

# त्रिकोणमिति

## [TRIGONOMETRY]

कुछ महत्त्वपूर्ण सूत्र :

$$1. \sin \theta = \frac{\text{लम्ब}}{\text{कर्ण}}$$

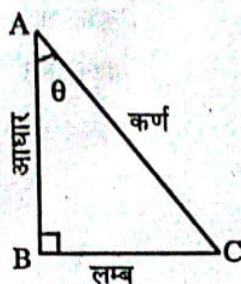
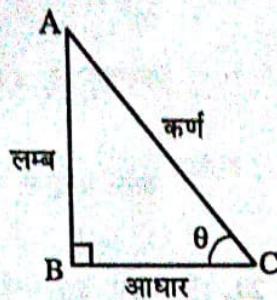
$$\cos \theta = \frac{\text{आधार}}{\text{कर्ण}}$$

$$\tan \theta = \frac{\text{लम्ब}}{\text{आधार}}$$

$$\operatorname{cosec} \theta = \frac{\text{कर्ण}}{\text{लम्ब}}$$

$$\sec \theta = \frac{\text{कर्ण}}{\text{आधार}}$$

$$\cot \theta = \frac{\text{आधार}}{\text{लम्ब}}$$



$$2. \sin \theta \cdot \operatorname{cosec} \theta = 1; \quad \sin \theta = \frac{1}{\operatorname{cosec} \theta};$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$3. \cos \theta \cdot \sec \theta = 1; \quad \cos \theta = \frac{1}{\sec \theta};$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$4. \tan \theta \cdot \cot \theta = 1; \quad \tan \theta = \frac{1}{\cot \theta};$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$5. \sin^2 \theta + \cos^2 \theta = 1$$

$$6. \tan \theta = \frac{\sin \theta}{\cos \theta}; \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$7. \sec^2 \theta - \tan^2 \theta = 1$$

$$8. \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$9. \sin(A + B) = \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$10. \sin(A - B) = \sin A \cdot \cos B - \cos A \cdot \sin B$$

$$11. \cos(A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$12. \cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

$$13. \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$14. \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$

$$15. \cot(A + B) = \frac{\cot A \cdot \cot B - 1}{\cot B + \cot A}$$

$$16. \cot(A - B) = \frac{\cot A \cdot \cot B + 1}{\cot B - \cot A}$$

$$17. \tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \cdot \tan B \cdot \tan C}{1 - \tan A \cdot \tan B - \tan B \cdot \tan C - \tan C \cdot \tan A}$$

$$18. \sin(A + B) \cdot \sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$$

$$19. \cos(A + B) \cdot \cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$$

$$20. \sin 2A = 2 \sin A \cdot \cos A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$21. \cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1 = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

22.  $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$
23.  $2 \sin^2 A = 1 - \cos 2A$
24.  $2 \cos^2 A = 1 + \cos 2A$
25.  $\sin 3A = 3 \sin A - 4 \sin^3 A$
26.  $\cos 3A = 4 \cos^3 A - 3 \cos A$
27.  $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$
28.  $\sin(A + B) + \sin(A - B)$   
 $= 2 \sin A \cdot \cos B$
29.  $\sin(A + B) - \sin(A - B)$   
 $= 2 \cos A \cdot \cos B$
30.  $\cos(A + B) + \cos(A - B)$   
 $= 2 \cos A \cdot \cos B$
31.  $\cos(A - B) - \cos(A + B)$   
 $= 2 \sin A \cdot \sin B$
32.  $\sin C + \sin D = 2 \sin \frac{C+D}{2} \cdot \cos \frac{C-D}{2}$
33.  $\sin C - \sin D = 2 \cos \frac{C+D}{2} \cdot \sin \frac{C-D}{2}$
34.  $\cos C + \cos D = 2 \cos \frac{C+D}{2} \cdot \cos \frac{C-D}{2}$
35.  $\cos C - \cos D = 2 \sin \frac{C+D}{2} \cdot \sin \frac{D-C}{2}$
36.  $\sin(\sin^{-1} x) = x, -1 \leq x \leq 1$   
 $\sin^{-1}(\sin x) = x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$   
 $\cos(\cos^{-1} x) = x, -1 \leq x \leq 1$   
 $\tan(\tan^{-1} x) = x, -\infty < x < \infty$   
 $\tan^{-1}(\tan x) = x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \text{ etc.}$
37.  $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, -1 \leq x \leq 1$   
 $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, -\infty < x < \infty$   
 $\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}, x \leq -1 \text{ or } x \geq 1$
38.  $\sin^{-1}(-x) = -\sin^{-1} x$   
 $\cos^{-1}(-x) = \pi - \cos^{-1} x$

$$\tan^{-1}(-x) = -\tan^{-1} x$$

39.  $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left[ \frac{x+y}{1-xy} \right]$  when  
 $x > 0, y > 0$  and  $xy < 1$

$$= \pi + \tan^{-1} \left[ \frac{x+y}{1-xy} \right] \text{ when } x > 0, y > 0$$

and  $xy > 1$

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left[ \frac{x-y}{1+xy} \right] \text{ if } xy > -1$$

$$\sin^{-1} x + \sin^{-1} y$$

$$= \sin^{-1} [x\sqrt{1-y^2} + y\sqrt{1-x^2}], x \geq 0,$$

$$y \geq 0, x^2 + y^2 \leq 1$$

$$\sin^{-1} x + \sin^{-1} y$$

$$= \pi - \sin^{-1} [x\sqrt{1-y^2} + y\sqrt{1-x^2}] \text{ when}$$

$$x > 0, y > 0 \text{ and } x^2 + y^2 > 1$$

40.  $2 \tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2}$  when  $-1 \leq x \leq 1$

$$= \pi - \sin^{-1} \frac{2x}{1+x^2}, \text{ when } x > 1$$

$$2 \tan^{-1} x = \cos^{-1} \frac{1-x^2}{1+x^2} \text{ when } x \geq 0$$

$$= -\cos^{-1} \frac{1-x^2}{1+x^2} \text{ when } x < 0$$

$$2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2} \text{ when } -1 \leq x \leq 1$$

$$= \pi - \tan^{-1} \frac{2x}{1-x^2} \text{ when } x > 1$$

41.  $\operatorname{cosec}^{-1} x = \sin^{-1} \frac{1}{x}, x \neq 0$

$$\cos^{-1} x = \sec^{-1} \frac{1}{x}, x \neq 0$$

$$\cot^{-1} x = \tan^{-1} \frac{1}{x}, x > 0$$

42.  $\sin^{-1} x = \cos^{-1} \sqrt{1-x^2}, 0 \leq x \leq 1$

$$\sin^{-1} x = -\cos^{-1} \sqrt{1-x^2}, -1 \leq x \leq 0$$

43. यदि  $0^\circ < \theta < 90^\circ$ , तो  $0 < \sin \theta < 1$

44. यदि  $0^\circ < \theta < 90^\circ$ , तो  $1 > \cos \theta > 0$