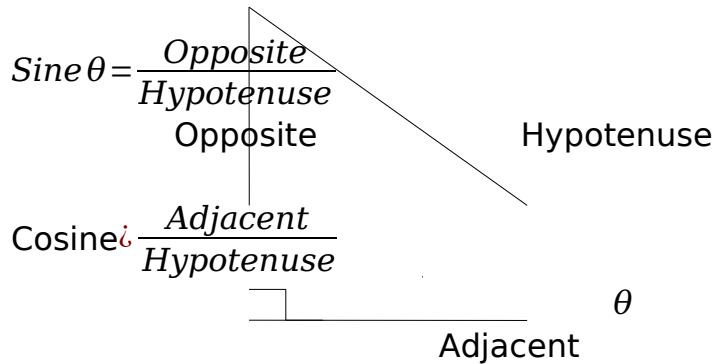


TRIGONOMETRY QUESTIONS

Sine, cosine, tangent

The trigonometric ratios sine, cosine and tangent are defined in terms of the hypotenuse, opposite and adjacent side of a right-angled triangle.



Sine rule

Sine rule is used when you are given

- two angle and one side
- two side and a non included angle

In obtuse-angled triangle $\sin \theta = \sin(180^\circ - \theta)$

Cosine rule

Cosine rule is used when you are given;

- two side and an included angle
- three side only

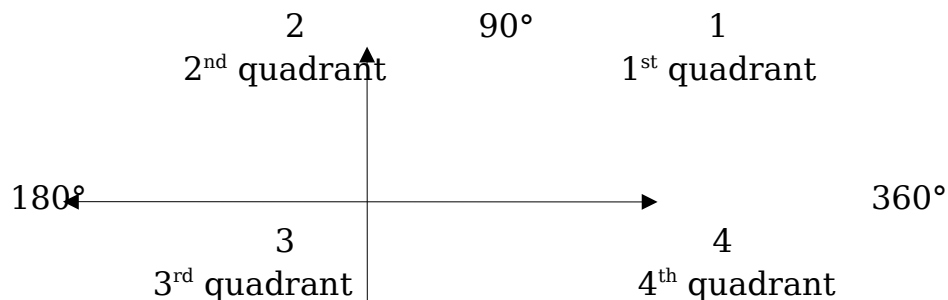
In obtuse-angled triangle $\cos \theta = -\cos(180^\circ - \theta)$

Pythagoras

In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the two adjacent sides.

TRIGONOMETRY ON THE CARTESIAN PLANE

- Determining the signs of the three trig ratios in the quadrants.

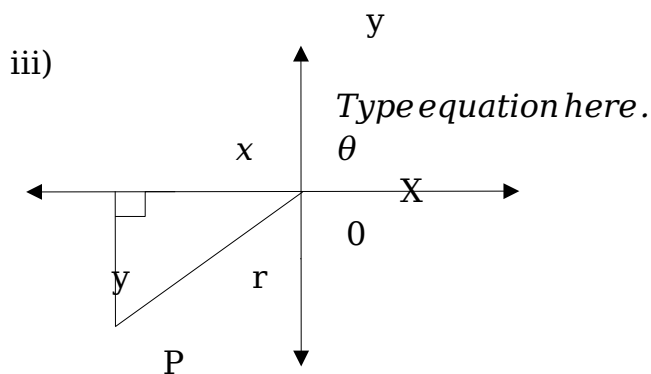
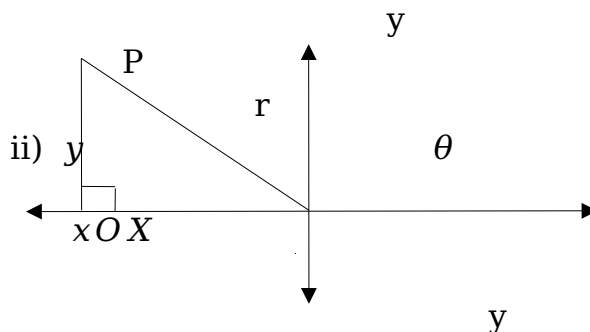
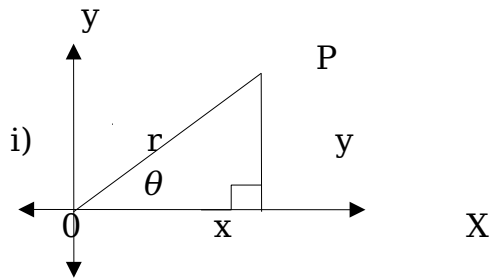


270°

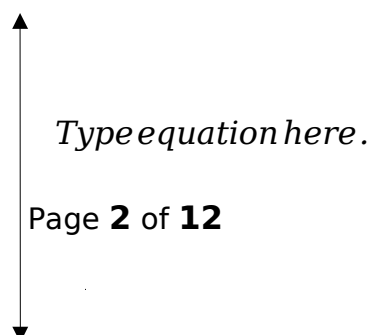
The quadrants are numbered 1 to 4 in an anti-clock wise direction.

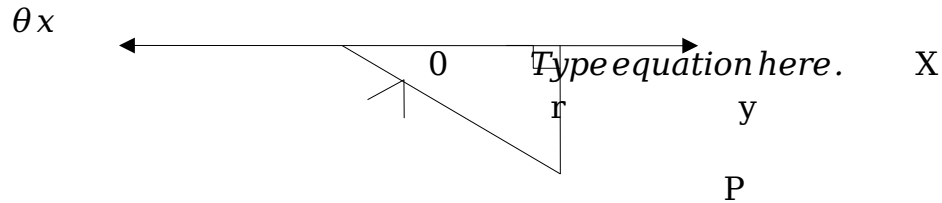
- i. angles between 0 and 90 fall in the first quadrant
- ii. angles between 90 and 180 fall in the second quadrant
- iii. angles between 180 and 270 fall in the third quadrant
- iv. angle between 270 and 360 fall in the fourth quadrant

We can find trig ratios on Cartesian plane given a point



iv)





- $P(x, y)$ is a point in each of the four quadrants of the Cartesian plane. The length $OP = r$, where $r > 0$ and $\angle POX = \theta$

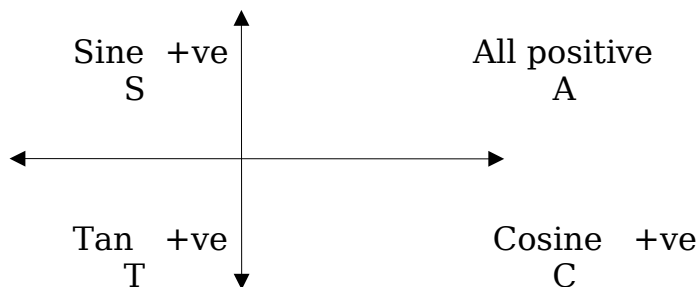
$$\sin \theta = \frac{y}{r}, \quad \cos \theta = \frac{x}{r} \quad \text{and} \quad \tan \theta = \frac{y}{x}$$

- The signs of x and y differ from quadrant to quadrant so the values of $\sin$, $\cos$ and $\tan$ will differ as well.
- θ is the angle between the line OP and the positive x -axis

TRIGONOMETRIC EQUATIONS

$\sin \theta$, $\cos \theta$ and $\tan \theta$ can be positive or negative depending on the quadrant within which θ fall.

The cartesian diagram

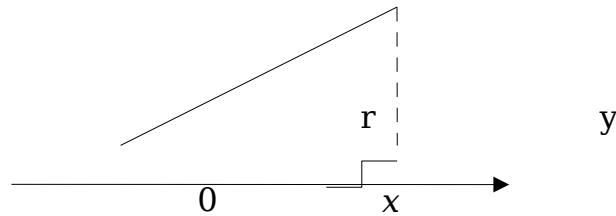


- The cast diagram is very useful when solving trig equations
- A trigonometric equations will usually have two solutions
- To solve equations of the form $\sin \theta = k$, $\cos \theta = k$ or $\tan \theta = k$
 - find the reference angle α in the 1st quadrant
 - determine in which two quadrants θ will lie
 - find the corresponding angle in the two quadrant

In the diagram below P has coordinates $(12, 5)$



P(12, 5)



Type equation here.

Find the value of;

- a) $\sin \angle XOP$
- b) $\cos \angle XOP$
- c) $\tan \angle XOP$

Solutions

First find side 'r' by using the Pythagoras theorem.

$$r^2 = y^2 + x^2$$

$$r^2 = 5^2 + 12^2$$

$$r^2 = 25 + 144$$

$$r^2 = 169$$

$$\sqrt{r^2} = \sqrt{169}$$

$$r = 13 \text{ units.}$$

$$\therefore (a) \sin \angle XOP = \frac{O}{H} (b) \cos \angle XOP = \frac{A}{H} (c) \tan \angle XOP = \frac{O}{A}$$

$$\therefore \frac{y}{r} = \frac{x}{r} = \frac{y}{x}$$

$$\therefore \frac{5}{13} = \frac{12}{13} = \frac{5}{12}$$

5. Solve the equation $\sin \theta = 0.766$ for $0^\circ \leq \theta \leq 180^\circ$.

Solution

$$\sin \theta = 0.766$$

$$\alpha = \sin^{-1} 0.766$$

$$\therefore 49.99603866^\circ$$

$$\therefore 50^\circ$$

Sin is positive in the 1st and 2nd quadrant

(i) 1st quadrant

$$\theta = 50^\circ$$

(ii) 2nd quadrant

$$\theta = 180^\circ - 50^\circ$$

$$\theta = 30^\circ$$

$$\therefore \theta = 50^\circ \wedge \theta = 130^\circ$$

6. Solve the equation $\tan\theta = -5.67$ for $0^\circ \leq \theta \leq 360^\circ$.

Solution

$$\tan\theta = -5.67$$

$$\alpha = \tan^{-1} 5.67$$

$$\approx 79.99^\circ$$

$$\approx 80^\circ$$

Tan is negative in the 2nd and 4th quadrants.

2nd quadrant

$$\theta = 180^\circ - 80^\circ$$

$$\approx 100^\circ$$

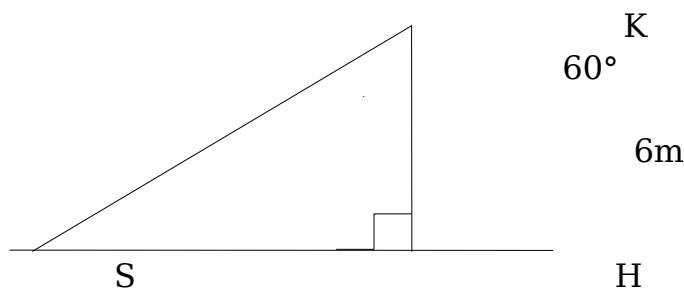
4th quadrant

$$\theta = 360^\circ - 80^\circ$$

$$\approx 280^\circ$$

$$\therefore \theta = 100^\circ \wedge \theta = 280^\circ$$

A kite (k) being flown is such that its vertical height HK is 6m and the angle formed between the vertical height and the string SK is attached to is 60° as shown in the diagram below.



If $\sin 60^\circ = 0.866$, $\cos 60^\circ = 0.5$ and $\tan 60^\circ = 1.73$, calculate the length of the string SK.

Solution

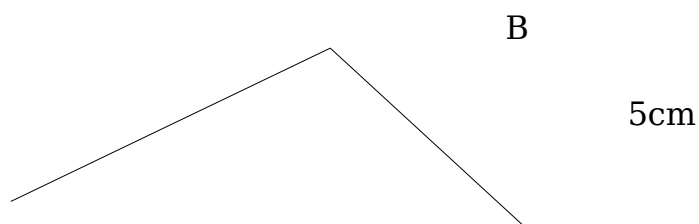
$$\cos\theta = \frac{A}{H}$$

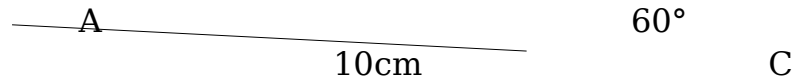
$$\cos 60^\circ = \frac{6}{SK}$$

$$6 = SK \cos 60^\circ$$

$$SK = \frac{6}{0.5} = 12 \text{ m.}$$

In the diagram below, $AC = 10\text{cm}$, $BC = 5\text{cm}$ and $\angle ACB = 60^\circ$. Given that $\sin 60^\circ = 0.866$, $\cos 60^\circ = 0.5 \wedge \tan 60^\circ = 1.73$. Calculate the value of $(AB)^2$.





Solution

$$(AB)^2 = (AC)^2 + (BC)^2 - 2 AB \times BC \times \cos C$$

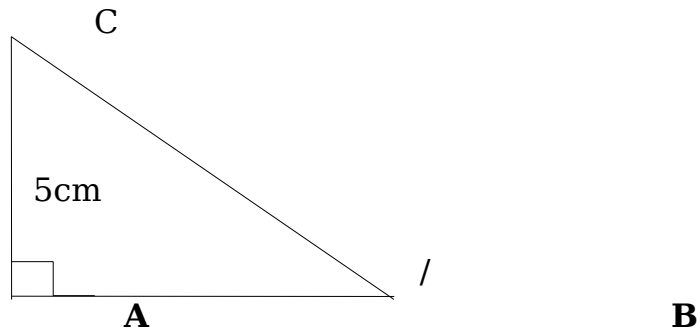
$$\hookrightarrow 10^2 + 5^2 - 2(10)(5)(0.5)$$

$$\hookrightarrow 100 + 25 - 50$$

$$(AB)^2 = 75 \text{ cm}$$

The figure below shows triangle ABC in which AC=5cm. Given that

$\sin B = 0.5$, $\cos B = 0.9$ $\wedge$ $\tan B = 0.6$. Calculate the length of BC;



Solution

$$\sin \theta = \frac{O}{H}$$

$$\sin B = \frac{AC}{BC}$$

$$0.5 = \frac{5}{BC}$$

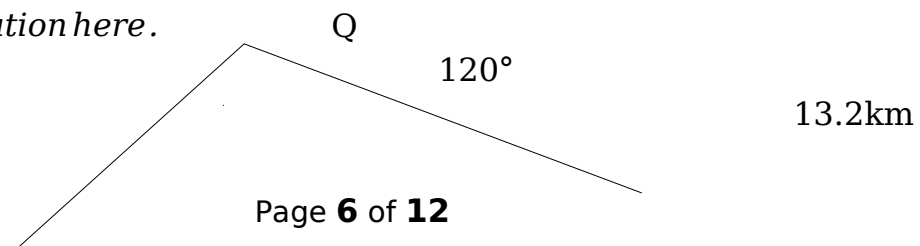
$$0.5 BC = 5$$

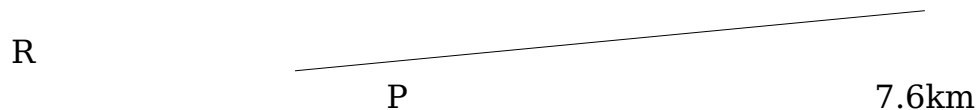
$$BC = 5 \div 0.5$$

$$\therefore BC = 10 \text{ cm}$$

2. PQ and R are fishing camps along the banks of lake kaliba joined by straight paths PQ, QR and RP. P is 7.6km from Q and Q is 13.2km from R and $\angle PQR = 120^\circ$.

Type equation here.





- a) Calculate;
- The distance PR
 - The area of triangle PQR
 - Find the shortest distance from Q to PR
- b) A fisherman takes 30 minutes to move from R to P. calculate his average speed in km/h.

Solution

a) (i) $(PR)^2 = (7.6)^2 + (13.2)^2 - 2 \times 7.6 \times 13.2 \cos 120^\circ$
 $= 232 - 2(-50.16)$
 $(PR)^2 = 332.32$
 $PR = 18.2 \text{ km (3 s.f.)}$

(ii) Area of triangle PQR $= \frac{1}{2} \times 7.6 \times 13.2 \sin 120^\circ$
 $= 43.4 \text{ km}^2$

(iii) in the formula, $A = \frac{1}{2}bh$ (h is a shortest distance)

$$A = 43.4 \text{ km}^2, b = PR = 18.2 \text{ km}$$

$$43.4 = \frac{1}{2} \times 18.2 \times h$$

$$h = \frac{43.4}{9.1}$$

$$h = 4.773 \text{ km}$$

$$h = 4.8 \text{ km}$$

$\therefore$ the shortest distance is 4.8 km

b) Average speed $= \frac{\text{Distance}}{\text{Time}}$ $D = 18.2 \text{ km}, T = \frac{30}{60} = 0.5 \text{ hrs}$
 $= \frac{18.2 \text{ km}}{0.5 \text{ hrs}}$

$$= 36.4 \text{ km/h}$$

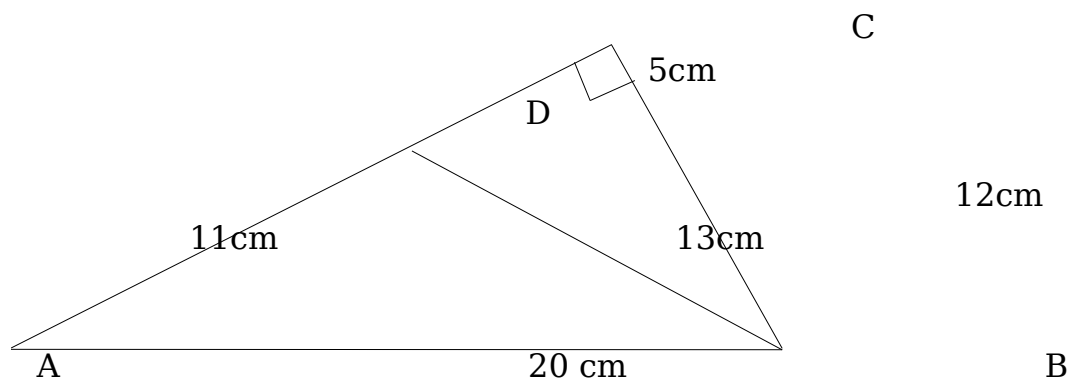
4) ABC is a triangle in which $\angle ACB = 90^\circ$, D is a point on AC, $AB = 20 \text{ cm}, BC = 12 \text{ cm}, CD = 5 \text{ cm}, BD = 13 \text{ cm}$ and $DA = 11 \text{ cm}$. Giving each answer as a fraction, find;

a) $\tan \angle CDB$

b) $\cos \angle CAB$

c) $\sin \angle ADB$

C



Solution

a) $\tan \angle CDB = \frac{BC}{DC}$

$$DC^2 = 13^2 - 12^2$$

$$= 169 - 144$$

$$DC^2 = 25$$

$$DC = 5 \text{ cm}$$

$$\therefore \tan \angle CDB = \frac{12}{5}$$

b) $\cos \angle CAB = \frac{16}{20}$

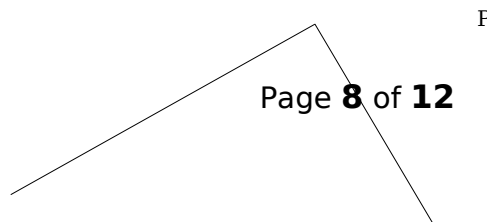
$$= \frac{4}{5}$$

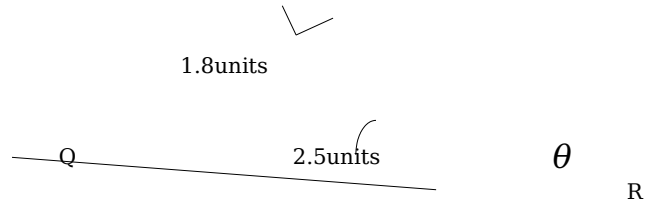
c) $\sin \angle ADB = ?$

$$\sin \angle BDC = \frac{12}{13}$$

$$\sin \angle ADB = \frac{-12}{13}$$

5) In the diagram below $\angle P = 1.8$ units, $QR = 2.5$ units, find the size of the angle marked θ .





Solution

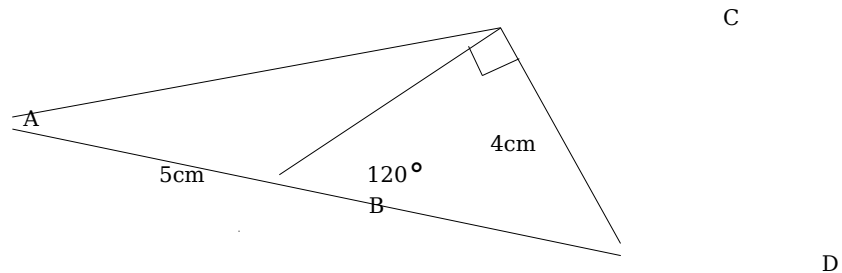
$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\sin \theta = \frac{1.8}{2.5}$$

$$\theta = \sin^{-1}\left(\frac{1.8}{2.5}\right)$$

$$\theta = 46^\circ$$

6)



ABC is a triangle with $AB=5\text{cm}$., $BC=4\text{cm}$ and angle $ABC=120^\circ$. AB is produced to D and angle $BCD=90^\circ$. Using as much information in the table below as necessary;

	sin	cos	tan
120°	0.87	-0.5	-1.73

Calculate,

- The area of the triangle ABC
- The length of BD

Solution

$$\text{a) Area of triangle } ABC = \frac{1}{2} \times 5 \times 4 \times \sin 120^\circ$$

$$= \frac{1}{2} \times 20 \times 0.87$$

$$= 8.7 \text{ cm}^2$$

$$\text{b) } BD = ? \quad \cos \angle CBD = \frac{BC}{BD}$$

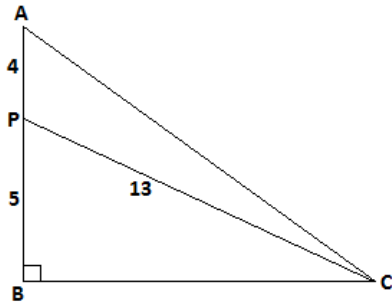
$$\cos 60^\circ = \frac{4}{BD}$$

$$0.5 = \frac{4}{BD}$$

$$\therefore BD = 8 \text{ cm.}$$

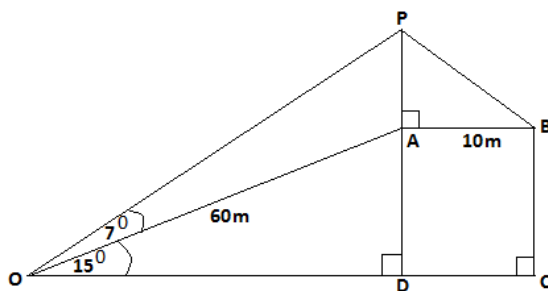
PRACTICE QUESTIONS:

- (1) In the right angled triangle ABC, P is a point on the side AB. Given that AP = 4 cm, PB = 5 cm, BC = 12 cm and PC = 13 cm, calculate



- (a) AC,
- (b) $\cos BPC$,
- (c) $\tan PAC$,
- (d) $\sin APC$.

- (2) ABCD represents a building with a vertical flagpole AP on the roof. The point O is on the same level as C and D.



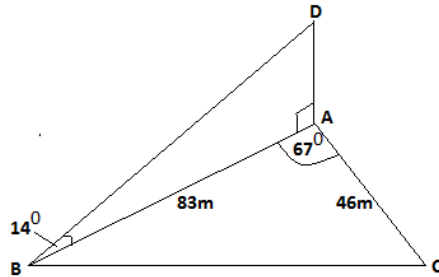
The angle of elevation of A from O is 15° , $OA = 60$ metres and $\angle POA = 7^\circ$

- (i) Calculate
 - (a) the height AD of the building,
 - (b) the height of the flagpole, AP.
- (ii) Given also that $AB = 10$ metres, calculate the angle of elevation of P from B.

(3) In the diagram, ABC represents a horizontal triangular field and AD represents

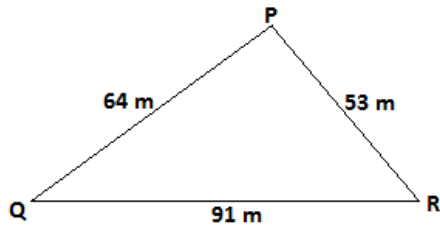
a vertical tree in the corner of the field. A path runs along the edge BC of the field.

AB = 83m, AC = 46m and angle BAC = 67° .



- The angle of elevation of the top of the tree when viewed from B is 14° . Calculate the height of the tree.
- Calculate the length of the path BC.
- Calculate the area of the field ABC.
- Calculate the shortest distance from A to the path BC.
- Calculate the greatest angle of elevation of the top of the tree when viewed from any point on the path.

(4) The diagram shows a triangular field PQR.



Calculate (a) angle QPR,

(b) the area of triangle PQR.

ANSWERS:

(1) (a) $AC = 15\text{cm}$, (b) $\cos \angle BPC = \frac{5}{13}$, (c) $\tan \angle PAC = \frac{4}{3}$,
 (d) $\sin \angle APC = \frac{12}{13}$.

(2) (i) (a) 15.53m (b) 7.89m
 (ii) 38.3° .

(3) (a) 20.7m , (b) 77.6m , (c) 1760m^2 , (d) 45.3m , (e) 24.6° .

(4) (a) 101.7° , (b) 1660.76m^2 .