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IIT JEE MATHEMATICS

Trigonometry Formula & Important Concept (Complete)



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Er. Gyanesh Rai

Address : L-3/87, Chauraha Purania, Sector D, Aliganj Lucknow

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TRIGONOMETRY FORMULA AND IMPORTANT CONCEPT :→

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1. Conversion of Angles :→

A. Radian to degree conversion

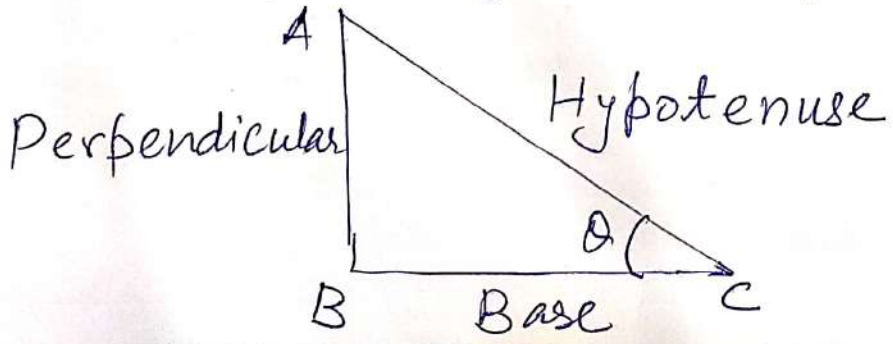
$$\text{Angle in Degree} = (\text{Angle in Radian}) \times \frac{180}{\pi}$$

B. Degree to Radian Conversion

$$\text{Angle in Radian} = (\text{Angle in Degree}) \times \frac{\pi}{180}$$

2. TRIGONOMETRY RATIO :→

ΔABC = Right angled Triangle



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A. $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotanuse}}$

B. $\cos\theta = \frac{\text{Base}}{\text{Hypotanuse}}$

C. $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

D. $\sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotanuse}}{\text{Base}}$

E. $\text{Cosec}\,\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotanuse}}{\text{Perpendicular}}$

F. $\cot\theta = \frac{1}{\tan\theta} = \frac{\text{Base}}{\text{Perpendicular}}$

3. Even and odd function :-

(A) Even function :→

if $f(-x) = f(x)$

$f(x) = \cos x$, $f(x) = \sec x$ → even functia

(B) odd function :→

If $f(-x) = -f(x)$

$f(x) = \tan x, \cot x, \sin x, \text{cosec}\,x$

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4. Trigonometrical Ratio Table :→

T.F ↓ \ Angle →	0°	30°	45°	60°	90°
Sin θ	0	1/2	1/√2	√3/2	1
Cos θ	1	√3/2	1/√2	1/2	0
Tan θ	0	1/√3	1	√3	not defined
Cosec θ	not defined	2	√2	2/√3	1
Sec θ	1	2/√3	√2	2	not defined
Cot θ	not defined	√3	1	1/√3	0

5. Sign of Trigonometric function in all Quadrant :→

I st Quadrant	II nd Quadrant	III rd Quadrant	IV rth Quadrant
A	S	T	C
A - All +ve	S - Sine & Cosec +ve	T - Tan & Cot → +ve	C - Cos & Sec +ve
Sin, Cos, Cot, Tan, Sec, Cosec } +ve	Sin, Cosec } +ve	Tan, Cot } +ve	Cos, Sec } +ve
	RULE - (1)		

Rule - ② Any Trigonometric function comes in any Quadrant

By two ways

		I st way	II nd way
I st Quadrant	→	$90-\theta$	$360+\theta$
II nd Quadrant	→	$180-\theta$	$90+\theta$
III rd Quadrant	→	$180+\theta$	$270-\theta$
IV rth Quadrant	→	$360-\theta$	$270+\theta$

Rule ③

④ If any Trigonometric function comes in any Quadrant (I st to IV rth) around Vertical axis So Trigonometric function will change

Sin → Cos
Cos → Sin
Tan → Cot
Cot → tan
Sec → Cosec
Cosec → Sec

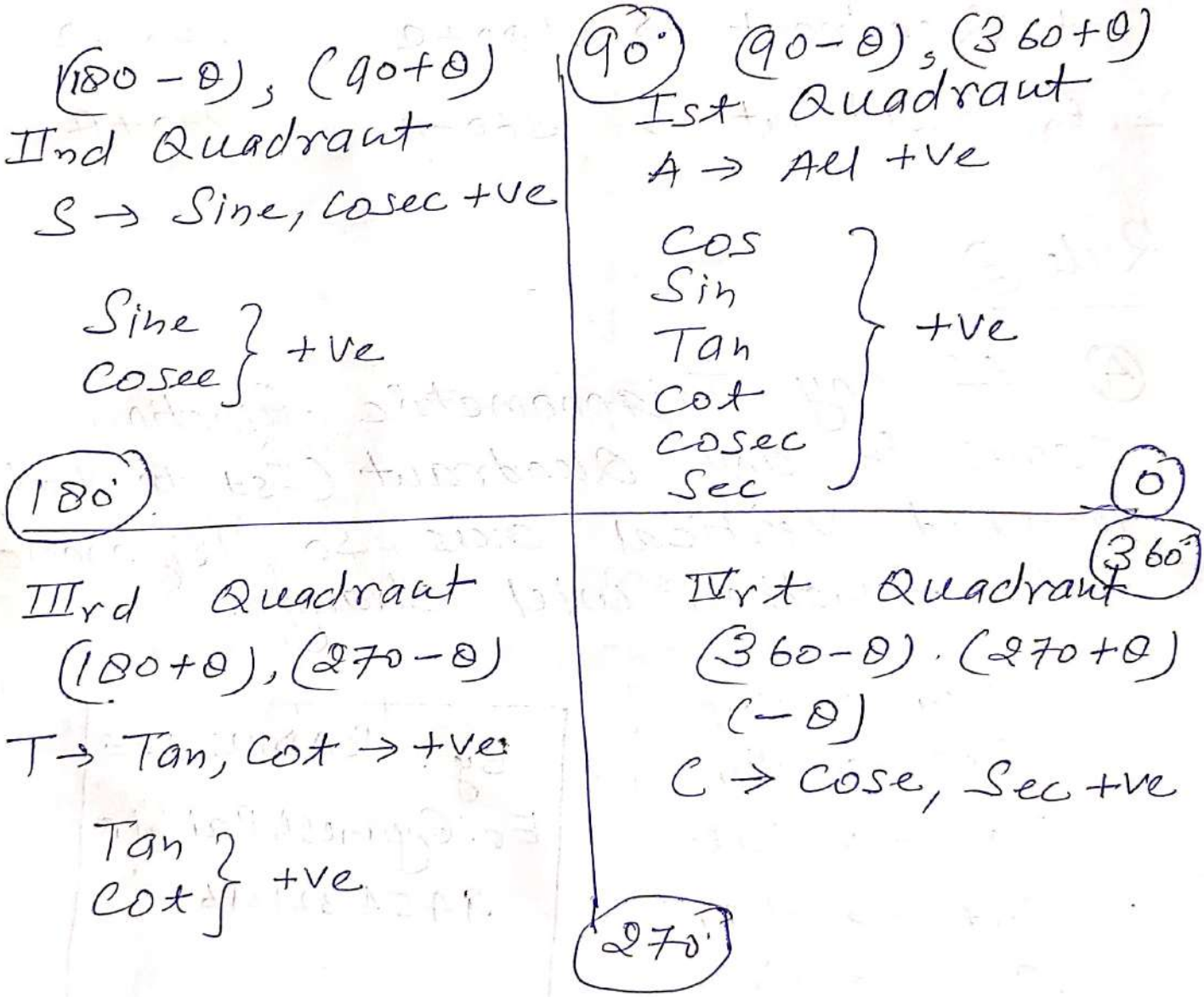
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⑤ if any Trigonometric function comes in any Quadrant around

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Horizontal axis, so Trigonometric function will not change

Sin → Sin	Tan → Tan
Cos → Cos	Sec → Sec
Cot → Cot	Cosec → Cosec



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For Ist Quadrant :→

(A) A → All Positive

(B) 90−θ ⇒ comes around vertical axis so trigonometric function will change

(C) 360+θ ⇒ Comes around Horizontal axis so trigonometric function will not change.

(90−θ) :→

$\sin(90-\theta) = \cos\theta$
$\cos(90-\theta) = \sin\theta$
$\tan(90-\theta) = \cot\theta$
$\cot(90-\theta) = \tan\theta$
$\sec(90-\theta) = \operatorname{cosec}\theta$
$\operatorname{cosec}(90-\theta) = \sec\theta$

} T.F. change

(360+θ) :→

$\sin(360+\theta) = \sin\theta$
$\cos(360+\theta) = \cos\theta$
$\tan(360+\theta) = \tan\theta$
$\cot(360+\theta) = \cot\theta$
$\sec(360+\theta) = \sec\theta$
$\operatorname{cosec}(360+\theta) = \operatorname{cosec}\theta$

} T.F. not Change.

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For IInd Quadrant :→

(A) S → Sine, Cosec → Positive

(B) 90+θ ⇒ Comes around vertical axis so trigonometric function will be change.

(C) 180 − θ ⇒ Comes around Horizontal axis so trigonometric function will not change

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(90+θ) :→

$\sin(90+\theta) = \cos\theta$
$\cos(90+\theta) = -\sin\theta$
$\tan(90+\theta) = -\cot\theta$
$\cot(90+\theta) = -\tan\theta$
$\sec(90+\theta) = -\operatorname{cosec}\theta$
$\operatorname{cosec}(90+\theta) = \sec\theta$

} T. F. change

(180−θ) :→

$\sin(180-\theta) = \sin\theta$
$\cos(180-\theta) = -\cos\theta$
$\tan(180-\theta) = -\tan\theta$
$\cot(180-\theta) = -\cot\theta$
$\sec(180-\theta) = -\sec\theta$
$\operatorname{cosec}(180-\theta) = \operatorname{cosec}\theta$

} T. F. not change

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For III rd Quadraut :→

(A) T → Tan and Cot Positive

(B) $\pi+\theta$ ⇒ comes around Horizontal axis so trigonometric function will not change

(C) $270-\theta$ ⇒ Comes around Vertical axis so trigonometric function will change

(180+θ) :→

$\sin(180+\theta) = -\sin\theta$
$\cos(180+\theta) = -\cos\theta$
$\tan(180+\theta) = +\tan\theta$
$\cot(180+\theta) = \cot\theta$
$\sec(180+\theta) = -\sec\theta$
$\operatorname{cosec}(180+\theta) = -\operatorname{cosec}\theta$

} T. F. Changed

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P, C, M, B

(270−θ) :-

$\sin(270-\theta) = -\cos\theta$
$\cos(270-\theta) = -\sin\theta$
$\tan(270-\theta) = \cot\theta$
$\cot(270-\theta) = \tan\theta$
$\sec(270-\theta) = -\operatorname{cosec}\theta$
$\operatorname{cosec}(270-\theta) = -\sec\theta$

} Trigonometric function (T. F.) Changed

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For IVrth Quadrant :→

(A) C → Cos, Sec Positive

(B) 360-θ ⇒ Comes around Horizontal axis so trigonometric function will not Changed

(C) 270+θ ⇒ Comes around vertical axis so trigonometric function will changed.

(D) -θ ⇒ Clockwise around Horizontal axis

(360-θ) :→

$$\begin{aligned}
\text{Sin}(360-\theta) &= -\text{Sin}\theta \\
\text{Cos}(360-\theta) &= \text{Cos}\theta \\
\text{Tan}(360-\theta) &= -\text{Tan}\theta \\
\text{Cot}(360-\theta) &= -\text{Cot}\theta \\
\text{Sec}(360-\theta) &= \text{Sec}\theta \\
\text{Cosec}(360-\theta) &= -\text{Cosec}\theta
\end{aligned}$$

(270+θ) :→

$$\begin{aligned}
\text{Sin}(270+\theta) &= -\text{Cos}\theta \\
\text{Cos}(270+\theta) &= \text{Sin}\theta \\
\text{Tan}(270+\theta) &= -\text{Cot}\theta \\
\text{Cot}(270+\theta) &= -\text{tan}\theta \\
\text{Sec}(270+\theta) &= +\text{Cosec}\theta \\
\text{Cosec}(270+\theta) &= -\text{Sec}\theta
\end{aligned}$$

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$(-\theta)$:→

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

$\sec(-\theta) = \sec\theta$

$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

6. Basic Formula in Trigonometry :→

1. $\sin^2\theta + \cos^2\theta = 1$

   $\cos^2\theta = 1 - \sin^2\theta$

   $\sin^2\theta = 1 - \cos^2\theta$

2. $1 + \tan^2\theta = \sec^2\theta$

3. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

4. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

5. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

6. $\tan\theta = \dfrac{1}{\cot\theta}$

7. $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

8. $\sec\theta = \dfrac{1}{\cos\theta}$

9. $\tan\theta = \dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$

10. $\sin\theta = \dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}$

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7. Trigonometric Ratios of Compound Angles :→

Compound Angles :⇒ An angle made by algebraic sum of two or more angles is called compound angles

$$\angle \alpha = \angle A + \angle B$$
$$\angle \beta = \angle \theta_1 + \angle \theta_2$$

1. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
2. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
3. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
4. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
5. $\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
   $= \dfrac{1}{\cot(\alpha+\beta)}$
6. $\tan(\alpha-\beta) = \dfrac{1}{\cot(\alpha-\beta)} = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$
7. $\sin(\alpha+\beta)\sin(\alpha-\beta) = \sin^2\alpha - \sin^2\beta$
   $= \cos^2\beta - \sin^2\alpha$
8. $\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\alpha - \sin^2\beta$
   $= \cos^2\beta - \sin^2\alpha$

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8. Trigonometric Ratios of multiples of an angle :→

1. $\sin 2\theta = 2\sin\theta\cos\theta$

2. $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

$= \dfrac{2\cot\theta}{1+\cot^2\theta}$

3. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1$

4. $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \dfrac{\cot^2\theta - 1}{\cot^2\theta + 1}$

5. $2\cos^2\theta = 1 + \cos 2\theta$

$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

6. $2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

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7. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \quad \theta \neq (2n+1)\frac{\pi}{4}$

8. $\tan \theta/2 = \frac{1-\cos\theta}{\sin\theta} \quad \theta \neq 2n\pi$

9. $\cot \theta/2 = \frac{1+\cos\theta}{\sin\theta} \quad \theta \neq (2n+1)\pi$

10. $\tan^2 \theta/2 = \frac{1-\cos\theta}{1+\cos\theta} \quad \theta \neq (2n+1)\pi$

11. $\cot^2 \theta/2 = \frac{1+\cos\theta}{1-\cos\theta} \quad \theta \neq 2n\pi$

12. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

13. $\sin 3\theta = 4\sin(60-\theta)\sin\theta\cdot\sin(60+\theta)$

14. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

15. $\cos 3\theta = 4\cos(60-\theta)\cdot\cos\theta\cdot\cos(60+\theta)$

16. $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

17. $\tan 3\theta = \tan(60-\theta)\cdot\tan\theta\cdot\tan(60+\theta)$

18. $\cos\theta\cdot\cos 2\theta\cdot\cos 2^2\theta\cdot\cos 2^3\theta \cdots \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n\sin\theta}$

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19. $1 + \sin 2\theta = (\sin\theta + \cos\theta)^2$

20. $1 - \sin 2\theta = (\sin\theta - \cos\theta)^2$

9. Sum of Trigonometric function in terms of Product :→

1. $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

2. $\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

3. $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

4. $\cos\alpha - \cos\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)$

5. $\tan\alpha + \tan\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$; $\alpha, \beta \neq n\pi + \frac{\pi}{2}$, $n \in Z$

6. $\tan\alpha - \tan\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha \cdot \cos\beta}$; $\alpha, \beta \neq n\pi + \frac{\pi}{2}$, $n \in Z$

7. $\cot\alpha + \cot\beta = \frac{\sin(\alpha+\beta)}{\sin\alpha \cdot \sin\beta}$; $\alpha, \beta \neq n\pi$, $n \in Z$

8. $\cot\alpha - \cot\beta = \frac{\sin(\beta-\alpha)}{\sin\alpha \cdot \sin\beta}$; $\alpha, \beta \neq n\pi$, $n \in Z$

9. $2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

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10. $2\cos\alpha \sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$

11. $2\cos\alpha \cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

12. $2\sin\alpha \sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$

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10. Sum of Trigonometric Ratios if angles are in A.P. :→

1. $\sin x + \sin(x+\alpha) + \sin(x+2\alpha) + \sin(x+3\alpha) - - - - \sin(x+(n-1)\alpha)$

$$= \frac{\sin\left(x+\frac{(n-1)\alpha}{2}\right).\sin\left(\frac{n\alpha}{2}\right)}{\sin(\alpha/2)}$$

2. $\cos x + \cos(x+\alpha) + \cos(x+2\alpha) + \cos(x+3\alpha) - - - - - \cos(x+(n-1)\alpha)$

$$= \frac{\cos\left(x+\frac{(n-1)\alpha}{2}\right)\sin\frac{n\alpha}{2}}{\sin(\alpha/2)}$$

11. Trigonometric Ratio of Sum of three or more Angles :→

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1. $\sin(A+B+C) = \sin[(A+B)+C]$

$= \sin(A+B)\cos C + \cos(A+B)\sin C$

$= \sin A \cos B \cos C + \cos A \sin B \cos C$

$+ \cos A \cos B \sin C - \sin A \sin B \sin C$

2. $\sin(A+B+C) =$

$\cos A \cos B \cos C [\tan A + \tan B + \tan C$

$- \tan A \tan B \tan C]$

3. $\cos(A+B+C) =$

$\cos A \cos B \cos C - \sin A \sin B \cos C - \sin A$

$\cos B \sin C - \cos A \sin B \sin C$

4. $\cos(A+B+C)$

$= \cos A \cos B \cos C [1 - \tan A \tan B$

$- \tan B \tan C - \tan C \tan A]$

5. $\tan(A+B+C)$

$$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \cdot \tan B - \tan B \cdot \tan C - \tan C \cdot \tan A}$$

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Generalized form :-

6. $\sin(\theta_1 + \theta_2 + \theta_3 \text{ ---- } \theta_n)$

$= \cos\theta_1 \cos\theta_2 \text{ ---- } \cos\theta_n (S_1 - S_3 + S_5 - S_7 \ldots)$

7. $\cos(\theta_1 + \theta_2 + \theta_3 \text{ ---- } \theta_n)$

$= \cos\theta_1 \cos\theta_2 \text{ ---- } \cos\theta_n (1 - S_2 + S_4 - S_6 \ldots)$

8. $\tan(\theta_1 + \theta_2 + \theta_3 \text{ --- } \theta_n)$

$$= \frac{S_1 - S_3 + S_5 - S_7 + \text{ ---}}{1 - S_2 + S_4 - S_6}$$

$S_1 = \tan\theta_1 + \tan\theta_2 + \text{ ----- } \tan\theta_n$

$S_2 = \tan\theta_1 \tan\theta_2 + \tan\theta_1 \cdot \tan\theta_3 \ldots$

$S_3 = \tan\theta_1 \cdot \tan\theta_2 \cdot \tan\theta_3 + \tan\theta_2 \tan\theta_3 \tan\theta_4 \text{ ----}$

If $\theta_1 = \theta_2 = \theta_3 \text{ ----- } = \theta_n$

$S_1 = n \tan\theta$

$S_2 = {}^nC_2 \tan^2\theta$

$S_3 = {}^nC_3 \tan^3\theta$

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9. $\sin n\theta = \cos^n\theta \left({}^nC_1 \tan\theta - {}^nC_3 \tan^3\theta + {}^nC_5 \tan^5\theta - \cdots \right)$

10. $\cos n\theta = \cos^n\theta \left(1 - {}^nC_2 \tan^2\theta + {}^nC_4 \tan^4\theta - \cdots \right)$

11. $\tan n\theta =$

$$= \frac{{}^nC_1 \tan\theta - {}^nC_3 \tan^3\theta + {}^nC_5 \tan^5\theta - \cdots}{1 - {}^nC_2 \tan^2\theta + {}^nC_4 \tan^4\theta - {}^nC_6 \tan^6\theta - \cdots}$$

12. $\sin n\theta + \cos n\theta$

$= \cos^n\theta \left(1 + {}^nC_1 \tan\theta - {}^nC_2 \tan^2\theta - {}^nC_3 \tan^3\theta + {}^nC_4 \tan^4\theta + {}^nC_5 \tan^5\theta - {}^nC_6 \tan^6\theta - \cdots \right)$

13. $\sin n\theta - \cos n\theta$

$= \cos^n\theta \left(-1 + {}^nC_1 \tan\theta + {}^nC_2 \tan^2\theta - {}^nC_3 \tan^3\theta - {}^nC_4 \tan^4\theta + {}^nC_5 \tan^5\theta + {}^nC_6 \tan^6\theta - \cdots \right)$

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14. $\sin(\theta) + \sin(\theta+\alpha) + \sin(\theta+2\alpha) + \sin(\theta+3\alpha) \text{ ------ } \sin(\theta+(n-1)\alpha)$

$$= \frac{\sin(n\alpha/2)}{\sin(\alpha/2)}\left[\sin\left(\theta+\frac{(n-1)\alpha}{2}\right)\right]$$

15. $\cos\theta + \cos(\theta+\alpha) + \cos(\theta+2\alpha) + \cos(\theta+3\alpha) \text{ ------ } \cos(\theta+(n-1)\alpha)$

$$= \frac{\sin(n\alpha/2)}{\sin(\alpha/2)}\left[\cos\left(\theta+\frac{(n-1)\alpha}{2}\right)\right]$$

16. $\left|\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right| = \sqrt{1+\sin\theta}$

$$= \begin{cases} +\sqrt{1+\sin\theta}, & 2n\pi+\frac{\pi}{4} \le \frac{\theta}{2} \le 2n\pi+\frac{3\pi}{4} \\ -\sqrt{1+\sin\theta}, & \text{otherwise} \end{cases}$$

17. $\left|\sin\frac{\theta}{2} - \cos\frac{\theta}{2}\right| = \sqrt{1-\sin\theta}$

$$= \begin{cases} +\sqrt{1-\sin\theta}, & 2n\pi+\frac{\pi}{4} \le \frac{\theta}{2} \le 2n\pi+\frac{5\pi}{4} \\ -\sqrt{1-\sin\theta}, & \text{otherwise} \end{cases}$$

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18. $\sin\frac{\theta}{2} = \sin\left[n\pi + (-1)^n\frac{\theta}{2}\right]$

19. $\cos\frac{\theta}{2} = \cos\left[2n\pi \pm \frac{\theta}{2}\right]$

20. $\tan\frac{\theta}{2} = \tan\left[n\pi + \frac{\theta}{2}\right]$

21. $\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$

22. $\operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}$

23. $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \cdot \operatorname{cosec}^2\theta$

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12.

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$
$\sin\theta$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\operatorname{cosec}\theta}$
$\cos\theta$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$
$\tan\theta$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\cot\theta$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\sqrt{\operatorname{cosec}^2\theta-1}$
$\sec\theta$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\sec\theta$	$\frac{\operatorname{cosec}\theta}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\operatorname{cosec}\theta$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\operatorname{cosec}\theta$

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13. Maximum and Minimum Value of Trigonometric function :→

TYPE 1. $f(\theta) = a\cos\theta$ or $a\sin\theta$

$-1 \le \sin\theta \le 1$ $\quad -1 \le \cos\theta \le 1$

$-a \le a\sin\theta \le a$ $\quad -a \le a\cos\theta \le a$

$f(\theta)_{max} = a$

$f(\theta)_{min} = -a$

TYPE 2

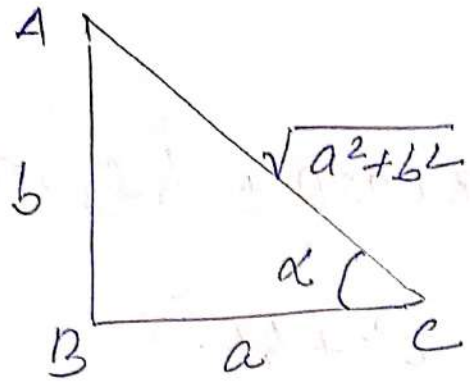
$f(\theta) = a\sin\theta \pm b\cos\theta$

$f(\theta)_{max} = \sqrt{a^2+b^2}$

$f(\theta)_{min} = -\sqrt{a^2+b^2}$

Let $f_1(\theta) = a\sin\theta + b\cos\theta$

Let $\triangle ABC$ is Right angle triangle



$$\cos\alpha = \frac{a}{\sqrt{a^2+b^2}} \quad ; \quad \sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$$

$$f_1(\theta) = a\sin\theta + b\cos\theta$$

$$= \sqrt{a^2+b^2}\left[\frac{a}{\sqrt{a^2+b^2}}\sin\theta + \frac{b}{\sqrt{a^2+b^2}}\cos\theta\right]$$

$$= \sqrt{a^2+b^2}\left[\cos\alpha\sin\theta + \sin\alpha\cos\theta\right]$$

$$= \sqrt{a^2+b^2}\left[\sin(\theta+\alpha)\right]$$

$$-1 \le \sin(\theta+\alpha) \le 1$$

$$-\sqrt{a^2+b^2} \le \sqrt{a^2+b^2}\sin(\theta+\alpha) \le \sqrt{a^2+b^2}$$

$$f_1(\theta)_{max} = \sqrt{a^2+b^2}$$

$$f_1(\theta)_{min} = -\sqrt{a^2+b^2}$$

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TYPE 3

$$f(\theta) = a\cos\theta \pm b\sin\theta$$

$$f(\theta)_{max} = \sqrt{a^2+b^2}$$

$$f(\theta)_{min} = -\sqrt{a^2+b^2}$$

TYPE 4

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$$f_1(\theta) = a\cos\theta + b\cos\theta$$
$$= (a+b)\cos\theta$$

$$f_1(\theta)_{max} = +(a+b)$$

$$f_1(\theta)_{min} = -(a+b)$$

$$f_2(\theta) = a\sin\theta + b\sin\theta$$
$$= (a+b)\sin\theta$$

$$f_2(\theta)_{max} = (a+b)$$

$$f_2(\theta)_{min} = -(a+b)$$

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TYPE 5

$$f(\theta) = a\sin^2\theta + b\cos^2\theta$$

① if $a > b$

$$f(\theta)_{max} = a$$

$$f(\theta)_{min} = b$$

② if $b > a$

$$f(\theta)_{max} = b$$

$$f(\theta)_{min} = a$$

TYPE 6

$$f(\theta) = a\sin^2\theta + b\,\mathrm{cosec}^2\theta$$

$$= a\cos^2\theta + b\sec^2\theta$$

$$= a\tan^2\theta + b\cot^2\theta$$

$$f(\theta)_{min} = 2\sqrt{ab}$$

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TYPE 7

$n \rightarrow$ even

$$f(\theta) = (\sin\theta\cos\theta)^n$$

$$f(\theta)_{max} = (1/2)^n$$

$$f(\theta)_{min} = 0$$

TYPE 8

$n \rightarrow$ odd

$$f(\theta) = (\sin\theta\cos\theta)^n$$

$$f(\theta)_{max} = (1/2)^n$$

$$f(\theta)_{min} = -(1/2)^n$$

TYPE 9

$$f(\theta) = r + a\cos\theta + b\sin\theta$$

$$f(\theta)_{max} = r + \sqrt{a^2+b^2}$$

$$f(\theta)_{min} = r - \sqrt{a^2+b^2}$$

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TYPE 10

1. $-1 \leq \sin\theta \leq 1$
2. $-1 \leq \cos\theta \leq 1$
3. $-\infty < \tan\theta < \infty$
4. $-\infty < \cot\theta < \infty$
5. $\operatorname{cosec}\theta \geq 1$ and $\operatorname{cosec}\theta \leq -1$
6. $\sec\theta \geq 1$ and $\sec\theta \leq -1$
7. $-1 \leq (\sin\theta)^{2n+1} \leq 1$
8. $-1 \leq (\cos\theta)^{2n+1} \leq 1$
9. $-\infty < (\tan\theta)^{2n+1} < \infty$
10. $-\infty < (\cot\theta)^{2n+1} < \infty$
11. $(\operatorname{cosec}\theta)^{2n+1} \geq 1$ $\quad (\operatorname{cosec}\theta)^{2n+1} \leq -1$
12. $(\sec\theta)^{2n+1} \geq 1$ $\quad (\sec\theta)^{2n+1} \leq -1$
13. $0 \leq (\sin\theta)^{2n} \leq 1$
14. $0 \leq (\cos\theta)^{2n} \leq 1$
15. $0 \leq (\tan\theta)^{2n} < \infty$
16. $0 \leq (\cot\theta)^{2n} < \infty$
17. $1 \leq (\operatorname{cosec}\theta)^{2n} < \infty$
18. $1 \leq (\sec\theta)^{2n} < \infty$

$n \in W$

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14. Trigonometric Equations and Inequations :→ ($n \in Z$)

1. $\sin\theta = 0$, $\theta = 0, \pm\pi, \pm 2\pi$ ----

$$\boxed{\theta = n\pi} \quad n \in Z$$

2. $\cos\theta = 0$, $\theta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$ ----

$$\boxed{\theta = (2n+1)\frac{\pi}{2}} \quad n \in Z$$

3. $\tan\theta = 0$, $\theta = 0, \pm\pi, +2\pi$

$$\boxed{\theta = n\pi} \quad n \in Z$$

4. $\sin\theta = \sin\alpha$ $\boxed{\theta = n\pi + (-1)^n \alpha}$

$\alpha \in [-\frac{\pi}{2}\ \frac{\pi}{2}]$ $n \in Z$

5. $\cos\theta = \cos\alpha$ $\boxed{\theta = 2n\pi \pm \alpha}$

$\alpha \in [0, \pi]$ $n \in Z$

6. $\tan\theta = \tan\alpha$ $\boxed{\theta = n\pi + \alpha}$ $n \in Z$

$\alpha \in (-\frac{\pi}{2}\ \frac{\pi}{2})$

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7. $\sin\theta = 1$ $\quad \theta = (4n+1)\frac{\pi}{2}$

8. $\cos\theta = 1$ $\quad \theta = 2n\pi$

9. $\tan\theta = 1$ $\quad \theta = (4n+1)\frac{\pi}{4}$

$n \in Z$

10. $\sin\theta = -1$ $\quad \theta = (4n+1)\frac{\pi}{2}$

11. $\cos\theta = -1$ $\quad \theta = (2n+1)\pi$

12. $\tan\theta = -1$ $\quad \theta = (4n-1)\frac{\pi}{4}$

13. $\sin^2\theta = \sin^2\alpha$ $\quad \theta = n\pi \pm \alpha$

14. $\cos^2\theta = \cos^2\alpha$ $\quad \theta = n\pi \pm \alpha$

15. $\tan^2\theta = \tan^2\alpha$ $\quad \theta = n\pi \pm \alpha$

16. $\left.\begin{array}{l} \sin\theta = \sin\alpha \\ \text{and} \\ \cos\theta = \cos\alpha \end{array}\right\} \Rightarrow \theta = 2n\pi + \alpha$

$\alpha \Rightarrow$ Principal value of angle

$n \Rightarrow$ Integer

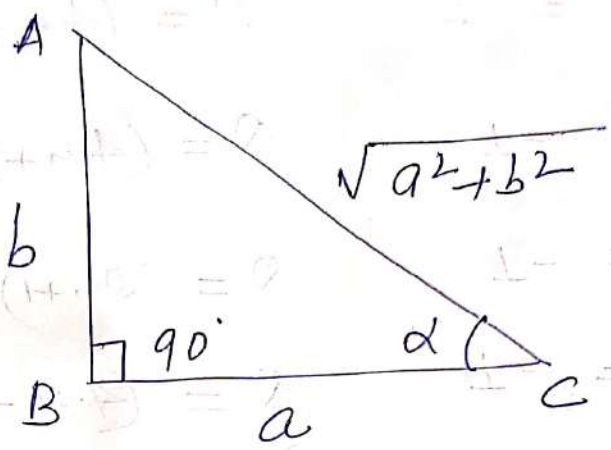
17. Trigonometric Equation of the form

$$a\cos\theta + b\sin\theta = c$$

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$$\frac{a}{\sqrt{a^2+b^2}}\cos\theta + \frac{b}{\sqrt{a^2+b^2}}\sin\theta = \frac{c}{\sqrt{a^2+b^2}}$$



in $\triangle ABC$

$$\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$$

$$\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$$

$$\cos\alpha\cos\theta + \sin\alpha\sin\theta = \frac{c}{\sqrt{a^2+b^2}}$$

$$\cos(\theta-\alpha) = \frac{c}{\sqrt{a^2+b^2}} \quad -(A)$$

Solution of Equation (A) is only possible when

$$-1 \le \frac{c}{\sqrt{a^2+b^2}} \le 1$$

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$$-\sqrt{a^2+b^2} \le c \le \sqrt{a^2+b^2}$$

$$\boxed{|c| \le \sqrt{a^2+b^2}}$$

Let $\dfrac{|c|}{\sqrt{a^2+b^2}} = \cos\beta$

from (A) $\cos(\theta-\alpha) = \cos\beta$

$$\theta-\alpha = 2n\pi \pm \beta$$

$$\boxed{\theta = 2n\pi \pm \beta + \alpha}$$

18. $\left.\begin{matrix}\sin\theta = \sin\alpha \\ \tan\theta = \tan\alpha\end{matrix}\right\}$ $\boxed{\theta = 2n\pi + \alpha}$

---

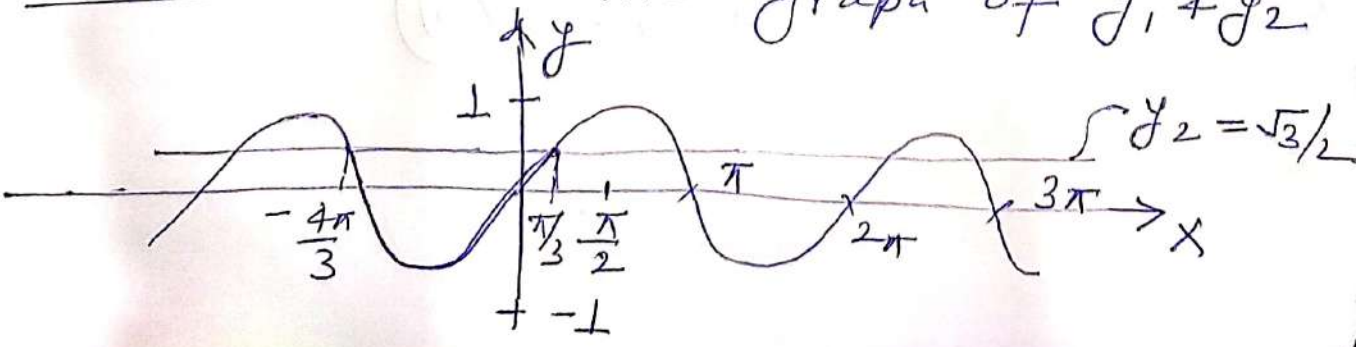
15. Trigonometric Inequations :→

Q.1 Solve $\sin x \le \frac{\sqrt{3}}{2}$

STEP 1 Let $y_1 = \sin x$

Let $y_2 = \sqrt{3}/2$

STEP 2 draw the graph of $y_1 + y_2$



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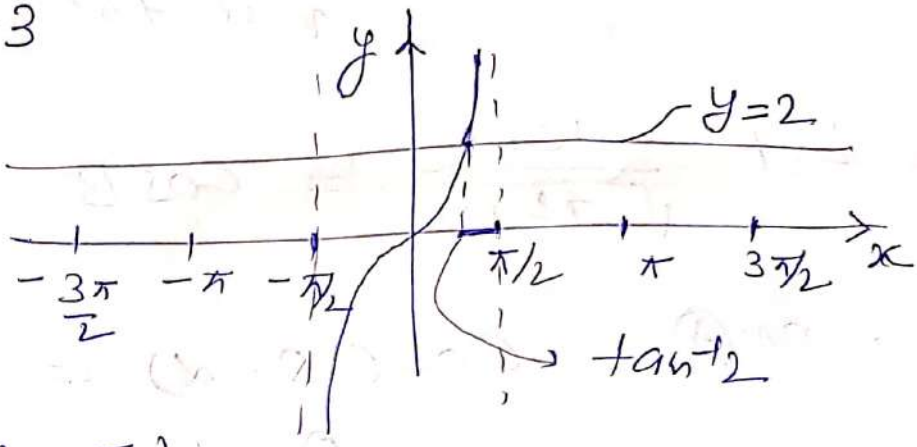
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STEP 3 $x \in \left[2n\pi - \frac{4\pi}{3}, 2n\pi + \frac{\pi}{3} \right]$

Q. 2 $\tan x > 3$



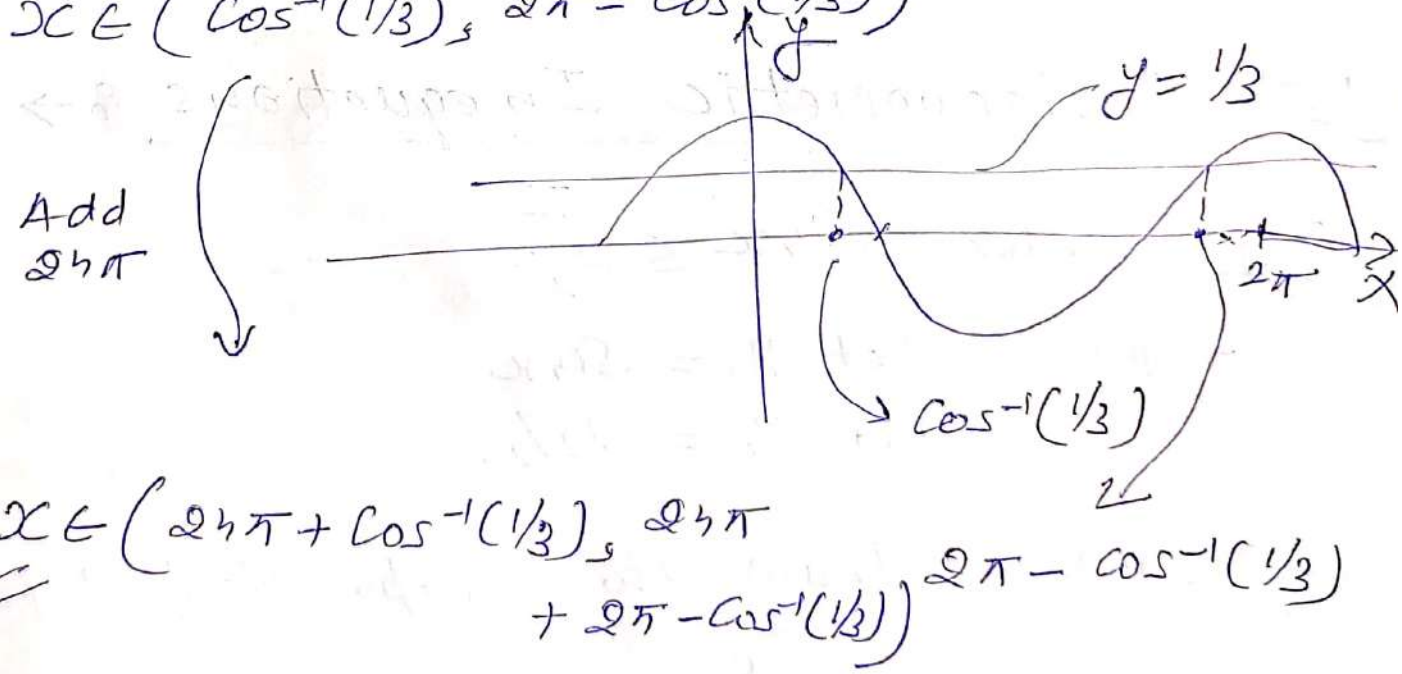
$x \in \left(\tan^{-1}2, \frac{\pi}{2} \right)$

Add $n\pi$

$x \in \left(n\pi + \tan^{-1}2, n\pi + \frac{\pi}{2} \right)$

Q 3 $\cos x < \frac{1}{3}$

$x \in \left(\cos^{-1}(1/3), 2\pi - \cos^{-1}(1/3) \right)$



$x \in \left(2n\pi + \cos^{-1}(1/3), 2n\pi + 2\pi - \cos^{-1}(1/3) \right)$

16. Value of Trigonometric function at an angle :→

1. $\sin 15^\circ = \sin(90-75^\circ) = \cos 75^\circ$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

2. $\cos 15^\circ = \cos(90-75^\circ) = \sin 75^\circ$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

3. $\tan 15^\circ = \tan(90-75^\circ) = \cot 75^\circ$

$= 2-\sqrt{3}$

4. $\cot 15^\circ = \cot(90-75^\circ) = \tan 75^\circ$

$= 2+\sqrt{3}$

5. $\sin 18^\circ = \sin(90-72) = \cos 72^\circ$

$= \frac{\sqrt{5}-1}{4}$

6. $\cos 18^\circ = \cos(90-72) = \sin 72^\circ$

$= \frac{\sqrt{10+2\sqrt{5}}}{4}$

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7. $\sin 22\frac{1}{2}^\circ = \sin(90 - 67\frac{1}{2}^\circ) = \cos 67\frac{1}{2}^\circ$

$= \frac{1}{2}(\sqrt{2-\sqrt{2}})$

8. $\cos 22\frac{1}{2}^\circ = \cos(90 - 67\frac{1}{2}^\circ) = \sin 67\frac{1}{2}^\circ$

$= \frac{1}{2}(\sqrt{2+\sqrt{2}})$

9. $\tan 22\frac{1}{2}^\circ = \tan(90 - 67\frac{1}{2}^\circ) = \cot 67\frac{1}{2}^\circ$

$= \sqrt{2} - 1$

10. $\cot 22\frac{1}{2}^\circ = \cot(90 - 67\frac{1}{2}^\circ) = \tan 67\frac{1}{2}^\circ$

$= \sqrt{2} + 1$

11. $\sin 9^\circ = \sin(90 - 81^\circ) = \cos 81^\circ$

$= \frac{\sqrt{3+\sqrt{5}} - \sqrt{5-\sqrt{5}}}{4}$

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12. $\cos 9^\circ = \cos(90 - 81) = \sin 81^\circ$

$= \frac{\sqrt{3+\sqrt{5}} - \sqrt{5-\sqrt{5}}}{4}$

13. $\sin 36^\circ = \sin(90 - 54) = \cos 54^\circ$

$= \sqrt{10 - 2\sqrt{5}}/4$

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14. $\cos 36^\circ = \cos(90-54^\circ) = \sin 54^\circ$

$= \frac{\sqrt{5}+1}{4}$

15. $\sin(105^\circ) = \sin(90+15^\circ) = \cos 15^\circ$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

16. $\cos(105^\circ) = \cos(90+15^\circ) = -\sin 15^\circ$

$= -\frac{(\sqrt{3}-1)}{2\sqrt{2}}$

17. $\tan(105^\circ) = \tan(90+15) = -\cot 15^\circ$

$= -(2+\sqrt{3})$

18. $\cot(105^\circ) = \cot(90+15^\circ) = -\tan 15^\circ$

$= -(2-\sqrt{3})$

19 $\cos 36^\circ - \cos 72^\circ = \frac{1}{2}$

20 $\cos 36^\circ \cdot \cos 72^\circ = \frac{1}{4}$

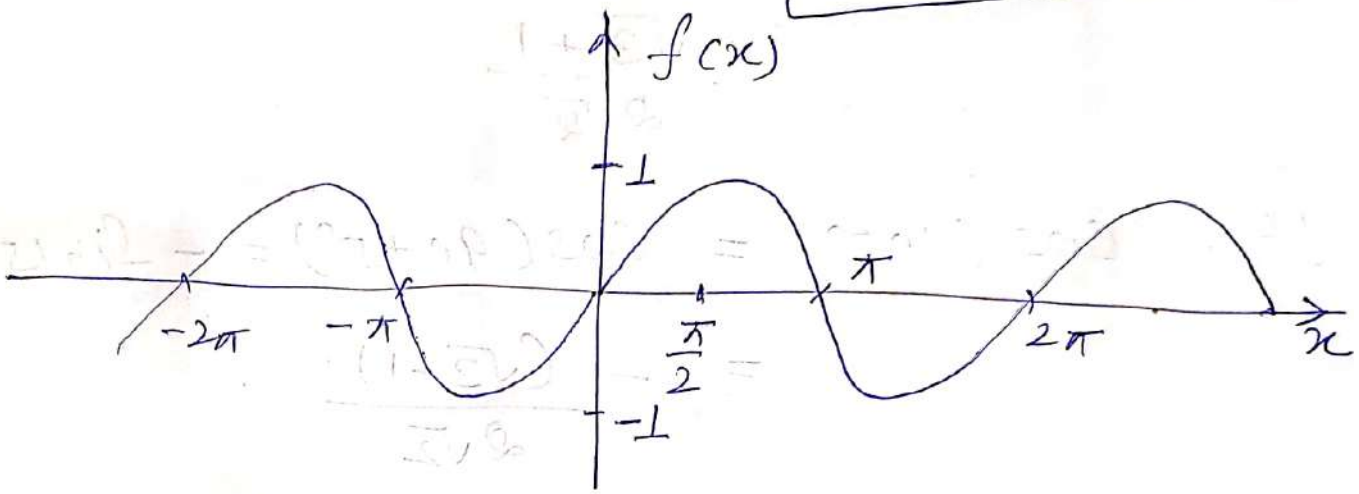
NOTE $\left\{ \begin{matrix} 165^\circ = 180-15^\circ \\ 195^\circ = 180+15^\circ \end{matrix} \right\}$ So We Can Calculate $\sin 165^\circ \sin 195^\circ$ ---

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17. Graph of Trigonometric functions →

1. $f(x) = \sin x$

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→ Domain, $x \in R$

$x \in (-\infty, \infty)$

→ Range, $f(x) \in [-1, 1]$

→ $f(x) = \sin x$ is an odd function because $f(-x) = -f(x)$

→ Period of $f(x) = \sin x \Rightarrow 2\pi$

Period of $f(x) = \sin ax \Rightarrow 2\pi/a$ $a \neq 0$

→ $\sin x = \sin \alpha$

$x = n\pi + (-1)^n \alpha \quad n \in Z$

→ $\sin x = 1 \Rightarrow x = (4n+1)\frac{\pi}{2}$

$x = (4n+1)\frac{\pi}{2}$

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→ $\sin x = -1$ ⇒ $x = (4n+1)\frac{\pi}{2}$

$x = (4n-1)\frac{\pi}{2}$

→ $\sin x = 0$ ⇒ $x = n\pi$, $n \in Z$

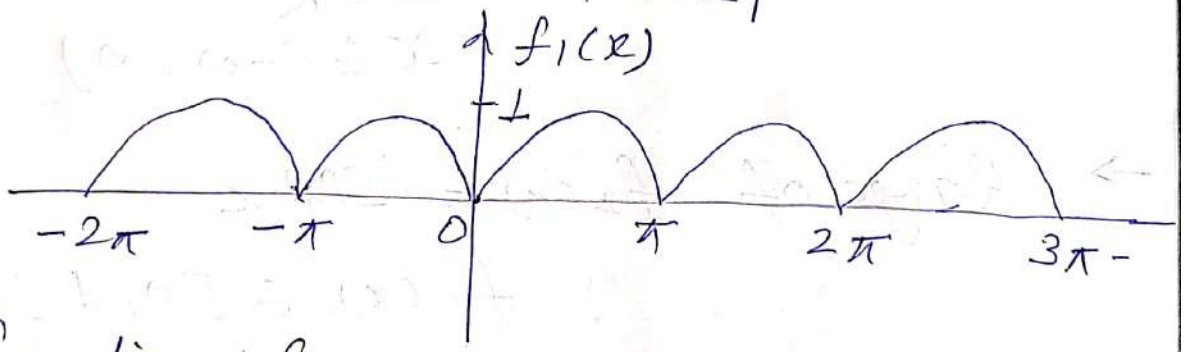
→ $\sin x > 0 \Rightarrow x \in (2n\pi, (2n+1)\pi)$

$n \in Z$

→ $\sin x < 0 \Rightarrow x \in ((2n-1)\pi, 2n\pi)$

$n \in Z$

→ Graph of $f_1(x) = |\sin x|$



→ function $f_1(x) = |\sin x|$ is not differentiable at $x = n\pi$ $(n \in z)$

→ Domain of $f_1(x) = |\sin x|$ will be

$x \in R$

$x \in (-\infty, \infty)$

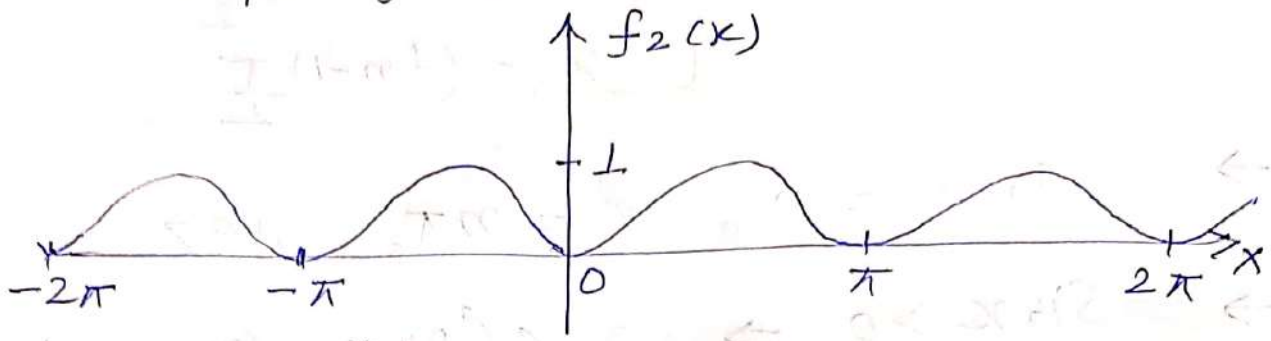
→ Range of $f_1(x) = |\sin x|$ will be

$f_1(x) \in [0, 1]$

→ function $f_1(x) = |\sin x|$ is a Periodic function of Period $= \pi$

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→ Graph of $f_2(x) = \sin^2 x$



→ Domain of $f_2(x) \Rightarrow x \in R$

$x \in (-\infty, \infty)$

→ Domain of $f_3(x) = \sin^2 x + 1$

$x \in R$

$x \in (-\infty, \infty)$

→ Range of $f_2(x) = \sin^2 x$

$f_2(x) \in [0, 1]$

→ function $f_2(x) = \sin^2 x$ is a differentiable function at $x = n\pi$

$n \in Z$

→ function $f_2(x) = \sin^2 x$ is a Periodic function of Period π

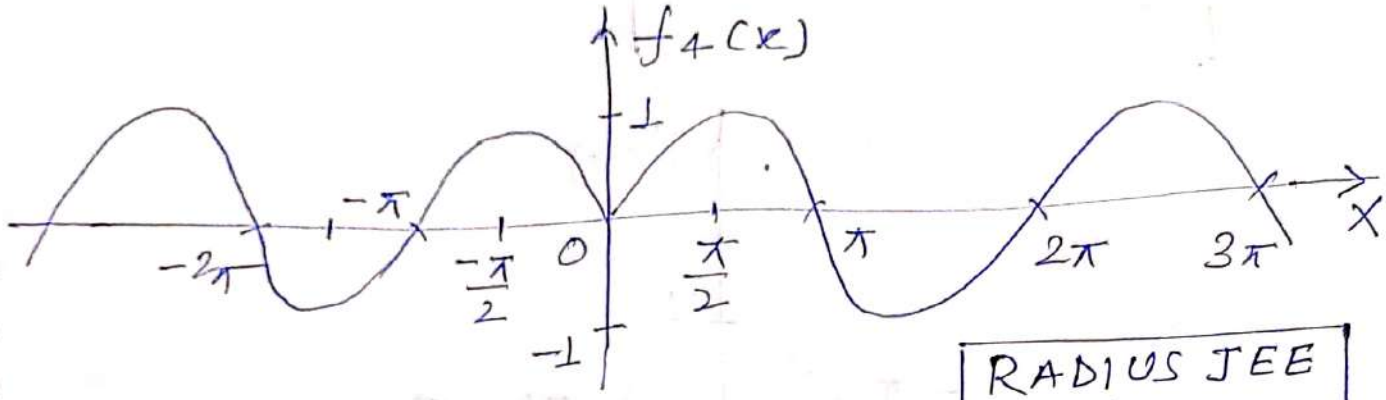
→ both $f_1(x) = |\sin x|$ & $f_2(x) = \sin^2 x$ are even function because

$f_1(-x) = f(x)$

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→ Graph of $f_4(x) = \sin|x|$



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→ $\frac{d}{dx}(\sin x) = \cos x$

→ $\int \sin x\, dx = -\cos x + C$

→ $\sin x = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \frac{x^7}{\lfloor 7} - - - -$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{\lfloor (2n+1)} x^{2n+1}$$

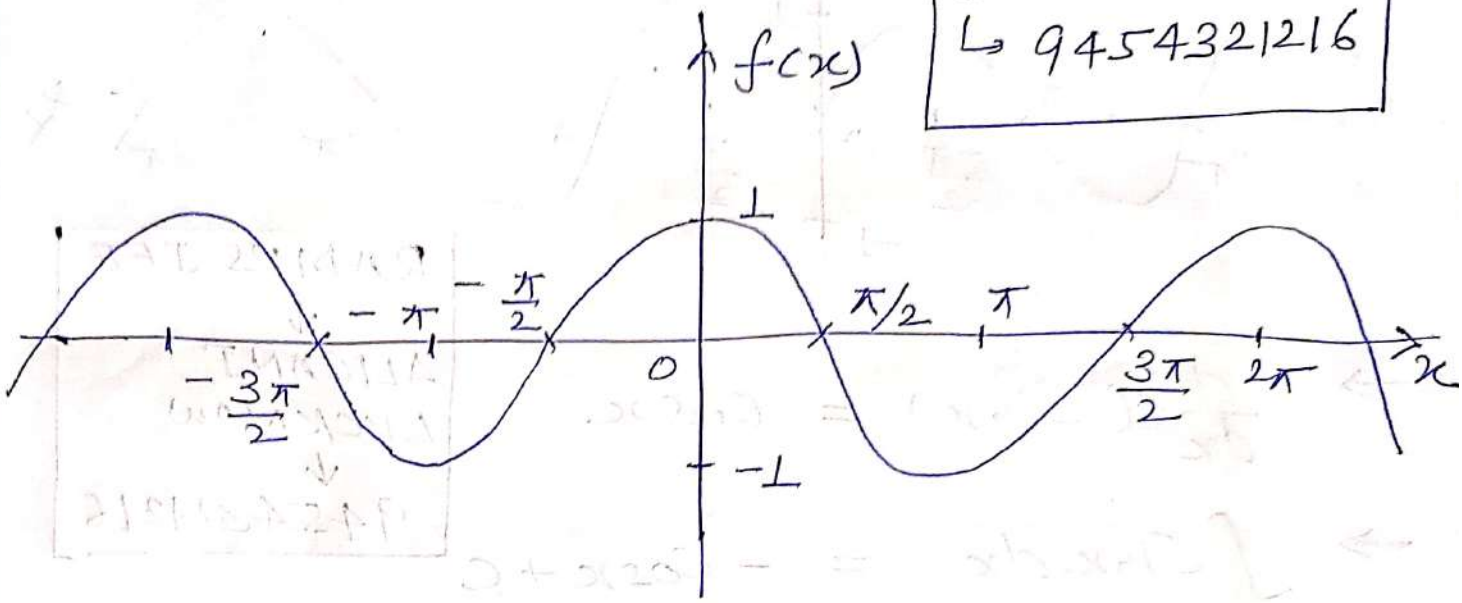
→ $f(x) = \sin x$ is a continous and differentiable function for all $x \in R$

→ function $f(x) = \sin x$ is Symmetric function with respect to origin $(0,0)$

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2. $f(x) = \cos x$

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→ Domain, $x \in R$
$x \in (-\infty, \infty)$

→ Range, $f(x) \in [-1, 1]$

→ $f(x) = \cos x$ is an even function because $f(-x) = f(x)$

→ $f(x) = \cos x$ is a Periodic function of Period $= 2\pi$

Period of $f(x) = \cos ax \Rightarrow 2\pi/a \quad a \neq 0$

→ $\cos x = \cos \alpha$ | $\alpha \in [0, \pi]$
$x = 2n\pi \pm \alpha$ | $n \in Z$

→ $\cos x = 1$
$x = 2n\pi$

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→ $\cos x = -1$

$x = (2n+1)\pi$

→ $\cos x = 0$

$x = (2n+1)\frac{\pi}{2}$

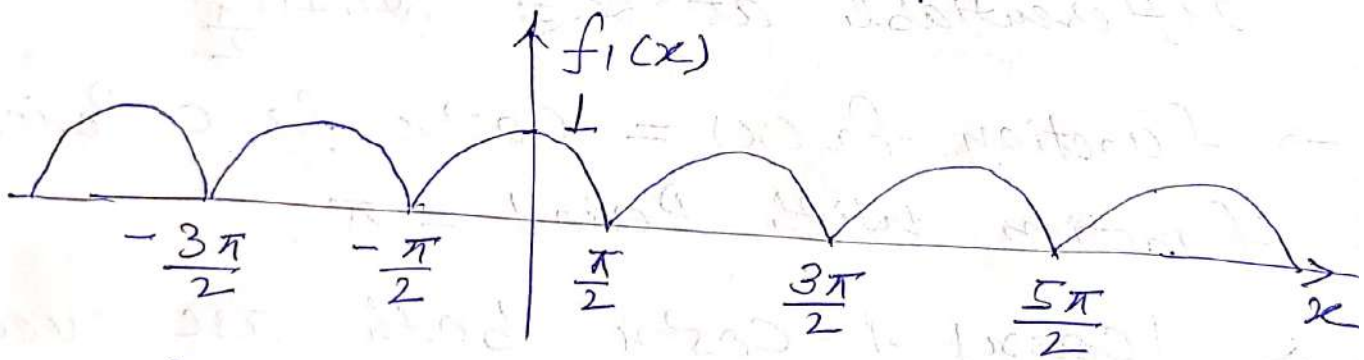
→ $\cos x > 0 \Rightarrow x \in \left((2n-1)\frac{\pi}{2}, (2n+1)\frac{\pi}{2}\right)$

$n \in Z$

→ $\cos x < 0 \Rightarrow x \in \left((2n+1)\frac{\pi}{2}, (2n+3)\frac{\pi}{2}\right)$

$n \in Z$

→ Graph of $f_1(x) = |\cos x|$



→ function $f_1(x) = |\cos x|$ is not differentiable at $x = (2n+1)\frac{\pi}{2}$ $n \in Z$

→ Domain of $f_1(x)$

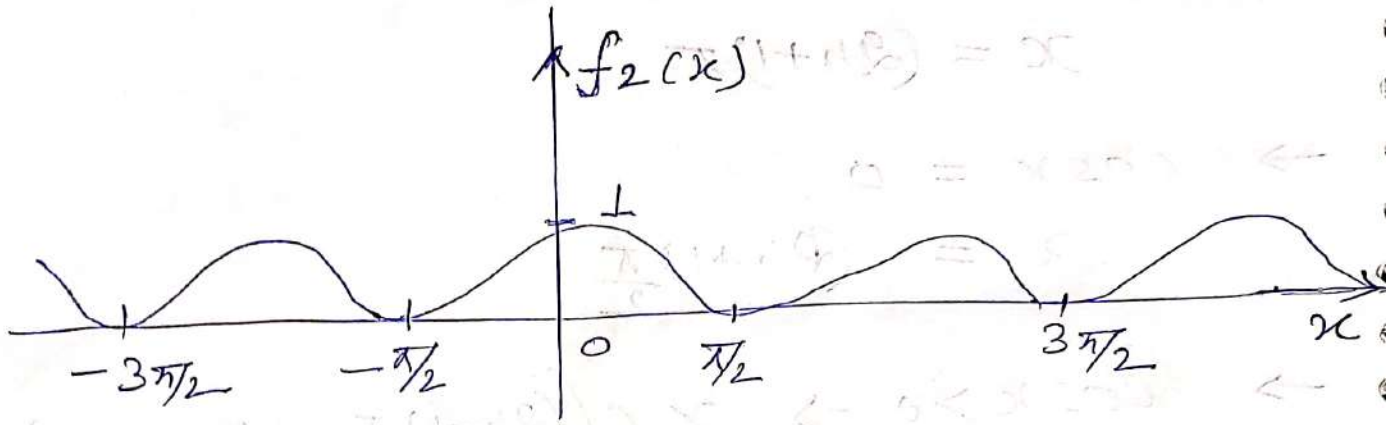
$x \in R$, $x \in (-\infty\ \infty)$

→ Range of $f_1(x) \in [0, 1]$

→ function $f_1(x) = |\cos x|$ is a Periodic function with Period π

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→ Graph of $f_2(x) = \cos^2 x$



→ Domain of $f_2(x) \Rightarrow$ $x \in R$
$x \in (-\infty, \infty)$

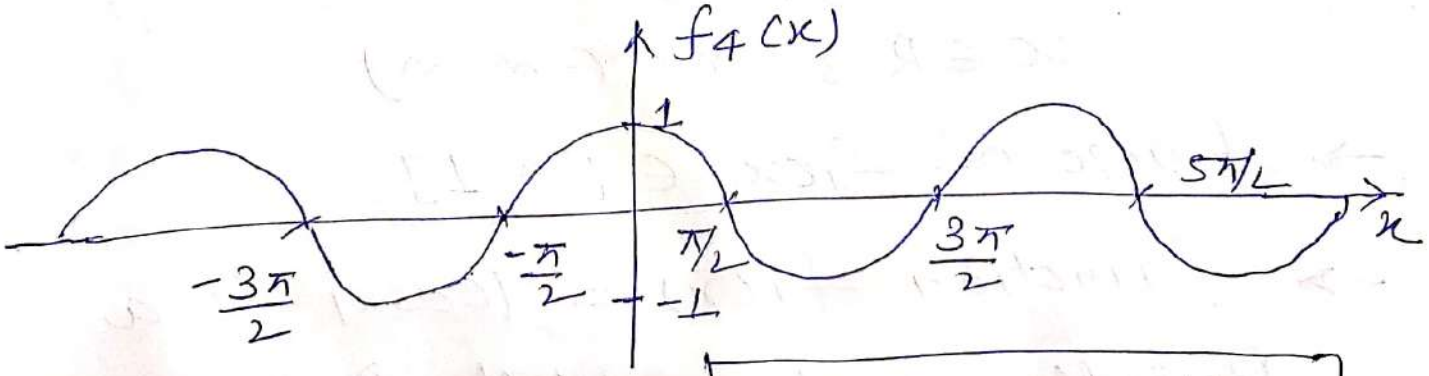
→ Range of $f_2(x) \Rightarrow$ $f_2(x) \in [0, 1]$

→ function $f_2(x) = \cos^2 x$ will be differentiable at $x = (2n+1)\frac{\pi}{2}$

→ function $f_2(x) = \cos^2 x$ is a Periodic function with Period $= \pi$

→ $|\cos x|$ & $\cos^2 x$ both are even function because $f(x) = f(-x)$

→ Graph of $f_4(x) = \cos|x|$



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→ $\frac{d}{dx}(\cos x) = -\sin x$

→ $\int \cos x\, dx = \sin x + c$

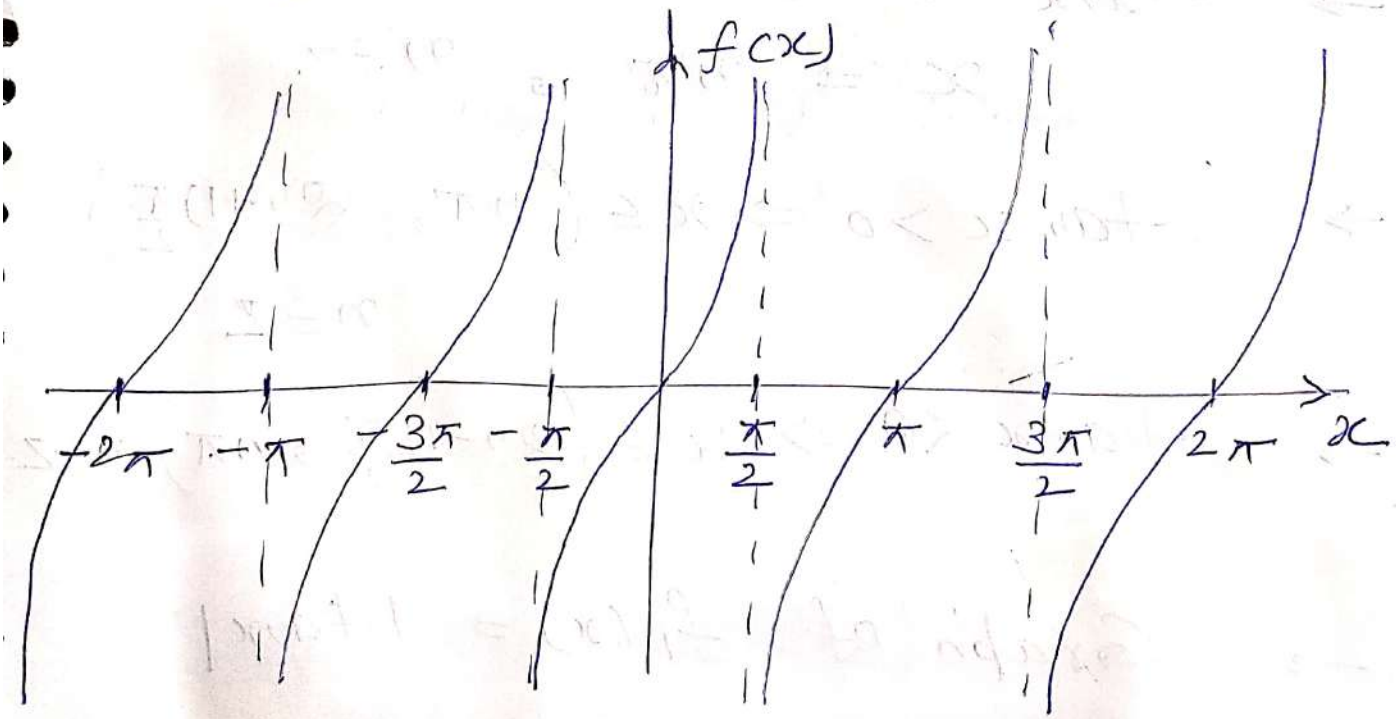
→ $\cos x = 1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} - \frac{x^6}{\lfloor 6} - \cdots$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{\lfloor 2n} x^{2n}$

→ $f(x) = \cos x$ is a continous and differentiable function for all $x \in R$

→ $f(x) = \cos x$, $|\cos x|$, $\cos^2 x$, $\cos|x|$ is symmetrical about y-axis.

3. $f(x) = \tan x$



→ Domain $x \in R - (2n+1)\frac{\pi}{2}$, $n \in Z$

→ Range $f(x) \in R$

→ $f(x) = \tan x$ is an odd function

→ Period of $f(x) = \tan x = \pi$

Period of $f(x) = \tan ax = \pi/a$

→ $\tan x = \tan \alpha$

$x = n\pi + \alpha \quad \alpha \in (-\frac{\pi}{2}, \frac{\pi}{2})$

→ $\tan x = 1$

$x = (4n+1)\frac{\pi}{4} \quad n \in Z$

→ $\tan x = -1$

$x = (4n-1)\frac{\pi}{4} \quad n \in Z$

→ $\tan x = 0$

$x = n\pi, \quad n \in Z$

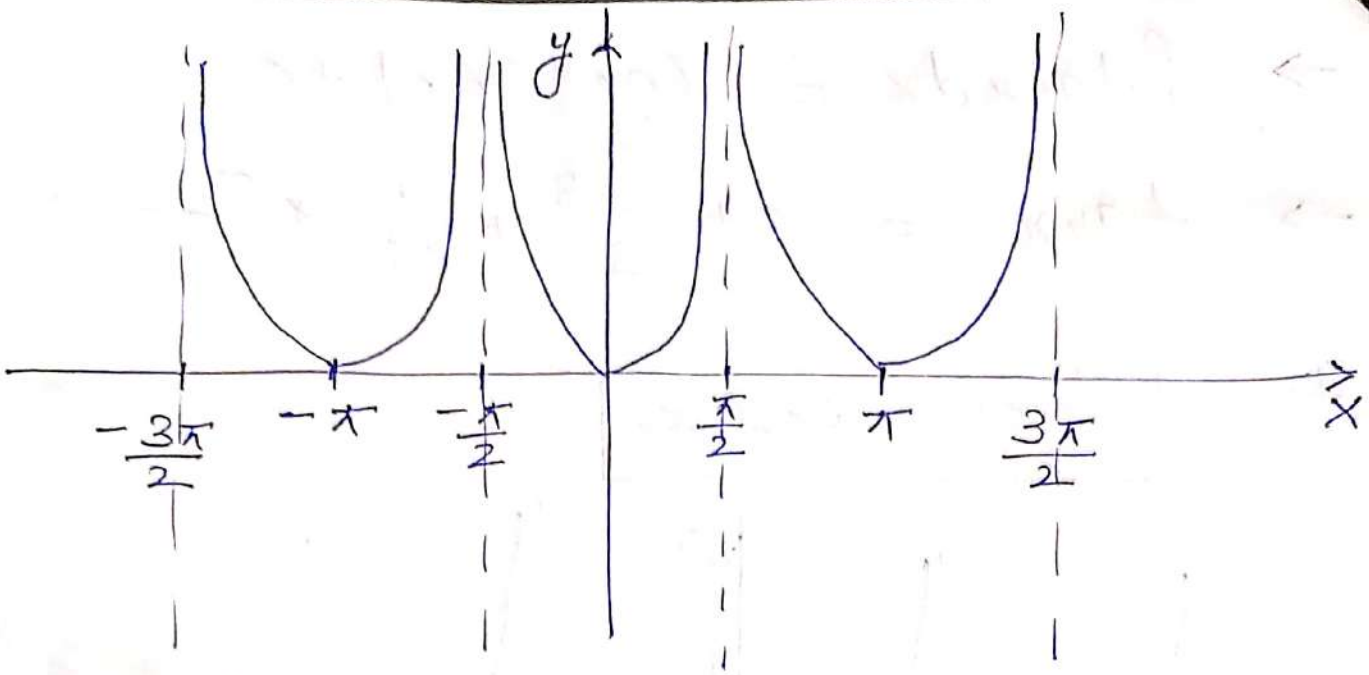
→ $\tan x > 0 \Rightarrow x \in (n\pi, (2n+1)\frac{\pi}{2})$

$n \in Z$

→ $\tan x < 0 \Rightarrow x \in ((2n-1)\frac{\pi}{2}, n\pi) \; n \in Z$

→ Graph of $f_1(x) = |\tan x|$

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→ $f_1(x) = |\tan x|$ is a Periodic function with Period $= \pi$

→ $f_1(x) = |\tan x|$ is a Periodic function with Period $= \pi$

→ $f_1(x) = |\tan x|$ is an even function an Symmetrical about y-axis

→ Domain of $f_1(x) = |\tan x|$

$$x \in R - (2n+1)\frac{\pi}{2}$$

→ Range of $f_2(x) = |\tan x|$

$$f_2(x) \in [0\ \infty)$$

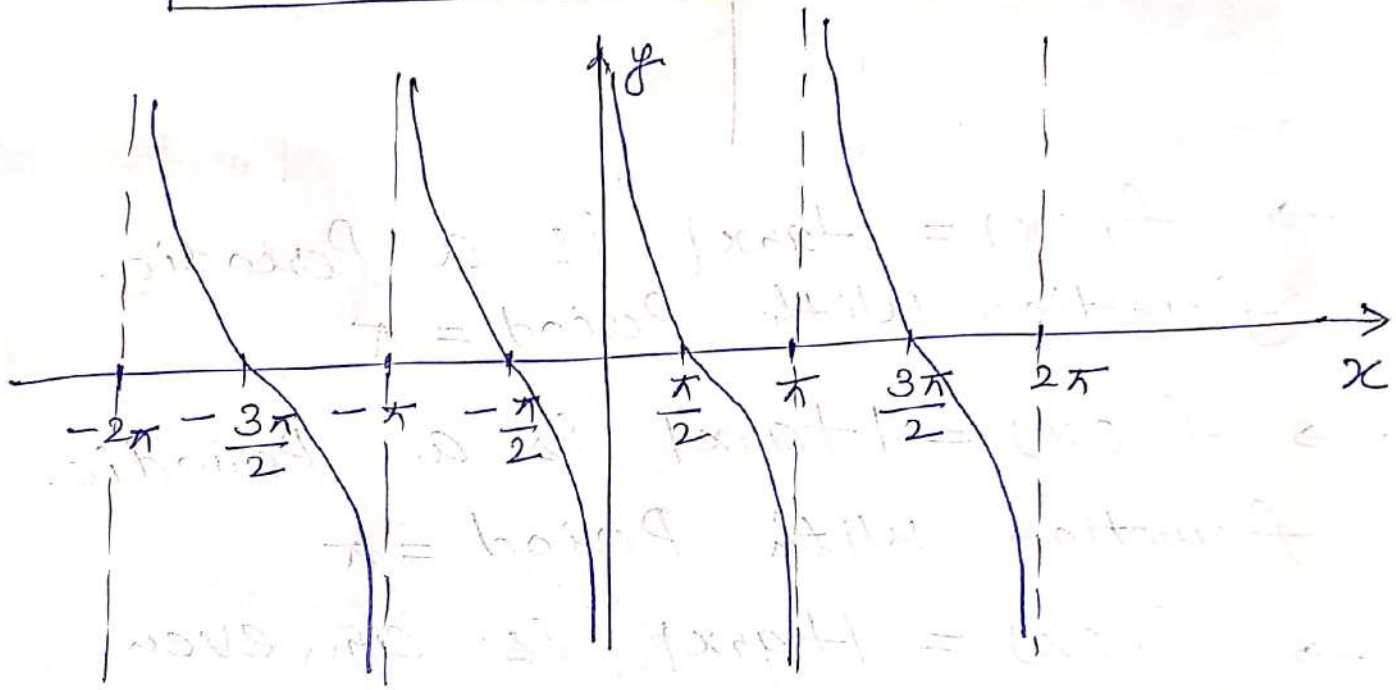
→ $\frac{d}{dx}(\tan x) = \sec^2 x$

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→ $\int \tan x\,dx = \log|\sec x| + C$

→ $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 ----$

4. $\boxed{f(x) = \cot x}$



→ Domain of function $f(x) = \cot x$

$x \in R - n\pi$

$x \in (-\infty\ \infty) - n\pi$

→ Range of function $f(x) = \cot x$

$f(x) \in R$ or $f(x) \in (-\infty\ \infty)$

→ $f(x) = \cot x$ is an odd function

→ $f(x) = \cot x$ is a Periodic function

With Period $= \pi$

→ Symmetrical about origin

→ monotonically decreasing function.

→ if $\cot x = \cot \alpha$

$x =$

→ $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

→ $\cot x = 1$ ⇒ $x = (4n+1)\frac{\pi}{4}$, $n \in Z$

→ $\cot x = -1$ ⇒ $x = (4n-1)\frac{\pi}{4}$ ⇒ $n \in Z$

→ $\cot x = 0$ ⇒ $x = (2n+1)\frac{\pi}{2}$ $n \in Z$

→ $\cot x \not< 0$

$\Rightarrow x \in \left((2n-1)\frac{\pi}{2}, n\pi\right)$ $n \in Z$

→ $\cot x > 0$

$\Rightarrow x \in \left(n\pi, (2n+1)\frac{\pi}{2}\right)$ $n \in Z$

→ $\int \cot x \, dx = \ln|\sin x| + C$

→ $\cot x = \frac{1}{x} + \sum_{n=0}^{\infty} \frac{2x}{x^2 - n^2\pi^2}$

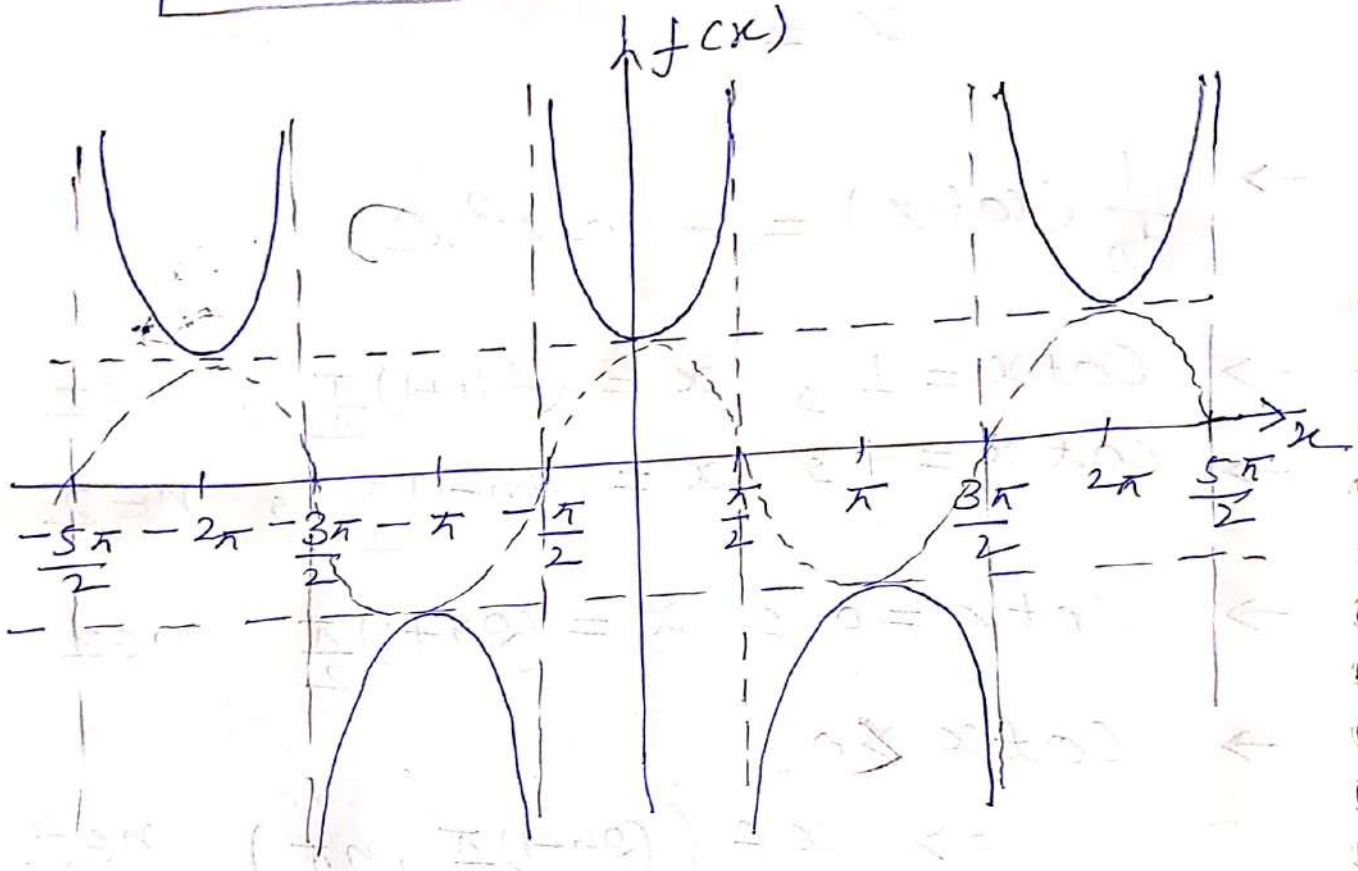
$= \frac{1}{x} - \frac{x}{3} - \frac{x3}{45} - \frac{2x5}{945} \cdots$

→ $f(x) = |\cot x|$, $\cot 2x$

Domain $x \in R - n\pi$, Range $\in [0\ \infty)$

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5. $f(x) = \sec x$



→ Domain $x \in R - (2n+1)\frac{\pi}{2}$ $n \in Z$

→ Range $f(x) \in (-\infty, -1] \cup [1, \infty)$

→ $f(x) = \sec x \rightarrow$ even function

→ Periodic function Period $= 2\pi$

→ Symmetrical about y-axis

→ Non-monotonic function

→ $\sec x = 1 \Rightarrow x = 2n\pi$, $n \in Z$

→ $\sec x = -1 \Rightarrow x = (2n+1)\pi$, $n \in Z$

→ $\sec x > 0 \Rightarrow x \in \left((4n-1)\frac{\pi}{2}, (4n+1)\frac{\pi}{2}\right)$

→ $\sec x < 0 \Rightarrow x \in \left((4n+1)\frac{\pi}{2}, (4n+3)\frac{\pi}{2}\right)$

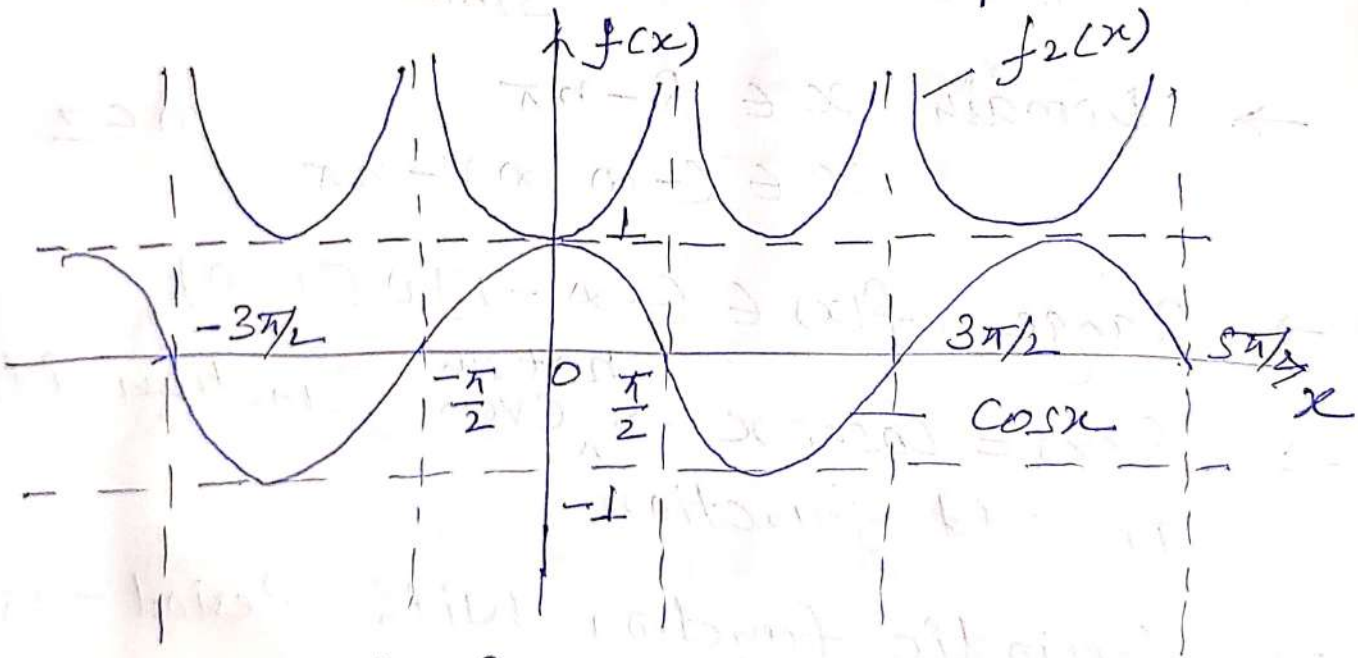
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→ $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

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→ $\int \sec x\,dx = \begin{cases} \ln|\sec x + \tan x| + C \\ \frac{1}{2}\ln\left|\frac{1+\sin x}{1-\sin x}\right| + C \\ \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| + C \end{cases}$

→ Graph of $f_2(x) = |\sec x|$



→ Domain of $f_2(x) = |\sec x|$

$x \in R - (2n+1)\frac{\pi}{2}$

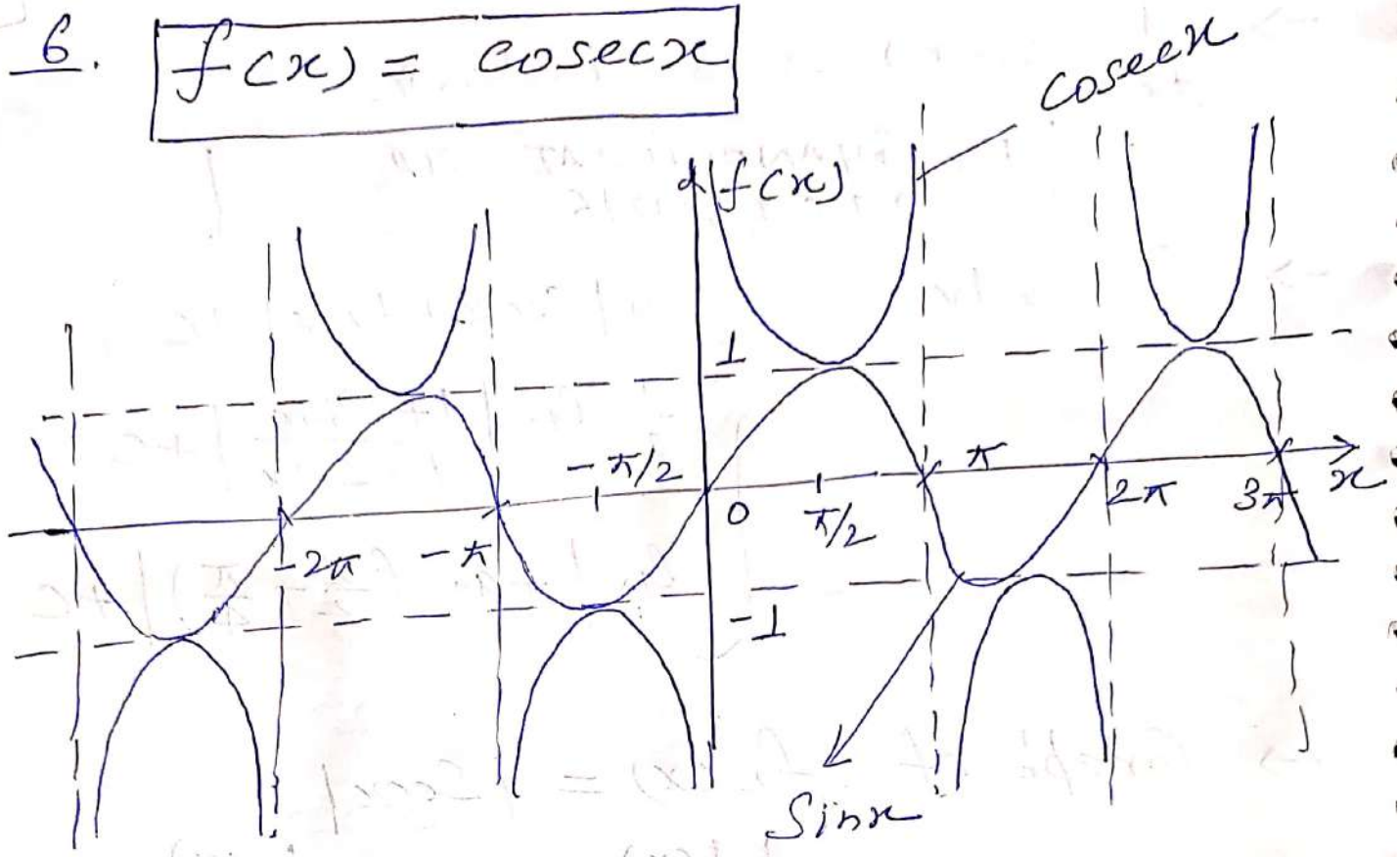
→ Range of $f_2(x) = |\sec x|$

$f_2(x) \in [1\ \infty)$

→ Period of $f_2(x) = |\sec x| \Rightarrow \pi$

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6. $f(x) = \text{cosec}\, x$



→ Domain $x \in R - n\pi$ $\quad n \in Z$

$x \in (-\infty\ \infty) - n\pi$

→ Range $f(x) \in (-\infty\ -1] \cup [1\ \infty)$

→ $f(x) = \text{cosec}\, x \rightarrow$ not an even function it is an odd function.

→ Periodic function with Period $= 2\pi$

→ Non-monotonic function

→ Symmetrical about origin

→ $\text{Cosec}\, x = 1 \Rightarrow x = (4n+1)\frac{\pi}{4}$, $n \in Z$

→ $\text{Cosec}\, x = -1 \Rightarrow x = (4n-1)\frac{\pi}{4}$, $n \in Z$

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→ $\text{Cosec}\, x > 0 \Rightarrow x \in (2n\pi, (2n+1)\pi)$
$n \in Z$

→ $\text{Cosec}\, x < 0 \Rightarrow x \in ((2n-1)\pi, 2n\pi)$
$n \in Z$

→ $\frac{d}{dx}(\text{Cosec}\, x) = -\cot x \,\text{Cosec}\, x$

→ $\int \text{Cosec}\, x\, dx = \begin{cases} \ln|\text{Cosec}\, x - \cot x| + C \\ -\ln|\text{Cosec}\, x + \cot x| + C \\ \frac{1}{2}\ln\left|\frac{\cos x - 1}{\cos x + 1}\right| + C \\ \ln\left|\tan\frac{x}{2}\right| + C \end{cases}$

→ $f(x) = |\text{Cosec}\, x|$
Domain $x \in R - n\pi$
Range $f(x) \in [1, \infty]$

→ Period of $f(x) = |\text{Cosec}\, x|$ is π

→ $\text{Cosec}\, x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^6}{15120} \cdots$

→ $\text{Sec}\, x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} \cdots$

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18. Domain and Range of Trigonometric function :→

T. function	Domain	Range
$f(x) = \sin x$	$x \in R$	$[-1, 1]$
$f(x) = \cos x$	$x \in R$	$[-1, 1]$
$f(x) = \tan x$	$x \in R - (2n+1)\frac{\pi}{2}$	R
$f(x) = \cot x$	$x \in R - n\pi$	R
$f(x) = \sec x$	$x \in R - (2n+1)\frac{\pi}{2}$	$(-\infty, -1] \cup [1, \infty)$
$f(x) = \operatorname{cosec} x$	$x \in R - n\pi$	$(-\infty, -1] \cup [1, \infty)$

19. Important formulas :→

1. $\tan\theta = \dfrac{\sin 2\theta}{1 + \cos 2\theta}$

2. $\cot\theta = \dfrac{\sin 2\theta}{1 - \cos 2\theta}$

3. $\cot\theta = \dfrac{1 + \sin 2\theta + \cos 2\theta}{1 + \sin 2\theta - \cos 2\theta}$

4. $\tan(\theta/2) = \dfrac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta}$

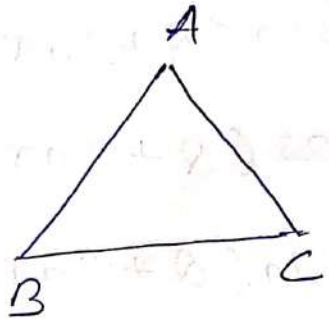
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5. $\tan\left(\frac{\pi}{4}-\theta\right) = \frac{\cos 2\theta}{1+\sin 2\theta}$

6. In $\triangle ABC$

$\angle A + \angle B + \angle C = 180^\circ$

(or) $= (n\pi)$



1 $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

2 $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{C}{2}\tan\frac{B}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

3 $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$

4. $\cos 2A + \cos 2B + \cos 2C$

$= -1 - 4\cos A \cdot \cos B \cdot \cos C$

7. $\sin A \cdot \sin\left(\frac{\pi}{3}-A\right)\sin\left(\frac{\pi}{3}+A\right)$

$= \frac{1}{4}\sin 3A$

8. $\cos A \cdot \cos\left(\frac{\pi}{3}-A\right)\cos\left(\frac{\pi}{3}+A\right)$

$= \frac{1}{4}\cos 3A$

9. $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

10. $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$

11. $\sin 6^\circ \cdot \sin 42^\circ \sin 66^\circ \sin 78^\circ = \frac{1}{16}$

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12. $\cos 24^\circ \cos 48^\circ \cos 96^\circ \cos 168^\circ = \frac{1}{16}$

13. $\sin(\theta + 2n\pi) = \sin\theta$

$\cos(\theta + 2n\pi) = \cos\theta$

$\tan(\theta + 2n\pi) = \tan\theta$

$\cot(\theta + 2n\pi) = \cot\theta$

$\sec(\theta + 2n\pi) = \sec\theta$

$\operatorname{cosec}(\theta + 2n\pi) = \operatorname{cosec}\theta$

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14. $\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{1}{\cos^2\theta \sin^2\theta}$

15. $1^\circ = \frac{\pi}{180}$ rad

$1 \text{ rad} = \left(\frac{180}{\pi}\right)^\circ$

16. if $\sec\theta \pm \tan\theta = x$

$\Rightarrow \sec^2\theta = \frac{x^2+1}{2x}$

17. $\sin\theta - \cos\theta = x$

$\sin\theta + \cos\theta = \sqrt{2 - x^2}$

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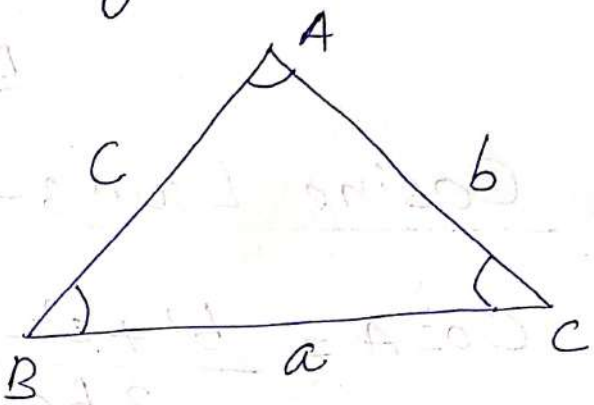
19. Solution of Triangles and Properties of Triangles :→

1 in $\triangle ABC$

$\angle ABC = \angle B$

$\angle BAC = \angle A$

$\angle AEB = \angle C$



$\left.\begin{matrix}\angle A \\ \angle B \\ \angle C\end{matrix}\right\}$ → Angles of $\triangle ABC$

Side opposite to $\angle A = a$

Side opposite to $\angle B = b$

Sid opposite to $\angle C = c$

→ Perimeter of $\triangle ABC = a+b+c$

→ Semi Perimeter of $\triangle ABC = \frac{(a+b+c)}{2}$

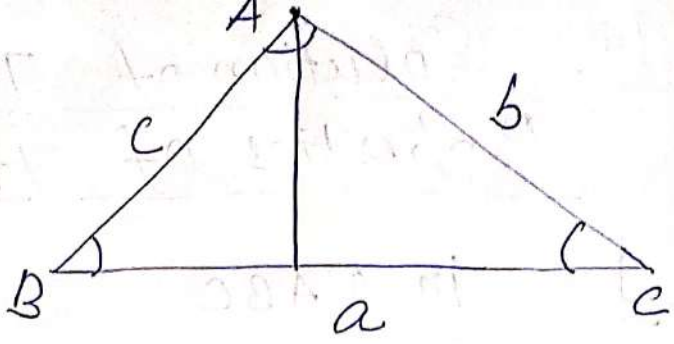
Semi Perimeter $(S) = \frac{(a+b+c)}{2}$

2 Sine Law :→

In $\triangle ABC$, Sides are proportional the Sines of opposite angles

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$

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3. Cosine Laws :-

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

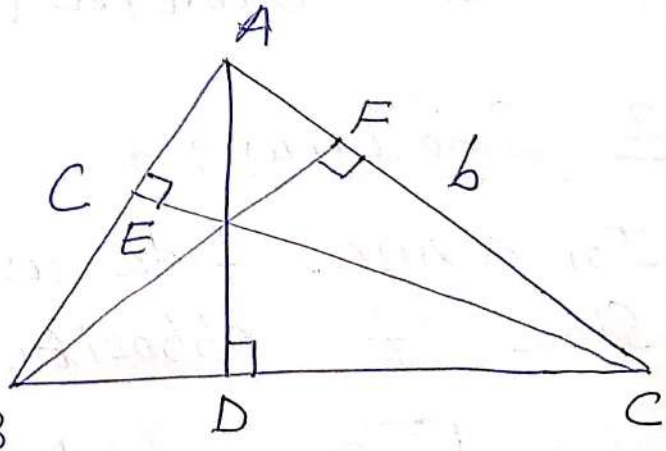
Cosine of angle can be represented in terms of sides

4. Projection formula :→ in ΔABC

$$a = BD + CD$$
$$= c\cos B + b\cos C$$

$$b = CF + AF$$
$$= a\cos C + c\cos A$$

$$c = AE + EB$$
$$= b\cos A + a\cos B$$



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$$a = b\cos C + c\cos B$$
$$b = c\cos A + a\cos C$$
$$c = a\cos B + b\cos A$$

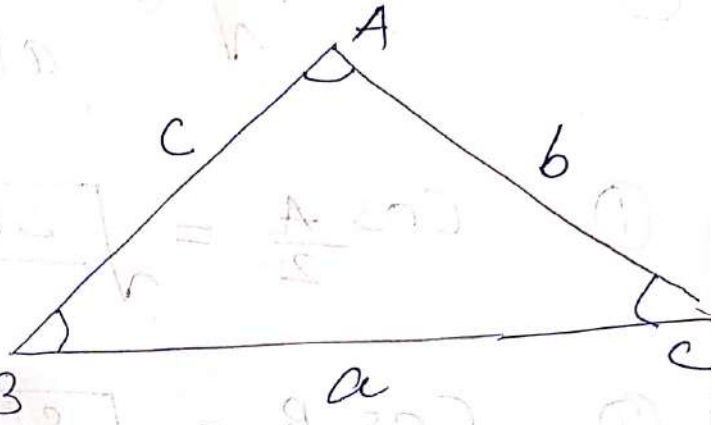
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5. Tangents Law (Napier's Analogy)

In Any ΔABC

$a, b, c \rightarrow$ Sides

$\left.\begin{matrix}\angle A\\ \angle B\\ \angle C\end{matrix}\right\}$ Angles

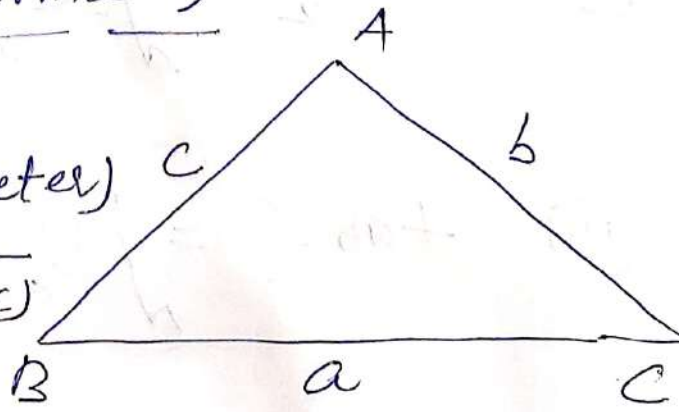


$$\tan\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{a+b}\right)\cot\frac{C}{2}$$

$$\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right)\tan\frac{A}{2}$$

$$\tan\left(\frac{C-A}{2}\right) = \left(\frac{c-a}{c+a}\right)\tan\frac{B}{2}$$

6. Half Angled formula (Auxiliary formula) in ΔABC



① $a+b+c = 2s$ (Perimeter)

② $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

↓

Area of ΔABC

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(I) $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

(II) $\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ac}}$

(III) $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

(I) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$

(II) $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$

(III) $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

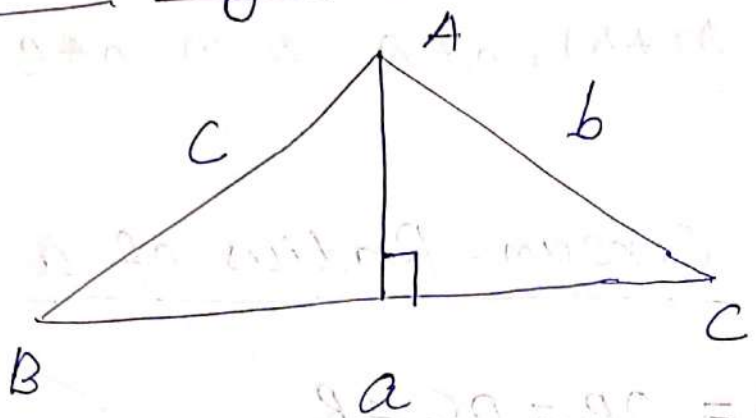
(I) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{\Delta}{s(s-a)}$

(II) $\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}} = \frac{\Delta}{s(s-b)}$

(III) $\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{\Delta}{s(s-c)}$

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7. Area of Triangle :→



Δ = Area of ΔABC

$$\Delta = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Delta = \frac{a^2}{2} \cdot \frac{\sin B \sin C}{\sin A}$$

$$= \frac{b^2}{2} \cdot \frac{\sin A \sin C}{\sin B}$$

$$= \frac{c^2}{2} \cdot \frac{\sin A \sin B}{\sin C}$$

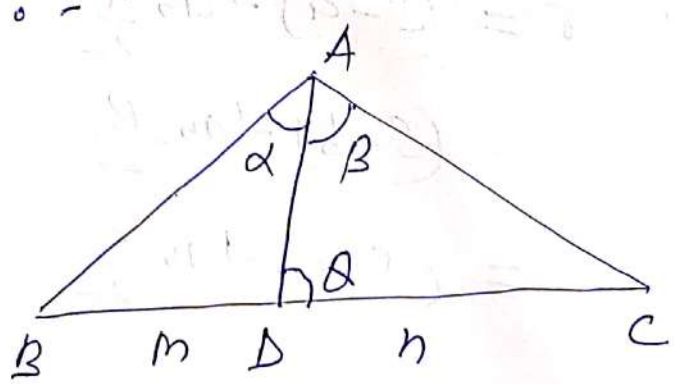
Q. m-n Theorem :-

In ΔABC, D is a point on side BC

$BD : DC = m : n$

$\angle DAC = \beta$

$\angle BAD = \alpha$ | $\angle ADC = \theta$

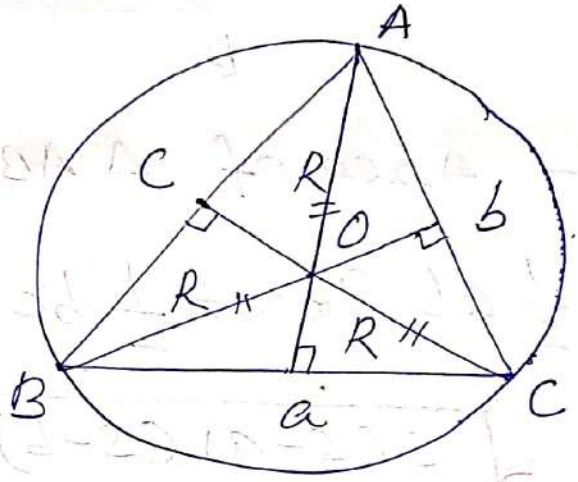


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(i) $(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

(ii) $(m+n)\cot\theta = n\cot B - m\cot C$

9. Circum-Radius of a ΔABC :→



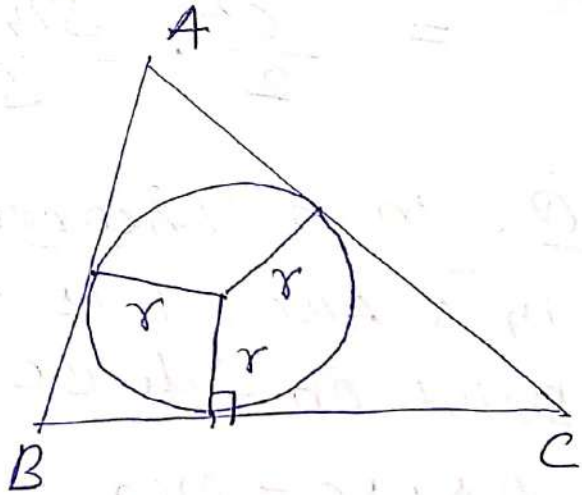
$OA = OB = OC = R$

R = Circum Radius

$$R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$$

$$R = \frac{abc}{4\Delta}$$ Δ = Area of ΔABC

10 Incircle of a Triangle ΔABC :→



(i) $r = \frac{\Delta}{s}$

(ii) $r = (s-a)\tan\frac{A}{2}$

$= (s-b)\tan\frac{B}{2}$

$= (s-c)\tan\frac{C}{2}$

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(III) $$r = \frac{a \sin\frac{B}{2} \sin\frac{C}{2}}{\cos\frac{A}{2}}$$

$$= \frac{b \sin\frac{C}{2} \sin\frac{A}{2}}{\cos\frac{B}{2}}$$

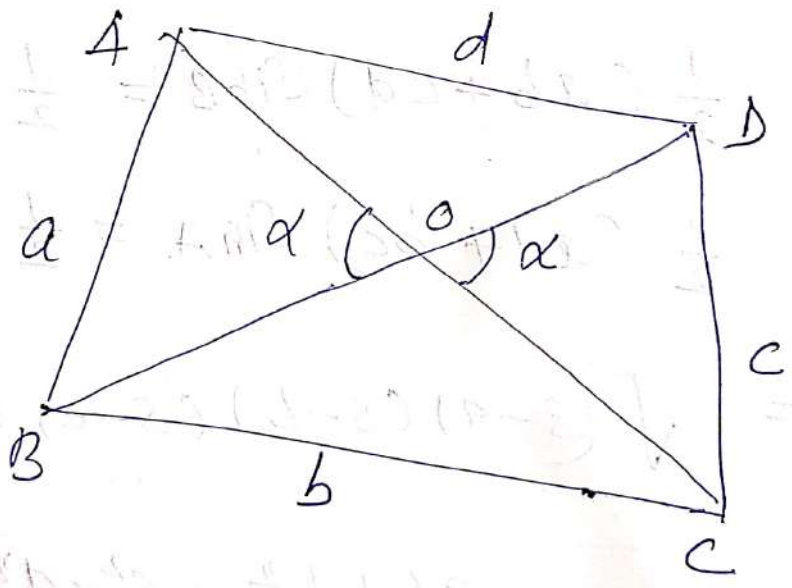
$$= \frac{c \sin\frac{A}{2} \sin\frac{B}{2}}{\cos\frac{C}{2}}$$

(IV) $$r = 4R \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$$

11. Area of Quadrilateral :→

ABCD
↓
Quadrilateral

AB = a
BC = b
CD = c
DA = d



∠DOC = α
∠AOB = α

Area of Quadrilateral ABCD
$= \frac{1}{2} AC \times BD \cdot \sin\alpha$

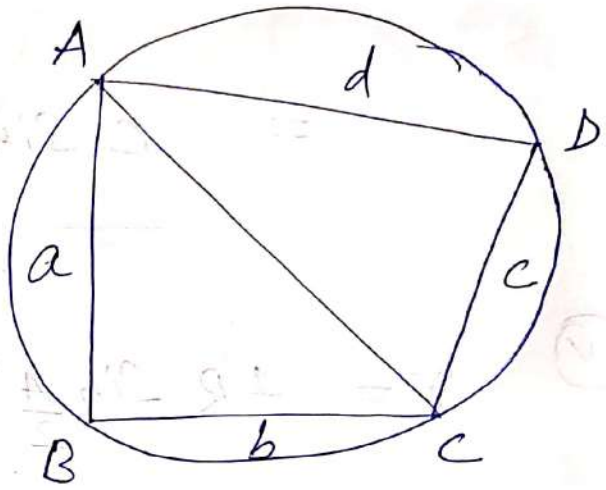
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Area of Quadrilateral ABCD

$$= \sqrt{(S-a)(S-b)(S-c)(S-d) - abcd\cos^2\alpha}$$

12. Area of Cyclic Quadrilateral :-

if Quadrilateral Vertices lies on a circle, it is called Cyclic Quadrilateral.



Δ = Area

$\angle B + \angle D = 180^\circ$

$\angle A + \angle C = 180^\circ$

$$\Delta = \frac{1}{2}(ab+cd)\sin B = \frac{1}{2}(ab+cd)\sin D$$

$$\Delta = \frac{1}{2}(ad+bc)\sin A = \frac{1}{2}(ad+bc)\sin C$$

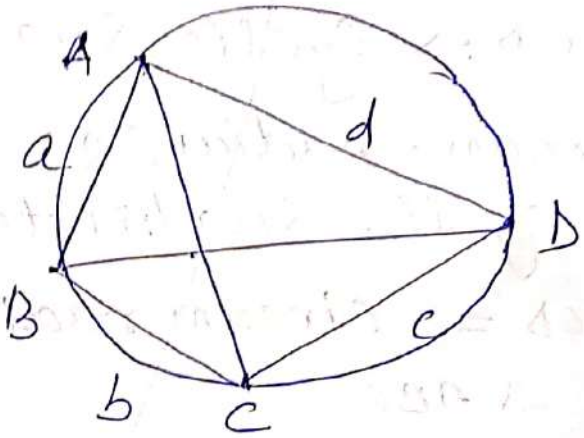
$$\Delta = \sqrt{(S-a)(S-b)(S-c)(S-d)}$$

$$\boxed{\cos B = \frac{a^2+b^2-c^2-d^2}{2(ab+cd)}}$$

→ In a Cyclic Quadrilateral ABCD

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13. Circum-radius of a Cyclic Quadrilateral :→



→ ABCD ⇒ Cyclic Quadrilateral

→ Circum radius (R) of a cyclic Quadrilateral ABCD = Circum radius of △ ABC

Circum-Radius of △ ABC

$$R = \frac{AC}{2 \sin B} \quad \text{— (4)}$$

using (1) $\Delta = \frac{1}{2}(ab+cd)\sin B$

$$\sin B = \frac{2\Delta}{(ab+cd)}$$

(2) $$AC = \frac{(ac+bd)\cdot(ad+bc)}{(ab+cd)}$$

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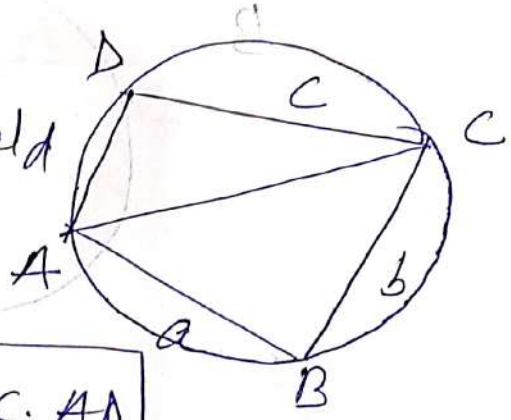
from (A), (I) & (II)

$$R = \frac{1}{4\Delta} \times \sqrt{(ac+bd)(ad+bc)(ab+cd)}$$

$$R = \frac{1}{4}\sqrt{\frac{(ac+bd)(ad+bc)(ab+cd)}{(s-a)(s-b)(s-c)(s-d)}}$$

14. Ptolemy's Thorem :→

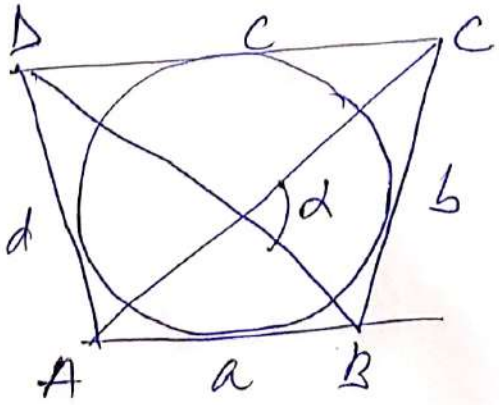
ABCD ⇒ Cyclic Quadrilateral



$$\boxed{AC \cdot BD = AB \cdot CD + BC \cdot AD}$$

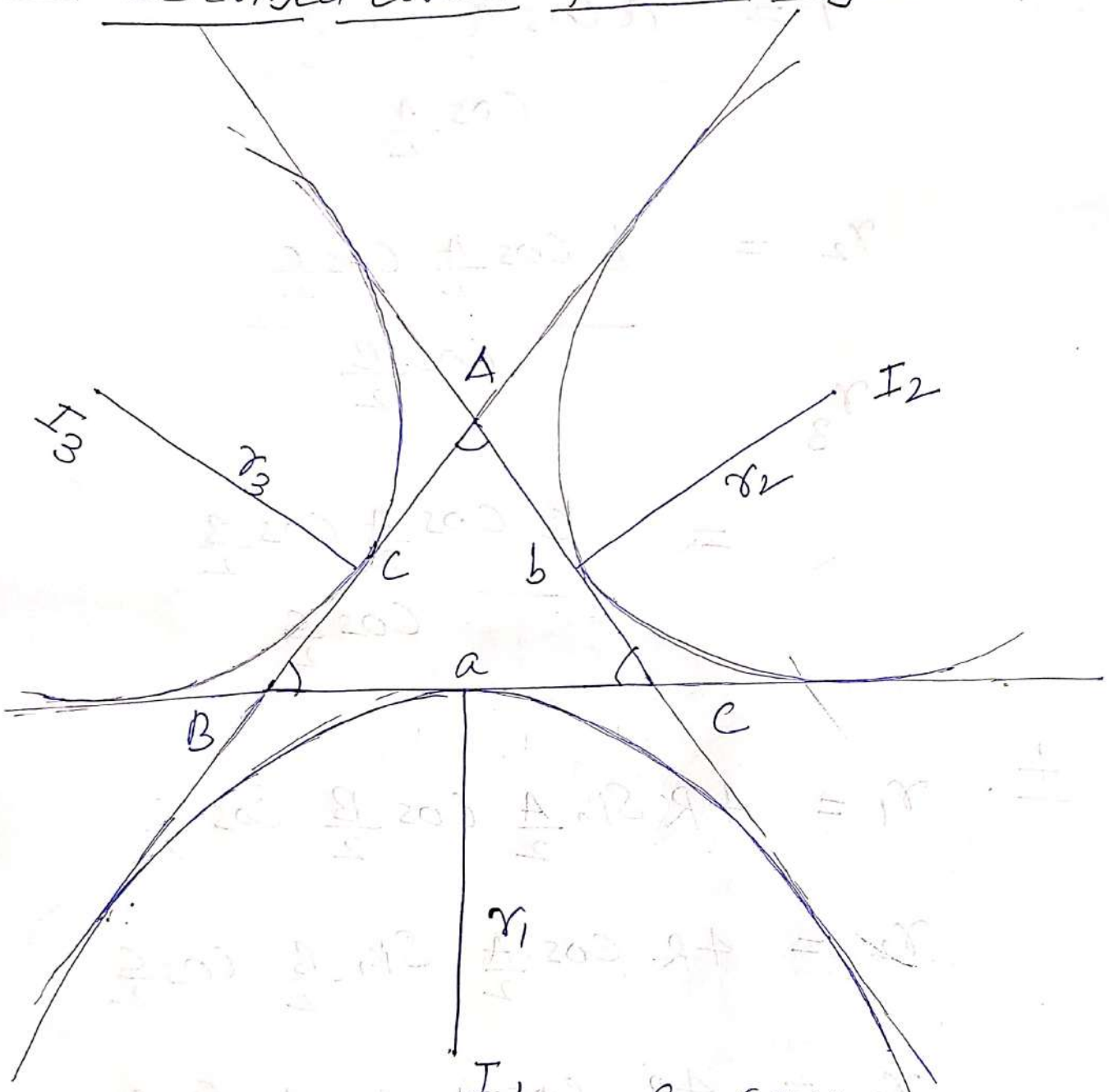
15. Area of a Quadrilateral with a circle inscribed in it :→

$$\boxed{\Delta = \sqrt{abcd}\,\sin\alpha}.$$



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16. Escribed circle of a Triangle :→



$$S = \frac{(a+b+c)}{2}$$

In ΔABC

1. $r_1 = \frac{\Delta}{S-a}$

$r_2 = \frac{\Delta}{S-b}$

$r_3 = \frac{\Delta}{S-c}$

2. $r_1 = S \tan \frac{A}{2}$

$r_2 = S \tan \frac{B}{2}$

$r_3 = S \tan \frac{C}{2}$

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3. $$r_1 = \frac{a \cos\frac{B}{2} \cos\frac{C}{2}}{\cos\frac{A}{2}}$$

$$r_2 = \frac{b \cos\frac{A}{2} \cos\frac{C}{2}}{\cos\frac{B}{2}}$$

$$r_3 = \frac{c \cos\frac{A}{2} \cos\frac{B}{2}}{\cos\frac{C}{2}}$$

4. $$r_1 = 4R \sin\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$$

$$r_2 = 4R \cos\frac{A}{2} \sin\frac{B}{2} \cos\frac{C}{2}$$

$$r_3 = 4R \cos\frac{A}{2} \cos\frac{B}{2} \sin\frac{C}{2}$$

17. Regular Polygons :-

→ if all Sides and each angles is equal of a Polygon — it is Called regular Polygon

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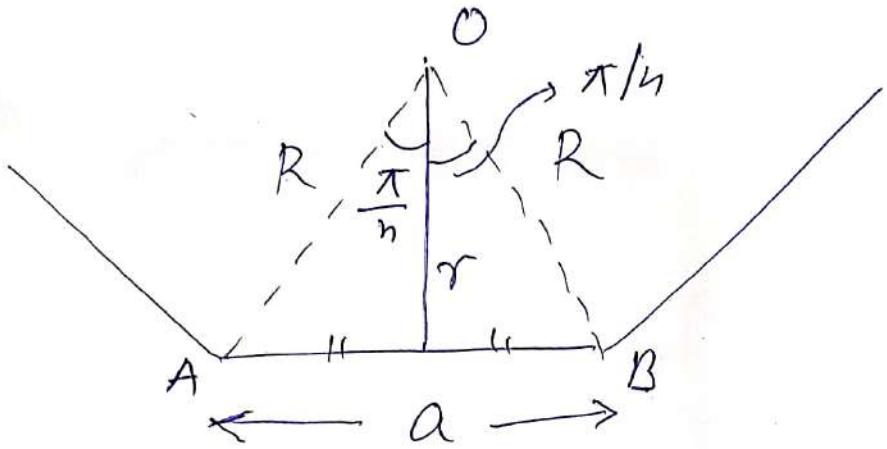
→ if Polygon has n sides, Sum of Internal angles is

$$= (n-2)\pi$$

→ Each angle value

$$= \frac{(n-2)\pi}{n}$$

→



① $R = \frac{a}{2} \operatorname{cosec}(\pi/n) \Rightarrow$ Circum-Radii

② $r = \frac{a}{2} \cot(\pi/n) \Rightarrow$ Inscribed Radi

③ Area of Polygon :-

$$= n \times \text{Area of } \triangle OAB$$

$$= \frac{na^2}{4} \cot(\pi/n)$$

(or) $= nr^2 \tan(\pi/n)$

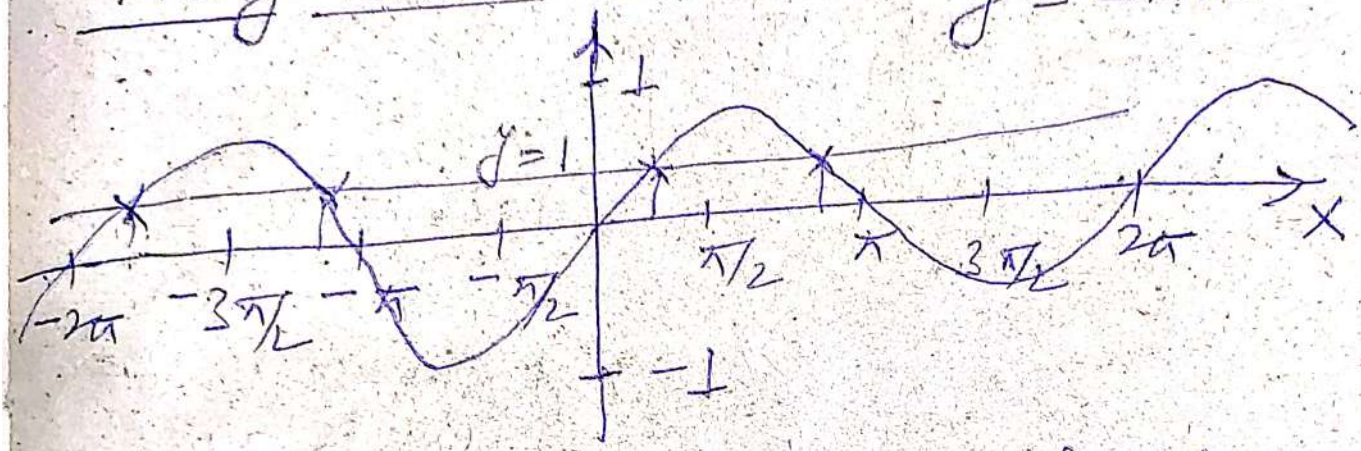
(or) $= \frac{nR^2}{2} \sin(2\pi/n)$

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(20) # Trigonometric Inverse function

→ We can not Take Inverse of Trigonometric function for Domain $x \in R$, $x \in R-(2n+1)\pi/2$, $x \in R-n\pi$ Because trigonometric functions are many to-one function

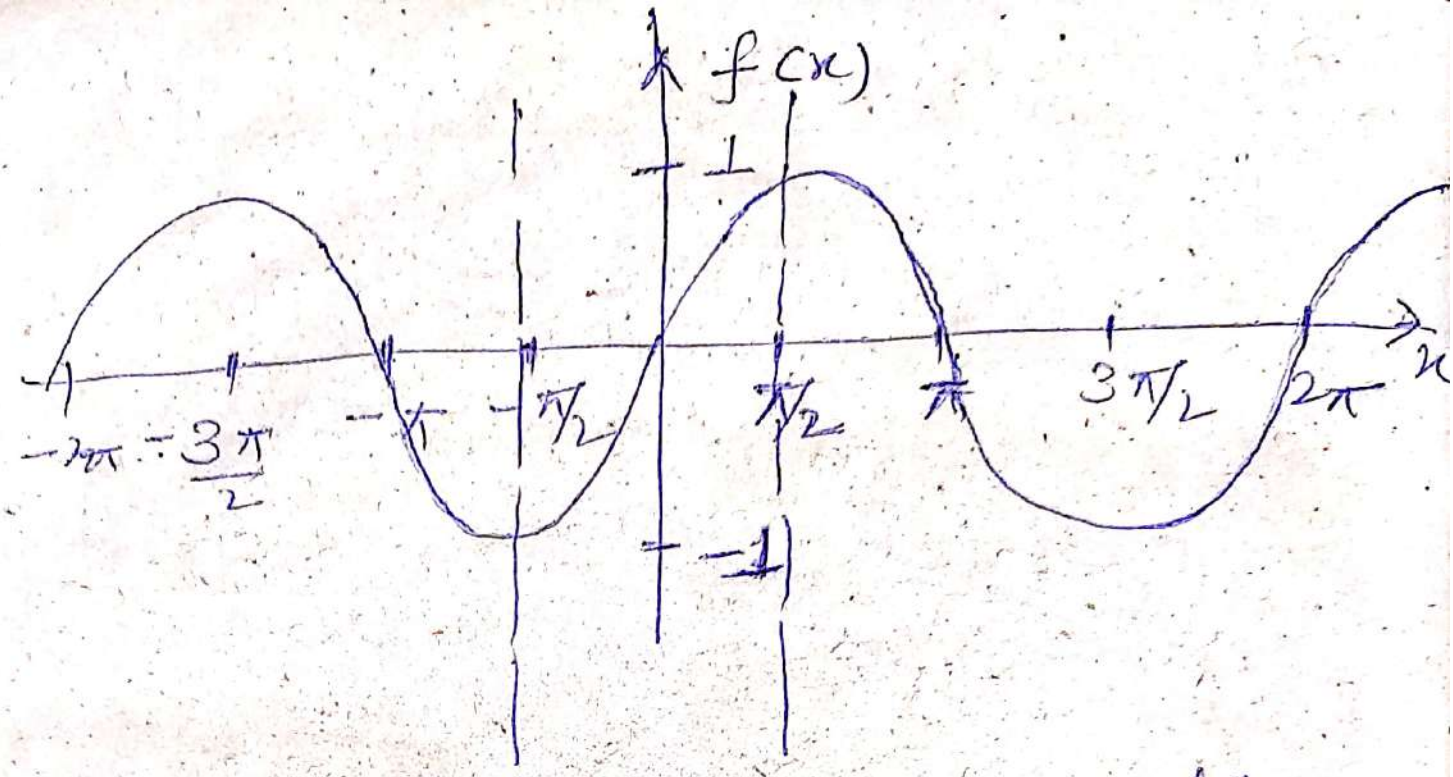
$y = \sin x$



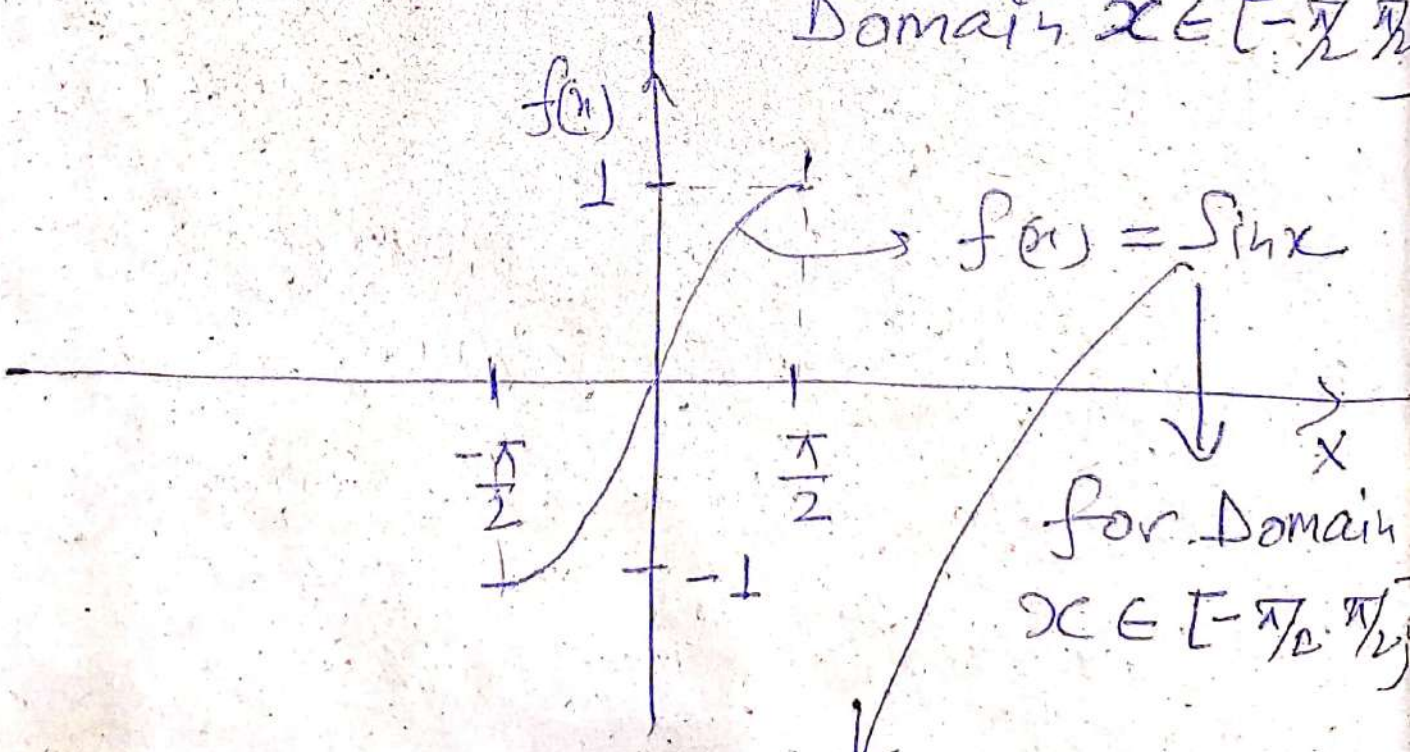
→ for Taking Inverse of trigono-metric function - We have to make them one-to-one function by restricting their domain

→ We can Take Inverse of one-to-one function.

→ If We restrict the domain of $f(x) = \sin x$ to $x \in [-\pi/2, \pi/2]$ So We can Take Inverse of $f(x)$



making this function from "many-one" to "one to one" by Restricting Domain $x \in [-\pi/2, \pi/2]$



for Domain $x \in [-\pi/2, \pi/2]$

One to-one fn

① Domain and Range of Inverse Trigonometric function :→

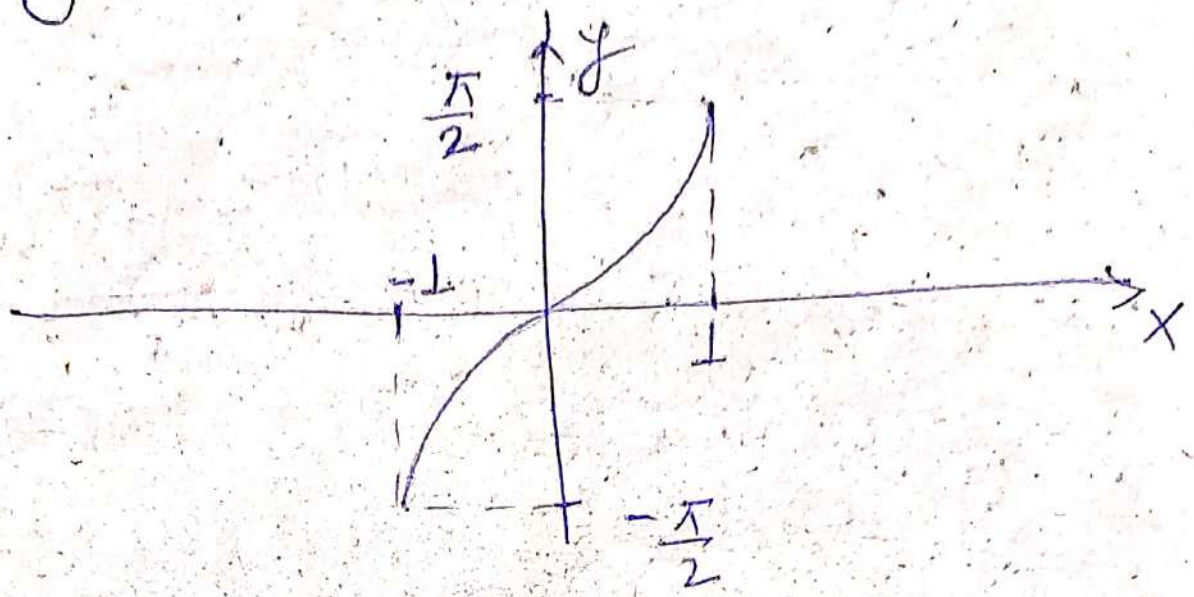
function	Domain	Range
① $f(x) = \sin^{-1}x$	$x \in [-1, 1]$	$f(x) \in [-\pi/2, \pi/2]$
② $f(x) = \cos^{-1}x$	$x \in [-1, 1]$	$f(x) \in [0, \pi]$
③ $f(x) = \tan^{-1}x$	$x \in R = (-\infty, \infty)$	$f(x) \in (-\pi/2, \pi/2)$
④ $f(x) = \cot^{-1}x$	$x \in R$	$f(x) \in (0, \pi)$
⑤ $f(x) = \sec^{-1}x$	$x \in (-\infty, -1] \cup [1, \infty)$	$f(x) \in [0, \pi] - \{\pi/2\}$
⑥ $f(x) = \operatorname{cosec}^{-1}x$	$x \in (-\infty, -1] \cup [1, \infty)$	$f(x) \in [-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$

② Graph of Trigonometric Inverse function (Inverse Trigonometric function) :→

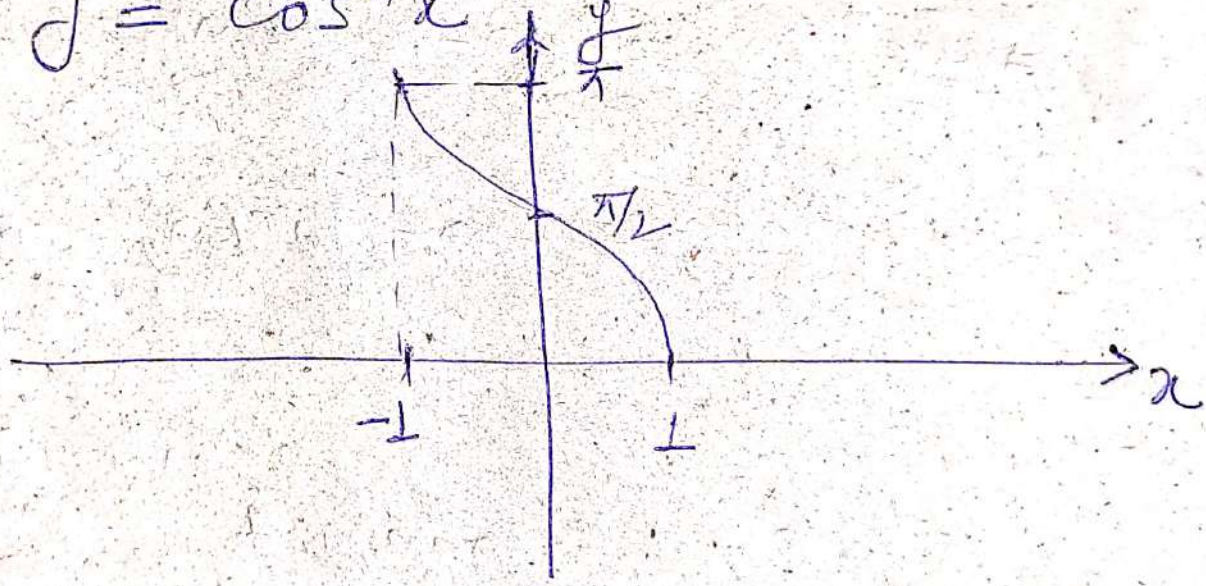
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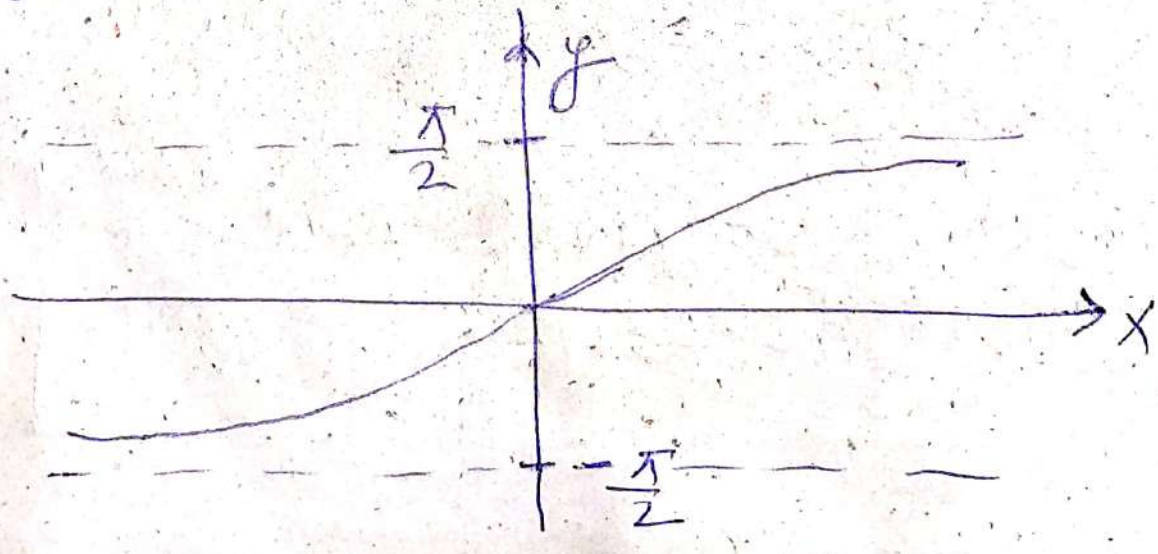
① $y = \sin^{-1}x$



② $y = \cos^{-1}x$

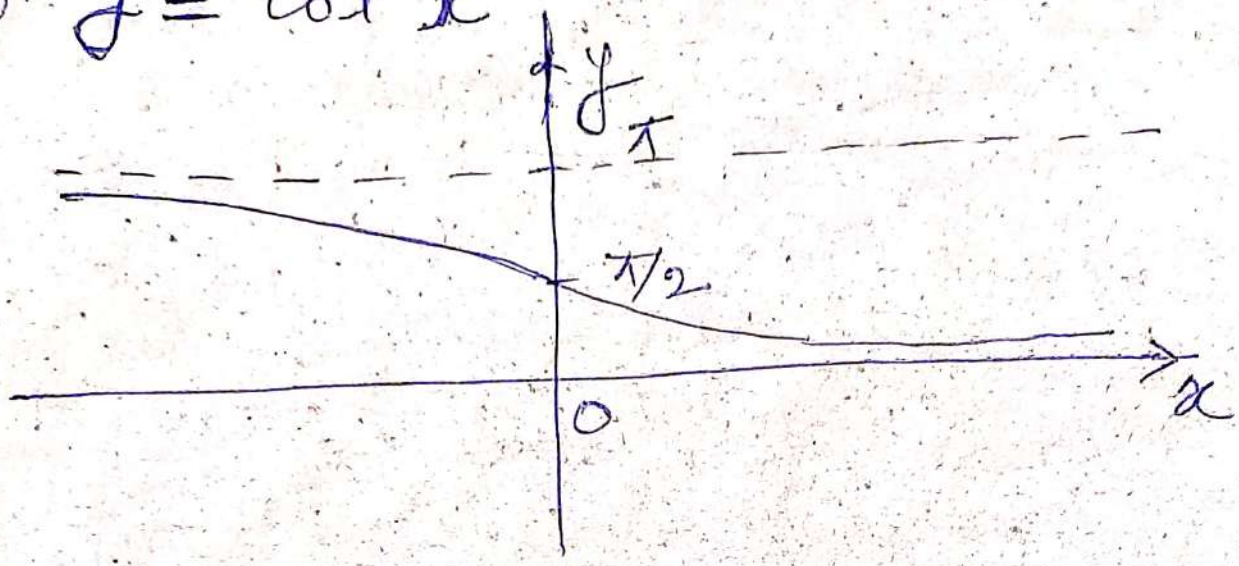


③ $y = \tan^{-1}x$

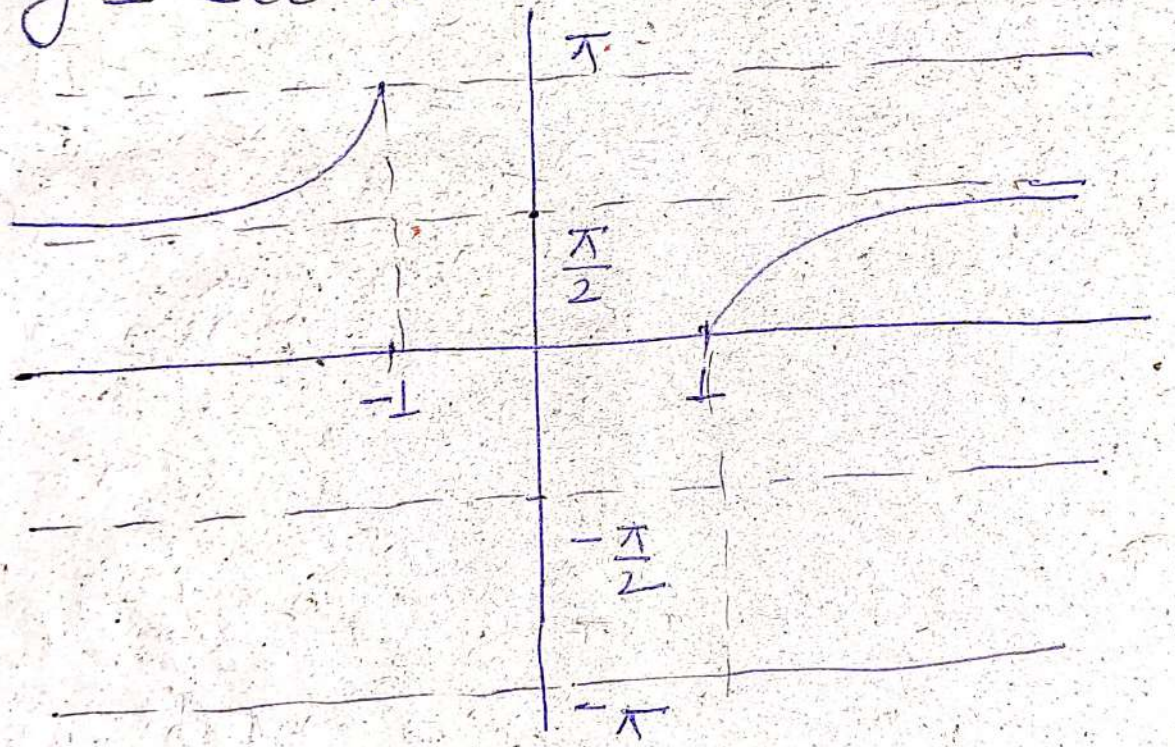


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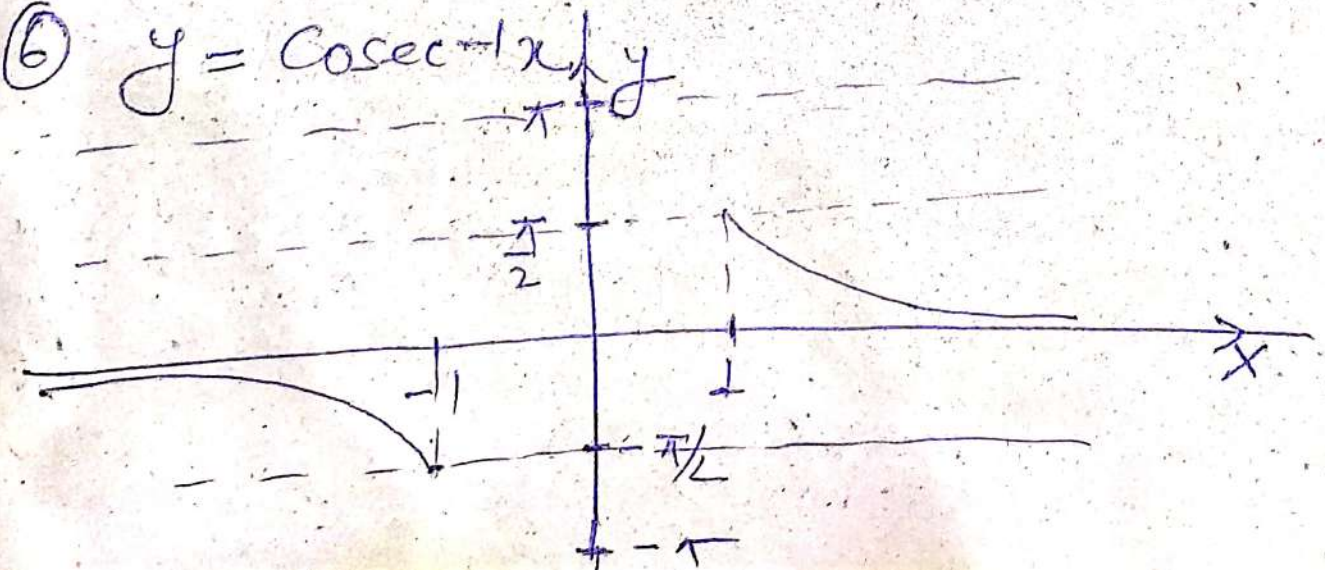
④ $y = \cot^{-1}x$



⑤ $y = \sec^{-1}x$



⑥ $y = \operatorname{cosec}^{-1}x$



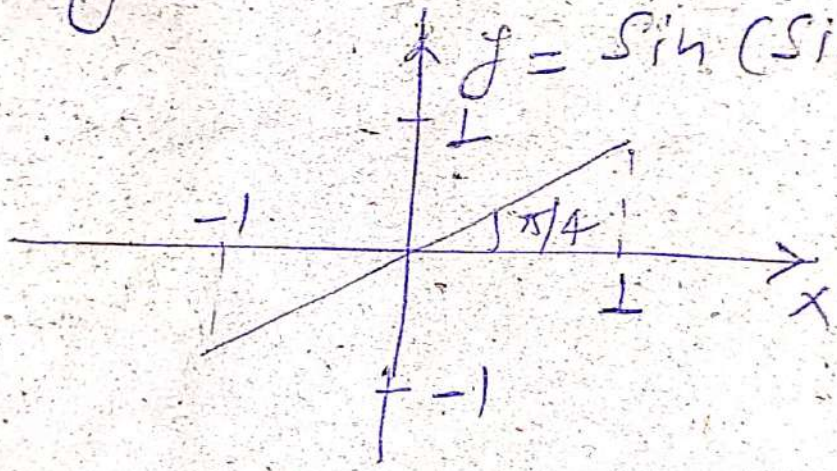
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(3) Properties of Inverse Trigonometric function :-

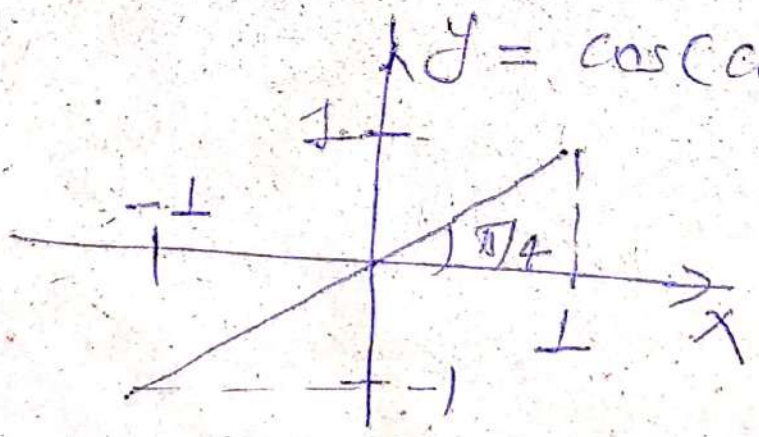
Property I

(A) $\sin(\sin^{-1}x) = x$, $x \in [-1, 1]$

So graph of $y = \sin(\sin^{-1}x)$ will be graph of $y = x$ for $x \in [-1, 1]$

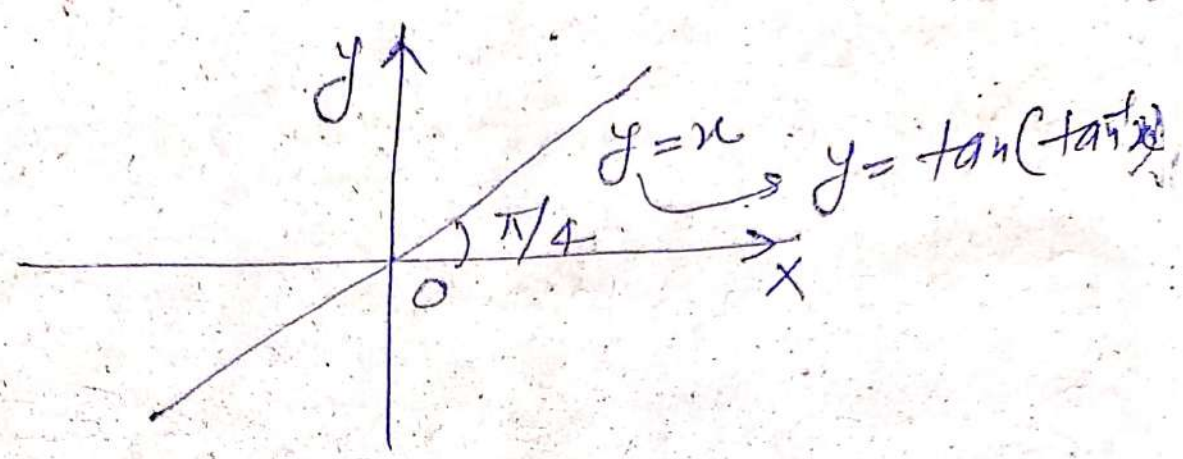


(B) $\cos(\cos^{-1}x) = x$, $x \in [-1, 1]$



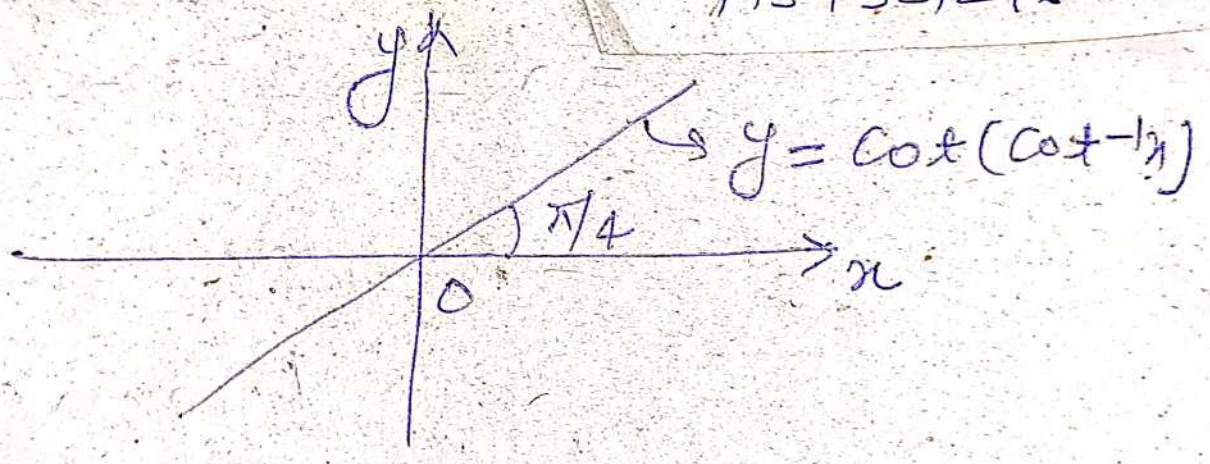
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(C) $\tan(\tan^{-1}x) = x \qquad x \in R$



(D) $\cot(\cot^{-1}x) = x \;, \quad x \in R$

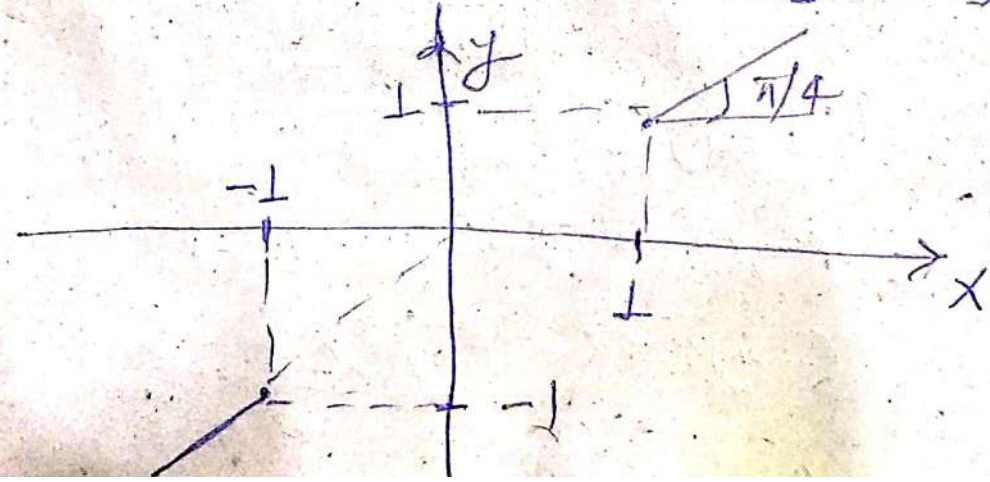
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(E) $\sec(\sec^{-1}x) = x$

$\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x$ }

$x \in (-\infty, -1] \cup [1, \infty)$



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Property II

(A) $\sin^{-1}(\sin\alpha) = \alpha$, for all $\alpha \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

(B) $\cos^{-1}(\cos\alpha) = \alpha$, $\alpha \in [0, \pi]$

(C) $\tan^{-1}(\tan\alpha) = \alpha$, $\alpha \in (-\frac{\pi}{2}, \frac{\pi}{2})$

(D) $\cot^{-1}(\cot\alpha) = \alpha$, $\alpha \in (0, \pi)$

(E) $\operatorname{Cosec}^{-1}(\operatorname{Cosec}\alpha) = \alpha$, $\alpha \in [-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$

(F) $\sec^{-1}(\sec\alpha) = \alpha$, $\alpha \in [0, \pi] - \{\pi/2\}$

- $\cos^{-1}(\cos\alpha) = \alpha$ for $\alpha \in [0, \pi]$
- $\cos^{-1}(\cos\alpha) \to$ defined for $\alpha \in [0, \pi]$
  When $\cos\alpha \in [-1, 1]$
  $\cos^{-1}\alpha \in [0, \pi]$

Property III

(A) $\sin^{-1}(-x) = -\sin^{-1}x$, $x \in [-1, 1]$

(B) $\cos^{-1}(-x) = \pi - \cos^{-1}x$, $x \in [-1, 1]$

(C) $\tan^{-1}(-x) = -\tan^{-1}x$ $x \in R$

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(D) $\cot^{-1}(-x) = \pi - \cot^{-1}x$

for all $x \in (-\infty, \infty)$

(E) $\operatorname{Cosec}^{-1}(-x) = -\operatorname{Cosec}^{-1}x$

$x \in (-\infty, -1] \cup [1, \infty)$

(F) $\sec^{-1}(-x) = \pi - \sec^{-1}x$

$x \in (-\infty, -1] \cup [1, \infty)$

Property IV

(A) $\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{Cosec}^{-1}x$

$x \in (-\infty, -1] \cup [1, \infty)$

(B) $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}x$

$x \in (-\infty, -1] \cup [1, \infty)$

(C) $\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1}x, & x > 0 \\ -\pi + \cot^{-1}x, & x < 0 \end{cases}$

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Property 5 :⇒

(A) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$, $x \in [-1, 1]$

(B) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$, $x \in R$

(C) $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

$x \in (-\infty, -1] \cup [1, \infty)$

Property 6

(A) $\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right), & \text{if } xy < 1 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) & \text{if } x>0,\ y>0,\ xy>1 \\ -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) & x<0,\ y<0,\ xy>1 \end{cases}$

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(B) $\tan^{-1}x - \tan^{-1}y = \begin{cases} \Rightarrow \tan^{-1}\left(\frac{x-y}{1+xy}\right) \\ \text{if } xy > -1, \\ \Rightarrow \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right) \\ \text{if } x > 0, y < 0, xy < -1 \\ \Rightarrow -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right) \\ \text{if } x < 0, y > 0, \& xy < -1 \end{cases}$

Property 7

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(A) $\sin^{-1}x + \sin^{-1}y =$

$= \begin{cases} \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right] \\ \text{if } -1 \le x, y \le 1 \\ x^2 + y^2 \le 1 \quad (\text{or}) \text{ if } xy < 0 \\ \& \; x^2 + y^2 > 1 \end{cases}$

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$$= \pi - \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$$

if $0 < x, y \leq 1$ and $x^2+y^2 > 1$

$$= -\pi - \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$$

if $-1 \leq x, y < 0$ and $x^2+y^2 > 1$

(B) $\sin^{-1}x - \sin^{-1}y$

$$= \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$$

if $-1 \leq x, y \leq 1$ and $x^2+y^2 \leq 1$

(or) $xy > 0$ and $x^2+y^2 > 1$

$$= \pi - \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$$

if $0 < x \leq 1$, $-1 \leq y \leq 0$ and

$x^2+y^2 > 1$

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$= -\pi - \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$

if $-1 \le x < 0$, $0 < y \le 1$ and $x^2+y^2 > 1$

Property 8

(A) $\cos^{-1}x + \cos^{-1}y \Rightarrow$

$= \left[\cos^{-1}\left[xy - \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right]\right.$

if $-1 \le x, y \le 1$ and $x+y \ge 0$

$= \left[2\pi - \cos^{-1}\left[xy - \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right]\right.$

if $-1 \le x, y \le 1$ & $x+y \le 0$

(B) $\cos^{-1}x - \cos^{-1}y \Rightarrow$

$= \left[\cos^{-1}\left[xy + \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right]\right.$

if $-1 \le x, y \le 1$ & $x \le y$

$= \left[-\cos^{-1}\left[xy - \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right]\right.$

if $-1 \le y \le 0$, $0 < x \le 1$ & $x \ge y$.

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Property 9

(A) $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$= \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

$= \operatorname{cosec}^{-1}(1/x)$

(B) $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

$= \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \sec^{-1}\left(\frac{1}{x}\right)$

$= \operatorname{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

(C) $\tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

$= \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

$= \cot\left(\frac{1}{x}\right) = \sec^{-1}\left(\sqrt{1+x^2}\right)$

$= \operatorname{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$

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Property 10

(A)

$$2\sin^{-1}x = \begin{cases} \sin^{-1}(2x\sqrt{1-x^2}), & -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}} \\ \pi - \sin^{-1}(2x\sqrt{1-x^2}) & \frac{1}{\sqrt{2}} \le x \le 1 \\ -\pi - \sin^{-1}(2x\sqrt{1-x^2}) & -1 \le x \le -\frac{1}{\sqrt{2}} \end{cases}$$

(B)

$$3\sin^{-1}x = \begin{cases} \sin^{-1}(3x-4x^3), & -\frac{1}{2} \le x \le \frac{1}{2} \\ \pi - \sin^{-1}(3x-4x^3), & \frac{1}{2} < x \le 1 \\ -\pi - \sin^{-1}(3x-4x^3), & -1 \le x < -\frac{1}{2} \end{cases}$$

Property 11

(A) $$2\cos^{-1}x = \begin{cases} \cos^{-1}(2x^2-1), & 0 \le x \le 1 \\ 2\pi - \cos^{-1}(2x^2-1), & -1 \le x \le 0 \end{cases}$$

(B) $$3\cos^{-1}x = \begin{cases} \cos^{-1}(4x^3-3x), & \frac{1}{2} \le x \le 1 \\ 2\pi - \cos^{-1}(4x^3-3x), & -\frac{1}{2} \le x \le \frac{1}{2} \\ 2\pi + \cos^{-1}(4x^3-3x), & -1 \le x \le -1/2 \end{cases}$$

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Property 12 :→

(A)

$$2\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{2x}{1-x^2}\right), & -1 < x \le 1 \\ \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), & x > 1 \\ -\pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), & x < -1 \end{cases}$$

(B)

$$3\tan^{-1}x$$

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$$= \begin{cases} \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), & -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), & x > 1/\sqrt{3} \\ -\pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), & \text{if } x < -\frac{1}{\sqrt{3}} \end{cases}$$

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Property 13

(A)

$$2\tan^{-1}x = \begin{cases} \sin^{-1}\left(\frac{2x}{1+x^2}\right), & -1 \le x \le 1 \\ \pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right), & x > 1 \\ -\pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right), & x < -1 \end{cases}$$

(B)

$$2\tan^{-1}x = \begin{cases} \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), & 0 \le x < \infty \\ -\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), & -\infty < x \le 0 \end{cases}$$

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Address : L-3/87, Chauraha Purania, Sector D, Aliganj Lucknow

Call Us : 05224312655, +919454321216

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