

Addis Ababa University
College of Natural and Computational Sciences
Department of Statistics
Probability and Statistics for Engineers (Stat2171): Worksheet/Assignment II

1. If the discrete random variable X is given with the following pmf of $p(x) = f(x)$,

X	1	2	4	6
P(x)	1/3	1/6	1/4	1/4

Then find the pmf of $y = 2x + 1$.

2. Suppose that X is uniformly distributed over $(-1, 1)$. Then given the pdf of x find the density function of

$$f(x) = \begin{cases} 1/2 & , \quad -1 \leq x \leq 1 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

a) $Y = 3x + 5$

b) $Y = x^3$

c) $Y = \frac{3}{(x+1)^2}$

d) $Y = 4 - x^2$

3. If X be the random variable with pmf given by:

X	-2	-1	0	1	2
P(x)	1/5	1/6	1/5	1/15	11/30

Then find the pmf of $y = x^2$.

4. If X be the random variable with pdf given by:

$$f(x) = \frac{3}{2} x^2 ; \quad -1 \leq x \leq 1.$$

Then find the pdf of $y = 3 - x$ and $y = x^2$

5. Xavier and Yvette are two real estate agents. Let X and Y denote the number of houses that Xavier and Yvette will sell next week, respectively. Suppose that there are only four houses for sale next week. The joint probability mass function $P(x_i, y_j)$ is given by:

Y \ X	0	1	2
0	0.12	0.42	0.06
1	0.21	0.06	0.03
2	0.07	0.02	0.01

Find: a). $P(X \leq 1, Y = 2)$ b). $P(X = 1)$ c). $P(Y = 2)$ d). $P(Y \geq 1)$ e). $P(X \geq 1, Y \geq 1)$.

6. The joint pdf of random variables X and Y is given by:

$$f(x, y) = 2e^{-x}e^{-2y} ; \quad 0 < x < \infty, \quad 0 < y < \infty.$$

Compute (a). $P\{X > 1, Y < 1\}$ and (b). $P\{X < Y\}$.

7. Suppose that the joint pdf of the two-dimensional random variable (X, Y) is given by

$$f(x, y) = \begin{cases} x^2 + \frac{xy}{3} & , \quad 0 < x < 1, \quad 0 < y < 2 \\ 0 & , \quad \text{otherwise.} \end{cases}$$

Compute the following.

(a) $P(X > 1/2)$

(b) $P(Y < X)$

(c) $P(Y < 1/2 | X < 1/2)$.

8. A candy company distributes boxes of chocolates with a mixture of creams, coffees, and nuts coated in both light and dark chocolate. For a randomly selected box, let X and Y , respectively, be the proportions of the light and dark chocolates that are creams and suppose that the joint density function is given by:

$$f(x, y) = \begin{cases} (2/5)(2x + 3y) & ; \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1. \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

a. Find $P[(X, Y) \in A]$, A is the region $A = \{(x, y) | 0 < x < 1/2, 1/4 < y < 1/2\}$.

b. Find $P[(X, Y) \in B]$, B is the region $B = \{(x, y) | 0 < x < y < 1\}$.

9. Suppose that a pair of random variables, X and Y have the same joint probability density function given by:

$$f(x, y) = \begin{cases} x(1 - y) & ; \quad 0 \leq x \leq 2, \quad 0 \leq y \leq 1. \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

Then find the marginal probability density functions for X and Y .

10. Quality audit records are kept on numbers of major and minor failures of circuit packs during-in of large electronic switching devices. They indicate that for a device of this type, the random variables X (the number of major failures) and Y (the number of minor failures) can be described at least approximately by the accompanying joint distribution.

X \ Y	0	1	2	3
0	0.15	0.1	0.1	0.1
1	0.05	0.1	0.14	0.13
2	0.01	0.04	0.04	0.04

Find the marginal probability mass functions for X and Y.

11. The joint pdf of X and Y is given by

$$f(x, y) = \begin{cases} kxy^2 & ; \quad 0 \leq x \leq y \leq 1. \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

- Find the value of the constant k.
- Find the marginal pdfs of X and Y.
- Calculate $P(X + Y < 1)$, $P(2X < Y)$, and $P(X = Y)$.

12. For the joint pmf of two random variables X and Y given below:

X \ Y	1	2	3
1	3/12	4/12	2/12
2	1/12	2/12	0

Find the conditional probability mass function of X given the value of $Y = 1$ and that of Y given the value of $X = 2$.

13. Let the joint pdf be given by:

$$f(x, y) = \begin{cases} \frac{12}{5}x(2 - x - y) & ; \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1. \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

- Compute the conditional pdf of X given $Y=y$.
- Compute the conditional pdf of X given $Y=1/2$.
- Calculate $P(X > 1/3 | Y = 1/2)$.

14. Suppose that the joint pdf of (X, Y) is given by

$$f(x, y) = \begin{cases} e^{-y} & , \quad \text{for } x > 0, y > x \\ 0 & , \quad \text{otherwise.} \end{cases}$$

- Find the marginal pdf of X.
- Find the marginal pdf of Y.
- Evaluate $P(X > 2 | Y < 4)$.

15. Let the joint pdf of X and Y is given by

$$f(x, y) = \begin{cases} \frac{xy}{16} & ; \quad 1 \leq x \leq 3, \quad 1 \leq y \leq 3. \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

- Show that the random variable X and Y are independent.
- Compute the $P(X \geq Y + 1/2)$.

16. From a lot containing 25 items, 5 of which are defective, 4 are chosen at random. Let X be the number of defectives found. Find $E(X)$.

17. Suppose that X is uniformly distributed over (1, 3). Find the expected values of the following r.v.s

(a) $Y = 3X + 4$

(b) $Z = e^x$

18. Suppose that the joint pdf of (X, Y) is given by

$$f(x, y) = \begin{cases} e^{-y} & , \quad x > 0, \quad y > x \\ 0 & , \quad \text{otherwise.} \end{cases}$$

Find $E(X | y)$ and $E(Y | x)$.

19. Suppose that the continuous random variable X has pdf

$$f(x) = 2xe^{-x^2}, x \geq 0$$

Let $Y = X^2$. Evaluate $E(Y)$:

(a) directly without first obtaining the pdf of Y, (b) By first obtaining the pdf of Y.

20. Suppose that X and Y are independent random variables with the following pdf's:

$$f(x) = 8/x^3, x > 2; \quad g(y) = 2y, 0 < y < 1$$

(a) Find the pdf of $Z = XY$.

(b) Obtain $E(Z)$ in two ways: (i) using the pdf of Z as obtained in (a).
(ii) Directly, without using the pdf of Z.

21. A coin is tossed three times. Let X be the number of heads. Find

a. The mean and variance of X. b. The probability of getting at least one head.

22. If a set of marks on mathematics examination are normally distributed with a mean of 75 and variance of 6.25, then what is the probability that the marks of the students will take a value:

a. Greater than 73.4? c. Between 67.95 and 71.9?
b. Between 78.6 and 80.6?

23. Two dice are rolled. Let X be a random variable denoting the sum of the numbers on the two dice.

a. Find the mean and variance of X.
b. Find the mean and variance of $Y = 2X + 1$.

24. A soft drink machine is regulated so that it discharges an average of 200 milliliters per cup. If the amount of a drink is normally distributed with a standard deviation equal to 15 milliliters

b) What fraction of the cups will contain more than 224 milliliters?
c) What is the probability that a cup contains between 191 and 209 milliliters?
d) How many cups over will probably overflow if 230 ml cups are used for the next 100 drinks?
e) Below what value do we get the smallest 25% of the drinks?

25. The scores for a mathematics course seem to be normally distributed with mean 70 & standard deviation of 8. If the instructor wishes to give an A grade to 10% of the students, what should be the dividing line between an A grade and a B grade?

26. A fair die is tossed 72 times. Given that X is the number of times six appears, evaluate $E(X^2)$.

27. Suppose that the two-dimensional random variable (X, Y) has pdf given by

$$f(x, y) = \begin{cases} ke^{-y} & , \quad 0 < x < y < 1 \\ 0 & , \quad \text{otherwise.} \end{cases}$$

Find the correlation coefficient ρ_{xy} .

28. If X has a Poisson distribution with parameter λ , and if $P(X = 0) = 0.2$, evaluate $P(X > 2)$.
29. Suppose that the probability that an item produced by a particular machine is defective equals 0.2. If 10 items produced from this machine are selected at random, what is the probability that not more than one defective is found?
30. Suppose that X has a Poisson distribution. If

$$P(X = 2) = \frac{2}{3}P(X = 1),$$

Evaluate $P(X = 0)$ and $P(X = 3)$.

31. Suppose that the bank customers arrive randomly and independently on an average of 3.2 customers every 4 minutes. What is the probability that:
- Exactly two customers arrive in every 4 minutes?
 - Exactly two customers will arrive in every 8 minutes interval?
 - One or more customers will arrive in every 12 minutes?
32. A random variable X has a uniform distribution over $(-3, 3)$, Then compute:
- a). $P(X > 2)$ b). $P(|X| < 2)$ c). $P(X > -1 \text{ and } X < 2)$ d). $P(X < 1 \text{ or } X > 2)$.
33. A normal distribution has mean 62.4, find its standard deviation if 20.05% of the area under the normal curve lies to the right of 72.9.
34. A random variable has a normal distribution with standard deviation 5. Find it's mean if the probability that the random variable will assume a value less than 52.5 is 0.6915.
35. Suppose that X is a discrete random variable with possible value 0, 1, 2, ... and such that its probability mass function is defined as

$$P(X = x) = \begin{cases} \frac{e^{-\lambda} \lambda^x}{x!} & x = 0, 1, 2, \dots \\ 0 & \text{elsewhere} \end{cases}$$

- Find the moment generating function of a random variable X.
 - Find the average and the variance of a random variable X.
 - If $Y = 3X + 5$
 - Find the moment generating function (MGF) of Y.
 - Find the mean and variance of Y.
 - Let a random variable X be the number of phone calls received at a real estate firm per hour with hypothesized mean of 6. Suppose that the secretary takes a 30 minutes lunch break. What is the probability that no calls will be received in this time?
36. The probability density function of a random variable X is given by

$$P(X = x) = \begin{cases} \frac{1}{20} & 0 < x < 20 \\ 0 & \text{elsewhere} \end{cases}$$

Find

a. $P(0 < X < 5)$

c. Variance of X

b. $E(X)$

d. Find the mean and variance of Y, if $Y = cX + d$ where c and d are constants.

37. Suppose X is a continuous random variable with probability density function defined as

$$P(X = x) = \begin{cases} \lambda e^{-\lambda x} & \lambda > 0 \quad \text{and } 0 < x < \infty \\ 0 & \text{elsewhere} \end{cases}$$

a. Find the moment generating function of the random variable X.

b. If $P(X \leq 1) = P(X > 1)$, find variance of X.

c. Find the r^{th} moment of the random variable X about the origin.

38. Suppose that the joint pdf of the two-dimensional random variable (X, Y) is given by

$$f(x, y) = \begin{cases} x^2 + \frac{xy}{3} & 0 < x < 1, 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

a. Find the marginal pdf of X.

b. Find the marginal pdf of Y.

c. Find the conditional pdf of Y given X.

d. $P\left(X > \frac{1}{2}\right)$

e. $P(Y < X)$

f. $P\left(Y > \frac{1}{2} \mid X < \frac{1}{2}\right)$

g. Find the mean and variance of X.

h. Find conditional expectation of X given Y (i.e., $E(X/Y)$).

i. Find the correlation coefficient of X and Y.

j. Are X and Y independent?

Note concerning group assignment:

Group formation: you can form groups with not more than 7 students.

Assignment Questions: 3, 4, 5, 7, 11, 12, 13, 31, 33, 35, 37, 38.

Due Date: before you sit for final exam.