
Institution: Mulungushi University

Course: MSM131 - Mathematics I

Instruction: Submit * marked questions (Typed/Scanned copies) to mathmustudents@gmail.com not later than, 18th September, 2020 (Compulsory)

1. Consider $x = 3^r$ and $y = 3^q$. Express the following in terms of x and/ or y .

(a) 9^{r+q} .

(b) 3^{4q} .

(c) 3^{r-3} .

(d) $* \frac{27^q}{9^{r+1}}$.

2. Given that

(a) $9^m \times 27^n = 3^k$, express k in terms of m and n .

$* (b) (\sqrt{3})^n = \frac{3^x}{27^y}$, express n in terms of x and y .

3. Solve the following equations:

(a) $270 = 2 \times 3^x \times 5$,

(b) $\sqrt{12} + \sqrt{108} = \sqrt{x}$,

(c) $5 \times 8^{3n+1} = 80$,

(d) $*(25\sqrt{5})^x = (\sqrt[3]{5})^{x+1}$.

4. (a) Express $\frac{(3^{-1}a^4b^{-3})^{-2}}{(6a^2b^{-1}c^{-2})^2}$ in the form $\frac{b^nc^m}{4a^r}$ where n, m and r are integers to be found.

(b) *Given that

$$\left(\frac{15m^3n^{-2}p^{-1}}{25m^{-2}n^{-4}} \right)^{-3} = \frac{125p^x}{27m^yn^z}. \quad (1)$$

Find the values of x, y and z .

5. (a) Find the value of b given that $N = 2^{0.15}$ and $N^b = 16$.

(b) *If $5^{10x} = 4900$ and $2^{\sqrt{y}} = 25$, what is the value of $\frac{(5^{x-1})^5}{4^{-\sqrt{y}}}$.

6. The binary operation $*$ defined on $\mathbb{Z}$ by $x * y = 1 - 2xy$. Determine whether $*$ is commutative and/ or associative.

7. An operation $\#$ is defined on the set $\{7, 9, 11\}$ in the table below as follows:

$a \# b$ is the result where the row along “ a ” and column along “ b ” meet. e.g $9 \# 11 = 9$.

$\#$	7	9	11
7	7	9	11
9	11	7	9
11	9	11	7

- (a) Is this operation a binary operation on set $\{7, 9, 11\}$?
 (b) Is this operation commutative?
 (c) Is this operation Associative?
 (d) Evaluate $11 \# (7 \# 9) \# (9 \# 11)$.
8. * An operation $\sim$ is defined on the set $\{1, 2, 3, 4\}$ in the table below as follows:
 $x \sim y$ is the result where the column along “ x ” and the row along “ y ” intersect.

$\sim$	1	2	3	4
1	4	3	2	1
2	3	1	4	2
3	2	4	1	3
4	1	2	3	4

- (a) What is the identity element for the operation $\sim$?
 (b) Is this operation a binary operation on set $\{1, 2, 3, 4\}$? Justify your answer.
 (c) Evaluate $3 \sim (2 \sim 4)$.
 (d) Is this operation commutative? Associative? Justify your answer.
9. If U is a universal set and A, B are its subsets. Where $U = \{1, 2, 3, 4, 5\}$. $A = \{1, 3, 5\}$, $B = \{x : x \text{ is a prime number}\}$, find $A^c \times B^c$.
10. Let $A = \{4, 6, 8, 10\}$, $B = \{2, 3, 4, 5\}$. R be a relation defined from A to B where $R = \{(a, b) : a \in A, b \in B \text{ and } a \text{ is a multiple of } b\}$. Find
- (a) R and R^{-1} in Roster form,
 (b) Domain of R ,
 (c) Range of R .
11. * Let R be a relation on $\mathbb{N}$ defined by

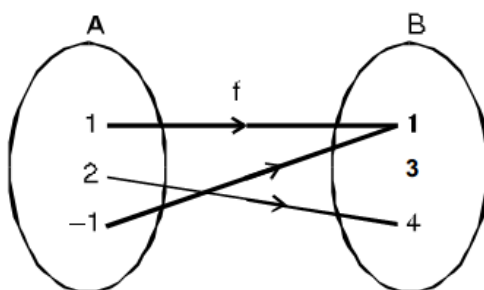
$$R = \{(x, x^2) : x \text{ is a prime number less than } 15\}.$$

Find

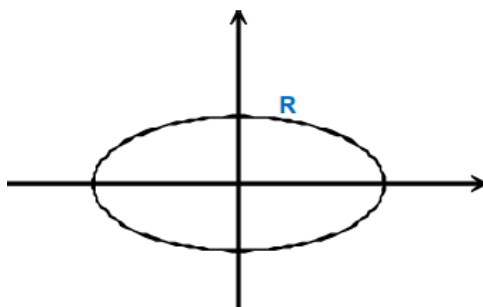
- (a) R and R^{-1} in Roster form,
 (b) Domain of R^{-1} ,
 (c) Range of R^{-1} ,
 (d) Co-domain of R .
12. State with a justification whether each of the following relations represent a function or not.

(a) $R = \left\{ \left(\sin \frac{\pi}{6}, \frac{1}{2} \right), \left(\cos \frac{\pi}{6}, \frac{\sqrt{3}}{2} \right), \left(\frac{1}{2}, -\frac{\sqrt{3}}{2} \right), \left(\tan \frac{\pi}{6}, \frac{1}{\sqrt{3}} \right), \left(\cot \frac{\pi}{6}, \sqrt{3} \right) \right\}$

(b) * $f : A \rightarrow B$ shown below:



- (c) $y - x \geq 0$
 (d) *The relation R below:



13. The function f and g are defined by $f(x) = x^2$ and $g(x) = \frac{1}{2}x + 1$ for all real values of x in the domain $0 \leq x \leq 3$.
- (a) Find $f^{-1}(x)$ and $g^{-1}(x)$.
 (b) Sketch, on the same diagram, the graphs of f , g , $f^{-1}(x)$ and $g^{-1}(x)$.
14. Find the compositions $(f \circ g)(x)$ and $(g \circ f)(x)$ for the functions given by:
- (a) $f(x) = x^2 - 9$ and $g(x) = \sqrt{9 - x^2}$.
 (b) * $f(x) = \sqrt{x}$ and $g(x) = 3 - x^2$.

(c) $f(x) = x^3$ and $g(x) = \frac{1}{x}$.

15. * The functions f and g are defined by $f(x) = \frac{25}{3x-2}$ and $g(x) = x^2$. Find the:

(a) domain and range of fg ,

(b) solution of the equation $fg(x) = \frac{2}{x-1}$.

16. Given

$$f(x) = \begin{cases} x^2 - 4, & x < 2, \\ 5, & x \geq 2. \end{cases}$$

Find

(a) $f(-1)$.

(b) $f(1)$.

(c) $f(2)$.

(d) * $f(3.5)$.

17. * A package of food is removed from the refrigerator. The number N of bacteria in the food is given by $N(T) = 10T^2 - 20T + 600$, $1 \leq T \leq 16$. Where T is the temperature (in degrees Celsius) of the food. The temperature of the food increases according to the function $T(t) = 3t + 1$, $0 \leq t \leq 5$, where t is the time in hours.

(a) Find the function $NT(t)$.

(b) Find the number of bacteria in the food after two hours.

(c) Find the time t when the bacteria count reaches 1500.

18. Which of the following functions are onto function if $f : \mathbb{R} \rightarrow \mathbb{R}$

(a) $f(x) = 115x + 49$

(b) $f(x) = |x|$

19. Determine which of the functions are one-to-one.

(a) $f(x) = -x + 7$.

(b) * $g(x) = \sqrt{2x-5}$.

(c) $h(x) = -x^2 + 3$.

(d) $k(x) = -1 + x^3$.

(e) $f : \{20, 21, 22\} \rightarrow \{40, 42, 44\}$ defined as $f(x) = 2x$

(f) * $f : \{1, 2, 4\} \rightarrow \{5\}$ defined as $f(x) = 5$.

20. Find the inverse of the functions in Question 19 whenever possible and confirm your answers by using the conditions $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$.

21. For each of the following functions state whether the function is odd, even or neither:

(a) $f(x) = 4 - 3x^2$.

(b) $g(x) = 3x^2 + x$.

(c) $h(x) = x - \frac{1}{x}$.

(d) $k(x) = 2 + x^2$.

END !