

Trigonometric Identities & Formulas

Reciprocal Identities

$$\sin x = \frac{1}{\csc x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\cot x = \frac{1}{\tan x}$$

Ratio or Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sin x = \cos x \tan x$$

$$\cos x = \sin x \cot x$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Note: there are only three, basic Pythagorean identities, the other forms are the same three identities, just arranged in a different order.

Pythagorean Identities in Radical Form

$$\sin x = \pm \sqrt{1 - \cos^2 x}$$

$$\tan x = \pm \sqrt{\sec^2 x - 1}$$

$$\cos x = \pm \sqrt{1 - \sin^2 x}$$

Confunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x$$

Odd-Even Identities

Also called negative angle identities

$$\sin(-x) = -\sin x \quad \csc(-x) = -\csc x$$

$$\cos(-x) = \cos x \quad \sec(-x) = \sec x$$

$$\tan(-x) = -\tan x \quad \cot(-x) = -\cot x$$

How to Find Reference Angles

Step 1: Determine which quadrant the angle is in

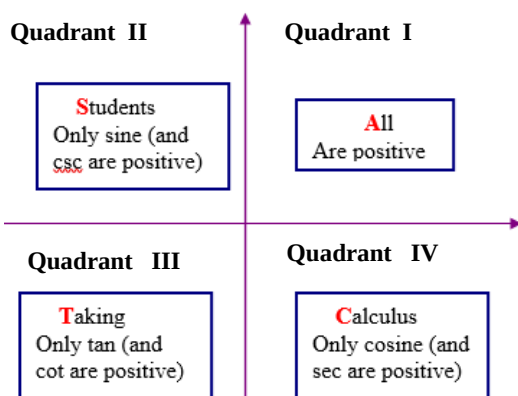
Step 2: Use the appropriate formula

$$\text{Quad I} = \text{is the angle itself}$$

$$\text{Quad II} = 180 - \theta \quad \text{or} \quad \pi - \theta$$

$$\text{Quad III} = \theta - 180 \quad \text{or} \quad \theta - \pi$$

$$\text{Quad IV} = 360 - \theta \quad \text{or} \quad 2\pi - \theta$$



Now we can use the reference angles to find the value of all the trigonometric functions. To help us doing this, we will remember the following rule ASTC (All Students Take Calculus)



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \csc x = \frac{1}{\sin x}$$

$$\cos x = \frac{1}{\sec x} \quad \sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{1}{\cot x} \quad \cot x = \frac{1}{\tan x}$$

Ratio or Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

$$\sin x = \cos x \tan x \quad \cos x = \sin x \cot x$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Pythagorean Identities in Radical Form

$$\sin x = \pm \sqrt{1 - \cos^2 x}$$

$$\tan x = \pm \sqrt{\sec^2 x - 1}$$

Note: there are only three, basic Pythagorean identities, the other forms are the same three identities, just arranged in a different order.

Confunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \quad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

Odd-Even Identities

Also called negative angle identities

$$\sin(-x) = -\sin x \quad \csc(-x) = -\csc x$$

$$\cos(-x) = \cos x \quad \sec(-x) = \sec x$$

$$\tan(-x) = -\tan x \quad \cot(-x) = -\cot x$$

Sum and Difference Formulas - Identities

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

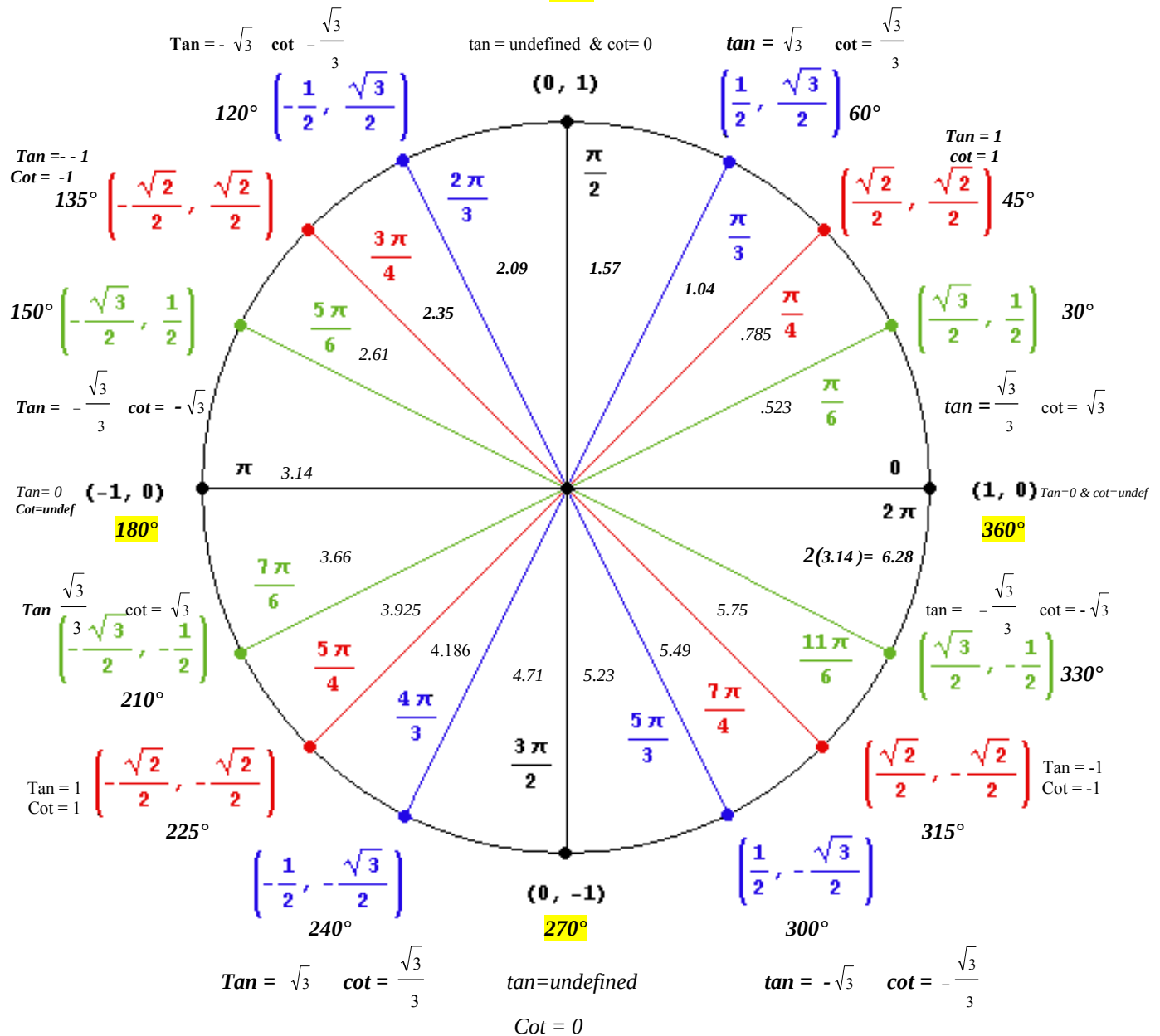
$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

The Unit Circle

90°



Definition of Trigonometric Functions concerning the Unit Circle

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r}$$

$$\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{y}$$

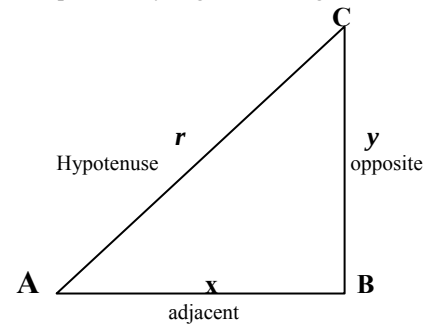
Right Triangle Definitions of Trigonometric Functions

Note: sin & cos are complementary angles, so are tan & cot and sec & csc, and the sum of complementary angles is 90 degrees.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} \quad \csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} \quad \sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} \quad \cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{y}$$



Adjacent = is the side adjacent to the angle in consideration. So if we are considering Angle A, then the adjacent side is CB

Trigonometric Values of Special Angles

Degrees	0°	30°	45°	60°	90°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sinθ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tanθ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

To Convert Degrees to Radians, Multiply by $\frac{\pi \text{ rad}}{180 \text{ deg}}$

To Convert Radians to Degrees, Multiply by $\frac{180 \text{ deg}}{\pi \text{ rad}}$

Vocabulary

- Cotangent Angles - are two angles with the same terminal side
- Reference Angle - is an acute angle formed by terminal side of angle(α) with x-axis

Double Angle Identities

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Half Angle Identities

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \frac{1 - \cos A}{\sin A}$$

$$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}$$

Power Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2} [\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2} [\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2} [\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2} [\sin(u + v) - \sin(u - v)]$$

Sum-to-Product Formulas

$$\sin x + \sin y = 2 \sin \left(\frac{x + y}{2} \right) \cos \left(\frac{x - y}{2} \right)$$

$$\sin x - \sin y = 2 \cos \left(\frac{x + y}{2} \right) \sin \left(\frac{x - y}{2} \right)$$

$$\cos x + \cos y = 2 \cos \left(\frac{x + y}{2} \right) \cos \left(\frac{x - y}{2} \right)$$

$$\cos x - \cos y = -2 \sin \left(\frac{x + y}{2} \right) \sin \left(\frac{x - y}{2} \right)$$

