ADVANCED THERMODYNAMICS AND COMBUSTION (MENG6102)

By

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CHAPTER 5
Entropy

Thermodynamics deals with stability of systems.

- What is state function?

A state function of a thermodynamic system that is dependent on initial and final state. (Mass, Energy – enthalpy, internal energy, Gibbs free energy, etc., Entropy, Pressure, Temperature, Volume, Chemical composition and Altitude)

- What is process function?

A process function is a thermodynamic term that is used to name a property whose value depends on the path taken to reach specific value. *Different paths have different values.*

when expressing the path function mathematically, multiple integrals and limits are required to integrate the path function. Eg. (Mechanical work, heat and arc length are process functions)

$\Delta U = q + w$ ⟹ ΔU = state function but w and q is process function.

Process function

No	Process Type	Constant Quantity	Q	W
1	Isobaric	P	$nC_P\Delta T$	$P\Delta V$
2	Isothermal	T	$nRTln\left({V_f}/{V_i}\right)$	
3	Isochoric	V	ΔU	
4	Adiabatic	PV^γ	0	$-\Delta U$

$$\Delta U = nC_v\Delta T$$

Second Law of Thermodynamics

- ***Many formulations. Consider the following two.***

1. Spontaneous (i.e. without external agent) transfer of heat from cold object to hot object is impossible.

2. No Perpetual Motion Machine of second type. (device that realizes full conversion of heat into useful work, without any "side effect", in contrast with real engines.)

In 1st Law of thermodynamics (for infinitesimal process) $\delta Q = dU + \delta W$

δQ = *not function of state, depends on process*

Entropy

There exist one more function of state = Entropy

$$dS = \frac{\delta Q}{T} \qquad \text{so that} \qquad \Delta S = \int_i^f \frac{\delta Q}{T}$$

- If T=constant : $\Delta Q = T\Delta S$
- $\Delta S = 0$ for adiabatic process
  $\Delta S \neq 0$ for non-adiabatic
- In thermodynamics there are 4 variable (P,V,T, S)

REVERSIBLE	IRREVERSIBLE
Series of quasi-static (equilibrium) steps No dissipation (e.g. no friction)	Series of complex, non- equilibrium steps Dissipation may be present
Any process that can be represented as a path on a PV diagram; e.g (Isochoric, Isobaric, Isothermal and Adiabatic)	Any process that is sudden, spontaneous, uncontrolled, or involves dissipation Eg (Free expansion, Explosion, Heat transfer between a hot, and a cold object, Volume changes of gas with a piston that has friction)

Entropy change for reversible process

Entropy change for adiabatic process? General process: (Ti, Vi, pi) --> (Tf, Vf, pf)

$$\Delta S = n\left[C_v ln\left({}^{T_f}\!/_{T_i}\right) + R ln\left({}^{V_f}\!/_{V_i}\right)\right]$$

- Intuitively: Entropy = Measure of Disorder

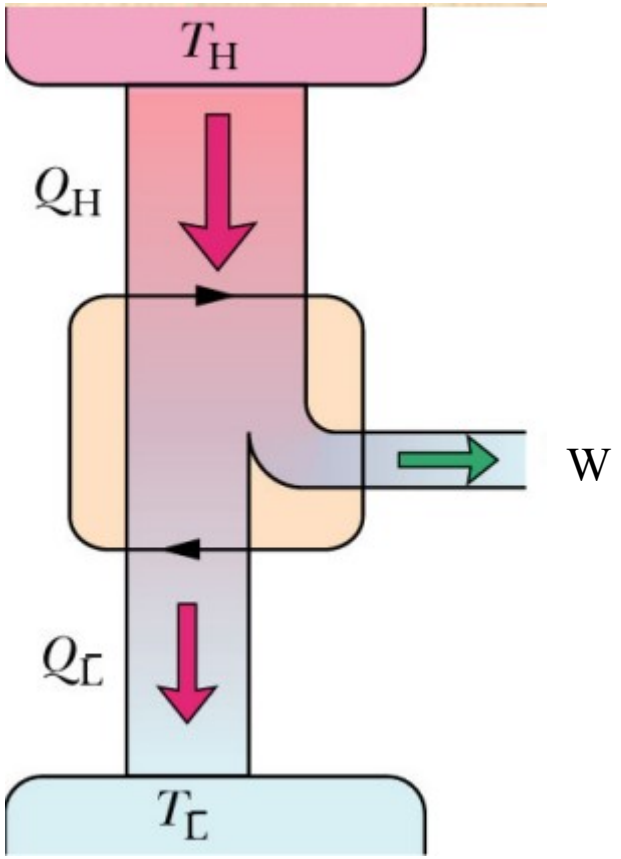
✓With time, entropy of isolated system: increases in irreversible process, remains the same in reversible process:

✓If you reach "thermal equilibrium" with untidy roommate, that would correspond to maximum possible disorder.

How can we extract heat? Heat Engines

Heat engine takes some substance (e.g. gas) through cyclic process during which:

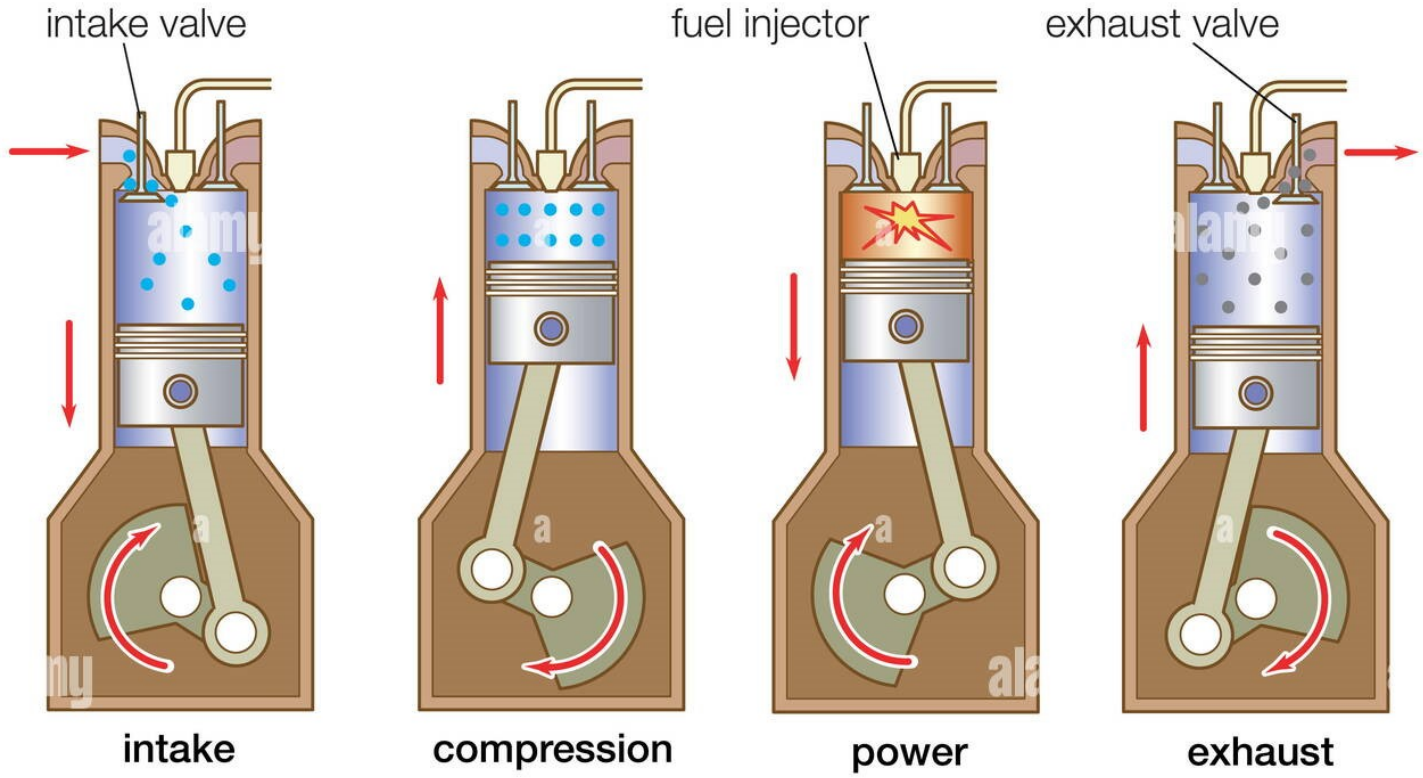
- Heat QH absorbed from hot reservoir
- Work W+ done by engine
- Heat QC dumped into cold reservoir
- "Negative" work W- done by engine to bring gas to original state.



In a cyclic process, it is impossible to convert all the energy extracted from an equilibrium system into work. Some part of this energy must be transferred to a colder system.

Internal combustion engines cycle consists 4 process

1. Burning of fuel (isothermal expansion)
2. Fast expansion (adiabatic)
3. Compression to original pressure (isothermal process)
4. Compression to original temperature (adiabatic process)

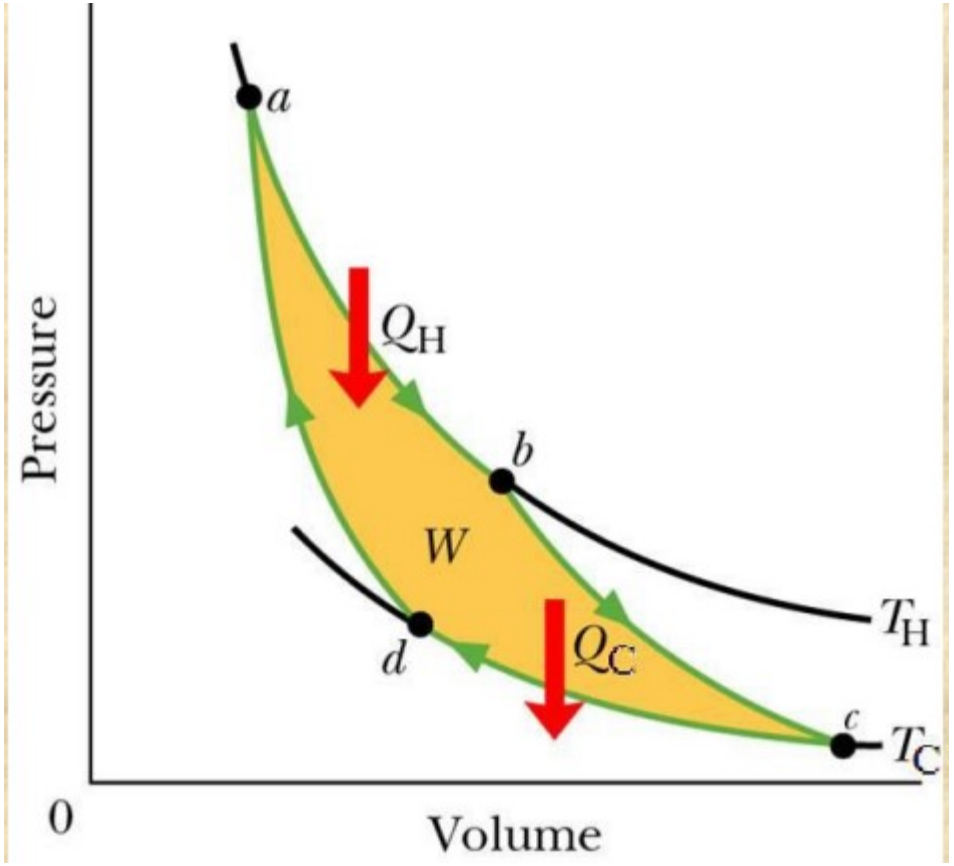


Carnot Heat Engine

Carnot heat engine takes gas through reversible cyclic process consisting of 4 steps:

1. Isothermal expansion at T= TH
2. Adiabatic expansion from T= TH to T= TC
3. Isothermal compression at T= TC
4. Adiabatic compression from T= TC to T= TH

$$K = 1 - \frac{T_C}{T_H} < 1$$



Reversible/Irreversible Processes

Entropy: $\Delta S \geq 0 \rightarrow \Delta S = nC_v \ln\left[\frac{T_f}{T_i}\right] + nR\ln\left[\frac{V_f}{V_i}\right]$

This is 2nd law of thermodynamics

Heat Engines (Carnot)

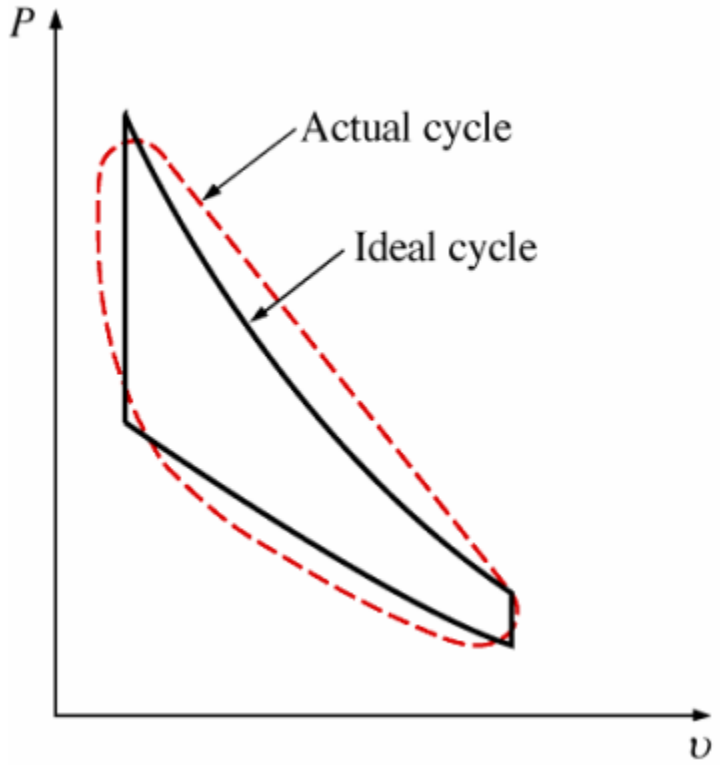
$$K = \frac{W}{Q_H} \rightarrow 1 - \frac{T_C}{T_H}$$

Chapter 6 Gas Power cycle

Gas Power Cycle: Heat engine in which working fluid remains in the gaseous system throughout the cycle.

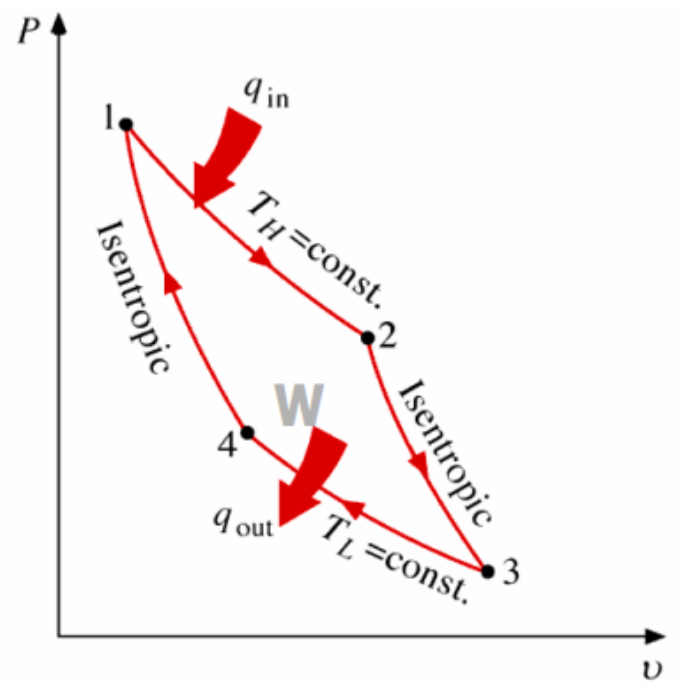
The performance of the cycle always measured in terms of efficiency. $\eta_{th} = \frac{W_{net}}{Q_{in}}$

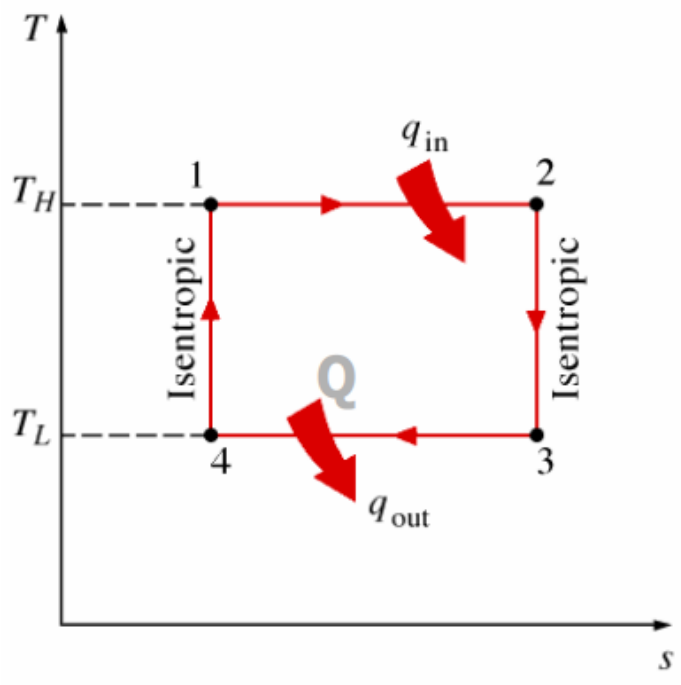
- Otto Cycle
- Spark Ignition
- Diesel Cycle
- Brayton Cycle
- Gas Turbine
- Rankine Cycle



- The most efficient heat engine that can operate between two fixed temperature TH and TC is called **Carnot engine**

Carnot Cycle





- 1 to 2 Isothermal heat addition
- 2 to 3 Isentropic expansion
- 3 to 4 isothermal heat rejection
- 4 to 1 Isentropic compression

Q-W=0 Q=W In addition, we know that the efficiency for a Carnot Cycle is:

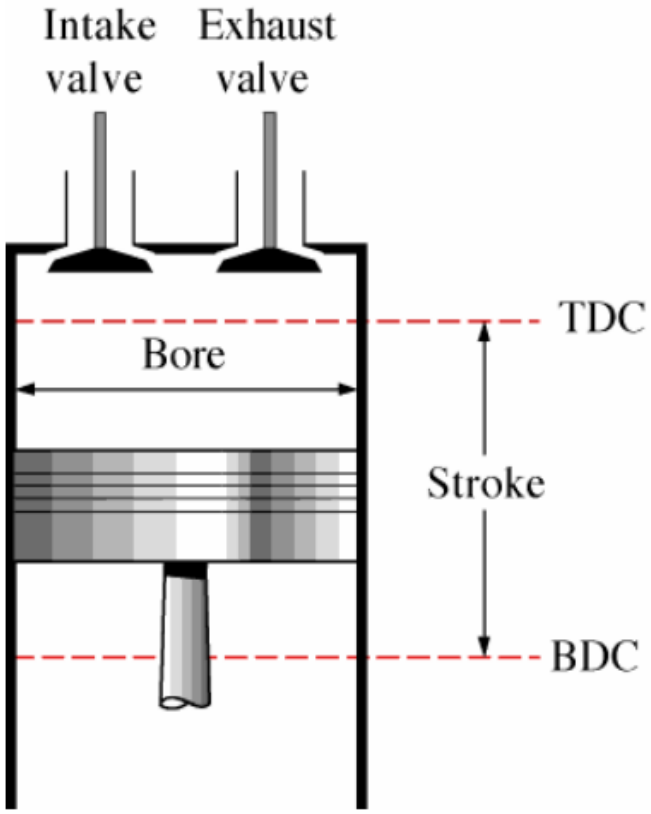
$$K\ carnot = 1 - \frac{T_L}{T_H}$$

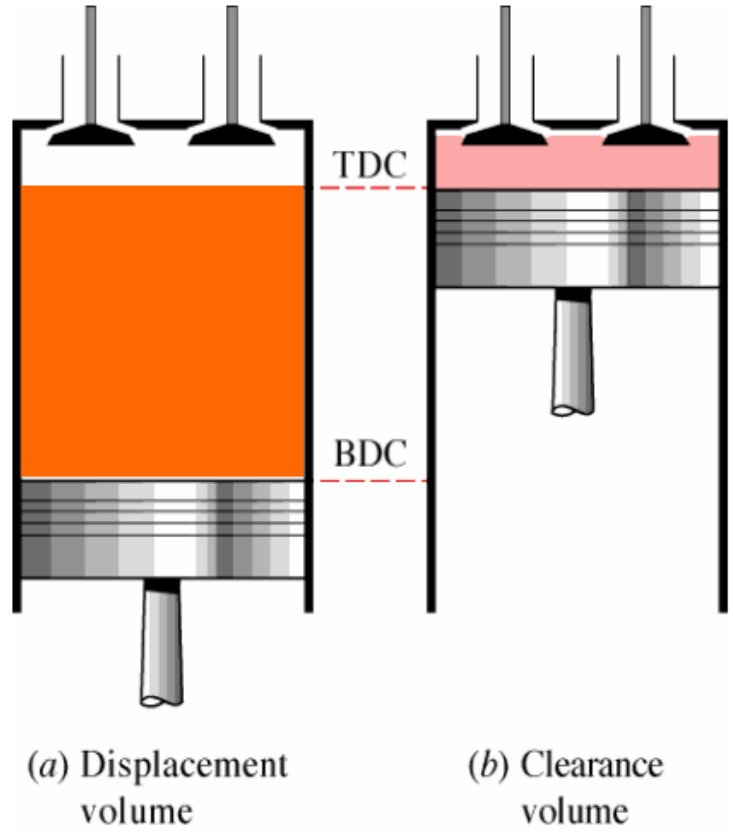
Where K is thermal efficiency of Carnot cycle

Air-Standard Assumptions

- Air continuously circulates in a closed loop and behaves as an ideal gas
- All the processes are internally reversible
- Combustion is replaced by a heat addition process from the outside
- Heat rejection replaces the exhaust process
- Also assume a constant value for Cp, evaluated at room temperature

Terminology for Reciprocating Devices

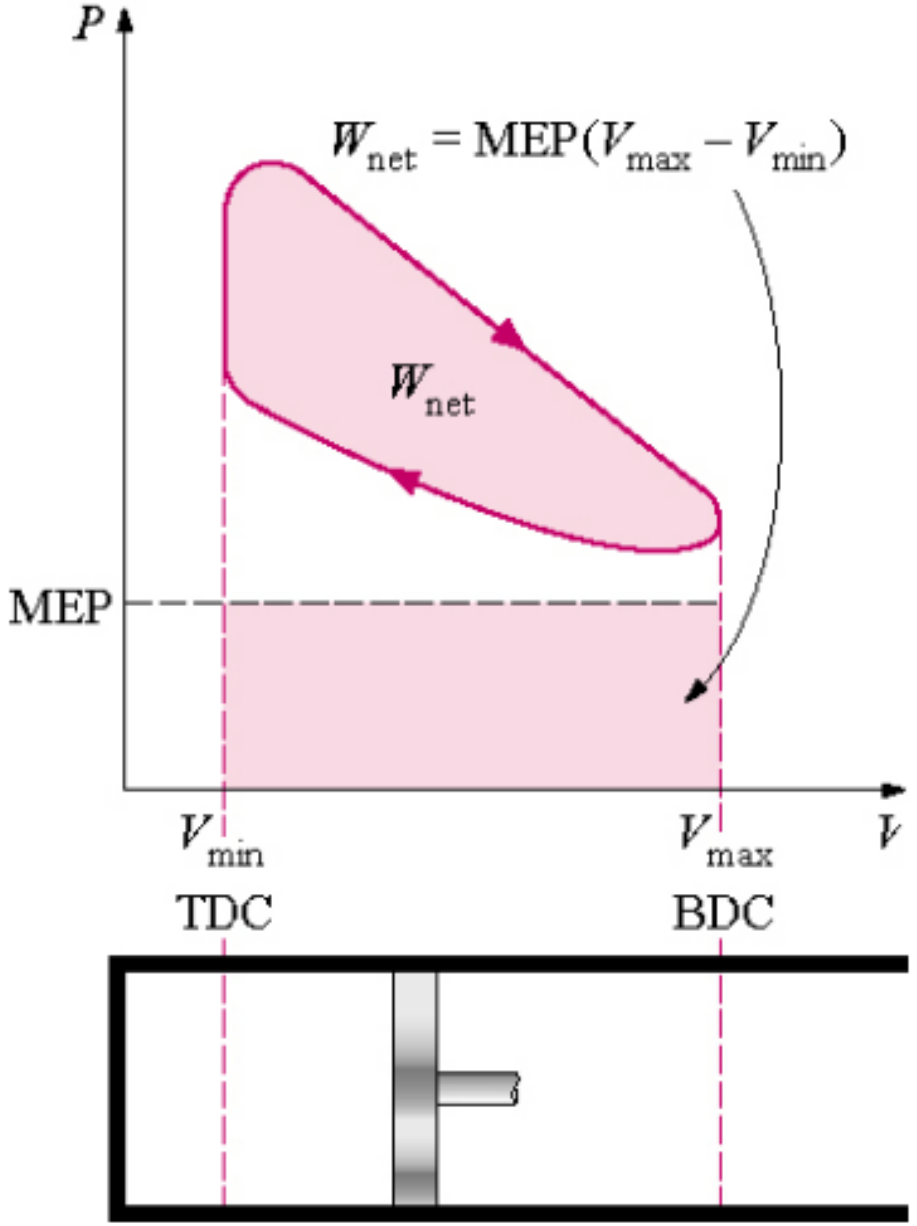


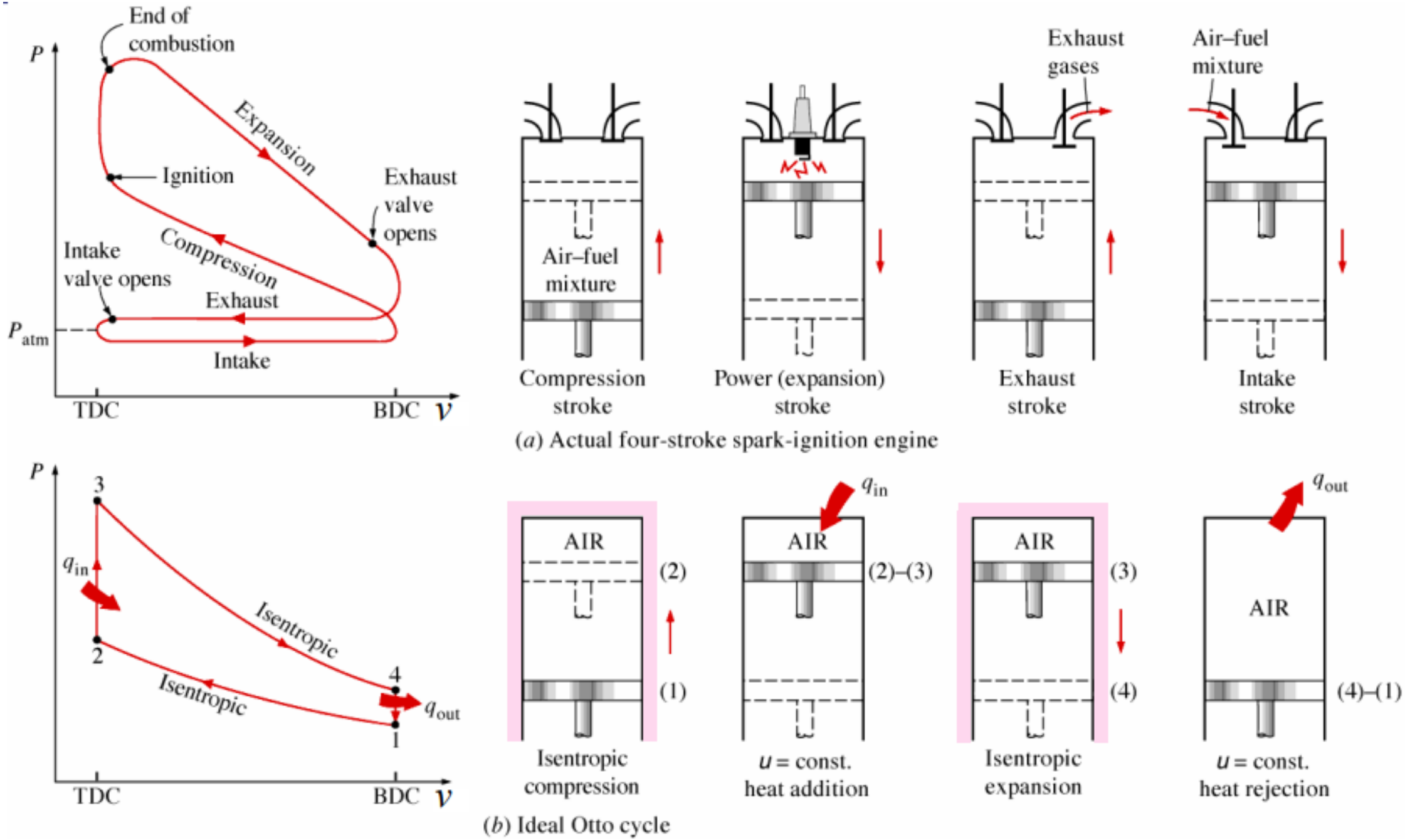


Compression Ratio = $r = \frac{V_{max}}{V_{min}} = \frac{V_{BDC}}{V_{TDC}}$

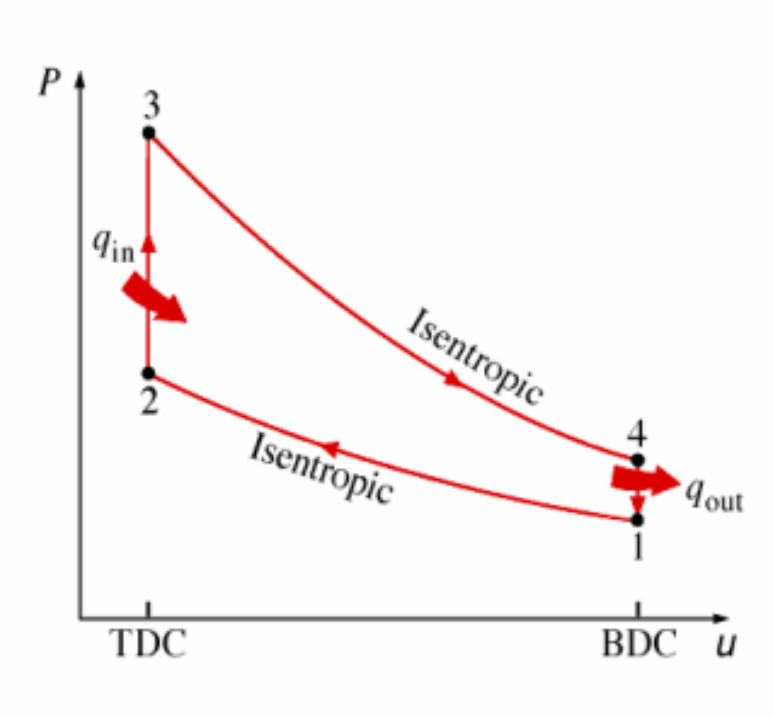
Mean Effective Pressure

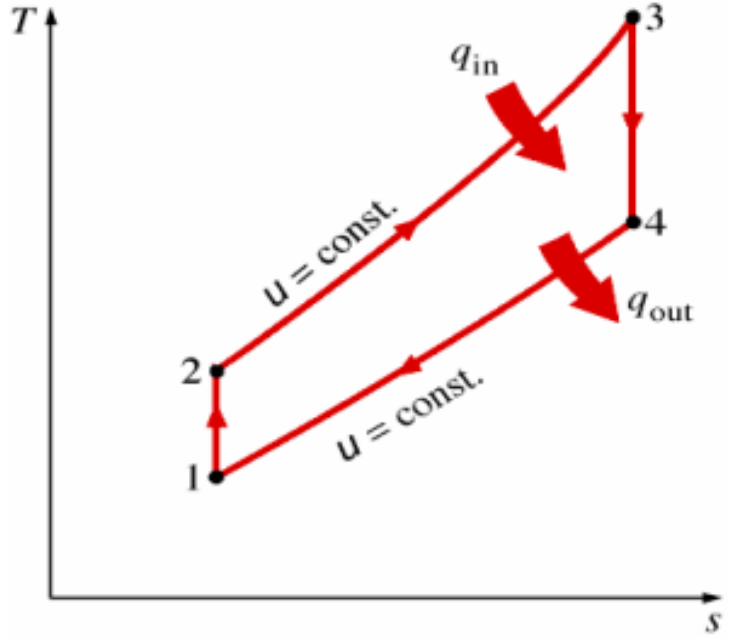
$$W = \int_1^2 P dV \qquad W = P\Delta V$$




End of combustion
P
Expansion
Ignition
Exhaust valve opens
Compression
Intake valve opens
Exhaust
P_{atm}
Intake
TDC
BDC
v
Air–fuel mixture
Compression stroke
Power (expansion) stroke
Exhaust gases
Exhaust stroke
Air–fuel mixture
Intake stroke
(a) Actual four-stroke spark-ignition engine
P
3
q_{in}
Isentropic
2
4
Isentropic
q_{out}
1
TDC
BDC
v
AIR
(2)
(1)
Isentropic compression
q_{in}
AIR
(2)–(3)
u = const. heat addition
AIR
(3)
(4)
Isentropic expansion
q_{out}
AIR
(4)–(1)
u = const. heat rejection
(b) Ideal Otto cycle

Otto Cycle





(a) 1-2 Isentropic Compression
(b) 2-3 Constant Volume Heat Addition
(c) 3-4 Isentropic Expansion
(d) 4-1 Constant Volume Heat Rejection

Thermal Efficiency of the Otto Cycle

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{Q_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Apply First Law Closed System to Process 2-3, V = Constant

$$Q_{net,23} - W_{net,23} = \Delta U_{23}$$

$$W_{net,23} = W_{other,23} + W_{b,23} = 0 + \int_2^3 PdV = 0$$

$$Q_{net,23} = \Delta U_{23}$$

$$Q_{net,23} = Q_{in} = mC_v(T_3 - T_2)$$

Apply First Law Closed System to Process 4-1,V = Constant

$$Q_{net,41} - W_{net,41} = \Delta U_{41}$$

$$W_{net,41} = W_{other,41} + W_{b,41} = 0 + \int_4^1 PdV = 0$$

$$Q_{net,41} = \Delta U_{41}$$

$$Q_{net,41} = -Q_{out} = mC_v(T_1 - T_4)$$

$$Q_{out} = -mC_v(T_1 - T_4) = mC_v(T_4 - T_1)$$

$$\eta_{th,\,Otto} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$= 1 - \frac{mC_v(T_4 - T_1)}{mC_v(T_3 - T_2)}$$

→

$$\eta_{th,\,Otto} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

Recall processes 1-2 and 3-4 are isentropic, so

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} \quad \text{and} \quad \frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{k-1}$$

$$v_3 = v_2 \text{ and } v_4 = v_1$$

→

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

or

$$\frac{T_4}{T_1} = \frac{T_3}{T_2}$$

$$\eta_{th,\,Otto} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

1

$$\eta_{th,\,Otto} = 1 - \frac{T_1}{T_2}$$

Efficiency of the Otto Cycle vs. Carnot Cycle

There are only two temperatures in the Carnot cycle

- Heat is added at TH
- Heat is rejected at TL

There are four temperatures in the Otto cycle

- Heat is added over a range of temperatures
- Heat is rejected over a range of temperatures

Since process 1-2 is isentropic,

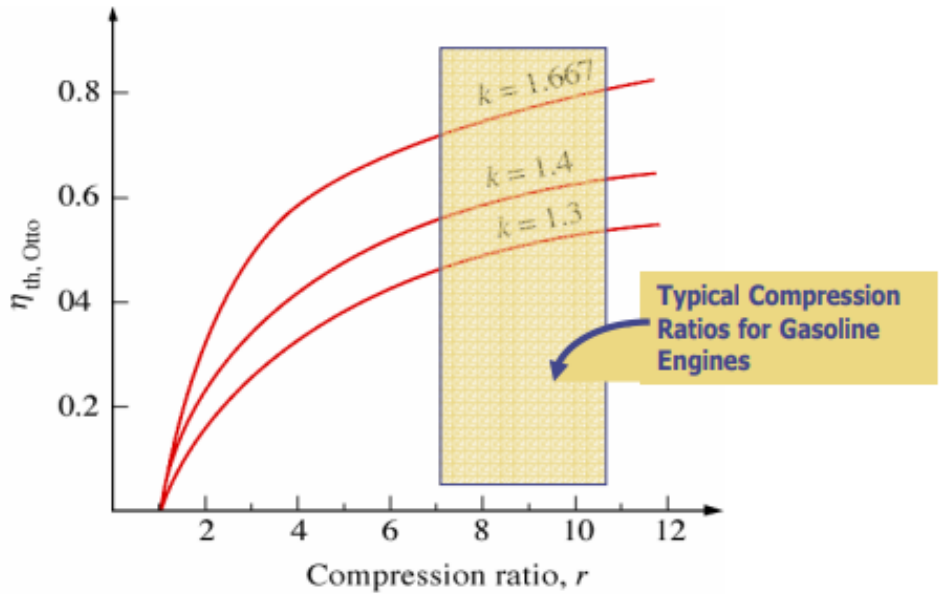
$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{k-1} = \frac{1}{r^{k-1}}$$

$$\eta_{th,\,Otto} = 1 - \frac{T_1}{T_2}$$

→

$$\eta_{th,\,Otto} = 1 - \frac{1}{r^{k-1}}$$

Increasing Compression Ratio Increases the Efficiency



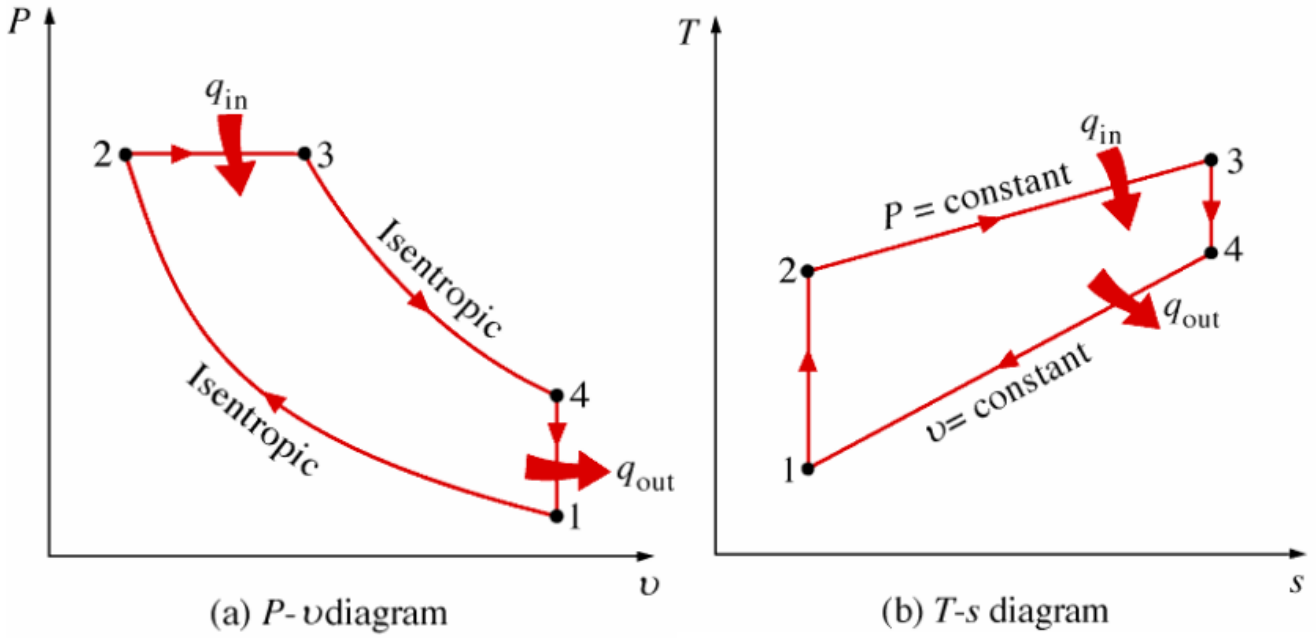
What happen if we use higher compression ratio to gasoline engine?

- Premature Ignition
- Causes "Knock"
- Reduces the Efficiency
- Mechanically need a better design

What is the fundamental behavior of diesel engines?

- No spark plug
- Fuel is sprayed into hot compressed air

Diesel Cycle



The only difference is in process 2-3

Consider Process 2-3

- This is the step where heat is transferred into the system
- We model it as constant pressure instead of constant volume

$$q_{in,23} - w_{b,out} = \Delta u = u_3 - u_2$$
$$q_{in,23} = \Delta u + P\Delta v = \Delta h = C_p\left(T_3 - T_2\right)$$

Consider Process 4-1

- This is where heat is rejected
- We model this as a constant v process
  - That means there is no boundary work

$$q_{41} - \cancel{w_{41}} = \Delta u \quad \Rightarrow \quad q_{41} = -q_{out} = \Delta u = C_v\left(T_1 - T_4\right)$$
$$q_{out} = C_v\left(T_4 - T_1\right)$$

For any heat engine thermal efficiency is given by

$$\eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

$$q_{out} = C_v(T_4 - T_1) \text{ and } q_{in} = C_p(T_3 - T_2)$$

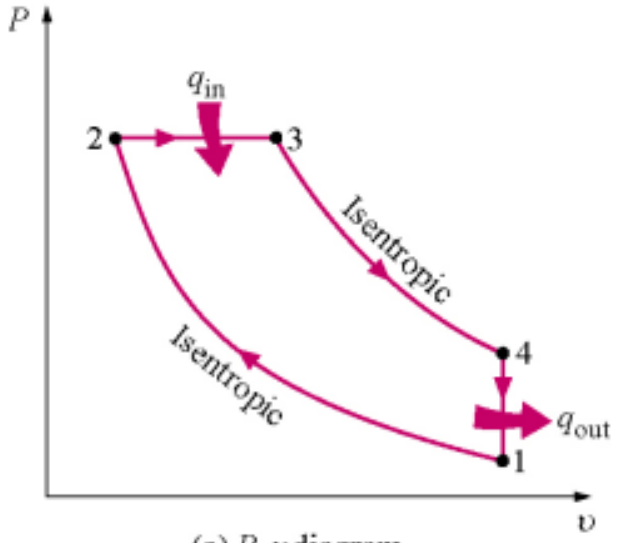
$$n_{th,diesel} = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{k(T_3 - T_2)}$$

$$n_{th,diesel} = 1 - \frac{T_1\left(T_4/T_1 - 1\right)}{kT_2\left(T_3/T_2 - 1\right)}$$

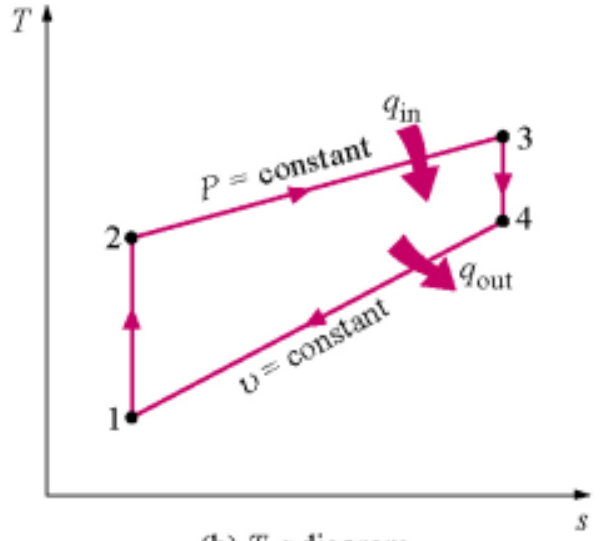
$$\frac{P_3V_3}{T_3} = \frac{P_2V_2}{T_2} \quad \text{where } P_3 = P_2$$

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c$$

rc is called the cutoff ratio – it's the ratio of the cylinder volume before and after the combustion process



(a) P-v diagram

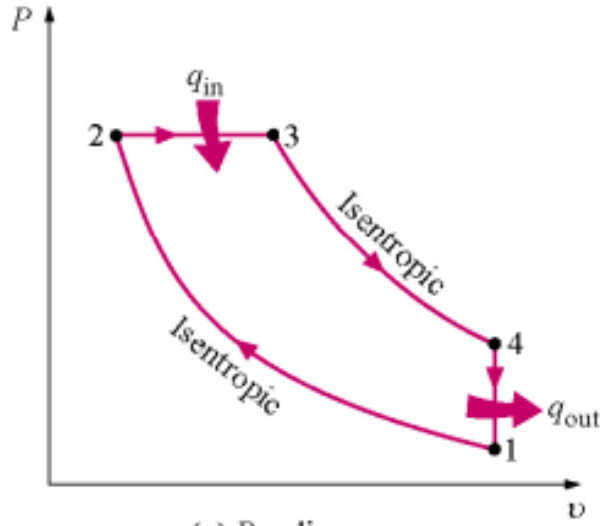


(b) T-s diagram

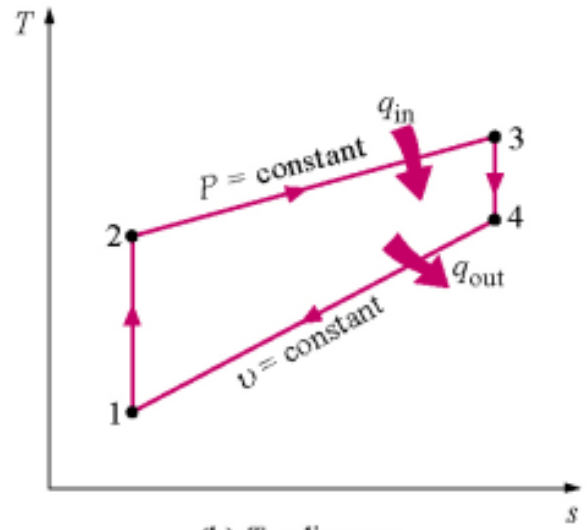
$$n_{th,diesel} = 1 - \frac{T_1\left(T_4/T_1 - 1\right)}{kT_2\left(r_c - 1\right)}$$

$$\frac{P_4V_4}{T_4} = \frac{P_1V_1}{T_1} \quad \text{where } V_4 = V_1$$

$$\frac{T_4}{T_1} = \frac{P_4}{P_1}$$



(a) P-υdiagram



(b) T-s diagram

$$n_{th,diesel} = 1 - \frac{T_1\left(P_4/P_1 - 1\right)}{kT_2\left(r_c - 1\right)}$$

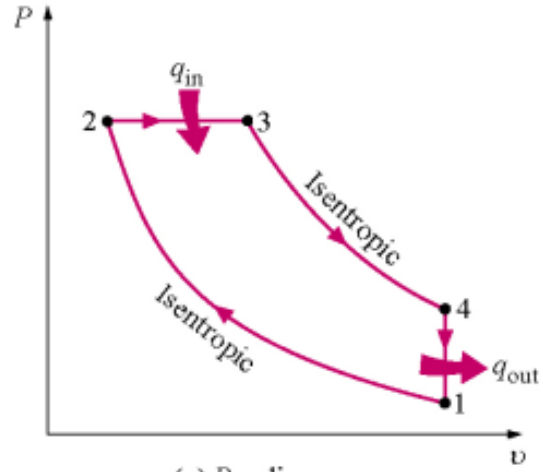
Since Process 1-2 and Process 3-4 are both isentropic

$$P_1 V_1^k = P_2 V_2^k \text{ and}$$

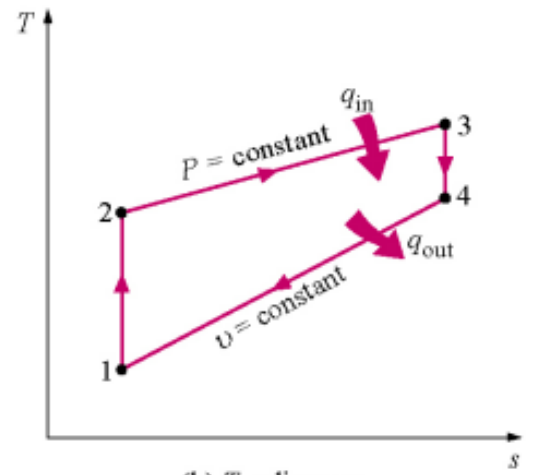
$$P_4 V_4^k = P_3 V_3^k$$

$$\frac{P_4}{P_1}\frac{V_4^k}{V_1^k} = \frac{P_3}{P_2}\frac{V_3^k}{V_2^k} = \left(\frac{V_3}{V_2}\right)^k = r_c^k$$

1 1



(a) P-υ diagram



(b) T-s diagram

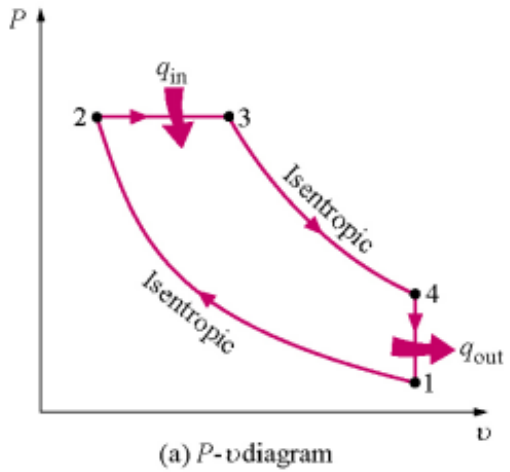
$$n_{th,diesel} = 1 - \frac{T_1\left(r_c^k - 1\right)}{kT_2\left(r_c - 1\right)}$$

Finally, Since process 1-2 is isentropic

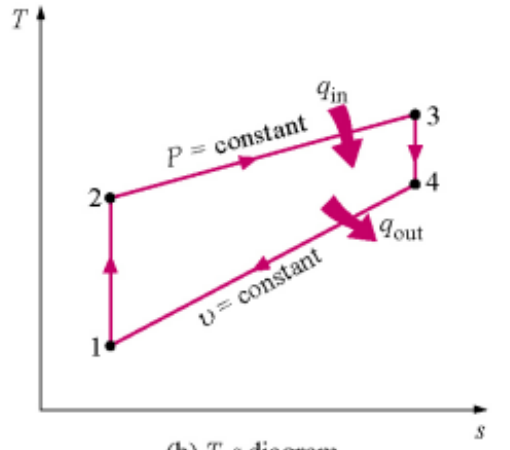
$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1}$$

The volume ratio from 1 to 2 is the compression ratio, r

$$\frac{T_1}{T_2} = \left(\frac{1}{r}\right)^{k-1}$$

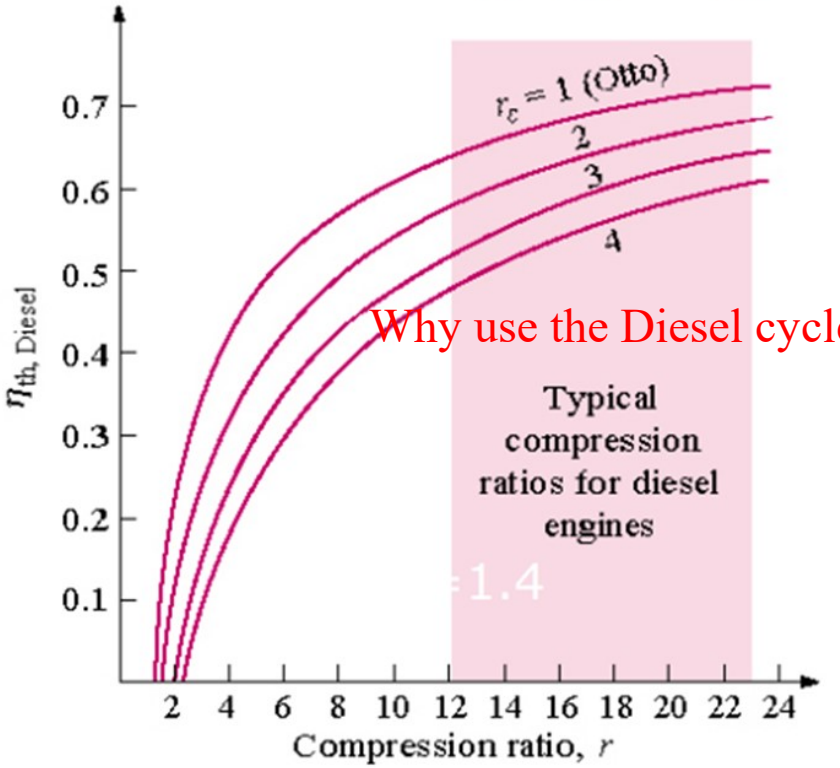


(a) P-v diagram



(b) T-s diagram

$$\eta_{th,diesel} = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k\left(r_c - 1\right)}\right]$$

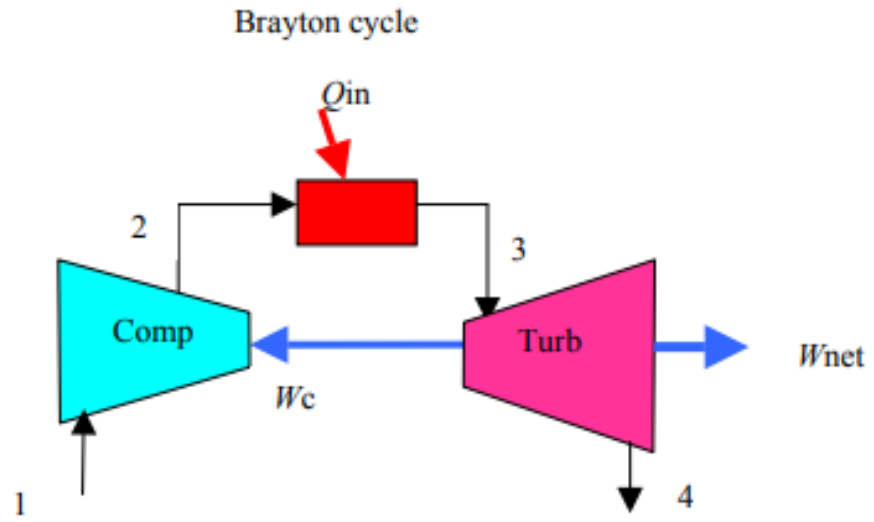


Why use the Diesel cycle?

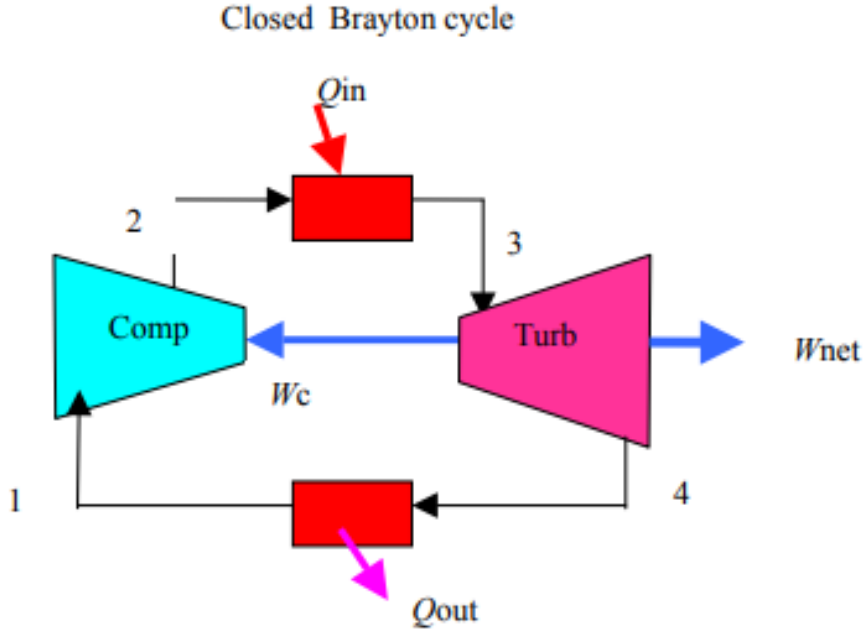
The efficiency of the Otto cycle is always higher than the Diesel cycle

Brayton Cycle

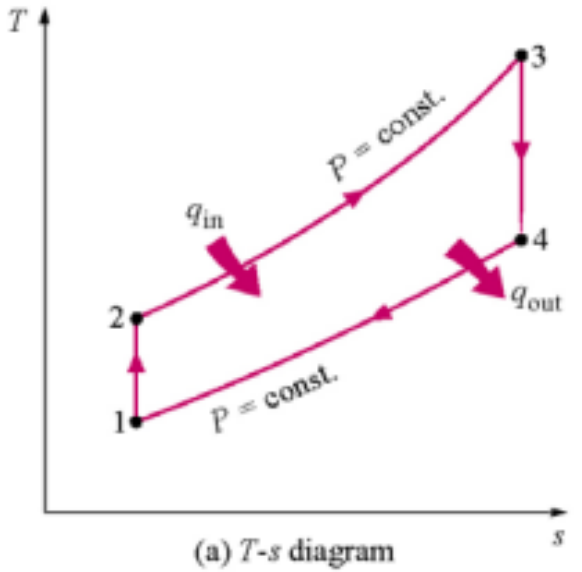
- The Brayton cycle is the air-standard ideal cycle approximation for gas-turbine engine.
- *Let us assume the working fluid is air and the specific heats are constant then we will consider cold-air-standard cycle*



The closed cycle gas-turbine engine



(a) 1-2 Isentropic Compression (in compressor)
(b) 2-3 Constant pressure Heat Addition
(c) 3-4 Isentropic Expansion (in turbine)
(d) 4-1 Constant pressure Heat Rejection



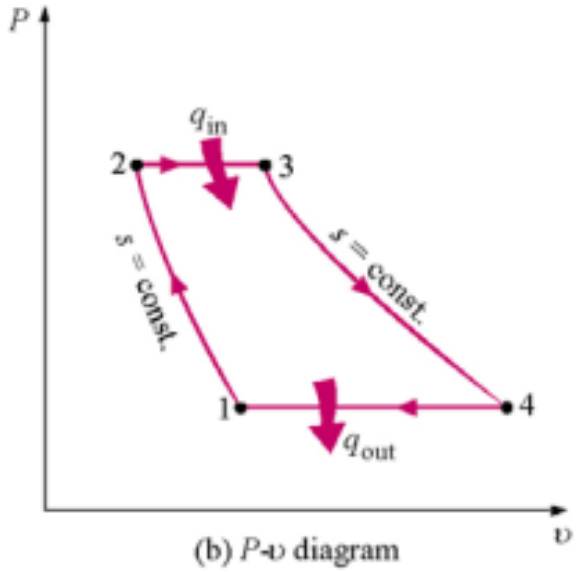
(a) *T-s* diagram

Thermal efficiency of Brayton cycle

$$\eta_{th,\,Brayton} = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Now to find Q_{in} and Q_{out}.

Apply the conservation of energy to process 2-3 for P = constant (no work), steady-flow, and neglect changes in kinetic and potential energies.



(b) *P-v* diagram

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_2 h_2 + \dot{Q}_{in} = \dot{m}_3 h_3$$

The conservation of mass gives

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\dot{m}_2 = \dot{m}_3 = \dot{m}$$

For constant specific heats, the heat added per unit mass flow is

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2)$$

$$\dot{Q}_{in} = \dot{m} C_p (T_3 - T_2)$$

$$q_{in} = \frac{\dot{Q}_{in}}{\dot{m}} = C_p (T_3 - T_2)$$

Conservation of energy for process 4-1 yields constant specific heats

$$\dot{Q}_{out} = \dot{m}(h_4 - h_1)$$

$$\dot{Q}_{out} = \dot{m} C_p (T_4 - T_1)$$

$$q_{out} = \frac{\dot{Q}_{out}}{\dot{m}} = C_p (T_4 - T_1)$$

Thermal efficiency becomes

$$\eta_{th,\,Brayton} = 1 - \frac{Q_{out}}{\dot{Q}_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

$$= 1 - \frac{C_p (T_4 - T_1)}{C_p (T_3 - T_2)}$$

$$\eta_{th,\,Brayton} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1 (T_4 / T_1 - 1)}{T_2 (T_3 / T_2 - 1)}$$

Recall processes 1-2 and 3-4 are isentropic, so

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \quad \textit{and} \quad \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k}$$

Since $P_3 = P_2$ and $P_4 = P_1$, we see that

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

The Brayton cycle efficiency becomes

$$\eta_{th,\,Brayton} = 1 - \frac{T_1}{T_2}$$

Is this the same as the Carnot cycle efficiency?

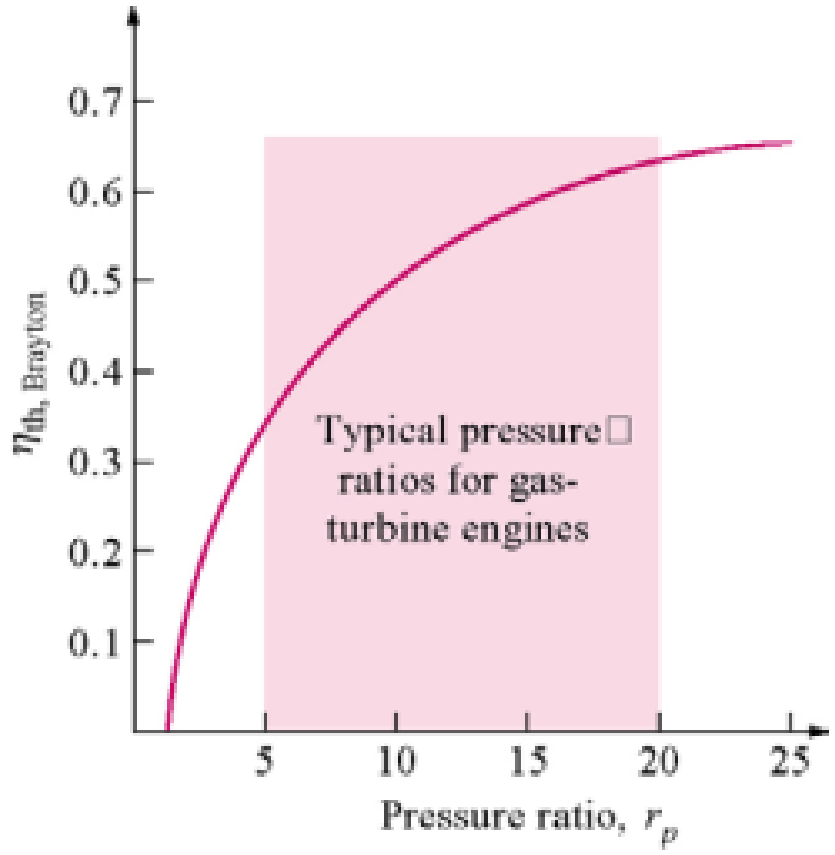
Since process 1-2 is isentropic,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = r_p^{(k-1)/k}$$

$$\frac{T_1}{T_2} = \frac{1}{r_p^{(k-1)/k}}$$

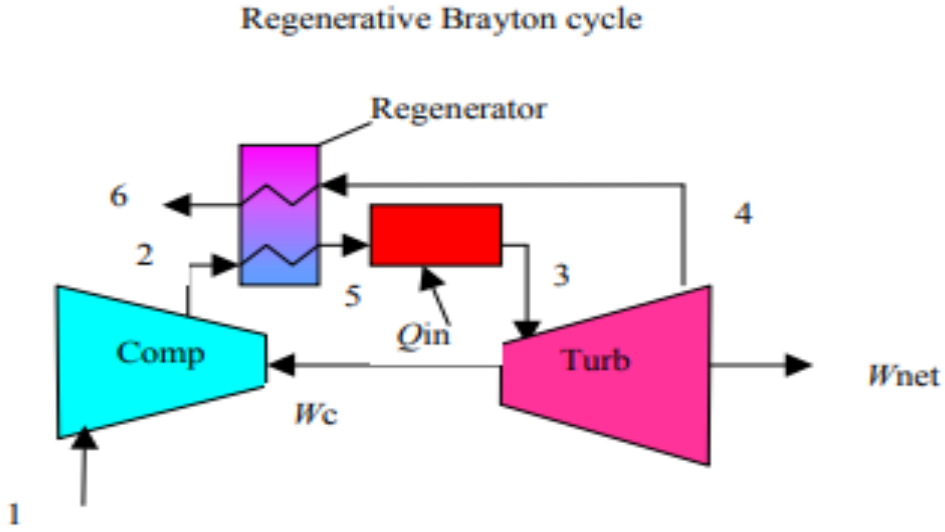
where the pressure ratio is $r_p = P_2/P_1$ and

$$\eta_{th,\,Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}}$$


0.7
0.6
0.5
0.4
0.3
0.2
0.1
$\eta_{th,\ Brayton}$
Typical pressure ratios for gas-turbine engines
5
10
15
20
25
Pressure ratio, r_p

Regenerative Brayton Cycle

- The Brayton cycle, the turbine exit temperature is greater than the compressor exit temperature. Therefore, a heat exchanger can be placed between the hot gas leaving the turbine and the cooler gases leaving the compressor. The heat exchange is called regenerator or recuperate

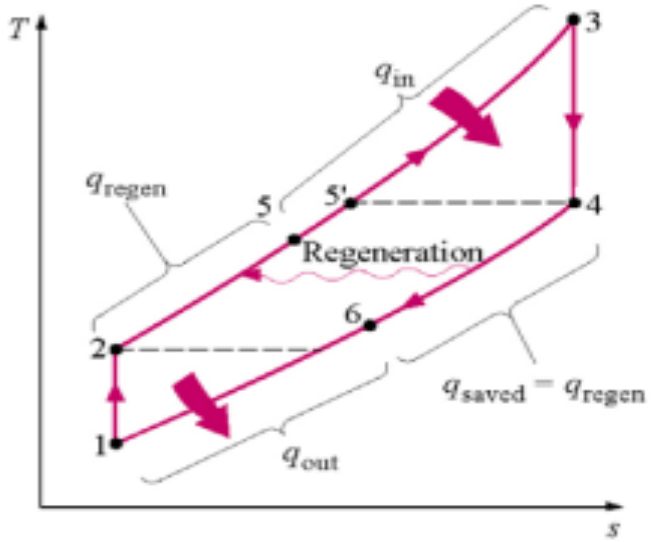


We define the regenerator effectiveness e_{regen} as the ratio of the heat transferred to the compressor gases in the regenerator to the maximum possible heat transfer to the compressor gases.

$$q_{regen,act} = h_5 - h_2$$

$$q_{regen,\max} = h_{5'} - h_2 = h_4 - h_2$$

$$\varepsilon_{regen} = \frac{q_{regen,act}}{q_{regen,\max}} = \frac{h_5 - h_2}{h_4 - h_2}$$



Regenerative Brayton Cycle

- For ideal gases using the cold-air-standard assumption with constant specific heats, the regenerator effectiveness becomes

$$\varepsilon_{regen} \cong \frac{T_5 - T_2}{T_4 - T_2}$$

Using the closed cycle analysis and treating the heat addition and heat rejection as steady-flow processes, the regenerative cycle thermal efficiency is

$$\eta_{th,\,regen} = 1 - \frac{q_{out}}{q_{in}}$$

$$= 1 - \frac{h_6 - h_1}{h_3 - h_5}$$

Notice that the heat transfer occurring within the regenerator is **not** included in the efficiency calculation because this energy is not a heat transfer acrossthe cycle boundary.

Assuming an ideal regenerator $e_{regen} = 1$ and constant specific heats, the thermal efficiency becomes (take the time to show this on your own)

Regenerative Brayton Cycle

$$\eta_{th,\,regen} = 1 - \frac{T_1}{T_3}\left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$= 1 - \frac{T_1}{T_3}(r_p)^{(k-1)/k}$$

When does the efficiency of the air-standard Brayton cycle equal the efficiency of the air-standard regenerative Brayton cycle? If we set $h_{th,Brayton} = h_{th,\,regen}$ then

$$\eta_{th,\,Brayton} = \eta_{th,\,regen}$$

$$1 - \frac{1}{(r_p)^{(k-1)/k}} = 1 - \frac{T_1}{T_3}(r_p)^{(k-1)/k}$$

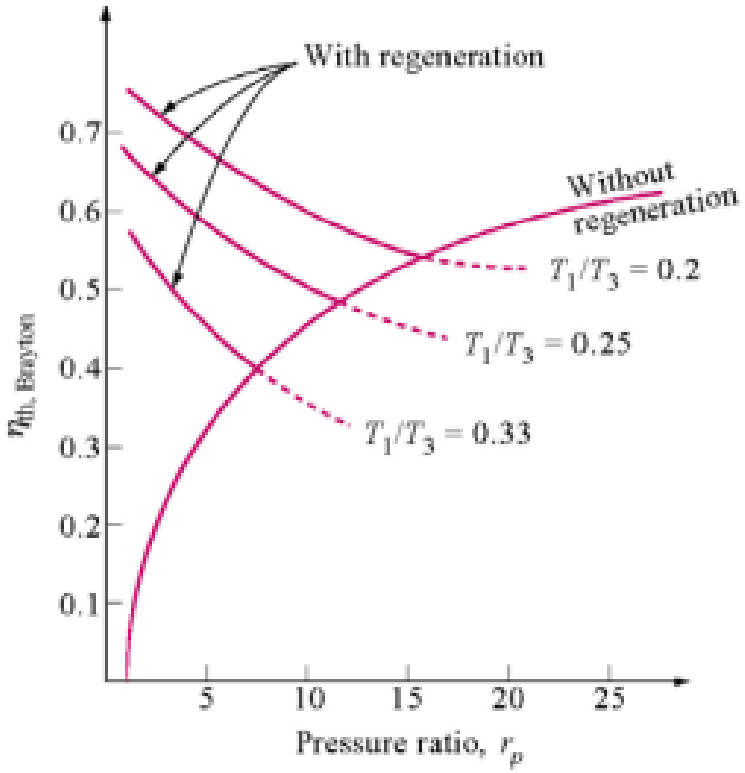
$$r_p = \left(\frac{T_3}{T_1}\right)^{k/[2(k-1)]}$$

- Recall that this is the pressure ratio that maximizes the net work for the simple Brayton cycle and makes T4 = T2. What happens if the regenerative Brayton cycle operates at a pressure ratio larger than this value?

- For fixed T3 and T1, pressure ratios greater than this value cause T4 to be less than T2, and the regenerator is not effective.

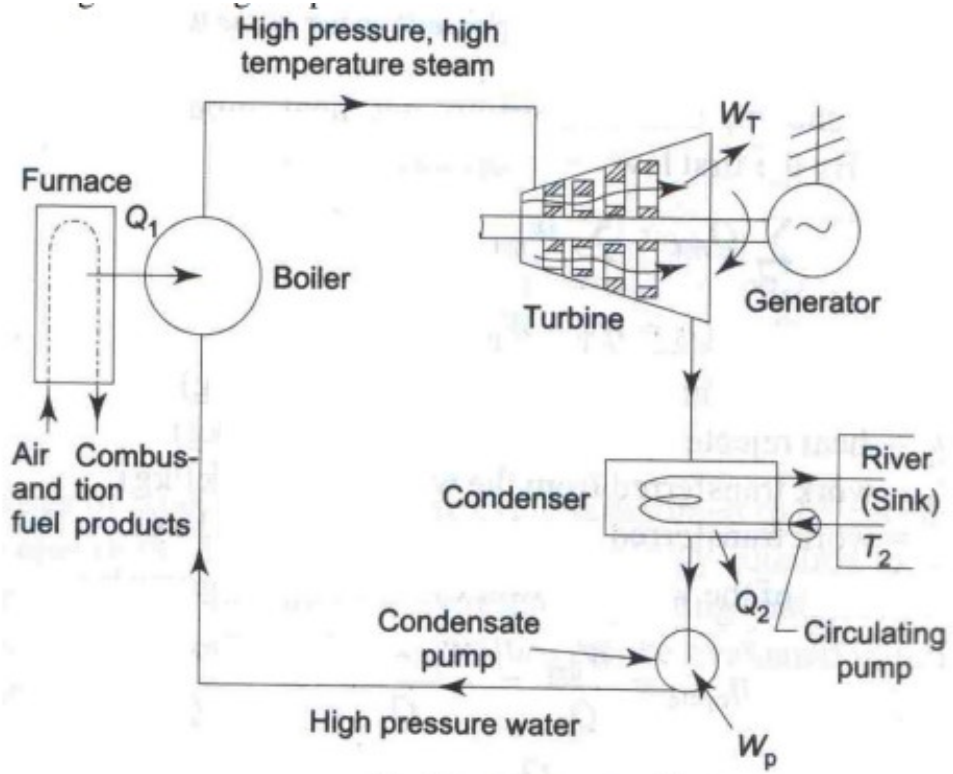
Regenerative Brayton Cycle

- The following shows a plot of the regenerative Brayton cycle efficiency as a function of the pressure ratio and minimum to maximum temperature ratio, T1/T3.



Chapter 7 Vapor Cycle

Carnot vapour power cycle, drawbacks as a reference cycle, simple Rankine cycle; description, T-s diagram, analysis for performance. Comparison of Carnot and Rankine cycles. Effects of pressure and temperature on Rankine cycle performance. Actual vapour power cycles. Ideal and practical regenerative Rankine cycles, open and closed feed water heaters. Reheat Rankine cycle.



Simple steam power plant

Application of vapor power cycle

- Vapor power cycles are used in steam power plants. In a power cycle heat energy (released by the burning of fuel) is converted into work (shaft work), in which a working fluid repeatedly performs a succession of processes. In a vapour power cycle, the working fluid is water, which undergoes a change of phase.

Chapter 7 Vapor Cycle

By the 1st law of Thermodynamics,

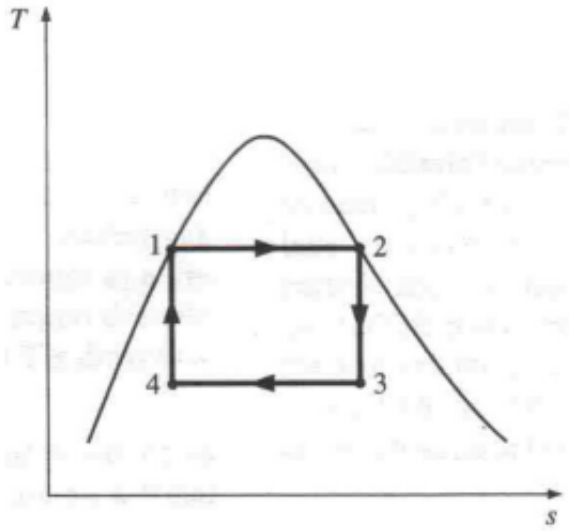
$$\sum_{cycle} Q_{net} = \sum_{cycle} W_{net}$$

Or $$Q_H - Q_L = W_T - W_P$$

Where Q_H = heat transferred to the working fluid (kJ/kg)
Q_L = heat rejected from the working fluid (kJ/kg)
W_T = work transferred from the working fluid (kJ/kg)
W_P = work transferred into the working fluid (kJ/kg)

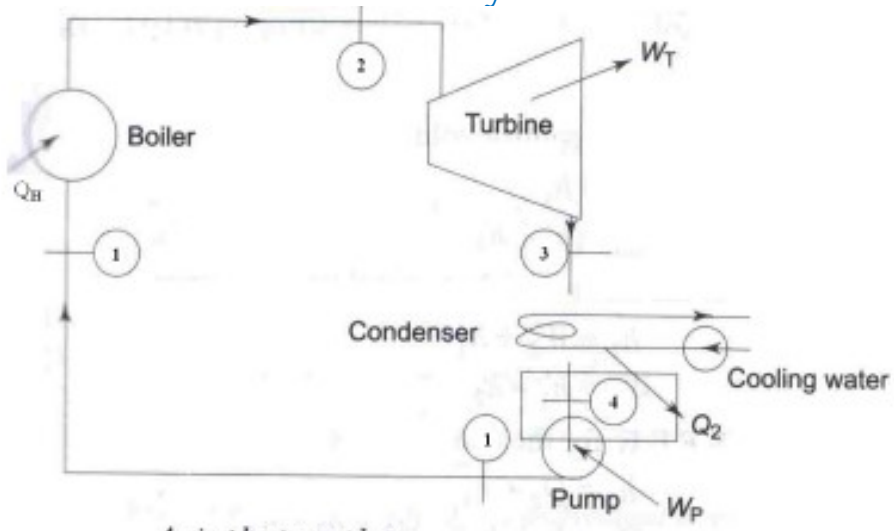
$$\therefore \quad \eta_{cycle} = \frac{W_{net}}{Q_H} = \frac{W_T - W_P}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

Idealized steam power cycles:



Rankine Cycle:

- The simplest way of overcoming the inherent practical difficulties of the Carnot cycle without deviating too much from it is to keep the processes 1-2 and 2-3 of the latter unchanged and to continue the process 3-4 in the condenser until all the vapour has been converted into liquid water. Water is then pumped into the boiler upto the pressure corresponding to the state 1 and the cycle is completed. Such a cycle is known as the Rankine cycle. This theoretical cycle is free of all the practical limitations of the Carnot cycle.



A simple steam plant

Chapter 7 Vapor Cycle

Rankine Cycle:

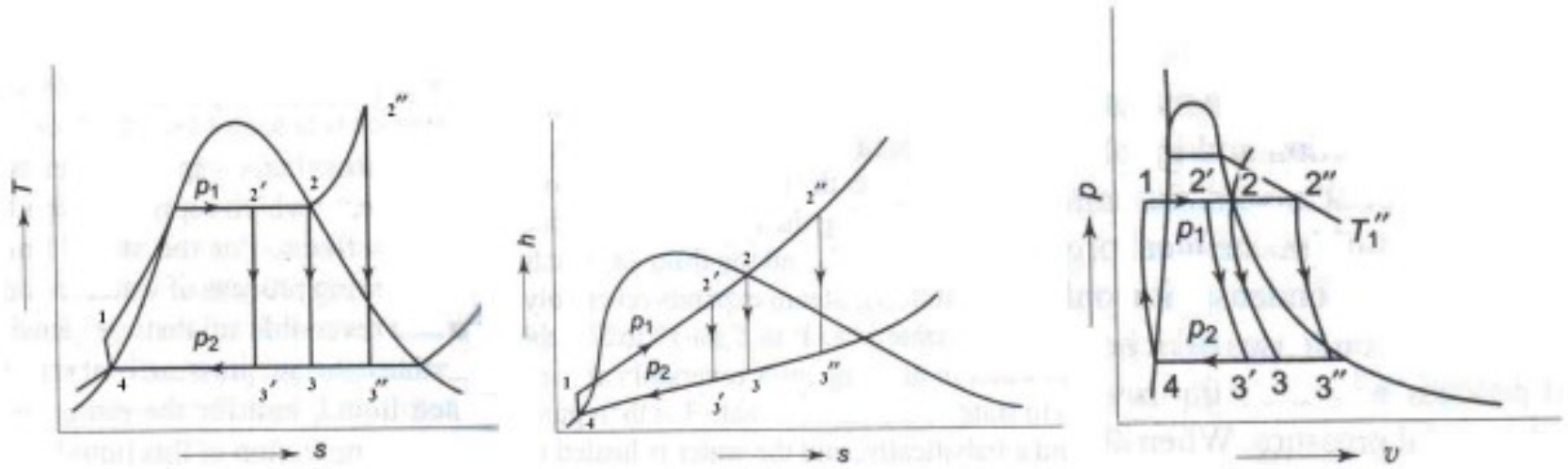


Figure (a) shows the schematic diagram for a simple steam power cycle which works on the principle of a Rankine cycle. Figure (b) represents the T-S diagram of the cycle.

The Rankine cycle comprises the following processes. Process

a) 1-2: Constant pressure heat transfer process in the boiler
b) Process 2-3: Reversible adiabatic expansion process in the steam turbine
c) Process 3-4: Constant pressure heat transfer process in the condenser and
d) Process 4-1: Reversible adiabatic compression process in the pump.

Chapter 7 Vapor Cycle

Rankine Cycle:

- Applying Steady flow energy equation (SFEE) to each of the processes on the basis of unit mass of fluid and neglecting changes in KE & PE, the work and heat quantities can be evaluated.
- For 1kg of fluid, the SFEE for the boiler as the CV, gives,
- h1 + QH = h2 i.e., QH = h2 – h1 --- (1)
- SFEE to turbine, h2 = WT + h3 i.e., WT = h2 – h3 --- (2)
- SFEE to condenser, h3 ± QL + h4 i.e., QL = h3 – h4 --- (3)
- SFEE to pump, h4 + WP = h1 i.e., WP = h1 – h4 --- (4)

The efficiency of Rankine cycle is $\eta = \frac{W_{net}}{Q_H} = \frac{W_T - W_P}{Q_H}$

i.e., $\eta = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$ or $\eta = \frac{(h_2 - h_1) - (h_3 - h_4)}{(h_2 - h_1)}$

The pump handles liquid water which is incompressible i.e., its density or specific volume undergoes little change with an increase in pressure.

For reversible adiabatic compression, we have Tds = dh – vdp; since ds = 0 . We have, dh = vdp

Since change in specific volume is negligible, Δh = v Δ P Or (h1 – h4) = v4 (P2 – P3)

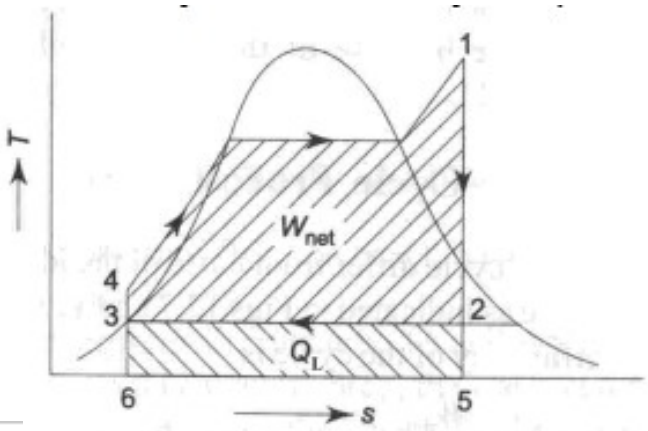
Usually the pump work is quite small compared to the turbine work and is sometimes neglected. In that case, h1 = h4

Chapter 7 Vapor Cycle

Rankine Cycle:

$$\eta_{rankine} \equiv \frac{(h_2 - h_3)}{(h_2 - h_1)} \equiv \frac{(h_2 - h_3)}{(h_2 - h_4)}$$

- The efficiency of the Rankine cycle is presented graphically in the T-S diagram



Q_H, W_{net} and Q_L are proportional to areas

Q_H α area 2-5-6-1,
Q_L α area 3-5-6-4

$W_{net} = (Q_H - Q_L)$ = area 1-2-3-4 enclosed by the cycle.

The capacity of the steam plant is expressed in terms of steam rate defined as the rate of steam flow (kg/h) required to produce unit shaft output (1kW)

$$\therefore \quad Steam\ rate = \frac{1}{W_T - W_P}\ \frac{kg}{kJ}\ \frac{1kJ/s}{1kW} = Specific\ Steam\ Consumption(SSC)$$

$$= \frac{1}{W_T - W_P}\ \frac{kg}{kWs} = \frac{3600}{W_T - W_P}\ \frac{kg}{kWh}$$

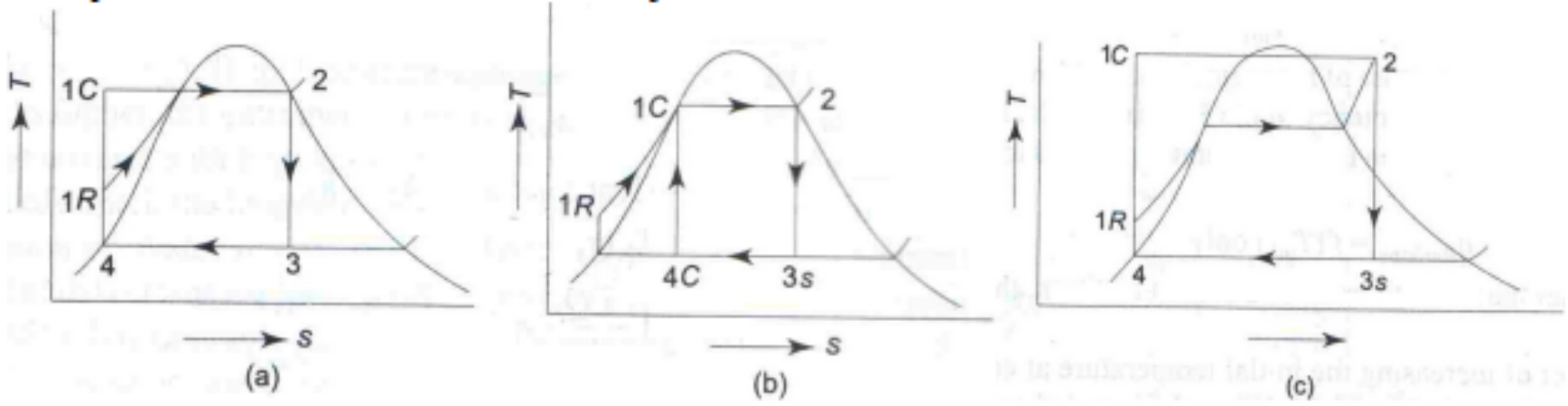
Chapter 7 Vapor Cycle

The cycle efficiency also expressed alternatively as heat rate which is the rate of heat input (Q_H) required to produce unit work output (1kW)

$$\text{Heat rate} = \frac{3600\,Q_H}{W_T - W_P} = \frac{3600}{\eta_{cycle}} \quad \frac{kJ}{kWh}$$

$$\text{Lastly, work ratio } r_w = \frac{\oint \delta W}{positive\ work} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_3)}$$

Comparison of Rankine and Carnot cycles



Chapter 7 Vapor Cycle

- Carnot cycle has the maximum possible efficiency for the given limits of temperature. But it is not suitable in steam power plants. Figure shows the Rankine and Carnot cycles on the T-S diagram.
- The reversible adiabatic expansion in the turbine, the constant temperature heat rejection in the condenser, and the Reversible adiabatic compression in the pump, are similar characteristic features of both the Rankine and Carnot cycles. But whereas the heat addition process in the Rankine cycle is reversible and at constant pressure, in the carnot cycle it is reversible and isothermal.

Mean temperature of Heat addition

- In the Rankine cycle, heat is added reversibly at a constant pressure, but at infinite temperatures. Let Tm1, is the mean temperature of heat addition, so that area under 1s and 2 is equal to the area under 5-6.

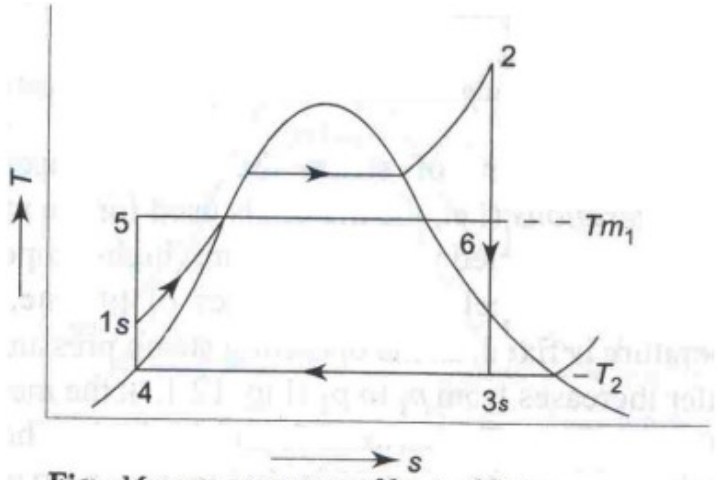


Fig. *Mean temperature of heat addition*

Heat added, $Q_H = h_2 - h_{1S} = T_{m1}\,(S_2 - S_{1S})$

$\therefore T_{m1}$ = Mean temperature of heat addition $= \dfrac{h_2 - h_{1S}}{S_2 - S_{1S}}$

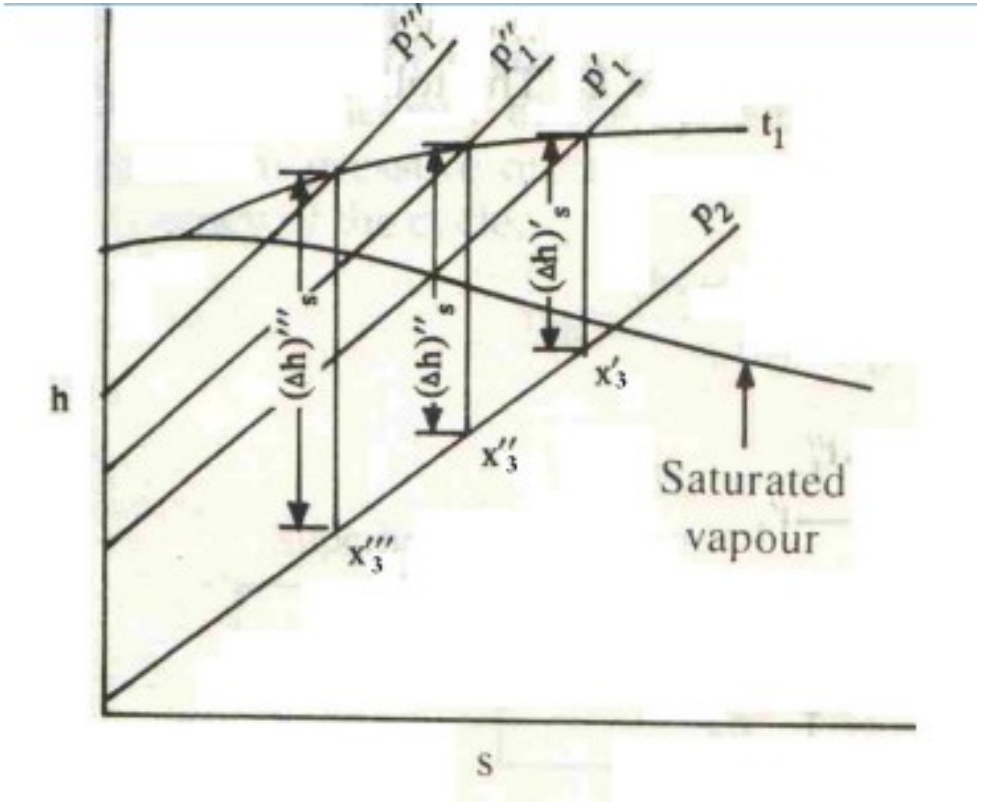
Q_L = heat rejected $= h_{3S} - h_4$
$= T_3\,(S_2 - S_{1S})$

$$\therefore \quad \eta_R = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_3(S_2 - S_{1S})}{T_{m1}(S_2 - S_{1S})}$$

$$\eta_R = 1 - \frac{T_3}{T_{m1}}$$ where T_3 = temperature of heat rejection.

Chapter 7 Vapor Cycle

- Effect of Boiler Pressure (Using Molliar Diagram i.e., h-s diagram)



i.e., Rankine cycle η depends on h2, h1 and ΔhS. From figure as P1‴ > P1″ > P1′ for the fixed maximum temperature of the steam t1 and condenser pressure P2, Isentropic heat drops increases with boiler pressure i.e., from the figure therefore it is evident that as boiler pressure increases, the isentropic heat drop (Δh)S increases, but the enthalpy of the steam entering the turbine decreases, with the result that the Rankine η increases. But quality of the steam at the exit of the turbine suffers i.e., x3‴ < x3 ″ < x3′, which leads to serious wear of the turbine blades.

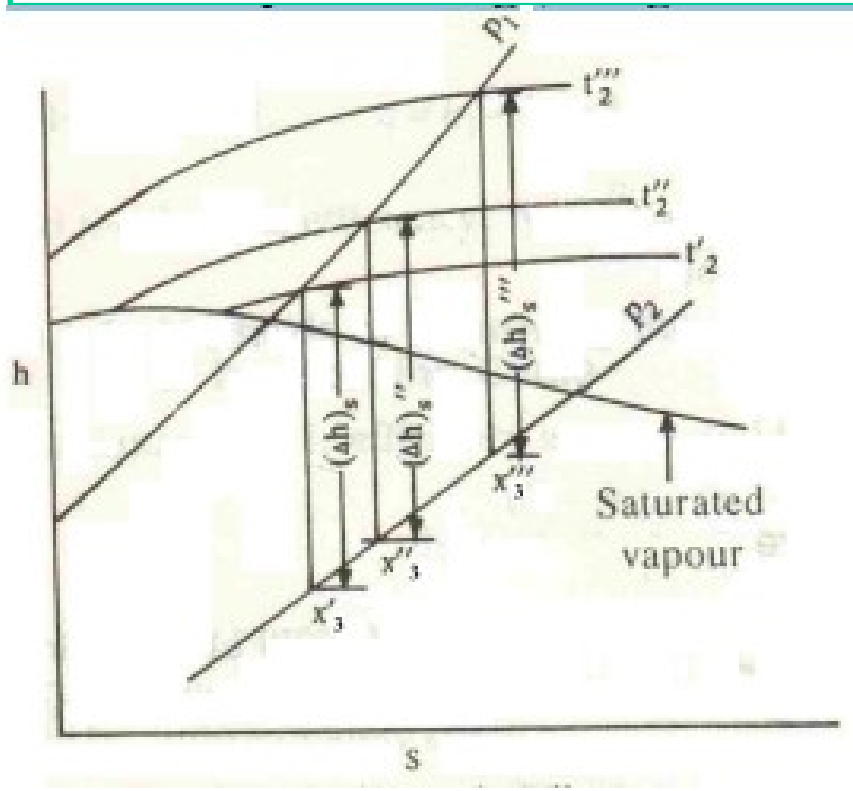
We have,

$$\eta_{th} = \frac{(h_2 - h_3) - (h_1 - h_4)}{h_2 - h_1} \text{ but } W_P << W_T$$

$$\therefore \eta_{th} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{(\Delta h)_S}{(h_2 - h_1)}$$

Chapter 7 Vapor Cycle

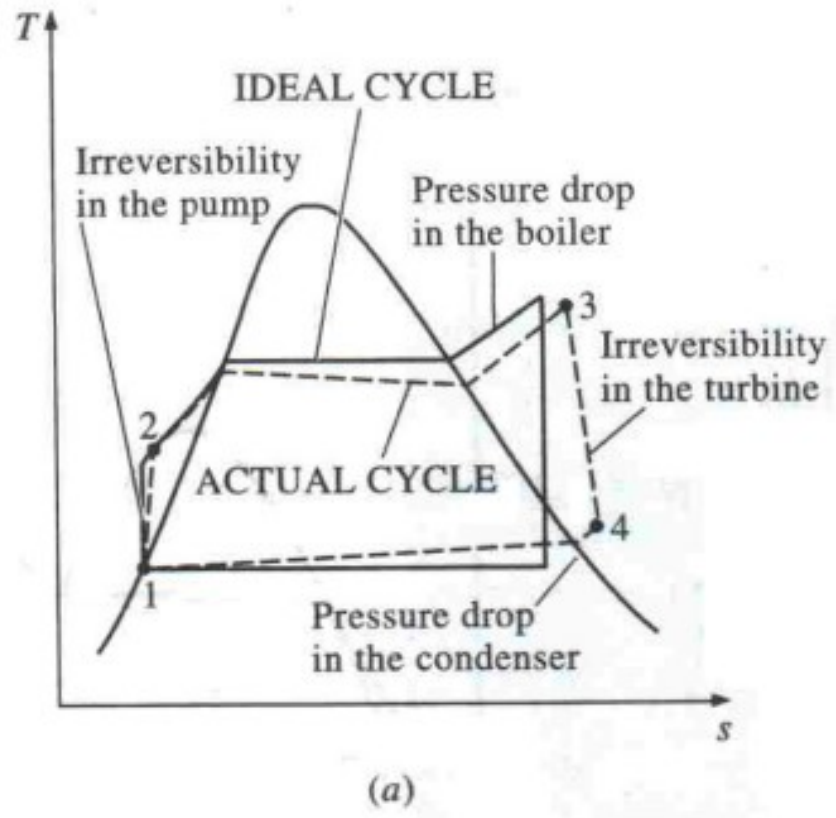
- Effect of Super Heating (Using Molliar Diagram i.e., h-s diagram)

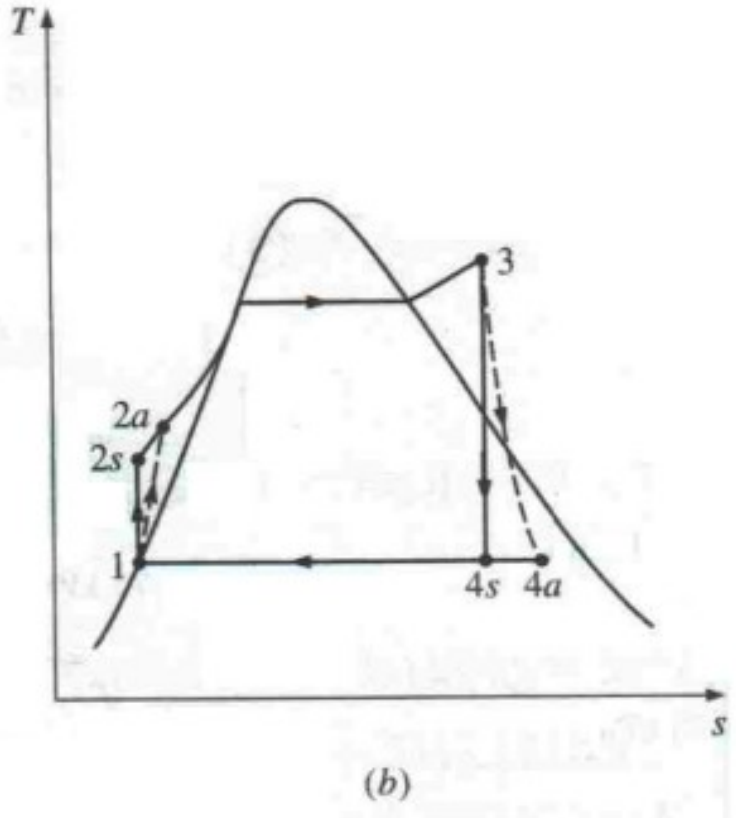


- The moisture in the steam at the end of the expansion may be reduced by increasing the super heated temperature of steam t2. This can be seen in figure where $t2''' > t2'' > t2'$, but $x3' < x3'' < x3'''$.
- It is, therefore, natural that to avoid erosion of the turbine blades, an increase in the boiler pressure must be accompanied by super heating at a higher temperature and since this raises the mean average temperature at which heat is transferred to the steam, the Rankine η increases.

Chapter 7 Vapor Cycle

- Deviation of Actual Vapour Power cycles from Ideal cycle





- The actual Vapour power cycle differs from the ideal Rankine cycle, as shown in figure, as a result of irreversibilities in various components mainly because of fluid friction and heat loss to the surroundings.

Chapter 7 Vapor Cycle

- Deviation of Actual Vapour Power cycles from Ideal cycle

- As a result of irreversibilities, a pump requires a greater work input, and a turbine produces a smaller work output. Under the ideal conditions, the flow through these devices are isentropic. The deviation of actual pumps and turbines from the isentropic ones can be accounted for by utilizing isentropic efficiencies, defined as

$$\eta_P = \frac{W_S}{W_a} = \frac{h_{1S} - h_4}{h_1 - h_4}$$

$$\text{And } \eta_t = \frac{W_a}{W_S} = \frac{h_2 - h_3}{h_2 - h_{3S}}$$

Ideal Reheat cycle: We know that, the efficiency of the Rankine cycle could be increased by increasing steam pressure in the boiler and superheating the steam. But this increases the moisture content of the steam in the lower pressure stages in the turbine, which may lead to erosion of the turbine blade. ∴The reheat cycle has been developed to take advantage of the increased pressure of the boiler, avoiding the excessive moisture of the steam in the low pressure stages. In the reheat cycle, steam after partial expansion in the turbine is brought back to the boiler, reheated by combustion gases and then fed back to the turbine for further expansion.

Chapter 7 Vapor Cycle

- In the reheat cycle the expansion of steam from the initial state (2) to the condenser pressure is carried out in two or more steps, depending upon the number of reheats used.
- In the first step, steam expands in HP turbine from state 2 to approximate the saturated vapour line (process 2-3s). The steam is then reheated (or resuperheated) at constant pressure in the boiler (or in a reheater) process 3s-4 and the remaining expansion process 4s-5 is carried out in the LP turbine.
- To protect the reheater tubes, steam is not allowed to expand deep into the two-phase region before it is taken for reheating, because in that case the moisture particles in steam while evaporating would leave behind solid deposits in the form of scale which is difficult to remove. Also a low reheat pressure may bring down Tm1 and hence cycle η. Again a high reheat pressure increases the moisture content at turbine exhaust. Thus reheat pressure is optimized. Optimum reheat pressure is about 0.2 to 0.25 of initial pressure.

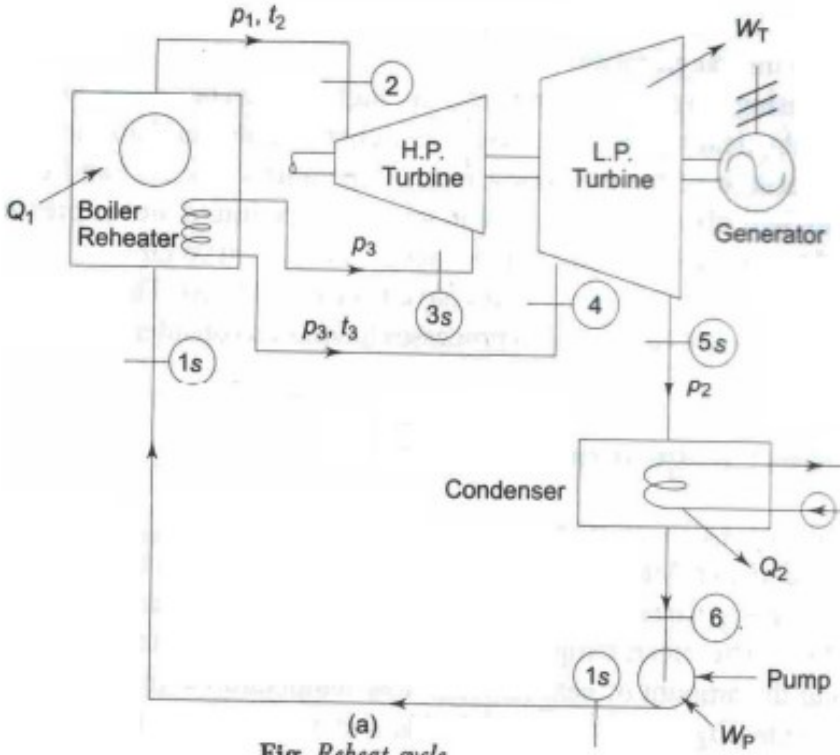
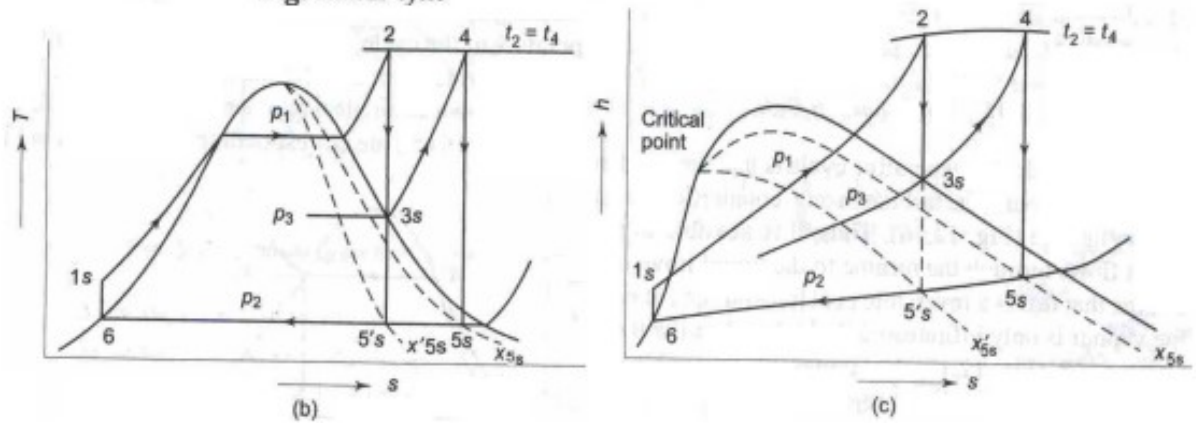


Fig. *Reheat cycle*



Chapter 7 Vapor Cycle

- In practice, the use of reheat gives a marginal increase in cycle η, but it increases the net work output by making possible the use of higher pressures, keeping the quality of steam at turbine exhaust within a permissible limit. The quality improves from X51 s to x5S by the use of reheat.

We have for 1 kg of steam

$Q_H = (h_2 - h_{1S}) + (h_4 - h_{3S})$; $Q_L = h_{5S} - h_6$

$W_T = (h_2 - h_{3S}) + (h_4 - h_{5S})$; $W_P = h_{1S} - h_6$

$$\therefore \quad \eta_R = \frac{W_T - W_P}{Q_H} ;$$

$$\text{Steam rate} = \frac{3600}{(W_T - W_P)} kg / kWh$$

Since higher reheat pressure is used, W_P work is appreciable.

Chapter 7 Vapor Cycle

- Ideal Regenerative cycle: The mean temperature of heat addition can also be increased by decreasing the amount of heat added at low temperatures. In a simple Rankine cycle (saturated steam entering the turbine), a considerable part of the total heat supplied is in the liquid phase when heating up water from 1 to 11 , at a temperature lower than T2, the maximum temperature of the cycle. For maximum η, all heat should be supplied at T2, and feed water should enter the boiler at 11 . This may be accomplished in what is known as an ideal regenerative cycle as shown in figures (a) and (b).

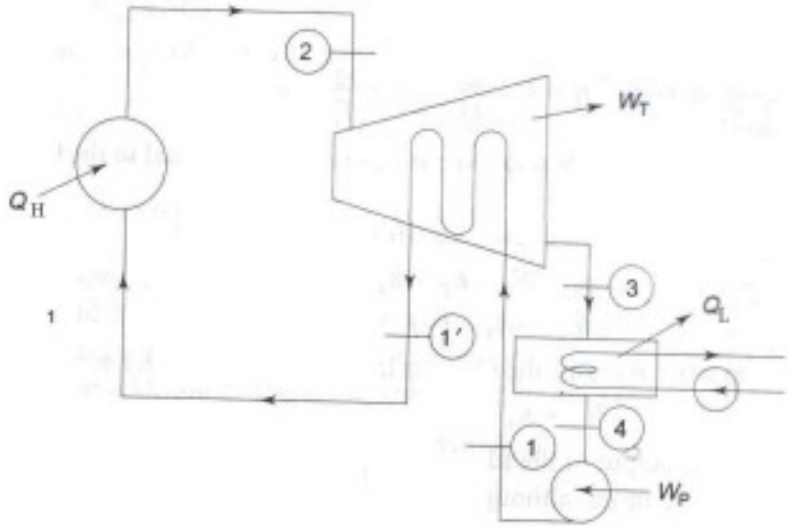


Fig. *Ideal regenerative cycle*

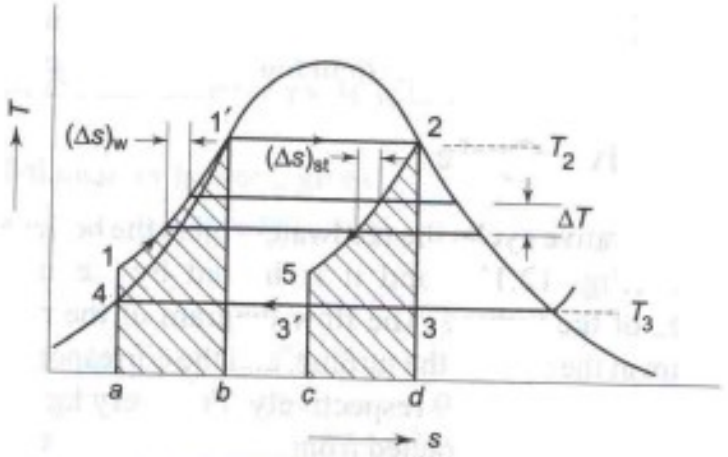


Fig. *Ideal regenerative cycle on T-s plot*

$\Delta T(\text{water}) = -\Delta T\,(\text{steam})$

Chapter 7 Vapor Cycle

- The unique feature of the ideal regenerative cycle is that the condensate, after leaving the pump circulates around the turbine casing, counter-flow to the direction of vapour flow in the turbine. Thus it is possible to transfer heat from the vapour as it flows through the turbine to the liquid flowing around the turbine. Let us assume that this is a reversible heat transfer i.e., at each point, the temperature of the vapour is only infinitesimally higher than the temperature of the liquid. ∴The process 2-31 represents reversible expansion of steam in the turbine with reversible heat rejection. i.e., for any small step in the process of heating the water ΔT (water) = - ΔT (steam) and (ΔS) water = (ΔS) steam. Then the slopes of lines 2-31 and 11 -4 will be identical at every temperature and the lines will be identical in contour. Areas 1-11 -b-a-1 and 31 -2-d-c-31 are not only equal but congruous. ∴, all heat added from external source (QH) is at constant temperature T2 and all heat rejected (QL) is at constant temperature T3, both being reversible

Then $Q_H = h_2 - h_1^{\,1} = T_2 (S_2 - S_1^{\,1})$

$Q_L = h_3^{\,1} - h_4 = T_3 (S_3^{\,1} - S_4)$

Since $S_1^{\,1} - S_4 = S_2 - S_3^{\,1}$ or $S_2 - S_1^{\,1} = S_3^{\,1} - S_4$

$\therefore \quad \eta_{Reg} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_3}{T_2}$ i.e., the η of ideal regenerative cycle is thus equal to the Carnot cycle η.

Writing SFEE to turbine,

$$h_2 + h_1 = W_T + h_1^1 + h_3^1$$

$$\text{i.e., } W_T = (h_2 - h_3^1) - (h_1^1 - h_1)$$

$$\text{or } W_T = (h_2 - h_3^1) - (h_1^1 - h_1) \quad \text{---} \quad (1)$$

and the W_P is same as simple rankine cycle i.e., $W_P = (h_1 - h_4)$

∴The net work output of the ideal regenerative cycle is less and hence its steam rate will be more. Although it is more efficient when compared to rankine cycle, this cycle is not practicable for the following reasons. 1) Reversible heat transfer cannot be obtained in finite time. 2) Heat exchanger in the turbine is mechanically impracticable. The moisture content of the steam in the turbine is high.

Regenerative cycle:

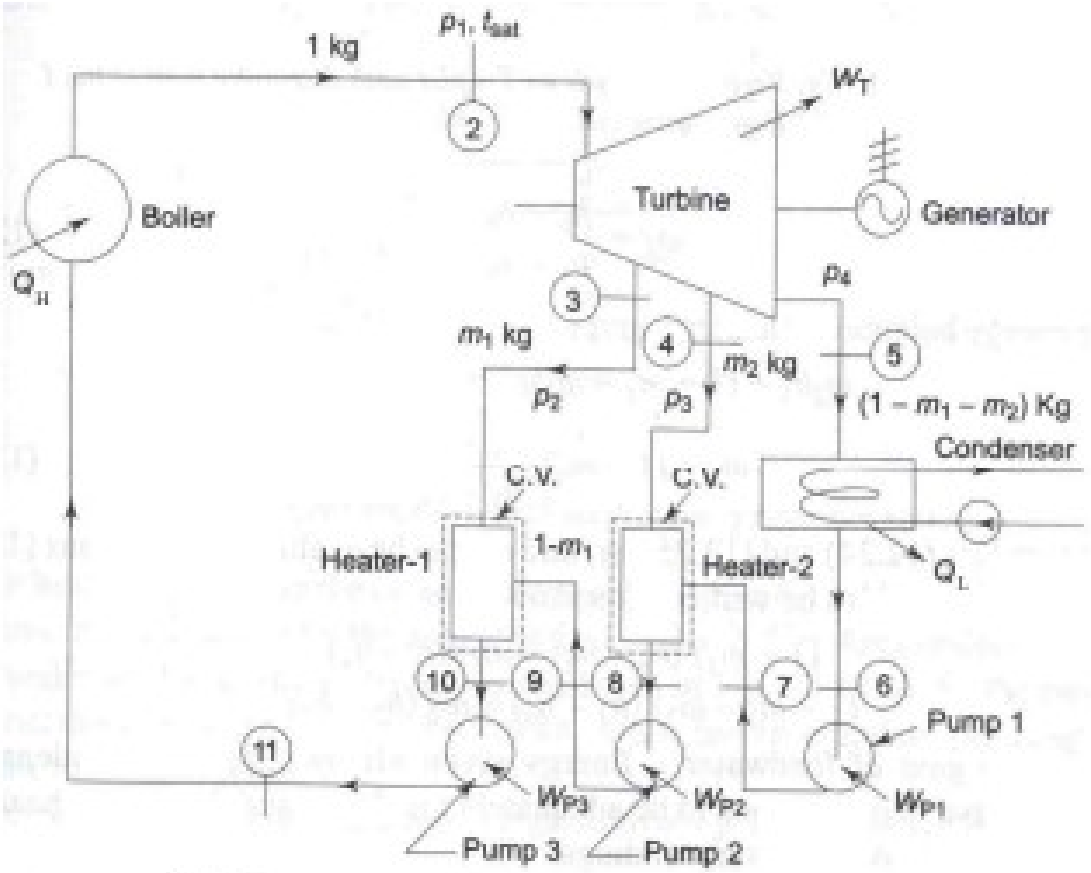
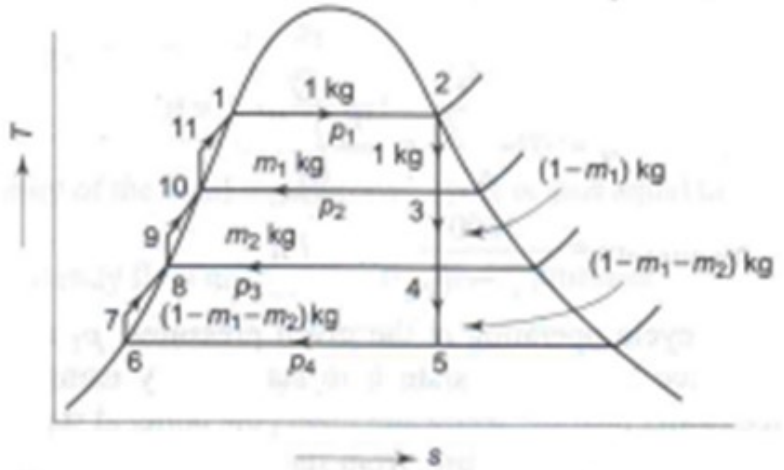
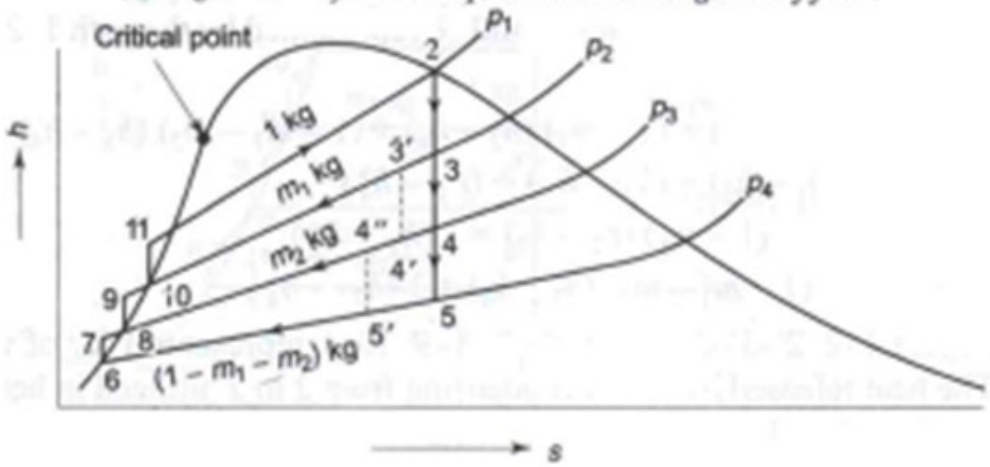


Fig. *Regenerative cycle flow diagram with two feedwater heaters*



(a) Regenerative cycle on T-s plot with decreasing mass of fluid



Regenerative cycle:

$\therefore$ Turbine work, $W_T = 1(h_2 - h_3) + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_4 - h_5)$

Pump work, $W_P = W_{P1} + W_{P2} + W_{P3}$

$= (1 - m_1 - m_2)(h_7 - h_6) + (1 - m_1)(h_9 - h_8) + 1\,(h_{11} - h_{10})$

$Q_H = (h_2 - h_{11})$; $\quad Q_L = (1 - m_1 - m_2)(h_5 - h_6)$

$\therefore$ Cycle efficiency, $\eta = \dfrac{Q_H - Q_L}{Q_H} = \dfrac{W_T - W_P}{Q_H}$

$$SSC = \frac{3600}{W_T - W_P}\, kg/kWh$$

In the Rankine cycle operating at the given pressure P1 and P4, the heat addition would have been from state 7 to state 2. By using two stages of regenerative feed water heating., feed water enters the boiler at state 11, instead of state 7, and heat addition is, therefore from state 11 to state 2.

Therefore $(T_{m1})_{\text{with regeneration}} = \dfrac{h_2 - h_{11}}{S_2 - S_{11}}$

And $(T_{m1})_{\text{without regeneration}} = \dfrac{h_2 - h_7}{S_2 - S_7}$

Since (Tm1)with regenerative > (Tm1)without regenerative, the η of the regenerative cycle will be higher than that of the Rankine cycle.

The energy balance for heater 1,

$m_1 h_3 + (1 - m_1) h_9 = 1\, h_{10}$

$$\therefore m_1 = \frac{h_{10} - h_9}{h_3 - h_9} \qquad \text{--- (1)}$$

Regenerative cycle:

The energy balance for heater 2,

$$m_2 h_4 + (1 - m_1 - m_2) h_7 = (1 - m_1) h_8$$

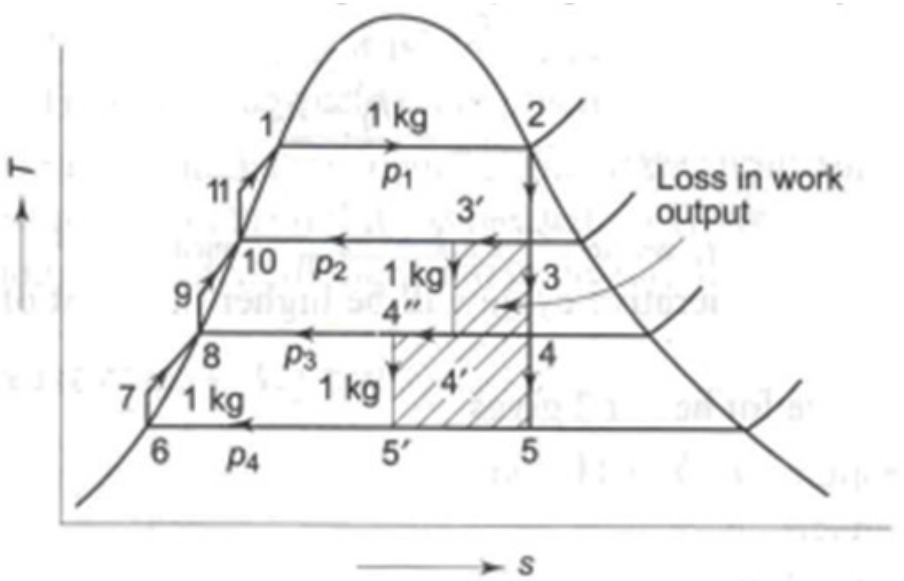
$$\text{Or } m_2 = (1 - m_1)\frac{(h_8 - h_7)}{(h_4 - h_7)} \quad \text{--- (2)}$$

Above equations (1) and (2) can also be written alternatively as

$$(1 - m_1)(h_{10} - h_9) = m_1 (h_3 - h_{10})$$

$$\text{and } (1 - m_1 - m_2)(h_8 - h_7) = m_2 (h_4 - h_8)$$

- Energy gain of feed water = energy given off by vapour in condensation. Heaters have been assumed to be adequately insulated and there is nc heat gain from, or heat loss to, the surroundings. In figure (a) path 2-3 5 represents the states of a decreasi mass of fluid. For 1kg of steam, the states would be represented by the path 2-31 -411-51 .



(b) Regenerative cycle on T-s plot for unit mass of fluid

We have $W_T = (h_2 - h_3) + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_4 - h_5)$

$= (h_2 - h_3) + (h_3^1 - h_4^1) + (h_4^{11} - h_5^1)$ [From Figure b] --- (3)

The cycle $2 - 3 - 3^1 - 4^1 - 4^{11} - 5^1 - 6 - 7 - 8 - 9 - 10 - 11 - 2$ represents 1kg of working fluid. The heat released by steam condensing from 3 to 3^1 is utilized in heating up the water from 9 to 10.

$\therefore \quad 1 (h_3 - h_3^1) = 1 (h_{10} - h_9)$ --- (4)

Similarly, $1 (h_4^1 - h_4^{11}) = 1 (h_8 - h_7)$ --- (5)