}$$

$$Re = \frac{\rho v \mathrm{L}}{\mu}$$

Where: ρ-density of the fluid

v- the flow speed

L- diameter of the pipe

M- dynamic viscousity of the fluid

If the value of Reynold's number

i. Lies between 0 to 2000, the flow of liquid is streamline or laminar.
ii. Lies between 2000 to 3000, the flow of liquid is unstable changing from streamline to turbulent flow.
iii. Above 3000, the flow of liquid is definitely turbulent.

In the case of flow through a straight pipe with a circular cross-section, Reynolds numbers of less than 2300 are generally considered to be of a laminar type. However, the Reynolds number at which laminar flows become turbulent is dependent upon the flow geometry.

Worked Example: 8.2.10

If the speed at which water flows through a long cylindrical pipe of radius 8mmis 10 cm/s, find the Reynolds number. [Density of water 0.1 g/cm, coefficient of viscosity of water 0.01 poise]

Solution

Data: $v_o = 10cm/s$, $\rho = 1\,{}^{g}/_{cm^3}$, r=8mm

$\Longrightarrow$ d=2r = 16mm=1.6cm , $\eta = 0.01poise$

The Reynolds number, $R_e = \frac{v_o \rho d}{\eta} = \frac{(10\text{cm})(1\,{}^{\text{g}}/_{\text{cm}^3})(1.6\text{cm})}{0.01\text{P}}$

$$R_e = 1600$$

Summary

In this section you have learnt that:

- Laminar flow occurs when liquids flow in parallel layers and there is no disruption between the layers.
- Turbulent flow occurs when there is disruption between the layer of fluid.
- The flow rate is the volume of liquid flowing past a given point per unit time.
- Factors that affect laminar flow are density, compressibility (how much the volume of the liquid can be reduced), temperature and viscosity.
- Density, compressibility and temperature of air become important at high speeds: on landing, a commercial aeroplane will feel cool to the touch.
- The Reynolds number is available in equations that describe whether conditions lead to laminar or turbulent flow.
- Bernovilli's principle states that a fluid travelling over the surface of an object exerts less pressure that if the fluid were still, for example, when aeroplanes fly.
- Bernovilli's equation ispρ+ gh+v22= constant.
- A simplified form is static pressure + dynamic pressure = constant
- Stoke's law is the viscous drag force Fthat acts on a sphere of radius r travelling at a velocity vthrough a fluid of viscosity η is given by F= 6πηrv.
- The equation of continuity states that the volume flow rate of an ideal fluid flowing through a closed system is the same at every point.

Review questions

1. Give examples of laminar flow.
2. Identify factors that affect the streamlining of cars, boats and planes.
3. Define the Reynolds number.
4. State Bernoulli's principle.
5. Use the simplified version of Bernoulli's equation to find the total pressure when the static pressure is 9 Pa and the dynamic pressure is 3 Pa.
6. State Stokes's law.

7. State the equation of continuity.
8. In Figure 8.33, water flows through a horizontal pipe and then out into the atmosphere at a speed v_1=15m/s. The diameters of the left and right sections of the pipe are 5.0 cm and 3.0cm. (a) Find the volume of water flows into the atmosphere during a 10 min period. In the left section of the pipe, what are (b) the speed v_2 and (c) the gauge pressure?

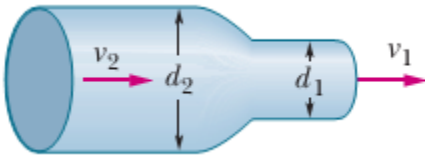


Figure 8.2.14

9. A water hose 2.50cm in diameter is used by a gardener to fill a 30.0-L bucket. The gardener notes that it takes 1.00min to fill the bucket. A nozzle with an opening of cross-sectional area 0.500cm^2 is then attached to the hose. The nozzle is held so that water is projected horizontally from a point 1.00m above the ground. Calculate the initial velocity vector of the projected water in the horizontal direction.
10. Calculate the terminal velocity of an air bubble with 0.8 mm in diameter which rises in a liquid of viscosity of 0.15 kg m^{-1}s^{-1}and density 0.9 g m^{-3}. What will be the terminal velocity of the same bubble while rising in water? For water $\eta = 10^{-2}$kgm^{-1}s^{-1}

8.3 Heat, temperature and thermal expansion

By the end of this section you should be able to:

- Distinguish between heat, temperature, internal energy and work.
- Describe the units for heat, heat capacity, specific heat capacity and latent heat.
- Define the terms calorimetry, phase, phase change, phase diagram, state variable, critical point, triple point, latent heat, heat capacity, specific heat capacity.
- Explain the factors that determine the rate of heat flow through a material.
- Describe the thermal expansion of solids in terms of the molecular theory of matter.
- Carry out calculations involving expansivity.
- Solve problems involving thermal conductivity.
- Describe experiments to measure latent heat.

We all have common sense notions of **heat** and **temperature**. Temperature is a measure of 'hotness' of a body. A kettle with boiling water is hotter than a box containing ice. In physics, we need to define the notion of heat, temperature, etc., more carefully. In this section, you will learn what heat is and how it is measured, and study the various proceses by which heat flows from one body to another. Along the way, you will find out why blacksmiths heat the iron ring before fitting on the rim of a wooden wheel of a horse cart and why the wind at the beach often reverses direction after the sun goes down. You will also learn what happens when water boils or freezes, and its

temperature does not change during these processes even though a great deal of heat is flowing into or out of it.

TEMPERATURE AND HEAT

We can begin studying thermal properties of matter with definitions of temperature and heat. Temperature is a relative measure, or indication of hotness or coldness. A hot utensil is said to have a high temperature, and ice cube to have a low temperature. An object that has a higher temperature than another object is said to be hotter. Note that hot and cold are relative terms, like tall and short. We can perceive temperature by touch. However, this temperature sense is somewhat unreliable and its range is too limited to be useful for scientific purposes.

We know from experience that a glass of ice-cold water left on a table on a hot summer day eventually warms up whereas a cup of hot tea on the same table cools down. It means that when the temperature of body, ice-cold water or hot tea in this case, and its surrounding medium are different, heat transfer takes place between the system and the surrounding medium, until the body and the surrounding medium are at the same temperature. We also know that in the case of glass tumbler of ice-cold water , heat flows from the environment to the glass tumbler , whereas in the case of hot tea, it flows from the cup of hot tea to the environment. So, we can say that heat is the form of energy transferred between two (or more) systems or a system and its surroundings by virtue of temperature difference. The SI unit of heat energy transferred is expressed in joule (J) while SI unit of temperature is Kelvin (K), and degree Celsius (^{o}C) is a commonly used unit of temperature.

When an object is heated, many changes may take place. Its temperature may rise, it may expand or change state. We will study the effect of heat on different bodies in later sections. the glass tumbler , whereas in the case of hot tea, it flows from the cup of hot tea to the environment. So, we can say that heat is the form of energy transferred between two (or more) systems or a system and its surroundings by virtue of temperature difference. The SI unit of heat energy transferred is expressed in joule (J) while SI unit of temperature is Kelvin (K), and degree Celsius (^{o}C) is a commonly used unit of temperature.

When an object is heated, many changes may take place. Its temperature may rise, it may expand or change state. We will study the effect of heat on different bodies in later sections.

TEMPERATURE SCALE CONVERSIONS

Temperature on one scale can be converted into other scale by using the following identity:

$$\frac{Reading\ on\ any\ scale - lower\ fixed\ point\ (LFP)}{Upper\ fixed\ point\ (UFP) - lower\ fixed\ point\ (LFP)} = constant\ for\ all\ scales$$

For example, for H_2O(water) LFP and UFP in C, F and K scales is as follows:

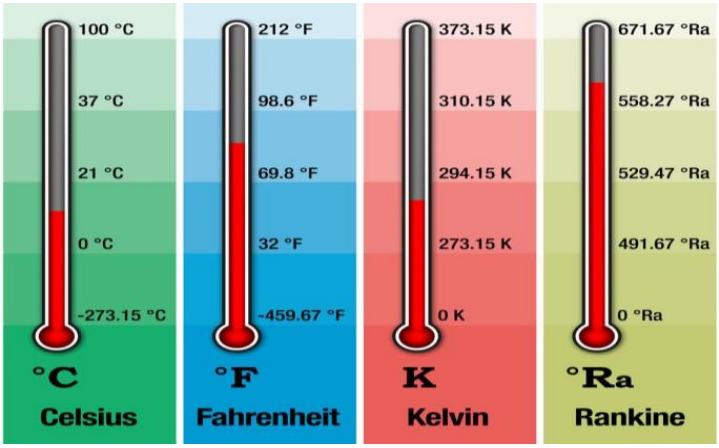


Figure 8.3.1 Comparison of the Celsius, Fahrenheit, Kelvin and Rankine temperature scales.

Name of the scale	Symbol for each degree	Lower fixed point (LFP)	Upper fixed point (UFP)	Number of divisions on the scale
Celsius	C	0 C	100 C	100
Fahrenheit	F	32 F	212 F	180
Kelvin	K	273.15 K	373.15 K	100

Hence , $\frac{C-0}{100-0} = \frac{F-32}{212-32} = \frac{K-273.15}{373.15-273.15}$

Worked Example: 8.3.1

On a day when the temperature reaches 50°F, what is the temperature in degrees Celsius and in kelvins?

Since, $\frac{C-0}{100-0} = \frac{F-32}{212-32} = \frac{K-273.15}{373.15-273.15}$

$$\Rightarrow T_c = \frac{5}{9}(F-32) = \frac{5}{9}(50-32) \Rightarrow T_c = 10℃$$

A convenient set of weather-related temperature equivalents to keep in mind is that 0°C is literally) freezing at 32°F, 10°C is cool at 50°F, 20°C is room temperature, 30°C is warm at 86°F,and 40°C is a hot day at 104°F.

Specific heat capacity

You learnt about specific heat capacity in Grade 9, Section 7.3, so this section should be revision. Accurate measurements suggest that for every gram of water we are heating and for every 1 K we are raising its temperature, we have to supply about 4.2 J of energy. If we took a thousand times as

much water, that is, a kilogram, we would have to provide a thousand times as much energy (4200 J) to raise the temperature of this larger mass of water by 1 K.

Other substances vary. Kerosene, for instance, is found to need only 2200 J of energy to warm 1 kg of it by 1 K. Thus a kettle containing a kilogram of kerosene instead of water would heat up nearly twice as fast, but as the heat escaped again it would cool down more quickly.

The specific heat capacity is the property of the substance which deter mines the change in the temperature of the substance (undergoing no phase change) when a given quantity of heat is absorbed (or given off) by it. It is defined as the amount of heat per unit mass absorbed or given off by the substance to change its temperature by one unit. It depends on the nature of the substance and its temperature.

$$c = \frac{Q}{m\Delta T}$$

The SI unit of specific heat capacity is J $kg^{-1}K^{-1}$.

The number of joules of heat energy required to raise the temperature of 1 kg of it by 1 K.

(Sometimes the specific heat capacity is defined as the amount of energy needed to raise the temperature of 1 gram of a substance by 1 K, but the definition using 1 kg is more correct.)

Therefore, to find the number of joules of energy needed:

1. Take the specific heat capacity c in J/kg/K or in J/g/K.
2. Multiply it by the number of kilograms (or grams) you are heating.
3. Multiply it again by the change in temperature in kelvin.

This process can be summed up by the formula: $Q = mc\Delta T$

where Q is the number of joules of energy to be supplied (if you are heating the substance), or the number of joules of thermal energy released (if it is cooling down).

Worked Example: 8.3.2

The specific heat capacity of water is 4200 J/kg/k. How much energy will be needed to warm 800 g of water from 17°C to 27°C?

Q(J)	m(kg)	C($Jkg^{-1}K^{-1}$)	ΔT
?	0.8	4,200	10

$Q = mc\Delta T$

The change in temperature $\Delta T = 27 - 17 = 10$ k

Because the specific heat capacity has been given in J/kg/k, we must take 'm' to be not 800 g but 0.8 kg.

$Q = mc\Delta T = 0.8 \times 4200 \times 10 = 33\,600$ J

Notice carefully in the above example that a rise in temperature from 17°C to 27°C is a gain of 10 k. Degrees Celsius and the kelvin are the same-sized interval of temperature. You do not need to add 273 to the 17°C and 27°C; you would still end up

with a 10 degree rise in temperature.

Worked Example: 8.3.3

Using a current of 1.5 A, a 12 V, 1.5 A heater was used to heat 100g of water for 15 minutes. The temperature rose from 21°C to 52°C. Based on these results, how much energy would be required to warm 1 kg of water by 1°C?

Power of heater (W)	Energy supplied (J)	ΔT (°C)
12×1.5=18	15×60×18 =16,200	52-21=31

Use $Q = mc\Delta T$

$c = \frac{Q}{m\Delta T} = \frac{16,200}{0.1\times31}$=5226 J/kg/k

Activity 8.3.1: Measuring the specific heat capacity of a liquid by electrical heating

To find the specific heat capacity of water you need to supply a measured number of joules of heat energy to a known number of kilograms of water, and see what temperature rise results. If you use an electric heater submerged under the water (an immersion heater), you can measure accurately its wattage. You work out the power of the heater in watts by multiplying the voltage drop across the
heating coil by the current in amperes which is passing through it (P = VI). This works because the voltage tells you how many joules of heat energy will be produced for each coulomb of charge that passes, and the current is the number of coulombs which pass every second. All this energy goes into the water, so we simply have to insulate the water's container in order to keep it there. Figure 8.31 shows the electrical circuit. Do not switch on until the heater is covered by water or you may
burn it out.

The rest of the apparatus is shown in Figure 8.32. A suitable container is an insulated cup of the sort drinks are sometimes sold in. The lid can be made from a small piece of plastic or hardboard. The thermometer can be supported by a tight-fitting collar cut from a length of rubber tubing. The stirrer must be made from a piece of insulated wire so that it does not short out the heater.

What we are trying to find out, remember, is how many joules of energy are needed to raise the temperature of 1 kg of water by 1°C.

A kilogram is a lot of water to heat, however, so it makes sense to take 100 g and scale the final answer up. A convenient way to measure the water out is to pour 100 cm3 into the cup from a measuring cylinder, since the density of water is 1 g/cm3.

You are now ready to start. Record the temperature of the cold water, then start a stopwatch and switch on the heater.

A temperature rise of 1°C is too small to measure accurately, so aim for a temperature rise somewhere between 10 and 30°C. When you have finished, switch off, note the time, stir thoroughly and take the final temperature of the water.

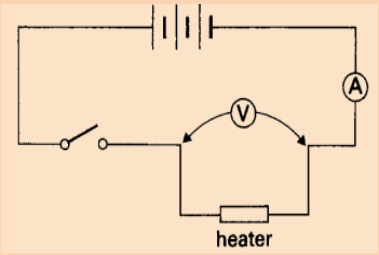


Figure 8.3.2 The circuit used to find the specific heat capacity of a liquid

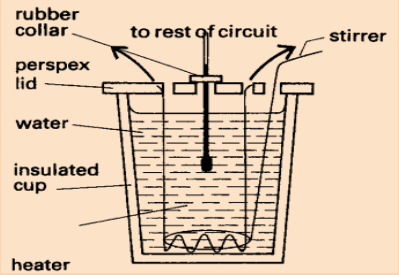


Figure 8.3.3 The rest of the apparatus used to find the specific heat capacity of a liquid

Calorimetry

A system is said to be isolated if no exchange or transfer of heat occurs between the system and its surroundings. When different parts of an isolated system are at different temperature, a quantity of heat transfers from the part at higher temperature to the part at lower temperature.

Calorimetry is the science of measuring the heat of chemical reactions or physical changes. Calorimetry involves the use of a calorimeter. The word calorimetry is derived from the Latin word calor, meaning heat. Scottish physician and scientist Joseph Black, who was the first to recognise the distinction between heat and temperature, is said to be the founder of calorimetry. A device in which heat measurement can be done is called a calorimeter. It consists of a metallic vessel and stirrer of the same material, like copper or aluminium. The vessel is kept inside a wooden jacket, which contains heat insulating material, like glass wool etc. The outer jacket acts as a heat shield and reduces the heat loss from the inner vessel. There is an opening in the outer jacket through which a mercury thermometer can be inserted into the calorimeter (Fig. 8.20). The following example provides a method by which the specific heat capacity of a given solid can be determinated by using the principle, heat gained is equal to the heat lost

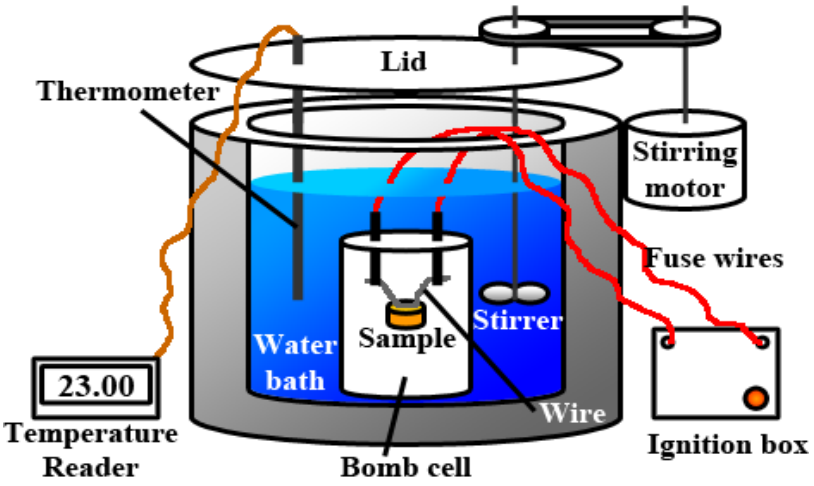


Figure 8.3.4 Calorimeter

Change of state

In Grade 9, you learnt about changes of state in Section 7.4. The three states of matter that you have considered are solid, liquid and gas. Each of these states of matter can be considered as a phase. A transition from one of these states to another is called **a change of state**. Two common changes of states are solid to liquid and liquid to gas (and, vice versa). These changes can occur when the exchange of heat takes place between the substance and its surroundings or when the kinetic energy of the molecules in the substance increases (which we usually record as an increase in temperature, because the kinetic energy of the molecules in a substance, internal energy, is related to the temperature of the substance) or decreases (which we usually record as a decrease in temperature). A phase change diagram such as the one in Figure 8.38 shows the three phases of a substance.

The diagram shows the triple point for the substance, which is the point where all three states coexist together. This occurs at 0°C for water.

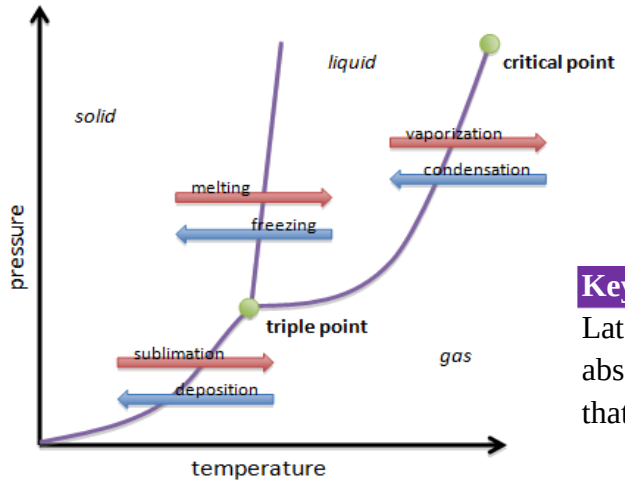


Figure 8.3.5 Phase change diagram

Activity 8.3.2: The phase

In a small group, discuss the phase diagram in Figure 8.38 and try to explain what it shows in your own words.

Key Words

Latent heat the amount of energy released or absorbed by a substance during a change of state that occurs without a change in temperature

The change of state from solid to liquid is called melting and from liquid to solid is called **fusion**. It is observed that the temperature remains constant until the entire amount of the solid substance melts. That is, both the solid and the liquid states of the substance coexist in thermal equilibrium during the change of states from solid to liquid. The temperature at which the solid and the liquid states of the substance is in thermal equilibrium with each other is called its melting point. It is characteristic of the substance. It also depends on pressure. The melting point of a substance at standard atmospheric pressure is called its normal **melting point**.

The change of state from liquid to vapor (or gas) is called vaporization. It is observed that the temperature remains constant until the entire amount of the liquid is converted into vapor . That is, both the liquid and vapor states of the substance coexist in thermal equilibrium, during the change of state from liquid to vapor. The temperature at which the liquid and the vapor states of the substance coexist is called its boiling point. At the critical point the liquid and gas phases become in distinguishable

Latent heat

If you heat a body, it will get hotter. That seems pretty obvious, so it may come as a surprise to discover that it is not necessarily true.

If you imagine being given a beaker containing water at 100°C and a Bunsen burner, you will begin to see why. You can continue to feed thermal energy into that water, but a thermometer would persist in reading 100°C. The extra energy you are supplying does not show itself as a rise in temperature, and it is given the name latent heat. '**Latent'** means **hidden**.

The water of course is boiling away and turning to steam. The energy goes not into making the molecules move faster, but instead it is used to separate the molecules from each other. Work has to be done in moving them apart against the attractive forces which hold them together as a liquid. Something similar happens as ice melts. All that time the heat energy it is gaining makes it no hotter, but instead it goes into breaking down the rigid structure of the solid.

Figure 8.39 shows what should happen if heat energy is fed into a block of ice in a steady and even way. Despite this constant input of heat energy the temperature twice stays fixed: once at 0°C as it melts and once at 100°C as it boils.

Melting and boiling are, of course, examples of changes of state, from the solid state to the liquid state or from liquid to gas. Whenever a substance changes state, latent heat is involved. When steam condenses or water freezes, this latent heat is given out again. A scald from steam at 100°C can prove much worse than a scald from water at that temperature. The steam condenses on the cool skin, giving out its considerable latent heat as it turns to water still at 100°C, and then the hot water gives out its heat in turn as it cools down.

Activity 8.3.3: Determining the amount of heat necessary t convert a known quantity of ice at 0°C to water at 0°C

In a small group, use the experience you have gained from the activities so far in this section to devise and carry out a investigation into the amount of heat necessary to convert a known quantity of ice to water at 0°C. Show your plans to your teacher before you begin the investigation. How will you measure how much heat is supplied? What measurements will you need to take?

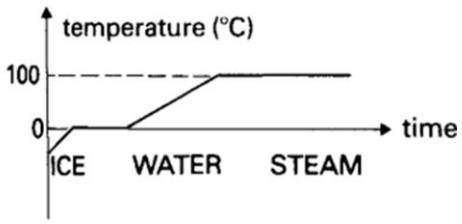


Figure 8.3.6: Why does the temperature stop rising at 0°C and 100°C?

Specific latent heat of fusion

The process of melting is sometimes called fusion (which is how an electrical fuse got its name). Likewise changing from a liquid to a gas is called vaporization, and includes evaporation as well as boiling.

To turn a 1 kg block of ice at 0°C into water still at 0°C needs about 340,000 J of energy (which may be expressed as 340 kilojoules, 340 kJ).
It is a very large figure, partly because the joule is quite a small unit but mainly because it does take a lot of energy to melt 1 kg of ice.
The figure is referred to as the specific latent heat of fusion (symbol 'L_f') of ice. The units are joules per kilogram (J/kg).
The specific latent heat of fusion of a substance is the thermal energy required to change 1 kg of the solid at its melting point into 1 kg of liquid at the same temperature.
If you tried to melt not 1 kg but 2 kg of ice, you would have to supply twice as much energy (i.e., 2 × 340 000 J).
In order to melt mkg of ice, the energy E_H needed is given by:

$$\mathbf{Q}_f = m\mathbf{L}_f$$

Activity 8.3.4: Measuring the specific latent heat of fusion of ice

The question you want to answer here is: how many joules of heat energy are needed to change 1 g (or 1 kg) of ice at 0°C into water still at 0°C?
If ice cubes on the point of melting are added to a drink, then it becomes chilled.
The heat energy removed from your drink has gone first into melting the ice and then into warming that 'molten ice' up from 0°C to the temperature of the rest of the chilled drink.
An experiment to discover how effective ice cubes are at cooling a drink should help us to find an answer to the question.
Figure 8.35 shows the idea. We assume the ice is at its melting point, but it is important to make sure the ice cubes are carefully dried with filter paper before they are added. It is therefore not feasible to weigh them before putting them in, so we find the number of grams of ice added by seeing at the end how much heavier the cup of water has become.

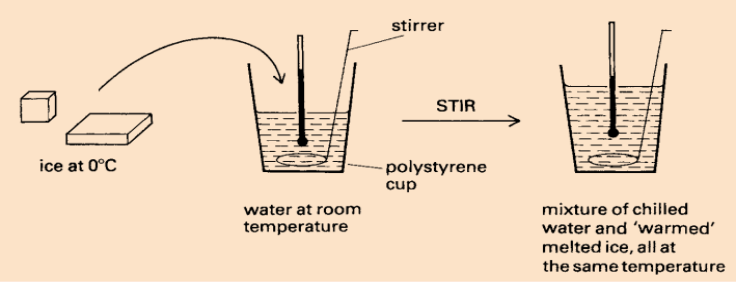


Figure 8.3.7: How much energy is needed to change the ice into water?

Finding the specific latent heat of fusion ice
Sample data: Mass of cup empty = 23 g
Mass of cup + 'room temperature' water = 123 g
Initial temperature of 'room temperature' water = 17°C
Final temperature of chilled water + 'molten ice' = 5°C
Total mass of cup, water + 'molten ice' at end = 139 g
Your calculations:

1. How many grams of 'room temperature' water were there at the start?
2. By how many degrees was this water eventually chilled?
3. How much energy must have been removed from this water? Take the specific heat capacity of water to be 4.2 J g–1K–1.
4. How many grams of ice at 0°C must there have been originally?
5. When, having melted, it warmed up from 0°C to the final temperature of the chilled mixture, how many joules of energy did it gain?
6. By comparing your answers to questions 3 and 5, how many joules of energy must the ice have taken in melting?
7. How much energy therefore was required to change 1 g of ice at 0°C into 1 g of water at 0°C?
8. Give your value for the specific latent heat of fusion of ice in both J/g and J/kg.

Specific latent heat of vaporization

The specific latent heat of vaporization ('L_v') of water is about 2,300,000 J/kg. In other words, if you are trying to turn 1 kg of water into steam, even when its temperature has reached 100°C, you still have a long way to go: another 2.3 million joules of energy have yet to be supplied just to separate the molecules and so change the liquid to a gas.

The specific latent heat of vaporization of a liquid is the thermal energy required to change 1 kg of the liquid at the boiling point into 1 kg of gas at the same temperature.

Again, to vaporise mkg the energy required is given by:

$$Q_v = mL_v$$

Worked Example: 8.3.4

Calculate the heat required to convert 3 kg of ice at –12 °C kept in a calorimeter to steam at 100 °C at atmospheric pressure. Given specific heat capacity of ice = 2,100 $Jkg^{-1}K^{-1}$, specific heat capacity of water = 4,186 $Jkg^{-1}K^{-1}$, latentheat of fusion of ice = $3.35 \times 10^5 J\ kg^{-1}$and latent heat of steam $2.256 \times 10^6 J\ kg^{-1}$

Q = heat required to convert 3 kg of ice at – 12 °C to steam at 100 °C

Q_1 = heat required to convert 3 kg of ice at – 12 °C to ice 0 °C

$= mc_{ice}\Delta T_1$ = (3kg)(2,100 $Jkg^{-1}K^{-1}$)(0 °C − (−12 °C)

=75,600J

Q_2 = heat required to melt ice at 0 °C

$= mL_f$ = (3kg)($3.35 \times 10^5 J\ kg^{-1}$)

=1,005,000J

Q_3 = heat required to convert water at 0 °C to water at 100 °C

$= mc_w\Delta T_2$ = (3kg)(4,186$Jkg^{-1}K^{-1}$)(100 °C − 0 °C)

$= 1{,}255{,}800J$

Q_4 = heat required to convert water at 100 °C to steam at 100 °C

$= mL_v$ = (3kg)($2.256 \times 10^6 J\ kg^{-1}$)

$= 6{,}768{,}000J$

So $Q = 75{,}600\text{J} + 1{,}005{,}000\text{J} + 1{,}255{,}800J + 6{,}768{,}000J$

$Q = 9.1 \times 10^6\text{J}$

Heat, temperature, internal energy and work

It is important that you are clear about the difference between heat and temperature. An example will show you that there is a difference. Suppose you have a race with a friend to see who can raise the temperature of some water from room temperature to 100°C the faster. To be fair, you each have an identical electric heater. There is just one point to mention, however: to be unselfish you give your friend a whole bucketful of water to warm up, and keep only a little in a cup for yourself. You know the result, of course. Both of you are doing the same thing: raising the temperature of some water by about 80 K. To do this you need to do work (feed in energy). One person has far more water than the other though, and she will have to feed in far more heat energy to do the job. Her heater must be switched on for a longer time, and her bill from the electricity company for energy supplied will be more.

Activity 8.3.5: Measuring the specific latent heat of vaporisation of water

In this activity you are trying to answer the following question: if we take 1 kg (or 1 g) of water which is already at 100°C, how many joules of heat energy must we supply to change it all into steam at 100°C? The answer will be expressed in J/kg (or J/g). In principle the method is straightforward. You must take some water at its boiling point, and find out how much energy has to be supplied so that 1 kg (or 1 g) shall be boiled away. In practice, how can this be done?

A method using a Bunsen burner is depicted in Figure 8.36.

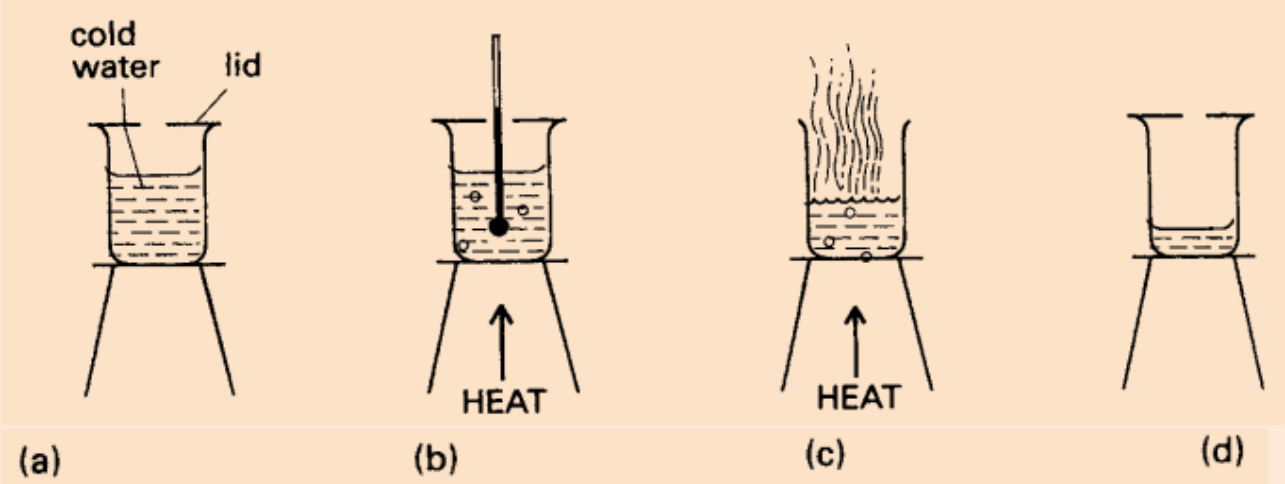


Figure 8.3.8 a) The mass of the cold water is known. b) The water is brought to the boil, using a Bunsen burner whose flame is not altered. Readings of temperature and time are taken. c) Still keeping the flame constant, the water is boiled for 10 minutes. d) The lid is replaced and the apparatus weighted to find out how much water has been boiled away.

Set up the apparatus as shown. The thermometer is there to measure how quickly the Bunsen burner heats the water. This enables you to work out how many joules of heat energy the Bunsen burner is supplying every second, so the flame must not be altered throughout the experiment.

Take readings and work out how much energy is needed to turn 1 g of water to steam.

Heat is a form of energy, measured in joules. When the water was heated and warmed up, the molecules of water moved faster. Thus the energy is stored in the hot water in the form of kinetic energy, the energy of motion of its molecules.
This energy is the internal energy of the water.
The word 'heat' is often used misleadingly in daily life. For example, some thermometers have 37°C marked as 'body heat'; what they should say is body temperature. When the weather is hot, this means that the temperature of the air outside is high.
Temperature, you may recall, may be described as the degree of hotness of a body, although that is rather a vague description. It is that which a thermometer records. A difference in temperature between two places can lead to a flow of heat energy between them, from the high temperature place to the lower temperature place.
Thus the hot body will cool down and the cold one will warm up, the exchange of energy stopping when they have both reached the same temperature.

Heat transfer: conduction

If a bowl of hot soup is left standing on a table, it will get cold. Evaporation must be one cause of this, but it is not the only one.
The table under the bowl becomes warm, and this is because heat has been conducted through the bowl. Conduction is the passage of heat through a material from molecule to molecule, without any movement of the material as a whole.
If a spoon is left in a cup of coffee, the handle gets warm. A hot drink served in a metal mug is good for warming your hands, though the drink is not going to stay hot for long. These are examples of the conduction of heat through a metal.
Gases are even poorer conductors of heat than liquids. Air is thus a very good insulator, and commercial insulating materials are effective because they have many tiny pockets of still air trapped in them. An example is expanded polystyrene, used in coolers to keep your food and drink chilled. A refrigerator is lined between its inner and outer surfaces with a kind of woolly mat made from glass fibres loosely packed together.
Sometimes the purpose of the insulation is to keep things hot, rather than to protect cool things from the heat of the day. An oven needs to have lining similar to that in a refrigerator. Hot water pipes and steam pipes should be covered with a jacket made of expanded polystyrene or something similar to prevent a wasteful loss of heat.

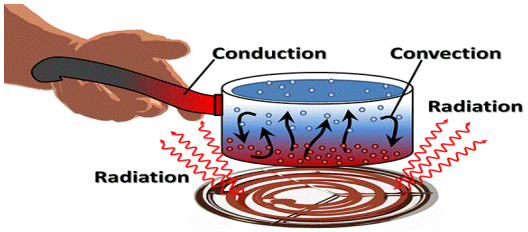


Figure 8.3.9: ways of transferring heat

Activity 8.3.6: Conduction in copper

Copper is a particularly good conductor of heat, and Figure 8.37 shows a demonstration of this.

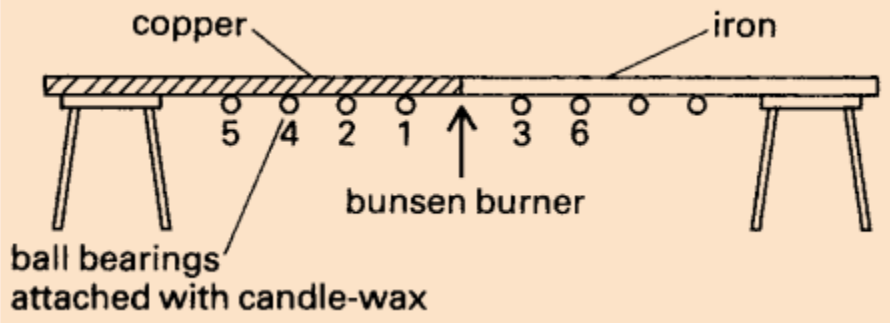


Figure 8.43 How to show that copper is a better conductor of heat than iron

Take a long bar made of copper at one end and iron at the other, joined together with a rivet. At regular intervals along it attach ball bearings with candle wax. Heat the bar strongly at its midpoint with a Bunsen burner. Record your observations.

Try to explain them before reading on. As heat is conducted along the bar in both directions, the wax melts and the ball bearings drop off. The iron conducts the heat, but not as well as the copper: the numbers on Figure 8.43 indicate a likely order in which the balls will fall.

Activity 8.3.7: Heat transfer in insulators

Metals generally are good conductors of heat, but most other substances are far poorer conductors (or, to look at it another way, are far better insulators).

A small electric heater used towarm a beaker of water can demonstrate this vividly. Use a low-value electrical resistor (perhaps 5 or 6 ohms if run off a 12-volt supply) as the heater, and arrange it so it is well clear of the bottom of the beaker (see Figure 8.44).

Do not switch on until the resistor is in the water or it will almost instantly burn out. Once in place, turn it on and leave it until the water has heated up. Now feel the outside of the beaker high up, just below the water line. It will be hot, maybe uncomfortably so. Run a finger down the beaker, and you will find that there is a sudden change in temperature as you go below the heater.

From that level downwards the water seems cold. Hot water is sitting on the top of cold water, yet hardly any conduction takes place.

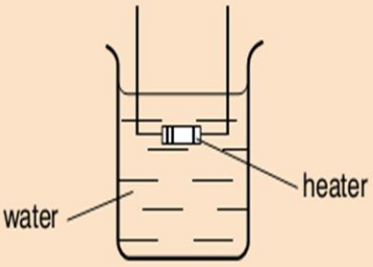


Figure 8.3.10 showing that water is a poor conductor of heat

Heat transfer: convection

If water is such a poor conductor, how is it that a kettle manages to heat its water up to boiling point? Why in Activity 8.28 was it only the water below the heater which was still cold?

The answer lies in thermal expansion. When water is heated it expands. This means that, comparing the same volumes, hot water is lighter than cold water. In other words, it is less dense.

In liquids and gases less dense substances are forced upwards. Oil floats on the top of water, for example, and hydrogen balloons rise when released in air. Therefore if some water is heated it expands, becomes less dense and so rises up to the surface. So long as the heating is being done at the bottom, this means that the water has a kind on inbuilt self-stirring mechanism. This is known as convection.

Activity 8.3.9: Observing convection

Convection can be observed in Figure 8.39 by taking a larger beaker of water and dropping in just one small crystal of potassium permanganate (a chemical that dissolves in water to give an amazingly deep purple coloring). Heat the beaker gently under the crystal and watch how the colored water rises to the surface in quite a fast and narrow stream.

Cold water sinks in a wider and gentler downdraught to take its place. The water in direct contact with the hot base gets heated by conduction. This water then rises, cold water replaces it and becomes warmed in its turn. These circulating currents are known as convection currents.

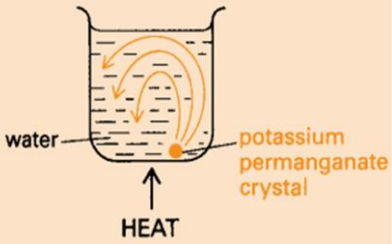


Figure 8.3.11 showing that water is a poor conductor of heat

Convection is the transfer of heat throughout a fluid (that is, a liquid or a gas) by means of bulk movement of the hot fluid.

Thermal conductivity

The rate at which heat energy will be conducted through a lagged bar is found to depend on three things:

- ✓ The material from which the bar is made.
- ✓ The area A of cross-section of the bar. Double the area, and heat is conducted through the bar at twice the rate.
- ✓ The temperature gradient along the bar – that is, how rapidly the temperature drops as you go along it. For a given conductor this takes into account both the temperature drop between its two ends and its total length. Using the notation in Figure 8.40, the temperature gradient will be $\frac{\Delta T}{L}$ (in Km^{-1}).

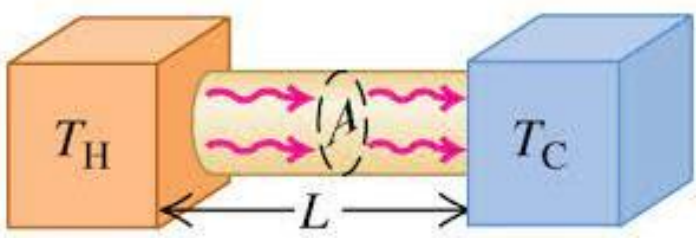


Figure 8.3.12: Conduction of energy through a uniform, insulated rod of length L

This means that the rate of flow of heat energy $\frac{Q}{t}$ in watts (J/s) along the bar will be given by:

$$\frac{Q}{t} = kA\frac{\Delta T}{L}$$

The constant k depends on which material the bar is made from. It is known as the material's thermal conductivity. The units of k work out to be W/m/K. Be sure you understand why

Worked Example: 8.3.5

The thermal conductivity of copper is 390 W/m/k. Calculate the rate of heat flow through a copper bar whose area is 4.0 cm^2 and whose length is 0.50 m, if a there is a temperature difference of 30°C maintained between its ends.

Start by putting down what you know. Here:

K9W/m/k)	A(m^2)	$\Delta\theta(k)$	x(m)	$\frac{Q}{t}(W)$
390	4.0×10^{-4}	30	0.5	?

$$\frac{Q}{t} = kA\frac{\Delta\theta}{x} = \frac{390\times4.0\times10^{-4}\times30}{0.5} = 2.3W$$

Worked Example: 8.3.6

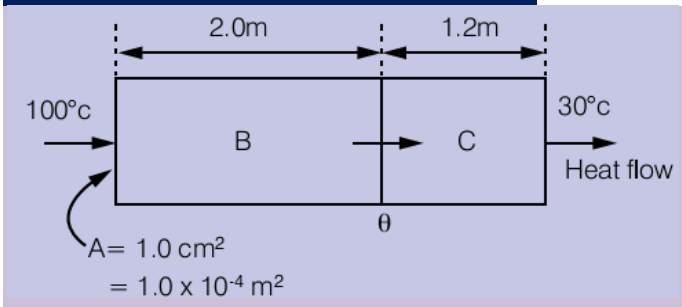


Figure 8.47 Thermal conductivity of copper

Suppose you had to work out the rate of flow of heat through a bar made of two materials in contact, as shown in the drawing.

Let the thermal conductivity of B be 400 W/m/k while that for C is 50 W/m/k.

Start by finding the temperature at the interface, θ say.

This rate must be the same through B and through C.

Equating them, $\frac{400\times A\times(100-\theta)}{2.0} = \frac{50\times A\times(\theta-30)}{1.2}$

The As cancel, and solving we get θ= 88°C.

You can then go back and substitute θ= 88°C into the flow equation for either bar.

Thus, considering bar B, rate of flow of heat $=\frac{400\times1.0\times10^{-4}\times(100-88)}{2.0} = 0.24W$

If you had chosen to substitute θ= 88°C for bar C instead, check that the result would have been the same.

Stefan–Boltzmann law

If you have the opportunity, look at the bare element of an electric heater or something else that is red hot. There are likely to be some spots on the surface which are glowing more brightly than others.
If so, remember where they are.
Let the body cool down and look again. You should find that the spots which glowed brightest are now the darkest parts of the surface – good emitters are good absorbers.
If we want to make a surface which will radiate freely to emit the maximum possible radiation at any given temperature, it will be a perfect absorber too. It will look matt black because no light will reflect from it: all is absorbed. We call this a black body.
The radiation a black body gives off depends on its temperature, and on nothing else. Figure 8.48 shows what will be emitted at a range of temperatures from 3000 K rising to 6000 K.

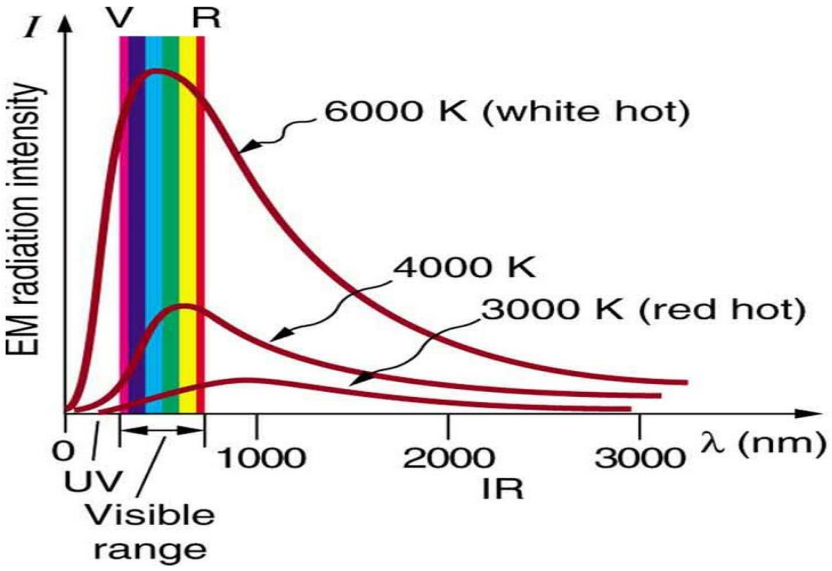


Figure 8.3.13: Black body radiation curve.

In Grade 10, you learnt that the wavelength of visible light ranges from about 750 nm (nanometres, × 10^{-9}m) at the red end of the spectrum down to 400 nm for the blue.
At 3500 K most of the radiation is in the infrared region, but just a little is visible light (mainly red). The surface is glowing red hot.
As it heats up further, two things happen: a greater amount of radiation is emitted in total, and the peak wavelength becomes smaller, so by 5500 K it is white hot. A pyrometer to measure temperatures might use either of those changes to estimate the temperature of the emitting surface.
The Stefan–Boltzmann law describes the total energy radiated by a black body. Consider a surface of area Aat a temperature T (which must be its absolute temperature in kelvin). The radiated power Pin watts (J/s) is given by:

$$P = \sigma A T^4$$

The constant σ (the Greek letter sigma) is a universal constant that does not depend on the material of the surface. All that is necessary is that it is behaving as a black body and so is radiating freely. Its value has been measured to be $5.67 \times 10^{-8} W/m^2/K^4$.
Thus all surfaces above absolute zero emit radiation, though at low temperatures this is very feeble and none is in the visible range. Because it depends on T^4, however, at very high temperatures radiation becomes a major source of energy loss. If the absolute temperature doubles, the radiated power increases sixteen fold; at ten times the temperature the power increases by a factor of 10^4– 10,000 times!

Newton's law of cooling

If some hot water is contained in a beaker, the presence of a lid should reduce the rate of cooling. This stops continual evaporation from the water's surface. To change from a liquid to a vapour needs energy (which you know is called latent heat), and this is obtained from the water that remains. Evaporation causes cooling.

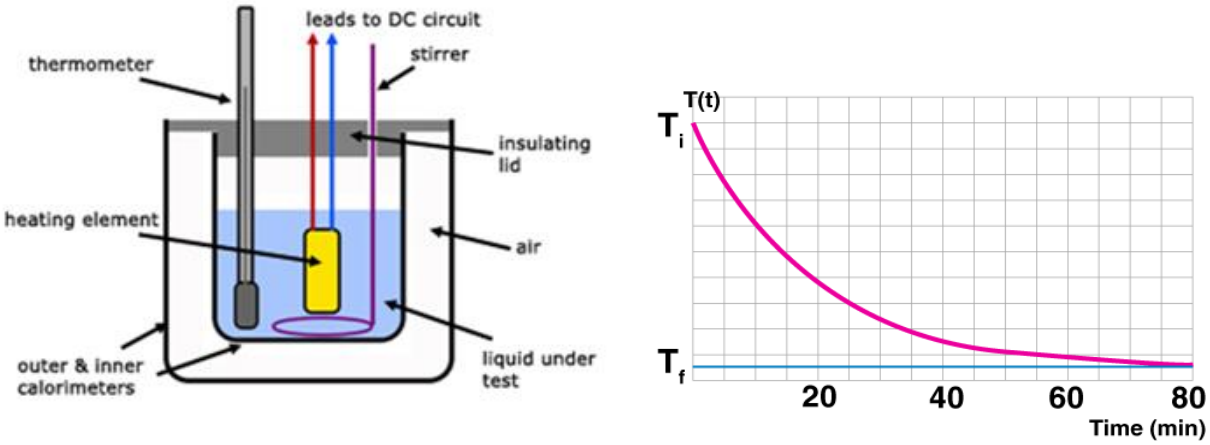


Figure 8. 3.14: Newton's law of cooling – Temperature vs Time

The cooling may be reduced further by lagging the beaker. What this means is that we put an insulating jacket round it to reduce the rate at which heat is being conducted to the outside world. The rate of cooling of a hot body was one of the many topics which Newton investigated. His experimental result is known as Newton's law of cooling. It states that:
The rate of loss of heat from a body is proportional to its excess temperature above its surroundings.
Mathematically, this is written as $\frac{dQ}{dt} = KA\,(T_{sum} - T_{obj})$

Q is heat energy, t is time, K is heat transfer coefficient (a constant)
A is surface area of heat being transferred T_{sum}-= sum is temperature of surroundings
T_{obj} - is temperature of body

If the body is 20 K warmer that the room, it will lose heat twice as fast than if it was only 10 K above its surroundings. It is approximately true for a body in air for excess temperatures up to a few tens of kelvin. The actual rate of that loss of heat may be increased if we subject the body to what we call forced convection – such as blowing cold air from a fan over it.

Worked Example: 8.3.7

A pan filled with hot food cools from 94°C to 86°C in 2 minutes when the room temperature is at 20°C. How long will it take to cool from 71°C to 69°C?

The average temperature of 94 °C and 86 °C is 90 °C, which is 70 °C above the room temperature. Under these conditions the pan cools 8 °C in 2 minutes.

$$\frac{\text{Change in temperature}}{\text{Time}} = \text{K}\Delta\text{T}$$

$$\frac{8°\text{C}}{2\text{min}} = \text{K}(70°\text{C})$$

The average of 69 °C and 71 °C is 70 °C, which is 50 °C above room temperature. K is the same for this situation as for the original.

When we divide above two equations, we have

$$\frac{2°\text{C}}{\text{Time}} = \text{K}(50°\text{C})$$

When we divide the above two equations, we have

$$\frac{8°\text{C}/2min}{2°\text{C}/time} = \frac{K(70°\text{C})}{K(50°\text{C})}$$

$$Time = 0.7min = 42sec$$

Global warming and the greenhouse effect

The greenhouse effect was first proposed in 1824. The earth's surface is a source of thermal radiation as it absorbs energy received from the Sun. The wavelength of this radiation lies in the long wavelength (infrared) region. But a large portion of this radiation is absorbed by greenhouse gases, namely, carbon dioxide (CO_2); methane (CH_4); nitrous oxide (N_2O); chlorofluorocarbon (CF_xCl_x); and tropospheric ozone (O_3). This heats up the atmosphere which, in turn, gives more energy to earth, resulting in warmer surface. This increases the intensity of radiation from the surface. The cycle of processes described above is repeated until no radiation is available for absorption. The net result is heating up of earth's surface and atmosphere. This is known as Greenhouse Effect. Without the Greenhouse Effect, the temperature of the earth would have been –18°C.

Concentration of greenhouse gases has enhanced due to human activities, making the earth warmer. According to an estimate, average temperature of earth has increased by 0.3 to 0.6°C, since the beginning of this century because of this enhancement. By the middle of the next century, the earth's

global temperature may be 1to 3°C higher than today. The increase in the average temperature of the Earth's near-surface air and oceans since the mid-2oth century is called global warming. This global warming may cause problem for human life, plants and animals. Because of global warming, ice caps are melting faster, sea level is rising, and weather pattern is changing. Many coastal cities are at the risk of getting submerged. The enhanced Greenhouse Effect may also result in expansion of deserts. All over the world, efforts are being made to minimize the effect of global warming.

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Summary

In this section you have learnt that:

- The three modes of heat transfer are conduction, convection and radiation.
- Calorimetry is the science of measuring the heat of chemical reactions or physical changes.
- A phase is a state of matter, e.g. solid, liquid, gas.
- A phase change occurs when a substance changes state.
- A phase diagram shows the three phases of a substance.
- State variables are properties such as temperature, pressure and internal energy.
- The critical point is where the liquid and gas phases become indistinguishable.
- The triple point is where all three states coexist together.
- Latent heat is energy that is supplied to a substance but does not result in a rise in temperature but does result in a change of phase (measured in J/kg).
- Specific heat capacity is the number of joules of heat energy required to raise the temperature of 1 kg of a substance by 1 k(measured J/kg/k).

- Heat capacity = mass specific × heat capacity (measured J/k)
- Internal energy is the energy possessed by the molecules of a substance.
- The rate at which heat energy will be conducted through a lagged bar of length L depends on: the material the bar is made from, the area of cross-section, A, of the bar, the temperature gradient along the bar, $\frac{\Delta T}{L}$
- The quantity of heat transferred by conduction is given by

$$Q = \frac{K(T_h - T_c)At}{d}$$

- Newton's Law of cooling states that the rate of cooling of a body is linearly proportional to the excess of temperature of the body above its surroundings

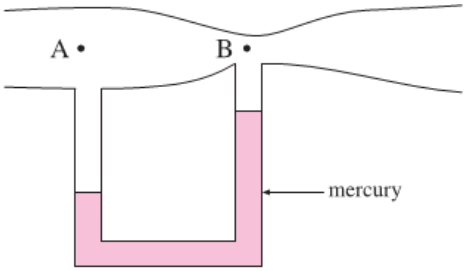
Review questions

1. Define the terms a) calorimetry, b) phase, c) phase change, d) phase diagram, e) state variable, f) latent heat, g) heat capacity, h) specific heat capacity.
2. Distinguish between heat, temperature, internal energy and work.
3. Give the units for heat, heat capacity, specific heat capacity and latent heat.
4. Explain the factors that determine the rate of heat flow through a material.
5. Work out the rate of flow of heat through the bar shown in Figure 8.43. Let the thermal conductivity of B be 450 W/m/K while that for C is 150 W/m/K.
6. Describe experiments to measure latent heat.
7. A pan of water is heated from 25°C to 80°C. What is the change in its temperature on the Kelvin scale and on the Fahrenheit scale?

End of unit questions

1. Construct a glossary of all the key terms in this unit. You could add it to the one you made for Units 1–7.
2. State Hooke's law.
3. A 0.50 kg mass is hung from the end of a wire 1.5 m long of diameter 0.30 mm. If Young's modulus for its material is 1.0×10^{11}Pa,
   a) calculate the extension produced b) hence find the strain energy.
4. Describe the concepts related to hydraulic and pneumatic systems.
5. Find the height of a column of methyl iodide in a soda-lime glass tube of radius 20 mm. The surface tension is 0.26, the contact angle is 29°, the density is 2.28.
6. Identify factors affecting laminar flow.
7. Explain applications of Bernoulli's principle.
8. A healthy section of artery of length 10 cm and radius r has a pressure difference of 120 mmHg. The volume flow rate of the blood in the artery is 100 cm3 the viscosity of blood is η.
   A diseased artery of the same length has a radius which is half that of the healthy artery. The volume flow rate of the blood in this artery is 6.3 cm3/min. What pressure difference is needed to bring the volume flow rate up to 100 cm3/min? (Assume the viscosity of the blood is the same as the healthy artery.)

9. Find the force on a ball bearing of radius 5 ×10−3m falling through a liquid of viscosity 0.985 at a velocity of 0.35 m/s.
10. Water flows at 20 m3/h of through a pipe with 150 mm inside diameter. The pipe is reduced to an inside dimension of 120 mm. Find the velocity of the water in each part of the pipe.
11. a) Draw a phase diagram for water b) Explain what is meant by i) critical point, ii) triple point.
12. The diagram shows two slabs of material in contact. A is 2.0 cm thick and its thermal conductivity is 3.0 W/m/K. B is 4.0 cm thick and of conductivity 1.5 W/m/K. The left hand face of A is kept at 0°C and the right hand face of B at 1000C.
    a) The slabs have been in place long enough for the temperatures within them to become steady. Which of the following quantities will have the same value in A and in B:
    i) temperature gradient ii) temperature difference between opposite faces iii) rate of flow of heat?
    b) Find the temperature at the junction of A and B.
    c) What is the rate of flow of heat through the slabs?
13. Describe an experiment to demonstrate conduction in copper.
14. The pressure of water inside a closed pipe is $8 \times 10^5\, N/m^2$. This pressure reduces to $2 \times 10^5\, N/m^2$on opening the valve of the pipe. Calculate the speed of water flowing through the pipe. [Density of water$1000\ {}^{kg}/_{m^3}$]
15. Water flows through a tube as shown in the given figure. Find the difference in mercury level, if the speed of flow of water at point A is 2 m/s and at point B is 5 m/s.



16. The speed of water through a pipe of internal diameter 10 cm is 2 m/s. What should be the internal diameter of nozzle of the pipe if the speed of water at nozzle is 4 m/s?
17. Determine the terminal velocity of an air bubble of 0.4mm diameter that rises in a liquid of viscosity 0.1Ns/m^2 and specific gravity (relative density) 0.9? Density of air is 1.29 kg/m.

End unit questions

1. A solid lead sphere of volume 0.50 m^3, dropped in the ocean, sinks to a depth of $2x10^3$ m (about 1 mile), where the pressure increases by $2x10^7$ Pa. Lead has a bulk modulus of 4.2 $x10^{10}$ Pa. What is the change in volume of the sphere?
2. An object of mass 15kg is hanging from one end of a steel wire. The wire without the mass has an unstretched length of 0.50 m. What are the resulting shear stress, shear strain and elongation of the wire? The cross-sectional area and Young's modulus(Y) for steel wire are $1.4\times10^{-2}cm^2$ and 20×10^{10}Pa.
3. An aluminum cube of each side 4cm is subjected to a tangential force. The top face of the cube is sheared 0.012cm with respect to the bottom. Find the
(a) Shearing strain (b) Shear stress (c) Shearing force
4. A metal wire of length 2.5m and area of cross-section $1.5x10^{-6}m^2$ is stretched through 2mm. Calculate the work done during starching the wire with Young's modulus $1.2x10^{11}N/m^2$.
5. When a wire is stretched, on amount of work is done. What is the amount of work done in stretching a wire through 0.1mm, if its length is 2m and area of cross-section $1x10^{-6}m^2$.($Y=2x10^{11}N/m^2$)
6. The elastic modulus of a certain metal is $1x10^{11}p_a$. If a force of 5000N is applied to a 1m long piece of this metal with a cross-sectional area of $1x10^{-6}m^2$, what is the new length of the wire?
7. A cord having an original length of 100cm is pulled by a force F. If the change in length of the cord and the Young's modulus of the cord are 2mm and $2.5x10^{11}N/m^2$respectively, determine the
(a) Strain (b) Stress (c) force applied perpendicular to a cross-section of $4mm^2$.
8. A string 4mm in diameter has original length of 2m. The string is pulled by a force of 200N. If the final length of the string is 2.02m, determine the (a) Stress (b) Strain (c) Young's Modulus.
9. A wire has a diameter of 1cm and the original length of 2m. The wire is pulled by a force of 200N. Determine the change in length of the wire. ($Y= 5x10^9N/m^2$)
10. Find the maximum load which may be placed on a tungsten wire of diameter 2mm so that the permitted stain is not exceed 1/1000.($Y= 3.5x10^{11}N/m^2$).
11. The Elastic limit is exceeded when the strain in a certain wire with Young's Modulus $14x10^{11}Pa$ exceeds 1/2000. If the cross-sectional area of the wire is $0.02cm^2$, find the maximum load that can be used for stretching the wire without causing a permanent set.
12. The elastic limit of a steel wire is exceeded when the stress on a given steel wire exceeds 8.26 $x10^8N/m^2$. Can the steel wire 2m long be stretched by 10mm without exceeding the elastic limit? ($Y= 2x10^{11}N/m^2$)
13. A wire of 9.68 $x10^{10}N/m^2$ with diameter 0.95mm is stretched by applying a certain force. What should be the limit of this force if the strain is not to exceed 1 in 1000?
14. The elastic limit of copper wire is 1.5 $x10^8N/m^2$. Suppose a 10kg mass is suspended at the free end of the wire and stretched. Find the minimum diameter the wire must have if the elastic limit is not to be exceeded?
15. What would be the greatest length of a steel wire which when fixed at one end can hang freely without breaking? Density of the steel is $7800kg/m^3$. Breaking stress for steel is 7.8 $x10^8N/m^2$.

16. A 2m long wire and cross-sectional area of 10^{-4}m2 is stretched by a load of 100kg. if the wire is stretched by 0.1cm, calculate the longitudinal stress, strain and Young's modulus of the wire.
17. A cylindrical rod of diameter 10mm and length 1m is fixed at one end. The other end is twisted by an angle of 100 by applying a torque. If the maximum shear strain in the rod is p x 10^{-3}, then what is the value of P?
18. The area of the upper face of a rectangular block is 0.5mx0.8m and the lower face is fixed. The height of the block is 2cm and a shearing force applied to the top face produces a displacement of 0.015mm. Find the (a) Strain (b) Stress (c) Shearing force. Modulus of rigidity is G= 4.5x10^{10}N/m^2.
19. A metallic cube of side 5 cm, has its lower surface fixed rigidly. When a tangential force of 9.8 x$10^4 N$ is applied to the upper surface, it is displacement through 0.03mm. Calculate the
(a) Shear stress (b) Shear strain (c) Shear Modulus of the material.
20. A metallic wire (Y= 20x10^{10}N/m^2, poison's ratio =$\delta = 0.26$) of length 3m and diameter of 0.10cm is stretched by a load of 10kg suspended at one end. Calculate the decrease in diameter of the wire.
21. A copper wire 3m long and 1mm^2 in cross-sections is fixed at one end and a weight of 10kg is attached at the free end. If Y for copper wire is 12.5x10^{10}N/$m^2$and Poisson's ratio =$\delta = 0.25$, find the extension, lateral strain and lateral compression produced in the wire.
22. Find the longitudinal stress to be subject to a wire to reduce its diameter uniformly by 1%. Poisson's ratio is 0.25, and Y = 2x 10^{11}N/m^2.
23. A solid rubber ball has its volume reduced by 14.4% when subjected to uniformly stress of 1.45x10^4N/m^2. Find the Bulk Modulus for the rubber.
24. What pressure should be applied to a lead block to reduce its volume by 10% if the Bulk Modulus for the lead is 6x10^9N/m^2?
25. A spherical ball contracts in volume by 0.01% when subjected to a normal uniform pressure of 10^8N/m^2. Find the Bulk Modulus of the material.
26. What is the maximum torque transmitted by a hollow shaft of external radius **R** internal radius **r** and maximum allowable Torsional shear stress ($\sigma_{\tau max}$)?
27. When a force of 100 N is applied to an object, it is compressed by 1.5 mm. Calculate the strain energy.
28. A rod of area 100 mm^2 has a length of 5 m. Determine the strain energy if the stress of 500 MPa is applied when stretched. Young's Modulus is taken to be 200 GPa.
29. Calculate the strain energy if the stress of 35 Pa is applied on a body of area of 12 m^2 and length of 4 m. The value of Young's modulus is given as 25Pa.
30**.** A 600 N weight is hanging from the end of a cable with a total volume of 0.5 m^3, causing a stress of 20,000 N/m^2 in the cable. If the Young's modulus of the cable is 36,000 N/m^2, how much strain energy is stored in the cable?
31. a) What is the maximum load which can be hung from a steel wire 5mm in diameter if its elastic limit is 250Mpa? Y=207,000Mpa for steel.
b) Determine the increase in length under this load if the original length is 0.90m.

32. A wire whose cross-sectional area is $4mm^2$ is stretched by 0.1mm by a certain weight. How far will a wire of the same material and length stretch if its cross-sectional area is 8mm2 and the same weight is attached?

33. The pressure of a medium is changed from $1.01x10^5$pa to $1.165x10^5$pa and changed in volume is 10% keeping temperature constant. Find the bulk modulus of the medium.

34.A steel bar 6m long and with rectangular cross-section of 5cmx2.5cm supports a mass of 2000kg. How much is the bar stretched? $Y=20x10^{10}N/m^2$. $g=10m/s^2$

35. Determine the bulk modulus of an oil if a volume of $1000cm^3$ shows a decrease in volume of $0.30cm^3$ when subjected to a pressure of 20atm. B=1700Mpa

36. A cube of copper 5cm on an edge is subjected to oppositely directed shearing forces along two of its faces. Each force is 900N. Determine the a) shearing stress b) shearing strain c) angle through which the cube has been sheared.