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TIFR-2015 PHYSICS

SOLVED PAPER

INSTITUTE FOR: NET/JRF, GATE, IIT-JAM, JEST, TIFR and GRE IN PHYSICS

TIFR - 2015

Q1. Which of the following vectors is parallel to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$?

(a) $-6\hat{i} - 2\hat{j} + 5\hat{k}$

(b) $+6\hat{i} + 2\hat{j} + 5\hat{k}$

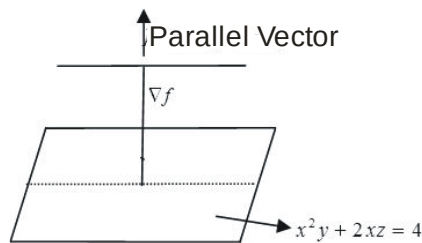
(c) $+6\hat{i} - 2\hat{j} + 5\hat{k}$

(d) $+6\hat{i} + 2\hat{j} + 5\hat{k}$

Ans.(c) $f = x^2y + 2xz - 4 = 0$

$$\nabla f = (2xy + 2z)\hat{i} + (x^2)\hat{j} + (2x)\hat{k}$$

At point $(2, -2, 3)$



$$\nabla f = (-8 + 6)\hat{i} + 4\hat{j} + 4\hat{k}$$

$$\nabla f = -2\hat{i} + 4\hat{j} + 4\hat{k}$$

When $(\nabla f) \cdot g = 0$

Then it is parallel to the surface

From options

(a) $-6\hat{i} - 2\hat{j} + 5\hat{k} = \vec{g}$

$$(\nabla f) \cdot g = (-2\hat{i} + 4\hat{j} + 4\hat{k}) \cdot (-6\hat{i} - 2\hat{j} + 5\hat{k})$$

$$12 - 8 + 20$$

$$32 - 8 \neq 0$$

Wrong option

(b) $\vec{g} = 6\hat{i} + 2\hat{j} + 5\hat{k}$

$$(\nabla f) \cdot \vec{g} = (-2\hat{i} + 4\hat{j} + 4\hat{k}) \cdot (6\hat{i} + 2\hat{j} + 5\hat{k})$$

$$-12 + 8 + 20 \neq 0$$

Wrong option

(c) $6\hat{i} - 2\hat{j} + 5\hat{k} = \vec{g}$

$$(\nabla f) \cdot \vec{g} = (-2\hat{i} + 4\hat{j} + 4\hat{k}) \cdot (6\hat{i} - 2\hat{j} + 5\hat{k})$$

$$-12 - 8 + 20 = 0$$

Correct option

(d) $6\hat{i} + 2\hat{j} + 5\hat{k} = \vec{g}$

$$(\nabla f) \cdot \vec{g} = (-2\hat{i} + 4\hat{j} + 4\hat{k}) \cdot (6\hat{i} + 2\hat{j} + 5\hat{k})$$

$$-12 + 8 + 20 \neq 0$$

Wrong option.



- Q2. A random number generator outputs +1 or -1 with equal probability every time it is run. After it is run 6 times, what is the probability that the sum of the answers generated is zero? Assume that the individual runs are independent of each other.

(a) $\frac{15}{32}$

(b) $\frac{5}{16}$

(c) $\frac{5}{6}$

(d) $\frac{1}{2}$

Ans. (b) = probability to getting output +1 = $1/2$

And probability to getting output -1 = $1/2$

Here : only one condition is that the sum of output will be zero when 6 time measures the output →

Three time +1 and three time -1

Then probability will be

$$= nC_r (p)^r q^{(n-r)}$$

$$= {}^6C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3 = \frac{20}{64} = \frac{5}{16}$$

Option (c)

Or

Since experiment given has only two results +1 and -1.

And it runs 6 times

The sum of the answer generated will be zero only when 3 times +1 and 3 times -1 occurs.

So

$$p(n, r) = \frac{nC_r}{2^n}$$

$$n = 6, r = 3$$

$$p = \frac{{}^6C_3}{2^6} = \frac{20}{64} = \frac{5}{16}$$

- Q3. It is required to construct the quantum theory of a particle of mass m moving in one dimension x under the influence of a constant force F . The characteristic length-scale in this problem is

(a) $\frac{mF}{\hbar^2}$

(b) $\frac{\hbar}{mF}$

(c) $\left(\frac{\hbar}{m^2 F}\right)^{1/3}$

(d) $\left(\frac{\hbar^2}{mF}\right)^{1/3}$

Ans.(d) We can solve it from dimension. If any option has dimension of length then it will be correct.

Now

$$m = \text{kg} = [M]$$

$$F = \text{Force} = ma = [MLT^{-2}]$$

$$\hbar = J.S. = [ML^2T^{-2}][T] = [ML^2T^{-1}]$$

From option



(a) $\frac{mf}{\hbar^2} = \frac{[M][MLT^{-2}]}{[M^2L^4T^{-2}]} = [L^3]$

This is dimension of volume of not length

Option wrong.

(b) $\frac{\hbar}{mF} = \frac{[ML^2T^{-1}]}{[M][MLT^{-2}]} = \left[\frac{LT}{M} \right] \neq [L]$

Wrong option

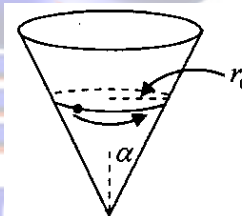
(c) $\left(\frac{\hbar}{m^2F} \right)^{1/3} = \left[\frac{[ML^2T^{-1}]}{[M^2][MLT^{-2}]} \right]^{1/3} = [M^2LT]^{1/3} \neq [L]$

Wrong option

(d) $\left(\frac{\hbar}{mF} \right)^{1/3} = \left[\frac{[M^2L^4T^{-2}]}{[M][MLT^{-2}]} \right]^{1/3} [L^3]^{1/3}$
 $= L \text{ (dimension of length)}$

Option is correct.

- Q4.** A particle slides on the inside surface of a frictionless cone. The cone is fixed with its tip on the ground and its axis vertical, as shown in the figure on the right. The semi-vertex angle of the cone is α . If the particle moves in a circle of radius r_0 , without slipping downwards, the angular frequency ω of this motion will be



(a) $\sqrt{\frac{g}{r_0 \tan \alpha}}$

(b) $\sqrt{\frac{g}{r_0 \cot \alpha}}$

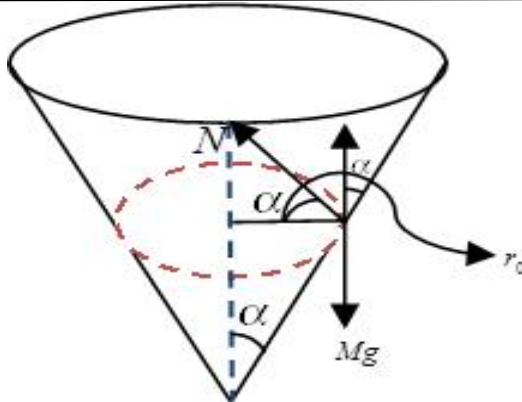
(c) $\sqrt{\frac{g}{r_0 \sin \alpha}}$

(d) $\sqrt{\frac{g}{r_0 \cos \alpha}}$

Ans. (a) Then

$$N \cos \alpha = \frac{mv^2}{r}$$

$\therefore$ for circular condition



$$v = r\omega$$

$$N \cos \alpha = m\omega_0^2 r_0 \quad \dots(1)$$

$$N \sin \alpha = mg \quad \dots(2)$$

$$\text{Eq}^n (2) / \text{eq}^n (1)$$

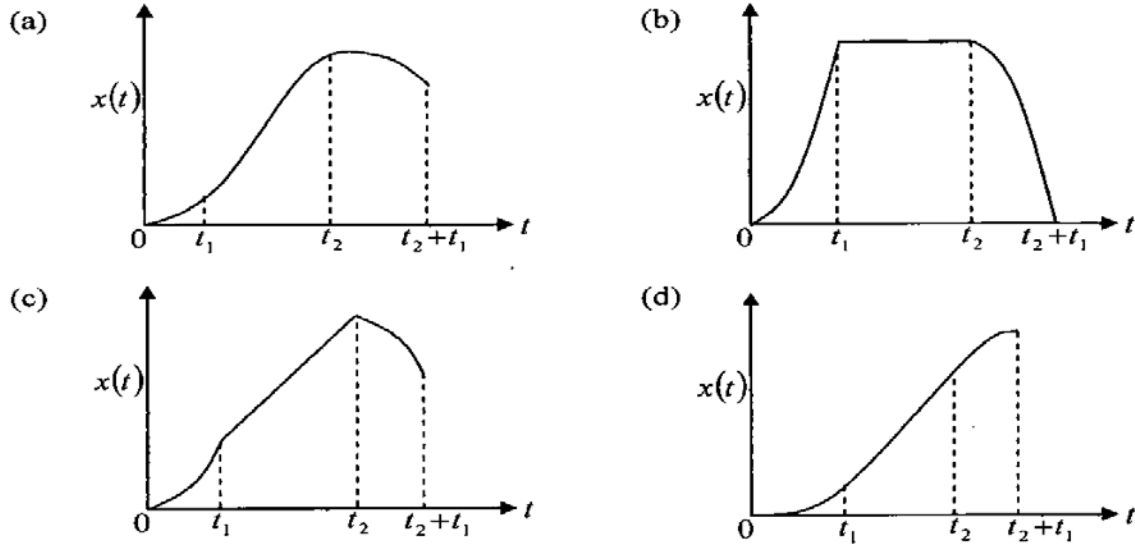
$$\frac{N \sin \alpha}{N \cos \alpha} = \frac{mg}{m\omega_0^2 r_0}$$

$$\tan \alpha = \frac{g}{\omega_0^2 r_0}$$

$$\omega_0^2 \frac{g}{r_0 \tan \alpha}$$

$$\omega_0 = \sqrt{\frac{g}{r_0 \tan \alpha}}$$

- Q5. A car starts from rest and accelerates under a force F increasing linearly in time as $F = at$ at where a is a constant. At time $t_1 > 0$, the force F is suddenly switched off. At a later time $t_2 > t_1$, brakes are applied resulting in a force F' whose magnitude increases linearly with time, $F' = a(t - t_2)$ where a is the same constant as before. Which of the following graphs would best represent the change in the position of the car $X(t)$ with time?



Ans. (d) at initially $F = at$

At $t_1 > 0 \rightarrow$ Force switched off

At $t_2 > t_1$ breaks are applied

Now

We know that when we are driving a car and we apply breaks then it will always go ahead and it does not come back
That is displacement does not reduce even when we applied breaks.

Option (a) $\rightarrow$

$t_1 > 0, x \uparrow$

But at $t_2 > t_1, x \downarrow$ (wrong)

Option (b) $\rightarrow$

$t_1 > 0, x \uparrow$

But $t_2 > t_1$ it will constant

And $t_2 > 0, x \downarrow$ (wrong)

Option (c)

At $t_1 > 0, x \uparrow$

At $t_2 > 0, x \downarrow$ (wrong)

Option (d)

At $t_1 > 0, x \uparrow$

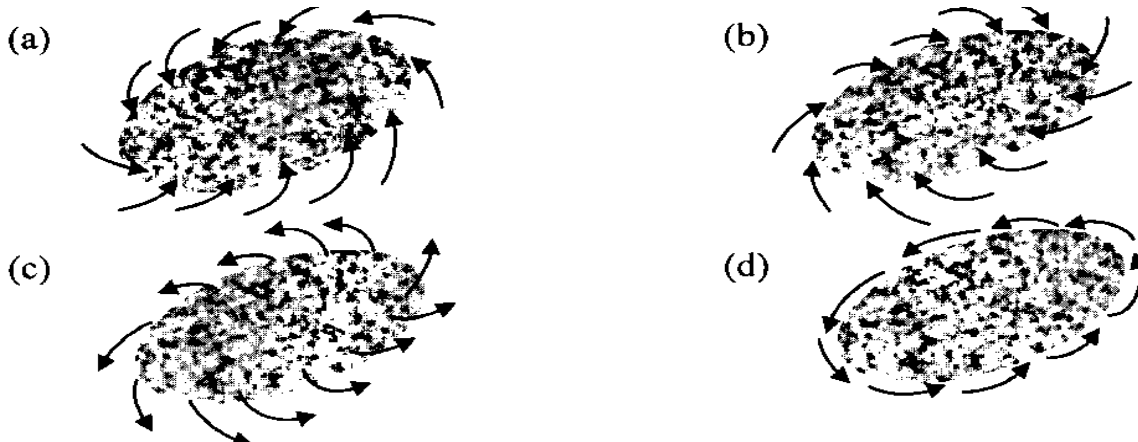
At $t_2 > t_1, x \uparrow$

And $t_2 > 0, x \uparrow$

Correct Option

Q6. In the Earth's atmosphere, a localised low-pressure region (shaded in diagrams) develops somewhere in the southern hemisphere. Which one of the

following diagrams represents the correct air flow pattern as observed from a satellite?



Ans. (b) In southern Hemisphere
Coriolis Force acts in clockwise directions
So
Option (a) → wrong
Option (c) → wrong
Option (d) → wrong
And
Option (b) → correct answer

Q7. The focal length in air of a thin lens made of glass of refractive index 1.5 is

When immersed in water (refractive index = $\frac{4}{3}$), its focal length becomes

(a) $\frac{3\ell}{4}$

(b) $\frac{4\ell}{3}$

(c) $\frac{\ell}{4}$

(d) 4ℓ

Ans. (d) $f_{\text{air}} = \text{Focal length of air} = \ell$

$$\mu_g = 1.5 \Rightarrow a^\mu g = 1.5 = \frac{3}{2}$$

$$\text{And } \mu_w = \frac{4}{3} = a^\mu w = \frac{4}{3}$$



$\therefore$ Focal length, when it is in air

$$\frac{1}{f_a} = (a^\mu g - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \quad \dots(1)$$

When lens is in water

$$\frac{1}{f_\omega} = (\omega^\mu g - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \quad \dots(2)$$

Eqⁿ(2)/Eqⁿ(1)

$$\frac{f_a}{f_\omega} = \frac{(\omega^\mu g - 1)}{(a^\mu g - 1)}$$

$$f_\omega = \frac{(a^\mu g - 1)}{(\omega^\mu g - 1)} f_a$$

We know that

$$a^\mu \omega \times \omega^\mu g \times g^\mu a = 1$$

$$\omega^\mu g = \frac{1}{\mu \times g^\mu a} \quad \therefore a^\mu g = \frac{1}{g^\mu a}$$

$$\omega^\mu g = \frac{a^\mu g}{a^\mu \omega}$$

$$\omega^\mu g = \frac{\mu_g}{\mu_\omega} = \frac{3}{2 \times \frac{4}{3}} = \frac{9}{8}$$

$$f_\omega = \left(\frac{\frac{3}{2} - 1}{\frac{9}{8} - 1} \right) \ell \Rightarrow f_\omega = 4\ell$$

Note :when lens put in water then its focus distance will be four times to the air

$$f_\omega = 4 f_a$$

- Q8.** A light beam of intensity I_0 passes at normal incidence through a flat plate of plastic kept in air. If reflection at the interface reduces the intensity by 20% and absorption on passing through the plate reduces the intensity by 2%, the intensity of the emergent beam will be about
- (a) $0.78 I_0$ (b) $0.65 I_0$ (c) $0.63 I_0$ (d) $0.60 I_0$

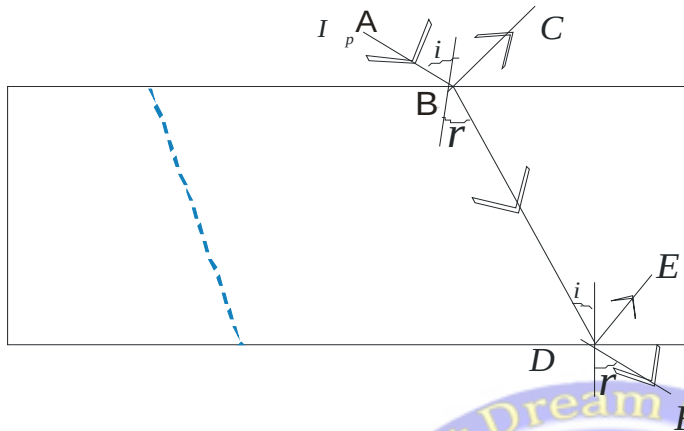
Ans. (b) AB $\rightarrow$ incident ray

BC $\rightarrow$ Reflected ray from 1st inter face

BD $\rightarrow$ Transitted ray (Absorb ray)

DE $\rightarrow$ Reflected ray from 2nd Interface

DF $\rightarrow$ Emergent ray =??



Transmitted ray Intensity
 $r = \text{reflected} = 0.2$

$$I_{BD} = \frac{I_0}{1 + \frac{4r^2}{(1+r^2)^2} \sin^2 \frac{\delta}{2}}$$

[BC & DF Reflected ray and refracted ray occur path difference of $\frac{\pi}{2}$

Then total path difference $= \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$$I_{BD} = \frac{I_0}{1 + \frac{4r^2}{(1+r^2)^2} \sin^2 \frac{\pi}{2}}$$

$$I_{BD} = \frac{I_0}{1 + 4(0.04)} = \frac{I_0}{(1+0.04)^2}$$

$$I_{BD} = 0.85I_0$$

$$I_{DF} = I_{BD} - I_{DE}$$

$$I_{DF} = 0.85I_0 - 0.2I_0$$

$$I_{DF} = 0.65I_0$$

Option (b)

- Q9. A light beam is propagating through a medium with index of refraction 1.5. If the medium is moving at constant velocity $0.7c$ in the same direction as the beam, what is the velocity of light in the medium as measured by an observer in the laboratory? (c = velocity of light in vacuum)
- (a) $0.96c$ (b) $0.98c$ (c) $0.93c$ (d) $0.90c$

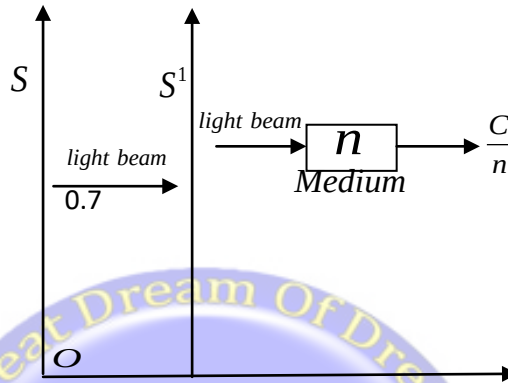
Ans.(c) $V_{(\text{due to Medium})} = \frac{c}{\eta}$

$$\left[\because \eta = \frac{c}{v} \right]$$

$$v = \frac{c}{1.5} = 0.67c$$

In center of mass frame

$$u_{x'} = \frac{u_x - v}{\left(\frac{1 - vu_x}{c^2} \right)}$$



But in lab frame

$$u = \frac{u_{x'} + v}{\left(\frac{1 + vu_{x'}}{c_2} \right)}$$

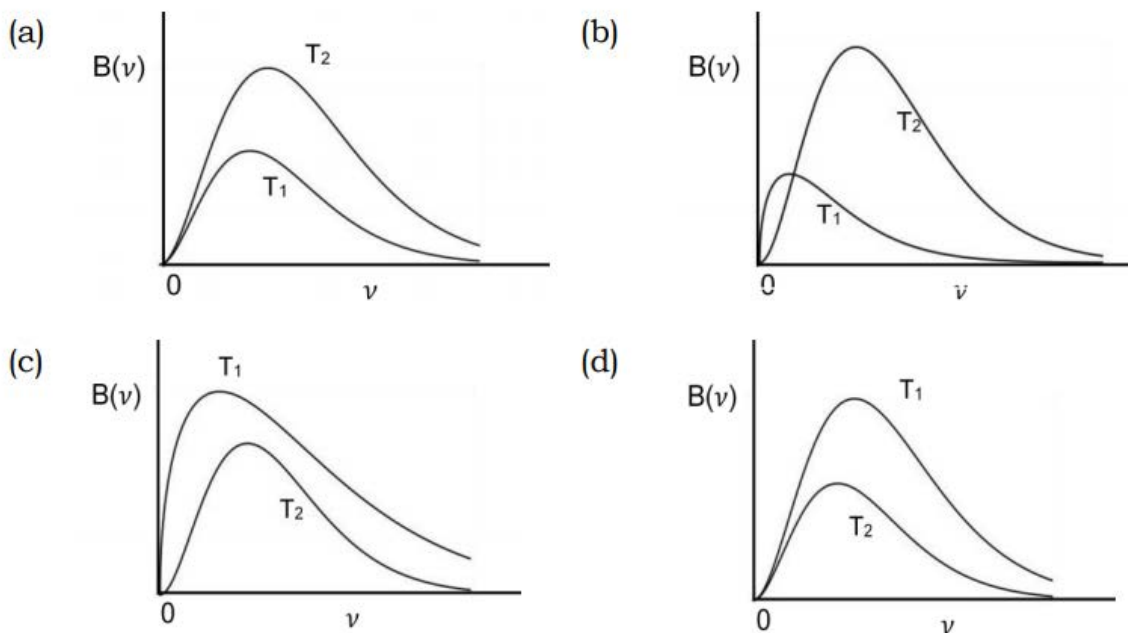
$$u = \frac{(0.67 + 0.7)c}{\left(\frac{1 + 0.7 \times 0.67 \frac{c_2}{c_2}}{c_2} \right)}$$

$$u = \frac{1.37c}{(1 + 0.7 \times 0.67)}$$

$$u = 0.93c$$

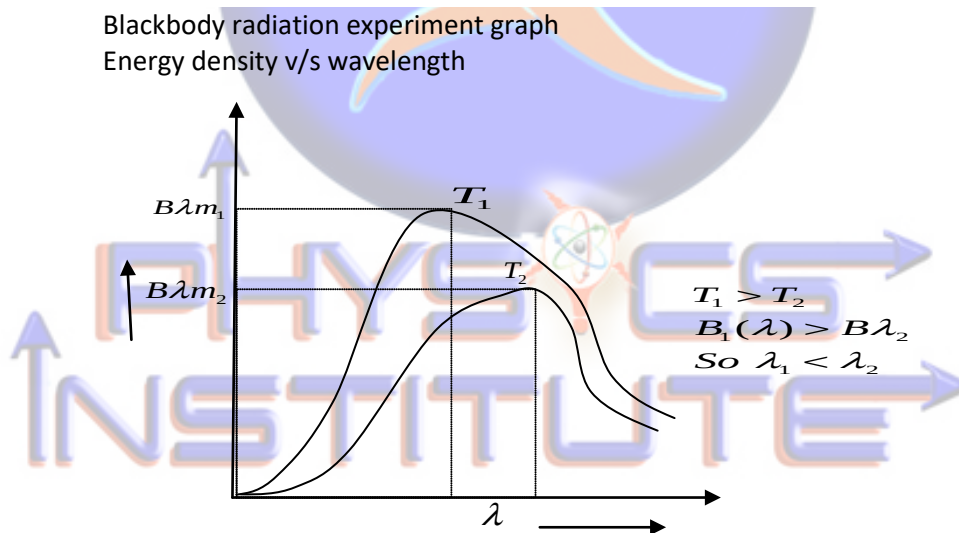
option (c)

Q10. Two blackbodies radiate energy at temperatures T_1 and T_2 ($T_1 > T_2$). The energy emitted per unit time per unit solid angle per unit surface area of a blackbody in the frequency range ν to $\nu + d\nu$ is given by $B(\nu)d\nu$. Which one of the following graphs has the correct form?



Ans.(d)

Blackbody radiation experiment graph
Energy density v/s wavelength



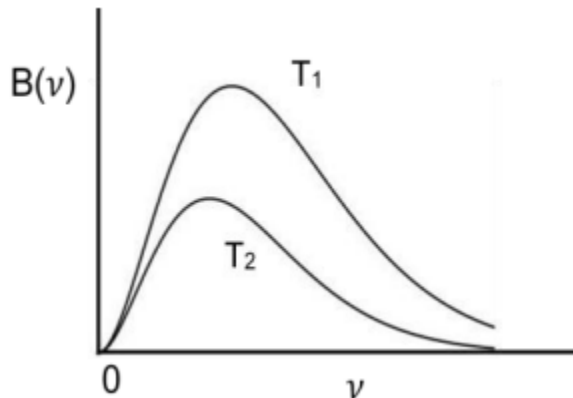
According to question graph is plotted b/w energy density and frequency

$$\therefore \text{ We know that } \nu = \frac{c}{\lambda} \Rightarrow \lambda = \frac{c}{\nu}$$

$$\therefore \lambda_2 > \lambda_1$$

Then $\boxed{\nu_1 > \nu_2}$

Then graph will be



$$\therefore T_1 > T_2$$

So $B_1 > B_2$ But Here $B_2 > B_1$

Option (a) $\rightarrow$

Here energy will be reduced/decreased when we increase the Temp while energy is increased. The temp.

So

Option (d)

This is correct option

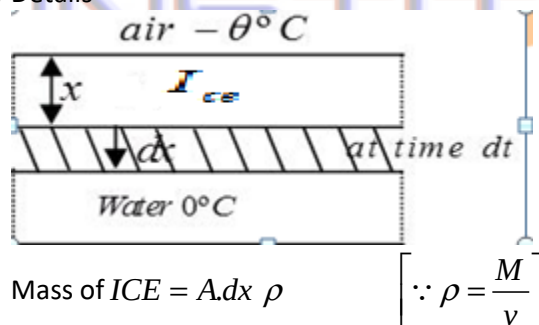
Because $T_1 > T_2$

$$B_1 > B_2$$

$$\nu_1 > \nu_2$$

- Q11.** In a cold country, in winter, a lake was freezing slowly. It was observed that it took 2 hours to form a layer of ice 2 cm thick on the water surface. Assuming a constant thermal conductivity throughout the layer, the thickness of ice would get doubled after
- (a) 8 more hours. (b) 4 more hours.
 (c) 2 more hours. (d) 6 more hours.

Ans.(d) Details



$$\text{Mass of ICE} = A \cdot dx \cdot \rho$$

$$\left[\because \rho = \frac{M}{V} \right]$$

Heat given up by water =

$$Q = ML$$

$$Q = A \cdot dx \cdot \rho L \quad \dots(1)$$

Where

$L \rightarrow$ Latent heat of fusion of I_{ce}

This amount of heat is conducted upwards through the *ICE* layer of thickness x in time t at k is the thermal conductivity

Heat conducted upwards

$$Q = ka \left[\frac{0 - (-\theta)}{x} \right] dt \quad \dots(2) \quad \left[\frac{Q}{A} = -k \frac{dT}{dx} \right]$$

From Eqⁿ (1) and Eqⁿ (2)

$$Adx \rho L = KA \frac{\theta}{x} dt$$

$$dt = \frac{\rho L}{k\theta} x dx$$

$$t = \int_{x_1}^{x_2} \frac{\rho L}{k\theta} x dx$$

Link with question $\rightarrow$

$$x_1 = 2 \text{ cm}$$

$$x_2 = 4 \text{ cm}$$

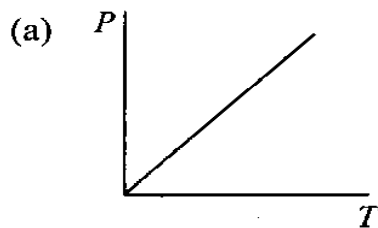
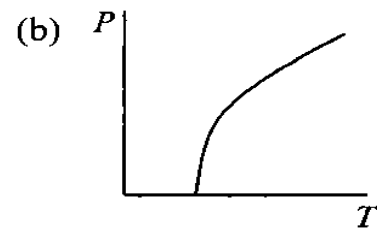
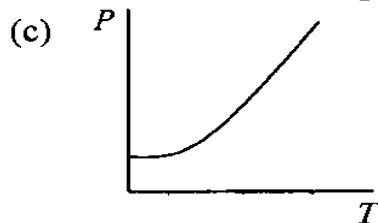
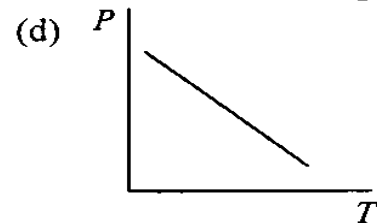
$$t = \frac{\rho L}{k\theta} \left[\frac{x^2}{2} \right]_{x_1}^{x_2}$$

$$\frac{\rho L}{k\theta} = 1 \text{ unit}$$

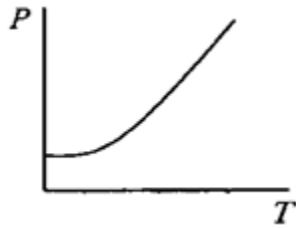
$$t = \frac{\rho L}{k\theta} \left[\frac{16 - 4}{2} \right] \text{ t}$$

$$t = 6 \text{ hour} \quad \text{option (d)}$$

Q12. Which of the following graphs qualitatively describes the pressure P of a gas of noninteracting fermions in thermal equilibrium at a constant volume as a function of temperature?



Ans.(c)

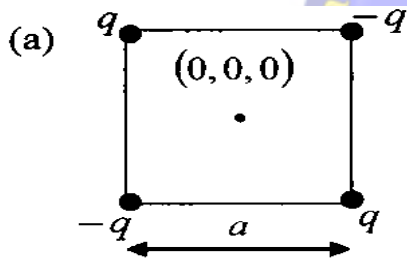
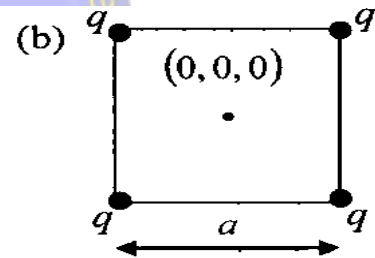
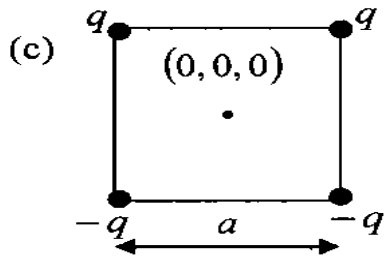
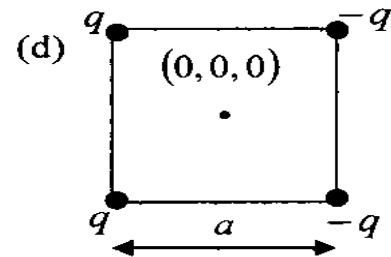


For fermions at absolute temp. $energy \neq 0 \Rightarrow velocity \neq 0$

So pressure $\neq 0$

Option (c) is correct

Q13. The electrostatic potential $\phi(r)$ of a distribution of point charges has the form $\phi(r) \propto r^{-3}$ at a distance r from the origin $(0,0,0)$, where $r \gg a$. Which of the following distributions can give rise to this potential?



Ans.(a) Ist Method :

If four charges are situated at alternative sequence $[+q, -q, +q, -q]$, in the same distance to each other then

Monopole moment = 0

Dipole moment = 0

Quadrupole moment $M \neq 0$

So

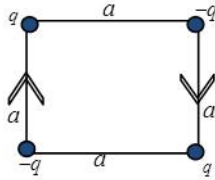
From option

Option (a) is correct answer

IInd Method ;

Use Frome Option

Option $\rightarrow$ (b)



Monopole moment

$$\Sigma Q_i = 0$$

So origin of dipole moment will be independent of choice of origin

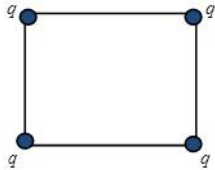
So dipole moment $p_1 + p_2 \because p_1 = -p_2$

$$= 0$$

Then Quadrupole moment will exist

Option (a) is correct answer

(b) Monopole moment



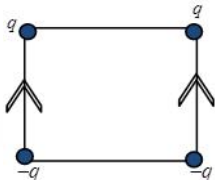
$$\Sigma Q_i = 4Q \neq 0$$

So

$$V = \phi(r) \propto \frac{1}{r}$$

So wrong option

(c) Monopole moment



$$\Sigma Q_i = 0$$

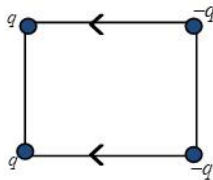
Then dipole moment origin will be independent choice of origin

$$\begin{aligned} \text{Dipole moment} &= p_1 + p_2 \quad \because p_1 = p_2 \\ &= 2p \neq 0 \end{aligned}$$

$$\phi(r) \propto \frac{1}{r^2}$$

Wrong Option

(d) Monopole moment



$$\Sigma Q_i = 0$$

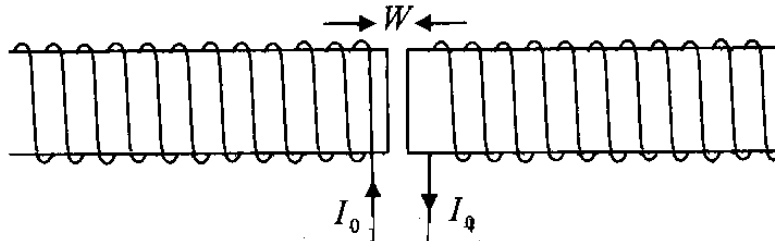
Then dipole moment's origin will be independent of choice of origin

$$p = p_1 + p_2 \quad \because p_1 = p_2$$

$$\phi(r) \propto \frac{1}{r^2}$$

Wrong options

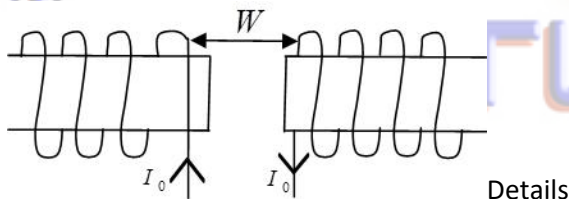
- Q14.** Two semi-infinite solenoids placed next to each other are separated by a small gap of width W as shown in the figure.



The current I_0 in the solenoids flows in the direction as shown. If the solenoids have a circular cross section of radius R and are filled with a magnetic material of permeability μ ($\mu > \mu_0$) then the magnetic energy densities u_i inside the solenoid and u_g in the gap are best related by

- (a) $u_g > cu_i$ (b) $u_g = cu_i$ (c) $u_g < u_i$ (d) $u_g > u_i$

Ans.(d)



$$\mu > \mu_0$$

$$\text{Given } \frac{\mu}{\mu_0} > 1$$

$\mu \rightarrow$ permeability of magnetic material

$\mu_0 \rightarrow$ permeability of gap

Magnetic energy density (μ_i) $\rightarrow$ inside the material

And $\mu_g \rightarrow$ gap's energy density

Magnetic energy density

$$\varepsilon = \frac{B^2}{2\mu}$$

$B \rightarrow$ Magnetic field

$u \rightarrow$ energy density

$\mu \rightarrow$ permeability of gap

Energy density

Inside solenoid

$$u_i = \frac{B^2}{2\mu} \quad \dots(1)$$

Energy density of gap

$$u_g = \frac{B^2}{2\mu_0} \quad \dots(2)$$

$\text{Eq}^n(2)/\text{Eq}^n(1)$

$$\frac{u_g}{u_i} = \frac{\mu}{\mu_0} > 1$$

$$u_g > u_i$$

Option (d)

- Q15.** A light source has a small filament at the centre of a spherical glass bulb of radius 5cm and negligible thickness. If this source emits 100 Watts of power in the form of spherical electromagnetic waves, the *r.m.s* electric field E at the surface of the bulb (in units of Volt/m) will be approximately
 (a) 1094 (b) 109.4 (c) 15.47 (d) 1547

Ans.(a) Power (P) = 10 watt

and $E - m$ wave shape $\rightarrow$ spherical

(radius) $= r = 5\text{cm} = 5 \times 10^{-2}\text{ meter}$

Intensity

$$I = \frac{P}{A} = S = \frac{1}{2} c \epsilon_0 E_0^2$$

$$E_0^2 = \left[\frac{2P}{4\pi r^2 c \epsilon_0} \right]$$

$$E_0 = \left[\frac{2 \times 100}{(4\pi \epsilon_0)(5 \times 10^{-2})^2 \times 3 \times 10^8} \right]^{1/2}$$

$$E_0 = \left[\frac{200 \times 9 \times 10^9}{25 \times 10^{-4} \times 3 \times 10^8} \right]^{1/2}$$

$$E_0 = \sqrt{\frac{60}{25}} \times 10^6$$

$$E_0 = 1.549 \times 10^3$$

$$\therefore E_{rms} = \frac{E_0}{\sqrt{2}} = 1.095 \times 10^3$$

$$E_{rms} = 1095 \text{ V / m}$$

Find method

$$I = \frac{P}{A} = S = E \times H$$

So,

$$E \times H = \frac{P}{A} = \frac{P}{4\pi r^2}$$

$$EH = \frac{10}{4\pi \times 25 \times 10^{-4}}$$

$$\text{We know, } \frac{E}{H} = 377 \Omega$$

$$\frac{E^2}{377} = \frac{10}{4\pi \times 25 \times 10^{-4}}$$

$$E = 10.95 \times 10^2$$

$$E = 1095 \text{ V / m}$$

Q16. A particle is moving in one dimension under a potential $V(x)$ such that, for large positive values of x , $V(x) \approx kx^\beta$ where $k > 0$ and $\beta \geq 1$. If the wavefunction in this region has the form $\psi(x) = \exp(-x^\lambda)$ which of the following is true?

- (a) $\lambda = \beta$ (b) $\lambda = \frac{\beta}{2} + 1$ (c) $\lambda = 2\beta - 2$ (d) $\lambda = \frac{\beta^2}{2}$

Ans.(b) $\psi(x) = \exp(-x^\lambda)$

$$\text{And } v(x) = kx^\beta$$

From schrodinger equation

$$\frac{d^2\psi}{dx^2} - \frac{2M}{\hbar^2} v\psi = -\frac{2M}{\hbar^2} E\psi$$

$$\psi(x) = \exp(-x^\lambda)$$

$$\frac{d\psi(x)}{dx} = \exp(-x^\lambda)(-\lambda x^{\lambda-1})$$

$$\frac{d^2(\psi(x))}{dx^2} = \exp(-x^\lambda)(-\lambda x^{\lambda-1})^2 - \lambda(\lambda-1)x^{\lambda-2} \exp(-x^\lambda)$$

$$\frac{d^2\Psi(x)}{dx^2} = \exp(-x^\lambda) [\lambda^2 x^{2(\lambda-1)} - \lambda(\lambda-1)x^{\lambda-2}]$$

$$\frac{d^2\Psi(x)}{dx^2} = [\lambda^2 x^{2\lambda} \cdot x^{-2} - \lambda(\lambda-1)x^{\lambda-2}] \psi(x)$$

Then

$$[\lambda^2 x^{2(\lambda-1)} - \lambda(\lambda-1)x^{\lambda-2}] \psi(x) - \frac{2M}{\hbar^2} kx\beta\psi = -\frac{2M}{\hbar^2} E\psi$$

Then

$$2(\lambda-1) = \beta$$

$$\lambda-2 = \beta$$

$$2(\lambda-1) = +\beta$$

$$\lambda = \beta + 2$$

$$\lambda - 1 = \frac{+\beta}{2}$$

$$\lambda = 1 + \beta/2$$

Option (b) is correct answer

Q17, The ground state energy of a particle of mass m in a three-dimensional cubical box of side ℓ is not zero but $\frac{3h^2}{8m\ell^2}$. This is because

- (a) the potential at the boundaries is not really infinite, but just very large.
- (b) this is the most convenient choice of the zero level of potential energy.
- (c) position and momentum cannot be exactly determined simultaneously.
- (d) the ground state has no nodes in the interior of the box.

Ans.(c) Energy $\neq 0$ (Due to uncertainty principal) Because according to quantum mechanics the particle does not exist in rest so, it has some momentum & energy

So option (c)

Note : But in classical mechanics there is no restriction on minimum energy

So, we call put the particle at the rest in the box. And giving zero energy.

Q18. A one-dimensional box contains a particle whose ground state energy is ϵ . It is observed that a small disturbance causes the particle to emit a photon of energy $h\nu = 8\epsilon$, after which it is stable. Just before emission, a possible state of the particle in terms of the energy eigenstates $\{\psi_1, \psi_2, \dots\}$ would be

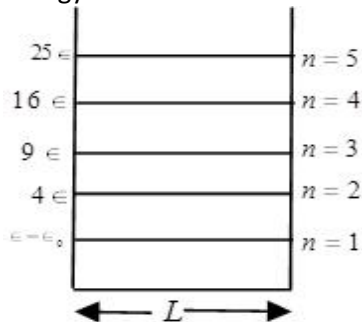
(a) $\frac{\sqrt{2}\psi_1 - 3\psi_2 + 5\psi_5}{6}$

(b) $\frac{\psi_2 + 2\psi_3}{\sqrt{5}}$

(c) $\frac{-4\psi_4 + 5\psi_5}{\sqrt{41}}$

(d) $\frac{\psi_1 - \psi_2}{\sqrt{2}}$

Ans.(b) Energy in one dimensional box



$$E_n = \frac{n^2 \pi^2 \hbar^2}{2ML^2}$$

$$E_n \propto n^2$$

From options

(a) $\frac{\sqrt{2}\psi_1 - 3\psi_2 + 5\psi_5}{6}$

<energy> = probability × energy

$$\frac{2}{36} \times \epsilon + \frac{9}{36} \times 4\epsilon + \frac{25}{36} \times 9\epsilon$$

$$\frac{\epsilon}{36} [2 + 9 \times 4 + 25 \times 9] =$$

$$= \frac{(2 + 36 + 225)}{36} \epsilon$$

$$= 7.30\epsilon \neq 8\epsilon \quad (\text{wrong answer})$$

(b) $\frac{\psi_2 + 2\psi_3}{\sqrt{5}}$

$$\langle E \rangle = \frac{1}{5} \times 4\epsilon + \frac{4}{5} \times 9\epsilon$$

$$\frac{1}{5} (4 + 36) \epsilon = 8\epsilon \quad (\text{correct answer})$$

(c) $\frac{-4\psi_4 + 5\psi_5}{\sqrt{41}}$

$$\langle E \rangle = \frac{16}{41} \times 16\epsilon + \frac{25}{41} \times 25\epsilon$$

$$\frac{(256 + 625)}{41} \epsilon \neq 8\epsilon \quad (\text{wrong})$$

(d) $\frac{\psi_1 - \psi_2}{\sqrt{2}}$

$$\langle E \rangle = \frac{1}{2} \times \epsilon_0 + \frac{1}{2} \times 4\epsilon$$

$$= \frac{5\epsilon}{2} \neq 8\epsilon \quad (\text{wrong option})$$

Q19. A sample of ordinary hydrogen (${}^1_1\text{H}$) gas in a discharge tube was seen to emit the usual Balmer spectrum. On careful examination, however, it was found that the α H line in the spectrum was split into two fine lines, one an intense line at 656.28nm , and the other a faint line at 656.04nm . From this, one can conclude that the gas sample had a small impurity of

(a) ${}^3_1\text{H}$

(b) ${}^2_1\text{H}$

(c) ${}^4_2\text{He}$

(d) H_2O

Ans.(a) $H\alpha$ - Line split into two fine line means L-S coupling will be occurs

Then ${}^1_1\text{H}^1$ [Magnetic moment from]

$$P=1 \rightarrow 2S_{1/2} \quad j=1/2 \quad \ell=0$$

$N=0 \rightarrow$ spin zero

Spin = $1/2$, Parity even

From options

(a) ${}^3_1\text{H}^3 \quad P=1 \rightarrow 2S'_{1/2} \quad J=1/2, \ell=0$

$N=2 \rightarrow$ spin zero

spin = $1/2$

Parity = even } option $\rightarrow$ correct

(b) ${}^2_1\text{H}^2$

$$P=1 \rightarrow 2S'_{1/2} = j_1 = 1/2, \ell_1 = 0$$

$$N=1 \quad 2S'_{1/2}, \quad j_2 = 1/2, \ell_2 = 0$$

$$\text{Northern rule} \rightarrow j_1 + j_2 + \ell_1 + \ell_2 = \frac{1}{2} + \frac{1}{2} + 0 + 0 = 1(\text{odd})$$

$$\text{Spin } (J_1 + j_2) = \frac{1}{2} + \frac{1}{2} = 1$$

Spin = 1 $\rightarrow$ (wrong option), Parity $\rightarrow$ even

(c) ${}^4_2\text{He}^4$

$P=2$ spin zero (wrong)

$N=2$

(d) $\text{H}_2\text{O} \rightarrow M=18g \rightarrow$ That is $1e^-$ free with 1 gm

So No of $e^- = 18$ (even)

So spin = 0 (wrong)



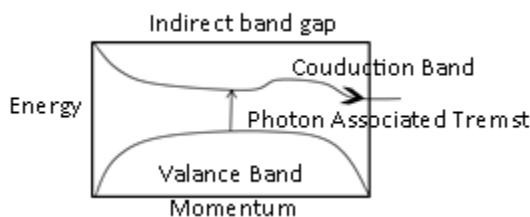
Option (a) is correct answer

- Q20.** An electron makes a transition from the valence band to the conduction band in an indirect band gap semiconductor. Which of the following is NOT true?
- (a) There is no momentum change in the electron.
 - (b) A phonon is involved in the process.
 - (c) A photon is absorbed in the process.
 - (d) The energy of the electron increases

Ans.(a) In semiconductor physics Band gap are two type

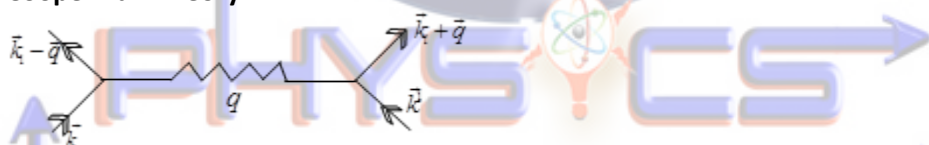
1. Band Gap (i) Direct Band gap (ii) Indirect Band gap

(i) Direct Band Gap : In this momentum of electrons and holes is the same in both the conduction band and valence band and electron can directly emit a photon. Ex. GaAs, InAs
 (ii) Indirect Band gap: Momentum of electrons & holes not the same in conduction band & valence band. In this band a photon can not be emitted because the electron must pass through an intermediate state and transfer momentum to the crystal lattice. Ex. Si, Ge, A/sb



Option (a) is correct answer

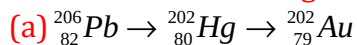
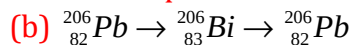
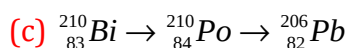
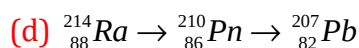
Cooper Pair Theory



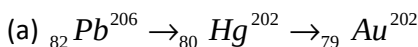
Electron-photon electron interaction

Option (a) correct

- Q21.** Which of the following radioactive decay chains is it possible to observe?



Ans.(c) If $\left. \begin{array}{l} \text{no. of } p = \text{even} \\ \text{no. of } N = \text{even} \end{array} \right\}$ Then stable (Not Radioactive decay)



P = 82, N = 124

(stable nuclei)

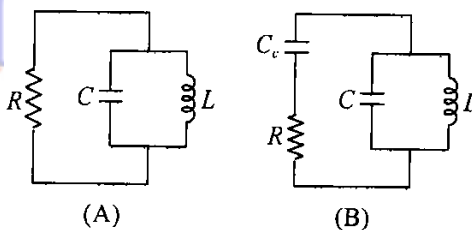
- | | | |
|-----|--|------------------------------|
| (b) | $ \begin{array}{lll} {}_{87}\text{Pb}^{206} \rightarrow & {}_{83}\text{Bi}^{206} \rightarrow & {}_{82}\text{Pb}^{206} \\ P = 87 & P = 83 & P = 82 \\ N = 119 & N = 123 & N = 124 \\ (\text{possible}) & (\text{Not possible}) & \end{array} $ | Wrong options in sir's paper |
| (c) | $ \begin{array}{lll} {}_{83}\text{Bi}^{210} \rightarrow & {}_{84}\text{Po}^{210} \rightarrow & {}_{82}\text{Pb}^{206} \\ P = 83 & P = 84 & P = 82 \\ N = 127 & N = 126 & N = 124 \\ (\text{possible}) & (\text{possible}) & (\text{possible}) \end{array} $ | (more Appropriate) |
| (d) | $ \begin{array}{lll} {}_{88}\text{Ra}^{214} \rightarrow & {}_{86}\text{Pn}^{210} \rightarrow & {}_{82}\text{Pb}^{206} \\ P = 88 & P = 86 & P = 82 \\ N = 126 & N = 124 & N = 125 \\ (\text{possible}) & (\text{possible}) & (\text{possible}) \end{array} $ | (Stable nuclide) |
- (b) ${}_{82}\text{Pb}^{206} \rightarrow {}_{80}\text{Hg}^{202} \rightarrow {}_{79}\text{Au}^{202}$] Tifr paper correct
Not possible

Q22. Which of the following is the best technique for measuring the effective mass of an electron in a semiconductor?

- (a) Millikan's oil drop experiment (b) Cyclotron resonance
 (c) X-ray diffraction experiment (d) Resistivity measurements

Ans.(b) Experimental accuracy is best for cyclotron resonance to measure effective mass of and e^-

Q23. Two LCR circuits (A) and (B) are shown below where $C_c \ll C$. At time $t = 0$, a charge Q is put on the capacitor C .



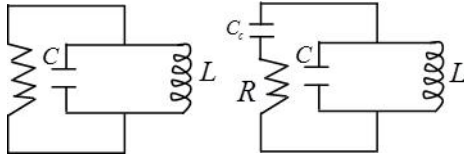
Which of the following statements is correct?

- (a) The charge Q will decay at the same rate in (A) and (B)
 (b) The charge Q will decay faster in (B)
 (c) The charge Q will decay faster in (A)

(d) The relative decay rates cannot be predicted without knowing the exact values of L, C, R and C_c

Ans.(c) It tells about Decay of charge

That is Discharging case of charge

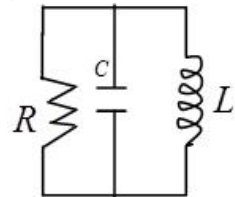


Discharging condition

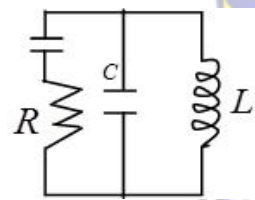
$$Q = Q_0 (1 - e^{-t/RC})$$

$$\tau = RC = \text{time constant}$$

(Parallel LCR circuit use only this loop



Us only this loop In this case C_c & C are in series



Then

$$\text{Ist } Q = Q_0 (1 - e^{-t/RC}) \quad \text{IInd } Q = Q_0 \left(1 - e^{-t \frac{(C_c + C)}{R(C_c C)}} \right)$$

It will understand by all example

At $R = 5k\Omega = \text{fix}$

$$\text{And } \left. \begin{array}{l} C = 10\mu F \\ C_c = 2\mu F \end{array} \right\} \because C_c \gg C$$

$$\text{I want to } \frac{t}{RC} = 0$$

So

$$RC = 5 \times 10 = 50$$

$$\therefore RC > R(C_c C)$$

$$(C_c + C)$$

And also here

$$\frac{t(C_c + C)}{R(C_c \cdot C)} = 0$$

$$\frac{R(C_c \cdot C)}{(C_c + C)}$$

$$\frac{5(10 \times 2)}{12} = \frac{100}{12}$$

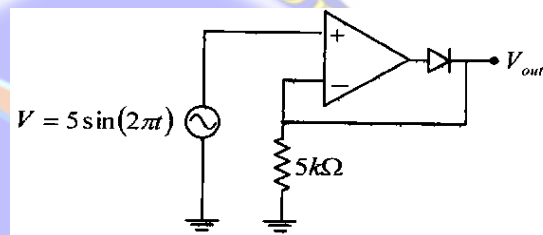
$$= 7.5$$

So

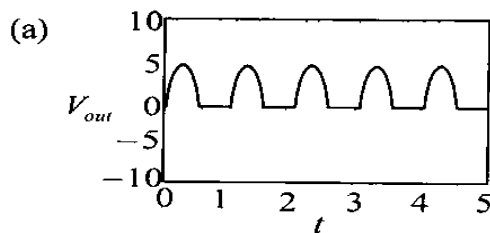
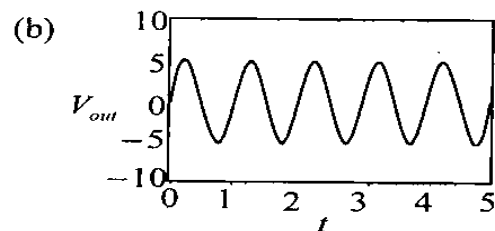
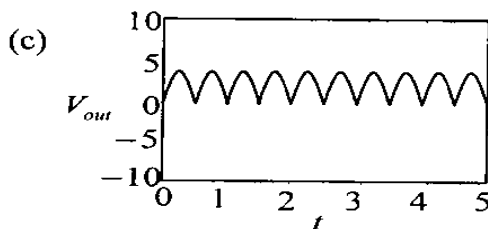
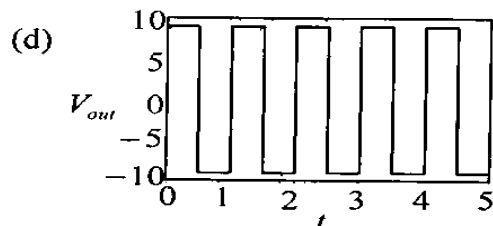
Q will be decay faster is case – 1st

Option (c) is correct answer

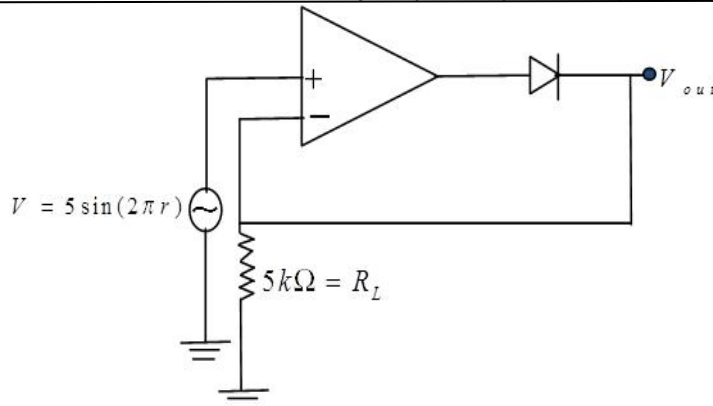
Q24. In the circuit shown below, the op-amp is powered by a bipolar supply of $\pm 10V$.



Which one of the following graphs represents V_{out} correctly?



Ans.(a)



op-Amp. Will work when diode

wire be short

Otherwise we obtained

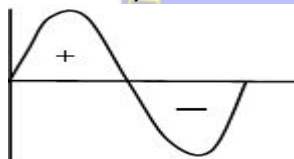
$$V_{out} = 0$$

→ when diode wire be short then output voltage drop across $R_L = 5k\Omega$.

Now,

Couditions

(i) +ve halft cycle



Here inverting terminal

(V_{out} के across)

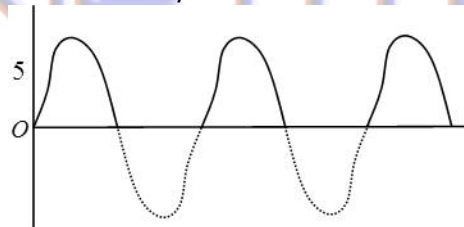
So output voltage obtained also -ive so diode in forward condition and wire be short

Or

In non-converting case for +ve half cycle diode in formed condition and

$$V_{out} = 5 \text{ volt}$$

(ii) For - ve half cycle



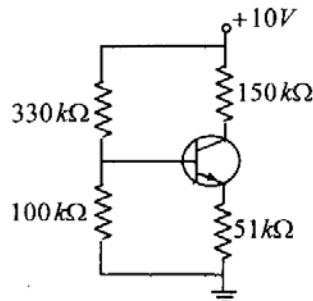
Diode will be in Reverse biase conditions and

$$V_{out} = 0$$

Then

Option (a)

Q25. All resistors in the circuit on the right have a tolerance of $\pm 5\%$. Assuming a diode drop of $0.7V$, which of the following is the lowest possible value of the collector voltage?



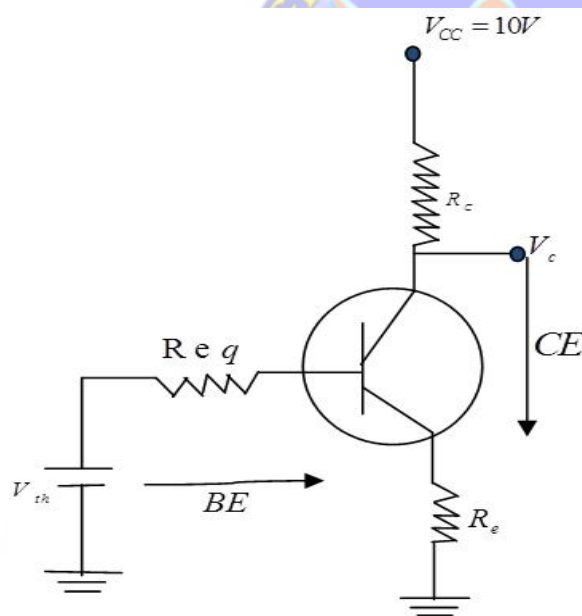
(a) $4.7V$

(b) $5.2V$

(c) $3.1V$

(d) $4.1V$

Ans.(d)



Tolerance = 5%

$$150 \times \frac{5}{100} = \frac{75}{10} = 7.5$$

$$(150 \pm 7.5)$$

$$(142.5 \text{ to } 157.5)$$

$$\text{And } 51 \times \frac{5}{100} = \frac{255}{100} = 2.55$$

$$(51 \pm 2.55)$$

$$(48.45 \text{ to } 53.55)$$

$$330 \times \frac{5}{100} = 16.5 (313.5 \text{ to } 346.5)$$

$$100 \times \frac{5}{100} = 5(95 \text{ to } 105)$$

$$V_{CC} - I_C R_C = V_C$$

V_C should be lowest

$$I_C \uparrow, R_C \uparrow, V_{CC} = 10 \text{ fix}$$

$$\uparrow V_p = \frac{R_2 V_{CC}}{R_1 + R_2} = \frac{V_{CC}}{1 + \frac{R_1}{R_2} \downarrow}$$

$$= \frac{10}{1 + \frac{313.5}{105}} = 2.5089$$

$$I_C \uparrow = \frac{V_p \uparrow - V_{BE} \downarrow}{R_E \downarrow} = \frac{2.5089 - 0.7}{48.45} = 0.0373 \text{ ma}$$

$$V_{CC} = R_C I_C + V_C$$

$$V_C = 10 - R_C I_C$$

$$V_C = 10 - 0.0373 \times 157.5$$

$$= 4.12 \text{ V}$$

GS-2015(Physics)

Section B

Q26. Consider the differential equation

$$\frac{d^2 y}{dx^2} = -4 \left(y + \frac{dy}{dx} \right)$$

with the boundary condition that $y(x) = 0$ at $x = \frac{1}{5}$. When plotted as a function

of x , for $x \geq 0$, we can say with certainty that the value of y

(a) first increases, then decreases to zero

(b) first decreases, then increases to zero

(c) has an extremum in the range $0 < x < 1$

(d) oscillates from positive to negative with amplitude decreasing to zero

Ans.(c) $\frac{d^2 y}{dx^2} = -4 \left(y + \frac{dy}{dx} \right)$

$$D^2 y = -4(y + Dy)$$

$$(D^2 + 4D + 4)y = 0$$

Auxiliary equation

$$M^2 + 4M + 4 = 0$$

$$M^2 + 2M + 2M + 4 = 0$$

$$M(M + 2) + 2(M + 2) = 0$$

$$(M + 2)(M + 2) = 0$$

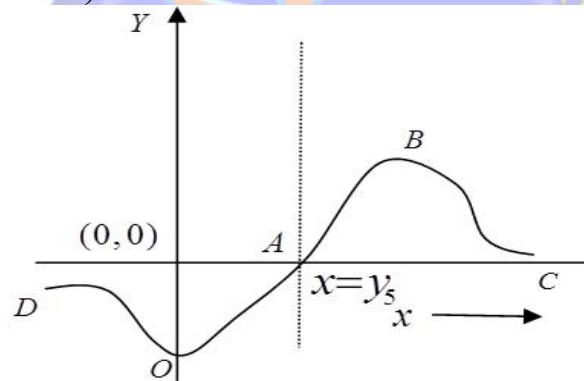
$$M = -2, -2$$

$$y = (C_1 + C_2 x)e^{-2x}$$

Boundary conditions

$$\text{At } x = \frac{1}{5}, Y = 0$$

$$0 = \left(C_1 + \frac{C_2}{5} \right) e^{-2/5}$$



$$C_1 = -\frac{C_2}{5}$$

$$Y = \left(-\frac{C_2}{5} + C_2 x \right) e^{-2x}$$

$$Y = C_2 \left[x - \frac{1}{5} \right] e^{-2x}$$

$$\frac{dy}{dx} = -\frac{1}{5} C_2 (-2) e^{-2x} + (1 - 2x) e^{-2x}$$

$$\text{For extremum } \frac{dy}{dx} = 0$$

$$2x = \frac{7}{5} \Rightarrow x = \frac{7}{10} = 0.7$$

$$= 0 < 0.7 < 1$$



Option (c) is correct

Q27. In a transmission diffraction grating, there are 10^4 lines/mm. Which of the following ranges of wavelength (in nm) will produce at least one principal maximum?

- (a) 1001-5000 (b) 501-1000 (c) 201-500 (d) 1- 200

Ans.(d) $(e+b)=1/10^4$ lines/mm

$$(e+b) = \frac{1}{10^4} \text{ mm}$$

$$(e+b) = 10^{-7} \text{ meter}$$

We know that about transmission diffraction grating

$$(e+b) \sin \theta = n\lambda$$

For Maximo $\theta = 90^\circ$, $\sin \theta = 1$

$$(e+b) = n\lambda$$

For Absent spectrum of n^{th} order

$$e \sin \theta = n\lambda$$

... (1) (differection Minima)

$$(e+b) \sin \theta = m\lambda$$

... (2) (Interference maxima)

$$\frac{(e+a) \sin \theta}{e \sin \theta} = \frac{m\lambda}{n\lambda}$$

$$\frac{(e+b)}{e} = \frac{m}{n}$$

Case Ist :

When $e = b$

$$\text{For } n = 1, 2, 3 \quad \frac{2e}{e} = \frac{m}{n} \quad \boxed{m = 2n}$$

$$\boxed{M = 2, 4, 6, \dots} \rightarrow \text{Missing}$$

I^{st} Maxima, at $m = 1$

$$(e+b) \sin \theta = m\lambda$$

$$\left(\frac{e+b}{m} \right) = \lambda$$

$$\lambda = 10^{-7}$$

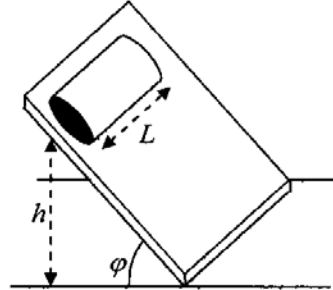
$$\boxed{\lambda = 100 \text{ nm}}$$

In the rangal $\boxed{1 - 200 \text{ nm}}$

Option (d) is correct answer.

Q28. Two cylinders A and B of the same length L and outer radius R were placed at the same height h on an inclined plane at an angle ϕ with the horizontal (see

figure). Starting from rest, each cylinder was allowed to roll down the plane without slipping. It was found that A reached the end of the inclined plane earlier than B . Which of the following possibilities could be true?



- (a) A is solid and made of copper; B is hollow and made of copper.
- (b) A is solid and made of copper; B is solid and made of aluminium.
- (c) A is hollow and made of aluminium; B is solid and made of aluminium.
- (d) A is hollow and made of copper; B is hollow and made of copper; B is heavier than A .

Ans.(a) $V = \left[\frac{2gh}{1 + \frac{k^2}{r^2}} \right]^{\frac{1}{2}}$

$V_A > V_B$

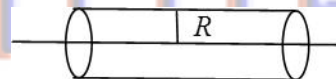
$\frac{V_A}{V_B} = \left\{ \frac{\left[1 + \frac{k^2}{r^2} \right]_B}{\left[1 + \frac{k^2}{r^2} \right]_A} \right\}^{1/2}$

Then $k_B > k_A$

So

Moment of inter of solid cylinder

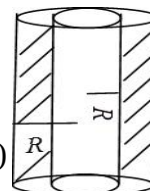
$$K_A = \frac{MR^2}{2}$$



Moment of inertia of Hollow cylinder $K_B = \frac{M}{2}(R^2 + r^2)$

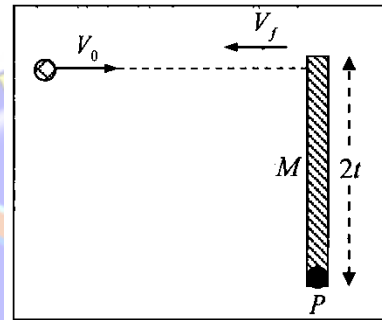
$\therefore k_B > k_A$

So



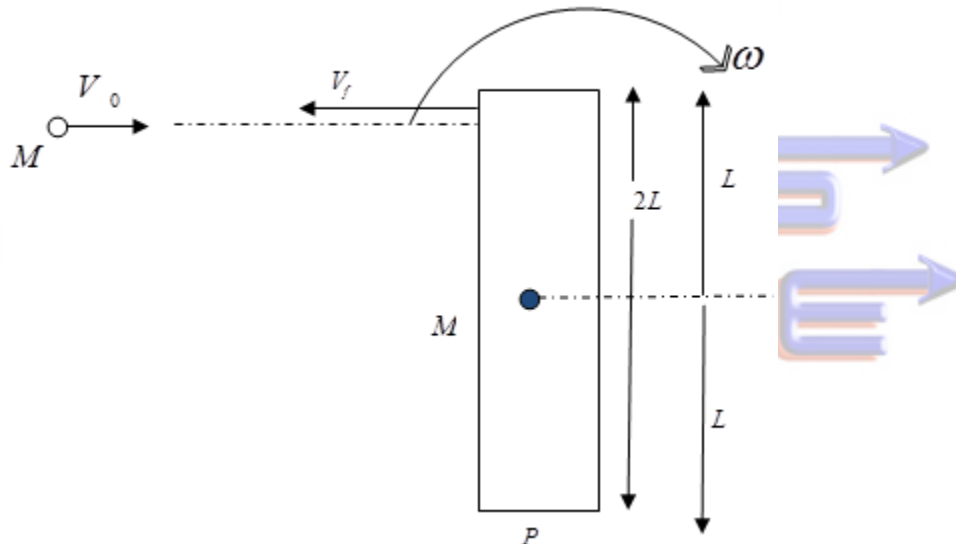
$A \rightarrow$ will be solid
 And $B \rightarrow$ will be hollow
 Option (a) will be correct answer

- Q29. A thin uniform rod of length $2l$ and mass M is pivoted at one end P on a horizontal plane (see figure). A ball of mass $m \ll M$ and speed v_0 strikes the free end of the rod perpendicularly and bounces back with velocity v_f along the original line of motion as shown in the figure. If the collision is perfectly elastic the magnitude of v_f is



- (a) $\frac{M+3m}{M-3m}v_0$ (b) $\frac{M-3m}{M+3m}v_0$ (c) $\frac{M-4m}{M+4m}v_0$ (d) $\frac{M+4m}{M-4m}v_0$

Ans.(b)



$\therefore$ collision is perfectly elastic
 Then K.E. & linear momentum will be conserved
 And also Angular momentum conserved
 Then

$$I_p = \frac{ML^2}{3}, L = 2l$$

From angular momentum conservation

$$mV_0(2L) = mV_f(2L) + I\omega$$

$$mV_0(2L) = mV_f(2L) + \frac{M(2L)^2}{3}\omega$$

$$mV_0 - mV_f = \frac{M(2L)}{3}\omega$$

$$\omega = \frac{3m}{2LM}(V_0 - V_f)$$

From energy conservation

$$\frac{1}{2}mV_0^2 = \frac{1}{2}mV_f^2 + \frac{1}{2}\frac{M(2L)^2}{3}\omega^2$$

$$m(V_0^2 - V_f^2) = \frac{4ML^2}{3} \left[\frac{9m^2(V_0 - V_f)^2}{4L^2M^2} \right]$$

$$m(V_0 - V_f)(V_0 + V_f) = \frac{3Mm^2}{M^2}(V_0 - V_f)^2$$

$$V_0 + V_f = \frac{3m}{M}(V_0 - V_f)$$

$$V_0 - \frac{3V_0m}{M} = \left(\frac{-3m}{M} + 1 \right) V_f$$

$$V_f \left(\frac{M + 3m}{M} \right) = \left(\frac{3m - M}{M} \right) V_0$$

$$V_f = \frac{(M - 3m)V_0}{(M + 3m)}$$

Option (b)

- Q30.** A collimated beam of pions originate from an accelerator and propagates in vacuum along a long straight beam pipe. The intensity of this beam was measured in the laboratory after a distance of 75m and found to have dropped to one-fourth of its intensity at the point of origin. If the proper half-life of a pion is 1.77×10^{-8} s, the speed of the pions in the beam, as measured in the laboratory, must be

(a) 0.99c

(b) 0.98c

(c) 0.96c

(d) 0.97c

Ans.(a) $L = 75$ m



$$\tau_0 = 1.77 \times 10^{-8} \text{ s}$$

$$\tau = \frac{75}{C} = \frac{75}{3 \times 10^8}$$

$$\tau = \frac{\tau_0}{\sqrt{1 - \frac{V^2}{C^2}}}$$

$$25 \times 10^{-8} = \frac{1.77 \times 10^{-8}}{\sqrt{1 - \frac{V^2}{C^2}}}$$

$$1 - \frac{V^2}{C^2} = \left(\frac{1.77}{25} \right)^2$$

$$1 - \left(\frac{1.77}{25} \right)^2 = \frac{V^2}{C^2}$$

$$(1 - 5.0126 \times 10^{-3}) = \frac{V^2}{C^2}$$

$$V^2 = 0.99 C^2$$

$$V = 0.99 C$$

Correct option (a)

Q31. The equation of state of a gas is given by $V = \frac{RT}{P} - \frac{b}{T}$

where R is the gas constant and b is another constant parameter. The specific heat at constant pressure C_p and the specific heat at constant volume C_v for this gas is related by $C_p - C_v = C$

(a) R (b) $R \left(1 + \frac{RT^2}{bP} \right)^2$ (c) $R \left(1 + \frac{bP}{RT^2} \right)^2$ (d) $R \left(1 - \frac{bP}{RT^2} \right)^2$

Ans.(i) $C_p - C_v = R \left[1 + \frac{2a}{VRT} \right]$ (for vander waul gas eqⁿ)

$$C_p - C_v = -T \left(\frac{\partial V}{\partial T} \right)_P^2 \left(\frac{\partial P}{\partial V} \right)_T \quad \dots(1)$$

$$C_p - C_v = T \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial T} \right)_P$$

Then

Eqⁿ of state

$$V = \frac{RT}{P} - \frac{b}{T} - (1) \Rightarrow \boxed{\frac{RT}{P} = V + \frac{b}{T}}$$

$$\frac{\partial V}{\partial T} = \frac{R}{P} + \frac{b}{T^2}$$

Form eqⁿ (1)

$$V + \frac{b}{T} = \frac{RT}{P}$$

$$P = \frac{RT}{\left(V + \frac{b}{T}\right)} \Rightarrow \left(\frac{\partial P}{\partial V}\right)_T = \frac{-RT(1)}{\left(V + \frac{b}{T}\right)^2}$$

From eqⁿ(2)

$$C_p - C_v = -T \left(\frac{R}{P} + \frac{b}{T^2} \right) \left(\frac{-RT}{\left(V + \frac{b}{T}\right)^2} \right)$$

$$C_p - C_v = \frac{RT^2 R^2 \left(1 + \frac{bP}{RT^2}\right)^2}{P^2 \left(\frac{RT}{P}\right)^2} \quad \{\text{From eq}^n (1)\}$$

$$\boxed{C_p - C_v = R \left(1 + \frac{bP}{RT^2}\right)^2}$$

Method IInd

$$C_p - C_v = \frac{VT\alpha^2}{k_T}$$

$$\alpha = + \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P$$

$$k_T = - \frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$$

On solution

$$C_p - C_v = R \left(1 + \frac{bP}{RT^2}\right)^2$$

Option (c) is correct answer

- Q32. An ideal diatomic gas is initially at a temperature $T = 0^\circ\text{C}$. Then it expands reversibly and adiabatically to 5 times its volume. Its final temperature will be approximately

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(a) 0°C

(b) -150°C

(c) -130°C

(d) -180°C

Ans.(c) Ideal gas $\rightarrow$ Diatomic

$$C_p = \frac{7}{2}R, C_v = \frac{5}{2}R$$

Initial volume, $\rightarrow v$

Final volume $\rightarrow 5v$

$$T_1 = 273 = 273\text{K}$$

$$T_2 = ??$$

For Adiabatic process

$$T_1 V_1^{r-1} = T_2 V_2^{r-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{r-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{r-1}$$

$$T_2 = 273 \left(\frac{V}{5V} \right)^{\frac{7}{5}-1}$$

$$T_2 = 273 \left(\frac{1}{5} \right)^{2/5}$$

$$T_2 = 273 \times 0.5253$$

$$T_2 = 143.4\text{K}$$

In $^\circ\text{C}$

$$T_2 = 143.4 - 273\text{K}$$

$$T_2 = -129.59^\circ\text{C}$$

$$T_2 = -130^\circ\text{C}$$

Option (c)

Q33. Consider an infinitely long cylinder of radius R , placed along the z -axis, which carries a static charge density $\rho(r) = kr$, where r is the perpendicular distance from the axis of the cylinder and k is a constant. The electrostatic potential $\phi(r)$ inside the cylinder is proportional to

(a) $-\frac{2}{3} \left(\frac{r^3}{R^3} - 1 \right)$

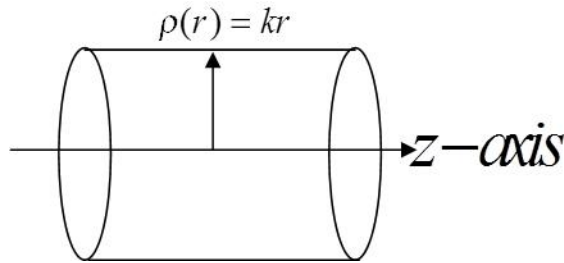
(b) $-2 \ln \left(\frac{r}{R} \right)$

(c) $-\frac{2}{3} \left(\frac{r^3}{R^3} + 1 \right)$

(d) $-2 \ln \left(\frac{R}{r} \right)$

Ans.(a) Electric field outside the cylinder

$$\int \vec{E} \cdot d\vec{s} = \frac{q_{en}}{\epsilon_0}$$



$$\int \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} \int \rho \cdot d\tau$$

$$E(2\pi r \ell) = \frac{1}{\epsilon_0} \int k r r dr \int d\phi \int dz$$

$$E(2\pi r \ell) = \frac{k}{\epsilon_0} \int_0^R r^2 dr (2\pi \ell)$$

$$E_{out} = \frac{k}{3\epsilon_0} \frac{R^3}{r}$$

Inside the electric field

$$\int \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \int \rho d\tau$$

$$E(2\pi r \ell) = \frac{1}{\epsilon_0} \int_0^r k r \cdot r dr \int d\phi \int dz$$

$$E(2\pi r \ell) = \left(\frac{2\pi \ell}{\epsilon_0} \right) k \left(\frac{r^3}{2} \right)$$

$$E_{in} = \frac{k}{3\epsilon_0} r^2$$

Electrostatic Potential

$$\phi(r) = \int_{R'}^R E_{out} \cdot dr - \int_R^r E_{in} \cdot dr$$

$$\phi(r) = \frac{KR^3}{3\epsilon_0} \int_{R'}^R \frac{1}{r} dr - \frac{k}{3\epsilon_0} \int_R^r r^2 dr$$

$$\phi(r) = \frac{KR^3}{3\epsilon_0} \ln\left(\frac{R}{R'}\right) - \frac{k}{3\epsilon_0} \left[\frac{r^3}{3} \right]_R^r$$

$$\phi(r) = \frac{KR^3}{3\epsilon_0} \ln\left(\frac{R}{R'}\right) - \frac{k}{3\epsilon_0} \left[\frac{r^3}{3} - \frac{R^3}{3} \right]$$

$$\therefore R' > R \quad \ln\left(\frac{R}{R'}\right) \approx 0$$

$$\phi(r) = \frac{k}{3\epsilon_0} \left[\frac{R^3}{3} - \frac{r^3}{3} \right]$$

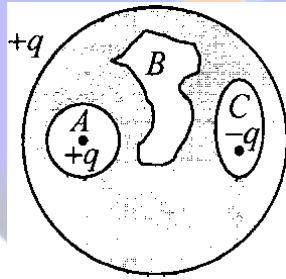
$$\phi(r) = \frac{k}{3\epsilon_0} \frac{R^3}{3} \left(1 - \frac{r^3}{R^3} \right)$$

$$\phi(r) = \frac{-k}{3\epsilon_0} R^3 \left(\frac{1}{3} \right) \left[\frac{r^3}{R^3} = 1 \right]$$

$$\phi(r) \propto - \left(\frac{r^3}{R^3} - 1 \right)$$

Option (a)

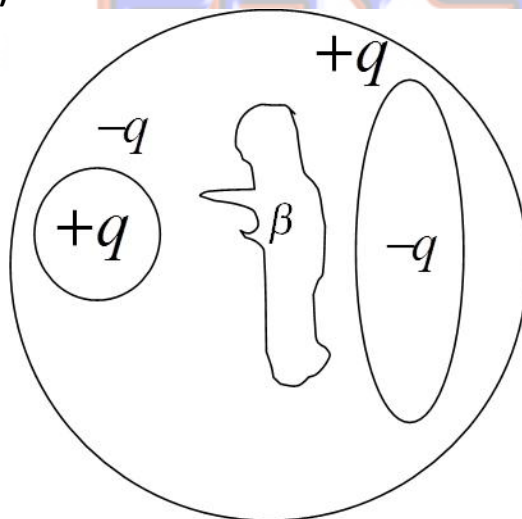
- Q34.** A solid spherical conductor encloses 3 cavities, a cross-section of which are as shown in the figure. A net charge $+q$ resides on the outer surface of the conductor. Cavities A and C contain point charges $+q$ and $-q$, respectively. The net charges on the surfaces of these cavities are



- (a) $A = -q, B = 0, C = -q$
 (c) $A = -q, B = 0, C = -q$

- (b) $A = -q, B = q, C = 0$
 (d) $A = -q, B = 0, C = +q$

Ans.(d)



So

$$A = -q$$

$$B = 0$$

$$\text{And } C = +q$$

Correct option (d)

- Q35. 1000 neutral spinless particles are confined in a one-dimensional box of length 100nm . At a given instant of time, if 100 of these particles have energy $4\epsilon_0$ and the remaining 900 have energy $225\epsilon_0$ then the number of particles in the left half of the box will be approximately
- (a) 441 (b) 100 (c) 500 (d) 625

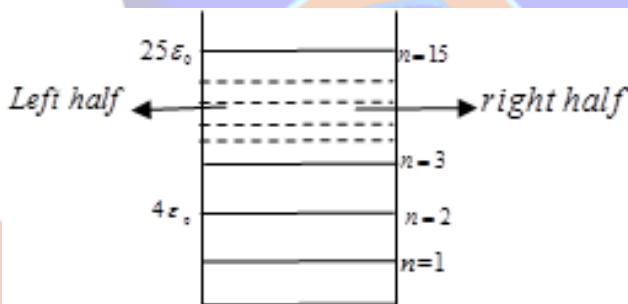
Ans.(c) Total no. of particles = 1000

$$a = 100\text{ nm}$$

$$100\text{ particles have energy} = 4\epsilon_0$$

$$900\text{ particles have energy} = 225\epsilon_0$$

For left half probability in any state is $\frac{1}{2}$ of $50 + 450 = 500$ particles will be there in left half of the box.



Correct option (c)

- Q36. A one-dimensional quantum harmonic oscillator is in its ground state

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}$$

Two experiments, [A] and [B] are performed on the system. In [A] the frequency ω of the oscillator is suddenly doubled, while in [B] the frequency ω is suddenly halved. If p_A and p_B denote the probability in each case that the system is found in its new ground state immediately after the frequency change, which of the following is true?

- (a) $p_A = 2p_B$ (b) $p_A = p_B$ (c) $2p_A = p_B$ (d) $p_A = \sqrt{2}p_B$

Ans.(d) $\psi_0(x) = \left(\frac{M\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{M\omega}{2\hbar}x^2\right)$



In experiment A

$$\omega \rightarrow 2\omega$$

$$\omega'_a(n) = \left(\frac{m(2\omega)}{\pi\hbar} \right)^{1/4} \exp\left(-\frac{m\omega}{\hbar} x^2 \right)$$

$$P_A = \left| \int \psi_0(n) \psi'_A(x) dx \right|^2$$

$$P_A = \left| 2^{1/4} \left(\frac{m\omega}{\pi\hbar} \right)^{1/2} \int_{-\infty}^{+\infty} \exp\left(-\frac{3}{2} \frac{m\omega}{\hbar} x^2 \right) dx \right|^2$$

$$\left| 2^{1/4} \left(\frac{m\omega}{\pi\hbar} \right)^{1/2} 2 \times \frac{1}{2} \frac{\sqrt{\frac{0+1}{2}}}{\left(\frac{3}{2} \frac{m\omega}{\hbar} \right)^{1/2}} \right|^2$$

$$= \left| 2^{1/4} \times 2^{1/2} / 3^{1/2} \right|^2 = \left| \frac{2^{3/4}}{3^{1/2}} \right|^2$$

$$P_A = \frac{2^{3/2}}{3}$$

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar} \right)^{1/4} \exp\left(-\frac{m\omega}{2\hbar} x^2 \right)$$

$$\psi_B(x) = \left(\frac{m\omega}{2\pi\hbar} \right)^{1/4} \exp\left(-\frac{m\omega}{4\hbar} x^2 \right)$$

$$P_B = \left| \frac{1}{2^{1/4}} \left(\frac{m\omega}{\pi\hbar} \right)^{1/2} \int_{-\infty}^{\infty} \exp\left(-\frac{3}{4} \frac{m\omega}{\hbar} x^2 \right) dx \right|^2$$

$$= \left| \frac{1}{2^{1/4}} 2 \times \frac{1}{2} \frac{\sqrt{\frac{0+1}{2}}}{\left(\frac{3}{4} \frac{m\omega}{\hbar} \right)^{1/2}} \left(\frac{m\omega}{\pi\hbar} \right)^{1/2} \right|^2$$

$$= \left| \frac{1}{2^{1/4}} \frac{2}{3^{1/2}} \right|^2$$

$$= \frac{2^{3/2}}{3} = P_B$$

$$P_A = P_B$$

Option (d)

Q37. In the basic band structure theory of crystalline solids, which of the following leads to energy gaps in the allowed electronic energy values?

- (a) Bragg reflection (b) Electron spin
(c) Electron-phonon interaction (d) Electron-electron interaction

Ans. (a)

Bragg reflection : $n\lambda = 2d \sin \theta$

$\therefore$ Energy gaps in crystalline solid can be given by Bragg reflection

Basic structure H of energy

Q38. The material inside a box is either a metal or a semiconductor. If $R (=1 \Omega)$ is the resistance of the material, which of the following experiments CANNOT distinguish whether it is a metal or a semiconductor?

- (a) Measurement of R at different temperatures.
(b) Measurement of R using power supplies of different frequencies.
(c) Measurement of absorption spectrum in the energy range $0.1 - 2eV$.
(d) Measurement of R in the presence of different magnetic fields.

Ans.(b) Measurement of R using power supplies of different frequencies.

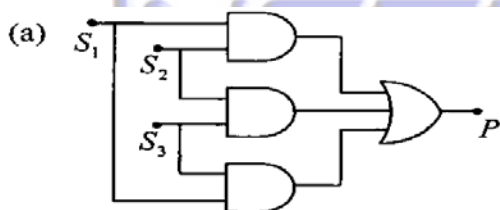
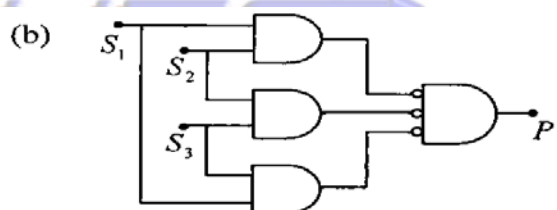
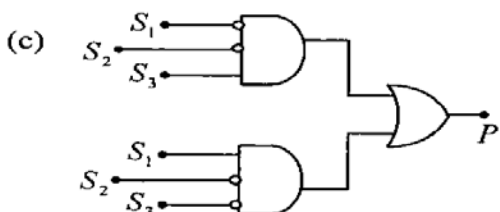
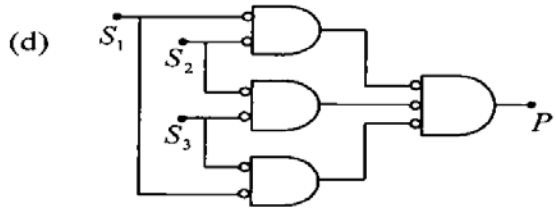
Correct Answer (b)

Q39. To measure the voltage in the range $0-5V$ with a precision of $5 mV$, the minimum number of bits required in a digital voltmeter is

- (a) 12 (b) 9 (c) 11 (d) 10

Ans.(d)

Q40. A building has three overhead water tanks, each fitted with a sensor (S_1, S_2, S_3) which goes to 0 when the water level in the tank falls below a set value and remains 1 otherwise. A common pump is used to raise water from an underground storage tank to these overhead tanks. Of the following circuits, which one will turn on ($P=1$) the pump only when at least two of the tanks have water level below the set value?



Ans.(b) According to questions

Two value will be below from pre set value

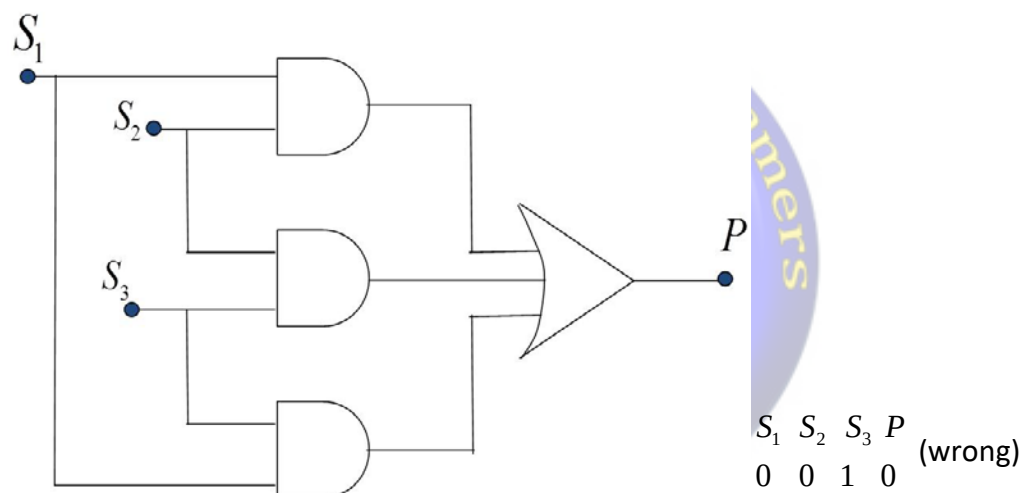
The there combination will be possible

S_1	S_2	S_3	
0	0	1	[and here output will = +1]
0	1	0	
1	0	0	

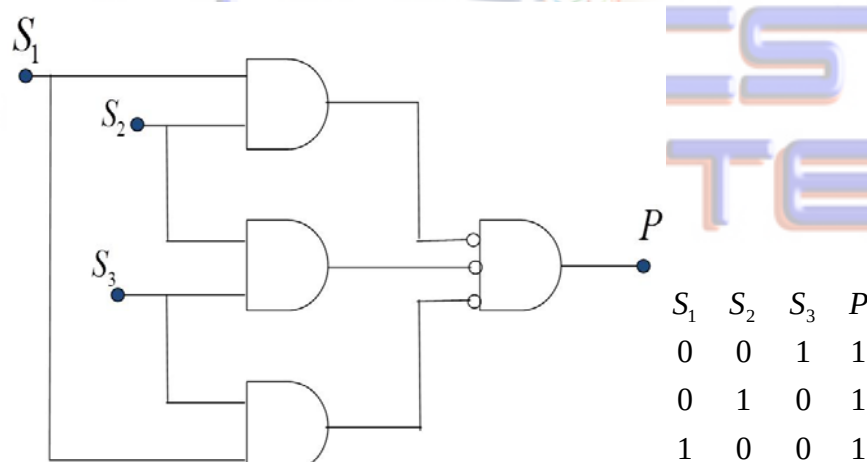
And other wise conditions are also possible and these condition P wire be O.

From Options :

(a)

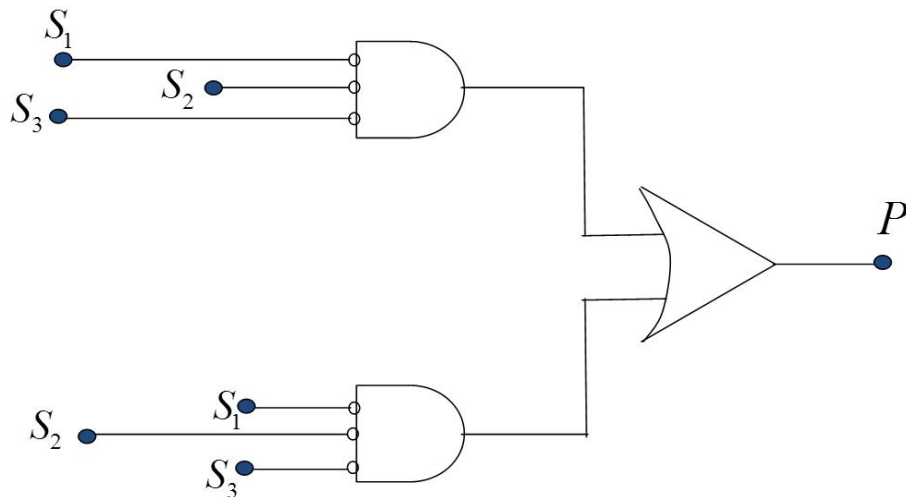


(b)

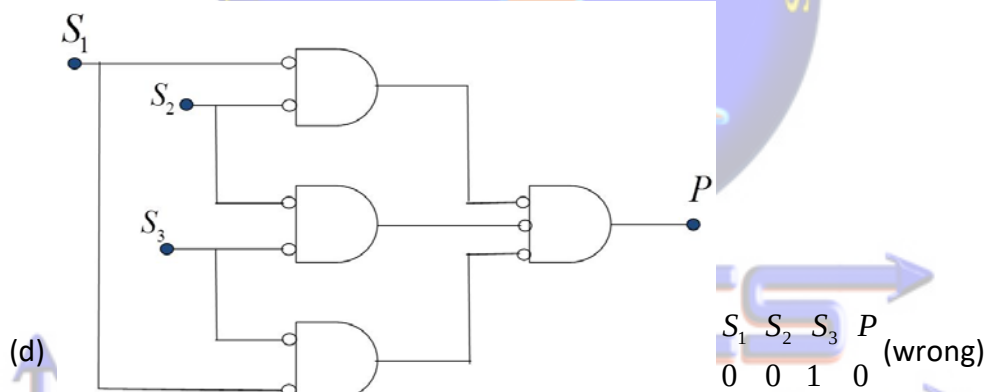


Option (b) may be correct answer

(c)



(wrong)



(d)

Then option (b) is correct answer

GS-2015-Y Physics Section C

Q41. The integral $\int_0^{2\pi} \frac{d\theta}{1-2a \cos \theta + a^2}$ where $0 < a < 1$, evaluate to

- (a) $\frac{2\pi}{1-a^2}$ (b) $\frac{2\pi}{1+a^2}$ (c) 2π (d) $\frac{4\pi}{1-a^2}$

Q42. The generating function for a set of polynomials in x is given by

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$$f(x, t) = (1 - 2xt + t^2)^{-1}$$

The third polynomial (order $2x$) in this set is

- (a) $2x^2 + 1$ (b) $4x^2 + 1$ (c) $2x^2 - 1$ (d) $4x^2 - 1$

Q43. A particle moves under the influence of a central potential in an orbit $r = k\theta^4$, where k is a constant and r is the distance from the origin. It follows that the angle θ varies with time t as

- (a) $\theta \propto t^{\frac{1}{7}}$ (b) $\theta \propto t^{\frac{1}{8}}$ (c) $\theta \propto t^{\frac{1}{9}}$ (d) $\theta \propto t^{\frac{1}{4}}$

Q44. In a system with two degrees of freedom, if (p, q) are the canonical coordinates, then which of the following transformations to (P, Q) is canonical?

- (a) $P = \frac{1}{2}(p^2 + q^2), Q = \tan^{-1} \frac{2q}{p}$ (b) $P = \frac{1}{2}(p^2 + q^2), Q = \cot^{-1} \frac{p}{q}$
 (c) $P = \frac{1}{2}(p^2 + q^2), Q = \sin^{-1} \frac{q}{2p}$ (d) $P = \frac{1}{2}(p^2 + q^2), Q = \cos^{-1} \frac{p}{q}$

Q45. In a monatomic gas, the first excited state is only $1.5eV$ above the ground state, while the other excited states are much higher up. The ground state is doubly-degenerate, while the first excited state has a four-fold degeneracy. If now, the gas is heated to a temperature of $7000K$, the fraction of atoms in the excited state will be approximately

- (a) 0.42 (b) 0.3 (c) 0.14 (d) 0.07

Q46. Measurement of the magnitudes of the electric field (E) and the magnetic field (B) in a plane-polarised electromagnetic wave in vacuum leads to the following results $\frac{\partial E}{\partial y} = \frac{\partial B}{\partial t}, \frac{\partial B}{\partial y} = -\frac{1}{c^2} \frac{\partial E}{\partial t}$ at all points where the measurement is made. In

this case the electric vector $\vec{E}$, the magnetic vector $\vec{B}$ and the wave vector $\vec{k}$ (with magnitude k) can be written in terms of the unit vectors $(\hat{x}, \hat{y}, \hat{z})$ along the Cartesian axes as

- (a) $\vec{E} = E\hat{x}, \vec{B} = B\hat{z}, \vec{k} = -k\hat{y}$ (b) $\vec{E} = E\hat{x}, \vec{B} = -B\hat{z}, \vec{k} = k\hat{y}$
 (c) $\vec{E} = E\hat{x}, \vec{B} = B\hat{y}, \vec{k} = k\hat{z}$ (d) $\vec{E} = -E\hat{y}, \vec{B} = -B\hat{z}, \vec{k} = -k\hat{x}$

Q47. A two-state quantum system has two observables A and B . It is known that the observable A has eigenstates $|\alpha_1\rangle$ and $|\alpha_2\rangle$ with eigenvalues a_1 and a_2 respectively, while B has eigenstates $|\beta_1\rangle$ and $|\beta_2\rangle$ with eigenvalues b_1 and b_2 respectively, and that these eigenstates are related by

$|\beta_1\rangle = \frac{3}{5}|\alpha_1\rangle - \frac{4}{5}|\alpha_2\rangle$ $|\beta_2\rangle = \frac{4}{5}|\alpha_1\rangle + \frac{3}{5}|\alpha_2\rangle$ Suppose a measurement is made of the observable A and a value a_1 is obtained. If the observable B is now measured, the probability of obtaining the value b_1 will be

(a) 0.80 (b) 0.64 (c) 0.60 (d) 0.36

Q48. An rigid rotator has the wave function

$$\psi(\theta, \varphi) = N(\theta, \varphi) + (2+i)Y_{2-1}(\theta, \varphi)$$

where $Y_{l,m}(\theta, \varphi)$ are the spherical harmonics, and N is a normalization constant. If L' is the orbital angular momentum operator, and $L_z = L_x \pm iL_y$ the expectation value of $L_+ L_-$ is

(a) $\frac{21\hbar^2}{9}$ (b) $\frac{23\hbar^2}{9}$ (c) $\frac{25\hbar^2}{9}$ (d) 0

Q49. In the ground state electronic configuration of nitrogen ($^{14}_7N$) the L, S and J quantum numbers are

(a) $L=1, S=\frac{1}{2}, J=\frac{3}{2}$ (b) $L=0, S=\frac{3}{2}, J=\frac{3}{2}$
(c) $L=0, S=\frac{1}{2}, J=\frac{1}{2}$ (d) $L=1, S=\frac{1}{2}, J=\frac{1}{2}$

Q50. Solar radiation tends to push any particle inside solar system away from the Sun. Consider a spherical dust particle of specific gravity 6.0 and no angular momentum about the Sun. What should be its minimum radius so that it does not escape from the solar system? Take the solar luminosity to be $3.8 \times 10^{26} W$.

(a) $0.1 \mu m$ (b) $10 \mu m$ (c) $0.01 \mu m$ (d) $10^{-6} \mu m$

Q51. Bosonic excitations of ferromagnets have a dispersion relation $e = \gamma k^2$ where e is the energy and k is the wavevector of the excitation. Assuming a system of such noninteracting bosonic excitations, at low temperature T , the specific heat C_V of a threedimensional ferromagnet will be proportional to

(a) T (b) $T^{\frac{5}{2}}$ (c) T^3 (d) $T^{\frac{3}{2}}$

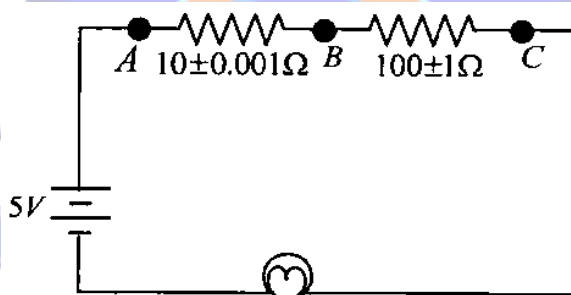
Q52. In the semi-empirical mass formula, the volume (V), surface (S), coulomb (C), and pairing (P) contributions to the binding energy of a nucleus ^A_ZX vary with mass number A as

- (a) $V \propto A$, $S \propto A^{\frac{-2}{3}}$, $C \propto A^{\frac{1}{3}}$, $P \propto A^{\frac{-3}{4}}$
- (b) $V \propto A$, $S \propto A^{\frac{1}{3}}$, $C \propto A^{\frac{-1}{3}}$, $P \propto A^{\frac{-3}{4}}$
- (c) $V \propto A$, $S \propto A^{\frac{2}{3}}$, $C \propto A^{\frac{-1}{3}}$, $P \propto A^{\frac{-3}{4}}$
- (d) $V \propto A^2$, $S \propto A^{\frac{2}{3}}$, $C \propto A^{\frac{-1}{3}}$, $P \propto A^{\frac{-3}{4}}$

Q53. In an experiment, $^{197}_{79}\text{Au}$ nuclei were bombarded with neutrons leading to formation of $^{198}_{79}\text{Au}$, which is unstable. The half-life of $^{198}_{79}\text{Au}$ was measured to be 2.25 days and it was found later that this measured half-life was an underestimate by 10%. The corresponding percentage error in the estimated population of $^{198}_{79}\text{Au}$ after 9 day's is

- (a) 2.5% (b) 10% (c) 15% (d) 25%

Q54. You are given the following circuit and two instruments: a voltmeter and an ammeter both with 0.001% accuracy in their readings. Which of the following methods will result in the most accurate reading for the current without interrupting the current in the circuit?



- (a) Use voltmeter to measure voltage across points A and B
- (b) Use the ammeter to measure current at point B
- (c) Use voltmeter to measure voltage across points B and C
- (d) Use voltmeter to measure voltage across points A and C

Q55. Consider the following reactions involving elementary particles:

- (A) $\pi^- + p \rightarrow K^- + \Sigma^+$
- (B) $K^- + p \rightarrow K^- + p^+$

Which of the following statements is true for strong interactions?

- (a) (A) and (B) are both forbidden
- (b) (B) is allowed but (A) is forbidden
- (c) (A) is allowed but (B) is forbidden
- (d) (A) and (B) are both allowed



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