



PHYSICS
INSTITUTE

PHYSICS INSTITUTE JAIPUR

TIFR-2016 PHYSICS
SOLVED PAPER

INSTITUTE FOR: NET/JRF, GATE, IIT-JAM, JEST, TIFR and GRE IN PHYSICS

GS-2016-X (PHYSICS)

Section A

Q1. If x is a continuous variable which is uniformly distributed over the real line from $x = 0$ to $x \rightarrow \infty$

according to the distribution $f(x) = \exp(-4x)$ then the expectation value of $\cos 4x$ is

- (a) zero (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{16}$

Ans. (b) x is any continuous variable which is uniformly distributed over the real line from 0 to ∞

$$F(x) = \exp(-4x)$$

Function

Firstly wave should be normalized

$$N^2 \int_0^{\infty} \exp(-4x) dx = 0$$

$$N^2 \int_0^{\infty} e^{-4x} dx = 0$$

$$N^2 \times \frac{1}{1} \sqrt{\frac{0+1}{4}} \quad \boxed{N=2}$$

They $f(x) = 2\exp(-4x)$

Then The expectation value-

$$\langle \cos 4x \rangle = 4 \int \exp(-4x) \cos 4x dx$$

$$4 \int_0^{\infty} e^{-4x} \frac{(e^{i4x} + e^{-i4x})}{2} dx$$

$$4 \int_0^{\infty} \frac{e^{4x(i-1)} + e^{-4x(i-1)}}{2} dx$$

$$2 \int_0^{\infty} e^{4x(i-1)} dx + 2 \int_0^{\infty} e^{4x(i+1)} dx$$



$$2 \left[\frac{\frac{1}{1} \sqrt{\frac{0+1}{1}}}{(1-i)} \right] + 2 \left[\frac{\frac{1}{1} \sqrt{\frac{0+1}{1}}}{(1-i)} \right]$$

$$= 2 \left[\frac{i+1+1-i}{4(i+1)} \right] \quad \frac{2}{4 \times 2} = \frac{1}{2}$$

2nd Method

$$\int_{x_1}^{x_2} e^{ax} \cos bx \cdot dx = \left[\frac{e^{ax}}{a^2 + b^2} (b \sin bx + a \cos bx) \right]_{x_1}^{x_2}$$

$$\int_{x_1}^{x_2} e^{-4x} \exp 4x \cdot dx = \left[\frac{e^{-4x}}{(4)^2 + (4)^2} (4 \sin 4x + (-4) \cos 4x) \right]_0^\infty$$

$$= e^{-\infty} - \frac{1}{32} [4 \sin 0 - 4 \cos 0]$$

$$= 0 + \frac{1}{32} \times 4 = \frac{1}{8}$$

Q2. If the eigenvalues of a symmetric 3x3 matrix A are 0,1,3 and the corresponding eigenvectors can be written as

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

respectively, then the matrix A^4 is

(a) $\begin{pmatrix} 41 & -81 & 40 \\ -81 & 0 & -81 \\ 40 & -81 & 41 \end{pmatrix}$

(b) $\begin{pmatrix} -82 & -81 & 79 \\ -81 & 81 & -81 \\ 79 & -81 & 83 \end{pmatrix}$

(c) $\begin{pmatrix} 14 & -27 & 13 \\ -27 & 54 & -27 \\ 13 & -27 & 14 \end{pmatrix}$

(d) $\begin{pmatrix} 14 & -13 & 27 \\ -13 & 54 & -13 \\ 27 & -13 & 14 \end{pmatrix}$

Ans. (c)

$$\lambda = 0$$

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$



$$\lambda_2 = 1 \quad x_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\lambda_3 = 3 \quad x_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Short method

1st Method

$$\text{Tr}(A) = \lambda_1 + \lambda_2 + \lambda_3 = 0 + 1 + 3 = 4$$

$$\text{Tr}(A^4) = \lambda_1^4 + \lambda_2^4 + \lambda_3^4 = 0 + 1 + 81 = 82$$

$$\text{Del}(A) = \lambda_1 \lambda_2 \lambda_3 = 0$$

$$\text{Del}(A^4) = 0$$

Option

$$(a) A^4 = \begin{bmatrix} 41 & -81 & 40 \\ -81 & 0 & -81 \\ 40 & -81 & 41 \end{bmatrix}$$

$$\text{Tr}(A^4) = 41 + 0 + 41 = 82$$

$$\text{Del}(A^4) = 41(0 + 8^2) + 81(-81 \times 41 + 81 \times 40) + 40(81^2)$$

$$81(41 \times 81) + 81^2(-41 + 40) + 81^2 \times 40$$

$$(81)^2[41 - 1 + 40] = 80 \times (81)^2 \neq 0 \quad (\text{wrong option})$$

$$(b) A^4 = \begin{bmatrix} -82 & -81 & 79 \\ -81 & 81 & -81 \\ 79 & -81 & 83 \end{bmatrix}$$

$$\text{Tr}(A^4) = -82 + 81 + 83 = 82$$

$$\text{Det}(A^4) = -82[81 \times 83 - 81 \times 81] + 81[-81 \times 83 + 81 \times 79] - 79(81^2 - 81 \times 79)$$

$$= -81 \times 82[83 - 81] + 81 \times 81[-83 + 79] - 79 \times 81(81 - 79)$$

$$= -81 \times 82 \times 2 + 81 \times 81 \times -4 - 79 \times 81 \times 2$$

$$= 81 \times 2[-82 + 81 \times -2 - 79] \neq 0$$

(wrong option)

$$(c) A^4 = \begin{bmatrix} 14 & -27 & 13 \\ -27 & 54 & -27 \\ 13 & -27 & 14 \end{bmatrix}$$

$$Tr(A^4) = 14 + 54 + 14 = 82$$

$$Det(A^4) = 14(54 \times 14 - 27 \times 27) + 27(-27 \times 14 + 27 \times 13) - 13(27 \times 27 - 54 \times 13)$$

$$= 14 \times 27(28 - 27) + 27 \times 27(-14 + 13) - 13 \times 27(27 - 28)$$

$$27(14 + 13 - 27) = 0$$

(correct)

$$(d) A^4 = \begin{bmatrix} 14 & -13 & 27 \\ -13 & 54 & -13 \\ 27 & -13 & 14 \end{bmatrix}$$

$$Tr(A^4) = 14 + 54 + 14 = 82$$

$$Det(A^4) = 14(54 \times 14 + 13 \times 13) + 13(-13 \times 14 + 13 \times 27) - 27(13^2 - 27 \times 54) \neq 0$$

(wrong option)

2nd Method

We know that

$$|A - \lambda I| x = 0$$

$$Ax = \lambda x$$

$\therefore A = \text{symmetric matrices}$

$$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$$

$$x_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \lambda_1 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\lambda_1 = 0$$

For $\lambda_1 = 0$

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$a+b+c=0 \dots\dots\dots(1)$$

$$b+d+e=0 \dots\dots\dots(2)$$

$$c+e+f=0 \dots\dots\dots(3)$$

And for $\lambda_2 = 1$,

$$x_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$a-c=1 \dots\dots\dots(4)$$

$$b-e=0 \Rightarrow 0 \dots\dots(5)$$

$$c-f=-1 \dots\dots\dots(6)$$

For $\lambda_3 = 3$

$$x_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$a-2b+c=3 \dots\dots\dots(7)$$

$$b-2d+e=-6 \dots\dots\dots(8)$$

$$c-2e+f=3 \dots\dots\dots(9)$$

NOW

Solving eq^o

From eqⁿ $b=e$

Them From eqⁿ(2)

$$b+d+e=0$$

$$2e+d=0 \dots\dots\dots(A)$$

From ϵq^n (8)

$$b - 2d + e = -6$$

$$2e - 2d = -6$$

$$2e - d = -3 \text{(B)}$$

From (A) & (B)

$$2e + d = 0$$

$$e - d = -3$$

$$3e = -3$$

$$\boxed{e = -1} \Rightarrow \boxed{b = e = -1}$$

From eq^n (7)

$$a - 2b + c = 3$$

$$a - 2(-1) + c = 3$$

$$a + c = 1 \text{(c)}$$

From eq^n & From ϵq^n (D)

$$a + c = 1$$

$$a - c = 1$$

$$2a = 2 \Rightarrow \boxed{a = 1}$$

From eq^n (c)

$$a + c = 1$$

$$\boxed{c = 0}$$

From eq^n (6)

$$c - f = -1$$

$$\boxed{f = 1}$$

Then symmetrix matrices

$$A = \begin{bmatrix} a & b & c \\ b & d & c \\ c & e & f \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

From clay Hamilton Theorem

$$A^4 = \alpha_{m-1} A^{m-1} + \alpha_{m-2} A^{m-2} + \dots \alpha_{m-m} A^{m-m}$$

Then $m = 3$

$$A^4 = \alpha_2 A^2 + \alpha_1 A + \alpha_0 I \text{(a)}$$

It fellow its own characteristic

$$\lambda^4 = \alpha_2 \lambda^2 + \alpha_1 \lambda + \alpha_0 \dots\dots\dots(b)$$

$$A^+ \boxed{\lambda=0} \Rightarrow \boxed{0 = \alpha_0}$$

(ii) $A^+ \boxed{\lambda=1}$

$$1 = \alpha_2 + \alpha_1 + \alpha_0$$

$$\alpha_2 + \alpha_1 = 1 \dots\dots\dots(c)$$

(iii) $A^+ \boxed{\lambda=3}$

$$81 = 9\alpha_2 + 3\alpha_1 \Rightarrow \alpha_1 + 3\alpha_2 = 27 \dots\dots\dots(d)$$

From eqⁿ (c) & (d)

$$27 = 3\alpha_2 + \alpha_1$$

$$1 = \alpha_2 + \alpha_1$$

$$\alpha_2 = 13$$

From (c) $\boxed{\alpha_1 = -12}$

Then From Eqⁿ (a)

$$A^4 = 13A^2 - 12A + 0$$

$$\boxed{A^4 = 13A^2 - 12A}$$

$$A^2 = \begin{bmatrix} 1 & -1 & 0 \\ -1 & -2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} +1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

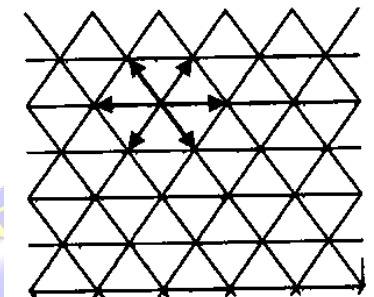
$$A^2 = \begin{bmatrix} 1+1+0 & -1-2+0 & 0+1+0 \\ -1-2+0 & 1+4+1 & 0-2-1 \\ 0+1+0 & 0-2-1 & 0+1+1 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 \\ -3 & 6 & -3 \\ 1 & -3 & 2 \end{bmatrix}$$

$$A^4 = 13 \begin{bmatrix} 2 & -3 & 1 \\ -3 & 6 & -3 \\ 1 & 6 & -3 \end{bmatrix} - 12 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 26-12 & -39+12 & 13-0 \\ -39+12 & 78-24 & 39+12 \\ 13-0 & -39+12 & 26-12 \end{bmatrix}$$

$$\boxed{A^4 = \begin{bmatrix} 14 & -27 & 13 \\ -27 & 54 & -27 \\ 13 & -27 & 14 \end{bmatrix}}$$

- Q3. In a triangular lattice a particle moves from a lattice point to any of its 6 neighbouring points with equal probability, as shown in the figure on the right. The probability that the particle is back at its starting point after 3 moves is



(a) $\frac{5}{18}$

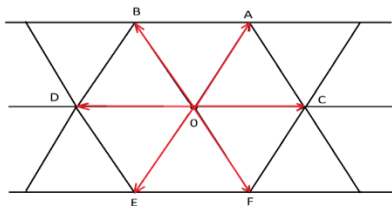
(b) $\frac{1}{6}$

(c) $\frac{1}{18}$

(d) $\frac{1}{36}$

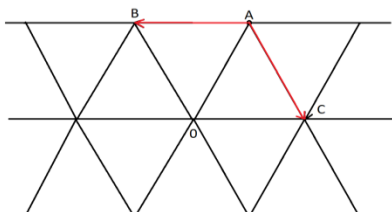
Ans. (C)

1ST - STEP



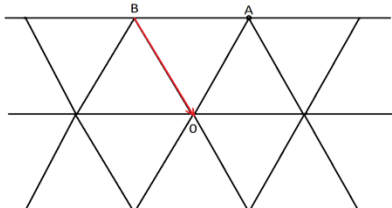
First step probability = $\frac{1}{6}$

2ND - STEP



second step probability = $\frac{2}{6}$

3RD - STEP

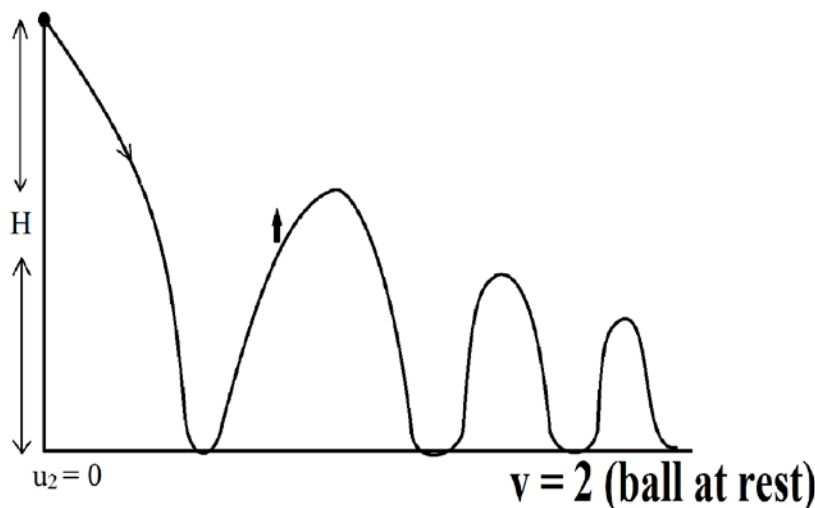


third step probability = $\frac{6}{6}$

$$\text{Then Probability} = \frac{1}{6} \times \frac{2}{6} \times \frac{6}{6} = \frac{2}{36} = \frac{1}{18}$$

- Q4.** A ball is dropped vertically from a height H on to a plane surface and permitted to bounce repeatedly along a vertical line. After every bounce, its kinetic energy becomes a quarter of its kinetic energy before the bounce. The ball will come to rest after time
- (a) infinity (b) $(2H/g)^{1/2}$ (c) $2(2H/g)^{1/2}$ (d) $3(2H/g)^{1/2}$

Ans. (d)



According to question

$$k_f = \frac{1}{4} k_i \Rightarrow \frac{1}{2} m v_1^2 = \frac{1}{4} \left(\frac{1}{2} m u_1^2 \right)$$

$$v_1^2 = \frac{1}{4} u_1^2$$

u_1 & v_1 are opposite

$$\text{so } u_1 = -2v_1$$

$$u_1 = \pm 2v_1$$

Elasticity coefficient of restitution

$$e = \frac{-(v_2 - v_1)}{(u_2 - u_1)} \Rightarrow e = \frac{-(0 - v_1)}{(0 - u_1)} = -\frac{v_1}{u_1} = \frac{-v_1}{-2v_1}$$

$$e = \frac{1}{2}$$

And we know that

$$t = \sqrt{\frac{2H}{g}} \left(\frac{1+e}{1-e} \right)$$

$$t = \sqrt{\frac{2H}{g}} \left(\frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} \right)$$

$$t = \sqrt{\frac{2H}{g}} \left[\frac{3/2}{1/2} \right]$$

$$t = 3\sqrt{\frac{2H}{g}}$$

- Q5. In a moving car, the wheels will skid if the brakes are applied too suddenly. This is because
- (a) the inertia of the car will carry it forward.
 - (b) the momentum of the car must be conserved.
 - (c) the impulsive retarding force exceeds the limiting force of static friction.
 - (d) the kinetic friction will suddenly get converted to static friction.

Ans. (a)

Wheel were skid that is $v < r\omega$
That is translation $K.E = 0$

And Rotational $K.E = \frac{1}{2} I \omega^2$

Option

The inertia of the car will carry it forward

(here will only Rotational K.E) → so only inertia will be responsible for it

So Inertia is responsible for motion

The Momentum of the car must be conserved In Rotational motion Linear Momentum not conserved (Inequality hold) angular Momentum → conserved

The Impulsive retarding force exceed the limiting force of static friction.

Static friction → (Due to static motion)

The kinetic friction will suddenly get converted to static friction.

- Q6. An aircraft, which weighs 12000 kg when unloaded, is on a relief mission, carrying 4000 food packets weighing 1kg each. The plane is gliding horizontally with its engines off at a uniform speed of 540kmph when the first food packet is dropped. Assume that the horizontal air drag can be neglected and the aircraft keeps moving horizontally. If one food packet is dropped every second, then the distance between the last two packet drops will be
- (a) 1.5km
 - (b) 200m
 - (c) 150m
 - (d) 100m

Ans. (c)

$$N_1 = 12000 \text{ kg (when unloaded)}$$

$$4000 \text{ kg food packet} = 1 \text{ kg}$$

Total weight

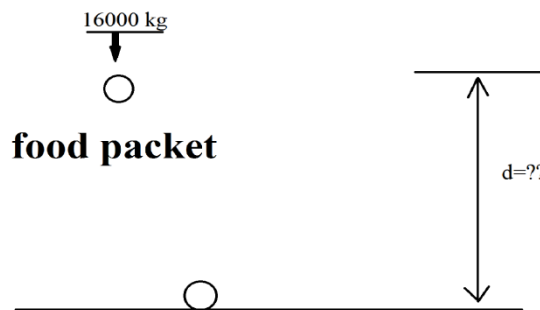
$$12000 + 4000 = 16000 \text{ kg}$$

$$v = 540 \frac{\text{km}}{\text{hr}} = 540 \times \frac{5}{18} \frac{\text{m}}{\text{sec}} = 150 \frac{\text{m}}{\text{sec}}$$

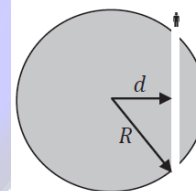
Distance = speed x time

$$= 150 \frac{\text{m}}{\text{sec}} \times \text{sec}$$

$$d = 150 M$$



- Q7. Imagine that a narrow tunnel is excavated through the Earth as shown in the diagram on the left and that the mass excavated to create the tunnel is extremely small compared to Earth's mass M . A person falls into the tunnel at one end. at time $t = 0$. Assuming that the tunnel is frictionless, the person will



- (a) fall straight through, escaping Earth's gravity at time $2\pi\sqrt{R^2 / GM}$
- (b) describe simple harmonic motion with period $2\pi(d / R)\sqrt{R^3 / GM}$
- (c) describe simple harmonic motion with period $2\pi\sqrt{(R-d)^3 / GM}$
- (d) describe simple harmonic motion with period $2\pi\sqrt{R^3 / GM}$

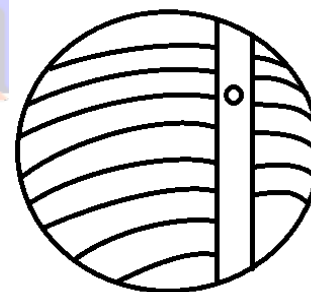
Ans. (d)

Describe simple Harmonic motion with period $2\pi\sqrt{\frac{R^3}{GM}}$

$$g = \frac{-Gmr}{R_e^3}$$

$$\frac{d^2 r}{dt^2} = \frac{-Gmr}{R_e^3}$$

$$\frac{d^2 r}{dt^2} + \frac{Gmr}{R_e^3} = 0$$



Compare

$$\frac{d^2 r}{dt^2} + \omega^2 r = 0$$

$$\omega = \sqrt{\frac{Gm}{R_e^3}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{Gm}{R_e^3}}$$

$$T = 2\pi \sqrt{\frac{R_e^3}{Gm}}$$

- Q8. A grounded conducting sphere of radius a is placed with its centre at the origin. A point dipole of dipole moment $\vec{p} = p\hat{k}$ is placed at a distance d along the x -axis, where $\hat{i}, \hat{k}$ are the units vector along the x and z -axes respectively. This leads to the formation of an image dipole of strength $\vec{p}'$ at a distance d' from the centre along the x -axis. If $d' = a^2/d$, then $\vec{p}' =$

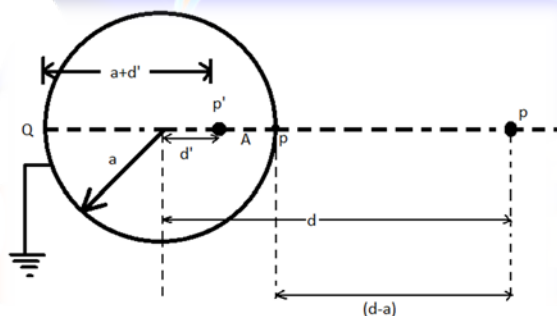
(a) $\frac{a^4 p}{d^4} \hat{k}$

(b) $-\frac{a^3 p}{d^3} \hat{k}$

(c) $-\frac{a^2 p}{d^2} \hat{k}$

(d) $-\frac{ap}{d} \hat{k}$

Ans. (b)



$\vec{p} = p\hat{k}$
Sphere $\rightarrow$
grounded conducting $\Rightarrow$
Potential on the sphere
surface = zero

At point P

$$V_p = 0$$

$$\therefore v = \frac{k \cdot \vec{p} \cdot \hat{r}}{r^2} \text{ (due to dipole)}$$

$$\Rightarrow V_p = \frac{kpz}{(d-a)^3} + \frac{kp'z}{(a-d')^3} = 0$$

$$\frac{kpz}{(d-a)^3} = -\frac{kp'z}{(a-d')^3}$$

$$\Rightarrow p' = \frac{(a-d')^3}{(d-a)^3} p \dots\dots\dots(1)$$

At point Q

$$v_Q = 0 = \frac{kpz}{(d+a)^3} + \frac{kp'z}{(a+d')^3} = 0 \dots\dots\dots(2)$$

Given $d' = a^2 / d$

$$p' = \frac{(a - a^2/d)^3}{(d-a)^3} p$$

$$p' = \frac{(ad - a^2)^3}{(d-a)^3} p$$

$$p' = \frac{a^3(1-a/d)^3}{-d^3(1-a/d)^3} p$$

$$p' = -\frac{a^3}{d^3} p \Rightarrow \vec{p}' = -\frac{a^3}{d^3} p \hat{k}$$

2nd Method

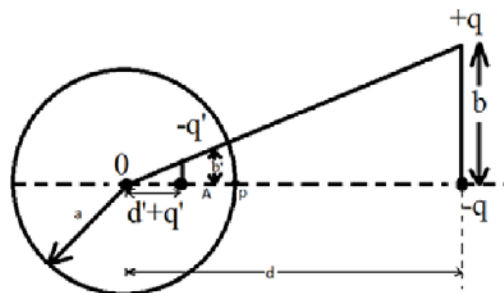
$$v = \frac{k\vec{p} \cdot \vec{r}}{r^3} = \frac{k\hat{p}z(x\hat{i} + y\hat{j} + z\hat{k})}{r^3}$$

$$v = \frac{kpz}{r^3}$$

$$\frac{kpz}{d^3} + \frac{kp'z}{a^3} = 0 \Rightarrow \frac{kpz}{d^3} = -\frac{kp'z}{a^3}$$

$$p' = -\frac{a^3}{d^3} p \quad p' = -\frac{a^3}{d^3} p \hat{k}$$

3rd Method



From similar Triangle

$$\frac{d'}{d} = \frac{b'}{b} \Rightarrow b' = \left(\frac{d'}{d}\right)b$$

$\therefore$ conductor

And for conductor (Method of image)

$$q'd = -qa \Rightarrow q' = -qa/d$$

$$p' = q'b'$$

$$p' = \frac{-qa}{d} b'$$

$$p' = \frac{-qa}{d} \left(\frac{d'}{d}\right)b$$

$$p' = \frac{-qa}{d^2} \left(\frac{a^2}{d}\right)b$$

$$p' = \frac{-qba^3}{d^3}$$

$$\therefore qb = p$$

$$p' = -\frac{pa^3}{d^3} \Rightarrow \vec{p}' = -\frac{pa^3}{d^3} \hat{k}$$

- Q9. A long, solid dielectric cylinder of radius a is permanently polarized so that the polarization is everywhere radially outward, with a magnitude proportional to the distance from the axis of the cylinder, i.e., $\vec{p} = \frac{1}{2} p_0 r \hat{r}$. The bound charge density in the cylinder is given by

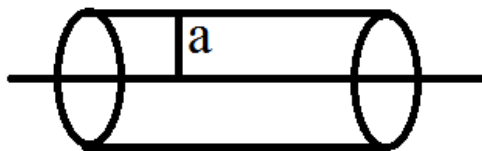
(a) $-p_0$

(b) p_0

(c) $-\frac{p_0}{2}$

(d) $\frac{p_0}{2}$

Ans. (a)



$$\vec{p} = \frac{1}{2} p_0 r \hat{r}$$

Bound charge density

$$\rho_b = -(\vec{\nabla} \cdot \vec{p})$$

$$\rho_b = -\frac{1}{r} \left[\frac{\partial}{\partial r} \frac{1}{2} p_0 r^2 \right]$$

$$\rho_b = -\frac{1}{2r} p_0 (2r)$$

$$\boxed{\rho_b = -p_0}$$

Q10. A circular loop of fine wire of radius R carrying a current I is placed in a uniform magnetic field B perpendicular to the plane of the loop. If the breaking tension of the wire is T_b , the wire will break when the magnetic field exceeds

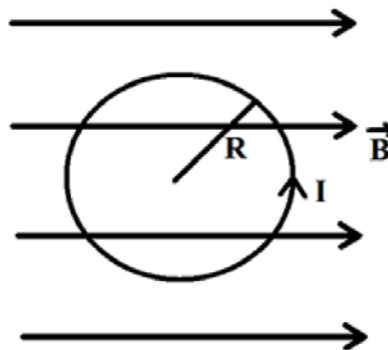
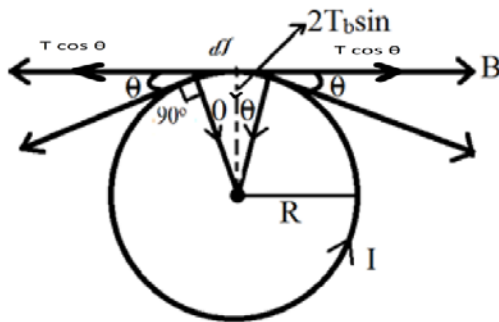
(a) $\frac{T_b}{IR}$

(b) $\frac{T_b}{2\pi IR}$

(c) $\frac{\mu_0 T_b}{2\pi IR}$

(d) $\frac{\mu_0 T_b}{4\pi IR}$

Ans. (a)



$\therefore \text{Angle} = \text{Arc}/\text{Radius}$

Arc = angle $\times$ Radius

$$dl = R (2\theta)$$

$$\therefore F = IlB$$

$$2T_b \sin \theta = IdlB$$

$$B = \frac{2T_b \sin \theta}{Idl}$$

$\theta \approx \text{small}$

$$\therefore \sin \theta \approx \theta$$



$$B = \frac{2T_b \sin \theta}{IR(2\theta)}$$

$$\boxed{B = \frac{2T_b}{2IR}} \Rightarrow \boxed{B = \frac{T_b}{IR}}$$

- Q11. A gas of non-interacting particles, each of rest mass 1MeV , is at a temperature $T = 2.0 \times 10^7 \text{ K}$ and has an average particle density $n = 2.7 \times 10^{34} \text{ cm}^{-3}$. We can obtain a reasonably correct treatment of this system
- (a) only by using special relativity as well as quantum mechanics.
 - (b) by neglecting quantum mechanics but not special theory of relativity.
 - (c) by neglecting special relativity but not quantum mechanics.
 - (d) by neglecting both special relativity and quantum mechanics.

Ans. (c)

If $\lambda > R$

Then Quantum mechanics valid

$$\lambda = \frac{h}{\sqrt{3mk_T}}$$

Given $m = \frac{1\text{MeV}}{C^2} = \frac{10^6 \times 1.6 \times 10^{-19}}{9 \times 10^{16}}$

$$T = 2 \times 10^7 \text{ K}$$

$$\lambda = \frac{6.62 \times 10^{-34} \times 3}{\sqrt{1.6 \times 10^{-35} \times 10^6 \times 2 \times 10^7 \times 1.38 \times 10^{-23}}}$$

$$\lambda = \frac{6.62 \times 10^{-34} \times 3}{\sqrt{1.6 \times 2 \times 1.38 \times 10^{-45} \times 3}}$$

$$\lambda = \frac{3 \times 6.62 \times 10^{-34} \times 10^{+23}}{\sqrt{1.6 \times 2 \times 1.38 \times 10 \times 3}} = \frac{2.98 \times 10^{-11}}{\sqrt{3}} = 0$$

$$= 1.72 \times 10^{-11}$$

$$= 0.172 \text{ \AA}$$

$$\therefore v = NR^3$$

$$R^3 = \left(\frac{V}{N} \right) \Rightarrow R = \left(\frac{V}{N} \right)^{1/3}$$

$$\therefore \text{density } \eta = 2.7 \times 10^{-34} \text{ cm}^{-3} = \frac{2.7 \times 10^{+34}}{\text{cm}^3}$$

$$R^3 = \left(\frac{V}{N} \right) = \left(\frac{1}{n} \right)$$

$$\therefore 100\text{cm} \rightarrow 1\text{m}$$

$$1\text{cm} = 10^{-2}\text{m}$$

$$\eta = \frac{2.7 \times 10^{+34} \times 10^6}{M^3}$$

$$R^3 = \left(\frac{1}{2.7 \times 10^{40}} \right)$$

$$R = \left(\frac{1}{27 \times 10^{39}} \right)^{1/3}$$

$$R = \frac{1}{3} \times 10^{-13}$$

$$R = 0.33 \times 10^{-13} < \lambda$$

$\therefore \lambda > R$ so use quantum mechanics

Now

S.T.R

If given $KE \approx m_0 c^2$

Then use S.T.R otherwise neglect

$$E = KE + m_0 c^2$$

$$KE = E - m_0 c^2$$

$$KE = \frac{hc}{\lambda} - m_0 c^2$$

$$KE = \frac{19.86 \times 10^{-26}}{0.172 \times 10^{-10}} - 1\text{mev}$$

$$KE = 72\text{kev} - 1\text{mev}$$

$$K.E = -ive \neq m_0 c^2$$

So neglect S.T.R and use quantum

Q12. A one-dimensional harmonic oscillator of mass m and natural frequency ω is in the quantum state $|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \frac{i}{\sqrt{3}}|1\rangle + \frac{i}{\sqrt{3}}|2\rangle$ at time $t = 0$, where n denotes the

eigenstate with eigenvalue $\left(n + \frac{1}{2}\right)\hbar\omega$. It follows that the expectation value (x) of the position operator $\hat{x}$ is

(a) $x(0) \left[\cos \omega t + \frac{1}{\sqrt{3}} \sin \omega t \right]$

(b) $x(0) [\cos \omega t - \sin \omega t]$



(c) $x(0) \left[\cos \omega t - \frac{1}{2} \sin \omega t \right]$

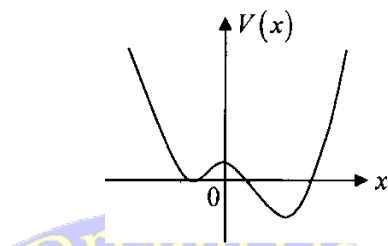
(d) $x(0) \left[\cos \omega t + \frac{1}{\sqrt{2}} \sin \omega t \right]$

Ans. (d)

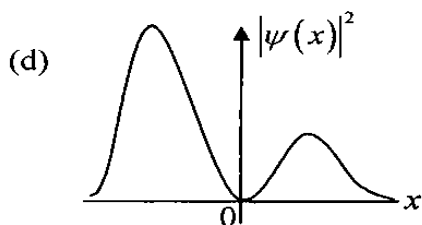
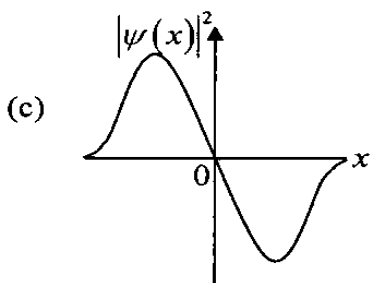
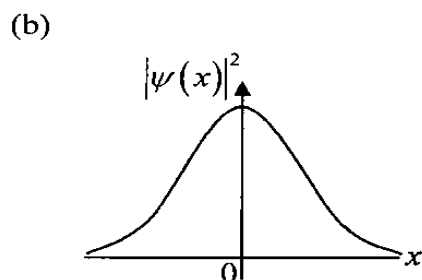
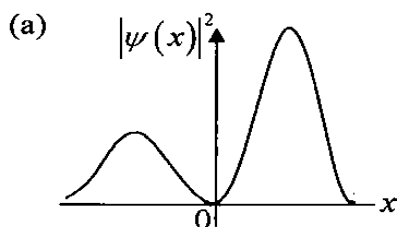
$$\begin{aligned}
 & \boxed{|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \frac{i}{\sqrt{3}}|1\rangle + \frac{i}{\sqrt{3}}|2\rangle} \\
 & \langle\psi|x|\psi\rangle_{at=0} = \left[\frac{1}{\sqrt{3}}\langle 0| - \frac{i}{\sqrt{3}}\langle 1| - \frac{i}{\sqrt{3}}\langle 2| \right] x \left[\frac{1}{\sqrt{3}}|0\rangle + \frac{i}{\sqrt{3}}|1\rangle + \frac{i}{\sqrt{3}}|2\rangle \right] \\
 & = \langle x \rangle_{t=0} = \left[\frac{1}{\sqrt{3}}\langle 0| - \frac{i}{\sqrt{3}}\langle 1| - \frac{i}{\sqrt{3}}\langle 2| \right] \left[\frac{1}{\sqrt{3}}x|0\rangle + \frac{i}{\sqrt{3}}x|1\rangle + \frac{i}{\sqrt{3}}x|2\rangle \right] \\
 & = \sqrt{\frac{\hbar}{2M\omega}} \left[\frac{1}{\sqrt{3}}\langle 0| - \frac{i}{\sqrt{3}}\langle 1| - \frac{i}{\sqrt{3}}\langle 2| \right] \left[\frac{1}{\sqrt{3}}(a^+ + a)|0\rangle + \frac{i}{\sqrt{3}}(a^+ + a)|1\rangle + \frac{i}{\sqrt{3}}(\hat{a}^+ + a)|2\rangle \right] \\
 & = \sqrt{\frac{\hbar}{2M\omega}} \left[\frac{1}{\sqrt{3}}\langle 0| - \frac{i}{\sqrt{3}}\langle 1| - \frac{i}{\sqrt{3}}\langle 2| \right] \left[\frac{1}{\sqrt{3}}(1)|1\rangle + 0 + \frac{i}{\sqrt{3}}\sqrt{2}|2\rangle + \frac{i}{\sqrt{3}}|a\rangle + \frac{i}{\sqrt{3}}\sqrt{3}|3\rangle + \frac{i}{\sqrt{3}}\sqrt{2}|2\rangle \right] \\
 & = \sqrt{\frac{\hbar}{2M\omega}} \left[\frac{1}{\sqrt{3}}\langle 0| - \frac{i}{\sqrt{3}}\langle 1| - \frac{i}{\sqrt{3}}\langle 2| \right] \left[\frac{1}{\sqrt{3}}|1\rangle + \sqrt{\frac{2}{3}}i|2\rangle + \frac{i}{\sqrt{3}}|1\rangle + \frac{i}{\sqrt{3}}\sqrt{2}|2\rangle + i|3\rangle \right] \\
 & = \sqrt{\frac{\hbar}{2M\omega}} \left[\frac{-i}{\sqrt{3}} + \frac{i}{\sqrt{3}} - \frac{\sqrt{2}}{3}i^2 - \frac{i^2\sqrt{2}}{3} \right] = \sqrt{\frac{\hbar}{2M\omega}} \left[\frac{+\sqrt{2}}{3} + \frac{\sqrt{2}}{3} \right] \\
 & \langle x \rangle_{t=0} = \sqrt{\frac{\hbar}{2m\omega}} \left[\frac{2\sqrt{2}}{3} \right] = \boxed{\frac{2}{3} \sqrt{\frac{\hbar}{2m\omega}}} = \langle x \rangle_{t=0} \\
 & \langle x \rangle_t = \langle \psi^*(x,t) | \hat{x} | \psi(x,t) \rangle \\
 & \langle x \rangle_t = \left[\frac{e^{-iE_0t/\hbar}}{\sqrt{3}}\langle 0| + \frac{i}{\sqrt{3}}e^{-iE_1t/\hbar}\langle 1| + \frac{i}{\sqrt{3}}e^{-iE_2t/\hbar}\langle 2| \right] \\
 & \langle x \rangle_t = \left[\frac{e^{iE_0t/\hbar}}{\sqrt{3}}\langle 0| - \frac{i}{\sqrt{3}}e^{iE_1t/\hbar}\langle 1| - \frac{i}{\sqrt{3}}e^{iE_2t/\hbar}\langle 2| \right] x \left[\frac{e^{-iE_0t/\hbar}}{\sqrt{3}}|0\rangle + \frac{i}{\sqrt{3}}e^{-iE_1t/\hbar}|1\rangle + \frac{i}{\sqrt{3}}e^{-iE_2t/\hbar}|2\rangle \right]
 \end{aligned}$$

$$\begin{aligned}
 \langle x \rangle_t &= \sqrt{\frac{\hbar}{2m\omega}} \left[\frac{e^{iE_0 t/\hbar}}{\sqrt{3}} \langle 0 | - \frac{i}{\sqrt{3}} e^{+iE_1 t/\hbar} \langle 1 | - \frac{i}{\sqrt{3}} e^{+iE_2 t/\hbar} \right] (a^+ + a) \left[\frac{e^{-iE_0 t/\hbar}}{\sqrt{3}} |0\rangle + \frac{i}{\sqrt{3}} e^{-iE_1 t/\hbar} |1\rangle + \frac{i}{\sqrt{3}} e^{-iE_2 t/\hbar} |2\rangle \right] \\
 \langle x \rangle_t &= \sqrt{\frac{\hbar}{2m\omega}} \left[\frac{e^{+iE_0 t/\hbar}}{\sqrt{3}} \langle 0 | - \frac{i}{\sqrt{3}} e^{+iE_1 t/\hbar} \langle 1 | - \frac{i}{\sqrt{3}} e^{+iE_2 t/\hbar} \langle 2 | \right. \\
 &\quad \left. \left[\frac{e^{-iE_0 t/\hbar}}{\sqrt{3}} |1\rangle + 0 + \frac{i}{\sqrt{3}} e^{-iE_1 t/\hbar} \sqrt{2} + \frac{i}{\sqrt{3}} e^{-iE_2 t/\hbar} |0\rangle + \frac{i}{\sqrt{3}} e^{-iE_2 t/\hbar} \sqrt{3} |3\rangle + \frac{i}{\sqrt{3}} e^{-iE_2 t/\hbar} \sqrt{2} |1\rangle \right] \right] \\
 \langle x \rangle_t &= \sqrt{\frac{\hbar}{2m\omega}} \left[\frac{ie^{i(E_0-E_1)t/\hbar}}{3} - \frac{ie^{i(E_1-E_0)t/\hbar}}{3} - \frac{i^2 \sqrt{2} e^{i(E_2-E_1)t/\hbar}}{3} - \frac{i^2 \sqrt{2} e^{i(E_1-E_2)t/\hbar}}{3} \right] \\
 \langle x \rangle_t &= \sqrt{\frac{\hbar}{2m\omega}} \left[\frac{ie^{-i\omega t/\hbar}}{3} - \frac{i}{3} e^{\frac{\hbar\omega t}{\hbar}} + \sqrt{2} e^{\frac{i\hbar\omega t}{\hbar}} + \frac{\sqrt{2} e^{-\frac{i\hbar\omega t}{\hbar}}}{3} \right] \\
 \langle x \rangle_t &= \sqrt{\frac{\hbar}{2m\omega}} \frac{1}{3} \left[ie^{-i\omega t} - ie^{i\omega t} + \sqrt{2} e^{i\omega t} + \sqrt{2} e^{-i\omega t} \right] \\
 &= \sqrt{\frac{\hbar}{2m\omega}} \frac{1}{3} \left[i(\cos \omega t - i \sin \omega t) - i(\cos \omega t + i \sin \omega t) + \sqrt{2}(\cos \omega t - i \sin \omega t) + \sqrt{2}i(\cos \omega t + i \sin \omega t) \right] \\
 &= \sqrt{\frac{\hbar}{2m\omega}} \frac{1}{3} \left[i \cos \omega t - i^2 \sin \omega t - i^2 \sin \omega t + \sqrt{2} \cos \omega t + \sqrt{2}i \sin \omega t + \sqrt{2} \cos \omega t - \sqrt{2}i \sin \omega t - i \cos \omega t \right] \\
 &= \sqrt{\frac{\hbar}{2m\omega}} \frac{1}{3} \left[+2 \sin \omega t + 2\sqrt{2} \cos \omega t \right] \\
 \langle x \rangle_t &= \sqrt{\frac{\hbar}{2m\omega}} \frac{2\sqrt{2}}{3} \left[\frac{1}{\sqrt{2}} \sin \omega t + \cos \omega t \right] \\
 \langle x \rangle_t &= x(0) \left[\cos \omega t + \frac{\sin \omega t}{\sqrt{2}} \right]
 \end{aligned}$$

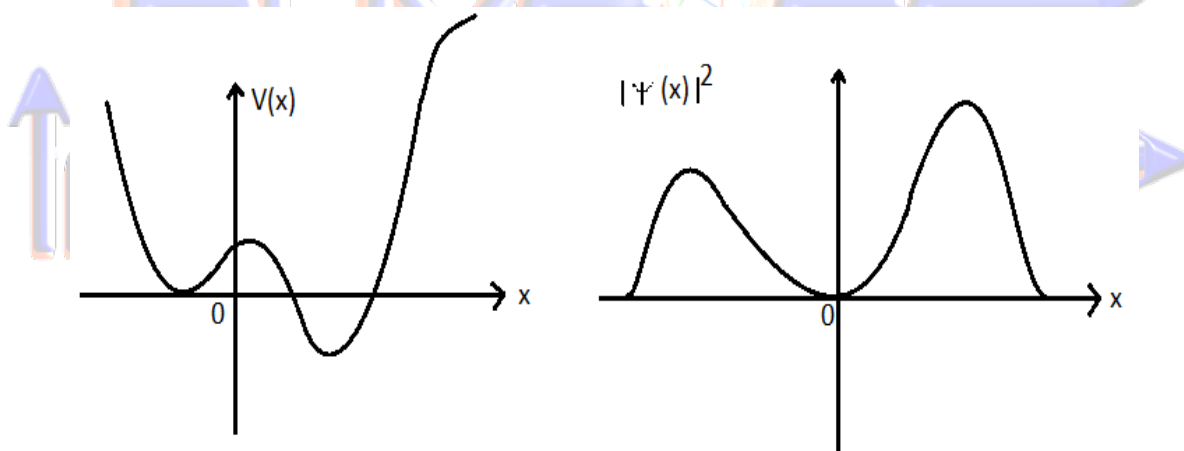
Q13. A particle is confined inside a one- dimensional potential well $V(x)$, as shown on the right. One of the possible probability distributions $|\psi(x)|^2$ for an energy eigenstate



for this particle is



Ans. (a)



$$|\psi(x)|^2 = \text{Probability}$$

Q14. The energy per oscillator of an isolated system of a large number of identical non-interacting fermions in a one-dimensional harmonic oscillator potential is $\frac{5\hbar\omega}{4}$,

where ω is the angular frequency of the harmonic oscillator. The entropy of the system per oscillator is given by

- (a) 0.25 (b) 0.56 (c) 0.63 (d) 0.75

Ans. (b)

Q15. In the temperature range 100 -1000 C, the molar specific heat of a metal varies with temperature T (measured in degrees Celsius) according to the formula

$C_p = \left(1 + \frac{T}{5}\right) J - \text{deg } C^{-1} \text{mol}^{-1}$. If 0.2 kg of the metal at 600 C is brought in thermal

contact with 0.1 kg of the same metal at 300 C, the final equilibrium temperature, in deg C, will be [Assume that no heat is lost due to radiation and/or other effects.]

- (a) 466 (b) 567 (c) 383 (d) 519

Ans. (d)

$$c_p = \left(1 + \frac{T}{5}\right) J / \text{deg}^\circ \text{C.Mol.}$$

$$\boxed{600^\circ \text{C}}$$

$$0.2 \text{ kg}$$

$$\boxed{300^\circ}$$

$$0.1 \text{ kg}$$

$\therefore$ Metal $\rightarrow$ same

So specific heat same

$$T_f = \frac{m_1 c_1 t_1 + m_2 c_2 t_2}{m_1 c_1 + m_2 c_2}$$

$$T_f = \frac{m_1 t_1 + m_2 t_2}{m_1 + m_2} = \frac{0.2 \times 873 + 0.1 \times 573}{0.2 + 0.1}$$

$$\boxed{T_f = 500^\circ \text{C}}$$

When use specific heat

2nd Method

$$T_f = \frac{m_1 c_1 t_1 + m_2 c_2 t_2}{m_1 c_1 + m_2 c_2}$$



$$T_f = \frac{0.2 \left(1 + \frac{873}{5} \right) \times 873 + 0.1 \left(1 + \frac{573}{3} \right) \times 573}{0.2 \left(1 + \frac{873}{5} \right) + 0.1 \left(1 + \frac{573}{3} \right)}$$

$$T_f = \frac{0.2(1+174.6) \times 873 + 0.1(1+114.6) \times 573}{0.2(1+174.6) + 0.1(1+114.6)}$$

$$T_f = \frac{0.2 \times 153298.8 + 0.1 \times 66238.8}{0.2 \times 175.6 + 0.1 \times 115.6}$$

$$T_f = \frac{30659.76 + 6623.88}{46.68}$$

$$T_f = 798.70 \text{ K}$$

$$T_f = 525.70^\circ \text{C}$$

$$T_f = 519^\circ \text{C}$$

- Q16. In a fixed target experiment, a proton of total energy 200 GeV is bombarded on a proton at rest and produces a nucleus ${}^Z_A N$ and its anti-nucleus ${}^Z_A \bar{N}$ $p + p \rightarrow {}^Z_A N + {}^Z_A \bar{N} + p + p$
The heaviest nucleus ${}^Z_A N$ that can be created has atomic mass number $A =$
(a) 15 (b) 9 (c) 5 (d) 4

Ans. (b)

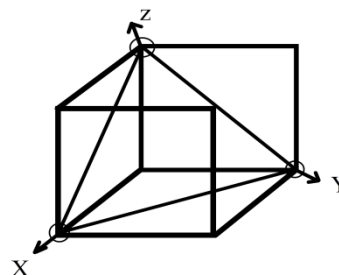
- Q17. In a simple cubic lattice of lattice constant 0.287 nm, the number of atoms per mm^2 along the 111 plane is
(a) 2.11×10^{13} (b) 1.73×10^{13} (c) 1.29×10^{13} (d) 1.21×10^{13}

Ans. (a)

For simple cubic
 $\frac{\text{no of atoms}}{\text{MM}^2} = \text{planer density}$

$$\text{No of atoms} = \frac{3 \times \frac{\pi}{3}}{\frac{2\pi}{3}} = \frac{3}{2}$$

$$\text{Area} = \frac{\sqrt{3}}{4} (2a)^2 = \frac{\sqrt{3}a^2}{2}$$



$$\rho = \frac{\sqrt{3}}{a^2}$$

Then
$$\rho = \frac{\sqrt{3}}{(0.287)^2} \times 10^{18} \times 10^{-6}$$

$$\rho = 21 \times 10^{12}$$

$$\boxed{\rho = 2.1 \times 10^{13}}$$

- Q18. In a glass fibre, light propagates by total internal reflection from the inner surface. A very short pulse of light enters a perfectly uniform glass fibre at $t = 0$ and emerges from the other end of the fibre with negligible losses. If the refractive index of the glass used in the fibre is 1.5 and its length is exactly 1.0 km , the time t at which the output pulse will have completely exited the fibre will be
- (a) $5.0 \mu\text{s}$ (b) $7.5 \mu\text{s}$ (c) 25 ns (d) 750 ns

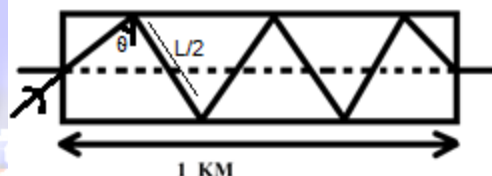
Ans. (b)

Let total internal reflection be η

$$nl \sin \theta_c = 1 \times 10^3$$

$$\sin \theta_c = \frac{1}{\mu} = \frac{2}{3}$$

$$\left(\mu = \frac{3}{2} \right)$$



So $nl \left(\frac{2}{3} \right) = 1 \times 10^3$

$$nl = \frac{10^3 \times 3}{2}$$

$$\eta = \frac{c}{v}$$

$$\eta = 3/2$$

$$v = \frac{c}{\eta}$$

$$\Rightarrow v = \frac{2c}{3}$$

$$t = \frac{d}{v}$$

$$\Rightarrow \frac{nL}{2c/3}$$

$$t = \frac{3 \times 3 \times 10^3}{2 \times 2 \times 3 \times 10^8}$$

$$t = 0.75 \times 10^{-5}$$

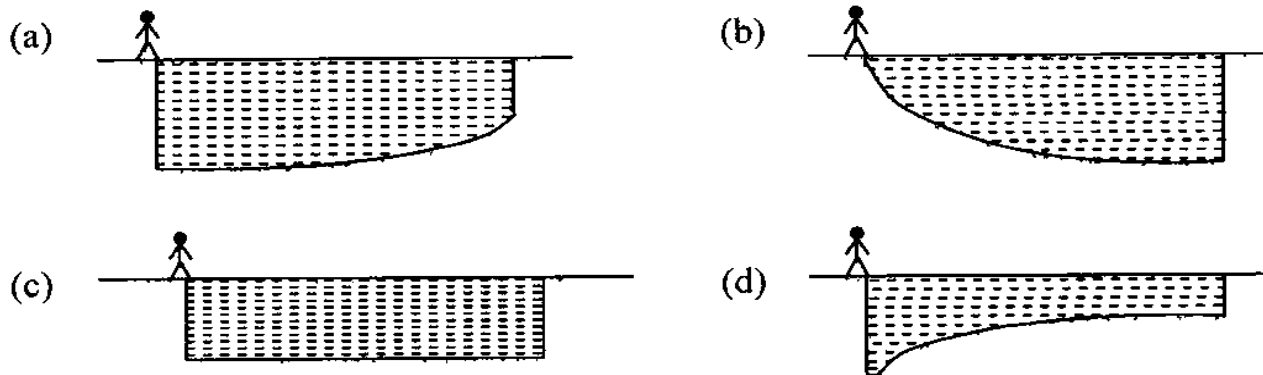
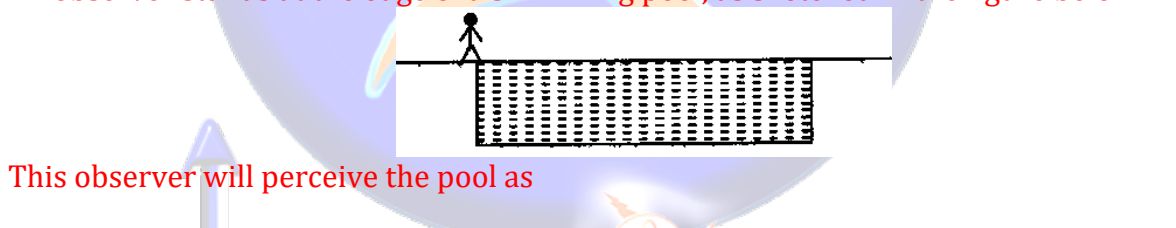
$$t = 7.5 \mu s$$

Q19. In an ionization experiment conducted in the laboratory, different singly- charged positive ions are produced and accelerated simultaneously using a uniform electric field along the x -axis. If we need to determine the masses of various ions produced, which of the following methods will NOT work

- (a) Detect them at a fixed distance from the interaction point along x -axis and measure their time of arrival.
- (b) Apply a uniform magnetic field along y -axis and measure the deviation.
- (c) Apply a uniform electric field along y -axis and measure the deviation.
- (d) Apply a uniform electric field along y -axis and a (variable) uniform magnetic field along z – axis simultaneously and note the zero deviation.

Ans. (c)

Q20. An observer stands at the edge of a swimming pool, as sketched in the figure below.



Ans. (a)

- Q21. Consider a process in which atoms of Actinium- $^{226}_{89}\text{Ac}$ get converted to atoms of Radium- $^{226}_{88}\text{Ra}$ and the yield of energy is 0.64 MeV per atom. This occurs through
- (a) Both $p \rightarrow n + e^+ + \nu_e$ and $p + e^- \rightarrow n + \nu_e$
 - (b) Both $p \rightarrow n + e^+ + \nu_e$ and $n \rightarrow p + e^- + \bar{\nu}_e$
 - (c) only $p \rightarrow n + e^+ + \nu_e$
 - (d) only $p + e^- \rightarrow n + \nu_e$

Ans. (d)

Here this energy yield is $\rightarrow 0.64 \text{ eV}$

Which is Q – value for β^- – decay & e^- – capture But β^- – decay need to exceed 1.02 MeV,

Below 1.02 eV e^- capture dominant

So, Here e^- -capture will be dominant

- Q22. Two homonuclear diatomic molecules produce different rotational spectra, even though the atoms are known to have identical chemical properties. This leads to the conclusion that the atoms must be
- (a) isotopes, i.e. with the same atomic number.
 - (b) isobars, i.e. with the same atomic weight.
 - (c) isotones, i.e. with the same neutron number.
 - (d) isomers, i.e. with the same atomic number and weight

Ans.(a)

Homo nuclear

That is O_2, H_2

Rotational spectra $\rightarrow$ different

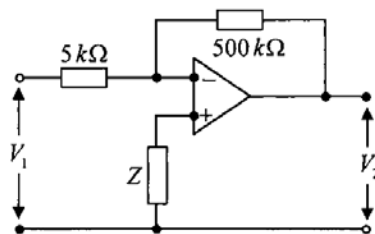
Have same chemical properties

Then it must be isotopes with same atomic number

That is HCL & DCL

H & D are Isotopes

- Q23. In the generalized operational amplifier circuit shown on the right, the op amp. has a very high input impedance ($Z > 50 \text{ M}\Omega$) and an open gain of 1000 for the frequency range under consideration. Assuming that the op . amp. draws negligible current, the voltage ratio $\frac{V_2}{V_1}$ is approximately



(a) -190

(b) -190

(c) -90

(d) 80

Ans.(c)

$$A_o L = 1000 \text{ given } \Rightarrow \& \Rightarrow Z_i \gg 50m\Omega$$

$$\frac{V_2}{V_1} = ??$$

That is $R_i = \infty$
and $R_o = 0$

So use
$$\frac{v_2}{v_1} = \frac{-R_F / R_1}{1 + \frac{R_F / R_1}{A_o L}}$$

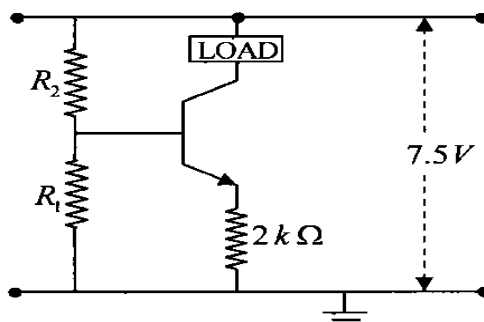
$$\frac{v_2}{v_1} = - \left[\frac{\frac{500k\Omega}{5k\Omega}}{1 + \frac{500k / 5k}{1000}} \right]$$

$$\frac{v_2}{v_1} = \frac{-100}{1 + \frac{100}{1000}}$$

$$\frac{v_2}{v_1} = - \frac{100}{11} \times 10$$

$$\boxed{\frac{v_2}{v_1} = -90.9}$$

Q24. In the transistor circuit shown on the right, assume that the voltage drop between the base and the emitter is 0.5 V. What will be the ratio of the resistances $\frac{R_2}{R_1}$ in order to make this circuit function as a source of constant current, $I = 1mA$?



(a) 4.5

(b) 3.0

(c) 2.5

(d) 2.0

Ans. (d)

From Be loop

$$V_{th} = I_B R_{eq} + I_E R_E + V_{BE}$$

$$V_{th} - V_{BE} = I_B R_{eq} + I_B R_E + R_E I_c$$

$$I_c = \frac{V_{th} - V_{BE}}{R_E}$$

$$R_E I_c = \frac{V_{cc} R_1}{R_1 + R_2} - 0.5$$

$$I_c = 1\text{mA}, V_{BE} = 0.5\text{volt}, R_E = 2\text{K}\Omega$$

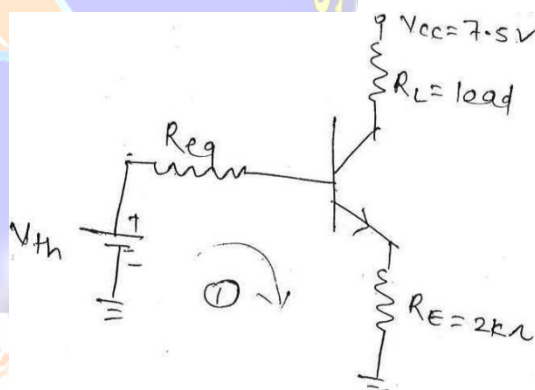
$$2 \times 10^{-3} \times 1 \times 10^{-3} = \frac{7.5 R_1}{R_1 + R_2} - 0.5$$

$$2R_1 + 2R_2 = 7.5R_1 - 0.5R_1 - 0.5R_2$$

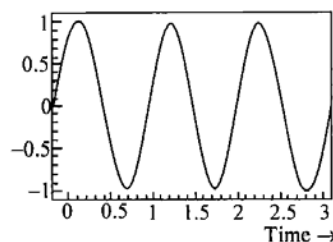
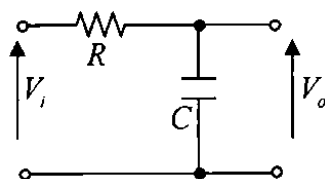
$$2R_1 + 2R_2 = 7R_1 - 0.5R_2$$

$$\frac{R_2}{R_1} = \frac{5}{2.5} \Rightarrow \frac{R_2}{R_1} = 2$$

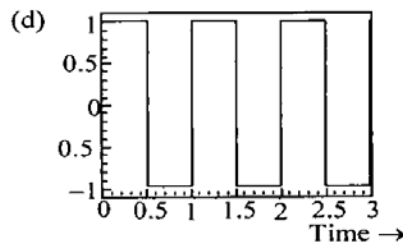
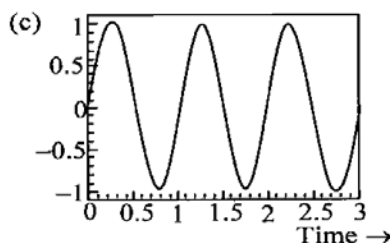
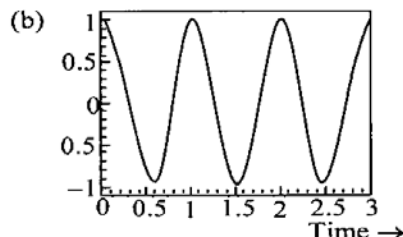
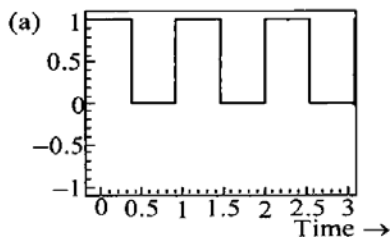
$\therefore I_B \approx 0$ (negligible)



Q25. For the circuit depicted on the right, the input voltage $i V$ is a simple sinusoid as shown below, where the time period is much smaller compared to the time constant of this circuit.



The voltage V across C is best represented by



Ans. (c)

This is high pass filter (differentiator)

$\therefore$ Here $RC \gg \tau$

Then $v_o \approx v_i$

Note:- If $RC \ll \tau$

$$\text{Then } V_o = -\int \frac{1}{RC} dv_i(t).dt$$

GS-2016-X (PHYSICS)

Section B

Q26. The integral $\int_0^{\infty} \frac{dx}{x} \left[\exp\left(-\frac{x}{\sqrt{3}}\right) - \exp\left(-\frac{x}{\sqrt{2}}\right) \right]$ evaluates to

(a) zero

(b) 2.03×10^{-2}

(c) 2.03×10^{-1}

(d) 2.03

Ans. (c)

$$\int_0^{\infty} \frac{dx}{x} \left[\exp\left(-\frac{x}{\sqrt{3}}\right) - \exp\left(-\frac{x}{\sqrt{2}}\right) \right]$$

$$\int_0^{\infty} \frac{1}{x} \exp\left(\frac{-x}{\sqrt{3}}\right) dx - \int_0^{\infty} \frac{1}{x} \exp\left(-\frac{x}{\sqrt{2}}\right) dx$$

From

$$L\left\{\frac{e^{-at}}{t}\right\} = \int_s^{\infty} \frac{1}{(s+a)} ds$$

$$L\left\{\frac{e^{-at}}{t}\right\} = \int_s^{\infty} f(s) ds$$

$$\int_0^{\infty} \frac{1}{x} e^{-x/\sqrt{3}} dx - \int_0^{\infty} \frac{1}{x} e^{-x/\sqrt{2}} dx$$

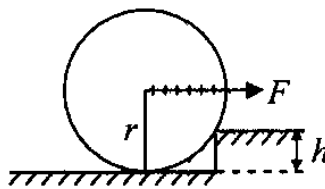
$$\int_0^{\infty} \frac{1}{\left(s + \frac{1}{\sqrt{3}}\right)} ds - \int_0^{\infty} \frac{1}{\left(s + \frac{1}{\sqrt{2}}\right)} ds$$

$$\left[\log\left(s + \frac{1}{\sqrt{3}}\right)\right]_0^{\infty} - \left[\log\left(s + \frac{1}{\sqrt{2}}\right)\right]_0^{\infty}$$

$$0 - \log\left(\frac{1}{\sqrt{3}}\right) - 0 + \log\left(\frac{1}{\sqrt{2}}\right)$$

$$-0.3465 + 0.549 = 0.20 = 2.03 \times 10^{-1}$$

Q27. A uniform solid wheel of mass M and radius r is halted at a step of height h as shown in the figure. The minimum force F , applied horizontally at the centre of the wheel, necessary to raise the wheel over this step is



(a) $Mg \frac{\sqrt{h(2r-h)}}{r+h}$

(b) $Mg \frac{\sqrt{h(2r+h)}}{r-h}$

(c) $Mg \frac{\sqrt{h(r+h)}}{r-h}$

(d) $Mg \frac{\sqrt{h(2r-h)}}{r-h}$

Ans. (d)

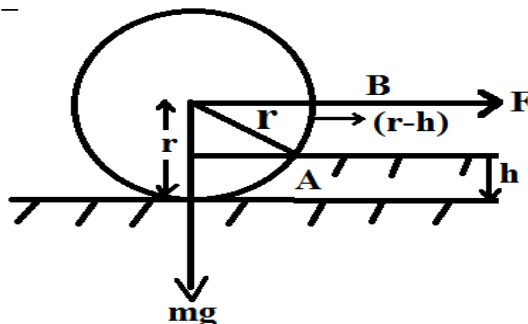
Torque = Force x perpendicular distance At Pt. A : –

$$F(r-h) = mg\sqrt{r^2 - (r-h)^2}$$

$$F(r-h) = mg\sqrt{r^2 - r^2 - h^2 + 2rh}$$

$$F(r-h) = mg\sqrt{h(2r-h)}$$

$$F = \frac{mg\sqrt{h(2r-h)}}{(r-h)}$$



- Q28. In a futuristic scenario, two spaceships, A and B, are running a race, where they start from the same point (marked START) but fly in opposite directions at constant speeds close to the speed of light. An observer fixed at the starting point observes that they both cross the points marked END, which are equidistant from the starting point, at the same time. Afterwards this observer receives messages from both spaceships.



Which of the following could be true?

- (a) Both A and B agree that A won the race.
- (b) A and B both claim to have won the race.
- (c) Both A and B agree that they crossed the end point simultaneously. A thinks B won the race while B thinks A won the race.

Ans. (b)

Both A and B are moving with opposite directions. If both calculate their velocities, with respect to each other, then they find their relative velocity which are

perpendicular their actual velocity $v = \frac{d}{t}$

$v \downarrow$ then $t \uparrow$

Both measured the time period of one another as so both claims that they won the race.

- Q29. The equation of state for a gas is given by $\left[p + \left(\frac{aN}{V} \right)^2 \right] (V - \beta N) = Nk_B T$ where P, V, T , N and k_B represent pressure, volume, temperature, number of atoms and the

Boltzmann constant, respectively, while α and β are constants specific to the gas. If the critical point C corresponds to a point of inflexion of the p - V curve, then the critical volume $C V$ and critical pressure p_c for this gas are given by

- (a) $V_c = 3\beta N, p_c = \alpha^2 / 3\beta^2$ (b) $V_c = 3\beta N, p_c = \alpha^2 / 27\beta^2$
 (c) $V_c = 3\beta N, p_c = 8\alpha^2 / 27\beta$ (d) $V_c = 3\beta N, p_c = \alpha^2 / 27\beta^2$

Ans. (d)

$$\left[p + \left(\frac{\alpha N}{v} \right)^2 \right] (v - \beta N) = Nk_B T \dots \dots \dots (1)$$

Comprise with Vander walls eqⁿ

$$\left(p + \frac{a}{v^2} \right) (v - b) = RT \dots \dots \dots (2)$$

$$\boxed{V_c = 3b}, \boxed{T_c = \frac{8a}{27Rb}}, \boxed{P_c = \frac{a^2}{27b^2}}$$

From eqⁿ (1) & (2)

$$a = \alpha N, b = \beta N$$

Then

$$\boxed{V_c = 3\beta N} \text{ \& } P_c = \frac{\alpha^2 N^2}{27\beta^2 N^2} \Rightarrow \boxed{P_c = \frac{\alpha^2}{27\beta^2}}$$

2nd method

eqⁿ of state

$$\left[p + \left(\frac{\alpha N}{v} \right)^2 \right] (v - \beta N) = Nk_B T$$

$$\left(\frac{dp}{dv} \right)_T = 0$$

$$\left(\frac{d^2 p}{dv^2} \right) = 0$$

Now

$$pv + v \frac{(\alpha N)^2}{v^2} - p\beta N - \frac{(\alpha N)^2}{v^2} \beta N = Nk_B T$$

At constant temp

$$\left(\frac{dp}{dv} \right)_T$$

$$P + V \left(\frac{dP}{dV} \right)_T + (\alpha N)^2 \left(-\frac{1}{V^2} \right) - \beta N \left(\frac{dP}{dV} \right)_T + \frac{2(\alpha N)^2}{V^3} \beta N = 0$$

Now $\left(\frac{dP}{dV} \right)_T = 0$

$$P + 0 + (\alpha N)^2 \left(-\frac{1}{V^2} \right) - 0 + \frac{2(\alpha N)^2}{V^3} \beta N = 0$$

$$P = \frac{(\alpha N)^2}{V^2} - \frac{2(\alpha N)^2}{V^3} \beta N \dots\dots\dots(A)$$

Again do diff. w.r to v at constant T of ϵq^n (1)

$$\left(\frac{dP}{dV} \right)_T + V \left(\frac{d^2P}{dV^2} \right)_T + \left(\frac{dP}{dV} \right)_T - (\alpha N)^2 \left(-\frac{2}{V^3} \right) - \beta N \left(\frac{d^2P}{dV^2} \right)_T + 2(\alpha N)^2 \beta N \left(\frac{-3}{V^4} \right) = 0$$

Now $\left(\frac{d^2P}{dV^2} \right)_T = 0$ and also $\left(\frac{dP}{dV} \right)_T = 0$

$$0 + 0 + 0 + \frac{2(\alpha N)^2}{V^3} - 0 - \frac{6}{V^4} (\alpha N)^2 \beta N = 0$$

$$\frac{2}{V^3} (\alpha N)^2 = \frac{36}{V^4} (\alpha N)^2 \beta N \quad \therefore V = V_C$$

$$V_C = 3\beta N$$

Put the value of V_C in ϵq^n (1)

Then $P = P_C$

$$P_C = \frac{(\alpha N)^2}{9\beta^2 N^2} - \frac{2(\alpha N)^2}{27(\beta N)}$$

$$P_C = \frac{(\alpha N)^2}{\beta^2 N^2} \left[\frac{1}{9} - \frac{2}{27} \right]$$

$$P_C = \frac{\alpha^2}{9\beta^2} \left[1 - \frac{2}{3} \right]$$

$$P_C = \frac{\alpha^2}{27\beta^2}$$

Q30. The lattice constant of a material is of the order of a μm , and its bond energies are of the order of an eV. The bulk modulus of such a material, in Pascals, is of the order of

Head office: 2nd Floor, Krishana Tower, Manav Ashram, Gopalpura Mode, Main Tonk Road, Jaipur-18

+91-8385049481/9782958100

www.physicsinstitute.org
physicsinstitute@gmail.com

www.facebook.com/physicsinstitute

www.youtube.com/c/physicsinstitutejaipur

(a) 10^{-1}

(b) 10^{-3}

(c) 10^{-6}

(d) 10^9

Ans. (a)

$$\text{Bulk modulus} = \frac{\text{stress}}{\text{strain}}$$

$$k = \frac{-F / A}{\Delta V / V \rightarrow \text{strain}}$$

$$\therefore K = \frac{-F}{A}$$

$$k = \frac{v}{x.A}$$

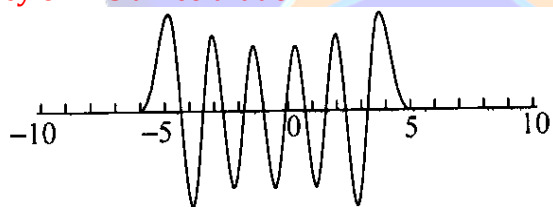
$$k = \frac{(ev)}{(10^{-6})(10^{-6})^2}$$

$$k = \frac{10^{-19}}{10^{-18}} \Rightarrow \boxed{k = 10^{-1}}$$

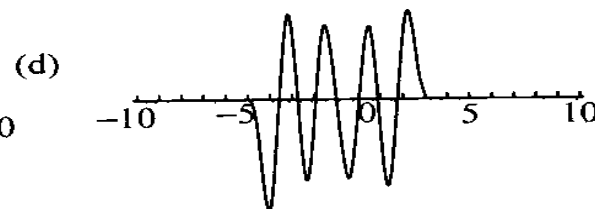
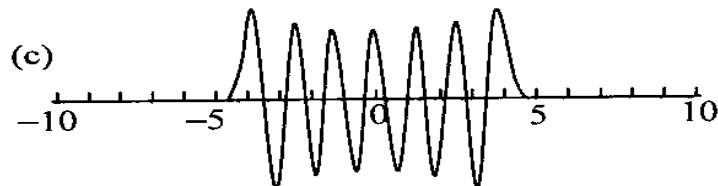
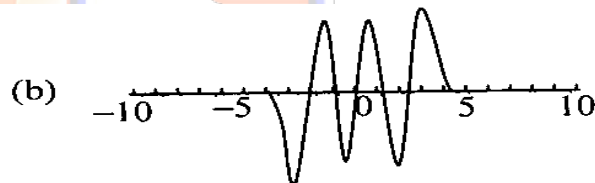
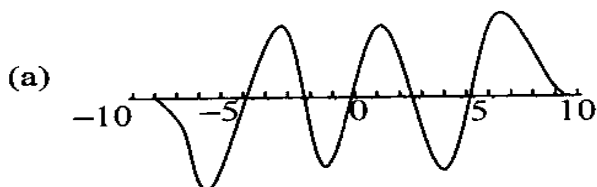
$$\therefore f = \frac{-dv}{dx}$$

No information about stress

- Q31. Two harmonic oscillators A and B are in excited eigenstates with the same excitation energy E , as measured from their respective ground state energies. The natural frequency of A is twice that of B .



If the wavefunction of B is as sketched in the above picture, which of the following would best represent the wavefunction of A ?



Ans. (b)

Here node point of wave function ($n=10$) excitation energy = same

Then option a will be wrong.

∴ According to question

$$E_A = E_B$$

$$\left(n_A + \frac{1}{2}\right)\hbar\omega_A = \left(n_B + \frac{1}{2}\right)\hbar\omega_B$$

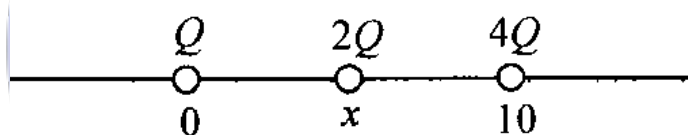
∴ $\omega_A = 2\omega_B$ (according to question)

$$\left(n_A + \frac{1}{2}\right)2\omega_B = \left(10 + \frac{1}{2}\right)\omega_B$$

$$2n_A + 1 = 10.5$$

$$2n_A = 9.5 \Rightarrow n_A = 4.75 \approx 5$$

Q32. Three positively charged particles lie on a straight line at positions 0, x and 10 as indicated in the figure below. Their charges are Q , $2Q$, and $4Q$ cm respectively.



If the charges at $x=10$ are fixed and the charge at x is movable, the system will be in equilibrium when $x =$

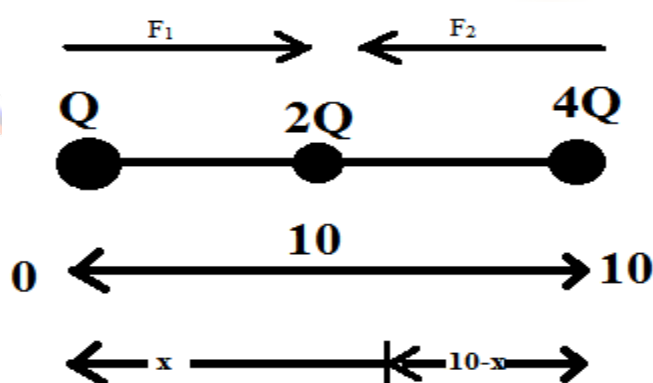
(a) 8

(b) 2

(c) $\frac{20}{3}$

(d) $\frac{10}{3}$

Ans. (d)



Ans.32

Given that system is in equilibrium

So

$$F_1 = F_2$$

$$F = \frac{KQ_1Q_2}{r^2}$$

$$\frac{kQ(2Q)}{x^2} = \frac{k(2Q)(4\theta)}{(10-x)^2}$$

$$\frac{1}{x^2} = \frac{4}{(10-x)^2}$$

$$(10-x)^2 = 4x^2$$

$$100 + x^2 - 20x - 4x^2 = 0$$

$$3x^2 + 20x - 100 = 0$$

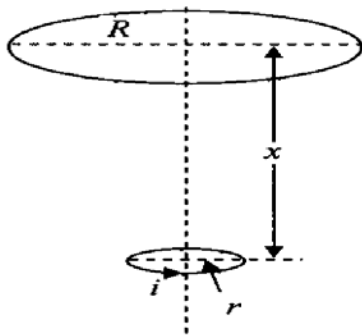
$$3x^2 - 10x + 30x - 100 = 0$$

$$x(3x-10) + 10(3x-10) = 0$$

$$(x+10)(3x-10) = 0$$

$$x = -10 \text{ not possible OR } 3x = 10 \Rightarrow \boxed{x = 10/3}$$

- Q33. Consider the following system. Two circular loops of wire are placed horizontally, having a common axis passing vertically through the centre of each coil (see figure). The lower loop has radius r and carries a current i as shown in the figure. The upper loop has a radius R ($R \gg r$) and is at a distance x ($x \gg r$) above it. If the lower loop is held fixed and the upper loop moves upwards with a uniform velocity $v = \frac{dx}{dt}$, then the induced e.m.f. and the direction of the induced current in this loop will be



(a) $3i\mu_0\pi^2r^2 \frac{R^2v}{2x^4}$; anti-clockwise

(b) $2i\mu_0\pi^2r^2 \frac{R^2v}{2x^4}$; clockwise

(c) $3i\mu_0\pi^2r^2 \frac{R^2v}{2x^3}$; anti-clockwise

(d) $2i\mu_0\pi^2r^2 \frac{R^2v}{2x^3}$; clockwise

Ans.(a)

$e = ??$

$\therefore e = -\frac{d\phi}{dt}$

$\therefore \phi = BA$

$\phi = B(\pi R^2)$

$e = -\pi R^2 \frac{dB}{dt}$

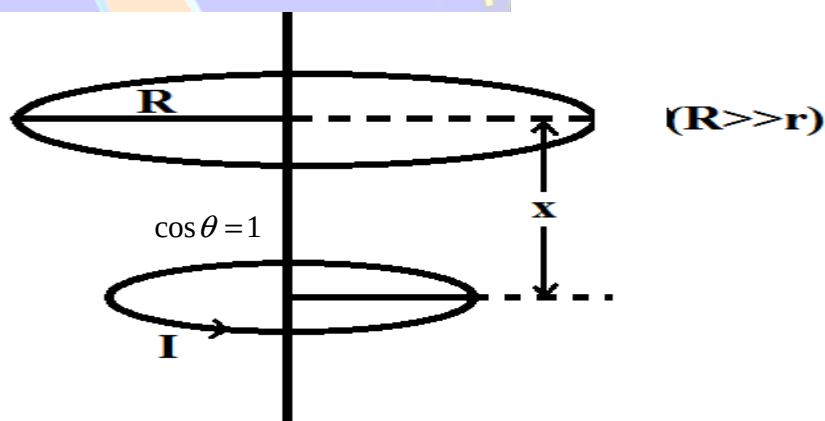
Magnetic field at axis due to circular coil

$$B = \frac{\mu_0 I r^2}{2(R^2 + x^2)^{3/2}}$$

Here

$$B = \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}$$

$$e = \pi R^2 \frac{d}{dt} \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}$$



$$e = \frac{-\pi R^2 \mu_0 I r^2}{2} \left[-\frac{3(r^2 + x^2)^{1/2}}{2(r^2 + x^2)^3} (2x) \cdot \frac{dx}{dt} \right]$$

$$e = \frac{-\pi R^2 \mu_0 I r^2}{2} \left[\frac{3x}{(r^2 + x^2)^{3/2}} \cdot \frac{dx}{dt} \right] \quad \because \frac{dx}{dt} = v$$

$$e = \frac{-3\mu_0 I r^2 V R^2 \pi}{2} \left[\frac{x}{x^5 \left[1 + \frac{r^2}{x^2} \right]^{5/2}} \right] \quad \because r \gg x$$

$$e = \frac{-3\mu_0 I r^2 V \pi R^2}{2x^4} \Rightarrow e = \frac{3I\mu_0 \pi r^2 R^2 V}{2x^4} \quad \frac{r^2}{x^2} = 1$$

- Q34. A student in the laboratory is provided with a bunch of standard resistors as well as the following instruments— Voltmeter accurate to 0.1 V — Thermometer accurate to 0.1 C — Ammeter accurate to 0.01 A — Stop watch accurate to 0.05 s — Constant current source (ideal) — Constant voltage source (ideal) Using this equipment (and nothing else), the student is expected to measure the resistance R of one of the given resistors. The least accurate result would be obtained by
- measuring the Joule heating.
 - passing a constant current and measuring the voltage across it.
 - measuring the current on application of a constant voltage across it.
 - the Wheatstone bridge method.

Ans. (a)

Joules law

$$H = I^2 RT$$

$$\frac{\Delta V}{V} = 0.1$$

$$\frac{\Delta T}{T} = 0.1$$

$$\frac{\Delta I}{I} = 0.01$$

$$\frac{\Delta T}{T} = 0.05$$

$$\because v = IR$$

$$H = I^2 RT$$

$$\frac{\Delta V}{V} = \frac{\Delta I}{I} = 0.1$$

$$= \sqrt{\left(2 \frac{\Delta I}{I} \right)^2 + \left(\frac{\Delta T}{T} \right)^2}$$

$$\frac{\Delta v}{v} = \frac{\Delta I}{I} = 0.1$$

(b)

$$V = IR$$



$$\frac{\Delta V}{V} = \frac{\Delta I}{I}$$

Here constant ideal voltage source

$$\text{So } \frac{\Delta I}{I} = 0$$

So Error = 0 (wrong option)

(c) $V = IR$

$$I = \frac{V}{R}$$

$$\frac{\Delta I}{I} = \frac{\Delta V}{V}$$

$$\frac{\Delta V}{V} = 0$$

$$\frac{\Delta I}{I} = 0$$

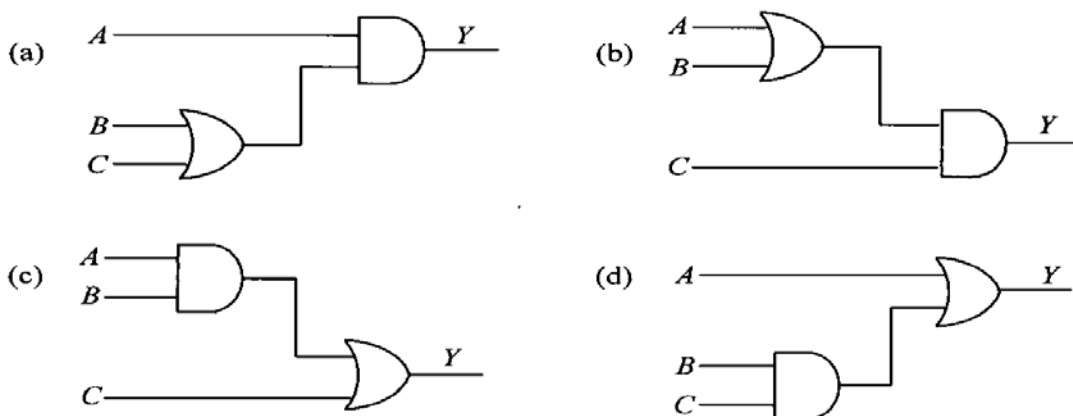
Here ideal current source (wrong option)

(d) Wheat stone bridge error=0
 $= 0.02 + 0.05$
 ≈ 0.07 (wrong option)

Q35. In a digital circuit for three input signals (A, B and C) the final output (Y) should be such that for inputs

A	B	C
0	0	0
0	0	1
0	1	0

the output (Y) should be low and for all other cases it should be high. Which of the following digital circuits will give such output?

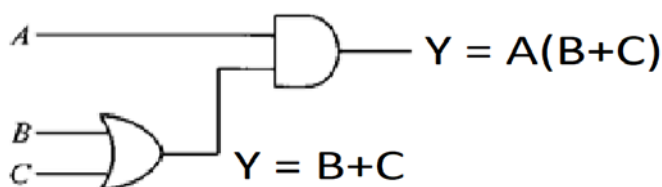


Ans. (d)

A	B	C
0	0	0
0	0	1
0	1	0

When we operate these input then we find output low (that is $y=0$).....(1)

Out otherwise we find $y=1$ (I.e high).....(2)



Truth table

A	B	C	Y'	Y
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0

(a) According to details- 1st this is correct option

But Details- II

A	B	C	Y'	Y
0	1	1	1	0

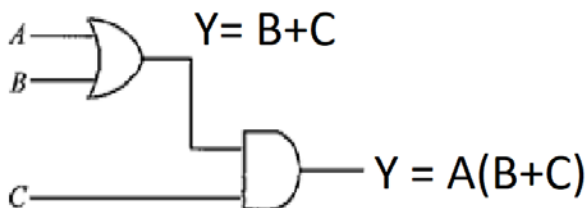
→ (high → According to question |)

This is wrong option – According to question output will be high (But here lo

(b)

Truth table

A	B	C	$Y' = (A+B)$	$Y = (A+B).C$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0

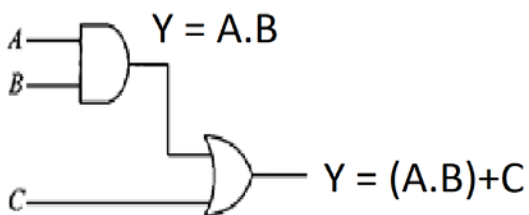


Details-II

A	B	C	$Y' = A+B$	$Y = C.(A+B)$
0	1	1	1	1
1	0	0	1	1
1	1	0	1	0

Wrong option

(c)



Truth table

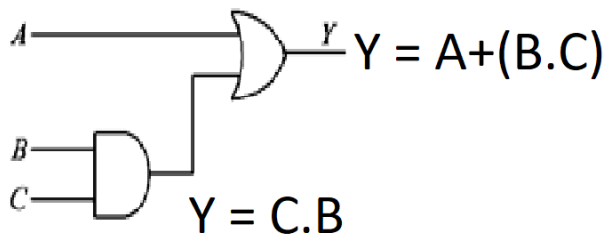
A	B	C	$Y' = A.B$	$Y = C + (A.B)$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0

Wrong option

(d)

Truth table

A	B	C	$Y' = B.C$	$Y = A + (B.C)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0



According to Detail-I this is correct

Detail-II

Truth table

A	B	C	$Y' = B.C$	$Y = A + (B.C)$
0	1	1	1	1
0	0	1	0	1
1	1	0	0	1

1	0	0	0	1
1	1	1	1	1

This is correct option

- Q36. On a planet having the same mass and diameter as the Earth, it is observed that objects become weightless at the equator. Find the time period of rotation of this planet in minutes (as defined on the Earth).

Ans.

At equator:-

$$g' = g - \omega^2 R \cos^2 \lambda$$

$$0 = g - \omega^2 R$$

$$\omega^2 R = g$$

$$\omega = \sqrt{g / R}$$

$$T = 2\pi \sqrt{\frac{R}{g}}$$

$$T = 2\pi \sqrt{\frac{6.4 \times 10^3 \times 10^3}{9.8}}$$

$$6400 \text{ km} = R$$

$$T = 2 \times 3.14 \times 0.8081 \times 10^3$$

$$T = 808.1 \times 2 \times 3.14$$

$$T = 5075.006 \text{ sec}$$

$$T = \frac{5075.006}{60} \text{ min}$$

$$T = 84.58 \text{ min}$$

$$T = 85 \text{ min}$$

Q37. The function $y(x)$ satisfies the differential equation $x \frac{dy}{dx} = y(\ln y - \ln x + 1)$ with the initial condition $y(1) = 3$. What will be the value of $y(3)$?

Ans.

$$x \frac{dy}{dx} = y(\ln y - \ln x + 1)$$

$$\frac{dy}{dx} = \frac{y}{x} \left[\ln \left(\frac{y}{x} \right) + 1 \right]$$

This is a homogeneous diff

$$y = vx$$

Let

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = v \left[\ln \left(\frac{y}{x} \right) + 1 \right]$$

$$v + x \frac{dv}{dx} = v \ln(v) + v$$

$$x \frac{dv}{dx} = v \ln v$$

$$\int \frac{dv}{v \ln v} = \int \frac{dx}{x}$$

$$\text{Let } \ln v = t$$

$$\frac{1}{v} dv = dt$$

$$\int \frac{1}{t} dt = \int \frac{dx}{x}$$

$$\ln t = \ln x + \ln c$$

$$\ln t = \ln(x.c)$$

Take Antilog

$$t = x.c$$

$$\ln v = xc \quad (\text{anti log})$$

$$v = e^{xc}$$

$$\frac{y}{x} = e^{xc} \Rightarrow \boxed{y = xe^{xc}}$$

Boundary condition

When $x=1$ Then

$$y = 3$$

$$\boxed{3 = e^c} \dots\dots\dots (1)$$

When

$$x = 3$$

Then $y = ??$

$$y = xe^{xc} \Rightarrow y = 3(e^c)^3$$

$$y = 3(3)^3 \Rightarrow y = 3(27)$$

$$\Rightarrow \boxed{y = 81}$$

- Q38.** Two containers are maintained at the same temperature and are filled with ideal gases whose molecules have mass m_1 and m_2 respectively. The mean speed of molecules of the second gas is 10 times the r.m.s. speed of the molecules of the first gas. Find the ratio of $\frac{m_1}{m_2}$ the nearest integer.

Ans.

According to question

Average speed (2nd gas) = 10 (rms speed of 1st gas)

$$\text{Average speed} = \sqrt{\frac{8kT}{\pi m}}$$

$$\text{rms speed} = \sqrt{\frac{3kT}{m}}$$

$$\sqrt{\frac{8kT}{\pi m_2}} = 10 \sqrt{\left(\frac{3kT}{m_1}\right)}$$

Do square on both side

$$\frac{8kT}{\pi m_2} = 100 \left(\frac{3kT}{m_1}\right)$$

$$\frac{m_1}{m_2} = \frac{300\pi}{8}$$

$$\frac{m_1}{m_2} = 12.5 \times 3 \times 3.14$$

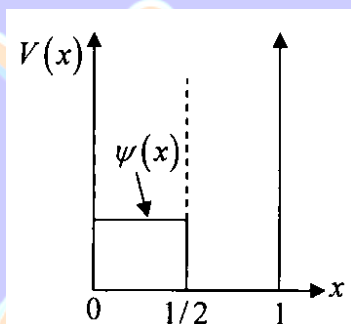
$$\frac{m_1}{m_2} = 37.5 \times 3.14$$

$$\Rightarrow \frac{m_1}{m_2} = 117.75 \approx 118$$

Q39. A particle is confined in a one-dimensional box of unit length, i.e. $L=1$, i.e. in a

potential $V(x) = \begin{cases} 0 & \text{if } 0 < x < 1 \\ \infty & \text{otherwise} \end{cases}$ The energy eigenvalues of this particle are denoted

$E_1, E_2, E_3, \dots$. In a particular experiment, the wavefunction of this particle, at $t=0$, is



given by

$$\psi(x) = \begin{cases} \sqrt{2} & \text{if } 0 < x < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

If, simultaneously, i.e. at $t=0$, a measurement of the energy of the particle is made, find $100 p_3$, where p_3 is the probability that the measurement will yield the energy E_3

Ans.

$$V(x) = \begin{cases} 0 & \text{if } 0 < x < 1 \\ \infty & \text{otherwise} \end{cases}$$

$$\psi(x) = \begin{cases} \sqrt{2} & \text{if } 0 < x < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Normalization condition

$$\int \psi^*(x) \psi(x) dx = 1 \Rightarrow 2A^2 \int_0^{\frac{1}{2}} dx = 1$$

$$2A^2 \left(\frac{1}{2} \right) = 1 \Rightarrow A^2 = 1 \Rightarrow \boxed{A=1}$$

Wave function already normalized.

$E_0 \rightarrow \text{ground } (n=1)$

$E_1 \rightarrow \text{second excited state } n=2$

$E_2 \rightarrow$ second excited state $n = 3$

$E_3 \rightarrow$ third excited state $n = 4$

$100 p_3 \rightarrow$ Calculation

So $\psi_3(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{4\pi x}{a}\right)$

$a = 1 = L$

$\psi_3(x) = \sqrt{2} \sin(4\pi x)$

Probability $\int \psi^* \psi_3(x) \cdot dx$

$\int_0^{\frac{1}{2}} \sqrt{2} \sqrt{2} \sin(4\pi x) \cdot dx$

$2 \int_0^{\frac{1}{2}} \sin(4\pi x) \cdot dx$

$2 [\cos(4\pi x)]_0^{\frac{1}{2}}$

$2 \left[-\cos\left(4\pi \times \frac{1}{2}\right) + \cos 0 \right]$

$2 [-\cos 2\pi + \cos 0]$

$P_3 = 0$

Q40. Consider a sawtooth waveform which rises linearly from 0 Volt to 1 Volt in 10 ns and then decays linearly to 0V over a period of 100 ns. Find the r.m.s. voltage in units of milliVolt?

Ans.

$V_{rms} = \sqrt{\langle v^2 \rangle}$
 $\langle v^2 \rangle = \frac{\int v^2 dt}{\int dt} = \int dt = 110 \times 10^{-9}$

Line oA slope = $\frac{1}{10ns}$

Line of slope AB = $\frac{-1}{100ns}$

$v = -10^7 t + c$

$$(110, 0) = 0 = -10^7 \times t + c$$

$$c = 1.1$$

$$v = 1.1 - 10^7 t$$

$$\langle v^2 \rangle = \int_0^{10^{-8}} (10^8 t)^2 dt + \int_{10^{-8}}^{11 \times 10^{-8}} (1.1 - 10^7 t)^2 dt$$

$$\langle v^2 \rangle = 0.33$$

$$V_{rms} = \sqrt{\langle v^2 \rangle} = 577 \text{ m/s}$$

GS-2016-X (PHYSICS) Section C

Q41. The value of the integral $\oint_C \frac{\sin z}{z^6} dz$ where C is the circle of centre $z = 0$ and radius = 1

(a) $i\pi$

(b) $\frac{i\pi}{120}$

(c) $\frac{i\pi}{60}$

(d) $\frac{-i\pi}{6}$

Ans. (c)

$$\oint \frac{\sin z}{z^6} dz$$

Pole $z = 0$
Order = 6

Then

$$R(z=0) = \lim_{z \rightarrow 0} \frac{1}{5!} \frac{d^5}{dz^5} \frac{\sin z}{z^6} (z-0)^6$$

$$\lim_{z \rightarrow 0} \frac{1}{5!} \frac{d^5}{dz^5} (\sin z)$$

$$\lim_{z \rightarrow 0} \frac{1}{5!} \frac{d^4}{dz^4} (\cos z)$$

$$\lim_{z \rightarrow 0} \frac{1}{5!} \frac{d^3}{dz^3} (-\sin z)$$

$$\lim_{z \rightarrow 0} \frac{1}{5!} \frac{d}{dz} (\sin z)$$

$$\lim_{z \rightarrow 0} \frac{1}{5!} (\cos z)$$

$$\frac{1}{5!} \cos(2) = \frac{1}{5!}$$

Then

$$\oint \frac{\sin z}{z^6} = \frac{2\pi i}{5!} = \frac{\pi i}{60}$$

- Q42. In a Rutherford scattering experiment, the number N of particles scattered in a direction θ , i.e. $\frac{dN}{d\theta}$, as a function of the scattering angle θ (in the laboratory frame) varies as

(a) $\cos^4 \frac{\theta}{2}$

(b) $\cos^2 \frac{\theta}{2} \cot \frac{\theta}{2}$

(c) $\cos^2 \frac{\theta}{2} \tan^2 \frac{\theta}{2}$

(d) $\sec^6 \frac{\theta}{2}$

Ans. (b)

- Q43. In a simple stellar model, the density ρ of a spherical star of mass M varies according to the distance r from the centre according to $\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2}\right)$

where R is the radius of the star. The gravitational potential energy of this star (in terms of Newton's constant G_N) will be

(a) $\frac{-G_N M^2}{4\pi R}$

(b) $\frac{-3G_N M^2}{5R}$

(c) $\frac{-5G_N M^2}{7R}$

(d) $\frac{-3G_N M^2}{5R}$

Ans. (c)

- Q44. A particle moving in one dimension is confined inside a rigid box located between $x = -\frac{a}{2}$ and $x = \frac{a}{2}$. If the particle is in its ground state $\psi_0(x) = \sqrt{\frac{2}{a}} \cos \frac{\pi x}{a}$ the quantum mechanical probability of its having a momentum p is given by

(a) $\frac{8h^4}{(\pi^2 h^2 - p^2 a^2)} \cos^2 \frac{pa}{2h}$

(b) $\frac{\pi^2 h^4}{(\pi^2 h^2 - p^2 a^2)} \sin^2 \frac{pa}{2h}$

(c) $\frac{2h^4}{(\pi^2 h^2 + p^2 a^2)} \cos^2 \frac{pa}{2h}$

(d) $\frac{16h^4}{(\pi^2 h^2 - p^2 a^2)^2}$

Ans. (a)

- Q45. Consider two spin- $\frac{1}{2}$ identical particles A and B , separated by a distance r , interacting through a potential $V(r) = \frac{V_0}{r} \vec{S}_A \cdot \vec{S}_B$ where V_0 is a positive constant and the spins are $\vec{S}_{AB} = \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ in terms of the Pauli spin matrices. The expectation values of this potential in the spin- singlet and triplet states are
- (a) Singlet: $-\frac{V_0}{3r}$, Triplet: $\frac{V_0}{r}$ (b) Singlet: $-\frac{3V_0}{r}$, Triplet: $\frac{V_0}{r}$
- (c) Singlet: $\frac{3V_0}{r}$, Triplet: $-\frac{V_0}{r}$ (d) Singlet: $-\frac{V_0}{r}$, Triplet: $\frac{3V_0}{r}$

Ans.(b)

$$V(r) = \frac{V_0}{r} \vec{S}_A \cdot \vec{S}_B \dots\dots\dots (A)$$

$$\therefore J = \boxed{L + S}$$

$$J^2 = L^2 + S^2 + 2\vec{L} \cdot \vec{S}$$

$$\vec{L} \cdot \vec{S} = \left[\frac{J^2 - L^2 - S^2}{2} \right]$$

As similairy

$$|\vec{S}_A + \vec{S}_B|^2 = S_A^2 + S_B^2 + 2\vec{S}_A \cdot \vec{S}_B$$

$$\vec{S}_A \cdot \vec{S}_B = \left[\frac{(S_A + S_B)^2 - S_A^2 - S_B^2}{2} \right] \dots\dots\dots (1)$$

According to question

Spin $\rightarrow$ singlet & triplet

So

Spin $\rightarrow$ singlet

$$S_A + S_B = 0$$

$\therefore$ Pauli malrics

So have spin $1/2$

$$S_A^2 = S_A(S_A + 1)\hbar^2$$

$$S_A = \frac{1}{2} \left(\frac{1}{2} + 1 \right) \hbar^2 = \frac{3}{4} \hbar^2$$

$$S_B = \frac{1}{2} \left(\frac{1}{2} + 1 \right) \hbar^2 = \frac{3}{4} \hbar^2 \quad \& \quad (S_A + S_B)^2 = 0(0+1)\hbar^2 = 0$$

Then

From $\mathcal{E}q^n \dots\dots\dots(1)$

$$\vec{S}_A \cdot \vec{S}_B = \left(\frac{0 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2}{2} \right) = -\frac{3}{4}\hbar^2$$

For singlet

From $\mathcal{E}q^n (A)$

$$v(r) = \frac{v_o}{r} \left(-\frac{3}{4}\hbar^2 \right)$$

$$v(r) = -\frac{3v_o}{4r}\hbar^2$$

So option (b) will be correct but we check second condition

2nd condition

$$|S_A + S_B| = 1$$

$$\text{Then } |S_A + S_B|^2 = 1(1+1)\hbar^2 = 2\hbar^2$$

From $\mathcal{E}q^n (1)$

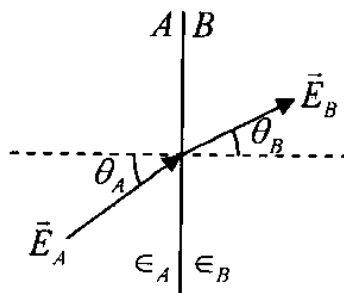
$$\vec{S}_A \cdot \vec{S}_B = \left[q - \frac{3}{4} - \frac{3}{4} \right] \hbar^2 = \left[\frac{2 - \frac{6}{4}}{2} \right] \hbar^2$$

$$\vec{S}_A \cdot \vec{S}_B = \frac{2}{2}\hbar^2 = (1)\hbar^2$$

Then

$$v(r) = \frac{v_o}{r}\hbar^2 \text{ Triplet}$$

Q46. Two semi-infinite slabs A and B of dielectric constant ϵ_A and ϵ_B meet in a plane interface, as shown in the figure below.



If the electric field in slab A makes an angle θ_A with the normal to the boundary and the electric field in slab B makes an angle θ_B with the same normal (see figure), then

(a) $\cos \theta_A = \frac{\epsilon_A}{\epsilon_B} \cos \theta_B$

(b) $\sin \theta_A = \frac{\epsilon_A}{\epsilon_B} \sin \theta_B$

(c) $\tan \theta_A = \frac{\epsilon_A}{\epsilon_B} \tan \theta_B$

(d) $\sin \theta_A = \frac{\epsilon_B}{\epsilon_A} \sin \theta_B$

Ans. (c)

Boundary condition

$$D_{above}^{11} - D_{below}^{11} = 0 \Rightarrow \epsilon_A E_A^{11} = \epsilon_B E_B^{11}$$

$$\epsilon_A E_A \cos \theta_A = \epsilon_B E_B \cos \theta_B \dots\dots\dots(1)$$

$$D_{above}^1 - D_{below}^1 = \sigma_f = 0$$

$$\because D = \epsilon E$$

$\epsilon \rightarrow$ same

$$\epsilon_A \sin \theta_A = \epsilon_B \sin \theta_B \dots\dots\dots(2)$$

$$\epsilon q^n(2) / \epsilon q^n(1)$$

$$\frac{\tan \theta_A}{\epsilon_A} = \frac{\tan \theta_B}{\epsilon_B}$$

$$\boxed{\tan \theta_A = \frac{\epsilon_A}{\epsilon_B} \tan \theta_B}$$

Q47. The Weizsacker semi-empirical mass formula for an odd nucleus with Z protons and A nucleons may be written as $M(Z, A) = \alpha_1 A + \alpha_2 A^{2/3} + \alpha_3 Z + \alpha_4 Z^2$ where the α_i are constants independent of Z, A . For a given A , if Z_A is the number of protons of the most stable isobar, the total energy released when an unstable nuclide undergoes a single β^- decay to (Z_A, A) is

(a) α_3

(b) α_4

(c) $\alpha_4 - \alpha_3$

(d) $\alpha_1 + \alpha_2$

Ans.47 (b)

β^- - decay

$$\eta \rightarrow p + e^- + \bar{\nu}_e$$

Then Released energy

i.e Q -value

$$Q = [\text{Mass of reactant} - \text{Mass of product}]C^2 \Rightarrow Q = [M(Z, A) - M(Z+1, A)]^2 C$$

Now or

$$Q = [\text{B.E of product} - \text{B.E of Reactant}]$$

Given semi-empirical mass formulas

$$Z^A = B.E = \alpha_1 A + \alpha_2 A^{2/3} + \alpha_3 z + \alpha_4 z^2 \dots (1)$$

$$(z+1)^A = (B.E)_p = \alpha_1 A + \alpha_2 A^{2/3} + \alpha_3 (z+1) + \alpha_4 (z+1)^2$$

$\therefore$ unstable nuclei

So have minimum B.E

i.e from $\frac{\partial B}{\partial z} = 0$ (1)

$$\frac{\partial B}{\partial z} = 0$$

From ... (1)

$$0 = \alpha_3 + \alpha_4 (2z)$$

$$z = -\frac{\alpha_3}{2\alpha_4}$$

Then

$$Q = [\alpha_1 A + \alpha_2 A^{2/3} + \alpha_3 z + \alpha_4 z^2 - \alpha_1 A - \alpha_2 A^{2/3} - \alpha_3 (z+1) + \alpha_4 (z+1)^2]$$

$$Q = [\alpha_3 z - \alpha_3 (z+1) + \alpha_4 z^2 - \alpha_4 (z+1)^2]$$

$$Q = [\alpha_3 z - \alpha_3 z - \alpha_3 + \alpha_4 z^2 - \alpha_4 (z^2 + 1 + 2z)]$$

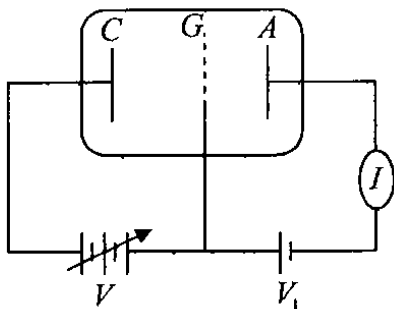
$$Q = [-\alpha_3 - \alpha_4 - 2\alpha_4 z]$$

$$Q = \left[-\alpha_3 - \alpha_4 - 2\alpha_4 \left(-\frac{\alpha_3}{2\alpha_4} \right) \right]$$

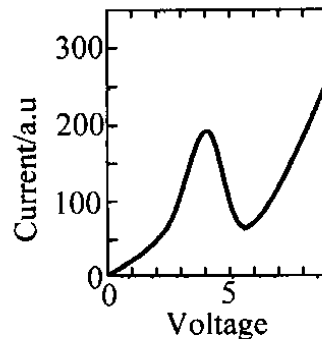
$$Q = -\alpha_4$$

Q48. In the experiment shown in figure (i) below, the emitted electrons from the cathode (C) are made to pass through the mercury vapor filled in the tube by accelerating

them using a grid (G) at potential V positive w.r.t. the cathode. The electrons are collected by the anode (A).



(i)



(ii)

The variation of electron current (I) as a function of V is given in figure (ii). The shape of this curve must be interpreted as due to

- (a) ionization of mercury atoms.
- (b) an emission line from mercury atoms.
- (c) attachment of electrons to mercury atoms.
- (d) resonant backscattering of electrons to cathode from grid.

Ans. (b)

- Q49. The dispersion relation for electrons in the conduction band of a n -type semiconductor has the form $E(k) = ak^2 + b$ where a and b are constants. It was observed that the cyclotron resonance frequency of such electrons is $\omega_0 = 1.8 \times 10^{11} \text{ rad s}^{-1}$, when placed in a magnetic field $B = 0.1 \text{ Wm}^{-2}$. It follows that the constant a must be about
- (a) 10^{-36} (b) 10^{-28} (c) 10^{-32} (d) 10^{-38}

Ans. (d)

Cyclotron condition

$$\frac{Mv^2}{r} = QvB$$

$$\frac{Mv}{r} = QB$$

$$\frac{Mr\omega}{r} = QB$$

$$\omega = \frac{QB}{M} \dots \dots (1)$$

$$[\because v = r\omega]$$

Now given E – K Relation then we calculate from it mass

$$M = \frac{\hbar^2}{d^2 E / dk^2}$$

$$E(k) = ak^2 + b$$

$$\frac{dE}{dk} = 2ak \Rightarrow \frac{d^2 E}{dk^2} = 2a$$

$$m^* = \frac{\hbar^2}{2a}$$

Now we calculate the value of a

So, from $\epsilon q^n \dots (1)$

$$\omega = \frac{QB}{m^*}$$

$$\omega = \frac{QB}{\hbar^2} (2a)$$

$$a = \frac{\omega \hbar^2}{2QB}$$

$$\omega_o = 1.8 \times 10^{11} \text{ rad/sec}$$

$$\hbar = 1.059 \times 10^{-34} \text{ J.sec}$$

$$Q \rightarrow 1.6 \times 10^{-19}$$

$$B \rightarrow 0.1 \text{ wb/m}^2$$

$$a = \frac{1.8 \times 10^{11} \times (1.059)^2 \times 10^{-68}}{2 \times 1.6 \times 10^{-19} \times 0.1}$$

$$a = 6.3 \times 10^{30} \times 10^{-68}$$

$$a = 6.3 \times 10^{30} \times 10^{-68}$$

$$a = 6.3 \times 10^{-38}$$

- Q50. Consider the hyperon decay (1) $A \rightarrow n + \pi^0$ it followed by (2) $\pi^0 \rightarrow \gamma\gamma$. If the isospin component, baryon number and strangeness quantum numbers are denoted by, z , I and S respectively, then which of the following statements is completely correct?
- (a) In (1) I_z is not conserved, B is conserved, S is not conserved; In (2) I_z is conserved, B is conserved, S is conserved.
- (b) In (1) I_z is conserved, B is not conserved, S is not conserved; In (2) I_z is conserved, B is conserved, S is conserved.



- (c) In (1) I_z is not conserved, B is conserved, S is not conserved; In (2) I_z is not conserved, B is conserved, S is conserved.
- (d) In (1) I_z is not conserved, B is conserved, S is conserved; In (2) I_z is conserved, B is conserved, S is conserved.

Ans.50 (a)

$\hat{\pi}^0 \rightarrow n + \pi^0$				
B	1	1	0	$\Delta B = 0$ (conserved)
$I_3 (I_z)$	0	$\frac{1}{2}$	0	$\Delta I_z \neq 0$ (not conserved)
S	-1	0	0	$\Delta S \neq 0$ (not conserved)
(ii) $\pi^0 \rightarrow \gamma + \gamma$				
B	0	0	0	$\Delta B = 0$
I_2	0	(not assigned)		$\Delta I_2 = 0$
S	0	0	0	$\Delta S = 0$
				} all one conserved

There are NO NEGATIVE MARKS for these questions.

Q51. Given the infinite series

$$y(x) = 1 + 3x + 6x^2 + 10x^3 + \dots + \frac{(n+1)(n+2)}{2} x^n + \dots$$

find the value of $y(x)$ for $x = \frac{6}{7}$

Ans.

$$y(x) = 1 + 3x + 6x^2 + 10x^3 + \dots + \frac{(n+1)(n+2)}{2} x^n + \dots$$

$$y(x) = (1+x)^n = (1-x)^{-n}$$

Comparison binomial expansion

$$y(x) = 1 + nx + \frac{n(n+1)x^2}{2!} + \frac{n(n+1)(n+2)x^3}{3!} + \dots$$

$$y(x) = 1 + 3x + 6x^2 + 10x^3 + \dots + \frac{(n+1)(n+2)}{2} x^n + \dots$$

Then, $n = 3$

At $x = 6/7$

$$y(x) = \left(1 - \frac{6}{7}\right)^{-3}$$



$$y(x) = \left(\frac{7-6}{7}\right)^{-3}$$

$$y(x) = (7)^3$$

$$y(x) = 343$$

- Q52.** A quantum mechanical plane rotator consists of two rigidly connected particles of mass m and connected by a massless rod of length d is rotating in the xy plane about their centre of mass. Suppose that the initial state of the rotor is given by $\psi(\theta, t=0) = A \cos^2 \phi$ where ϕ is the angle between one mass and the x axis, while A is a normalization constant. Find the expectation value of $3\hat{L}_z^2$ in this state, in units of $\hbar^2$

Ans.

$$\psi(\phi) = A \cos^2 \phi$$

Normalization condition

$$\int_{\text{all space}} \psi^* \psi dx = 1$$

$$A^2 \int \cos^4 \phi d\phi = 1$$

$$A^2 \int \frac{(1 + \cos 2\phi)^2}{4} d\phi = 1$$

$$\frac{A^2}{4} \int (1 + \cos^2 2\phi + 2 \cos 2\phi) d\phi = 1$$

$$\frac{A^2}{4} \int \left(\frac{1+1+\cos 4\phi}{2} + 2 \cos 2\phi \right) d\phi = 1$$

$$\frac{A^2}{8} \int_0^{2\pi} (2+1+\cos 4\phi + 4 \cos 2\phi) d\phi = 1$$

$$\frac{A^2}{8} \int_0^{2\pi} (3 + 4 \cos 2\phi + \cos 4\phi) d\phi = 1$$

$$\frac{A^2}{8} \left[3\phi + \frac{4 \sin 2\phi}{2} + \frac{\sin 4\phi}{4} \right]_0^{2\pi} = 1$$

$$\frac{A^2}{8} [6\pi + 0 + 0 - 0 -] = 1$$

Now

$$A^2 = 8/6\pi \Rightarrow A^2 = 4/3\pi$$

$$\langle L_z^2 \rangle = \int \psi^* L_z^2 \psi \cdot d\phi \quad \because L_z = -i\hbar \frac{\partial}{\partial \phi}$$

$$\langle L_z^2 \rangle = \int \cos^2 \phi \left(-\hbar^2 \frac{\partial^2}{\partial \phi^2} \right) \cos^2 \phi \cdot d\phi \quad L_z^2 = \left(-i\hbar \frac{\partial}{\partial \phi} \right)^2$$

$$\langle L_z^2 \rangle = -\hbar^2 \int \left[\cos^2 \phi \frac{\partial^2}{\partial \phi^2} \cos^2 \phi \right] d\phi$$

$$\frac{\partial}{\partial \phi} \cos^2 \phi = -2 \cos \phi \sin \phi = -\sin 2\phi$$

$$\frac{\partial^2}{\partial \phi^2} \sin^2 \phi = -\cos 2\phi \cdot 2$$

$$\frac{4}{3\pi} \hbar^2 \int_0^{2\pi} \cos^2 \phi (+2 \cos 2\phi) \cdot d\phi$$

$$\frac{8}{3\pi} \hbar^2 \int_0^{2\pi} \left(\frac{1 + \cos 2\phi}{2} \right) (\cos 2\phi) \cdot d\phi$$

$$\frac{8}{3\pi} \hbar^2 \int_0^{2\pi} (\cos 2\phi + \cos^2 2\phi) \cdot d\phi$$

$$\frac{4}{3\pi} \hbar^2 \int_0^{2\pi} \left[\cos 2\phi + \frac{1 + \cos 4\phi}{2} \right] \cdot d\phi$$

$$\frac{1}{2} \left(\frac{4\hbar^2}{3\pi} \right) \int_0^{2\pi} (2 \cos 2\phi + 1 + \cos 4\phi) \cdot d\phi$$

$$\frac{2\hbar^2}{3\pi} \left[\frac{2 \sin 2\phi}{2} + \phi + \frac{\sin 4\phi}{4} \right]_0^{2\pi}$$

$$\frac{2\hbar^2}{3\pi} [0 + 2\pi + 0 - 0 - 0 - 0]$$

$$\langle L_z^2 \rangle = \frac{4\hbar^2}{3}$$

$$\langle 3L_z^2 \rangle = 4\hbar^2$$

- Q53.** A continuous monochromatic ($\lambda = 600 \text{ nm}$) laser beam is chopped into 0.1 ns pulses using some sort of shutter. Find the resultant line-width $\Delta\lambda$ of the beam in units of 10^{-3} nm .

Ans.

$$\therefore v = \frac{c}{\lambda}$$

$$\Delta v = \frac{c}{\lambda^2} \Delta\lambda$$

$$\therefore \Delta t = 0.1 \text{ ns}$$

$$\Delta\lambda = \frac{\lambda^2}{c} \Delta v$$

$$\Delta\lambda = \frac{\lambda^2}{c(\Delta t)}$$

$$\Delta\lambda = \frac{(6 \times 10^{-7})^2}{3 \times 10^8 \times 0.1 \times 10^{-9}}$$

$$\Delta\lambda = \frac{3.6 \times 10^{-14}}{3 \times 10^{-1} \times 0.1}$$

$$\Delta\lambda = 12 \times 10^{-12}$$

$$\Delta\lambda = 12 \times 10^{-3} \text{ nm}$$

$$\Delta\lambda = 12$$

- Q54.** N distinguishable particles are distributed among three states having energies $E = 0, k_B T$ and $2 k_B T$ respectively. If the total equilibrium energy of the system is $138.06 k_B T$, find the number N of particles.

Ans.

For equilibrium state $2kT$

$$n_1 + n_2 + n_3 = N \text{ (1) } kT$$

And

$$n_1 \epsilon_1 + n_2 \epsilon_2 + n_3 \epsilon_3 = 138.06 kT$$

$$n_1 (0) + n_2 (kT) + n_3 (2kT) = 138.06 kT$$

$$n_2 + 2n_3 = 138.06 \text{ (2)}$$

For distinguishable Maxwell Boltzmann statistics

$$n_i = A e^{-\beta \epsilon_i} = A e^{-\beta \epsilon_i}$$

$$n_1 = A e^0 = A$$

$$n_2 = Ae^{-\beta(kT)} = Ae^{-1}$$

$$n_3 = Ae^{-\beta(2kT)} = Ae^{-2}$$

From eqⁿ (2)

$$Ae^{-1} + 2Ae^{-2} = 138.06$$

$$A(0.367) + 2A \times (0.135) = 138.06$$

$$A[0.367 + 0.270] = 138.06$$

$$A = 217.06$$

$$n_1 + n_2 + n_3 = N$$

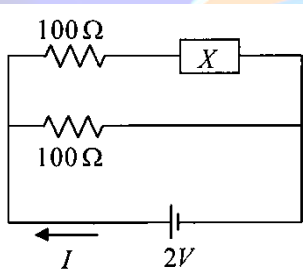
$$A + Ae^{-1} + Ae^{-2} = N$$

$$A \left[1 + \frac{1}{e} + \frac{1}{e^2} \right] = N$$

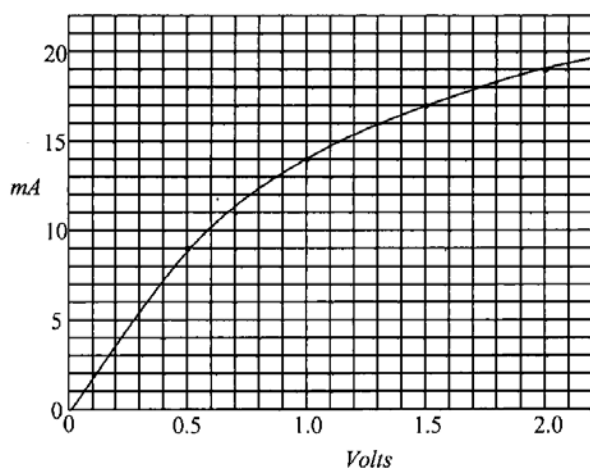
$$217.06[1 + 0.367 + 0.135] = N$$

$$N \approx 325$$

Q55. The circuit shown below contains an unknown device X



The current-voltage characteristics of the device X were determined and shown in the plot given below.



Determine the current I (in mA) flowing through the device X .

Ans. 012