EXERCISE # (S)

1. In system called the star system we have 1 star kilogram = 10^{20} kg. 1 starmetre = 10^8 m, 1 starsecond = 10^3 second then calculate the value of 1 joule in this system.

Ans. 10^{-30} star joule

Sol. 1 Joule = x Joule e*

$ML^2T^{-2} = x\,[ML^2T^{-2}]^*$

$2kg\ 1m^2 1s^{-2} = x\ kg^x\ m^{2x}\ T^{-2*}$

$x = \frac{1}{10^{20}} \times \frac{1}{10^8} \times \frac{1}{10^3}$

$\boxed{x = 10^{-31*}\text{Star joule}}$

2. A particle is in a uni-directional potential field where the potential energy (U) of a particle depends on the x-coordinate given by $U_x = k\,(1 - \cos ax)$ & k and 'a' are constants. Find the physical dimensions of 'a' & k.

Ans. L^{-1}, ML^2T^{-2}

Sol. $Ux = k - k\cos ax$

We Know that

$U_x = k$ …(i) $W_x = k\cos ax$ …(ii)

From (i) & (ii)

$\underbrace{\cos ax}_{\text{Dimenstion less}} = 1$

$\therefore$ $ax = 1$

$a = \frac{1}{x}$ $\boxed{a = L^{-1}}$

3. The equation for the speed of sound in a gas states that $v = \sqrt{\frac{\gamma k_B T}{m}}$. Speed v is measured in m/s, γ is a dimensionless constant, T is temperature in kelvin (K), and m is mass in kg. Find the SI unit for the Boltzmann constant, k_B?

Ans. $kg.m^2.s^{-2}.K^{-1}$

Sol. $K_B = \frac{V^2 m}{T} = \frac{L^2T^{-1}M}{K} = ML^2T^{-2}K^{-2}$

$= kg\ M^1S^{-2}K^{-1}$

4. If Energy (E), velocity (v) and time (T) are fundamental units. What will be the dimension of surface tension?

Ans. $[S] = Ev^{-2}T^{-2}$

Sol. $S = KE^aV^bT^c$ $\because \left(S = \frac{F}{L}\right) \Rightarrow MT^{-2}$

$MT^{-2} = K[ML^2T^{-2}]^a [LT^{-2}]^b [T]$

$MT–2 = KM^aL^{2a}T^{-2a}\ L^a\ T^{-b}\ T^c$

Compare power

$\boxed{a = 1}$ $\qquad 2a + b = 0$

$\boxed{b = -2}$

$-2\times1 + 0 + c = -2$

$\boxed{c = -2}$

$\therefore \qquad \boxed{S_T = \dfrac{KE}{V^2T^2}}$

5. The time period (T) of a spring mass system depends upon mass (m) & spring constant (k) & length of the spring (ℓ) $\left[k = \dfrac{Force}{length}\right]$. Find the relation among. (T), (m), (ℓ) & (k) using dimensional method.

Ans. $T = a\sqrt{\dfrac{m}{k}}$

Sol. $T \propto m^a k^b \ell^c$

$T = (R)\ m^a k^b \ell^c$ Proptionality R = Constant

$T1 = M^a\ M^b\ T^{-2b}\ L^c$

$a + b = 0 \qquad -2b = 1 \qquad \boxed{c = 0}$

$\boxed{b = -\dfrac{1}{2}}$

$\boxed{a = \dfrac{1}{2}}$

$\boxed{T = R\sqrt{\dfrac{m}{k}}}$

6. The distance moved by a particle in time t from centre of a ring under the influence of its gravity is given by x = a sinωt, where a &ω are constants. If ω is found to depend on the radius of the ring (r), its mass (m) and universal gravitational constant (G), find using dimensional analysis an expression for ω in terms of r, m and G.

Ans. $\omega = K\sqrt{\dfrac{GM}{r^3}}$

Sol. $W \propto r^a\ m^b G^c$

$W = Kr^a\ m^b\ G^c$

$T^{-1} = K L^a m^b M^{-c} L^{3C} T^{-C}$

$b - c = 0$ …..(1) $3C + a = 0$ …(2) $-2C = 1$ ….(3)

$\boxed{c = -\frac{1}{2}}$ $\boxed{b = \frac{1}{2}}$ $\boxed{c = \frac{3}{2}}$

$\boxed{w = K\sqrt{\frac{r^3}{mG}}}$

7. A satellite is orbiting around a planet. Its orbital velocity (v_0) is found to depend upon
(a) Radius of orbit (R) (b) Mass of planet (M) (c) Universal gravitation constant (G)
Using dimensional analysis find an expression relating orbital velocity (v_0) to the above physical quantities.

Ans. $v_0 = k\sqrt{\frac{GM}{R}}$

Sol. $V_0 \propto R^a M^b G^c$

$V_0 = KR^a M^b G^c$

$LT^{-1} = KL^a M^b M^{-c} L^{3c} T^{-2c}$

$b - c = 0$ ….(1) $3c + a = 1$ …(2) $2c = 1$ …(3)

$\boxed{a = \frac{-3}{2}}$ $\boxed{b = \frac{1}{2}}$ $\boxed{c = \frac{1}{2}}$

8. Assume that the largest stone of mass 'm' that can be moved by a flowing river depends upon the velocity of flow v, the density d & the acceleration due to gravity g. If 'm' varies as the K^{th} power of the velocity of flow, then find the value of K.

Ans. K = 6

Sol. $m = KV^{k+h} d^b g^c$

$M^1 = KL^{Kth} T^{-Kth} M^b L^{-3b} L^c T^{-2c}$

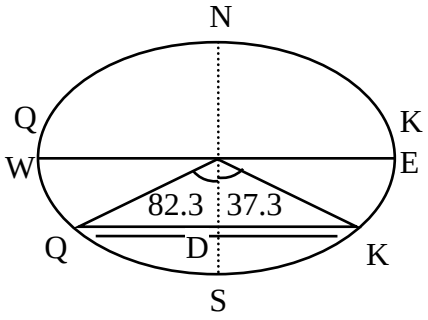
$-3b + K^{th} + c = 0$ …(1) $-2c - K^{th} = 0$ …(2)

$\boxed{b = 1}$ $\boxed{K^{th} = 6}$ $\boxed{c = -3}$

9. Quito, a city in Ecuador and Kampala, a city situated in Uganda both lie on the Equator. The longitude of Quito is 82°30' W and that of Kampala is 37°30' E. What is the distance from Quito to Kampala.
(A) along the shortest surface path
(B) along a direct (through-the-Earth) path? (The radius of the Earth is 6.4×10^6 m)

Ans. (A) $\frac{2\pi}{3} \times 6.4 \times 10^6 \text{m}$; (B) $\sqrt{3} \times 6.4 \times 10^6 \text{m}$

Sol. $\theta = 120°$ (Approx)



(A) A_{xc} = O × Radius of Earth

$$= \frac{2\pi}{3} \times 6.4 \times 10^6 \text{m}$$

(B) $D = 2r\sin\theta = \sqrt{3} \times 6.4 \times 10^6 \text{m}$

10. Use the approximation $(1+x)^n \approx 1 + nx$, $|x| << 1$, to find approximate value for (A) $\sqrt{99}$ (B) $\frac{1}{1.01}$

Ans. (A) 9.95, (B) 0.99

Sol. (A) $\sqrt{99} \Rightarrow \sqrt{100-1} \Rightarrow (100-1)^{1/2} \Rightarrow 100^{1/2}\left[1-\frac{1}{100}\right]^{1/2}$ $\frac{1}{100} <<< 1$

$$\therefore \quad 100^{1/2}\left[1-\frac{1}{200}\right] \Rightarrow 10\left[\frac{199}{200}\right] = 9.95.$$

(B) $\frac{1}{1.01} \Rightarrow 1(1.01)^{-1} \Rightarrow 1\left[1+\frac{1}{100}\right]^{-1} \Rightarrow \left[1-\frac{1}{100}\right] = 99.$

11. Use the small angle approximations to find approximate values for (A) sin 8° and (B) tan 5°

Ans. 2π/45 ,π/36

Sol. SAA is valid only in 'Radius' on Angle

(a) $8 \Rightarrow 8 \times \frac{\pi}{180}$

$$= \frac{2\pi}{45}$$

$$\sin\frac{2\pi}{45} \approx \frac{2\pi}{45} \Rightarrow \frac{2\times 3.14}{45} = 0.14$$

Similarly

(b) $\tan 5° \Rightarrow 5 = 5 \times \frac{\pi}{180} = \frac{\pi}{36}$

$$\frac{\sin 5}{\cos 5} = \frac{\sin\frac{\pi}{36}}{\cos\frac{\pi}{36}} = \frac{\frac{\pi}{36}}{1} = \frac{\pi}{36} = 0.087 = 0.09$$

12. Given $\vec{F} = \frac{\vec{a}}{t}$ where symbols have their usual meaning. The dimensions of a is.

Ans. $[MLT^{-1}]$

Sol. $\vec{a} = \vec{F}t \Rightarrow$ (Impulse)

$\vec{a} = ML^{-2}T^{-1}$

$\boxed{\vec{a} = MLT^{-1}}$

13. The position of a particle at time t is given by the relation

$x(t) = \left(\frac{v_0}{\alpha}\right)(1 - e^{-\alpha t})$ where v_0 is a constant and $\alpha > 0$. Find the dimensions of v_0 and α.

Ans. $[v_0] = [M^0L^1T^{-1}]$, $[\alpha] = [M^0L^0T^{-1}]$

Sol. $x(d) = \frac{V_0}{2}(1 - \alpha^{-\alpha t})$

$x(t) = \frac{V_0}{2}$ ...(1) $x(t) = \frac{V_0}{2}\alpha^{-\alpha t}$ …(2) $\alpha t = 1$

$V_0 = L^1T^{-1}$ from (i) $\boxed{\alpha = T^{-1}}$ from (ii)

14. The equation of state for a real gas at high temperature is given by $P = \frac{nRT}{V-b} - \frac{a}{T^{1/2}V(V+b)}$

where n, P, V & T are number of moles, pressure, volume & temperature respectively & R is the universal gas constant. Find the dimensions of constant *a* in the above equation.

Ans. $ML^5T^{-2}K^{1/2}$

Sol. $P = \frac{nRT}{V-b} - \frac{a}{T^{1/2}V(V+b)}$

$V - b = \frac{nRT}{p} \Rightarrow b = V - \frac{nRT}{p}$ $\boxed{b = V}$

$a = PT^{1/2}V(V + V)$

$a = 2PT^{1/2}V^2$ $\Rightarrow \frac{ML^0T^{-2}}{L^2}K^{1/2}L^6$

$\boxed{a = ML^2T^{-2}K^{1/2}}$

HCV Exercises (Chapter No.1 :- 1,5,7,8,10,14,16,17,19)

EXERCISE # (O)

1. The dimensional formula for which of the following pair is not the same

(A) impulse and momentum (B) torque and work

(C) stress and pressure (D) momentum and angular momentum

Ans. (D)

Sol. (a) $Ft = mv$ (b) $F \times r = F.d$ Same

$\boxed{F = \frac{mv}{t}}$ Samepeir

(c) stress = press (d) $mx = t \times V$

$\frac{F}{A} = \frac{F}{A}$ $\boxed{t = m}$ Not same

$\boxed{t = \text{Turgul}}$

2. Which of the following can be a set of fundamental quantities

(A) length, velocity, time (B) momentum, mass, velocity

(C) force, mass, velocity (D) momentum, time, frequency

Ans. (C)

Sol. See all four option and check which one is derived to each other and Independent each other

3. Which of the following functions of A and B may be performed if A and B possess different dimensions?

(A) $\frac{A}{B}$ (B) A + B (C) A – B (D) None of these

Ans. (A)

Sol. In addition & subtraction they passes same dimension but in multiplication and dividation not.

4. Density of wood is 0.5 g/ cc in the CGS system of units. The corresponding value in MKS units is

(A) 500 (B) 5 (C) 0.5 (D) 5000

Ans. (A)

Sol. CGS $\Rightarrow$ MKS

$0.5 g/cc \Rightarrow \frac{5}{10} \times \frac{1000000}{1000}$

$\Rightarrow 500 \text{ kg/m}^3$

5. If area (A), velocity (v), and density (ρ) are base units, then the dimensional formula of force can be represented as

(A) $Av\rho$ (B) $Av^2\rho$ (C) $Av\rho^2$ (D) $A^2v\rho$

Ans. (B)

Sol. $F = KA^a V^b e^c$

$MLT^{-2} = L^{2a} L^b T^{-b} M^c L^{-3c}$

$\boxed{c = 1}$

$2a + b - 3c = 1$ $-b = -3$ $\boxed{b = 2}$ $\boxed{F = AV^2e}$

$2a = 1 + 3 = 2$ $\boxed{a = 1}$

6. The velocity v of a particle at time t is given by $v = at + \frac{b}{t+c}$, where a, b and c are constants. The dimensions of a, b and c are respectively :
(A) LT^{-2}, L and T (B) L^2, T and LT^2 (C) LT^2, LT and L (D) L, LT and T^2

Ans. (A)

Sol. $V = at + \frac{b}{t+c}$

$At = v$ …(1) $a = \frac{V}{t} \Rightarrow LT^{-2}$

(b) $Vt + Vc = b$ $\boxed{b = Vt \Rightarrow L}$

(c) $Vc = Vt$ $\boxed{c = T}$

7. If force, acceleration and time are taken as fundamental quantities, then the dimensions of length will be: (A) FT^2 (B) $F^{-1}A^2T^{-1}$ (C) FA^2T (D) AT^2

Ans. (A)

Sol. $L = F^a a^b t^c$

$L' = M^a L^a T^{-2a} L^b T^{-2b} T^c$

$\boxed{a = 0}$ ...(1) $a + b = 1$ …(2) $-2a + c = 0$ …(3)

$\boxed{b = 1}$ $\boxed{c = 2}$

$\boxed{L = AT^2}$

8. In a particular system the units of length, mass and time are chosen to be 10 cm, 10 g and 0.1 s respectively. The unit of force in this system will be equal to
(A) 0.1 N (B) 1 N (C) 10 N (D) 100 N

Ans. (A)

Sol. Let the system 'x' N

$F = MLT^{-2}$

$= \frac{10}{1000} \times \frac{10}{100} \times 100$

$\boxed{F = 0.1N}$

9. In a certain system of units, 1 unit of time is 5 sec, 1 unit of mass is 20 kg and unit of length is 10 m. In this system, one unit of power will correspond to

(A) 16 watts (B) $\frac{1}{16}$ watts (C) 25 watts (D) None of these

Ans. (A)

Sol. New Power (P) $= \frac{ML^2}{T^3}$

$= \frac{20 \times 100}{125} = 16$ Watt

10. If the unit of length be doubled then the numerical value of the universal gravitation constant G will become (with respect to present value)
(A) Double (B) Half (C) 8 times (D) 1/8 times

Ans. (D)

Sol. $G = L^3$ ….(1) If double New $G' = (2L)^3$
$= 8L^3$ ...(2)

Divide (1) & (2)

$$\frac{G}{G'} = \frac{L^3}{8L^3}$$

$$\boxed{G = \frac{1}{8}G'}$$

11. If in a system, the force of attraction between two point masses of 1 kg each situated 1 km apart is taken as a unit of force and is called Newton (Newton written in reverse order) & if $G=6.67\times10^{-11}$N-m^2 kg^{-2} in SI units then which of the following is true?
(A) 1 Newton = 6.67×10^{-11} Newton (B) 1 Newton = 6.67×10^{-17} Newton
(C) 1 Newton = 6.67×10^{-17} Newton (D) 1 Newton = 6.67×10^{-12} Newton

Ans. (C)

Sol. New Unit (1 Newton) = $\frac{G}{1000000}$

1 Newton = 6.67×10^{-17} Newton

12. The units of three physical quantities x, y and z are gcm^2 s^{-5}, gs^{-1} and cms^{-2} respectively. The relation between x, y and z is
(A) $z = x^2y$ (B) $y^2 = xz$ (C) $x = yz^2$ (D) $x = y^2z$

Ans. **(C)**

Sol. $x = gcm^2s^{-5}$
$y = g\ s^{-1}$
$z = cms^{-1}$ Same ring 'z' & multiply y2 which
$z^2 = cm^2s^{-4}$ is equal to 'x'

$$\boxed{x = yz^2}$$

13. In a book, the answer for a particular question is expressed as $b = \frac{ma}{k}\left[\sqrt{1+\frac{2k\ell}{ma}}\right]$ here m represents mass, a represents acceleration, ℓ represents length. The unit of b should be
(A) m/s (B) m/s^2 (C) meter (D) /sec

Ans. (C)

Sol. $b = \frac{ma}{K}\left[\sqrt{1+\frac{2k\ell}{ma}}\right]$

Squaring both side and Apply dimension Rule

$b^2 = \frac{m^2a^2}{K^2}$ $\qquad b^2 = \frac{2\ell ma}{K}$ …(2)

$b = \frac{ma}{K}$ …(1) $\qquad b^2 = 2\ell b$

Putting (2) $\qquad b = 2\ell$ $\qquad \boxed{b = \text{meter}}$

$\boxed{b = L}$

14. In two different systems of units an acceleration is represented by the same number, while its velocity is represented by numbers in the ratio 1:3. The ratios of unit of length and time are respectively

(A) $\frac{1}{3}, \frac{1}{9}$ (B) $\frac{1}{9}, \frac{1}{3}$ (C) 1, 1 (D) None of these

Ans. (B)

Sol. $a_1 = a_2$

$L_1T_1^{-2} = L_2T_2^{-2}$ …(1)

$\frac{L_1T_1^{-1}}{L_2T_2^{-2}} = \frac{1}{3}$ …(2)

$3V_1 = V_2$ …(3)

for two different system

$\frac{n_1}{n_2} = \frac{v_1}{v_2}$ [when n_1 & n_2 ® Numerical value v_1 & $v_2 \Rightarrow$ unit]

$\frac{n_1}{n_2} = \frac{L_1T_1^{-2}}{L_2T_2^{-2}} \frac{L_1T_1^{-2}}{L_2T_2^{-2}} = 1$ ….(4) $\qquad \frac{L_2T_2^{-1}}{L_1T_1^{-1}} = \frac{1}{3}$ …(5)

From (5) $\left(\frac{L_1T_1^{-1}}{L_2T_2^{-1}}\right)\frac{T_1^{-1}}{T_2^{-1}} = 1$ $\qquad \boxed{\frac{T_2}{T_1} = \frac{1}{3}} \Rightarrow$ Putting (5)

$\boxed{\frac{L_2}{L_1} = \frac{1}{9}}$

15. **Statement 1 :**Method of dimensions cannot tell whether an equation is correct.
and
Statement 2 :A dimensionally incorrect equation may be correct.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

Ans. (C)

Sol. Method of dimension is to find the dimension formula. If is not given that equations Never be correct.

$\therefore$ Statement 1 – true
Statement 2 – false

16. **Statement 1 :**Whenever change the unit of measurement of a quantity, its numerical value changes.
and
Statement 2 : Smaller the unit of measurement smaller is its numerical value.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

Ans. (C)

Sol. Numerical value is Inversely proportional to unit.

$$\boxed{N \propto \frac{1}{V}}$$

S-I true
S-II false $\therefore$ (C)

17. Due to some unknown interaction, force F experienced by a particle is given by the following equation.

$$\vec{F} = -\frac{A}{r^3}\vec{r}$$

Where A is positive constant and r distance of the particle from origin of a coordinate system. Dimensions of constant A are
(A) ML^2T^{-2} (B) ML^3T^{-2} (C) ML^4T^{-2} (D) ML^0T^0

Ans. (B)

Sol. $F = \frac{-A}{r^3}\vec{r}$

$$A = \left|\frac{Fr^3}{\vec{r}}\right|$$

$A = Fr^2 = ML^3T^{-2}$ (B)

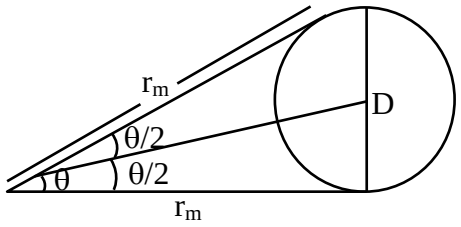
18. The angle subtended by the moon's diameter at a point on the earth is about 0.50°. Use this and the fact that the moon is about 384000 km away to find the approximate diameter of the moon.



(A) 192000 km (B) 3350 km
(C) 1600 km (D) 1920 km

Ans. (B)

Sol.



$$\text{Sin}\ \frac{\theta}{2} = \frac{D}{2\times r_m}$$

$$D = 2r_m \sin\frac{\theta}{2} \qquad \frac{\theta}{2} \text{ <<<}$$

$$= 2 \times 384000 \times \frac{1}{2}\times\frac{\pi}{180} \qquad \therefore \sin\frac{\theta}{2} = \frac{\theta}{2}$$

$$= 3349.33$$

$$= 3350 \text{ km}$$

19. Match the following :

Physical quantity	**Dimension**	**Unit**
(1) Gravitational constant 'G'	(P) $M^1L^1T^{-1}$	(a) N.m
(2) Torque	(Q) $M^{-1}L^3T^{-2}$	(b) N.s
(3) Momentum	(R) $M^1L^{-1}T^{-2}$	(c) Nm^2/kg^2
(4) Pressure	(S) $M^1L^2T^{-2}$	(d) pascal

Ans. (1) → (Q) → (c) , (2) → (S) → (a) , (3) → (P) → (b) , (4) → (R) → (d)

Sol. Refer Relevant formula which is in theory .

20. Match the following :

Physical quantity	**Dimension**	**Unit**
(i) Stefan's constant 'σ'	(P) $M^1L^1T^{-2}A^{-2}$	(a) W/m^2
(ii) Wien's constant 'b'	(Q) $M^1L^oT^{-3}K^{-4}$	(b) K.m.
(iii) Coefficient of viscosity 'η'	(R) $M^1L^oT^{-3}$	(c) tesla .m/A
(iv) Emissive power of radiation (Intensity emitted)	(S) $M^oL^1T^oK^1$	(d) $W/m^2.K^4$
(v) Mutual inductance 'M'	(T) $M^1L^2T^{-2}A^{-2}$	(e) poise
(vi) Magnetic permeability 'μ_0'	(U) $M^1L^{-1}T^{-1}$	(f) henry

Ans. (i) → (Q) → (d)
(ii) → (S) → (b)
(iii) → (U) → (e)
(iv) → (R) → (a)
(v) → (T) → (f)
(vi) → (P) → (c)

Sol. Refer Relevant formula which is in theory .

21. In a new system of units known as RMP, length is measured in 'retem', mass is measured in 'marg' and time is measured in 'pal'.

100 retem = 1.0 meter
1.0 marg = 10^{-3} kilogram
10 pal = 1.0 second

In the given table some unit conversion factors are given. Suggest suitable match.

Column-I	**Column-II**
(A) One SI unit of force	(P) 10^2 units of RMP
(B) One SI unit of potential energy	(Q) 10^3 units of RMP
(C) One SI unit of power	(R) 10^4 units of RMP
(D) One SI unit of momentum	(S) 10^5 units of RMP
	(T) 10^6 units of RMP

Ans. **(A) → (Q); (B) → (S); (C) → (R); (D) → (R)**

Sol. 100 retern = 1 meter

1000 marg = 1 kg.

10 pal = 1 sec.

(A) $F = MLT^{-2} = 1000 \times 100 \times \frac{1}{100} = 10^3$ RMP

(B) $V = ML^2T^{-2} = 1000 \times 100000 \times \frac{1}{100} = 10^5$ RMP

(C) $P = \frac{W}{t} = 1000 \times 10000 \times \frac{1}{1000} = 10^4$ RMP

(D) $P = mV = 1000 \times 100 \times \frac{1}{10} = 10^4$ RMP.

Comprehension # 1

The Vander waal equation for 1 mole of a real gas is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

where P is the pressure, V is the volume, T is the absolute temperature, R is the molar gas constant and a, b are Vander waal constants.

22. The dimensions of a are the same as those of

(A) PV (B) PV^2 (C) P^2V (D) P/V

Ans. (B)

23. The dimensions of b are the same as those of

(A) P (B) V (C) PV (D) nRT

Ans. (B)

24. The dimensional formula for ab is

(A) $ML^2 T^{-2}$ (B) $ML^4 T^{-2}$ (C) $ML^6 T^{-2}$ (D) $ML^8 T^{-2}$

Ans. (D)

Sol. **(22 to 24)**

$$\left(P + \frac{a}{V^2}\right)(V - B) = RT$$

$$PV - Pb + \frac{a}{V} - \frac{ab}{V^2} = RT$$

$PV = RT$ …(1) $Pb = RT$ …(2) $a = VRT$ …(3)

$$\frac{ab}{V^2} = RT \quad \ldots(4)$$

(22) From (1) & (3) $\boxed{a = PV^2}$

(23) From (1) & (2) $\boxed{b = V}$

(24) From (i) & (4) $ab = PV^3$

$$\boxed{ab = ML^8T^{-2}}$$

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EXERCISE # (JM)

1. Identify the pair whose dimensions are equal **[AIEEE-2002, 4/300]**
(1) Torque and work (2) Stress and energy
(3) Force and stress (4) Force and work
Ans. (1)

2. The physical quantities not having same dimensions are - **[AIEEE-2003, 4/300]**
(1) stress and Young's modulus (2) speed and $(\mu_0 \in_0)^{-1/2}$
(3) torque and work (4) momentum and Planck's constant
Ans. (4)

3. Dimensions of $1/(\mu_0 \in_0)$ where symbols have their usual meaning are - **[AIEEE-2003, 4/300]**
(1) L^2T^{-2} (2) L^2T^2 (3) $L^{-1}T$ (4) LT^{-1}
Ans. (1)

4. The dimensions of the coefficient of viscosity are - **[AIEEE-2004, 4/300]**
(1) $ML^{-1}T^{-1}$ (2) MLT (3) $M^{-1}L^{-1}T^{-1}$ (4) $M^0L^0T^0$
Ans. (1)

5. Out of the following pairs, which one does not have identical dimensions? **[AIEEE-2005, 4/300]**
(1) Angular momentum and Planck's constant
(2) Impulse and momentum
(3) Moment of inertia and moment of a force
(4) Work and torque
Ans. (3)

6. Which of the following units denotes the dimensions ML^2/Q^2, where Q denotes the electric charge? **[AIEEE-2006, 3/180]**
(1) H/m^2 (2) Weber (Wb) (3) Wb/m^2 (4) Henry (H)
Ans. (4)

7. The dimension of magnetic field in M, L, T and C (Coulomb) is given as **[AIEEE-2008, 3/105]**
(1) MT^2C^{-2} (2) $MT^{-1}C^{-1}$ (3) $MT^{-2}C^{-1}$ (4) $MLT^{-1}C^{-1}$
Ans. (2)

8. Let $[\in_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then : **[JEE-MAIN - 2013, 4/120,–1]**
(1) $[\in_0] = [M^{-1} L^{-3} T^2 A]$ (2) $[\in_0] = [M^{-1} L^{-3} T^4 A^2]$
(3) $[\in_0] = [M^{-1} L^2 T^{-1} A^{-2}]$ (4) $[\in_0] = [M^{-1} L^2 T^{-1} A]$
Ans. (2)

9. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion withh frequency.

[JEE Mains - 2013]

(1) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (2) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$ (3) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$ z

Ans. **(3)**

10. Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to : **[JEE Main-2019]**

(1) $\sqrt{\frac{Gh}{c^3}}$ (2) $\sqrt{\frac{Gh}{c^5}}$ (3) $\sqrt{\frac{hc^5}{G}}$ (4) $\sqrt{\frac{c^5}{Gh}}$

Ans. **(2)**

Sol. Let $t = G^P h^Q C^R$

$\Rightarrow$ $[T] = [M^{-1}L^3T^{-2}]^P [ML^2T^{-1}]^Q [LT^{-1}]^R$

$[T] = M^{-P+Q} L^{3P+2Q+R} T^{-2P-Q-R}$

$\Rightarrow$ $-P + Q = 0$, $3P + 2Q + R = 0$ and $-2P - Q - R = 1$

Solving above,

$\Rightarrow$ $P = Q = \frac{1}{2}$ and $P = \frac{-5}{2}$

$\Rightarrow$ $t \propto \sqrt{\frac{Gh}{c^5}}$

11. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is : **[JEE Main-2019]**

(1) 640 (2) 16 (3) 410 (4) 40

Ans. **(4)**

Sol. nu = constant (n $\rightarrow$ numerical value, u $\rightarrow$ unit)

$n_1u_1 = n_2u_2$

$n_1[M_1L_1^{-3}] = n_2[M_2L_2^{-3}]$

$128\ [kg \times (m)^{-3}] = n_2\ [(50g)\ (25\ cm)^{-3}]$

$n_2 = 128\left(\frac{kg}{50g}\right)^1 \times \left(\frac{m}{25cm}\right)^{-3}$

$= 128\left(\frac{1000}{50}\right) \times \left(\frac{100}{25}\right)^{-3}$

= 40 units

12. The force of interaction between two atoms is given by $F = \alpha\beta \exp\left(-\frac{x^2}{\alpha kt}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is : **[JEE Main-2019]**

(1) $M^2L^2T^{-2}$ (2) $M^0L^2T^{-4}$ (3) MLT^{-2} (4) M^2LT^{-4}

Ans. **(4)**

Sol. $\frac{X^2}{\alpha KT} = M^0L^0T^0$

$\Rightarrow \alpha = M^{-1}T^2$

$[F] = \alpha\beta$

$MLT^{-2} = M^{-1}T^2[\beta]$

$\Rightarrow \beta = M^2LT^{-4}$

13. If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be :-

[JEE Main-2019]

(1) $V^{-2}A^2F^{-2}$ (2) $V^{-4}A^{-2}F$ (3) $V^{-2}A^2F^2$ (4) $V^{-4}A^2F$

Ans. **(2)**

Sol. By Dimensional Analogies

$[Y] \Rightarrow ML^{-1}T^{-2}$

$F \Rightarrow MLT^{-2}$; $V = LT^{-1}$; $d = LT^{-2}$

With help equation

$Y \Rightarrow F^x \cdot A^y \cdot V^z$

$x \Rightarrow 1$; $y = 2$; $z = -4$

Thus $\boxed{Y \Rightarrow F.A^2V^{-4}}$

14. Let ℓ, r, c and v represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{\ell}{rcv}$ in SI units will be:

[JEE Main-2019]

(1) $[A^{-1}]$ (2) [LTA] (3) $[LA^{-2}]$ (4) $[LT^2]$

Ans. **(1)**

Sol. $\frac{\ell}{rcv} = r \times \frac{\ell}{r} \times \frac{1}{rc} \times \frac{1}{v}$

$= r \times t \times \frac{1}{t} \times \frac{1}{v}$

$= \frac{r}{v} = [A^{-1}]$

15. In SI units, the dimensions of $\sqrt{\frac{\epsilon_0}{\mu_0}}$ is: **[JEE Main-2019]**

(1) $AT^2M^{-1}L^{-1}$ (2) $A^{-1}TML^3$ (3) $AT^{-3}ML^{3/2}$ (4) $A^2T^3M^{-1}L^{-2}$

Ans. **(4)**

Sol. Dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$

$[\varepsilon_0]= [M^{-1}L^{-3}T^4A^2]$ and $[\mu_0] = [MLT^{-2}A^{-2}]$

Dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}} = \left[\frac{M^{-1}L^{-3}T^4A^2}{MLT^{-2}A^{-2}}\right]^{\frac{1}{2}}$

$=[M^{-1}L^{-2}T^3A^2]$

16. If Surface tension (S), Moment of Inertia (I) and Planck's constant (h), where to be taken as the fundamental units, the dimensional formula for linear momentum would be : **[JEE Main-2019]**

(1) $S^{3/2}I^{1/2}h^0$ (2) $S^{1/2}I^{3/2}h^{-1}$ (3) $S^{1/2}I^{1/2}h^0$ (4) $S^{1/2}I^{1/2}h^{-1}$

Ans. **(3)**

Sol. $p = k\, s^a I^b h^c$

where k is dimensionless constant

$MLT^{-1} = (MT^{-2})^a (ML^2)^b (ML^2T^{-1})^c$

$a + b + c = 1$

$2b + 2c = 1$

$-2a - c = -1$

$a = \frac{1}{2}$ $b = \frac{1}{2}$ $c = 0$

$s^{1/2} I^{1/2}h^0$

17. In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units ? **[JEE Main-2019]**

(1) $[M^{-3} L^{-2} T^8 A^4]$ (2) $[M^{-1} L^{-2} T^4 A^2]$

(3) $[M^{-2} L^{-2} T^6 A^3]$ (4) $[M^{-2} L^0 T^{-4}A^{-2}]$

Ans. **(1)**

Sol. $C = 5\, Y\, B^2$

$U = \frac{Q^2}{2C}$

$\Rightarrow [C] = \left[\frac{Q^2}{U}\right] = \left[\frac{I^2t^2}{U}\right]$

$F = BIL$

$\Rightarrow [B] = \left[\frac{F}{IL}\right]$

So $[Y] = \left[\frac{C}{B^2}\right] = \left[\frac{I^2t^2 \times I^2L^2}{UF^2}\right]$

$= \frac{A^4T^2L^2}{ML^2T^{-2}M^2L^2T^{-4}} = M^{-3}L^{-2}T^8A^4$

18. Which of the following combinations has the dimension of electrical resistance (ϵ_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum)? **[JEE Main-2019]**

(1) $\sqrt{\frac{\epsilon_0}{\mu_0}}$ (2) $\frac{\mu_0}{\epsilon_0}$ (3) $\frac{\epsilon_0}{\mu_0}$ (4) $\sqrt{\frac{\mu_0}{\epsilon_0}}$

Ans. **(4)**

Sol. $[\varepsilon_0] = M^{-1}L^{-3}T^4A^2$

$[\mu_0] = M\,L\,T^{-2}A^{-2}$

$[R] = M\,L^2\,T^{-3}A^{-2}$

$[R] = \left[\sqrt{\frac{\mu_0}{\varepsilon_0}}\right]$

19. The dimension of stopping potential V_0 in photoelectric effect in units of Planck's constant 'h', speed of light 'c' and Gravitational constant 'G' and ampere A is : **[JEE Main-2020]**

(1) $h^{-2/3}c^{-1/3}G^{4/3}A^{-1}$

(2) $h^{2/3}c^{5/3}G^{1/3}A^{-1}$

(3) $h^{1/3}G^{2/3}c^{1/3}A^{-1}$

(4) $h^2G^{3/2}c^{1/3}A^{-1}$

Ans. **Bonus, Answer given by NTA is (4) but none of the option matches**

Sol. $v_0 = h^x c^y G^z A^w$

$\frac{ML^2T^{-2}}{AT} = (ML^2T^{-1})^x (LT^{-1})^y (M^{-1}L^3T^{-2})^z A^w$

$\Rightarrow w = -1$

$(x - z = 1)$

$2x + y + 3x = 2$

$-x - y - 2z = -3$

Solving, we get

$x = 0$

$z = -1$

$y = 5$

$\Rightarrow v_0 = h^0 c^5 G^{-1} A^{-1}$

20. A quantity f is given by $f = \sqrt{\frac{hc^5}{G}}$ where c is speed of light, G universal gravitational constant and h is the Planck's constant. Dimension of f is that of : **[JEE Main-2020]**

(1) area (2) momentum (3) energy (4) volume

Ans. **(3)**

Sol. $[h]\ M^1L^2T^{-1}$

$[C] = L^1T^{-1}$

$[G] = M^{-1}L^3T^{-2}$

$$[f] = \sqrt{\frac{M^1L^2T^{-1} \times L^5T^{-5}}{M^{-1}L^3T^{-2}}} = M^1L^2T^{-2}\ b$$

EXERCISE # (JA)

1. A quantity X is given by $\varepsilon_0 L\frac{\Delta V}{\Delta t}$, where ε_0 is the permittivity of free space, L is length, ΔV is potential difference and Δt is time interval. The dimensional formula for X is the same as that of **[JEE (Scr.), 2001' 3/35, –1]**

(A) resistance (B) charge (C) voltage (D) current

Ans. **(D)**

Sol. $X = \varepsilon_0 L\frac{\Delta v}{\Delta t}$

$[X] = [\varepsilon_0][L]\left[\frac{\Delta v}{\Delta t}\right]$

$[\Delta v]$ = [Electric field] [Length

$= \frac{[\text{Force}]}{[\text{charge}]}$ [Length] $= \frac{MLT^{-2}L}{Q} = MQ^{-1}L^2T^{-2}$

$[\varepsilon_0] = M^{-1}L^{-3}T^4I^2$ (as in question no. 6)

$= M^{-1}L^{-3}Q^2T^2$

$\therefore [X] = M^{-1}L^{-3}Q^2T^2\ L\ \frac{MQ^{-1}L^2T^{-2}}{T}$

$= QT^{-1} = I$

$\therefore$ [x] is that of current

2. In the relation ; $P = \frac{\alpha}{\beta}e^{-\frac{\alpha Z}{k\theta}}$ **[JEE - 2004(Scr.) 3/84, –1]**

P is pressure, Z is distance, k is Boltzman constant and θ is the temperature. The dimensions of β will be:

(A) $[M^0 L^2 T^0]$ (B) $[ML^2 T]$ (C) $[ML^0 T^{-1}]$ (D) $[M^0 L^2 T^{-1}]$

Ans. **(A)**

Sol. $\left[\frac{\alpha Z}{k\theta}\right] = [M^0 L^0 T^0] \Rightarrow [\alpha] = \left[\frac{k\theta}{Z}\right]$

Further, $[P] = \left[\frac{\alpha}{\beta}\right] \Rightarrow [\beta] = \left[\frac{\alpha}{P}\right] = \left[\frac{k\theta}{ZP}\right]$

Dimensions of $k\theta$ are that of energy. Hence,

$$[\beta] = \left[\frac{ML^2T^{-2}}{LML^{-1}T^{-2}}\right] = [M^0L^2T^0]$$

Therefore, the correct option is (A).

3. Which of the following set have different dimensions? **[JEE-2005(Scr.); 3/84, –1]**

(A) Pressure, Young's modulus, Stress

(B) Emf, Potential difference, Electric potential

(C) Heat, Work done, Energy

(D) Dipole moment, Electric flux, Electric field

Ans. **(D)**

Sol. [Dipole moment] = LIT, $[\phi_E] = ML^3/IT^3$ $[E] = ML/IT^3$.

4. Some physical quantities are given in **Column I** and some possible SI units in which these quantities may be expressed are given in **Column II**. Match the physical quantities in **Column I** with the units in **Column II.** **[IIT-JEE-2007; 6/184]**

	Column I	**Column II**
(A)	GM_eM_s G - universal gravitational constant, M_e - mass of the earth, Ms - mass of the Sun	(p) (volt) (coulomb) (metre)
(B)	$\frac{3RT}{M}$ R - universal gas constant, T - absolute temperature, M - molar mass	(q) (kilogram) $(metre)^3$ $(second)^{-2}$
(C)	$\frac{F^2}{q^2B^2}$ F - force, q - charge, B - magnetic field	(r) $(metre)^2$ $(second)^{-2}$
(D)	$\frac{GM_e}{R_e}$ G - universal gravitational constant, M_e - mass of the earth R_e - radius of the earth	(s) (farad) $(volt)^2$ $(kg)^{-1}$

Ans. **(A) → (p),(q); (B) → (r), (s); (C) → (r), (s); (D) → (r), (s)**

Sol. [Dipole moment] = LIT, $[\phi_E] = ML^3/IT^3$ $[E] = ML/IT^3$.

4.** Some physical quantities are given in **Column I** and some possible SI units in which these quantities may be expressed are given in **Column II**. Match the physical quantities in **Column I** with the units in **Column II.** **[IIT-JEE-2007; 6/184]**

	Column I		**Column II**
(A)	GM_eM_s G - universal gravitational constant M_e - mass of the earth M_s - mass of the Sun	(p)	(volt) (coulomb) (metre)
(B)	$\frac{3RT}{M}$ R - universal gas constant T - absolute temperature M - molar mass	(q)	(kilogram) $(metre)^3$ $(second)^{-2}$

(C)	$\frac{F^2}{q^2B^2}$ F^2 - force Q^2 - charge B^2 - magnetic field	(r)	$(\text{metre})^2\ (\text{second})^{-2}$
(D)	$\frac{GM_e}{R_e}$ G - universal gravitational constant M_e - mass of the earth R_e - radius of the earth	(s)	$(\text{farad})\ (\text{volt})^2\ (\text{kg})^{-1}$

Ans. (A) → (p),(q); (B) → (r), (s); (C) → (r), (s); (D) → (r), (s)

Sol. (A) $\frac{GM_eM_s}{R_e^2} = \text{Force}$

$[GM_eM_s] = [\text{Force}]\,[R_e^2]$

$= MLT^{-2}\,L^2 = ML^3T^{-2}$

Hence SI unit of GM_eM_s, will be (kilogram) $(\text{meter}^3)(\text{sec}^{-2})$
ie same as (volt) (coulomb) (metre)

(B) $\sqrt{\frac{3RT}{M}} = V_{R.M.S.}$

$\left[\frac{3RT}{M_0}\right] = [V_{R.M.S.}]^2 = L^2T^{-2}$

Hence SI unit will be $(\text{metre})^2\ (\text{second})^{-2}$ i.e. same as $(\text{farad})\ (\text{volt})^2\ (\text{kg})^{-1}$

(C) $\frac{[F^2]}{[q^2B^2]} = \frac{[q^2v^2B^2]}{[q^2B^2]} = [V^2] = L^2T^{-2}$

Hence SI unit $(\text{metre})^2\ (\text{second})^{-2}$ i.e. same as $(\text{farad})\ (\text{volt})^2\ (\text{kg})^{-1}$

(D) $\left[\frac{GM_e}{R_e}\right] = \frac{[\text{Force}][R_e]}{[\text{Mass}]} = \frac{MLT^{-2}L}{M} = L^2T^{-2}$

Hence SI unit will be $(\text{meter})^{-2}\ (\text{second})^{-2}$ i.e. same as $(\text{farad})\ (\text{volt})^2\ (\text{kg})^{-1}$

5. Match List Iwith List IIand select the correct answer using the codes given below the lists :

[JEE (Advanced) 2013; 3/60]

	List I		**List II**
P.	Boltzmann constant	1.	$[ML^2T^{-1}]$
Q.	Coefficient of viscosity	2.	$[ML^{-1}T^{-1}]$
R.	Planck constant	3.	$[MLT^{-3}K^{-1}]$
S.	Thermal conductivity	4.	$[ML^2T^{-2}K^{-1}]$

Codes :

P Q R S

(A)	3	1	2	4
(B)	3	2	1	4
(C)	4	2	1	3
(D)	4	1	2	3

Ans. **(C)**

Sol. (p) $U = \frac{1}{2}kT \Rightarrow ML^2T^{-2} = [k]\,K \Rightarrow [K] = ML^2T^{-2}K^{-1}$

(q) $F = \eta A\frac{dv}{dx} \Rightarrow [\eta] = \frac{MLT^{-2}}{L^2LT^{-1}L^{-1}} = ML^{-1}T^{-1}$

(r) $E = h\nu \Rightarrow ML^2T^2 = [h]\,T^{-1} \Rightarrow [h] = ML^2T^{-1}$

(s) $\frac{dQ}{dt} = \frac{kA\Delta\theta}{\ell} \Rightarrow [k] = \frac{ML^2T^{-3}L}{L^2K} = MLT^{-3}K^{-1}$

6. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer find that d is proportional to $S^{1/n}$. The value of n is: **[JEE Advanced-2014; P-1, 3/60]**

Ans. **3**

Sol. $d = k\ (\rho)^a\ (S)^b\ (f)^c$

$\left[\frac{M}{L^3}\right]^a \left[\frac{M^1L^2T^{-2}}{L^2T}\right]^b \left[\frac{1}{T}\right]^c$

$0 = a + b$

$1 = -3a \Rightarrow a = -\frac{1}{3}$ So $b = \frac{1}{3}$

$0 = -3b + c$

So $n = 3$

7. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement(s) is/are correct?

[JEE Advanced–2019]

(1) The dimension of force is L^{-3}
(2) The dimension of linear momentum is L^{-1}
(3) The dimension of energy is L^{-2}
(4) The dimension of power is L^{-5}

Ans. **(1,2,3)**

Sol. [M] = 1

Angular moment $\Rightarrow ML^2T^{-1}$

$[ML^2T^{-1}] = 1 : [M] = 1$

$1.L^2T^1 = 1$

$L^2 = T$

(1) $[F] = MLT^{-2}$

$= 1.L \cdot L^{-4}$

$[F] = L^{-3}$

(2) $P = M \cdot L \cdot T^{-1}$

$\Rightarrow 1 \cdot L \cdot L^{-2}$

$\Rightarrow L^{-1}$

(3) E_{net} $\Rightarrow M L^2 T^{-2}$
$= 1 \cdot L^2 L^{-4}$
$= L^{-2}$

(4) power $= ML^2T^{-3}$
$= ML^2 \cdot L^{-6}$
Power $= L^{-4}$

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ANSWER KEY

EXERCISE # (S)

1. 10^{-30} star joule
2. L^{-1}, ML^2T^{-2}
3. $kg.m^2.s^{-2}.K^{-1}$
4. $[S] = Ev^{-2}T^{-2}$
5. $T = a\sqrt{\frac{m}{k}}$
6. $\omega = K\sqrt{\frac{GM}{r^3}}$
7. $v_0 = k\sqrt{\frac{GM}{R}}$
8. K = 6
9. (A) $\frac{2\pi}{3}\times 6.4\times 10^6 m$; (B) $\sqrt{3}\times 6.4\times 10^6 m$
10. (A) 9.95, (B) 0.99
11. $2\pi/45$, $\pi/36$
12. $[MLT^{-1}]$
13. $[v_0] = [M^0L^1T^{-1}]$, $[\alpha] = [M^0L^0T^{-1}]$
14. $ML^5T^{-2}K^{1/2}$

EXERCISE # (O)

1.	D	**2.**	C	**3.**	A	**4.**	A	**5.**	B	**6.**	A	**7.**	D
8.	A	**9.**	A	**10.**	D	**11.**	C	**12.**	C	**13.**	C	**14.**	B
15.	C	**16.**	C	**17.**	B	**18.**	B						

19. (1) → (Q) → (c) , (2) → (S) → (a) , (3) → (P) → (b) , (4) → (R) → (d)
20. (i) → (Q) → (d)
(ii) → (S) → (b)
(iii) → (U) → (e)
(iv) → (R) → (a)
(v) → (T) → (f)
(vi) → (P) → (c)
21. **(A) → (Q); (B) → (S); (C) → (R); (D) → (R)**
22. B **23.** B **24.** D

EXERCISE # (JM)

1.	(1)	**2.**	(4)	**3.**	(1)	**4.**	(1)	**5.**	(3)	**6.**	(4)	**7.**	(2)
8.	(2)	**9.**	(3)	**10.**	(2)	**11.**	(4)	**12.**	(4)	**13.**	(2)	**14.**	(1)
15.	(4)	**16.**	(3)	**17.**	(1)	**18.**	(4)						

EXERCISE # (JA)

1. (D) **2.** (A) **3.** (D)
4. (A) → (p),(q); (B) → (r), (s); (C) → (r), (s); (D) → (r), (s)
5. (C) **6.** 3