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# Spectrum Sharing Between Radars and Communication Systems

## A MATLAB Based Approach

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# Spectrum Sharing Between Radars and Communication Systems

A MATLAB Based Approach

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# Preface

Radio frequency spectrum is a scarce and critical natural resource that is utilized for many services including surveillance, navigation, communication, and broadcasting. Recent years have seen tremendous growth in the use of spectrum especially by commercial cellular operators. Ubiquitous use of smartphones and tablets is one of the reasons behind an all-time high utilization of spectrum. As a result, cellular operators are experiencing a shortage of radio spectrum to meet bandwidth demands of users. On the other hand, spectrum measurements have shown that much spectrum not held by cellular operators is underutilized even in dense urban areas. This has motivated shared access to spectrum by secondary systems with no or minimal impact on incumbent systems. Spectrum sharing is a promising approach to solve the problem of spectrum congestion as it allows cellular operators access to more spectrum in order to satisfy the ever-growing bandwidth demands of commercial users.

The US spectrum regulatory bodies, the Federal Communications Commission (FCC) and the National Telecommunications and Information Administration (NTIA), are working on an initiative to share 150 MHz of spectrum, held by federal agencies, in the band 3550–3700 MHz with commercial wireless operators. This band is primarily used by the Department of Defense for air-, ground-, and ship-borne radar systems that are critical to national defense. Field tests have shown that spectrum sharing between radars and communication systems require large separation distance in order to protect them from harmful interference. This excludes large population of the US from reaping the benefits of spectrum sharing and has little to no business value for commercial operators. Thus, novel methods are required to ensure spectrum sharing between the two systems without the need of large protect distances.

In order to efficiently share spectrum between radars and communication systems at the same time and in the same geographical area, this book proposes novel methods that transform radar signal in such a way that it does not interfere with communication systems. This is accomplished by projecting the radar signal onto null space of the wireless channel between radar and communication system. Basically, nulls are formed in radar beam pattern in the direction of communication

systems. The proposed signal shaping/design approach not only meets radar objectives but also meets spectrum sharing objectives. However, there is a trade-off as signal shaping or designing new radar signal, with spectrum sharing constraint, results in some performance degradation for radars. Therefore, it is of interest to study the impact of projection on radar performance. Performance metrics such as probability of target detection, Cramer–Rao bound and maximum likelihood estimate of target's angle of arrival, and beam pattern of radar are studied for performance degradation. The results show minimal degradation in radar performance and significant reduction in exclusion zones, thus, showing the efficacy of the proposed approach.

Spectrum sharing between radars and communication systems is an important cross-disciplinary research area of federal and commercial interest. The methods proposed for spectrum sharing in this book allow radars and communication systems to operate in a geographical area without the need for exclusion zones, required to protect both systems from each other interference, and promise highly sought-after capacity gains for communication systems.

Arlington, VA, USA  
September 2016

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# Chapter 1

## Introduction

**Awais Khawar, Jasmin Mahal and Chowdhury Shahriar**

Cellular network operators are predicting a  $1000\times$  increase in capacity to keep pace with the tremendous growth of aggregate and per-subscriber data traffic [1]. Increased investments in infrastructure, e.g., larger number of small cells, and more spectrally efficient technologies, e.g., LTE-Advanced, can help meet this challenge partially. The Federal Communications Commission (FCC) is considering a number of options including incentive auctioning and sharing of federal spectrum to meet the commercial spectrum requirements. Of these two, spectrum sharing is a quite promising technology due to the large number of underutilized federal spectrum bands that can be shared with commercial cellular operators to satisfy their growing demands. But spectrum sharing is associated with its inherent set of challenges because the incumbents need to be protected from the harmful interference that can arise due to the operation of other systems in the shared bands.

Spectrum sharing between wireless communication systems and radars is an emerging area of research. In the past, spectrum has been shared primarily between wireless communication systems using opportunistic approaches by users equipped with cognitive radios [2]. This type of spectrum sharing has been made possible with the use of spectrum sensing [3], or geolocation databases [4], or a combination of both in the form of radio environment maps (REM) [5]. Some recent efforts have explored co-channel sharing approaches among secondary network entities, please see [6] and reference therein. However, in contrast, co-channel spectrum sharing between wireless systems and radars has received little attention thus far because of regulatory concerns.

Spectrum policy regulators, in the past, have not allowed commercial wireless services in radar bands, except in few cases, due to the fear of harmful interference from these services to radar systems [7]. Recently, in the United States (U.S.), the Federal Communications Commission (FCC), has proposed to use the 3550–3650 MHz band for commercial broadband use [8]. The incumbents in this band are radar and satellite systems [9]. The Commission has proposed that incumbents share

this band with commercial communication systems. The Commission's spectrum sharing initiative is motivated by many factors including the President's National Broadband plan, which called to free up to 500 MHz of federal-held spectrum by 2020 [10]; surge in consumers' demand for access to mobile broadband, which operators cannot meet with current spectrum allocations; the report on efficient spectrum utilization by President's Council of Advisers on Science and Technology (PCAST), which emphasized to share 1000 MHz of government-held spectrum [11], and the low utilization of the 3550–3650 MHz band by federal incumbents [12].

In future, when radio frequency (RF) spectrum will be shared among many different systems, e.g., radars and cellular systems, it is important to access the interference scenario. Of course, radars will cause interference to communication systems and vice versa if proper interference-mitigation methods and novel sharing algorithms are not employed. In a study conducted by the National Telecommunications and Information Administration (NTIA), it was observed that in order to protect commercial cellular communication systems, from high-power radar signal, large exclusion zones are required [12]. These exclusion zones cover a large portion of the U.S. where majority of the population lives and, thus, does not make a business case for commercial deployment in radar bands. In order to share radar bands for commercial operation we have to address interference-mitigation techniques at both the systems. In this work, we focus on interference caused by radar systems to communication system and propose methods to mitigate this interference.

Protecting the primary user (PU) or the incumbent Federal user from the harmful interferences from the secondary user (SU) or the commercial user and modifying the SU signal accordingly is the approach adopted by the first two generations of spectrum sharing between a Federal radar and a commercial communication system. In this approach, government users are entirely passive as they were not designed for sharing in mind which places the entire burden on commercial system to maintain an extremely high confidence level of interference protection for Federal users. However, the potential for such sharing with very conservative criteria is naturally so limited that this approach has made only modest progress in the U.S. to date [13]. The basic issue is addressed by Michael J. Marcus as presented in Fig. 1.1 [13]. The plot shows the percentage of idle spectrum that can be used as a function of the required confidence of no or negligible interference to the PU. If the PU is not designed with any sharing in mind as is the case with current incumbents then interference protections have to consider all possible worst case scenarios. This results in the very conservative sharing criteria. However, if the PU is originally designed with the anticipation of sharing and can cooperate with the new user, then a significantly higher fraction of idle spectrum can be utilized. Consequently, as Marcus suggested a third generation of sharing could be based upon the innovative design of the Federal systems with the intention of sharing.

The federal-commercial spectrum sharing is not a new practice. In fact, in the past, commercial wireless systems have shared government bands on a low-transmit power basis, in order to protect incumbents from interference. An example of such a scenario is wireless local area network (WLAN) in the 5250–5350 and 5470–5725 MHz radar bands [14]. So, the Commission's latest initiative to share the 3.5 GHz radar band

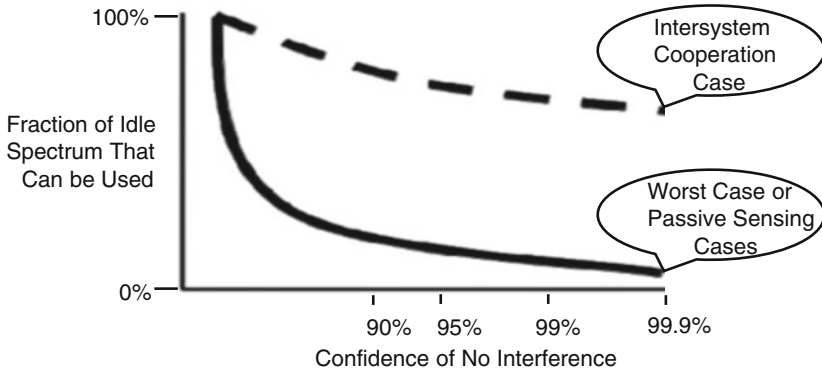


**Fig. 1.1** Fraction of idle spectrum that can be used under different cases [13]

with small cells, i.e., wireless base stations operating on a low power, is in harmony with previous practices [8].

On going research efforts have shown that there are numerous ways to share spectrum between radars and communication systems. Cooperative sensing based spectrum sharing approaches can be utilized where radar's allocated bandwidth is shared with communication systems [15–17]. A joint communication-radar platform can be envisioned in which spectrally agile radar performs an additional task of spectrum sensing and upon finding of unused frequencies it can change its operating frequency. In addition of spectrum sharing such a setting can enable colocated radar and communication system platforms for integrated communications and radar applications [18–21]. Radar waveforms can be shaped in a way that they do not cause interference to communication systems [22–26]. Moreover, database-aided sensing at communication systems [27] and beamforming approaches at MIMO radars can also be realized for spectrum sharing [28].

In this book, we address various aspects of spectrum sharing between different types of radars and communication systems. We specifically look at the performance of spectrum sharing radar systems when they are subject to spectrum sharing constraints. In Chap. 2, the authors discuss spectrum sharing between a multiple-input multiple-output (MIMO) ship-borne radar system and a commercial MIMO cellular network. They design spectrum sharing algorithms and then analyze performance of radar system. They address the problem of target parameter estimation, detection, and tracking which lies at the heart of radar signal processing. This problem becomes critically important when we talk about sharing radar spectrum with other systems, say cellular systems. In Chap. 3, the authors address the specific problem of spectrum sharing between a MIMO radar system and a commercial MIMO cellular network consisting of clusters of cooperative base stations, commonly known as Coordinated Multi-Point (CoMP) system. The authors design the radar precoder for two modes of operation: cooperation/cognition mode and interference-mitigation mode. During the cooperation/cognition mode, the radar not only broadcast information

to the communication system using the designed precoder but also, utilizing its sensing capability, the radar collects information about the BS clusters, estimates the channel either by using training symbols sent by the communication system or by blind null-space learning. By processing these information, the radar decides on the best base station cluster for spectrum sharing during interference-mitigation mode. In Chap. 4, the authors share cellular spectrum with a collocated overlapped multiple-input multiple-output (Overlapped-MIMO) radar system. The proposed antenna design origins lesser interference to communication systems along with retaining high degree performance of MIMO radar (such as retaining improved side-lobe suppression in the beampattern and achieving higher SNR gain). On the other hand, the radar-centric projection algorithm to share spectrum avoids interference to communications system by projecting the radar signal onto the null space of the communications channel.

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## Chapter 2

# A Projection-Based Approach to Spectrum Sharing

Awais Khawar, Ahmed Abdelhadi and T. Charles Clancy

Spectrum sharing is a new way forward to solve spectrum scarcity problem. In this chapter, we first propose a spatial approach for spectrum sharing between a MIMO radar and an LTE cellular system with multiple base stations (BS). The MIMO radar and LTE share multiple interference channels. We propose projecting the radar signal onto the null space of interference channel between the MIMO radar and LTE using our proposed interference-channel-selection algorithm, in order to have zero-interference from the MIMO radar. We select interference channel with the maximum null space and project the radar signal onto the null space of this channel. Our proposed spatial spectrum sharing algorithm is radar-centric such that it causes minimum loss in radar performance by carefully selecting the interference channel and at the same time protects the  $i^{\text{th}}$  LTE BS from the radar interference. Through our analytical and simulation results we show that the loss in the radar performance is less when the proposed interference-channel-selection algorithm is used to select the channel onto which radar signals are projected. Second, we address the problem of target detection by radars that project waveform onto the null space of interference channel in order to mitigate interference to cellular systems. We consider a multiple-input multiple-output (MIMO) radar and a MIMO cellular communication system with multiple base stations (BS). We consider two spectrum sharing scenarios. In the first scenario the degrees of freedom (DoF) available at the radar are not sufficient enough to simultaneously detect target and mitigate interference to multiple BSs. For this case, we select one BS among many BSs for waveform projection

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on the basis of guaranteeing minimum waveform degradation. For the second case, the radar has sufficient DoF to simultaneously detect target and mitigate interference to all BSs. We study target detection capabilities of null-space projected (NSP) waveform and compare it with the orthogonal waveform. We derive the generalized likelihood ratio test (GLRT) for target detection and derive detector statistic for NSP and orthogonal waveform. The target detection performance for both waveforms is studied theoretically and via Monte Carlo simulations.

This chapter is organized as follows. Section 2.1 discuss MIMO radar, target channel, orthogonal waveforms, interference channel, and our cellular system model. Moreover, it also discusses modeling and statistical assumptions. Section 2.2 discusses spectrum sharing between MIMO radar and cellular system and introduces sharing architecture and projection algorithms. Section 2.3 discusses target parameter estimation performance for spectrum sharing radars followed by numerical results. Section 2.4.3 presents the generalized likelihood ratio test (GLRT) for target detection and derives detector statistic for NSP and orthogonal waveform followed by numerical results. Section 2.5 concludes the chapter.

## 2.1 System Model

In this section, we introduce preliminaries of MIMO radar, point target in far-field, orthogonal waveforms, interference channel, and cellular system model. Moreover, we also discuss modeling and statistical assumptions along with RF environment assumptions used throughout the chapter.

### 2.1.1 Radar Model

The radar we consider in this chapter is a colocated MIMO radar with  $M$  transmit and receive antennas and is mounted on a ship. The colocated MIMO radar has antennas that have spacing on the order of half the wavelength. Another class of MIMO radar is widely spaced MIMO radar where elements are widely spaced which results in enhanced spatial diversity [1]. The colocated radar gives better spatial resolution and target parameter identification as compared to the widely-spaced radar [2].

### 2.1.2 Target Model/Channel

In this chapter, we consider a point target model which is defined for targets having a scatterer with infinitesimal spatial extent. This model is a good assumption and is widely used in radar theory for the case when radar elements are colocated and there exists a large distance between the radar array and the target as compared to

inter-element distance [3]. The signal reflected from a point target with unit radar cross-section (RCS) is mathematically represented by the Dirac delta function.

### 2.1.3 Signal Model

Let  $\mathbf{x}(t)$  be the signal transmitted from the  $M$ -element MIMO radar array, defined as

$$\mathbf{x}(t) = [x_1(t)e^{j\omega_c t} \ x_2(t)e^{j\omega_c t} \ \dots \ x_M(t)e^{j\omega_c t}]^T \quad (2.1)$$

where  $x_k(t)e^{j\omega_c t}$  is the baseband signal from the  $k^{\text{th}}$  transmit element,  $\omega_c$  is the carrier angular frequency,  $t \in [0, T_o]$ , with  $T_o$  being the observation time. We define the transmit steering vector as

$$\mathbf{a}_T(\theta) \triangleq [e^{-j\omega_c \tau_{T_1}(\theta)} \ e^{-j\omega_c \tau_{T_2}(\theta)} \ \dots \ e^{-j\omega_c \tau_{T_M}(\theta)}]^T. \quad (2.2)$$

Then, the transmit-receive steering matrix can be written as

$$\mathbf{A}(\theta) \triangleq \mathbf{a}_R(\theta)\mathbf{a}_T^T(\theta). \quad (2.3)$$

Since, we are considering  $M$  transmit and receive elements, we define  $\mathbf{a}(\theta) \triangleq \mathbf{a}_T(\theta) \triangleq \mathbf{a}_R(\theta)$ . The signal received from a single point target, in far-field with constant radial velocity  $v_r$ , at an angle  $\theta$  can be written as

$$\mathbf{y}(t) = \alpha e^{-j\omega_D t} \mathbf{A}(\theta) \mathbf{x}(t - \tau(t)) + \mathbf{n}(t) \quad (2.4)$$

where  $\tau(t) = \tau_{T_k}(t) + \tau_{R_l}(t)$ , denoting the sum of propagation delays between the target and the  $k^{\text{th}}$  transmit element and the  $l^{\text{th}}$  receive element, respectively;  $\omega_D$  is the Doppler frequency shift,  $\alpha$  represents the complex path loss including the propagation loss and the coefficient of reflection, and  $\mathbf{n}(t)$  is the zero-mean complex Gaussian noise.

### 2.1.4 Modeling Assumptions

In order to keep the analysis tractable we have made the following assumptions about our signal model:

- The path loss  $\alpha$  is assumed to be identical for all transmit and receive elements, due to the far-field assumption [4].
- The angle  $\theta$  is the azimuth angle of the target.
- After compensating the range-Doppler parameters, we can simplify Eq. (2.4) as

$$\mathbf{y}(t) = \alpha \mathbf{A}(\theta) \mathbf{x}(t) + \mathbf{n}(t). \quad (2.5)$$

### 2.1.5 Statistical Assumptions

We make the following assumptions for our received signal model in Eq. (2.5):

- $\theta$  and  $\alpha$  are deterministic unknown parameters representing the target's direction of arrival and the complex amplitude of the target, respectively.
- The noise vector  $\mathbf{n}(t)$  is independent, zero-mean complex Gaussian with known covariance matrix  $\mathbf{R}_n = \sigma_n^2 \mathbf{I}_M$ , i.e.,  $\mathbf{n}(t) \sim \mathcal{N}^c(\mathbf{0}_M, \sigma_n^2 \mathbf{I}_M)$ , where  $\mathcal{N}^c$  denotes the complex Gaussian distribution.
- With the above assumptions, the received signal model in Eq. (2.5) has independent complex Gaussian distribution, i.e.,

$$\mathbf{y}(t) \sim \mathcal{N}^c(\alpha \mathbf{A}(\theta) \mathbf{x}(t), \sigma_n^2 \mathbf{I}_M). \quad (2.6)$$

### 2.1.6 Orthogonal Waveforms

In this chapter, we consider orthogonal waveforms transmitted by MIMO radars, i.e.,

$$\mathbf{R}_x = \int_{T_o} \mathbf{x}(t) \mathbf{x}^H(t) dt = \mathbf{I}_M. \quad (2.7)$$

The transmission of orthogonal signals gives MIMO radar advantages in terms of digital beamforming at the transmitter in addition to receiver, improved angular resolution, extended array aperture in the form of virtual arrays, increased number of resolvable targets, lower sidelobes [5], and lower probability of intercept as compared to coherent waveforms [4].

### 2.1.7 Communication System

In this chapter, we consider a MIMO cellular system, with  $\mathcal{K}$  base stations, each equipped with  $N^{\text{BS}}$  transmit and receive antennas, with  $i^{\text{th}}$  BS supporting  $\mathcal{L}_i^{\text{UE}}$  user equipment (UE). The UEs are also multi-antenna systems with  $N^{\text{UE}}$  transmit and receive antennas. If  $\mathbf{s}_j^{\text{UE}}(t)$  is the signals transmitted by the  $j^{\text{th}}$  UE in the  $i^{\text{th}}$  cell, then the received signal at the  $i^{\text{th}}$  BS receiver can be written as

$$\mathbf{r}_i(t) = \sum_j \mathbf{H}_j^{N^{\text{BS}} \times N^{\text{UE}}} \mathbf{s}_j^{\text{UE}}(t) + \mathbf{w}(t) \quad 1 \leq j \leq \mathcal{L}_i^{\text{UE}} \quad (2.8)$$

where  $\mathbf{w}(t)$  is the additive white Gaussian noise.

### 2.1.8 Interference Channel

In this section, we characterize the interference channel that exists between MIMO cellular base station and MIMO radar. In our chapter, we are considering  $\mathcal{K}$  cellular BSs that is why our model has  $\mathbf{H}_i$ ,  $i = 1, 2, \dots, \mathcal{K}$ , interference channels, where the entries of  $\mathbf{H}_i$  are denoted by

$$\mathbf{H}_i = \begin{bmatrix} h_i^{(1,1)} & \dots & h_i^{(1,M)} \\ \vdots & \ddots & \vdots \\ h_i^{(N^{\text{BS}},1)} & \dots & h_i^{(N^{\text{BS}},M)} \end{bmatrix} \quad (N^{\text{BS}} \times M) \quad (2.9)$$

where  $h_i^{(l,k)}$  denotes the channel coefficient from the  $k^{\text{th}}$  antenna element at the MIMO radar to the  $l^{\text{th}}$  antenna element at the  $i^{\text{th}}$  BS. We assume that elements of  $\mathbf{H}_i$  are independent, identically distributed (i.i.d.) and circularly symmetric complex Gaussian random variables with zero-mean and unit-variance, thus, having a i.i.d. Rayleigh distribution. A more thorough treatment of interference channel modeling between radar and cellular system, including two- and three-dimensional channel models, can be found in [6–9].

### 2.1.9 Cooperative RF Environment

In the wireless communications literature, it is usually assumed that the transmitter (mostly BS) has channel state information (CSI) either by feedback from the receiver (mostly UE), in FDD systems [10], or transmitters can reciprocate the channel, in TDD systems [10]. The feedback and reciprocity are valid and practical as long as the feedback has a reasonable overhead and coherence time of the RF channel is larger than the two-way communication time, respectively.

In the case of radars sharing their spectrum with communications systems one way to get CSI is that radar estimates  $\mathbf{H}_i$  based on the training symbols sent by communication receivers (or BSs in this case) [11]. Another approach is that radar aids communication systems in channel estimation, with the help of a low-power reference signal, and they feed back the estimated channel to radar [12]. Since, radar signal is treated as interference at communication system, we can characterize the channel as interference channel and refer to information about it as interference-channel state information (ICSI).

Spectrum sharing between radars and communications systems can be envisioned in two domains: military radars sharing spectrum with military communication systems, we call it *Mil2Mil* sharing; another possibility is military radars sharing spectrum with commercial communication systems, we call it *Mil2Com* sharing. In *Mil2Mil* sharing, ICSI can be acquired by radars fairly easily as both systems belong to military. In *Mil2Com* sharing, ICSI can be acquired by giving incentives to com-

mercial communication system. The biggest incentive in this scenario is null-steering and protection from radar interference. Thus, regardless of the sharing scenario, *Mil2Mil* or *Mil2Com*, we have ICSI for the sake of mitigating radar interference at communication systems.

## 2.2 Radar-Cellular System Spectrum Sharing

After introducing our radar and cellular system models we can now discuss the spectrum sharing scenario between radar and cellular system. In our sharing architecture, MIMO radar and cellular systems are the co-primary users of the 3550–3650 MHz band under consideration. In the following sections, we will discuss the architecture of spectrum sharing problem which is followed by our spectrum sharing algorithm.

### 2.2.1 Architecture

We illustrate our coexistence scenario in Fig. 2.1 where the maritime MIMO radar is sharing  $\mathcal{K}$  interference channels with the cellular system. Considering this scenario, the received signal at the  $i^{\text{th}}$  BS receiver can be written as

$$\mathbf{r}_i(t) = \mathbf{H}_i^{N^{\text{BS}} \times M} \mathbf{x}(t) + \sum_j \mathbf{H}_j^{N^{\text{BS}} \times N^{\text{UE}}} \mathbf{s}_j^{\text{UE}}(t) + \mathbf{w}(t). \quad (2.10)$$

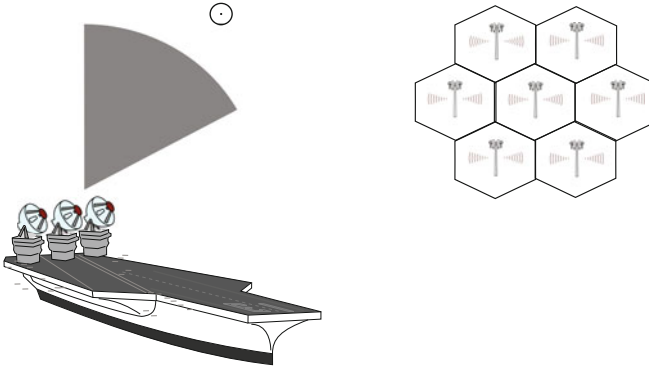


**Fig. 2.1** Spectrum sharing scenario: A seaborne MIMO radar detecting a point target while simultaneously sharing spectrum with a MIMO cellular system without causing interference to the cellular system

The goal of the MIMO radar is to map  $\mathbf{x}(t)$  onto the null-space of  $\mathbf{H}_i$  in order to avoid interference to the  $i^{\text{th}}$  BS, i.e.,  $\mathbf{H}_i \mathbf{x}(t) = \mathbf{0}$ , so that  $\mathbf{r}_i(t)$  has Eq.(2.8) instead of Eq.(2.10).

## 2.3 Spectrum Sharing Algorithms for Small MIMO Radar

In this section, we discuss performance metrics of spectrum sharing MIMO radars when a MIMO radar has a smaller antenna array as compared to a cellular base station (BS), i.e.,  $M_T \leq N_R$ . We consider the spectrum sharing between a MIMO radar and an LTE cellular system with  $\mathcal{K}$  BSs. The MIMO radar and LTE share  $\mathcal{K}$  interference channels, i.e.,  $\mathbf{H}_i, i = 1, 2, \dots, \mathcal{K}$ . We propose projecting the radar signal onto the null space of interference channel between the MIMO radar and LTE using our proposed interference-channel-selection algorithm, in order to have zero-interference from the MIMO radar. We select interference channel with the maximum null space, i.e.,  $\arg\max_{1 \leq i \leq \mathcal{K}} \dim[\mathcal{N}(\mathbf{H}_i)]$  and project the radar signal onto the null space of this channel. Our proposed spatial spectrum sharing algorithm is radar-centric such that it causes minimum loss in radar performance by carefully selecting the interference channel and at the same time protects the  $i^{\text{th}}$  LTE BS from the radar interference. Through our analytical and simulation results we show that the loss in the radar performance is less when the proposed interference-channel-selection algorithm is used to select the channel onto which radar signals are projected.

### 2.3.1 Performance Metrics

We choose the Cramèr Rao bound (CRB) and maximum likelihood (ML) estimate of the target's angle of arrival as our performance metric for the MIMO radar system. We are interested in studying the degradation in the estimate of the target's angle of arrival due to null-space projection of the radar waveform. The CRB for a single target, no-interference case, is given as in [4],

$$\text{CRB}(\theta) = \frac{1}{2 \text{SNR}} \left( M_R \dot{\mathbf{a}}_T^H(\theta) \mathbf{R}_x^T \dot{\mathbf{a}}_T(\theta) + \mathbf{a}_T^H(\theta) \mathbf{R}_x^T \mathbf{a}_T(\theta) \|\dot{\mathbf{a}}_R(\theta)\|^2 - \frac{M_R |\mathbf{a}_T^H(\theta) \mathbf{R}_x^T \dot{\mathbf{a}}_T(\theta)|^2}{\mathbf{a}_T^H(\theta) \mathbf{R}_x^T \mathbf{a}_T(\theta)} \right)^{-1} \quad (2.11)$$

and the ML for the case of no interference and a single target can be written as in [4],

$$(\hat{\theta}, \hat{\tau}_r, \hat{\omega}_D)_{\text{ML}} = \arg \max_{\theta, \tau_r, \omega_D} \frac{|\mathbf{a}_R^H(\theta) \mathbf{E}(\tau_r, \omega_D) \mathbf{a}_T^*(\theta)|^2}{M_R \mathbf{a}_T^H(\theta) \mathbf{R}_x^T \mathbf{a}_T(\theta)} \quad (2.12)$$

where

$$\begin{aligned}\dot{\mathbf{a}}_R(\theta) &= \frac{d\mathbf{a}_R(\theta)}{d\theta} \\ \dot{\mathbf{a}}_T(\theta) &= \frac{d\mathbf{a}_T(\theta)}{d\theta} \\ \mathbf{R}_x &= \int_{T_0} \mathbf{x}(t) \mathbf{x}^H(t) dt \\ \mathbf{E}(\tau_r, \omega_D) &= \int_{T_0} \mathbf{y}(t) \mathbf{x}^H(t - \tau_r) e^{j\omega_D t} dt\end{aligned}$$

$\tau_r$  is the propagation delay, two-way, between the target and the reference point, and  $\omega_D$  is the Doppler frequency shift.

In addition to performance metrics like CRB and ML, we are also interested in the changes in beampatterns of the MIMO radar due to null-space projection of the radar waveform. Beampattern is a measure of beamformer's response to a target at direction  $\theta$  given by, as in [4],

$$G(\theta, \theta_D) = \Gamma \frac{|\mathbf{a}_T^H(\theta) \mathbf{R}_x^T \mathbf{a}_T(\theta_D)|^2}{\mathbf{a}_T^H(\theta_D) \mathbf{R}_x^T \mathbf{a}_T(\theta_D)} \frac{|\mathbf{a}_R^H(\theta) \mathbf{a}_R(\theta_D)|^2}{M_R} \quad (2.13)$$

where  $\Gamma$  is the normalization constant and  $\theta_D$  represents the digital steering direction of the main beam.

### 2.3.2 Interference-Channel-Selection Algorithm

In this section, we propose our interference-channel-selection algorithm, shown in Algorithm 1, which selects interference channel onto which radar signals are projected using NSP method, i.e., Algorithm 2. We assume there exist  $\mathcal{K}$  interference channels, i.e.,  $\mathbf{H}_i, i = 1, 2, \dots, \mathcal{K}$ , between the MIMO radar and the LTE system and we seek to select the best interference channel, defined as

$$\begin{aligned}i_{\max} &\triangleq \underset{1 \leq i \leq \mathcal{K}}{\operatorname{argmax}} \dim[\mathcal{N}(\mathbf{H}_i)] \\ \mathbf{H}_{\text{Best}} &\triangleq \mathbf{H}_{i_{\max}}\end{aligned}$$

and we seek to avoid the worst channel, defined as

$$\begin{aligned}i_{\min} &\triangleq \underset{1 \leq i \leq \mathcal{K}}{\operatorname{argmin}} \dim[\mathcal{N}(\mathbf{H}_i)] \\ \mathbf{H}_{\text{Worst}} &\triangleq \mathbf{H}_{i_{\min}}\end{aligned}$$

where null space of  $\mathbf{H}_i^{N_R \times M_T}$  is defined as

$$\mathcal{N}(\mathbf{H}_i) \triangleq \{\mathbf{x} \in \mathbb{C}^{M_T} : \mathbf{H}_i \mathbf{x} = \mathbf{0}\}$$

and then null of  $\mathbf{H}_i^{N_R \times M_T}$  is defined as

$$\text{null } \mathbf{H}_i \triangleq \dim[\mathcal{N}(\mathbf{H}_i)]$$

where ‘dim’ is the number of linearly independent columns in null space of  $\mathbf{H}_i^{N_R \times M_T}$ .

At the MIMO radar, we first estimate the channel state information (CSI) of the  $\mathcal{K}$  interference channels using a blind null-space learning algorithm [13]. This is followed by the calculation of null space of these  $\mathcal{K}$  interference channels via Algorithm 2. Once Algorithm 1 receives null space of interference channels, it selects channel with the maximum null space as the candidate channel, i.e.,  $\check{\mathbf{H}}$  and sends it to Algorithm 2 for NSP of radar signals. Our interference-channel-selection algorithm, i.e., Algorithm 1 guarantees minimum degradation in radar performance and at the same time assures zero-interference to the candidate BS.

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**Algorithm 1** Interference-Channel-Selection Algorithm

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```

loop
  for  $i = 1 : \mathcal{K}$  do
    Estimate CSI of  $\mathbf{H}_i$ .
    Send  $\mathbf{H}_i$  to Algorithm 2 for null space computation.
    Receive  $\dim[\mathcal{N}(\mathbf{H}_i)]$  from Algorithm 2.
  end for
  Find  $i_{\max} = \underset{1 \leq i \leq \mathcal{K}}{\text{argmax}} \dim[\mathcal{N}(\mathbf{H}_i)]$ .
  Set  $\check{\mathbf{H}} = \mathbf{H}_{i_{\max}}$  as the candidate interference channel.
  Send  $\check{\mathbf{H}}$  to Algorithm 2 to get NSP radar waveform.
end loop

```

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### 2.3.3 Modified-Null-Space Projection (NSP) Algorithm

In this section, we explain the projection of radar signals onto null space of interference channel selected using Algorithm 1. As mentioned earlier, the CSI of  $\mathcal{K}$  interference channels is estimated using a blind null-space learning algorithm [13]. After getting CSI estimates of  $\mathcal{K}$  interference channels, from Algorithm 1, the next step is to find null space of each  $\mathbf{H}_i^{N_R \times M_T}$  using Algorithm 2. This step is performed using the singular value decomposition (SVD) theorem according to our modified-NSP projection algorithm, as shown in Algorithm 2. For the complex  $i^{\text{th}}$  interference channel matrix the SVD is given as

$$\begin{aligned}
\mathbf{H}_i^{N_R \times M_T} &= \mathbf{U}_i \boldsymbol{\Sigma}_i^{N_R \times M_T} \mathbf{V}_i^H \\
&= \mathbf{U}_i \begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & 0 \\ & & \ddots & \\ 0 & & & \sigma_{j \in \min(N_R, M_T)} \end{pmatrix} \mathbf{V}_i^H
\end{aligned}$$

where  $\mathbf{U}_i$  is the complex unitary matrix,  $\boldsymbol{\Sigma}_i$  is the diagonal matrix of singular values, and  $\mathbf{V}_i^H$  is the complex unitary matrix. If the SVD analysis do not yield any zero singular values we resort to a numerical approach to calculate null space. In order to do that, in Algorithm 2, we set a threshold  $\delta$  and select singular values below the threshold value. Then the number of singular values below the threshold serves as the dimension of null space.

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**Algorithm 2** Modified-Null-Space Projection (NSP)

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```

if  $\mathbf{H}_i$  received from Algorithm 1 then
  Perform SVD on  $\mathbf{H}_i$  (i.e.,  $\mathbf{H}_i = \mathbf{U}_i \boldsymbol{\Sigma}_i \mathbf{V}_i^H$ )
  if  $\sigma_j \neq 0$  (i.e.,  $j^{\text{th}}$  singular value of  $\boldsymbol{\Sigma}_i$ ) then
     $\dim[\mathcal{N}(\mathbf{H}_i)] = 0$ 
    Use pre-specified threshold  $\delta$ 
    for  $j = 1 : \min(N_R, M_T)$  do
      if  $\sigma_j < \delta$  then
         $\dim[\mathcal{N}(\mathbf{H}_i)] = \dim[\mathcal{N}(\mathbf{H}_i)] + 1$ 
      else
         $\dim[\mathcal{N}(\mathbf{H}_i)] = 0$ 
      end if
    end for
  else
     $\dim[\mathcal{N}(\mathbf{H}_i)] = \text{The number of zero singular values}$ 
  end if
  Send  $\dim[\mathcal{N}(\mathbf{H}_i)]$  to Algorithm 1.
end if
if  $\tilde{\mathbf{H}}$  received from Algorithm 1 then
  Perform SVD on  $\tilde{\mathbf{H}} = \mathbf{U} \boldsymbol{\Sigma} \mathbf{V}$ 
  if  $\sigma_j \neq 0$  then
    Use pre-specified threshold  $\delta$ 
     $\sigma_{\text{Null}} = \{ \}$  {An empty set to collect  $\sigma$ s below threshold  $\delta$ }
    for  $j = 1 : \min(N_R, M_T)$  do
      if  $\sigma_j < \delta$  then
        Add  $\sigma_j$  to  $\sigma_{\text{Null}}$ 
      end if
    end for
     $\tilde{\mathbf{V}} = \sigma_{\text{Null}}$  corresponding columns in  $\mathbf{V}$ .
  end if
  Setup projection matrix  $\mathbf{P}_{\tilde{\mathbf{V}}} = \tilde{\mathbf{V}} \tilde{\mathbf{V}}^H$ .
  Get NSP radar signal via  $\tilde{\mathbf{x}} = \mathbf{P}_{\tilde{\mathbf{V}}} \mathbf{x}$ .
end if

```

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Once the null space of all interference channels is determined we seek to find the best channel  $\check{\mathbf{H}}$ , the one with the maximum null space, which according to our Algorithm 1 is given as

$$i_{\max} = \underset{1 \leq i \leq \mathcal{K}}{\operatorname{argmax}} \dim \mathcal{N}(\mathbf{H}_i)$$

$$\check{\mathbf{H}} = \mathbf{H}_{i_{\max}}$$

Algorithm 1 sends  $\check{\mathbf{H}}$  to Algorithm 2 for null-space computation, where after SVD the right singular vectors corresponding to vanishing singular are collected in  $\check{\mathbf{V}}$  for the formation of projection matrix. Once this is done, we project the radar signal onto the null space of  $\mathbf{H}_{\text{Best}}$  via a modified version of our projection algorithm [5, 14]. The proposed NSP algorithm removes redundancy from previous algorithm and is computationally efficient. The modified-NSP algorithm is given as

$$\mathbf{P}_{\check{\mathbf{V}}} = \check{\mathbf{V}}\check{\mathbf{V}}^H.$$

The radar waveform projected onto null space of  $\check{\mathbf{H}}$  can be written as

$$\check{\mathbf{x}} = \mathbf{P}_{\check{\mathbf{V}}} \mathbf{x}. \quad (2.14)$$

By inserting the projected signal, as in Eq. (2.14), into the Cramer–Rao bound (CRB) for the single target no interference case, Eq. (2.11), we get the CRB for the NSP projected radar waveform as

$$\text{CRB}_{\text{NSP}}(\theta) = \frac{1}{2 \text{SNR}} \left( M_R \dot{\mathbf{a}}_T^H(\theta) \mathbf{R}_{\check{\mathbf{x}}}^T \dot{\mathbf{a}}_T(\theta) + \mathbf{a}_T^H(\theta) \right. \\ \left. \mathbf{R}_{\check{\mathbf{x}}}^T \mathbf{a}_T(\theta) \|\dot{\mathbf{a}}_R(\theta)\|^2 - \frac{M_R |\mathbf{a}_T^H(\theta) \mathbf{R}_{\check{\mathbf{x}}}^T \dot{\mathbf{a}}_T(\theta)|^2}{\mathbf{a}_T^H(\theta) \mathbf{R}_{\check{\mathbf{x}}}^T \mathbf{a}_T(\theta)} \right)^{-1}. \quad (2.15)$$

Similarly, Eq. (2.14) can be substituted in (2.12) to get the ML estimate of angle arrival for the NSP projected radar waveform as

$$(\hat{\theta}, \hat{\tau}_r, \hat{\omega}_D)_{\text{MLNSP}} = \arg \max_{\theta, \tau_r, \omega_D} \frac{|\mathbf{a}_R^H(\theta) \mathbf{E}(\tau_r, \omega_D) \mathbf{a}_T^*(\theta)|^2}{M_R \mathbf{a}_T^H(\theta) \mathbf{R}_{\check{\mathbf{x}}}^T \mathbf{a}_T(\theta)}. \quad (2.16)$$

In order to analyze the beampattern of the NSP projected waveform we can substitute Eq. (2.14) in Eq. (2.13) to get

$$G_{\text{NSP}}(\theta, \theta_D) = \Gamma \frac{|\mathbf{a}_T^H(\theta) \mathbf{R}_{\check{\mathbf{x}}}^T \mathbf{a}_T(\theta_D)|^2 |\mathbf{a}_R^H(\theta) \mathbf{a}_R(\theta_D)|^2}{\mathbf{a}_T^H(\theta_D) \mathbf{R}_{\check{\mathbf{x}}}^T \mathbf{a}_T(\theta_D) M_R}. \quad (2.17)$$

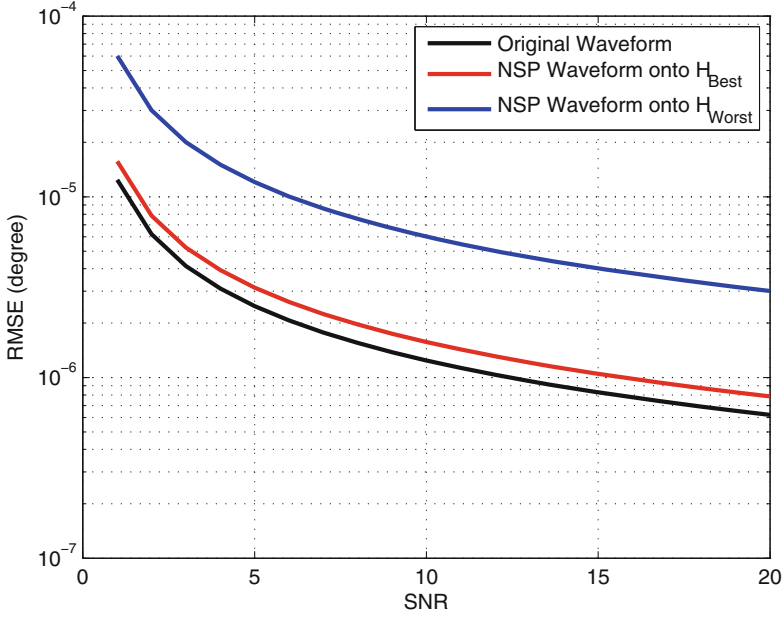


**Fig. 2.2** CRB on target direction estimation RMSE as a function of the SNR.  $H_{Best}$  and  $H_{Worst}$  channels are selected using Algorithms 1 and 2

### 2.3.4 Simulation Results

In this section, we simulate our MIMO radar-LTE sharing scenario and study its impact on the performance of radar.

The CRB for the target's angle of arrival is given by Eqs. (2.11) and (2.15) for the original radar waveform and the NSP radar waveform, respectively. We are interested in understanding the effects of NSP on the radar waveform. In Fig. 2.2, we compare the root-mean-square-error (RMSE) of different radar waveforms. We compare the performance of original radar waveform with the NSP waveform projected onto  $H_{Best}$  and  $H_{Worst}$ . Note that by using Algorithms 1 and 2 we are able to minimize degradation in the radar performance as the NSP waveform onto  $H_{Best}$  is closer to the original radar waveform in RMSE sense than the NSP waveform onto  $H_{Worst}$ . Thus, by an appropriate selection of the interference-channel degradation in the radar performance, due to the NSP of its waveform, can be minimized.

Similar to the CRB, the ML estimate of the target's angle of arrival is given by Eqs. (2.12) and (2.16) for the original radar waveform and the NSP radar waveform, respectively. We are interested in the estimation error of the angle due to the NSP of radar waveform. In Fig. 2.3, we compare original angles and estimated angles using ML estimation for different radar waveforms. Using Algorithms 1 and 2 we can achieve almost similar ML results for original waveform and the NSP waveform

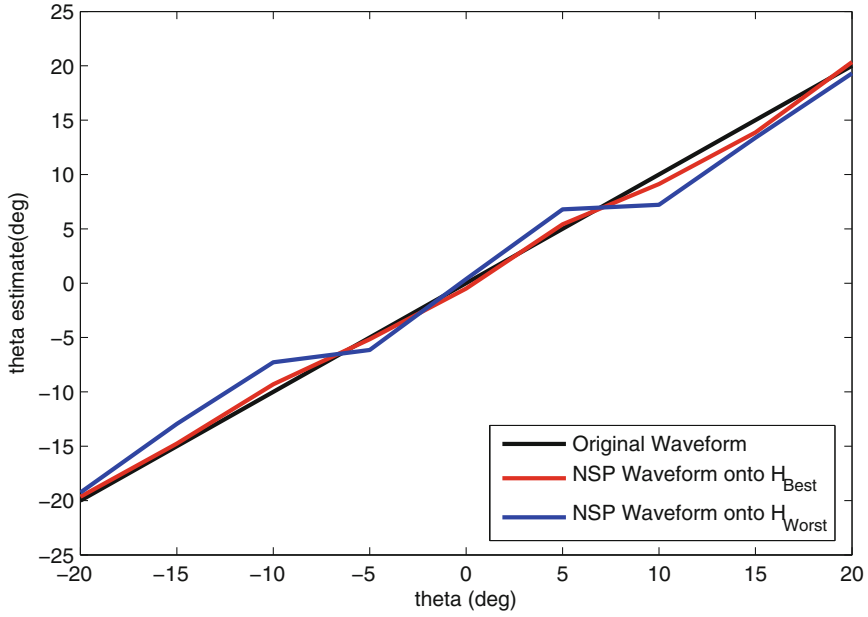


**Fig. 2.3** ML on target direction estimation.  $H_{Best}$  and  $H_{Worst}$  channels are selected using Algorithms 1 and 2

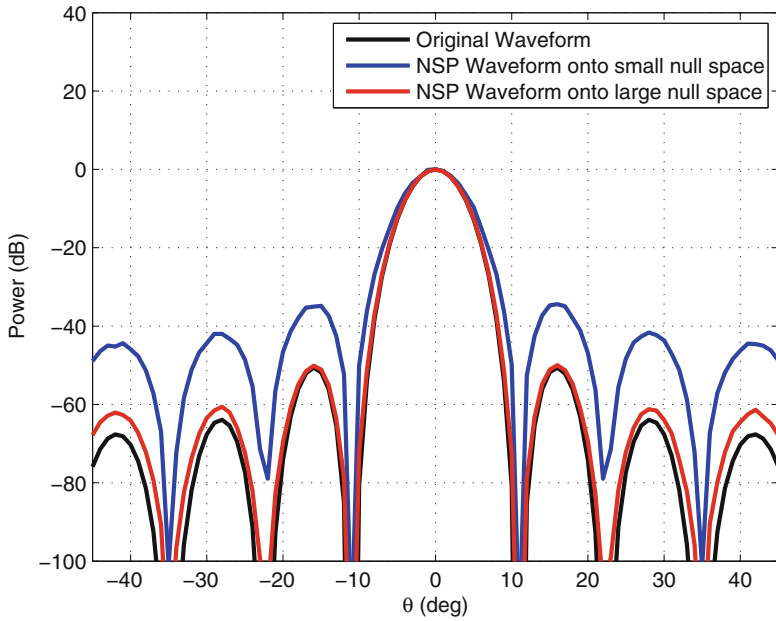


**Fig. 2.4** Beampattern of MIMO radar when different values of threshold are used to calculate the null space of interference channels in Algorithm 2

which shows that by choosing  $\mathbf{H}_{\text{Best}}$  to project we can cause minimum degradation in radar performance. Note that the ML estimate for the NSP waveform onto  $\mathbf{H}_{\text{Worst}}$  is much degraded from the original waveform and the NSP waveform onto  $\mathbf{H}_{\text{Best}}$ .

In Algorithm 2, we describe an approach to numerically calculate null space of interference channels. This is an important approach in the presence of rounding errors and fuzzy data. We select singular values below a certain threshold and take the corresponding columns of  $\mathbf{V}_i^H$  for our NSP equation. Thus, the value of threshold can be a limitation parameter in the projection algorithm, since, the bigger the value of threshold the bigger the null space and the better the performance of the NSP radar waveform. This can be easily noticed from Fig. 2.4, where we compare the beampattern of original radar waveform with the NSP waveform when we choose a larger and a smaller value of threshold. The larger value of threshold corresponds to the best channel and the smaller value corresponds to the worst channel, according to our definitions in Sect. 2.3.2. Note that by increasing or decreasing the value of threshold we can manipulate the magnitude of sidelobes. Thus, for the best radar performance, it is desirable to select interference channel with the maximum null space, i.e., according to Algorithms 1 and 2.

## 2.4 Spectrum Sharing Algorithms for Large MIMO Radar

In this section, we address the problem of target detection by radars that project waveform onto the null space of interference channel in order to mitigate interference to cellular systems. We consider a multiple-input multiple-output (MIMO) radar and a MIMO cellular communication system with  $\mathcal{K}$  base stations (BS). We consider two spectrum sharing scenarios. In the first scenario, the degrees of freedom (DoF) available at the radar are not sufficient enough to simultaneously detect target and mitigate interference to  $\mathcal{K}$  BSs. For this case, we select one BS among  $\mathcal{K}$  BSs for waveform projection on the basis of guaranteeing minimum waveform degradation. For the second case, the radar has sufficient DoF to simultaneously detect target and mitigate interference to all  $\mathcal{K}$  BSs. We study target detection capabilities of null-space projected (NSP) waveform and compare it with the orthogonal waveform. We derive the generalized likelihood ratio test (GLRT) for target detection and derive detector statistic for NSP and orthogonal waveform. The target detection performance for both waveforms is studied theoretically and via Monte Carlo simulations.

We consider two spectrum sharing scenarios which are discussed as follows.

**Case 1** ( $M \ll \mathcal{K}N^{\text{BS}}$  but  $M > N^{\text{BS}}$ ): Consider a scenario in which a MIMO radar has a very small antenna array as compared to the combined antenna array of  $\mathcal{K}$  BSs, i.e.,  $M \ll \mathcal{K}N^{\text{BS}}$ , but is larger than individual BS antenna array, i.e.,  $M > N^{\text{BS}}$ . In such a scenario, it is not possible for the MIMO radar to simultaneously mitigate interference to all the  $\mathcal{K}$  BSs present in the network because of insufficient degrees of freedom (DoF) available. However, the available DoF allow simultaneous target detection and interference mitigation to one of the BS among  $\mathcal{K}$  BSs. The choice of BS selection depends upon the performance metric which radar wants to optimize.

In this chapter, our performance metric is minimum degradation of radar waveform in a minimum norm sense.

A drawback of this approach is that interference is not mitigated to  $\mathcal{K} - 1$  BSs present in the network and the radar has to utilize higher transmit power to achieve the same performance level which can increase the level of interference at BSs not part of the mitigation scheme. This drawback is addressed in the literature by moving  $\mathcal{K} - 1$  BSs to non-radar frequency bands by using resource allocation and carrier aggregation techniques [15, 16].

It is worth mentioning that when  $M \ll \mathcal{K}N^{\text{BS}}$  traditional colocated MIMO radar architecture is not suitable for mitigation of interference by using NSP approaches because sufficient DoF are not available and doing so will result in performance degradation of radar systems. However, the MIMO radar architecture can be modified into an overlapped-MIMO radar architecture, where the transmit array of colocated MIMO radar is partitioned into a number of subarrays that are allowed to overlap. The overlapped-MIMO radar architecture increases the DoF and enjoys the advantages of the MIMO radar while mitigating interference to communication systems without sacrificing the main desirable characteristics for its own transmission.

**Case 2** ( $M \gg \mathcal{K}N^{\text{BS}}$ ): Consider a scenario in which a MIMO radar has a very large antenna array as compared to the combined antenna array of  $\mathcal{K}$  BSs, i.e.,  $M \gg \mathcal{K}N^{\text{BS}}$ . In such a scenario, it is feasible for the MIMO radar to simultaneously mitigate interference to all the  $\mathcal{K}$  BSs present in the network while reliably detecting targets. This is because sufficient degrees of freedom are available for both the tasks. In such a scenario, the combined interference channel that the MIMO radar shares with  $\mathcal{K}$  BSs in the networks is given as

$$\mathbf{H} = [\mathbf{H}_1 \ \mathbf{H}_2 \ \cdots \ \mathbf{H}_{\mathcal{K}}]. \quad (2.18)$$

### 2.4.1 Projection Matrix

In this section, we introduce formation of projection matrices for Case 1 and Case 2.

**Projection for Case 1** ( $M \ll \mathcal{K}N^{\text{BS}}$  but  $M > N^{\text{BS}}$ ): In this section, we define the projection algorithm for ‘Case 1’ which projects radar signal onto the null space of interference channel  $\mathbf{H}_i$ . Assuming, the MIMO radar has channel state information of all  $\mathbf{H}_i$  interference channels, through feedback, in *Mil2Mil* or *Mil2Com* scenario, we can perform singular value decomposition (SVD) to find the null space and then construct a projector matrix. We proceed by first finding SVD of  $\mathbf{H}_i$ , i.e.,

$$\mathbf{H}_i = \mathbf{U}_i \mathbf{\Sigma}_i \mathbf{V}_i^H. \quad (2.19)$$

Now, let us define

$$\tilde{\mathbf{\Sigma}}_i \triangleq \text{diag}(\tilde{\sigma}_{i,1}, \tilde{\sigma}_{i,2}, \dots, \tilde{\sigma}_{i,p}) \quad (2.20)$$

where  $p \triangleq \min(N^{\text{BS}}, M)$  and  $\tilde{\sigma}_{i,1} > \tilde{\sigma}_{i,2} > \dots > \tilde{\sigma}_{i,q} > \tilde{\sigma}_{i,q+1} = \tilde{\sigma}_{i,q+2} = \dots = \tilde{\sigma}_{i,p} = 0$  are the singular values of  $\mathbf{H}_i$ . Next, we define

$$\tilde{\Sigma}'_i \triangleq \text{diag}(\tilde{\sigma}'_{i,1}, \tilde{\sigma}'_{i,2}, \dots, \tilde{\sigma}'_{i,M}) \quad (2.21)$$

where

$$\tilde{\sigma}'_{i,u} \triangleq \begin{cases} 0, & \text{for } u \leq q, \\ 1, & \text{for } u > q. \end{cases} \quad (2.22)$$

Using above definitions we can now define our projection matrix, i.e.,

$$\mathbf{P}_i \triangleq \mathbf{V}_i \tilde{\Sigma}'_i \mathbf{V}_i^H. \quad (2.23)$$

In order to show that  $\mathbf{P}_i$  is a valid projection matrix we prove two results on projection matrices below.

**Property 2.1**  $\mathbf{P}_i \in \mathbb{C}^{M \times M}$  is a projection matrix if and only if  $\mathbf{P}_i = \mathbf{P}_i^H = \mathbf{P}_i^2$ .

*Proof* Let's start by showing the 'only if' part. First, we show  $\mathbf{P}_i = \mathbf{P}_i^H$ . Taking Hermitian of Eq. (2.23) we have

$$\mathbf{P}_i^H = (\mathbf{V}_i \tilde{\Sigma}'_i \mathbf{V}_i^H)^H = \mathbf{P}_i. \quad (2.24)$$

Now, squaring Eq. (2.23) we have

$$\mathbf{P}_i^2 = \mathbf{V}_i \tilde{\Sigma}'_i \mathbf{V}_i^H \times \mathbf{V}_i \tilde{\Sigma}'_i \mathbf{V}_i^H = \mathbf{P}_i \quad (2.25)$$

where above equation follows from  $\mathbf{V}_i^H \mathbf{V}_i = \mathbf{I}$  (since they are orthonormal matrices) and  $(\tilde{\Sigma}'_i)^2 = \tilde{\Sigma}'_i$  (by construction). From Eqs. (2.24) and (2.25) it follows that  $\mathbf{P}_i = \mathbf{P}_i^H = \mathbf{P}_i^2$ . Next, we show  $\mathbf{P}_i$  is a projector by showing that if  $\mathbf{v} \in \text{range}(\mathbf{P}_i)$ , then  $\mathbf{P}_i \mathbf{v} = \mathbf{v}$ , i.e., for some  $\mathbf{w}$ ,  $\mathbf{v} = \mathbf{P}_i \mathbf{w}$ , then

$$\mathbf{P}_i \mathbf{v} = \mathbf{P}_i (\mathbf{P}_i \mathbf{w}) = \mathbf{P}_i^2 \mathbf{w} = \mathbf{P}_i \mathbf{w} = \mathbf{v}. \quad (2.26)$$

Moreover,  $\mathbf{P}_i \mathbf{v} - \mathbf{v} \in \text{null}(\mathbf{P}_i)$ , i.e.,

$$\mathbf{P}_i (\mathbf{P}_i \mathbf{v} - \mathbf{v}) = \mathbf{P}_i^2 \mathbf{v} - \mathbf{P}_i \mathbf{v} = \mathbf{P}_i \mathbf{v} - \mathbf{P}_i \mathbf{v} = \mathbf{0}. \quad (2.27)$$

This concludes our proof.

**Property 2.2**  $\mathbf{P}_i \in \mathbb{C}^{M \times M}$  is an orthogonal projection matrix onto the null space of  $\mathbf{H}_i \in \mathbb{C}^{N^{\text{BS}} \times M}$ .

*Proof* Since  $\mathbf{P}_i = \mathbf{P}_i^H$ , we can write

$$\mathbf{H}_i \mathbf{P}_i^H = \mathbf{U}_i \tilde{\mathbf{\Sigma}}_i \mathbf{V}_i^H \times \mathbf{V}_i \tilde{\mathbf{\Sigma}}_i' \mathbf{V}_i^H = \mathbf{0}. \quad (2.28)$$

The above results follows from noting that  $\tilde{\mathbf{\Sigma}}_i \tilde{\mathbf{\Sigma}}_i' = \mathbf{0}$  by construction.

For ‘Case 1’ we are dealing with  $\mathcal{K}$  interference channels. Therefore, we need to select the interference channel which results in least degradation of radar waveform in a minimum norm sense, i.e.,

$$i_{\min} \triangleq \underset{1 \leq i \leq \mathcal{K}}{\operatorname{argmin}} \left\| \mathbf{P}_i \mathbf{x}(t) - \mathbf{x}(t) \right\|_2 \quad (2.29)$$

$$\check{\mathbf{P}} \triangleq \mathbf{P}_{i_{\min}}. \quad (2.30)$$

Once we have selected our projection matrix it is straight forward to project radar signal onto the null space of interference channel via

$$\check{\mathbf{x}}(t) = \check{\mathbf{P}} \mathbf{x}(t). \quad (2.31)$$

The correlation matrix of our NSP waveform is given as

$$\mathbf{R}_{\check{\mathbf{x}}} = \int_{T_o} \check{\mathbf{x}}(t) \check{\mathbf{x}}^H(t) dt \quad (2.32)$$

which is no longer identity, because the projection does not preserve the orthogonality, and its rank depends upon the rank of the projection matrix.

**Projection for Case 2** ( $M \gg \mathcal{K} N^{\text{BS}}$ ): In this section, we define the projection algorithm for ‘Case 2’ which projects radar signal onto the null space of combined interference channel  $\mathbf{H}$ . The SVD of  $\mathbf{H}$  is given as

$$\mathbf{H} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H. \quad (2.33)$$

Now, let us define

$$\tilde{\mathbf{\Sigma}} \triangleq \operatorname{diag}(\tilde{\sigma}_1, \tilde{\sigma}_2, \dots, \tilde{\sigma}_p) \quad (2.34)$$

where  $p \triangleq \min(N^{\text{BS}}, M)$  and  $\tilde{\sigma}_1 > \tilde{\sigma}_2 > \dots > \tilde{\sigma}_q > \tilde{\sigma}_{q+1} = \tilde{\sigma}_{q+2} = \dots = \tilde{\sigma}_p = 0$  are the singular values of  $\mathbf{H}$ . Next, we define

$$\tilde{\mathbf{\Sigma}}_i' \triangleq \operatorname{diag}(\tilde{\sigma}'_1, \tilde{\sigma}'_2, \dots, \tilde{\sigma}'_M) \quad (2.35)$$

where

$$\tilde{\sigma}'_u \triangleq \begin{cases} 0, & \text{for } u \leq q, \\ 1, & \text{for } u > q. \end{cases} \quad (2.36)$$

Using above definitions we can now define our projection matrix, i.e.,

$$\mathbf{P} \triangleq \mathbf{V} \tilde{\Sigma}' \mathbf{V}^H. \quad (2.37)$$

It is straightforward to see that  $\mathbf{P}$  is a valid projection matrix by using Properties 1 and 2.

### 2.4.2 Spectrum Sharing and Projection Algorithms

In this section, we explain spectrum sharing and projection algorithms for ‘Case 1’ and ‘Case 2’.

**Algorithms for Case 1** ( $M \ll \mathcal{K}N^{\text{BS}}$  but  $M > N^{\text{BS}}$ ): For this case, the process of spectrum sharing by forming projection matrices and selecting interference channels is executed with the help of Algorithms 3 and 4. First, at each pulse repetition interval (PRI), the radar obtains ICSI of all  $\mathcal{K}$  interference channels. This information is sent to Algorithm 4 for the calculation of null spaces and formation of projection matrices. Algorithm 3 process  $\mathcal{K}$  projection matrices, received from Algorithm 4, to find the projection matrix which results in least degradation of radar waveform in a minimum norm sense. This step is followed by the projection of radar waveform onto the null space of the selected BS, i.e., the BS to the corresponding selected projection matrix, and waveform transmission.

---

#### Algorithm 3 Spectrum Sharing Algorithm for Case 1

---

```

loop
  for  $i = 1 : \mathcal{K}$  do
    Get CSI of  $\mathbf{H}_i$  through feedback from the  $i^{\text{th}}$  BS.
    Send  $\mathbf{H}_i$  to Algorithm 4 for the formation of projection matrix  $\mathbf{P}_i$ .
    Receive the  $i^{\text{th}}$  projection matrix  $\mathbf{P}_i$  from Algorithm 4.
  end for
  Find  $i_{\min} = \underset{1 \leq i \leq \mathcal{K}}{\operatorname{argmin}} \left\| \mathbf{P}_i \mathbf{x}(t) - \mathbf{x}(t) \right\|_2$ .
  Set  $\check{\mathbf{P}} = \mathbf{P}_{i_{\min}}$  as the desired projector.
  Perform null space projection, i.e.,  $\check{\mathbf{x}}(t) = \check{\mathbf{P}} \mathbf{x}(t)$ .
end loop

```

---

**Projection for Case 2** ( $M \gg \mathcal{K}N^{\text{BS}}$ ): For this case, the process of spectrum sharing is executed with the help of Algorithms 5 and 6. First, at each pulse repetition interval (PRI), the radar obtains ICSI of all  $\mathcal{K}$  interference channels. This information is sent to Algorithm 6 for the calculation of null space of  $\mathbf{H}$  and the formation of projection matrix  $\mathbf{P}$ . The projection of radar waveform onto the null space of  $\mathbf{H}$  is performed by Algorithm 5.

**Algorithm 4** Projection Algorithm for Case 1

---

```

if  $\mathbf{H}_i$  received from Algorithm 3 then
  Perform SVD on  $\mathbf{H}_i$  (i.e.,  $\mathbf{H}_i = \mathbf{U}_i \boldsymbol{\Sigma}_i \mathbf{V}_i^H$ )
  Construct  $\tilde{\boldsymbol{\Sigma}}_i = \text{diag}(\tilde{\sigma}_{i,1}, \tilde{\sigma}_{i,2}, \dots, \tilde{\sigma}_{i,p})$ 
  Construct  $\tilde{\boldsymbol{\Sigma}}'_i = \text{diag}(\tilde{\sigma}'_{i,1}, \tilde{\sigma}'_{i,2}, \dots, \tilde{\sigma}'_{i,M})$ 
  Setup projection matrix  $\mathbf{P}_i = \mathbf{V}_i \tilde{\boldsymbol{\Sigma}}'_i \mathbf{V}_i^H$ .
  Send  $\mathbf{P}_i$  to Algorithm 3.
end if

```

---

**Algorithm 5** Spectrum Sharing Algorithm for Case 2

---

```

loop
  Get CSI of  $\mathbf{H}$  through feedback from  $\mathcal{K}$  BSs.
  Send  $\mathbf{H}$  to Algorithm 6 for the formation of projection matrix  $\mathbf{P}$ .
  Receive the projection matrix  $\mathbf{P}$  from Algorithm 6.
  Perform null space projection, i.e.,  $\tilde{\mathbf{x}}(t) = \mathbf{P}\mathbf{x}(t)$ .
end loop

```

---

**Algorithm 6** Projection Algorithm for Case 2

---

```

if  $\mathbf{H}$  received from Algorithm 5 then
  Perform SVD on  $\mathbf{H}$  (i.e.,  $\mathbf{H} = \mathbf{U} \boldsymbol{\Sigma} \mathbf{V}^H$ )
  Construct  $\tilde{\boldsymbol{\Sigma}} = \text{diag}(\tilde{\sigma}_1, \tilde{\sigma}_2, \dots, \tilde{\sigma}_p)$ 
  Construct  $\tilde{\boldsymbol{\Sigma}}'_i = \text{diag}(\tilde{\sigma}'_1, \tilde{\sigma}'_2, \dots, \tilde{\sigma}'_M)$ 
  Setup projection matrix  $\mathbf{P} = \mathbf{V} \tilde{\boldsymbol{\Sigma}}'_i \mathbf{V}^H$ .
  Send  $\mathbf{P}$  to Algorithm 5.
end if

```

---

**2.4.3 Statistical Decision Test for Target Detection**

In this section, we develop a statistical decision test for target illuminated with the orthogonal radar waveforms and the NSP projected radar waveforms. The goal is to compare performance of the two waveforms by looking at the test decision on whether the target is present or not in the range-Doppler cell of interest.

For target detection and estimation, we proceed by constructing a hypothesis test where we seek to choose between two hypothesis: the null hypothesis  $\mathcal{H}_0$  which represents the case when the target is absent or the alternate hypothesis  $\mathcal{H}_1$  which represents the case when the target is present. The hypothesis for a single target model in Eq. (2.5) can be written as

$$\mathbf{y}(t) = \begin{cases} \mathcal{H}_1 : \alpha \mathbf{A}(\theta) \mathbf{x}(t) + \mathbf{n}(t), & 0 \leq t \leq T_o, \\ \mathcal{H}_0 : \mathbf{n}(t), & 0 \leq t \leq T_o. \end{cases} \quad (2.38)$$

Since,  $\theta$  and  $\alpha$  are unknown, but deterministic, we use the generalize likelihood ratio test (GLRT). The advantage of using GLRT is that we can replace the unknown parameters with their maximum likelihood (ML) estimates. The ML estimates of  $\alpha$

and  $\theta$  are found for various signal models, targets, and interference sources in [4, 17] when using orthogonal signals. In this chapter, we consider a simpler model with one target and no interference sources in order to study the impact of NSP on target detection in a tractable manner. Therefore, we present a simpler derivation of the ML estimation and GLRT.

The received signal model in Eq. (2.5) can be written as

$$\mathbf{y}(t) = \mathbf{Q}(t, \theta)\alpha + \mathbf{n}(t) \quad (2.39)$$

where

$$\mathbf{Q}(t, \theta) = \mathbf{A}(\theta)\mathbf{x}(t). \quad (2.40)$$

We use Karhunen–Loève expansion for derivation of the log-likelihood function for estimating  $\theta$  and  $\alpha$ . Let  $\Omega$  denote the space of the elements of  $\{\mathbf{y}(t)\}$ ,  $\{\mathbf{Q}(t, \theta)\}$ , and  $\{\mathbf{n}(t)\}$ . Moreover, let  $\psi_z$ ,  $z = 1, 2, \dots$ , be an orthonormal basis function of  $\Omega$  satisfying

$$\langle \psi_z(t), \psi_{z'}(t) \rangle = \int_{T_0} \psi_z(t) \psi_{z'}^*(t) dt = \delta_{zz'} \quad (2.41)$$

where  $\delta_{zz'}$  is the Kronecker delta function. Then, the following series can be used to expand the processes,  $\{\mathbf{y}(t)\}$ ,  $\{\mathbf{Q}(t, \theta)\}$ , and  $\{\mathbf{n}(t)\}$ , as

$$\mathbf{y}(t) = \sum_{z=1}^{\infty} \mathbf{y}_z \psi_z(t) \quad (2.42)$$

$$\mathbf{Q}(t, \theta) = \sum_{z=1}^{\infty} \mathbf{Q}_z(\theta) \psi_z(t) \quad (2.43)$$

$$\mathbf{n}(t) = \sum_{z=1}^{\infty} \mathbf{n}_z \psi_z(t) \quad (2.44)$$

where  $\mathbf{y}_z$ ,  $\mathbf{Q}_z$ , and  $\mathbf{n}_z$  are coefficients in the Karhunen–Loève expansion of the considered processes obtained by taking the corresponding inner product with basis function  $\psi_z(t)$ . Thus, an equivalent discrete model of Eq. (2.39) can be obtained as

$$\mathbf{y}_z = \mathbf{Q}_z(\theta)\alpha + \mathbf{n}_z, \quad z = 1, 2, \dots \quad (2.45)$$

For white circular complex Gaussian processes, i.e.,  $\mathbb{E}[\mathbf{n}(t)\mathbf{n}^*(t - \tau(t))] = \sigma_n^2 \mathbf{I}_M \delta(\tau(t))$ , the sequence  $\{\mathbf{n}_z\}$  is i.i.d. and  $\mathbf{n}_z \sim \mathcal{N}^c(\mathbf{0}_M, \sigma_n^2 \mathbf{I}_M)$ . Thus, we can express the log-likelihood function as

$$L_{\mathbf{y}}(\theta, \alpha) = \sum_{z=1}^{\infty} \left( -M \log(\pi \sigma_n^2) - \frac{1}{\sigma_n^2} \|\mathbf{y}_z - \mathbf{Q}_z(\theta)\alpha\|^2 \right). \quad (2.46)$$

Maximizing Eq. (2.46) with respect to  $\alpha$  yields

$$L_{\mathbf{y}}(\theta, \hat{\alpha}) = \Gamma - \frac{1}{\sigma_n^2} \left( E_{\mathbf{y}\mathbf{y}} - \mathbf{e}_{\mathbf{Q}\mathbf{y}}^H \mathbf{E}_{\mathbf{Q}\mathbf{Q}}^{-1} \mathbf{e}_{\mathbf{Q}\mathbf{y}} \right) \quad (2.47)$$

where

$$\Gamma \triangleq -M \log(\pi \sigma_n^2) \quad (2.48)$$

$$E_{\mathbf{y}\mathbf{y}} \triangleq \sum_{z=1}^{\infty} \|\mathbf{y}_z\|^2 \quad (2.49)$$

$$\mathbf{e}_{\mathbf{Q}\mathbf{y}} \triangleq \sum_{z=1}^{\infty} \mathbf{Q}_z^H \mathbf{y}_z \quad (2.50)$$

$$\mathbf{E}_{\mathbf{Q}\mathbf{Q}}^{-1} \triangleq \sum_{z=1}^{\infty} \mathbf{Q}_z^H \mathbf{Q}_z. \quad (2.51)$$

Note that, in Eq. (2.47), apart from the constant  $\Gamma$ , the remaining summation goes to infinity. However, due to the noncontribution of higher order terms in the estimation of  $\theta$  and  $\alpha$  the summation can be finite. Using the identity

$$\int_{T_o} \mathbf{v}_1(t) \mathbf{v}_2^H(t) dt = \sum_{z=1}^{\infty} \mathbf{v}_{1z} \mathbf{v}_{2z}^H \quad (2.52)$$

for  $\mathbf{v}_i(t) = \sum_{z=1}^{\infty} \mathbf{v}_{1z} \psi_z(t)$ ,  $i = 1, 2$ , Eqs. (2.49)–(2.51) can be written as

$$E_{\mathbf{y}\mathbf{y}} \triangleq \int_{T_o} \|\mathbf{y}(t)\|^2 dt \quad (2.53)$$

$$\mathbf{e}_{\mathbf{Q}\mathbf{y}} \triangleq \int_{T_o} \mathbf{Q}^H(t, \theta) \mathbf{y}(t) dt \quad (2.54)$$

$$\mathbf{E}_{\mathbf{Q}\mathbf{Q}} \triangleq \int_{T_o} \mathbf{Q}^H(t, \theta) \mathbf{Q}(t, \theta) dt. \quad (2.55)$$

Using the definition of  $\mathbf{Q}(t, \theta)$  in Eq. (2.40), we can write the  $f^{\text{th}}$  element of  $\mathbf{e}_{\mathbf{Q}\mathbf{y}}$  as

$$[\mathbf{e}_{\mathbf{Q}\mathbf{y}}]_f = \mathbf{a}^H(\theta_f) \mathbf{E}^T \mathbf{a}(\theta_f) \quad (2.56)$$

where

$$\mathbf{E} = \int_{T_o} \mathbf{y}(t) \mathbf{x}^H(t) dt. \quad (2.57)$$

Similarly, we can write the  $fg^{\text{th}}$  element of  $\mathbf{E}_{\mathbf{Q}\mathbf{Q}}$  as

$$[\mathbf{E}_{\mathbf{Q}\mathbf{Q}}]_{fg} = \mathbf{a}^H(\theta_f)\mathbf{a}(\theta_g)\mathbf{a}^H(\theta_f)\mathbf{R}_x^T\mathbf{a}(\theta_g). \quad (2.58)$$

Since,  $\mathbf{e}_{\mathbf{Q}\mathbf{y}}$  and  $\mathbf{E}_{\mathbf{Q}\mathbf{Q}}$  are independent of the received signal, the sufficient statistic to calculate  $\theta$  and  $\alpha$  is given by  $\mathbf{E}$ . Using Eqs. (2.56)–(2.58) we can write the ML estimate in matrix-vector form as

$$L_{\mathbf{y}}(\hat{\theta}_{\text{ML}}) = \underset{\theta}{\operatorname{argmax}} \frac{\left| \mathbf{a}^H(\hat{\theta}_{\text{ML}})\mathbf{E}\mathbf{a}^*(\hat{\theta}_{\text{ML}}) \right|^2}{M\mathbf{a}^H(\hat{\theta}_{\text{ML}})\mathbf{R}_x^T\mathbf{a}(\hat{\theta}_{\text{ML}})}. \quad (2.59)$$

Then, the GLRT for our hypothesis testing model in Eq. (2.38) is given as

$$L_{\mathbf{y}} = \max_{\theta, \alpha} \{ \log f_{\mathbf{y}}(\mathbf{y}, \theta, \alpha; \mathcal{H}_1) \} - \log f(\mathbf{y}; \mathcal{H}_0) \stackrel{\mathcal{H}_1}{\underset{\mathcal{H}_0}{\gtrless}} \delta \quad (2.60)$$

where  $f_{\mathbf{y}}(\mathbf{y}, \theta, \alpha; \mathcal{H}_1)$  and  $f(\mathbf{y}; \mathcal{H}_0)$  are the probability density functions of the received signal under hypothesis  $\mathcal{H}_1$  and  $\mathcal{H}_0$ , respectively. Hence, the GLRT can be expressed as

$$L_{\mathbf{y}}(\hat{\theta}_{\text{ML}}) = \underset{\theta}{\operatorname{argmax}} \frac{\left| \mathbf{a}^H(\hat{\theta}_{\text{ML}})\mathbf{E}\mathbf{a}^*(\hat{\theta}_{\text{ML}}) \right|^2}{M\mathbf{a}^H(\hat{\theta}_{\text{ML}})\mathbf{R}_x^T\mathbf{a}(\hat{\theta}_{\text{ML}})} \stackrel{\mathcal{H}_1}{\underset{\mathcal{H}_0}{\gtrless}} \delta. \quad (2.61)$$

The asymptotic statistic of  $L(\hat{\theta}_{\text{ML}})$  for both the hypothesis is given by [18]

$$L(\hat{\theta}_{\text{ML}}) \sim \begin{cases} \mathcal{H}_1 : \chi_2^2(\rho), \\ \mathcal{H}_0 : \chi_2^2, \end{cases} \quad (2.62)$$

where

- $\chi_2^2(\rho)$  is the noncentral chi-squared distributions with two degrees of freedom,
- $\chi_2^2$  is the central chi-squared distributions with two degrees of freedom,
- and  $\rho$  is the noncentrality parameter, which is given by

$$\rho = \frac{|\alpha|^2}{\sigma_n^2} |\mathbf{a}^H(\theta)\mathbf{R}_x^T\mathbf{a}(\theta)|^2. \quad (2.63)$$

For the general signal model, we set  $\delta$  according to a desired probability of false alarm  $P_{\text{FA}}$ , i.e.,

$$P_{\text{FA}} = P(L(\mathbf{y}) > \delta | \mathcal{H}_0) \quad (2.64)$$

$$\delta = \mathcal{F}_{\chi_2^2}^{-1}(1 - P_{\text{FA}}) \quad (2.65)$$

where  $\mathcal{F}_{\chi_2^2}^{-1}$  is the inverse central chi-squared distribution function with two degrees of freedom. The probability of detection is given by

$$P_D = P(L(\mathbf{y}) > \delta | \mathcal{H}_1) \quad (2.66)$$

$$P_D = 1 - \mathcal{F}_{\chi_2^2(\rho)} \left( \mathcal{F}_{\chi_2^2}^{-1}(1 - P_{FA}) \right) \quad (2.67)$$

where  $\mathcal{F}_{\chi_2^2(\rho)}$  is the noncentral chi-squared distribution function with two degrees of freedom and noncentrality parameter  $\rho$ .

### 2.4.3.1 $P_D$ for Orthogonal Waveforms

For orthogonal waveforms  $\mathbf{R}_x^T = \mathbf{I}_M$ , therefore, the GLRT can be expressed as

$$L_{\text{Orthog}}(\hat{\theta}_{\text{ML}}) = \frac{\left| \mathbf{a}^H(\hat{\theta}_{\text{ML}}) \mathbf{E} \mathbf{a}^*(\hat{\theta}_{\text{ML}}) \right|^2}{M \mathbf{a}^H(\hat{\theta}_{\text{ML}}) \mathbf{a}(\hat{\theta}_{\text{ML}})} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \delta_{\text{Orthog}} \quad (2.68)$$

and the statistic of  $L(\hat{\theta}_{\text{ML}})$  for this case is

$$L_{\text{Orthog}}(\hat{\theta}_{\text{ML}}) \sim \begin{cases} \mathcal{H}_1 : \chi_2^2(\rho_{\text{Orthog}}), \\ \mathcal{H}_0 : \chi_2^2, \end{cases} \quad (2.69)$$

where

$$\rho_{\text{Orthog}} = \frac{M^2 |\alpha|^2}{\sigma_n^2}. \quad (2.70)$$

We set  $\delta_{\text{Orthog}}$  according to a desired probability of false alarm  $P_{\text{PF-Orthog}}$ , i.e.,

$$\delta_{\text{Orthog}} = \mathcal{F}_{\chi_2^2}^{-1}(1 - P_{\text{PF-Orthog}}) \quad (2.71)$$

and then the probability of detection for orthogonal waveforms is given by

$$P_{D\text{-Orthog}} = 1 - \mathcal{F}_{\chi_2^2(\rho_{\text{Orthog}})} \left( \mathcal{F}_{\chi_2^2}^{-1}(1 - P_{\text{PF-Orthog}}) \right). \quad (2.72)$$

### 2.4.3.2 $P_D$ for NSP Waveforms

For spectrum sharing waveforms  $\mathbf{R}_x^T = \mathbf{R}_x^T$ , therefore, the GLRT can be expressed as

$$L_{\text{NSP}}(\hat{\theta}_{\text{ML}}) = \frac{\left| \mathbf{a}^H(\hat{\theta}_{\text{ML}}) \mathbf{E} \mathbf{a}^*(\hat{\theta}_{\text{ML}}) \right|^2}{M \mathbf{a}^H(\hat{\theta}_{\text{ML}}) \mathbf{R}_x^T \mathbf{a}(\hat{\theta}_{\text{ML}})} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \delta_{\text{NSP}} \quad (2.73)$$

and the statistic of  $L(\hat{\theta}_{\text{ML}})$  for this case is

$$L_{\text{NSP}}(\hat{\theta}_{\text{ML}}) \sim \begin{cases} \mathcal{H}_1 : \chi_2^2(\rho_{\text{NSP}}), \\ \mathcal{H}_0 : \chi_2^2, \end{cases} \quad (2.74)$$

where

$$\rho_{\text{NSP}} = \frac{|\alpha|^2}{\sigma_n^2} |\mathbf{a}^H(\theta) \mathbf{R}_{\mathbf{x}}^T \mathbf{a}(\theta)|^2. \quad (2.75)$$

We set  $\delta_{\text{NSP}}$  according to a desired probability of false alarm  $P_{\text{PF-NSP}}$ , i.e.,

$$\delta_{\text{NSP}} = \mathcal{F}_{\chi_2^2}^{-1}(1 - P_{\text{PF-NSP}}) \quad (2.76)$$

and then the probability of detection for orthogonal waveforms is given by

$$P_{\text{D-NSP}} = 1 - \mathcal{F}_{\chi_2^2(\rho_{\text{NSP}})}\left(\mathcal{F}_{\chi_2^2}^{-1}(1 - P_{\text{PF-NSP}})\right). \quad (2.77)$$

#### 2.4.4 Numerical Results

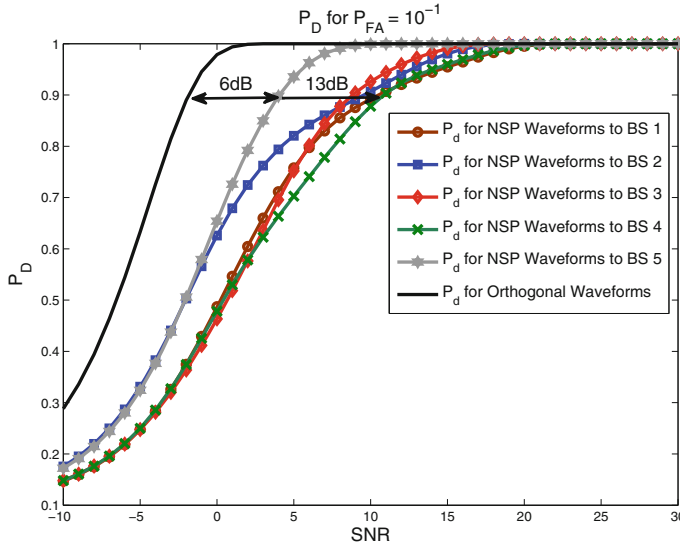
In order to study the detection performance of spectrum sharing MIMO radars, we carry out Monte Carlo simulation using the radar parameters mentioned in [19].

##### 2.4.4.1 Analysis of Case 1

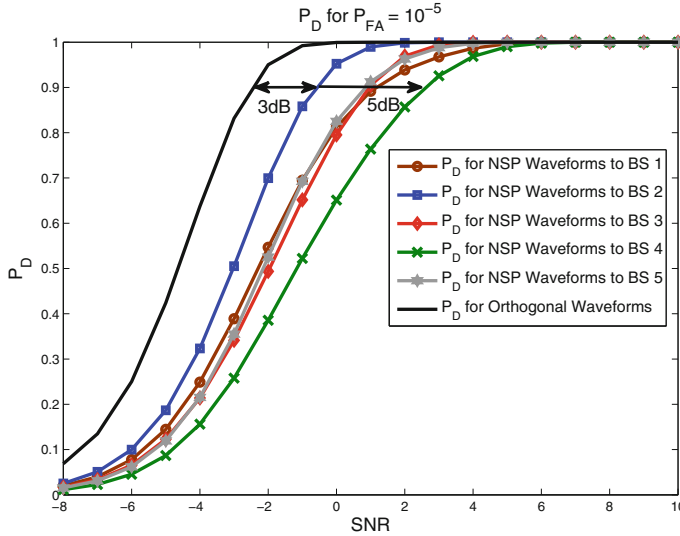
For this case, at each run of Monte Carlo simulation we generate  $\mathcal{K}$  Rayleigh interference channels each with dimensions  $N^{\text{BS}} \times M$ , calculate their null spaces and construct corresponding projection matrices using Algorithm 4, determine the best channel to perform projection of radar signal using Algorithm 3, transmit NSP signal, estimate parameters  $\theta$  and  $\alpha$  from the received signal, and calculate the probability of detection for orthogonal and NSP waveforms.

**Performance of Algorithms 3 and 4:** In Fig. 2.5, we demonstrate the use of Algorithms 3 and 4 in improving target detection performance when multiple BSs are present in detection space of radar and the radar has to reliably detect target while not interfering with communication system of interest. As an example, we consider a scenario with five BSs and the radar has to select a projection channel which minimizes degradation in its waveform, thus, maximizing its probability of detection of the target.

In Fig. 2.5a, we consider the case when  $\dim \mathcal{N}(\mathbf{H}_i) = 2$ . We show detection results for five different NSP signals, i.e., radar waveform projected onto five different BSs. Note that, in order to achieve a detection probability of 90%, we need 6 to 13 dB more gain in SNR as compared to the orthogonal waveform, depending upon which channel we select. Using Algorithms 3 and 4 we can select interference channel that results in minimum degradation of radar waveform and results in enhanced



(a) Probability of detection when  $\dim \mathcal{N}(\mathbf{H}_i) = 2$ . Note that 6 dB to 13 dB of additional gain in SNR is required to detect target with 90% probability, depending upon the NSP waveform transmitted.



(b) Probability of detection when  $\dim \mathcal{N}(\mathbf{H}_i) = 6$ . Note that 3 dB to 5 dB of additional gain in SNR is required to detect target with 90% probability, depending upon the NSP waveform transmitted.

**Fig. 2.5** Case 1 – Performance of Algorithms 3 and 4: Using our spectrum sharing and projection algorithms, we can select interference channel for radar signal projection to maximize detection probability and minimize gain in SNR required as a result of NSP of radar waveforms. For example, Algorithms 3 and 4 select BS#5 and BS#2 for  $\dim \mathcal{N}(\mathbf{H}_i) = 2$  and  $\dim \mathcal{N}(\mathbf{H}_i) = 6$  cases, respectively, as they require minimum additional gain in SNR

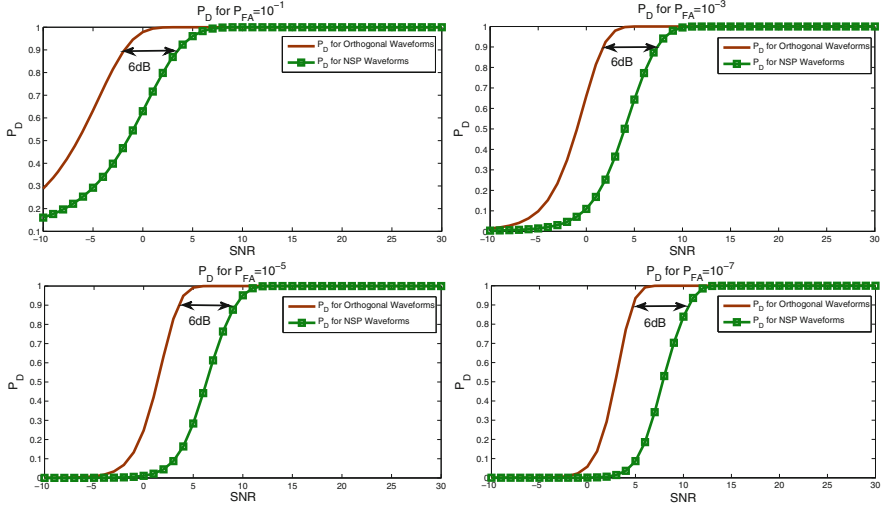


**Fig. 2.6** ‘Case 1(a):  $\dim \mathcal{N}(\mathbf{H}_i) = 2$ ’:  $P_D$  as a function of SNR for various values of probability of false alarm  $P_{FA}$ , i.e.,  $P_{FA} = 10^{-1}, 10^{-3}, 10^{-5}$  and  $10^{-7}$ . The interference channel  $\mathbf{H}_i$  has dimensions  $2 \times 4$ , i.e., the radar has  $M = 4$  antennas and the communication system has  $N^{BS} = 2$  antennas, thus, we have a null-space dimension of ‘ $\dim \mathcal{N}(\mathbf{H}_i) = 2$ ’. Note that we need 6 dB more gain in SNR for the NSP waveform to get the same result produced by the orthogonal waveform

target detection performance with the minimum additional gain in SNR required. For example, Algorithms 3 and 4 would select BS#5 because in this case NSP waveform requires least gain in SNR to achieve a detection probability of 90% as compared to other BSs.

In Fig. 2.5b, we consider the case when  $\dim \mathcal{N}(\mathbf{H}_i) = 6$ . Similar to Fig. 2.5a we show detection results for five different NSP signals but now MIMO radar has a larger array of antennas as compared to the previous case. In this case, in order to achieve a detection probability of 90%, we need 3 to 5 dB more gain in SNR as compared to the orthogonal waveform. As in the previous case, using Algorithms 3 and 4 we can select interference channel that results in minimum degradation of radar waveform and results in enhanced target detection performance with the minimum additional gain in SNR required. For example, Algorithms 3 and 4 would select BS#2 because in this case NSP waveform requires least gain in SNR to achieve a detection probability of 90% as compared to the other BSs.

The above two examples demonstrate the importance of Algorithms 3 and 4 in selecting interference channel for radar signal projection to maximize detection probability and minimize gain in SNR required as a result of NSP of radar waveforms for spectrum sharing.

**Case 1(a):  $\dim \mathcal{N}(\mathbf{H}_i) = 2$ :** In Fig. 2.6, we plot the variations of probability of detection  $P_D$  as a function of signal-to-noise ratio (SNR) for various values of probability of false alarm  $P_{FA}$ . Each subplot represents the  $P_D$  for a fixed  $P_{FA}$ . We choose to evaluate  $P_D$  against  $P_{FA}$  values of  $10^{-1}, 10^{-3}, 10^{-5}$  and  $10^{-7}$  when the interference

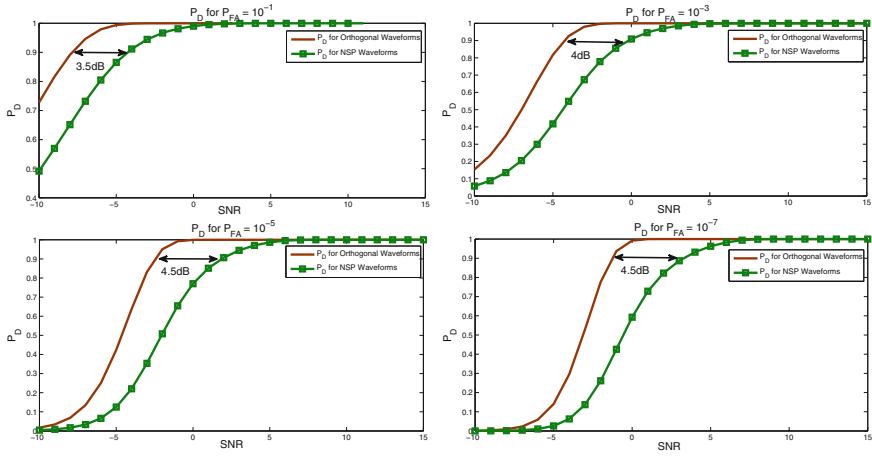


**Fig. 2.7** ‘Case 1(b):  $\dim \mathcal{N}(\mathbf{H}_i) = 6$ ’:  $P_D$  as a function of SNR for various values of probability of false alarm  $P_{FA}$ , i.e.,  $P_{FA} = 10^{-1}, 10^{-3}, 10^{-5}$  and  $10^{-7}$ . The interference channel  $\mathbf{H}_i$  has dimensions  $2 \times 8$ , i.e., the radar has  $M = 8$  antennas and the communication system has  $N^{BS} = 2$  antennas, thus, we have a nullspace dimension of ‘ $\dim \mathcal{N}(\mathbf{H}_i) = 6$ ’. Note that we need 3.5 to 4.5 dB more gain in SNR for the NSP waveform to get the same result produced by the orthogonal waveform

channel  $\mathbf{H}_i$  has dimensions  $2 \times 4$ , i.e., the radar has  $M = 4$  antennas and the communication system has  $N^{BS} = 2$  antennas, thus, we have a null-space dimension of ‘ $\dim \mathcal{N}(\mathbf{H}_i) = 2$ ’. When we compare the detection performance of two waveforms we note that in order to get a desired  $P_D$  for a fixed  $P_{FA}$  we need more SNR for NSP than orthogonal waveforms. For example, say we desire  $P_D = 0.9$ , then according to Fig. 2.6 we need 6 dB more gain in SNR for NSP waveform to get the same result produced by the orthogonal waveform.

**Case 1(b):  $\dim \mathcal{N}(\mathbf{H}_i) = 6$ :** In Fig. 2.7, similar to Fig. 2.6, we do an analysis of  $P_D$  against the same values of  $P_{FA}$  but for interference channel  $\mathbf{H}_i$  having dimensions  $2 \times 8$ , i.e., now the radar has  $M = 8$  antennas and the communication system has  $N^{BS} = 2$  antennas, thus, we have a null-space dimension of ‘ $\dim \mathcal{N}(\mathbf{H}_i) = 6$ ’. Similar to Case 1, when we compare the detection performance of two waveforms we note that in order to get a desired  $P_D$  for a fixed  $P_{FA}$  we need more SNR for NSP than the orthogonal waveforms. For example, say we desire  $P_D = 0.9$ , then according to Fig. 2.7 we need 3.5 to 4.5 dB more gain in SNR for the NSP waveform to get the same result produced by the orthogonal waveform.

**Comparison of Case 1(a) and Case 1(b):** As expected, when SNR increases detection performance increases for both waveforms. However, when we compare the two waveforms at a fixed value of SNR, the orthogonal waveforms perform much better than the NSP waveform in detecting target. This is because our transmitted waveforms are no longer orthogonal and we lose the advantages promised by orthogonal waveforms when used in MIMO radars as discussed in Sect. 2.1.6, but, we ensure

zero-interference to the BS of interest, thus, sharing radar spectrum at an increased cost of target detection in terms of SNR.

In Case 1(a), in order to achieve a desired  $P_D$  for a fixed  $P_{FA}$  we need more SNR for NSP as compared to Case 1(b). This is because we are using more radar antennas, while the antennas at the BS remain fixed in Case 1(b) which increases the dimension of the null space of the interference channel. This yields better detection performance even for NSP waveform. So, in order to mitigate the effect of NSP on radar performance one way is to employ a larger array at the radar transmitter.

#### 2.4.4.2 Analysis of Case 2

For this case, at each run of Monte Carlo simulation we generate  $\mathcal{K}$  Rayleigh interference channels, combine them into one interference channel with dimensions  $\mathcal{K}N^{BS} \times M$ , calculate its null space and construct corresponding projection matrix using Algorithm 6, perform projection of radar signal using Algorithm 5, transmit NSP signal, estimate parameters  $\theta$  and  $\alpha$  from the received signal, and calculate the probability of detection for orthogonal and NSP waveforms.

In Fig. 2.8, we consider the case when the radar has a very large antenna array as compared to the combined antenna array of  $\mathcal{K}$  BSs. In such a scenario, we have enough degrees of freedom at the radar for reliable target detection and simultaneously nulling out interference to all the BSs present in the network. As an example, in

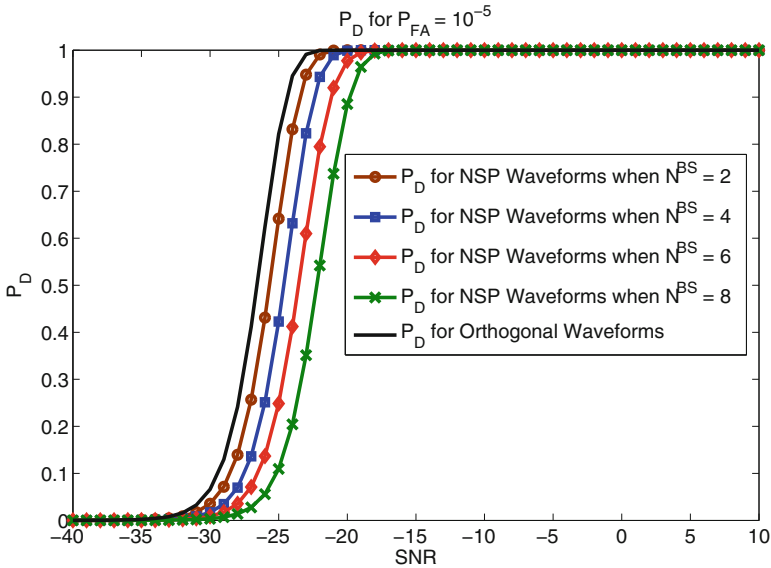


**Fig. 2.8** Case 2:  $P_D$  as a function of SNR for  $P_{FA} = 10^{-5}$ . The MIMO radar mitigates interference to all the BSs in the network. As an example, we consider  $M = 100$ ,  $\mathcal{K} = 5$ , and  $N^{BS} = \{2, 4, 6, 8\}$

Fig. 2.8, we consider  $M = 100$ ,  $\mathcal{K} = 5$ , and  $N^{\text{BS}} = \{2, 4, 6, 8\}$ . We do an analysis of  $P_D$  against  $P_{\text{FA}} = 10^{-5}$  for the combined interference channel  $\mathbf{H}$  having dimensions  $\mathcal{K}N^{\text{BS}} \times M$ . When we compare the detection performance of original waveform and NSP waveform onto the combined channel we note that in order to get a desired  $P_D$  for a fixed  $P_{\text{FA}}$  we need more SNR for NSP than the orthogonal waveforms. For example, say we desire  $P_D = 0.95$ , then according to Fig. 2.8 we need 1, 2, 3.5, and 4.5 dB more gain in SNR for the NSP waveform when  $N^{\text{BS}}$  is 2, 4, 6, and 8, respectively, to get the same result produced by the orthogonal waveforms.

## 2.5 Conclusion

In future, radar RF spectrum will be shared with wireless communication systems to meet the growing bandwidth demands and mitigate the effects of spectrum congestion for commercial wireless services. In this chapter, we analyzed a similar spectrum sharing scenario between radars and cellular systems. We proposed a spatial approach for interference mitigation of radar signals at an LTE cellular system. We focused on a radar-centric interference-mitigation approach where our goal was to manipulate radar signals such that they were not a source of interference to a chosen LTE BS. We extended the idea to project radar signals onto the null space of interference channel for a single system to a cellular system with many base stations. We evaluated target parameter estimation and detection performance of spectrum sharing MIMO radars. We formulated the statistical detection problem for target detection and used generalized likelihood ratio test to decide about the presence of target when using orthogonal waveforms and null-space projected (NSP) waveforms. We proposed novel spectrum sharing algorithms for various cases in which MIMO radar is sharing spectrum with cellular system with minimal degradation in its performance metrics.

## 2.6 MATLAB Code

This section presents MATLAB code for target detection for spectrum sharing MIMO radars.

```
% Compares Pd for Multi BS side-to-side with orthogonal waveforms
% Select Mt number
% Select BS number

%% Define Parameters
% Speed of light
c = 3*10^8;
% Nr Comm Receivers
Nr=2;
% Mt Radar Transmitters
Mt=8;
% Mr Radar Receivers
```

```

Mr=Mt;
% Radial velocity of 2000 m/s
v_r = 2000;
% Radar reference point
r_0 = 500*10^3;
% Carrier frequency 3.5GHz
f_c = 3.5*10^9;
% Angular carrier frequency
omega_c=2*pi*f_c;
lambda = (2*pi*c)/omega_c;
theta=0;

%% Steering vector and Transmit Signal Correlation Matrix
% Transmit/Receive Steering vector (Mt x 1)
a = [1 exp(1i * pi * (1:Mt-1) * sin(theta))]';
% Transmit Correlation Matrix (Mt x Mt) for Orthonormal Waveforms
Rs = eye(Mt);

%% Define SNR for ROC (Reciever Operating Characteristics)

SNR_db=-8:1:10;
SNR_mag=10.^(SNR_db./10);
%Probability of false alarm values
P_FA = [10^-5];
%% Monte-Carlo iterations
MC_iter=10;
Pd_orthog_cell=cell(1,MC_iter);
Pd_NSP_cell=cell(1,MC_iter);
for i=1:MC_iter
%% Interference channel matrix H generation and null space computation

% Generate Cellular Channels and Find NS of them and select
% Channel with Min NS
BS =5;
% Make a cell to store matrices
BS_channels = cell(1,BS);
% Make a cell to store Projectors for every BS
Proj_matrix = cell(1,BS);
for b =1:BS
    BS_channels{b} = (randn(Nr,Mt)+1i*randn(Nr,Mt));
    Proj_matrix{b} = null(BS_channels{b}) * ...
        transpose(null(BS_channels{b}));
    Rs_null{b} = Proj_matrix{b} * Rs * Proj_matrix{b}';

% Non-centrality parameter of chi-square
for z = 1:length(SNR_mag)

    rho_orthog(b) = SNR_mag(z)*(abs(a'*Rs.'*a))^2;
    rho_NSP(b) = SNR_mag(z)*(abs(a'*Rs_null{b}.'*a))^2;

% Creates threshold values for a desired Pfa for
% an inverse central-chi-square w/ 2
% degrees of freedom

delta = chi2inv(ones(1,length(P_FA)) - P_FA, repmat(2,1,length(P_FA)));

% rows = SNR, cols = P_FA %
% ncx2cdf = Noncentral chi-square cumulative distribution function
Pd_orthog(z,:) = ones(1,length(P_FA))- ...
    ncx2cdf(delta, repmat(2,1,length(P_FA)), ...
    repmat(rho_orthog(b),1,length(P_FA)));

Pd_NSP(z,:) = ones(1,length(P_FA))- ...
    ncx2cdf(delta, repmat(2,1,length(P_FA)), ...
    repmat(rho_NSP(b),1,length(P_FA)));

end
Pd_orthog_cell{b}=Pd_orthog;

```

```

Pd_NSP_cell{b}=Pd_NSP;

end
Pd_orthog_cell_multiBS{i}=Pd_orthog_cell;
Pd_NSP_cell_multiBS{i}=Pd_NSP_cell;
Pd_orthog_cat(:, :, i) = cell2mat(Pd_orthog_cell_multiBS{i});
Pd_NSP_cat(:, :, i) = cell2mat(Pd_NSP_cell_multiBS{i});

end
Pd_orthog_cat_mean = mean(Pd_orthog_cat,3);
Pd_NSP_cat_mean = mean(Pd_NSP_cat,3);

%% Plots Probability of detection curves for given
% Probability of false alarm
figure
plot(SNR_db', Pd_NSP_cat_mean(:,1), 'g', 'LineWidth', 2.5)
hold on
plot(SNR_db', Pd_NSP_cat_mean(:,2), 'b', 'LineWidth', 2.5)

plot(SNR_db', Pd_NSP_cat_mean(:,3), 'r', 'LineWidth', 2.5)

plot(SNR_db', Pd_NSP_cat_mean(:,4), 'm', 'LineWidth', 2.5)

plot(SNR_db', Pd_NSP_cat_mean(:,5), 'y', 'LineWidth', 2.5)

plot(SNR_db', Pd_orthog_cat_mean(:,1), 'k', 'LineWidth', 2.5)

xlabel('SNR', 'fontsize', 14)
ylabel('P_D', 'fontsize', 14)
title('P_D for P_{FA} = 10^{-5}', 'fontsize', 14)
legend('P_D for NSP Waveforms to BS 1', 'P_D for NSP Waveforms to BS 2', ...
'P_D for NSP Waveforms to BS 3', 'P_D for NSP Waveforms to BS 4', ...
'P_D for NSP Waveforms to BS 5', 'P_D for Orthogonal Waveforms', 14)

```

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# Chapter 3

## Coloacted MIMO Radar and CoMP Cellular System

Jasmin Mahal, Awais Khawar, Ahmed Abdelhadi and T. Charles Clancy

In this chapter, the authors address the specific problem of spectrum sharing between a multiple-input multiple-output (MIMO) ship-borne defense radar system and a commercial MIMO cellular network consisting of clusters of cooperative base stations (BS), commonly known as Coordinated Multi-Point (CoMP) system. The CoMP system coordinates simultaneous transmissions from multiple BSs to User Equipment (UE) in the downlink and perform joint decoding of UE signals at multiple BSs in the uplink. Thus, the CoMP system utilizes coordination among BSs to effectively turn otherwise harmful inter-cell interference (ICI) into useful signals. This results in improved coverage, throughput, and efficiency for overall cellular system as well as for the cell-edge users. Due to these advantages of the CoMP system over traditional cellular system, 3GPP LTE-Advanced Release 11 and beyond consider CoMP as an enabling technology for 4G mobile systems [1].

We design the radar precoder for two modes of operation: cooperation/cognition mode and interference mitigation mode. During the cooperation/cognition mode, the radar not only broadcasts information to the communication system using the designed precoder but also, utilizing its sensing capability, the radar collects information about the BS-clusters, estimates the channel either by using training symbols sent by the communication system or by blind null-space learning. By processing these information, the radar decides on the best BS- cluster for NSP/SSVSP during interference mitigation mode.

Building upon the works in [2, 3], the authors of this chapter have extended the solution approach to address a MIMO ship-borne radar and a MIMO commercial CoMP communication system coexistence scenario, which is applicable for the LTE-Advanced system. Our contributions are summarized as follows:

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- Precoder Design for Interference Mitigation:** In order to mitigate interference to CoMP system we derive two radar precoders using subspace projection methods in this paper. In the first scheme, the radar projects its signals onto the CoMP BS-clusters and selects the cluster with the maximum null space. This scheme is known as Switched Null-Space Projection (SNSP) as radar looks for the optimal cluster at each pulse repetition interval (PRI), and thus switches among clusters at each PRI depending upon its mission requirements. In the newly proposed second scheme, called Switched Small Singular Value Space Projection (SSSVSP), the projection space has been expanded to include the subspace corresponding to the small nonzero singular values under a specified threshold in addition to the null space. The precoder is designed based on the knowledge of a composite interference channel matrix between the radar and a particular BS-cluster.
- Precoder Design for Cooperation/Cognition:** We derive radar precoders for communicating with the CoMP system based on zero forcing (ZF) and minimum mean square error (MMSE) criteria. The purpose of this mode is information exchange between the radar and the CoMP system. This mode is further divided into two phases. In the first phase, the radar is in cognition mode to sense the training symbols sent by the communication system to perform channel estimation. The CoMP system also informs the radar of its clustering information through a control channel. Consequently, this first phase is associated with signal design on the CoMP side rather than the radar side. In the second phase, the radar in broadcast mode informs the CoMP system about which cluster it has selected for interference mitigation. Obviously, the second phase is associated with radar precoder design for effective radar signal detection at the communication system. The cooperation mode is vital for the success of interference mitigation mode coming right after it. Without this sort of information exchange, spectrum sharing would not be successful.
- Precoder Performance Analysis:** In order to evaluate the performance of radar precoders, we perform detailed theoretical analysis. Although theories are developed based on the assumption of perfect channel knowledge, simulations are carried out considering channel estimation errors [4]. We look at the target localization and interference mitigation capabilities of the radar precoders. The results indicate that while the precoder nullifies the radar interference to the clusters, it degrades the radar performance by introducing correlation in the probing signals. We show that this performance loss can be compensated for by two means either by increasing the number of radar antennas or by utilizing small singular value space projection rather than using null-space projection. Our results show that between the two compensation schemes, the former is more effective.

This chapter is structured as follows. Section 3.1 details the radar/communication system spectral-coexistence model. Design of radar precoder for different operating modes is described in Sect. 3.2. The newly proposed SSSVSP algorithms are presented in Sect. 3.3. Section 3.4 analyzes the performance of the designed radar precoders theoretically. Simulation results are explained in Sect. 3.5. Section 3.6 concludes the chapter with final remarks.

### 3.1 Radar/CoMP System Spectral-Coexistence Models

In this section, we introduce CoMP communication system model, MIMO radar signal model, and our radar-communication system spectrum sharing scenario.

In this chapter, the vectors and matrices are denoted by lowercase and uppercase boldface letters, respectively (e.g.,  $\mathbf{j}$  and  $\mathbf{J}$ ). The rank, null space, transpose, and Hermitian transpose of  $\mathbf{J}$  are represented by  $\text{rank}\{\mathbf{J}\}$ ,  $\mathcal{N}\{\mathbf{J}\}$ ,  $\mathbf{J}^T$  and  $\mathbf{J}^*$ , respectively. The subspace spanned by a set of vectors  $S$  is denoted by  $\text{Span}\{S\}$ . The  $N$  by  $N$  identity matrix is presented by  $\mathbf{I}_N$ .  $\|\cdot\|_2$  and  $\|\cdot\|_F$  denote L-2 norm and Frobenius norm respectively. Moreover,  $(X)^+ = \max(X, 0)$ .

#### 3.1.1 Coordinated Multi-point (CoMP) System

The CoMP reception can be mainly characterized into two classes. One is coordinated scheduling and/or beamforming (CoMP-CS) and the other is joint processing/reception (CoMP-JP). We consider the later for this paper. In this class of joint processing/reception, data from a single UE is detected by multiple BSs to improve the detected signal quality and/or cancel actively interference from other UEs. On the other hand, these BSs can serve a single UE as in the CoMP Single-user MIMO (CoMP-SU-MIMO) mode, or serve multiple UEs simultaneously as in the CoMP Multi-user MIMO (CoMP-MU-MIMO) mode using the same frequency at the same time, as shown in Fig. 3.1.

We consider a CoMP system with a total of  $M$  base stations forming  $C_T$  nonoverlapping sets of BSs with  $i^{\text{th}}$  BS-cluster denoted by  $\mathcal{M}_i = \{1, 2, \dots, m_i, \dots, M_i\}$ , serving a total of  $K$  users forming  $C_T$  disjointed sets of UEs with  $i^{\text{th}}$  UE-cluster denoted by  $\mathcal{K}_i = \{1, 2, \dots, k_i, \dots, K_i\}$ , where  $i = \{1, 2, \dots, C_T\}$ . As  $i^{\text{th}}$  BS-cluster is paired with  $i^{\text{th}}$  UE-cluster, the  $k_i^{\text{th}}$  UE in  $i^{\text{th}}$  UE-cluster receives its message from a cluster of  $M_i$  base stations  $\mathcal{M}_i \subseteq \mathcal{M}$ , where  $\mathcal{M} = \{1, 2, \dots, M\}$  or the  $m_i^{\text{th}}$  BS in  $i^{\text{th}}$  BS-cluster is provided with the messages of its assigned users set  $\mathcal{K}_i \subseteq \mathcal{K}$ , where  $\mathcal{K} = \{1, 2, \dots, K\}$ . Each BS is equipped with  $N_{\text{BS}}$  antennas and each UE is equipped with  $N_{\text{UE}}$  antennas. Consequently, the  $i^{\text{th}}$  CoMP group consists of  $M_i$  cooperative BSs to serve  $K_i$  UEs which are paired and jointly scheduled in the same frequency at the same time. In the uplink/downlink, the  $i^{\text{th}}$  CoMP cluster can form a  $(M_i N_{\text{BS}}) \times (K_i N_{\text{UE}})$  virtual MIMO system.

Let  $\mathbf{d}_{k_i} \in \mathbb{C}^{N_{\text{UE}} \times 1}$  be the vector of transmit symbols intended for the  $k_i^{\text{th}}$  UE. All the BSs in the cluster  $\mathcal{M}_i$  are informed about the data streams  $\mathbf{d}_{k_i}$ . This can be realized by utilizing the backhaul links among the BSs and the central switching unit. For user  $k_i$ , a linear transmit precoding matrix,  $\mathbf{F}_{k_i} \in \mathbb{C}^{M_i N_{\text{BS}} \times N_{\text{UE}}}$ , which transforms the data vector  $\mathbf{d}_{k_i}$  to the  $(M_i N_{\text{BS}} \times 1)$  transmitted vector  $\mathbf{F}_{k_i} \mathbf{d}_{k_i}$ , is employed by the BSs. The received signal vector  $\mathbf{y}_{k_i} \in \mathbb{C}^{N_{\text{UE}} \times 1}$  at the  $k_i^{\text{th}}$  UE on the downlink is given by

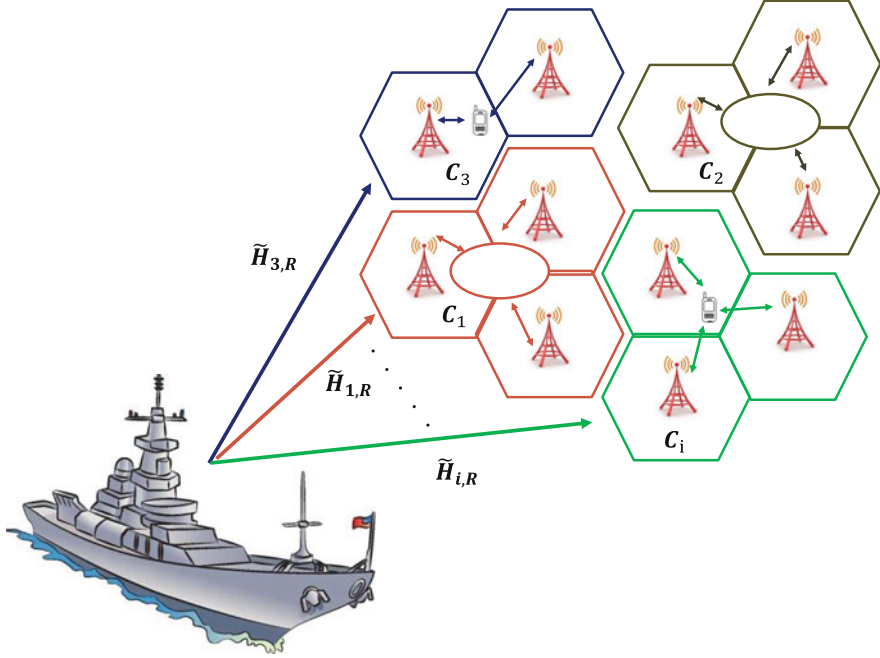


**Fig. 3.1** Spectral coexistence of a littoral MIMO radar (for example, an AN/SPN-43C air traffic control (ATC) radar used by navy in the 3.5 GHz band) with a stationary coordinated multi-point (CoMP) system on the shore. An uplink–downlink model of the CoMP system while sharing radar bands is illustrated

$$\mathbf{y}_{k_i} = \mathbf{G}_{k_i} \mathbf{F}_{k_i} \mathbf{d}_{k_i} + \mathbf{n}_{k_i} \quad (3.1)$$

where  $\mathbf{G}_{k_i} \in \mathbb{C}^{N_{\text{UE}} \times M_i N_{\text{BS}}}$  is the channel matrix between the  $i^{\text{th}}$  BS-cluster and  $k_i^{\text{th}}$  user and  $\mathbf{n}_{k_i} \in \mathbb{C}^{N_{\text{UE}} \times 1}$  accounts for noise and non-radar interference terms. The network channel for the  $i^{\text{th}}$  CoMP cluster is defined as  $\mathbf{G}_i = [\mathbf{G}_1^T, \mathbf{G}_2^T, \dots, \mathbf{G}_{K_i}^T]^T$ , and the corresponding set of signals received by all  $K_i$  users  $\mathbf{y}_i \in \mathbb{C}^{K_i N_{\text{UE}} \times 1}$  is expressed by

$$\mathbf{y}_i = \mathbf{G}_i \mathbf{F}_i \mathbf{d}_i + \mathbf{n}_i. \quad (3.2)$$

Details of the variables and matrices are as follows:  $\mathbf{y}_i = [\mathbf{y}_1^T, \mathbf{y}_2^T, \dots, \mathbf{y}_{K_i}^T]^T$ ,  $\mathbf{F}_i = [\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_{K_i}]$ ,  $\mathbf{d}_i = [\mathbf{d}_1^T, \mathbf{d}_2^T, \dots, \mathbf{d}_{K_i}^T]^T$ ,  $\mathbf{n}_i = [\mathbf{n}_1^T, \mathbf{n}_2^T, \dots, \mathbf{n}_{K_i}^T]^T$ . The precoding matrix  $\mathbf{F}_i$  is designed based on channel information in order to improve performance of the Cooperative-MIMO system.

Similarly, the signal received by the  $m_i^{\text{th}}$  BS in the  $i^{\text{th}}$  CoMP cluster  $\mathbf{b}_{m_i} \in \mathbb{C}^{N_{\text{BS}} \times 1}$  and the received signal vector at the whole cluster  $\mathbf{b}_i \in \mathbb{C}^{M_i N_{\text{BS}} \times 1}$  on the uplink are respectively given by

$$\mathbf{b}_{m_i} = \tilde{\mathbf{G}}_{m_i} \tilde{\mathbf{F}}_{m_i} \mathbf{a}_{m_i} + \tilde{\mathbf{n}}_{m_i} \quad (3.3)$$

$$\mathbf{b}_i = \tilde{\mathbf{G}}_i \tilde{\mathbf{F}}_i \mathbf{a}_i + \tilde{\mathbf{n}}_i \quad (3.4)$$

where  $\mathbf{a}_{m_i} \in \mathbb{C}^{N_{\text{BS}} \times 1}$  is the signal vector intended for the  $m_i^{\text{th}}$  BS and  $\tilde{\mathbf{n}}_{m_i} \in \mathbb{C}^{N_{\text{BS}} \times 1}$  accounts for the corresponding noise and non-radar interference terms. Moreover  $\tilde{\mathbf{G}}_{m_i} \in \mathbb{C}^{N_{\text{BS}} \times K_i N_{\text{UE}}}$  is the channel matrix between the  $i^{\text{th}}$  UE-cluster and  $m_i^{\text{th}}$  BS, and  $\tilde{\mathbf{F}}_{m_i} \in \mathbb{C}^{K_i N_{\text{UE}} \times N_{\text{BS}}}$  is the precoding matrix. Also  $\tilde{\mathbf{G}}_i = [\tilde{\mathbf{G}}_1^T, \tilde{\mathbf{G}}_2^T, \dots, \tilde{\mathbf{G}}_{M_i}^T]^T$ ,  $\mathbf{b}_i = [\mathbf{b}_1^T, \mathbf{b}_2^T, \dots, \mathbf{b}_{M_i}^T]^T$ ,  $\tilde{\mathbf{F}}_i = [\tilde{\mathbf{F}}_1, \tilde{\mathbf{F}}_2, \dots, \tilde{\mathbf{F}}_{M_i}]$ ,  $\mathbf{a}_i = [\mathbf{a}_1^T, \mathbf{a}_2^T, \dots, \mathbf{a}_{M_i}^T]^T$ ,  $\tilde{\mathbf{n}}_i = [\tilde{\mathbf{n}}_1^T, \tilde{\mathbf{n}}_2^T, \dots, \tilde{\mathbf{n}}_{M_i}^T]^T$ . Assuming reciprocity of the channel,

$$\tilde{\mathbf{G}}_i = \mathbf{G}_i^*. \quad (3.5)$$

### 3.1.2 Clustering Algorithms

Joint transmission/reception in the CoMP system requires additional signaling overhead and robust backhaul channels. Due to this reason, only a limited number of BSs cooperate to form a cluster [5]. Cluster formation is an important aspect of the CoMP system in order to exploit benefits promised by the CoMP system. In general, static and dynamic clustering algorithms are proposed. Each scheme has its own merits which are discussed as follows [6]:

- **Static Clustering:** Static clusters do not change over time and are designed based on the time-invariant network parameters such as the geography of BSs and surroundings. As the neighboring BSs interfere the most with each other on average, the clusters are formed by the adjacent BSs only [5].
- **Dynamic Clustering:** Dynamic clusters continuously adapt to changing parameters in the network such as UE locations and RF conditions. Dynamic clustering schemes exploit the effect of changing channel conditions, which leads to much higher performance gains without a significant overhead increase [7]. So the dynamic clustering algorithms cluster the BSs not based on their geographical proximity, but rather based on their level of interference in the absence of any cooperation [8].

As the narrow radar beam will illuminate a specific geographical area, we use static nonoverlapping clustering algorithms with the clusters formed by adjacent cells. We assume that mobile network operators are performing the task of BS clustering and the clustering information is communicated to the radar during the first phase of the cooperation mode.

### 3.1.3 Colocated MIMO Radar

MIMO radar is an emerging area of research and a possible upgrade option of legacy radar systems. Unlike the standard phased-array radar which transmits scaled versions of a single waveform, MIMO radars transmit multiple probing signals that can be chosen freely. This gives MIMO radars significant additional degrees of freedom compared to phased-array radars, allowing them to track more targets with better performance, while simultaneously better eliminating clutter and interference. For colocated transmit and receive antennas, the MIMO radar paradigm is shown to offer better target parameter identifiability, and higher spatial resolution [9–11] which is very critical for radar operation. For orthogonal signals, MIMO configuration extends the array aperture by virtual sensors. This phenomenon of virtual aperture extension enables MIMO radar to obtain higher DOA estimation accuracy, narrower beams and therefore higher angular resolution, and better detection performance.

In this chapter, we consider a colocated MIMO radar with equal number of transmit and receive antenna elements,  $M_R$ . If we denote the samples of baseband equivalent of  $M_R$ -dimensional transmitted radar signals as  $\{\mathbf{x}_R(n)\}_{n=1}^L$ , the signal coherence matrix can be written as [12]

$$\mathbf{R}_x = \frac{1}{L} \sum_{n=1}^L \mathbf{x}_R(n) \mathbf{x}_R^*(n) = \begin{bmatrix} 1 & \beta_{12} & \dots & \beta_{1M_R} \\ \beta_{21} & 1 & \dots & \beta_{2M_R} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{M_R1} & \beta_{M_R2} & \dots & 1 \end{bmatrix} \quad (3.6)$$

where  $n$  is the time index,  $L$  is the total number of time samples, and  $\beta_{oc}$  denotes the correlation coefficient between  $o^{\text{th}}$  and  $c^{\text{th}}$  signals ( $1 \leq o, c \leq M_R$ ). The phases of the set of correlation coefficients,  $\{\beta_{oc}\}$ , direct the beam to the angle of interest. If  $\beta_{oc} = 0$  for  $o \neq c$ , then  $\mathbf{R}_x = \mathbf{I}_{M_R}$ , i.e., orthogonal waveforms. This corresponds to omnidirectional transmission.

The signal received from a single point target at an angle  $\theta$  as shown in array configuration in Fig. 3.2 can be written as [13]

$$\mathbf{y}_R(n) = \alpha \mathbf{A}(\theta) \mathbf{x}_R(n) + \mathbf{w}(n) \quad (3.7)$$

where  $\alpha$  represents the complex path loss including the propagation loss and the coefficient of reflection and  $\mathbf{A}(\theta)$  is the transmit–receive steering matrix defined as

$$\mathbf{A}(\theta) \triangleq \mathbf{a}_t(\theta) \mathbf{a}_r^T(\theta). \quad (3.8)$$

Denoting the propagation delay between the target and the  $p^{\text{th}}$  transmit element by  $\tau_{t,p}(\theta)$  and the delay between the target and the  $l^{\text{th}}$  receive element by  $\tau_{r,l}(\theta)$ , the

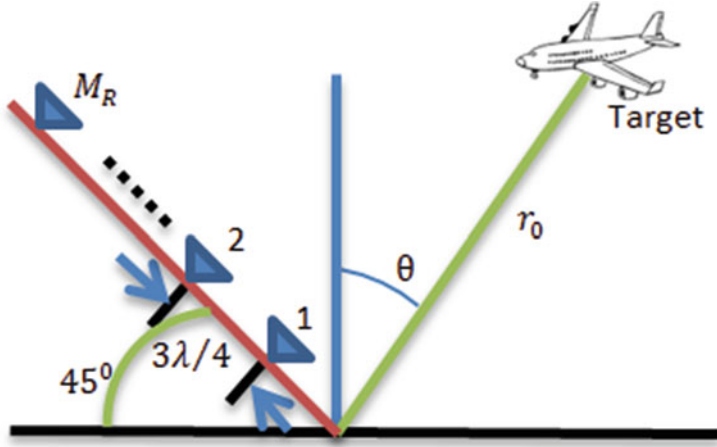


Fig. 3.2 Configuration of radar antenna array

total propagation delay between the  $p^{\text{th}}$  transmit element and the  $l^{\text{th}}$  receive element is given by

$$\tau_{pl}(\theta) = \tau_{t,p}(\theta) + \tau_{r,l}(\theta). \quad (3.9)$$

Based on these notations of propagation delays, the transmit steering vector,  $\mathbf{a}_t(\theta)$  is defined as

$$\mathbf{a}_t(\theta) \triangleq [e^{-j\omega_c\tau_{t,1}(\theta)} \dots e^{-j\omega_c\tau_{t,p}(\theta)} \dots e^{-j\omega_c\tau_{t,M_R}(\theta)}]^T, \quad (3.10)$$

and the receive steering vector,  $\mathbf{a}_r(\theta)$  is defined as

$$\mathbf{a}_r(\theta) \triangleq [e^{-j\omega_c\tau_{r,1}(\theta)} \dots e^{-j\omega_c\tau_{r,l}(\theta)} \dots e^{-j\omega_c\tau_{r,M_R}(\theta)}]^T. \quad (3.11)$$

Using this model, the Cramér–Rao bound (CRB) for target direction estimation is given in Eq.(3.12) [11]. In Eq.(3.12),  $\dot{\mathbf{a}}_t(\theta) = \frac{d\mathbf{a}_t}{d\theta}$  and  $\dot{\mathbf{a}}_r(\theta) = \frac{d\mathbf{a}_r}{d\theta}$ . Assuming all other parameters fixed, the performance of target direction estimation in terms of maximum likelihood or Cramér–Rao bound is optimal for orthogonal probing signals [11, 14, 15].

$$\text{CRB}(\theta) = \frac{1}{2 \text{ SNR}} \left( M_R \dot{\mathbf{a}}_t^*(\theta) \mathbf{R}_x^T \dot{\mathbf{a}}_t(\theta) + \mathbf{a}_t(\theta)^* \mathbf{R}_x^T \mathbf{a}_t(\theta) \|\dot{\mathbf{a}}_r(\theta)\|^2 - \frac{M_R |\mathbf{a}_t^*(\theta) \mathbf{R}_x^T \dot{\mathbf{a}}_t(\theta)|^2}{\mathbf{a}_t^*(\theta) \mathbf{R}_x^T \mathbf{a}_t(\theta)} \right)^{-1}. \quad (3.12)$$

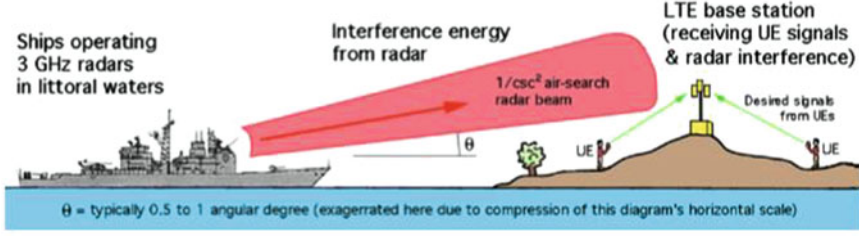


**Fig. 3.3** Schematic diagram showing coupling scenario between littoral radar transmitters and possible future 3.5 GHz LTE systems. By geometry, coupling from radar transmitters should occur more into LTE base station receivers than into UE [16]

### 3.1.4 Spectral-Coexistence Scenario

The communication system shares the spectrum with a monostatic ship-borne MIMO radar system as shown in Fig. 3.1. The composite interference channel between radar transmitter and the  $i^{\text{th}}$  BS-cluster  $\mathcal{M}_i$  is denoted by  $\tilde{\mathbf{H}}_{i,R} \in \mathbb{C}^{M_i N_{\text{BS}} \times M_R}$ . Channels are assumed to be block faded and quasi-static. The signal received by the  $i^{\text{th}}$  BS-cluster  $\mathcal{M}_i$  on the uplink, in the presence of radar, is given simply by

$$\mathbf{b}_i = \tilde{\mathbf{G}}_i \tilde{\mathbf{F}}_i \mathbf{a}_i + \tilde{\mathbf{n}}_i + \tilde{\mathbf{H}}_{i,R} \mathbf{x}_R \quad (3.13)$$

where  $\tilde{\mathbf{H}}_{i,R} \mathbf{x}_R$  is the composite interfering signal from the MIMO radar to the  $i^{\text{th}}$  BS-cluster which we want to mitigate by designing radar precoders in the next section.

We consider the composite interference channel matrix between the radar and the BS-clusters rather than between the radar and the UEs. The handheld UEs are usually at or near ground level that are expected to communicate with BSs typically located at greater heights than the UEs. The BSs are mounted on the sides of buildings or on building rooftops. As antenna beams for the ship-borne radar systems are tilted somewhat above local horizons, typically about 0.5–1 angular degrees, they will couple more strongly into BS receivers than into the UE receivers as shown schematically in Fig. 3.3 [16].

## 3.2 Signal Design for Spectral Coexistence

For amicable spectral coexistence between the radar and the communication system, we design the radar signal so that the interference term, the last terms in (3.13) is forced to zero. The goal is to construct the precoded radar signal  $\tilde{\mathbf{x}}_R$  such that its interference at all of the BSs of the optimally chosen cluster of the CoMP system is either zero-forced or minimized during interference-free mode and the radar keeps switching the beam across the optimal clusters with time. For cooperation mode

when radar is communicating information to the BSs in the second phase, the radar is precoded to ensure that the bit error rate (BER) of the received signal at the BSs is low enough for effective detection. During the first phase of the cooperation mode, all of the BSs in the cluster coordinate in their choices of training symbols and power transmission so that the radar can estimate the composite interference channel between itself and the BS-cluster optimally.

### 3.2.1 Radar Precoder Design for Interference Mitigation

Radar interference to the  $i^{\text{th}}$  BS-cluster  $\mathcal{M}_i$  can be mitigated by designing the radar precoding matrix  $\mathbf{P}_{R,i} \in \mathbb{C}^{M_R \times M_R}$  as

$$\mathbf{H}_{m_i,R} \mathbf{P}_{R,i} \mathbf{x}_R = 0, \quad \forall m_i \in \mathcal{M}_i, \quad (3.14)$$

$$\mathbf{H}_{m_i,R} \tilde{\mathbf{x}}_R = 0, \quad \forall m_i \in \mathcal{M}_i, \quad (3.15)$$

where the precoded radar signal,  $\tilde{\mathbf{x}}_R = \mathbf{P}_{R,i} \mathbf{x}_R$  and  $\mathbf{H}_{m_i,R}$  is the channel matrix between the radar and the  $m_i^{\text{th}}$  BS in  $\mathcal{M}_i$ . The above criteria is satisfied by the following condition:

$$\mathbf{P}_{R,i} \mathbf{x}_R \in \mathcal{N}(\mathbf{H}_{m_i,R}), \quad \forall m_i \in \mathcal{M}_i. \quad (3.16)$$

Therefore, the transmitted radar signal must lie in the null space of  $\mathbf{H}_{m_i,R}$  for all  $m_i$ . We can rewrite Eq. (3.16) as

$$\mathbf{P}_{R,i} \mathbf{x}_R \in \mathcal{N}(\mathbf{H}_{1,R}) \cap \mathcal{N}(\mathbf{H}_{2,R}) \cdots \cap \mathcal{N}(\mathbf{H}_{M_i,R}). \quad (3.17)$$

Using the equality  $\mathcal{N}(\mathbf{A}) \cap \mathcal{N}(\mathbf{B}) = \mathcal{N}(\mathbf{C})$  where  $\mathbf{C} = [\mathbf{A}^* \mathbf{B}^*]^*$ , the precoder must satisfy that

$$\mathbf{P}_{R,i} \mathbf{x}_R \in \mathcal{N}(\tilde{\mathbf{H}}_{i,R}) \quad (3.18)$$

where,

$$\tilde{\mathbf{H}}_{i,R} = [(\mathbf{H}_{1,R})^* (\mathbf{H}_{2,R})^* \cdots (\mathbf{H}_{M_i,R})^*]^*. \quad (3.19)$$

To find the null space, we first obtain the Singular Value Decomposition (SVD) of direct communication link [17] which diagonalizes the composite channel matrix  $\tilde{\mathbf{H}}_{i,R} \in \mathbb{C}^{M_i N_{\text{BS}} \times M_R}$ . The regular form of the SVD of  $\tilde{\mathbf{H}}_{i,R}$  is,  $\tilde{\mathbf{H}}_{i,R} = \tilde{\mathbf{U}}_i \tilde{\mathbf{S}}_i \tilde{\mathbf{V}}_i^*$ , where  $\tilde{\mathbf{U}}_i \in \mathbb{C}^{M_i N_{\text{BS}} \times M_i N_{\text{BS}}}$  is a unitary matrix,  $\tilde{\mathbf{S}}_i \in \mathbb{C}^{M_i N_{\text{BS}} \times M_R}$  is a rectangular diagonal matrix with nonnegative real numbers on the diagonal, and  $\tilde{\mathbf{V}}_i^* \in \mathbb{C}^{M_R \times M_R}$  is another unitary matrix. The diagonal entries of  $\tilde{\mathbf{S}}_i$  are known as the singular values of  $\tilde{\mathbf{H}}_{i,R}$ . The  $M_i N_{\text{BS}}$  columns of  $\tilde{\mathbf{U}}_i$  and the  $M_R$  columns of  $\tilde{\mathbf{V}}_i$  are called the left-singular vectors ( $\mathbf{u} \in \mathbb{C}^{M_i N_{\text{BS}} \times 1}$ ) and right-singular vectors ( $\mathbf{v} \in \mathbb{C}^{M_R \times 1}$ ) of  $\tilde{\mathbf{H}}_{i,R}$  respectively.

Using SVD, null space of  $\tilde{\mathbf{H}}_{i,R}$  is  $\text{Span}\{\tilde{\mathbf{V}}_i\}$  where  $\tilde{\mathbf{V}}_i$  comprises of the columns of  $\tilde{\mathbf{V}}_i$  corresponding to zero singular values of  $\tilde{\mathbf{H}}_{i,R}$ . The precoder  $\mathbf{P}_{R,i}$  is the projection matrix into  $\text{Span}\{\tilde{\mathbf{V}}_i\}$ ,  $\mathbf{P}_{R,i} = \tilde{\mathbf{V}}_i (\tilde{\mathbf{V}}_i^* \tilde{\mathbf{V}}_i)^{-1} \tilde{\mathbf{V}}_i^*$ .

Assuming that the matrix  $\tilde{\mathbf{H}}_{i,R}$  is full row rank, the nullity, i.e., the dimension of null space of  $\tilde{\mathbf{H}}_{i,R}$  is greater than the difference metric,  $(M_R - M_i N_{BS})$ . In mathematical terms,

$$\text{null}[\tilde{\mathbf{H}}_{i,R}] = \dim[\mathcal{N}(\tilde{\mathbf{H}}_{i,R})] = (M_R - M_i N_{BS})^+. \quad (3.20)$$

The necessary condition for a nontrivial precoder ( $\mathbf{P}_{R,i} \neq 0$ ) to exist is that the number of radar transmit antennas is greater than sum of the requested degree of freedom (DoF) of all the base stations in a cluster. In that case, we must have,  $M_R > M_i N_{BS}$  to have a nonzero nullity for  $\tilde{\mathbf{H}}_{i,R}$  and hence a nonzero precoder.

In SVD, the singular values are always arranged in decreasing order:  $\sigma_{M_i N_{BS}} < \sigma_{M_i N_{BS}-1} < \dots < \sigma_1$ . These singular values are in fact measures of the transmitted power in the directions of their corresponding right singular vectors. Consequently, projecting the radar transmitted signal onto a space spanned by the right singular vectors corresponding to singular values smaller than a specific threshold,  $\sigma_{th}$ , will minimize radar interference to the cluster rather than eliminating them completely. This compromise is expected to mitigate the performance loss in radar target detection capability to some extent. The value of  $\sigma_{th}$  is dependent on the power constraints of the communication system. With,  $\sigma < \sigma_{th}$ , i.e., when the projection space is formed by the right singular vectors corresponding to the nonzero small singular values under a threshold in addition to the zero singular values, the interference would be reduced to a great extent for the current channel condition. This kind of projection could be called Small Singular Value Space Projection (SSVSP). For SSVSP, using SVD of  $\tilde{\mathbf{H}}_{i,R}$ , the projection space is  $\text{Span}\{\tilde{\mathbf{V}}_{s,i}\}$  where  $\tilde{\mathbf{V}}_{s,i}$  comprises of the columns of  $\tilde{\mathbf{V}}_i$  corresponding to the small nonzero singular values under  $\sigma_{th}$  in addition to the zero singular values. As a result, the precoder  $\mathbf{P}_{R,i}$  is given by,  $\mathbf{P}_{R,i} = \tilde{\mathbf{V}}_{s,i} (\tilde{\mathbf{V}}_{s,i}^* \tilde{\mathbf{V}}_{s,i})^{-1} \tilde{\mathbf{V}}_{s,i}^*$ .

Under switched beam condition, the radar keeps scanning for the best composite interference channel between itself and the clusters, and projects its beam towards that cluster. With Switched Null-Space Projection (SNSP), the best and the worst composite interference channels are selected in the following way [2]

$$\tilde{\mathbf{H}}_{best} = (\tilde{\mathbf{H}}_{i,R})_{i_{max}} \quad (3.21)$$

$$\text{where, } i_{max} = \arg \max_{1 \leq i \leq C_T} \dim[\mathcal{N}(\tilde{\mathbf{H}}_{i,R})] \quad (3.22)$$

$$\text{and, } \tilde{\mathbf{H}}_{worst} = (\tilde{\mathbf{H}}_{i,R})_{i_{min}} \quad (3.23)$$

$$\text{where, } i_{min} = \arg \min_{1 \leq i \leq C_T} \dim[\mathcal{N}(\tilde{\mathbf{H}}_{i,R})]. \quad (3.24)$$

For newly proposed Switched Small Singular Value Space Projection (SSSVSP), the best and the worst composite interference channels are chosen as follows:

$$\tilde{\mathbf{H}}_{best} = (\tilde{\mathbf{H}}_{i,R})_{i_{min}} \quad (3.25)$$

$$\text{where, } i_{min} = \arg \min_{1 \leq i \leq C_T} \|\mathbf{P}_{R_s,i} \mathbf{x}_R - \mathbf{x}_R\|_2 \quad (3.26)$$

$$\text{and, } \tilde{\mathbf{H}}_{worst} = (\tilde{\mathbf{H}}_{i,R})_{i_{max}} \quad (3.27)$$

$$\text{where, } i_{max} = \arg \max_{1 \leq i \leq C_T} \|\mathbf{P}_{R_s,i} \mathbf{x}_R - \mathbf{x}_R\|_2. \quad (3.28)$$

### 3.2.1.1 Singular Value Threshold $\sigma_{th}$

The singular value threshold  $\sigma_{th}$  is determined based on the  $P_{rad}$ , which is the allowed radar power level to the communication system so that it does not drive the communication receiver amplifier into saturation. Assuming  $\max \{P_{com}\}$  to be the maximum power allowed at the BS, the corresponding power constraint is given by

$$E \left[ \|\mathbf{P}_{R_s,i} \mathbf{x}_R\|^2 \right] = \text{Tr} \{ \mathbf{P}_{R_s,i} \mathbf{P}_{R_s,i}^* \} \leq \sigma_{th}^2 P_{rad} \quad (3.29)$$

$$\text{therefore, } \sigma_{th} = \sqrt{\frac{\max \{P_{com}\}}{P_{rad}}}. \quad (3.30)$$

As  $\sigma_{th}$  becomes larger,  $\tilde{\mathbf{V}}_{s,i}$  approaches  $\tilde{\mathbf{V}}_i$  and consequently,  $\mathbf{P}_{R_s,i}$  approaches  $\mathbf{I}_{M_R}$ . Finally, with  $\sigma_{th} \geq \sigma_1$  we have  $\tilde{\mathbf{V}}_{s,i} = \tilde{\mathbf{V}}_i$  and  $\mathbf{P}_{R_s,i} = \mathbf{I}_{M_R}$ , which essentially means no precoding at radar with consequent interferences at the communication system but best target detection and localization capability of radar.

### 3.2.1.2 Composite Channel Matrix $\tilde{\mathbf{H}}_{i,R}$ and Singular Values

To make the composite channel matrix full row rank, we assume that the elements of  $\tilde{\mathbf{H}}_{i,R}$  are independent, identically distributed (i.i.d.) and drawn from a continuous Gaussian distribution. According to random matrix theory, this makes the rows of  $\tilde{\mathbf{H}}_{i,R}$  linearly independent [18]. Moreover, this also represents the Rayleigh-faded multipath environment of typical cellular system. Now as the condition number of composite channel matrix,  $Z(\tilde{\mathbf{H}}_{i,R}) = \sigma_{max}/\sigma_{min}$  increases, i.e., the more spread out the singular values are, SSSVSP results in less leakage of radar power to the communication system with improved CRB.

### 3.2.2 Radar Precoder Design for Cooperation

In the second phase of this mode, the radar communicates with the CoMP system. The radar informs the communication system of the channel matrices corresponding to base station locations that will be affected by its transmission and the locations that will have a suppressed interference due to the null space or small singular value space projection. In this phase, radar operates in broadcast (BC) mode on downlink, i.e., from radar to the BSs. Linear precoding (LP) techniques such as zero forcing (ZF) and minimum mean square error (MMSE) precoders are considered for this MIMO BC mode. By inverting the channel matrix at the transmitter, the zero-forcing precoder can completely eliminate the interference among the BSs. However, a price of high transmission energy has to be paid especially for near singular matrix channels. This problem can be partially mitigated by using the MMSE linear precoder. This kind of precoder balances the transmission energy and interference level to achieve the minimum detection error. In this mode, radar signal is given by

$$\mathbf{x}_R = [x_{R,1} \dots x_{R,D_R}]^T \in \mathbb{C}^{D_R}$$

where  $D_R$  is the number of independent information streams intended for the BSs and  $D_R \leq \min(MN_{BS}, M_R)$ . Using ZF criterion, the precoding matrix is given by [19]

$$\mathbf{P}_R = \tilde{\mathbf{H}}_R^* \left( \tilde{\mathbf{H}}_R \tilde{\mathbf{H}}_R^* \right)^{-1} \quad (3.31)$$

where,  $\tilde{\mathbf{H}}_R$  is the composite channel between the radar and all the BSs in the communication system and is given by

$$\tilde{\mathbf{H}}_R = [(\mathbf{H}_{1R})^* (\mathbf{H}_{2R})^* \dots (\mathbf{H}_{MR})^*]^*. \quad (3.32)$$

The inverse in Eq.(3.31) can be performed only when the condition,  $M_R \geq D_R$ , is satisfied. Similarly, using MMSE criterion, the precoder is as follows [20]

$$\mathbf{P}_R = \tilde{\mathbf{H}}_R^* \left( \tilde{\mathbf{H}}_R \tilde{\mathbf{H}}_R^* + D_R \sigma^2 \mathbf{I} \right)^{-1}. \quad (3.33)$$

### 3.2.3 CoMP Signal Design for Interference Mitigation

Due to the assumption of MIMO structure on both sides, the communication signal can also be precoded so that they are in the null space of the radar. If we had done the precoding on the communication side, it would have been performed in the following way. The signal received by the radar while the CoMP system is transmitting on the downlink, is given by

$$\mathbf{y}_R(n) = \alpha \mathbf{A}(\theta) \mathbf{x}_R(n) + \mathbf{w}(n) + \sum_{i=1}^{C_T} \tilde{\mathbf{H}}_{i,R}^* \mathbf{F}_i \mathbf{d}_i \quad (3.34)$$

where  $\tilde{\mathbf{H}}_{i,R}^* \mathbf{F}_i \mathbf{d}_i$  is the composite interfering signal from the  $i^{\text{th}}$  BS-cluster to the radar which we want to mitigate by designing communication system precoders. As each BS-cluster has its own signal, the goal is to precode the signal of each BS-cluster so that its corresponding interference to the radar is zero. Consequently, each of the  $C_T$  interference terms in (3.34) is forced to zero during interference mitigation mode. This can be realized by designing the associated precoding matrix  $\mathbf{F}_i \in \mathbb{C}^{M_i N_{BS} \times K_i N_{UE}}$  as

$$\tilde{\mathbf{H}}_{i,R}^* \mathbf{F}_i \mathbf{d}_i = 0. \quad (3.35)$$

The above criteria is satisfied by the following condition

$$\mathbf{F}_i \mathbf{d}_i \in \mathcal{N}(\tilde{\mathbf{H}}_{i,R}^*). \quad (3.36)$$

Therefore, the transmitted communication signal must lie in the left null space of  $\tilde{\mathbf{H}}_{i,R}$  for  $1 \leq i \leq C_T$ .

### 3.2.3.1 Feasibility of Null-Space Projection Precoding on both Radar and Communication Side

As MIMO structure is available on both sides, it naturally calls for a possibility of exploiting null-space projection precoding on both sides so that there is zero mutual interference. The following proposition explains the feasibility of this option.

**Proposition 1** *In a spectral-coexistence scenario of a MIMO radar and a MIMO communication system, null-space projection precoding to avoid interference is possible on either side but not the both.*

*Proof* It is shown in Sect. 3.2.1 that a radar precoder which eliminates its interference to the  $i^{\text{th}}$  CoMP-cluster reduces to the projection matrix into null space of an effective interference channel ( $\tilde{\mathbf{H}}_{i,R}$ ). The necessary condition for a nontrivial precoder ( $\mathbf{P}_{R,i} \neq 0$ ) to exist is that the number of radar transmit antennas is greater than sum of the requested DoF of all the base stations in the cluster, i.e., we must have

$$M_R > M_i N_{BS}.$$

On the other hand, it is shown in Sect. 3.2.3 that the  $i^{\text{th}}$  CoMP precoder which eliminates its interference to the radar is the projection matrix into the left null space of the effective interference channel ( $\tilde{\mathbf{H}}_{i,R}$ ). The necessary condition for a nontrivial precoder ( $\mathbf{F}_i \neq 0$ ) to exist is that the number of radar transmit antennas is less than sum of the requested DoF of all the base stations in the cluster, i.e., we must have

$$M_R < M_i N_{BS}.$$

As we end up in contradictory requirements to have null-space projection precoding accomplished on both sides, that option is not viable.

### 3.2.4 CoMP Signal Design for Cooperation

In the first phase of this mode the CoMP system basically helps radar in the estimation of  $\tilde{\mathbf{H}}_{i,R}$  for all the  $C_T$  BS-clusters. We adopt the channel estimation technique as described in [21]. The estimation is carried out by having all the BSs in the  $i^{\text{th}}$  cluster  $\mathcal{M}_i$  send training symbols to the radar and the radar utilizes the received signal to estimate  $\tilde{\mathbf{H}}_{i,R}$  and find  $\mathbf{P}_{R,i}/\mathbf{P}_{R,i}$ . As this is a CoMP system, BSs in the  $i^{\text{th}}$  cluster  $\mathcal{M}_i$  can cooperate in their choices of training symbols and transmission power. Further assuming channel reciprocity, the channel from  $m_i^{\text{th}}$  BS to the radar transmitter is  $(\mathbf{H}_{m_i,R})^*$ . The composite channel from all of the BSs in the cluster to the radar transmitter is given by

$$\tilde{\mathbf{H}}_{i,R} = [(\mathbf{H}_{1,R})^* (\mathbf{H}_{2,R})^* \cdots (\mathbf{H}_{M_i,R})^*]. \quad (3.37)$$

We can see that,  $\tilde{\mathbf{H}}_{i,R} = (\tilde{\mathbf{H}}_{i,R})^*$ . The coordination among BSs reduces  $\tilde{\mathbf{H}}_{i,R}$  to a standard MIMO channel. So, by using standard MIMO channel estimation algorithm, the equations for estimation of composite interference channel is as shown below [21]

$$\mathbf{y}_e = \sqrt{\frac{\rho}{M_i N_{BS}}} \tilde{\mathbf{H}}_{i,R} \mathbf{s}_e + \mathbf{w}_e, \quad 1 \leq e \leq L_t. \quad (3.38)$$

Here,  $L_t$  is the fixed period at the beginning of each block of  $L$  channel uses where the estimation is performed;  $\mathbf{y}_e$  and  $\mathbf{w}_e$  are respectively the  $M_R$ -dimensional received signal vector and the noise vector at time  $e$ ;  $\mathbf{s}_e$  is a  $(M_i N_{BS})$ -dimensional vector containing the concatenation of training symbols sent by all of the communication BSs in the  $i^{\text{th}}$  cluster at time  $e$  and  $\rho$  is the average SNR at each receiving antenna. The maximum likelihood (ML) estimation of the  $\tilde{\mathbf{H}}_{i,R}$  is given by [21]

$$\hat{\tilde{\mathbf{H}}}_{i,R} (ML) = \sqrt{\frac{M_i N_{BS}}{\rho}} \mathbf{Y} \mathbf{S}^* (\mathbf{S} \mathbf{S}^*)^{-1} \quad (3.39)$$

where the matrices are:  $\mathbf{Y} = [\mathbf{y}_1 \mathbf{y}_2 \cdots \mathbf{y}_{L_t}]$ ,  $\mathbf{W} = [\mathbf{w}_1 \mathbf{w}_2 \cdots \mathbf{w}_{L_t}]$ , and  $\mathbf{S} = [\mathbf{s}_1 \mathbf{s}_2 \cdots \mathbf{s}_{L_t}]$ . The optimal training symbols that minimizes the mean square error is chosen so that  $\mathbf{S} \mathbf{S}^* = L_t \mathbf{I}_{M_i N_{BS}}$ . Cooperation among the BSs is required to choose the optimal training sequence. So, the estimation of  $\tilde{\mathbf{H}}_{i,R}$  is given by

$$\hat{\mathbf{H}}_{i,R} = \left[ \hat{\mathbf{H}}_{i,R} (ML) \right]^*. \quad (3.40)$$

### 3.2.5 The Impact of Ship's Motion on Radar Precoder Design

In [22], the authors investigate this issue and derive coherence time of channel between ship-borne radar and stationary communication system. In addition to ship's horizontal motion (speed), they consider ship's vertical motion (bob) induced by sea and derive the expression for channel coherence time. Considering an AN/SPN-43C air traffic control (ATC) radar, used by navy in the 3.5 GHz band, with PRI of 1 millisecond (ms) and mounted on ships that typically move with a top speed of 32 knots, and also assuming the radar transmits fixed-frequency carrier wave pulse modulated waveform and swept-frequency carrier wave pulse modulated waveform (referred to as P0N and Q3N, respectively, in NTIA reports), it is observed that the PRI of radar is much smaller than the coherence time. The coherence time is more than 2.5 ms while the PRIs are 1 ms or less. Therefore, the designed precoder and NSP/SSVSP will be working perfectly even with a moving shipborne radar without any concerns for CSI being outdated.

### 3.2.6 The Two Modes of Operation and the PRI of Radar

The radar will keep switching among two modes of operation and its target detection interval during its Pulse Repetition Interval ( $PRI = 1/f_p$ ) where  $f_p$  repetition. During the first part of its PRI, it will operate in cooperation/cognition mode for a short period. But this short period is very critical for the proper operation of the radar for the rest of the PRI. In the first phase of the cooperation/cognition mode, it will be basically in cognition mode collecting all the information about the BS clustering of the CoMP and estimating the composite interference channels between itself and the cellular network. The second phase of this mode is associated with the radar informing the communication system of the channel matrices. Based on the vital information it has gathered in the cooperation/cognition mode, it will enter next into the interference mitigation mode. After this mode, it will remain in its target detection mode for the rest of the PRI. Then the same cycle will repeat as shown in Fig. 3.4.

## 3.3 Spectrum Sharing Algorithms

Based on the background theories presented in the last section about the interference mitigation mode, we propose two new algorithms to implement Switched Small

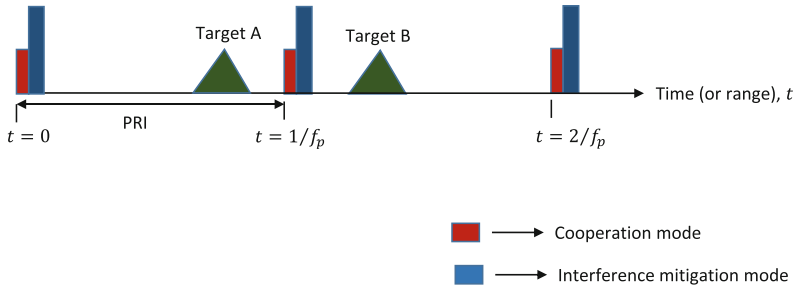


**Fig. 3.4** Schematic diagram presenting the different modes of operation and the PRI of the radar. The cooperation is for brief period to take care of the operational security concerns of the defense radar. The two phases of the cooperation mode are not shown

Singular Value Space Projection. As algorithmic implementation of Switched Null-Space Projection is already available in literature [2, 23], that part is omitted here.

### 3.3.1 Optimal Cluster Selection Algorithm

Our optimal cluster selection algorithm, shown in Algorithm 7, selects the best composite interference channel based on the optimality criteria as mentioned in Eqs.(3.25) and (3.26). Channel State Information (CSI)s are obtained using the channel estimation technique as described in the last section. The SSVSP projection matrices of all the clusters, available at that time interval are then found using Algorithm 8. Algorithm 8 also calculates the difference between the precoded and the original radar signal and returns it to Algorithm 7. Once Algorithm 7 receives the difference metrics of all the composite interference channels, it selects the best composite interference channel associated with the optimal cluster and sends it to Algorithm 8 for SSVSP radar signal. This is the cognitive algorithm component of the overall system.

### 3.3.2 Small Singular Value Space Projection (SSVSP) Algorithm

In this section, we present our proposed Small Singular Value Space Projection algorithm. On the first cycle, Algorithm 8 gets the CSI estimates of the composite interference channels from Algorithm 7 and finds the corresponding small singular value space projection matrices using the singular value decomposition (SVD) theorem. It also calculates the associated difference metrics and return them to Algorithm 7. On the second cycle, after receiving the best composite interference

**Algorithm 7** Optimal Cluster Selection Algorithm

---

```

loop
  for  $i = 1 : C_T$  do
    Estimate CSI of  $\tilde{\mathbf{H}}_{i,R}$ .
    Send  $\tilde{\mathbf{H}}_{i,R}$  to Algorithm 8 for small singular value space computation.
    Receive  $\|\mathbf{P}_{R_s,i} \mathbf{x}_R - \mathbf{x}_R\|_2$  from Algorithm 8.
  end for
  Find  $i_{min} = \arg \min_{1 \leq i \leq C_T} \|\mathbf{P}_{R_s,i} \mathbf{x}_R - \mathbf{x}_R\|_2$ .
  Set  $\check{\mathbf{H}} = (\tilde{\mathbf{H}}_{i,R})_{i_{min}}$  as the best composite interference channel associated with the optimal cluster.
  Send  $\check{\mathbf{H}}$  to Algorithm 8 to get SSVSP radar waveform.
end loop

```

---

channel matrix associated with the optimal cluster from Algorithm 7, it finally calculates the precoded SSVSP radar signal by performing another SVD.

**Algorithm 8** Small Singular Value Space Projection (SSVSP) Algorithm

---

```

if  $\tilde{\mathbf{H}}_{i,R}$  received from Algorithm 7 then
  Perform SVD on  $\tilde{\mathbf{H}}_{i,R}$  (i.e.  $\tilde{\mathbf{H}}_{i,R} = \tilde{\mathbf{U}}_i \tilde{\mathbf{S}}_i \tilde{\mathbf{V}}_i^*$ ).
  Find small singular value space projection matrix  $\mathbf{P}_{R_s,i} = \tilde{\mathbf{V}}_{s,i} \tilde{\mathbf{V}}_{s,i}^*$ .
  Calculate  $\|\mathbf{P}_{R_s,i} \mathbf{x}_R - \mathbf{x}_R\|_2$ .
  Send  $\|\mathbf{P}_{R_s,i} \mathbf{x}_R - \mathbf{x}_R\|_2$  to Algorithm 7.
end if
if  $\check{\mathbf{H}}$  received from Algorithm 7 then
  Perform SVD on  $\check{\mathbf{H}}$  (i.e.  $\check{\mathbf{H}} = \check{\mathbf{U}} \check{\mathbf{S}} \check{\mathbf{V}}^*$ ).
  Find small singular value space projection matrix  $\check{\mathbf{P}}_{R_s} = \check{\mathbf{V}}_s \check{\mathbf{V}}_s^*$ .
  Calculate SSVSP radar signal  $\check{\mathbf{x}}_{R_s} = \check{\mathbf{P}}_{R_s} \mathbf{x}_R$ .
end if

```

---

### 3.4 Theoretical Performance Analysis of the Radar Precoder

This section lays the theoretical foundation of the designed radar precoder performance. As is shown in Sect. 3.2.1, null space of  $\tilde{\mathbf{H}}_{i,R}$  is  $\text{Span}\{\tilde{\mathbf{V}}_i\}$  where  $\tilde{\mathbf{V}}_i$  comprises of the  $M_R - M_i N_{BS}$  right-most columns of  $\tilde{\mathbf{V}}_i$  corresponding to zero singular values of  $\tilde{\mathbf{H}}_{i,R}$ . As  $\tilde{\mathbf{V}}_i$  is a unitary matrix, columns of it form an orthonormal set and rows of it also form an orthonormal set. This results in  $\tilde{\mathbf{V}}_i$  consisting of a set of orthonormal columns but a set of non-orthonormal rows because the rows of the  $\tilde{\mathbf{V}}_i$  are actually part of the orthonormal rows of the corresponding unitary matrix as shown in Fig. 3.5. Consequently, the precoding matrix  $\mathbf{P}_{R,i}$  reduces from

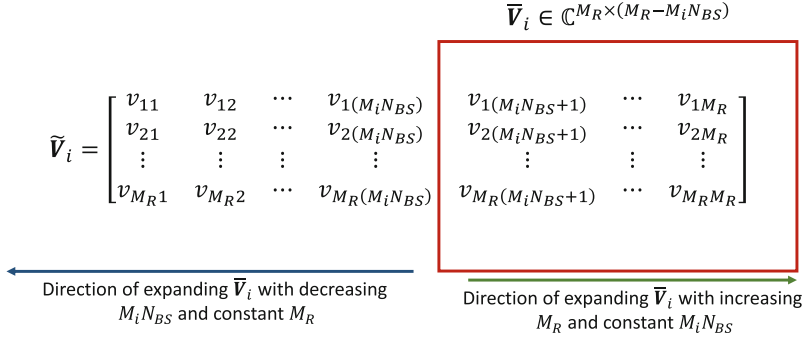


**Fig. 3.5** Schematic diagram presenting how the matrix  $\bar{\mathbf{V}}_i$  changes with the variations in  $M_R$ ,  $M_i$ , and  $N_{BS}$ . This results in variation in Null space with consequent changes in CRB

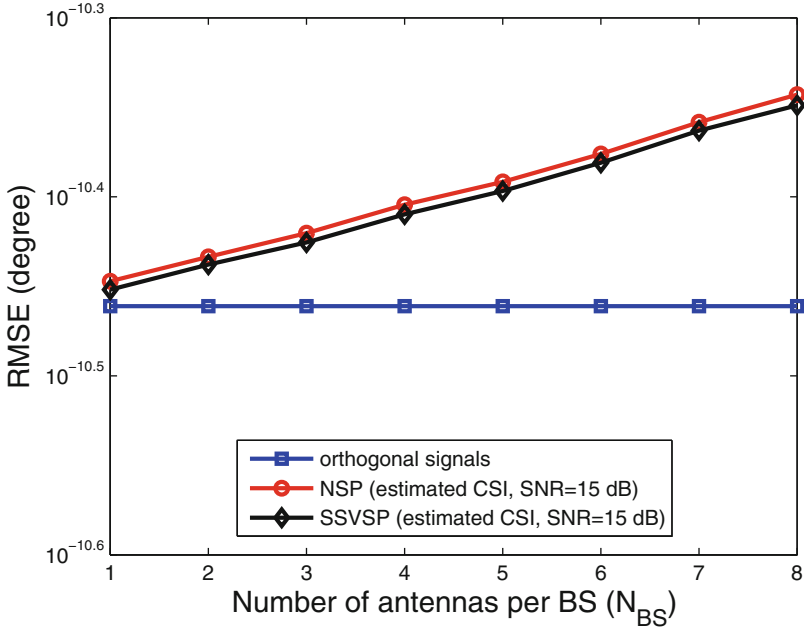


**Fig. 3.6** CRB on direction estimation of the target as a function of antennas employed by the BS when using NSP and SSVSP in a spectrum sharing scenario between MIMO radar and cellular system. A cluster size of  $M_i = 3$  is used while having  $M_R = 100$  radar antenna elements

$(\bar{\mathbf{V}}_i (\bar{\mathbf{V}}_i^* \bar{\mathbf{V}}_i)^{-1} \bar{\mathbf{V}}_i^*)$  to  $(\bar{\mathbf{V}}_i \bar{\mathbf{V}}_i^*)$  because being the correlation matrix of the orthonormal columns,  $(\bar{\mathbf{V}}_i^* \bar{\mathbf{V}}_i) = \mathbf{I}_{M_R - M_i N_{BS}}$ .

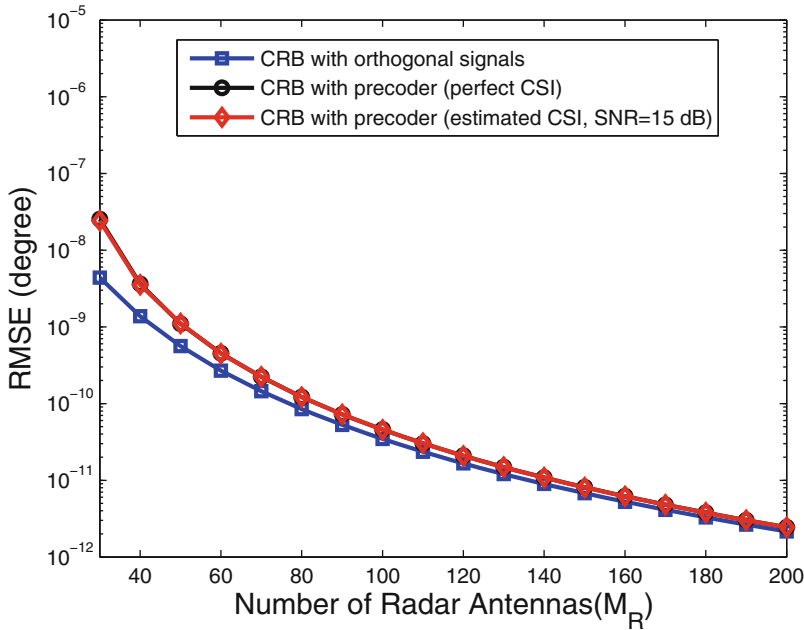


**Fig. 3.7** Effect of number of radar antennas ( $M_R$ ) on the CRB( $\theta$ ) performance of radar. A cluster size of  $M_i = 3$  is used and  $N_{BS} = 8$

On the other hand, being the correlation matrix of the non-orthonormal rows of  $\tilde{\mathbf{V}}_i$ , the precoding matrix comprises of real diagonal elements but nonzero off-diagonal elements. This turns the orthogonal original radar signal  $\mathbf{x}_R$  into non-orthogonal precoded radar signal  $\tilde{\mathbf{x}}_R$ . Although this precoder eliminates the radar interference to all the BSs in the  $i^{\text{th}}$  cluster  $\mathcal{M}_i$ , it affects the spatial correlation of radar probing signals and change their coherence matrix from an identity matrix into one with nonzero off-diagonal elements. For precoded radar signal, the coherence matrix,  $\mathbf{R}_{\mathbf{x},i} = E[\tilde{\mathbf{x}}_R \tilde{\mathbf{x}}_R^*] = \mathbf{P}_{R,i} E[\mathbf{x}_R \mathbf{x}_R^*] \mathbf{P}_{R,i}^* = \mathbf{P}_{R,i} \mathbf{P}_{R,i}^*$  as  $E[\mathbf{x}_R \mathbf{x}_R^*] = \mathbf{I}_{M_R}$ .

Figure 3.5 also reflects the fact that as  $M_R - M_i N_{BS}$  increases with decreasing  $M_i N_{BS}$  keeping  $M_R$  constant, the rows of  $\tilde{\mathbf{V}}_i$  approach closer to the orthonormality as it covers more elements of the rows of the corresponding unitary matrix. As a result, the wider null space with more orthonormal row vectors results in less correlated precoded radar signal with consequent improvement in CRB. Same series of events occurs as  $M_R - M_i N_{BS}$  increases with increasing  $M_R$  keeping  $M_i N_{BS}$  constant. With SSVSP, the  $\tilde{\mathbf{V}}_i$  gets expanded into  $\tilde{\mathbf{V}}_{s,i}$  in a similar way.  $\tilde{\mathbf{V}}_{s,i}$  has more columns and greater part of the rows of the  $\tilde{\mathbf{V}}_i$  as compared to  $\tilde{\mathbf{V}}_i$ , which results in improved CRB as compared to NSP.

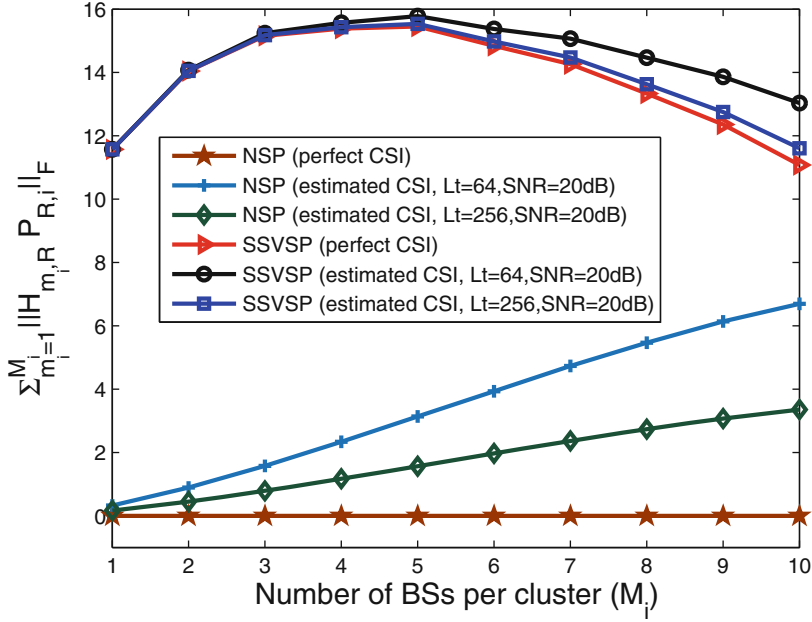


**Fig. 3.8** Variation of radar interference on BS-clusters with cluster size ( $M_i$ ).  $N_{BS} = 6$  BS antenna elements are used while having  $M_R = 100$  radar antenna elements

### 3.5 Simulation Results

In this section, the performance of MIMO radar is compared in terms of CRB for target direction estimation with and without radar precoder and as a function of number of antennas per BS, number of BSs per cluster, and number of radar antennas with either NSP or SSVSP. We have also explored the effect of null-space estimation error on target direction estimation and radar interference to clusters. The distance of target to radar array and the radar inter-element spacing are assumed to be  $r_0 = 5$  km and  $3\lambda/4$ , respectively. The frequency of operation is 3.5 GHz. The signal-to-noise ratio between the radar and its target is denoted by SNR while the same parameter between the radar and the cluster of BSs of CoMP system is represented by  $\rho$ . The target direction is assumed to be at  $\theta = 0^\circ$ .

#### 3.5.1 Performance Analysis of Interference Mitigating Precoder

Figure 3.6 shows the radar CRB performance for orthogonal radar signals as well as precoded radar signals for NSP and SSVSP with estimated Channel State Information

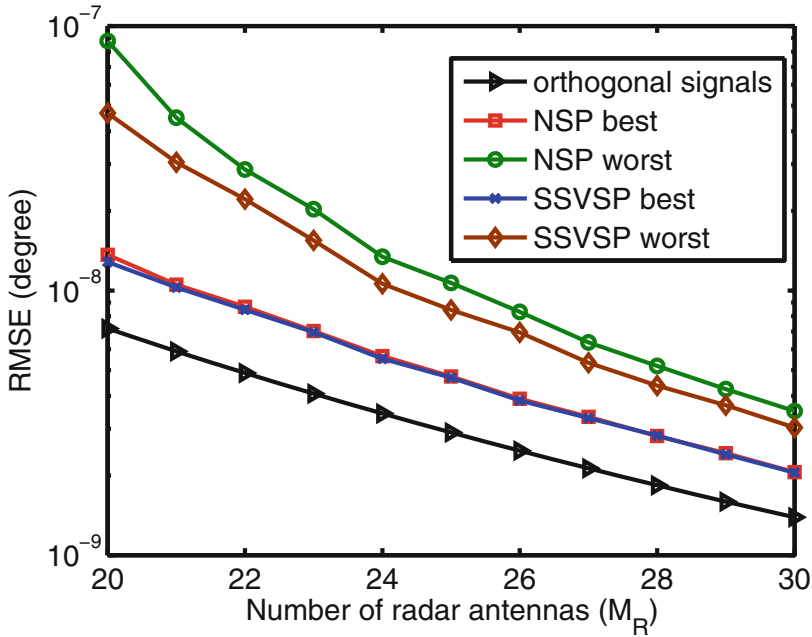


**Fig. 3.9** CRB on direction estimation of the target as a function of number of antenna elements employed by the radar ( $M_R$ ) with switched NSP and SSVSP while in a spectrum sharing scenario between MIMO radar and cellular system. A cluster size of  $M_i = 3$  is used while having estimated CSI. Radar beam is swept across 4 clusters ( $C_T = 4$ ) and the nullity of the clusters is varied by varying the number of antenna elements per BS,  $[N_{BS} = \{6, 5, 4, 3\}]$

(CSI). The figure reflects the fact that SSVSP performs better than NSP from the perspective of radar target detection capability as predicted. This plot also shows that as  $N_{BS}$  increases, the target localization performance of radar degrades. Because with the increase in  $N_{BS}$ ,  $(M_R - M_i N_{BS})$  decreases for constant  $M_R$  and  $M_i$ . So, null space of  $\tilde{\mathbf{H}}_{i,R}$  shrinks with consequent impact on the precoder which in turn degrades  $\text{CRB}(\theta)$ .

In Fig. 3.7, we have investigated the effect of number of radar antennas ( $M_R$ ) on the CRB performance of radar. As obvious from Eq. (3.12) that CRB explicitly depends on  $M_R$  and improves when  $M_R$  increases. For precoded radar signals, increase in  $M_R$  leads to increase in the nullity of  $\tilde{\mathbf{H}}_{i,R}$  which impacts the choice of  $\mathbf{P}_{R,i}$  and results in improved CRB performance. Results in this plot indicate that increasing the number of radar antennas can compensate for the performance degradation in target direction estimation due to correlation in the precoded radar signals. Consequently, for a given target RMSE, number of radar antennas must increase when radar employs a precoder to zero-force or minimize its interference at the clusters.

In Fig. 3.8, the effect of interference at the clusters is measured using the Frobenius metric,  $\sum_{m_i=1}^{M_i} \|\mathbf{H}_{m_i,R} \mathbf{P}_{R,i}\|_F$ . Interference becomes less significant as the duration

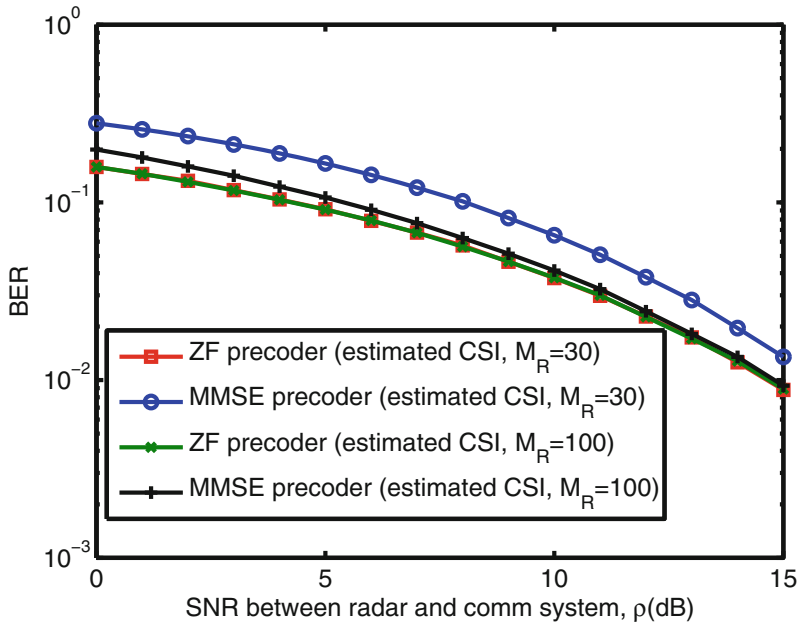


**Fig. 3.10** BER performance during cooperation mode using QPSK waveform. A total of 4 BSs ( $M = 4$ ) is used

of training phase,  $L_t$  increases while it will be completely eliminated with perfect knowledge of  $\tilde{\mathbf{H}}_{i,R}$ . So, estimated CSI approaches perfect CSI as  $L_t$  increases. This plot also reflects the fact that from the perspective of interference at the clusters, NSP outperforms SSVSP to a great extent. With NSP, as  $M_i$  increases,  $(M_R - M_i N_{BS})$  decreases for constant  $M_R$  and  $N_{BS}$ . So, null space of  $\tilde{\mathbf{H}}_{i,R}$  shrinks monotonically with consequent monotonic increase in interference at the communication system. With SSVSP, to keep the interference minimum at the communication system, we performed the simulation by always expanding the null space by incorporating sub-space corresponding to the smallest singular value. Consequently, as  $M_i$  increases keeping  $M_R$  and  $N_{BS}$  constant, we ended up having gradually decreasing smallest singular value which in turn led to monotonically decreasing interference leakage to the communication system. The concave curve reflect the fact that as  $M_i$  increases, up to certain point gradual shrinking of null space dominates over monotonic decrease in smallest singular value with consequent increase in interference. After that with further increase in  $M_i$ , the decrease in smallest singular value dominates over gradual shrinking of null space with consequent decrease in interference.

Figure 3.9 plots radar target estimation capability with switched NSP and SSVSP. It is obvious that both of the beam sweeping methods improve radar performance when the beam is projected on to the optimal cluster associated either with the maximum null space or minimum deviation of the precoded radar signal from the original one. For the worst-case scenarios, SSVSP outperforms NSP.

### 3.5.2 Performance Analysis of Information Exchange Precoder

Figure 3.10 shows the spectrum sharing scenario during the second phase of the cooperation mode. The plot represents the performance of the BS-cluster in terms of BER. During simulations, we make sure that  $M_R \geq D_R$  which results in nonsingular channel matrices. Consequently, ZF outperforms MMSE precoding. The increase in the number of radar antenna is the obvious solution to close the gap between the performance of these two precoding methods.

## 3.6 Conclusion

In this chapter, the authors design precoders of a ship-borne radar in a scenario where a MIMO radar coexists spectrally with a CoMP commercial communication system. We investigate the design of precoder for two modes of radar operation: interference mitigation mode, when the radar attempts to avoid interference with the communication system and cooperation/cognition mode, when it exchanges information with the CoMP system. For the interference mitigation mode, the radar steers its beam towards the optimal clusters of base stations and keeps switching the beam from one optimal cluster to another. In case of traditional SNSP, the optimality is decided upon the maximum nullity of the cluster. A new space projection and switching method has been proposed where the beam is steered to the small singular value space of the optimal cluster whose optimality is chosen based on the minimum difference between the precoded and original radar signal. SSSVSP proves to perform better in terms of sacrificing the radar's target localization ability by intentionally allowing some leakage of radar power in the direction of communication system within the power constraints of preventing the communication system from burnout or saturation.

For cooperation mode, the radar precoder is designed based on the zero-forcing and MMSE criteria to reduce BER at the BSs for effective detection. Channel estimation error is taken into account for both modes of operation. Although the introduction of precoding at the radar tries to mitigate the radar interference at the BS-clusters during interference-free mode, in doing so it causes the radar probing signals lose their orthogonality with consequent performance loss in radar's target parameter estimation capability. SSSVSP mitigates that effect to some extent but increase in the number of radar antenna elements has stronger impact.

Moreover, by modifying the incumbent's signal rather than the secondary user, we attempted to push out the boundaries of the traditions of the first two generations of spectrum sharing. We tried to tread our own path by designing the future federal systems to enable and actively facilitate more efficient spectrum sharing. This increased sharing could in turn stimulate new economic growth.

## 3.7 MATLAB Code

In this section, we provide MATLAB code to generate Fig. 3.6 through Fig. 3.10 that present the performance of designed radar precoder in this chapter.

### 3.7.1 Interference Mitigation Mode

The following scripts produce Fig. 3.6 through Fig. 3.9 that analyze performance of interference mitigation radar precoder.

#### 3.7.1.1 Figure 3.6

```

1  % CRB versus Number of antennas per BS
2  K=3; % No of BS in the cluster
3  M_k=8*ones(1,K); % No of antenna elements at kth transmitter
4  N_k=8*ones(1,K); % No of antenna elements at kth receiver
5  M_R=100; % No of transmit antenna elements on RADAR
6  N_R=100; % No of receive antenna elements on RADAR
7  r_o=5000; % distance of target to radar array in meter
8  fc=3.5e9; % frequency of operation in Hz
9  c=3e8; % speed of light in m/s
10 lamda=c/fc;
11 d=3*lamda/4; % inter-element spacing on radar
12 theta=0; % target direction in degrees
13 L_b=1024; % block length
14 L_t=512; % part of block used for channel estimation, has to be
15 % power of 2 for using 'hadamard'
16 SNR=15; % SNR of received radar signal from target in dB
17 snr=10^(SNR/10);
18 DoF=[1:8];
19 rho_dB=[20]; % average SNR at the radar from comm system
20 rho_ratio=10.^(rho_dB/10);
21 noise_v=sqrt(1./rho_ratio);
22 %% steering vectors and their differentiations
23 a_t=steering_ULA(theta,M_R,fc,d);
24 a_r=steering_ULA(theta,N_R,fc,d);
25 a_tdiff=diff_steering_ULA(theta,M_R,fc,d);
26 a_rdiff=diff_steering_ULA(theta,N_R,fc,d);
27
28 % *****
29 % GENERATE DATA FROM RADAR FOR TARGET DETECTION
30 % *****
31 % Generation of pulses: Orthogonal waveforms S (M_RxL)
32 % such that (1/L)*S*S' is identity matrix
33 s=(1/sqrt(2))*(hadamard(L_b)+1i*hadamard(L_b));
34 s=s(1:M_R,:);
35 Rs=(1/L_b)*s*s';
36
37 % *****

```

```

38 % ESTIMATION OF COMPOSITE CHANNEL FROM COMMUNICATION RECEIVER
39 % TO RADAR TRANSMITTER
40 % *****
41 for n=1:8
42     for x=1:100
43         d_k=n*ones(1,K); %DoF
44         d_ksum=sum(d_k);
45
46         % training symbol generation
47         S=(1/sqrt(2))*(hadamard(L_t)+1i*hadamard(L_t)); % L_t>d_ksum
48         S=S(1:d_ksum,:);
49
50         % composite channel generation
51         H_bar=zeros(M_R,d_ksum);
52         for a=1:K
53             H_kR=randn(d_k(a),M_R)+1i*randn(d_k(a),M_R);
54             if (a==1)
55                 dim=1:d_k(a);
56             else
57                 dim=(sum(d_k(1:a-1))+1):(sum(d_k(1:a)));
58             end
59             H_bar(:,dim)=H_kR';
60         end
61         H_real=H_bar';
62
63         %% Null-space computation for perfect CSI
64         [U_per,B_per,V_per] = svd(H_real); % SVD of H
65
66         % Select eigenvectors corresponding to SVs below threshold sigma
67         if n==1
68             B1 = B_per(n,n);
69         else
70             B1=diag(B_per)';
71         end
72         [row1,col1] = find(B1==0); % Find cols with SV < threshold
73         col=[col1 d_ksum+1:M_R];
74         V_tilda1 = V_per(:,col); % Pick corresponding cols of V
75         P_R1 = V_tilda1*V_tilda1';
76         Rs_null_per = P_R1*P_R1';
77         cb(x) = CRB(Rs, a_t,a_tdiff, a_rdiff, N_R, snr);
78         cb_per(x) = CRB(Rs_null_per, a_t,a_tdiff, a_rdiff, N_R, snr);
79
80         %%SSVSP
81         ssl=length(B1);
82         colms=[ssl d_ksum+1:M_R];
83         V_tilda1ms = V_per(:,colms); % Pick corresponding cols of V
84         P_R1ms = V_tilda1ms*V_tilda1ms';
85         Rs_null_perms = P_R1ms*P_R1ms';
86         cb_perms(x) = CRB(Rs_null_perms, a_t,a_tdiff, a_rdiff, N_R, snr);
87         for r=1:length(noise_v)
88             % AWGN generation
89             W=sqrt(noise_v(r))*(randn(M_R,L_t)+1i*randn(M_R,L_t));
90
91             % signal received at radar for the whole training period
92             Y=zeros(M_R,L_t);
93             for b=1:L_t
94                 Y(:,b)=sqrt(rho_ratio(r)/d_ksum)*H_bar*S(:,b)+W(:,b);
95             end
96

```

```

97 % ML estimation of composite channel
98 H_barest=sqrt(d_ksum/rho_ratio(r))*Y*S'*inv(S*S');
99 H_est=H_barest';
100
101 %% Null-space computation for estimated CSI
102 [U_es,A_es,V_es] = svd(H_est); % SVD of H
103 % Select eigenvectors corresponding to SVs below threshold sigma
104 if n==1
105 A1 = A_es(n,n);
106 else
107     A1=diag(A_es)';
108 end
109 [row2,col2] = find(A1==0); % Find cols with SV < threshold
110 co2=[col2 d_ksum+1:M_R];
111 V_tilda2 = V_es(:,co2); % Pick corresponding cols of V
112 P_R2 = V_tilda2*V_tilda2';
113 Rs_null_wchest = P_R2*P_R2';
114 cb_wchest(r,x) = CRB(Rs_null_wchest, a_t,a_tdiff, a_rdiff,...
115 N_R, snr);
116
117 %SSVSP
118 ss2=length(A1);
119 co2ms=[ss2 d_ksum+1:M_R];
120 V_tilda2ms = V_es(:,co2ms); % Pick corresponding cols of V
121 P_R2ms = V_tilda2ms*V_tilda2ms';
122 Rs_null_wchestms = P_R2ms*P_R2ms';
123 cb_wchestms(r,x) = CRB(Rs_null_wchestms, a_t,a_tdiff,...
124 a_rdiff, N_R, snr);
125 end
126 end
127 crb(n)=mean(cb);
128 crb_null_wchest1(n) = mean(cb_wchest(1,:));
129 crb_ms_wchest1s(n) = mean(cb_wchestms(1,:));
130 crb_null_per(n) = mean(cb_per);
131 crb_ms_per(n) = mean(cb_perms);
132 end
133
134 %% This produces CRB
135 figure(1)
136 semilogy(Dof,crb,'-bs','LineWidth',2)
137 axis([1 8 10^(-10.6) 10^(-10.30)])
138 hold on
139 semilogy(Dof,abs(crb_null_wchest1),'-ro','LineWidth',2)
140 hold on
141 semilogy(Dof,abs(crb_ms_wchest1s),'-kd','LineWidth',2)
142 xlabel('Number of antennas per BS (N_{BS})','fontsize',14)
143 ylabel('RMSE (degree)','fontsize',14)
144 legend('Location','Best','orthogonal signals',...
145 'NSP (estimated CSI, SNR=15 dB)','SSVSP (estimated CSI, SNR=15 dB)')
146 hold off

```

### 3.7.1.2 Figure 3.8

```

1 % Radar Interference versus number of BSs/cluster
2
3 K=(1:10); % No of BSs per cluster

```

```

4  M_k=6*ones(1,length(K));
5  N_k=6*ones(1,length(K));
6  M_R=100; % No of transmit antenna elements on RADAR
7  N_R=100; % No of receive antenna elements on RADAR
8  L_b=512;% block length
9  L_t=[64 256];% part of block used for channel estimation, has to
   be
10 % power of 2 for using 'hadamard'
11 SNR=20; % SNR of received radar signal from target in dB
12 snr=10^(SNR/10);
13 rho_dB=20;
14 rho_ratio=10^(rho_dB/10);
15 noise_v=sqrt(1/rho_ratio);
16
17 % *****
18 % ESTIMATION OF COMPOSITE CHANNEL FROM COMMUNICATION RECEIVER
19 % TO RADAR TRANSMITTER
20 % *****
21 for n=1:length(K)
22     d_k=6*ones(1,n); %DoF for kth user
23     d_ksum=sum(d_k);
24     for x=1:100
25
26         % *****
27         % composite channel generation
28         % *****
29         H_kR=zeros(N_k(1),n*M_R);
30         H_bar=zeros(M_R,d_ksum);
31         for a=1:n
32             H_kR(:,(a-1)*M_R+1:a*M_R)=randn(N_k(a),M_R)+1i*randn(N_k(a),M_R)
33             ;
34             if (a==1)
35                 dim=1:d_k(a);
36             else
37                 dim=(sum(d_k(1:a-1))+1):(sum(d_k(1:a)));
38             end
39             H_bar(:,dim)=(H_kR(:,(a-1)*M_R+1:a*M_R))';
40         end
41         H_real=H_bar';
42
43         % *****
44         %% Null-space computation for perfect CSI
45         % *****
46         [U_per,B_per,V_per] = svd(H_real); % SVD of H
47
48         % Select eigenvectors corresponding to SVs below threshold sigma
49         if d_ksum==1
50             B = B_per(d_ksum,d_ksum);
51         else
52             B=diag(B_per)';
53         end
54         [row1,col1] = find(0 == B); % Find cols with SV < threshold
55         col=[col1 d_ksum+1:M_R];
56         V_tilda1 = V_per(:,col); % Pick corresponding cols of V
57         P_Real = V_tilda1*V_tilda1';
58         frob_per(x)=0;
59         for z=1:n
60             interf_per=(H_kR(:,(z-1)*M_R+1:z*M_R))*P_Real;
61             frob_per(x) = frob_per(x)+norm(interf_per,'fro');

```

```

61 end
62
63 %%SSVSP
64 ssl=length(B);
65 cols=[ssl d_ksum+1:M_R];
66 V_tilda1ms = V_per(:,cols); % Pick corresponding cols of V
67 P_R1ms = V_tilda1ms*V_tilda1ms';
68 frob_perss(x)=0;
69 for zs=1:n
70 interf_perss=(H_kR(:,(zs-1)*M_R+1:zs*M_R))*P_R1ms ;
71 frob_perss(x) = frob_perss(x)+norm(interf_perss,'fro');
72 end
73
74 for r=1:length(L_t)
75
76 %*****
77 % training symbol generation
78 %*****
79 S=(1/sqrt(2))*(hadamard(L_t(r))+1i*hadamard(L_t(r))); % L_t>d_ksum
80 S=S(1:d_ksum,:);
81
82 % AWGN generation
83 W=sqrt(noise_v)*(randn(M_R,L_t(r))+1i*randn(M_R,L_t(r)));
84
85 % signal received at radar for the whole training period
86 Y=zeros(M_R,L_t(r));
87 for b=1:L_t(r)
88 Y(:,b)=sqrt(rho_ratio/d_ksum)*H_bar*S(:,b)+W(:,b);
89 end
90
91 % ML estimation of composite channel
92 H_barest=sqrt(d_ksum/rho_ratio)*Y*S'*inv(S*S');
93 H_est=H_barest';
94
95 %*****
96 %% Null-space computation for estimated CSI
97 %*****
98 [U_es,A_es,V_es] = svd(H_est); % SVD of H
99
100 % Select eigenvectors corresponding to SVs below threshold sigma
101 if d_ksum==1
102 A = A_es(d_ksum,d_ksum);
103 else
104 A=diag(A_es)';
105 end
106 [row2,col2] = find(0 == A); % Find cols with SV < threshold
107 co2=[col2 d_ksum+1:M_R];
108 V_tilda2 = V_es(:,co2); % Pick corresponding cols of V
109 P_Rest = V_tilda2*V_tilda2';
110 frob_wchest(r,x)=0;
111 for z=1:n
112 interf_wchest=(H_kR(:,(z-1)*M_R+1:z*M_R))*P_Rest;
113 frob_wchest(r,x) =frob_wchest(r,x)+norm(interf_wchest,'fro');
114 end
115
116 %%SSVSP
117 ss2=length(A);
118 co2ms=[ss2 d_ksum+1:M_R];
119 V_tilda2ms = V_es(:,co2ms); % Pick corresponding cols of V

```

```

120 P_R2ms = V_tilda2ms*V_tilda2ms';
121 frob_wchestss(r,x)=0;
122 for zs=1:n
123 interf_wchestss=(H_kR(:,(zs-1)*M_R+1:zs*M_R))*P_R2ms ;
124 frob_wchestss(r,x) = frob_wchestss(r,x)+norm(interf_wchestss,'fro
    ');
125 end
126 end
127 end
128
129 frob_null_wchest1(n) = mean(frob_wchest(1,:));
130 frob_null_wchest2(n) = mean(frob_wchest(2,:));
131 frob_null_per(n) = mean(frob_per);
132 frob_ssvsp_wchest1(n) = mean(frob_wchestss(1,:));
133 frob_ssvsp_wchest2(n) = mean(frob_wchestss(2,:));
134 frob_ssvsp_per(n) = mean(frob_per);
135 end
136
137 %% This produces CRB
138 figure (1)
139 plot(K,frob_null_per,'-mp','LineWidth',2)
140 axis([1 10 -1 16])
141 hold on
142 plot(K,frob_null_wchest1,'-c+','LineWidth',2)
143 hold on
144 plot(K,frob_null_wchest2,'-gd','LineWidth',2)
145 hold on
146 plot(K,frob_ssvsp_per,'-r.','LineWidth',2)
147 hold on
148 plot(K,frob_ssvsp_wchest1,'-ko','LineWidth',2)
149 hold on
150 plot(K,frob_ssvsp_wchest2,'-bs','LineWidth',2)
151 xlabel('Number of BSs per cluster (M_i)','fontsize',14)
152 ylabel('\Sigma_{m_i=1}^{M_i} ||H_{m_i,R} P_{R,i}||_F','fontsize
    ',14)
153 legend('Location','Best','NSP (perfect CSI)',...
154 'NSP (estimated CSI, Lt=64, SNR=20dB)',...
155 'NSP (estimated CSI, Lt=256, SNR=20dB)',...
156 'SSVSP (perfect CSI)', 'SSVSP (estimated CSI, Lt=64, SNR=20dB)',...
157 'SSVSP (estimated CSI, Lt=256, SNR=20dB)')
158 hold off

```

### 3.7.1.3 Figure 3.9

```

1 % NSP SSVSP best worst versus number of radar antennas (M_R)
2 C=4; % No of clusters
3 K=3; % No of BS in the cluster
4 M_k=[6 5 4 3];
5 N_k=[6 5 4 3];
6 M_R=(20:30); % No of transmit antenna elements on RADAR
7 N_R=(20:30); % No of receive antenna elements on RADAR
8 L_b=512;% block length
9 L_t=256;
10 SNR=[20]; % SNR of received radar signal from target in dB
11 snr=10.^(SNR/10);
12 DoF=[1:M_k];
13 rho_dB=[15] ;

```

```

14 rho_ratio=10.^(rho_dB/10);
15 noise_v=sqrt(1./rho_ratio);
16
17 %*****
18 % ESTIMATION OF COMPOSITE CHANNEL FROM COMMUNICATION RECEIVER
19 % TO RADAR TRANSMITTER
20 %*****
21 for n=1:length(M_R)
22 % steering vectors and their differentiations
23 a_t=steering_ULA(theta,M_R(n),fc,d);
24 a_r=steering_ULA(theta,N_R(n),fc,d);
25 a_tdiff=diff_steering_ULA(theta,M_R(n),fc,d);
26 a_rdiff=diff_steering_ULA(theta,N_R(n),fc,d);
27 s=(1/sqrt(2))*(hadamard(L_b)+1i*hadamard(L_b));
28 s1=s(1:M_R(n),:);
29 Rs=(1/L_b)*s1*s1';
30 for x=1:100
31     H_mem=zeros(K*max(M_k),C*M_R(n));
32     for p=1:C
33 d_k=M_k(p)*ones(1,K); %DoF
34 d_ksum=sum(d_k);
35
36 % composite channel generation
37 H_bar=zeros(M_R(n),d_ksum);
38 for a=1:K
39     H_kR=(randn(d_k(a),M_R(n))+1i*randn(d_k(a),M_R(n)));
40     if (a==1)
41         dim=1:d_k(a);
42     else
43         dim=(sum(d_k(1:a-1))+1):(sum(d_k(1:a)));
44     end
45     H_bar(:,dim)=H_kR';
46 end
47 H_real=H_bar';
48 H_mem(1:K*M_k(p),(p-1)*M_R(n)+1:p*M_R(n))=H_bar';
49
50 %% Null-space computation for perfect CSI
51 [U_per,B_per,V_per] = svd(H_real); % SVD of H
52 % Select eigenvectors corresponding to SVs below threshold sigma
53 if d_ksum==1
54 B1 = B_per(d_ksum,d_ksum);
55 else
56     B1=diag(B_per)';
57 end
58 [row1,col1] = find(B1==0); % Find cols with SV < threshold
59 col=[col1 d_ksum+1:M_R(n)];
60 nnty(p)=length(col);
61
62 %SSVSP
63 ssl=length(B1);
64 cols=[ssl d_ksum+1:M_R(n)];
65 V_tilda1ms = V_per(:,cols); % Pick corresponding cols of V
66 P_Rlms = V_tilda1ms*V_tilda1ms';
67 sig_rad=s(1:M_R(n),1);
68 diff=P_Rlms*sig_rad-sig_rad;
69 metric(p)=norm(diff);
70 end
71
72 %%%nsp

```

```

73 [best_nsp,I_best_nsp]=max(nlty);
74 H_best=H_mem(1:M_k(I_best_nsp),...
75 (I_best_nsp-1)*M_R(n)+1:I_best_nsp*M_R(n));
76 [worst_nsp,I_worst_nsp]=min(nlty);
77 H_worst=H_mem(1:M_k(I_worst_nsp),...
78 (I_worst_nsp-1)*M_R(n)+1:I_worst_nsp*M_R(n));
79
80 %% Null-space computation for perfect CSI
81 [U_bn,B_bn,V_bn] = svd(H_best); % SVD of H
82 %%
83 % Select eigenvectors corresponding to SVs below threshold sigma
84 d_k=M_k(I_best_nsp)*ones(1,K); %DoF
85 d_ksum=sum(d_k);
86 if d_ksum==1
87 B1_bn = B_bn(d_ksum,d_ksum);
88 else
89 B1_bn=diag(B_bn)';
90 end
91
92 [rowbn,colbn] = find(B1_bn==0); % Find cols with SV < threshold
93 cobn=[colbn d_ksum+1:M_R(n)];
94 V_tildabn = V_bn(:,cobn); % Pick corresponding cols of V
95 P_Rbn = V_tildabn*V_tildabn';
96 Rs_bn = P_Rbn*P_Rbn';
97 cb(x) = CRB(Rs, a_t,a_tdiff, a_rdiff, M_R(n), snr);% orthogonal
    signals
98 cb_best_nsp(x) = CRB(Rs_bn, a_t,a_tdiff, a_rdiff, M_R(n), snr);
99
100 %% Null-space computation for perfect CSI
101 [U_wn,B_wn,V_wn] = svd(H_worst); % SVD of H
102 %%
103 % Select eigenvectors corresponding to SVs below threshold sigma
104 d_k=M_k(I_worst_nsp)*ones(1,K); %DoF
105 d_ksum=sum(d_k);
106 if d_ksum==1
107 B1_wn = B_wn(d_ksum,d_ksum);
108 else
109 B1_wn=diag(B_wn)';
110 end
111 [rowwn,colwn] = find(B1_wn==0); % Find cols with SV < threshold
112 cown=[colwn d_ksum+1:M_R(n)];
113 V_tildawn = V_wn(:,cown); % Pick corresponding cols of V
114 P_Rwn = V_tildawn*V_tildawn';
115 Rs_wn = P_Rwn*P_Rwn';
116 cb_worst_nsp(x) = CRB(Rs_wn, a_t,a_tdiff, a_rdiff, M_R(n), snr);
117
118 %%%ssvsp
119 [best_vsp,I_best_vsp]=min(metric);
120 H_best_v=H_mem(1:M_k(I_best_vsp),...
121 (I_best_vsp-1)*M_R(n)+1:I_best_vsp*M_R(n));
122 [worst_vsp,I_worst_vsp]=max(metric);
123 H_worst_v=H_mem(1:M_k(I_worst_vsp),...
124 (I_worst_vsp-1)*M_R(n)+1:I_worst_vsp*M_R(n));
125 %% ssv space computation for perfect CSI
126 [U_bv,B_bv,V_bv] = svd(H_best_v); % SVD of H
127 %%
128 % Select eigenvectors corresponding to SVs below threshold sigma
129 d_k=M_k(I_best_vsp)*ones(1,K); %DoF
130 d_ksum=sum(d_k);

```

```

131 if d_ksum==1
132 B1_bv = B_bv(d_ksum,d_ksum);
133 else
134     B1_bv=diag(B_bv)';
135 end
136
137 [rowbv,colbv] = find(B1_bv==0); % Find cols with SV < threshold
138 ssl_bv=length(B1_bv);
139 cobv=[ssl_bv d_ksum+1:M_R(n)];
140 V_tildabv = V_bv(:,cobv); % Pick corresponding cols of V
141 P_Rbv = V_tildabv*V_tildabv';
142 Rs_bv = P_Rbv*P_Rbv';
143 cb_best_vsp(x) = CRB(Rs_bv, a_t,a_tdiff, a_rdiff, M_R(n), snr);
144
145 %% Null-space computation for perfect CSI
146 [U_wv,B_wv,V_wv] = svd(H_worst_v); % SVD of H
147 %%
148 % Select eigenvectors corresponding to SVs below threshold sigma
149 d_k=M_k(I_worst_vsp)*ones(1,K); %DoF
150 d_ksum=sum(d_k);
151 if n==1
152 B1_wv = B_wv(n,n);
153 else
154     B1_wv=diag(B_wv)';
155 end
156 [rowwv,colwv] = find(B1_wv==0); % Find cols with SV < threshold
157 ssl_wv=length(B1_wv);
158 cowv=[ssl_wv d_ksum+1:M_R(n)];
159 V_tildawv = V_wv(:,cowv); % Pick corresponding cols of V
160 P_Rwv = V_tildawv*V_tildawv';
161 Rs_wv = P_Rwv*P_Rwv';
162 cb_worst_vsp(x) = CRB(Rs_wv, a_t,a_tdiff, a_rdiff, M_R(n), snr);
163 end
164
165 crb(n)=mean(cb);
166 crb_null_best(n) = mean(cb_best_nsp);
167 crb_null_worst(n) = mean(cb_worst_nsp);
168 crb_ssvsp_best(n) = mean(cb_best_vsp);
169 crb_ssvsp_worst(n) = mean(cb_worst_vsp);
170 end
171
172 %% This produces CRB
173 figure(1)
174 semilogy(M_R,crb,'-kh','LineWidth',2)
175 hold on
176 semilogy(M_R,abs(crb_null_best),'-rs','LineWidth',2)
177 hold on
178 semilogy(M_R,abs(crb_null_worst),'-go','LineWidth',2)
179 hold on
180 semilogy(M_R,abs(crb_ssvsp_best),'-bx','LineWidth',2)
181 hold on
182 semilogy(M_R,abs(crb_ssvsp_worst),'-cd','LineWidth',2)
183 xlabel('Number of radar antennas (M_R)','fontsize',14)
184 ylabel('RMSE (degree)','fontsize',14)
185 legend('Location','Best','orthogonal signals','NSP best',...
186 'NSP worst','SSVSP best','SSVSP worst')
187 hold off

```

### 3.7.2 Cooperation Mode

The following script produces Fig. 3.10 that analyzes performance of radar precoder in cooperation mode.

#### 3.7.2.1 Figure 3.10

```

1  % BER versus SNR in cooperation mode with channel estimation
2  K=4; % No of transmitter-receiver pair in the comm system
3  ind=1;
4  if ind==1
5      M_k=6*ones(1,K);
6      N_k=6*ones(1,K);
7      M_R=30; % No of transmit antenna elements on RADAR
8      N_R=30; % No of receive antenna elements on RADAR
9  else
10     M_k=8*ones(1,K);
11     N_k=8*ones(1,K);
12     M_R=100; % No of transmit antenna elements on RADAR
13     N_R=100; % No of receive antenna elements on RADAR
14 end
15 L_b=256;% block length
16 L_t=128;
17 SNR=20; % SNR of received radar signal from target in dB
18 snr=10^(SNR/10);
19 rho_dB=(0:15);
20 rho_ratio=10.^(rho_dB/10);
21 noise_v=sqrt(1./rho_ratio);
22 noit=10^4;
23
24 for n=1:length(rho_ratio)
25     errCount_BERZF=0;
26     errCount_BERMM=0;
27     for x=1:noit
28         d_k=M_k; %DoF
29         d_ksum=sum(M_k);
30
31         %Generate random data block using QPSK modulation.
32         tmp=round(rand(2,d_ksum));
33         tmp=tmp*2-1;
34         input=(tmp(1,:)+1i*tmp(2,:))/sqrt(2);
35         u=input.';
36
37         % composite channel generation
38         H_bar=zeros(M_R,d_ksum);
39         for a=1:K
40             H_kR=randn(d_k(a),M_R)+1i*randn(d_k(a),M_R);
41             if (a==1)
42                 dim=1:d_k(a);
43             else
44                 dim=(sum(d_k(1:a-1))+1):(sum(d_k(1:a)));
45             end
46             H_bar(:,dim)=H_kR';

```

```

47 end
48 H_r=H_bar';
49
50 % training symbol generation
51 S=(1/sqrt(2))*(hadamard(L_t)+1i*hadamard(L_t));
52 S=S(1:d_ksum,:);
53
54 % AWGN generation
55 W=sqrt(noise_v(n))*(randn(M_R,L_t)+1i*randn(M_R,L_t));
56
57 % signal received at radar for the whole training period
58 Y=zeros(M_R,L_t);
59 for b=1:L_t
60     Y(:,b)=sqrt(rho_ratio(n)/d_ksum)*H_bar*S(:,b)+W(:,b);
61 end
62
63 % ML estimation of composite channel
64 H_barest=sqrt(d_ksum/rho_ratio(n))*Y*S'*inv(S*S');
65 H_real=H_barest';
66
67 %% ZF precoder for perfect CSI
68 P_R1 = H_real'*inv(H_real*H_real');
69
70 %%MMSE precoder for perfect CSI
71 P_R1s = H_real'*inv(H_real*H_real'+d_ksum*(1/rho_ratio(n))...
72 *eye(d_ksum));
73
74 % AWGN generation
75 W=sqrt(noise_v(n)/2)*(randn(d_ksum,1)+1i*randn(d_ksum,1));
76 Y_ZF=H_real*P_R1*u+W;
77 EstSymbols_ZF=sign(real(Y_ZF))+1i*sign(imag(Y_ZF));
78 EstSymbols_ZF=EstSymbols_ZF/sqrt(2);
79 I_br_ZF=find((real(u)-real(EstSymbols_ZF))==0);
80 errCount_BERZF=errCount_BERZF+(d_ksum-length(I_br_ZF));
81 I_bi_ZF=find((imag(u)-imag(EstSymbols_ZF))==0);
82 errCount_BERZF=errCount_BERZF+(d_ksum-length(I_bi_ZF));
83
84 Y_MMSE=H_real*P_R1s*u+W;
85 EstSymbols_MM=sign(real(Y_MMSE))+1i*sign(imag(Y_MMSE));
86 EstSymbols_MM=EstSymbols_MM/sqrt(2);
87 I_br=find((real(u)-real(EstSymbols_MM))==0);
88 errCount_BERMM=errCount_BERMM+(d_ksum-length(I_br));
89 I_bi=find((imag(u)-imag(EstSymbols_MM))==0);
90 errCount_BERMM=errCount_BERMM+(d_ksum-length(I_bi));
91 end
92
93 % Calculate the bit error rate (BER)
94 BER_ZF(1,n)=errCount_BERZF/(2*d_ksum*noit);
95 BER_MM(1,n)=errCount_BERMM/(2*d_ksum*noit);
96 end

```

### 3.7.3 Several Functions for both Modes

In this section, we provide MATLAB code for several functions used for interference mitigation as well as cooperation mode.

```

2 function [ crb ] = CRB( Rs, a_t,a_tdiff, a_rdiff, N_R, snr)
3 %This function calculates Cramer Rao Bound
4 T1=N_R*a_tdiff'*Rs.*a_tdiff;
5 T2=a_t'*Rs.*a_t*((norm(a_rdiff,2))^2);
6 T3=N_R*abs(a_t'*Rs.*a_tdiff).^2;
7 T4=a_t'*Rs.*a_t;
8 T=T1+T2-(T3./T4);
9 crb=1/(2*snr*T);
10 end
11
12 function a = steering_ULA(theta,M,fc,d)
13 % generates a steering vector for Uniform Linear Array (ULA)
14 % theta: angle of arrival with the vertical (in degrees)
15 % M: number of antennas
16 % fc: carrier frequency (in Hz)
17 % d: inter-element spacing (in meter)
18
19 c=3e8;           % speed of light in meter/sec
20 Lambda=c/fc;    % wavelength in meter
21 m=0:M-1;
22 a=exp(-1i*2*pi*m*d*sind(theta-45)/Lambda);
23 a=a.';
24 end
25
26 function a = diff_steering_ULA(theta,M,fc,d)
27 b=exp(-1i*2*pi*m*d*sind(theta-45)/Lambda);
28 a=(-1i*2*pi*d*m.*b*cosd(theta-45))/Lambda;
29 a=a.';
30 end

```

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## Chapter 4

# Overlapped-MIMO Radar and MIMO Cellular System

Chowdhury Shahriar, Ahmed Abdelhadi and T. Charles Clancy

In this chapter, we explore a colocated overlapped multiple-input multiple-output (Overlapped-MIMO) radar waveform design, which is combined with an algorithm to share spectrum through null space projection (NSP) for the coexistence of radar and communications system. In this chapter, we extend the previous works [1–3], which consider a coexistence scenario where colocated MIMO radar operates in the same band as MIMO communications system, to a spectrum sharing topology of Overlapped-MIMO radar and MIMO communications system. The antenna array of colocated Overlapped-MIMO radar is separated into multiple subarrays for transmitting signals in this antenna orientation. The antenna elements are overlapped among the subarrays, where the elements of transmitting subarrays comprise coherent signals and inter-subarray signals are orthogonal to each other.

The proposed antenna design origins lesser interference to communication systems along with retaining high degree performance of MIMO radar (such as retaining improved sidelobe suppression in the beampattern and achieving higher SNR gain). On the other hand, the radar-centric projection algorithm to share spectrum avoids interference to communications system by projecting the radar signal onto the null space of the communications channel. Numerical analysis offered here demonstrates the outcome of the proposed orientation in terms of overall beampattern, sidelobe levels of the radar waveform, and SNR gain.

The concepts of MIMO radar are receiving attention nowadays as they can have better performance than legacy radar systems to identify targets with higher angular resolution [4]. In MIMO radar, multiple waveforms are transmitted via multiple

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transmit antenna elements and reflected signals from the targets are received by multiple receive antennas. In [5], authors have proposed a different kind of MIMO radar that they called ‘Phased-MIMO’ radar. In Phased-MIMO radar, waveforms are transmitted from a MIMO radar where antenna elements are partitioned into multiple subarrays, and the elements are allowed to overlap among subarrays. The profit of this construction over conventional MIMO radar is its higher coherent processing gain and overall suppressed sidelobes.

The notion of projecting signals onto the null space of an interference channel, in order to elude interference, is a well-studied theme in the cognitive radio research community [6, 7]. An interference channel’s null space is calculated at the transmitter either by exploiting channel reciprocity using its second order statistics [6] or by blindly estimating the null space, if no cooperation exists between resource sharing nodes [7]. However, for MIMO radar systems this idea of null space projection (NSP) was first proposed in [1], which was followed by an array of papers [2, 3, 8] where authors studied the NSP-based spectrum sharing approach for various radar-communications scenarios to avoid interference.

The remainder of this chapter is organized as follows. Section 4.1 builds the foundation of spectrum sharing architecture between MIMO radar and MIMO communications system by providing the channel model. Section 4.2 discusses the preliminaries of colocated MIMO radar. Section 4.3 introduces the proposed Overlapped-MIMO radar architecture. In Sect. 4.4, we discuss the performance of the proposed radar architecture in terms of beam pattern and SNR gain. Section 4.5 derives the optimum subarray size for colocated Overlapped-MIMO radar. Section 4.6 presents the radar-centric spectrum sharing algorithm that is called the null space projection (NSP) algorithm. In Sect. 4.7, a general discussion on the assumptions and limiting factors of NSP algorithm is presented. Section 4.8 discusses the simulation setup and provides quantitative results along with discussion. Section 4.9 concludes the chapter.

## 4.1 System Model for Coexistence

In this section, we build the foundation for coexistence of radar and communications system. We start that by describing the model of both radar and communications system. We define the channel model for spectrum sharing too. On top of that, we also state the assumptions made on the interference channel.

### 4.1.1 Radar Model

The radars assumed to be employed here are the variants of colocated MIMO radar that comprises  $M_T$  transmit and  $M_R$  receive antenna elements. The antennas of the colocated MIMO radars are uniform linear array (ULA) and elements are spaced at

least a half wavelength apart (or at the order of half wavelength). The use of colocated radar is beneficial as this provides superior spatial resolution and target parameter identification than other antenna architecture such as widely spaced radar [9].

### 4.1.2 Communications System Model

We assume that the communications system is either wireless broadband or cellular system with MIMO antennas. The MIMO communications system have  $N_T$  transmit and  $N_R$  receive antennas. The communication nodes can be either base stations or user equipments.

### 4.1.3 Coexistence Channel Model

Let us now look into the channel model for this spectrum sharing scenario. If we look from the communication systems perspective, then the received signal at the receiver terminal of the communications system can be written as

$$\mathbf{y}_C(t) = \mathbf{H}_I^{N_R \times M_T} \mathbf{x}_{\text{Radar}}(t) + \mathbf{H}^{N_R \times N_T} \mathbf{x}_C(t) + \mathbf{n}(t) \quad (4.1)$$

where  $\mathbf{x}_{\text{Radar}}(t)$  is transmitted radar signal,  $\mathbf{x}_C(t)$  is transmitted communications signal,  $\mathbf{H}_I$  is  $N_R \times M_T$  interference channel between radar and communications system,  $\mathbf{H}$  is  $N_R \times N_T$  channel between transmitter and receiver of the communications system,  $\mathbf{n}(t)$  is AWGN.

The interference channel  $\mathbf{H}_I$  can be denoted as

$$\mathbf{H}_I = \begin{bmatrix} h^{(1,1)} & \dots & h^{(1,M_T)} \\ \vdots & \ddots & \vdots \\ h^{(N_R,1)} & \dots & h^{(N_R,M_T)} \end{bmatrix} \quad (N_R \times M_T) \quad (4.2)$$

where  $h_i^{(n,m)}$  is the coefficient of the channel between  $m^{\text{th}}$  antenna element of the MIMO radar to the  $n^{\text{th}}$  antenna element of the MIMO communications system. The elements of the  $\mathbf{H}_I$  are assumed to be independent, identically distributed (i.i.d.) and circularly symmetric complex Gaussian random variables with zero-mean and unit-variance (also known as Rayleigh fading).

### 4.1.4 Key Assumptions

We assume that both the radar and communications system are working in an approachable and cooperative RF environment, cooperating with each other and sharing

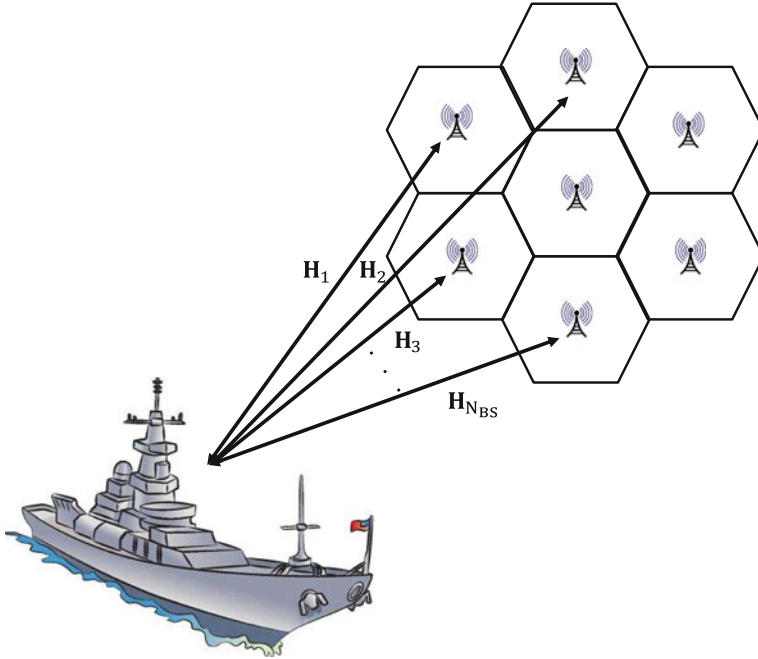


**Fig. 4.1** A possible spectrum sharing scenario between a radar mounted on a ship and an on shore communications system

various information. Information is shared under an arrangement that each system will seek to avoid causing interference to the other. In this chapter, we investigate a radar-centric design approach. In our radar-centric design, we assume that the interference channel state information (CSI) of the communications system is available at the radar terminal and the goal of the radar is then to develop its own radar waveform that will avoid interference to the communications system. A typical coexistence scenario is shown in Fig. 4.1.

On top of that, the following assumptions are used throughout this chapter to make the analysis tractable and intuitive:

- A point target/source is considered, which is defined for targets/sources having a scatterer with infinitesimal spatial extent.
- Both  $\theta$  and  $\alpha$  are deterministic unknown parameters representing the target's direction of arrival and the complex amplitude of the target, respectively.
- The path loss  $\alpha$  is assumed to be identical for all transmit and receive elements, due to the far-field assumption.
- The angle  $\theta$  is the azimuth angle of the target.

## 4.2 Collocated MIMO Radar

In this section, we develop the preliminary mathematical foundations of collocated MIMO radar. This derivation will enable us to further understand the proposed overlapped radar architecture in the later sections. The MIMO radar considered in this work is assumed to be collocated. The term ‘collocated’ refers a radar system where the transmit and the receive antennas are geographically located closely in space (often they are the same array) [4]. In this case, let us assume the number of antenna elements in the transmit arrays and the receive arrays are  $M_T$  and  $M_R$ , respectively. Let  $\theta$  be the location parameter of generic target.

Let  $\phi(t)$  be the waveform emitted from collocated MIMO radar, which can be defined as

$$\phi(t) = [\phi_1(t) \ \phi_2(t) \ \cdots \ \phi_{M_T}(t)]^T \quad (4.3)$$

where  $t$  is the time dimension of a radar pulse, and  $(\cdot)^T$  is the transpose of a vector/matrix. The signal  $\phi_m(t)$ , the  $m$ th element of the vector  $\phi(t)$ , is the waveform emitted by the  $m$ th transmit antenna of the MIMO radar. In this MIMO radar case, signal transmitted by each elements are assumed to be orthogonal to each other. As a result, the overall waveform satisfies the orthogonality principle, which infers

$$\mathbf{R}_x = \int_{T_0} \phi(t) \phi^H(t) dt = \mathbf{I}_{M_T} \quad (4.4)$$

where  $T_0$  stands for the radar pulse width,  $(\cdot)^H$  denotes the Hermitian transpose and  $\mathbf{I}_{M_T}$  is the  $M_T \times M_T$  identity matrix. The orthogonal signal transmission has many advantages. For example, beamforming is possible at transmitter in addition to receiver, improved angular resolution can be achieved, extended array aperture is available in the form of virtual arrays, sidelobes are lower than usual, and the number of resolvable targets are increased.

In the transmitter, the waveform is steered toward the direction of a particular target (or source) during transmission. If the target (or source) direction is assumed to be  $\theta$  and  $M_T \times 1$  transmit steering vector is assumed to be  $\mathbf{a}(\theta)$ , then for a uniform linear array (ULA), the transmit steering vector  $\mathbf{a}(\theta)$  can be expressed as

$$\begin{aligned} \mathbf{a}(\theta) &= [a_1(\theta) \ a_2(\theta) \ \cdots \ a_{M_T}(\theta)]^T \\ &= [1 \ e^{-j2\pi d_T \sin \theta} \ \cdots \ e^{-j2\pi d_T (M_T-1) \sin \theta}]^T \end{aligned} \quad (4.5)$$

where the first element of vector  $\mathbf{a}(\theta)$  is considered as the reference element, which is set as  $a_1(\theta) = 1$ , the  $m$ th element is set as  $a_m(\theta) = e^{-j2\pi d_T (M_T-1) \sin \theta}$ , and the inter-element space for array is denoted as  $d_T$ , which is measured in terms of wavelength.

Hence, the initial waveform is multiplied (or steered) with the steering vector and final output of the radar transmitter can be expressed in compact vector form as

$$\begin{aligned}
\mathbf{x}_{\text{Radar}}(t) &= \mathbf{a}(\theta) \odot \phi(t) \\
&= [a_1(\theta)\phi_1(t) \ a_2(\theta)\phi_2(t) \ \cdots \ a_{M_T}(\theta)\phi_{M_T}(t)] \\
&= [x_1(t) \ x_2(t) \ \cdots \ x_{M_T}(t)]
\end{aligned} \tag{4.6}$$

where  $\odot$  denotes the Hadamard (elementwise) product.

The snapshot vector of size  $M_R \times 1$  received by the collocated MIMO radar receive antenna array can be expressed as

$$\mathbf{y}_{\text{Radar}}(t) = \mathbf{y}_s(t) + \mathbf{y}_i(t) + \mathbf{n}(t) \tag{4.7}$$

where  $\mathbf{y}_s(t)$  is the signal from the target/source,  $\mathbf{y}_i(t)$  is the jamming/interference signal, and  $\mathbf{n}(t)$  is the AWGN.

If single point target/source is assumed, then the received signal at the radar becomes

$$\mathbf{y}_s(t) = \beta_s (\mathbf{a}^T(\theta_s) \phi(t)) \mathbf{b}(\theta_s) \tag{4.8}$$

where  $\theta_s$  is the direction of the target/source,  $\beta_s$  is the complex-valued reflection coefficient of the focal point  $\theta_s$  (that takes account of channel effect and propagation loss), and  $\mathbf{b}(\theta)$  is the receive steering vector of size  $M_R \times 1$  for the direction  $\theta$ , which can be expressed as

$$\begin{aligned}
\mathbf{b}(\theta) &= [b_1(\theta) \ b_2(\theta) \ \cdots \ b_{M_R}(\theta)]^T \\
&= [1 \ e^{-j2\pi d_T \sin \theta} \ \cdots \ e^{-j2\pi d_T (M_R-1) \sin \theta}]^T
\end{aligned} \tag{4.9}$$

The signal returned from  $m$ th transmitted waveform can be recovered by implementing a matched-filter at the receiver of the radar. The matched-filter would contain each of the waveforms  $\{\phi_m(t)\}_{m=1}^{M_T}$  and will be matched with received signal as following

$$\mathbf{y}_m(t) = \int_{T_0} \mathbf{y}_{\text{Radar}}(t) \phi_m^*(t) dt \quad m = 1, \dots, M_T. \tag{4.10}$$

One of the key differences of collocated MIMO radar is having more degrees of freedom (DoF) by the emergence of virtual array. Notice that, the transmitting signals from the single transmitter of the radar are different. As a result, the echo signals can be reassigned to the source. This in return gives an enlarged virtual receive aperture. Then, the size of the virtual data vector will be  $M_T M_R \times 1$  and it can be expressed as

$$\begin{aligned}
\mathbf{y}_v &= [\mathbf{y}_1^T \mathbf{y}_2^T \cdots \mathbf{y}_{M_T}^T]^T \\
&= \beta_s \mathbf{a}(\theta_s) \otimes \mathbf{b}(\theta_s) + \mathbf{y}_{i+n}
\end{aligned} \tag{4.11}$$

where  $\otimes$  denotes the Kronker product operator and  $\mathbf{y}_{\mathbf{i}+\mathbf{n}}$  denotes the combined component of interference and noise. Hence, the target/source signal component can be written as

$$\mathbf{y}_s = \beta_s \mathbf{v}(\theta_s) \quad (4.12)$$

where  $\mathbf{v} = \mathbf{a}(\theta_s) \otimes \mathbf{b}(\theta_s)$  is the virtual steering vector of size  $M_T M_R \times 1$ , which is associated with a virtual array of  $M_T M_R$  elements.

For ULA, the  $(m_t M_R + m_r)$ th entry of virtual array steering vector  $\mathbf{v}(\theta)$  is given by

$$\mathbf{v}_{[m_t M_R + m_r]}(\theta) = e^{-j2\pi(m_t d_T \sin \theta + m_r d_R \sin \theta)} \quad (4.13)$$

where  $m_t = 0, \dots, M_T - 1$  and  $m_r = 0, \dots, M_R - 1$ . For  $d_T = M_R d_R$ , the virtual array steering vector simplifies to [10]

$$\mathbf{v}_{[\varsigma]}(\theta) = e^{-j2\pi\varsigma d_R \sin \theta} \quad (4.14)$$

where  $\varsigma = m_t M_R + m_r = 0, 1, \dots, M_T M_R - 1$ , which infers that an  $M_T M_R$  effective aperture array can be achieved by employing  $M_T + M_R$  antennas [5]. Here, the resulting virtual array is a ULA of  $M_T M_R$  elements spaced and  $d_R$  wavelength apart. For collocated MIMO radar, aperture size increases due to virtual array, which is resulted from the use of orthogonal signals in the antenna elements. This size extension is referred as *waveform diversity*.

### 4.3 Overlapped-MIMO Radar

In this section, we present a novel formulation of antenna arrays in the MIMO radar that we named ‘Overlapped-MIMO’ radar. In this formulation, the antenna elements of the array are partitioned into multiple overlapped subarrays. Among many, one of the key advantage of this formulation is that it allows to beamform in both transmit and receive arrays. The key idea behind this formulation is to partition the transmit arrays into  $K$  subarrays where  $1 \leq K \leq M_T$ , which are allowed to overlap [5]. The Overlapped-MIMO radar formulation is shown in Fig. 4.2.

The complex envelope of the signals at the output of the  $k$ th subarray can be expressed as

$$s_k(t) = \sqrt{\frac{M_T}{K}} \phi_k(t) \tilde{\mathbf{w}}_k \quad k = 1, \dots, K \quad (4.15)$$

where  $\tilde{\mathbf{w}}_k$  is a  $M_T \times 1$  unit-norm complex vector with  $M_k$  beamforming weights corresponding to the active antenna elements in  $k$ th subarray and  $M_T - M_k$  zero weights corresponding to the inactive antennas.

As transmitted signal, the frequency spaced signals can be adopted [11], which is orthogonal if the frequency increment  $\Delta f = f_{k+1} - f_k$  between the waveform  $\phi_{k+1}$  to  $\phi_k$  satisfies  $\Delta f \gg 1/T_0$ . The orthogonal waveform  $\phi(t)_k$  can be modeled as

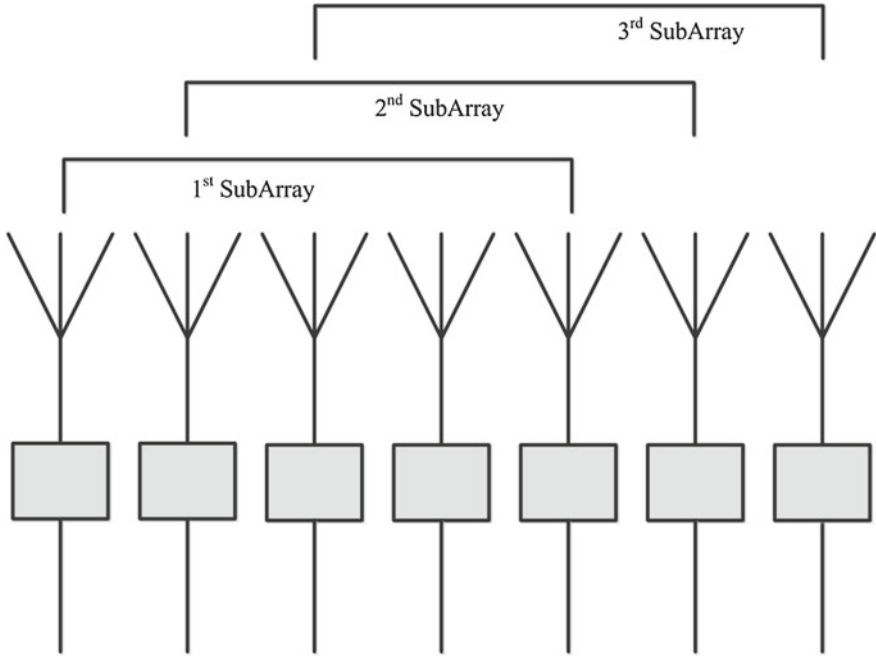


**Fig. 4.2** A block diagram of the overlapped-MIMO radar formulation

$$\phi_k(t) = Q(t)e^{j2\pi k(\Delta f)t} \quad (4.16)$$

where  $Q(t)$  is the pulse shape of duration  $T_0$ , where  $0 < t < T_0$ , and  $k = 1, \dots, K$  [12].

The energy of  $s_k(t)$  within one radar pulse can be written as

$$E_k = \int_{T_0} s_k^H(t)s_k(t)dt = \frac{M_T}{K} \quad (4.17)$$

which infers that the total transmitted power is equal to  $M_T$ .

The reflected signal from the target/source at the direction  $\theta$  in the far-field can be expressed as

$$\begin{aligned} r(t, \theta) &\triangleq \sqrt{\frac{M_T}{K}}\beta(\theta) \sum_{k=1}^K \tilde{\mathbf{w}}_k^H \tilde{\mathbf{a}}_k(\theta) \phi_k(t) \\ &= \sqrt{\frac{M_T}{K}}\beta(\theta) \sum_{k=1}^K \mathbf{w}_k^H \mathbf{a}_k(\theta) e^{-j\tau_k(\theta)} \phi_k(t) \end{aligned} \quad (4.18)$$

where  $\beta(\theta)$  is the reflection coefficient,  $\mathbf{w}_k$  and  $\mathbf{a}_k(\theta)$  are the  $M_k \times 1$  beamforming vectors and steering vector, respectively. The  $\tilde{\mathbf{a}}_k$  is a  $M_T \times 1$  vector with  $M_k$  steering vector corresponding to the active antenna elements in  $k$ th subarray and  $M_T - M_k$  zero corresponding to the inactive antennas. Finally,  $\tau_k(\theta)$  is the time of propagation required for the wave to travel from the first element to the next element.

The Eq. (4.18) can be rewritten as

$$r(t, \theta) = \sqrt{\frac{M_T}{K}} \beta(\theta) (\mathbf{c}(\theta) \odot \mathbf{d}(\theta))^T \phi_K(t) \quad (4.19)$$

where the waveform vector is

$$\phi_K(t) \triangleq [\phi_1(t), \dots, \phi_K(t)],$$

with dimension  $K \times 1$ , the transmit coherent processing vector is  $\mathbf{c}(\theta) \triangleq [\mathbf{w}_1^H \mathbf{a}_1(\theta), \dots, \mathbf{w}_K^H \mathbf{a}_K(\theta)]$  with dimension  $K \times 1$ , and the waveform diversity vector is  $\mathbf{d}(\theta) \triangleq [e^{-j\tau_1(\theta)}, \dots, e^{-j\tau_K(\theta)}]$  with dimension  $K \times 1$ .

The received complex vector of the array observation can be written as

$$\mathbf{y}_{\text{Radar}}(t) = r(t, \theta_s) \mathbf{b}(\theta_s) + \sum_i^D r(t, \theta_i) \mathbf{b}(\theta_i) + \mathbf{n}(t), \quad (4.20)$$

where  $D$  is the number of interfering signals (or scatterers),  $\mathbf{b}(\theta)$  is the receive steering vector of size  $M_R \times 1$  associated with direction  $\theta$ , and  $\mathbf{n}(t)$  is AWGN.

Following Eqs. (4.10) and (4.11), by match-filtering  $\mathbf{y}_{\text{Radar}}(t)$  to each of the waveforms  $\{\phi_k\}_{k=1}^K$  we can obtain  $K M_R \times 1$  virtual data vectors as

$$\begin{aligned} \mathbf{y}_v &= [\mathbf{y}_{\text{Radar},1}^T(t), \dots, \mathbf{y}_{\text{Radar},K}^T(t)]^T \\ &= \sqrt{\frac{M_T}{K}} \beta_s \mathbf{u}(\theta_s) + \sum_i^D \sqrt{\frac{M_T}{K}} \beta_i \mathbf{u}(\theta_i) + \mathbf{n}, \end{aligned} \quad (4.21)$$

where  $\mathbf{u}(\theta) \triangleq [(\mathbf{c}(\theta) \odot \mathbf{d}(\theta)) \otimes \mathbf{b}(\theta)]$  is the  $K M_R \times 1$  virtual steering vector,  $\beta_s = \beta(\theta_s)$  and  $\beta_i = \beta(\theta_i)$  are the reflection coefficients of the target/source and interference, respectively. This overlapped subarray formulation collapses to a phased-array when  $K = 1$  is chosen. In this case, only one waveform is emitted. This results lower angular resolution but higher coherent processing gain, like phased-array radar. On the other hand, when  $K = M_T$  is chosen, the formulation becomes a conventional MIMO without array partition. This completely diminishes coherent processing gain, but results higher angular resolution.

#### 4.4 Performance Metrics for Overlapped-MIMO Radar

In this section, we develop performance metrics of the proposed null space projected Overlapped-MIMO radar waveform. We evaluate the performance of the proposed architecture based on beampattern and SNR gain calculation.

We start with calculating the corresponding beamformer weight vector. In the case of nonadaptive beamforming, the corresponding beamformer weight vectors are given for the  $k$ th transmitting subarray as

$$\mathbf{w}_k = \frac{\mathbf{a}_k(\theta_s)}{\|\mathbf{a}_k(\theta_s)\|} = \frac{\mathbf{a}_k(\theta_s)}{\sqrt{M_T - K + 1}} \quad (4.22)$$

where  $k = 1, 2, \dots, K$ . The beamforming weight vector of size  $KN \times 1$  for the receiving subarrays can be written as

$$\mathbf{w}_d \triangleq \mathbf{u}(\theta_s) = [\mathbf{c}(\theta_s) \odot \mathbf{d}(\theta_s)] \otimes \mathbf{b}(\theta_s). \quad (4.23)$$

##### 4.4.1 Beampattern Improvement

Let  $G(\theta)$  be the normalized overall beampattern for Overlapped-MIMO

$$G(\theta) = \frac{|\mathbf{w}_d^H \mathbf{u}(\theta)|^2}{|\mathbf{w}_d^H \mathbf{u}(\theta_s)|^2} = \frac{|\mathbf{u}^H(\theta_s) \mathbf{u}(\theta)|^2}{\|\mathbf{u}(\theta_s)\|^4} \quad (4.24)$$

For the special case of a ULA, we have  $\mathbf{a}_1^H(\theta) \mathbf{a}_1(\theta_s) = \dots = \mathbf{a}_K^H(\theta) \mathbf{a}_K(\theta_s)$ . Using Eq.(4.24), the beampattern of the Overlapped-MIMO radar for a ULA with overlapped partitioning of  $K$  transmit subarrays can be expressed as

$$G_O(\theta) = \frac{|\mathbf{a}_K^H(\theta_s) \mathbf{a}_K(\theta) [(\mathbf{d}(\theta_s) \otimes \mathbf{b}(\theta_s))^H (\mathbf{d}(\theta) \otimes \mathbf{b}(\theta))]|^2}{\|\mathbf{a}_K^H(\theta_s)\|^4 \|\mathbf{d}(\theta_s) \otimes \mathbf{b}(\theta_s)\|^4}, \quad (4.25)$$

As we know that  $\|\mathbf{a}_K(\theta_s)\|^2 = M_T - K + 1$ ,  $\|\mathbf{d}(\theta_s)\|^2 = K$ , and  $\|\mathbf{b}(\theta_s)\|^2 = M_R$ , the overall beampattern can be expressed as

$$\begin{aligned} G_O(\theta) &= \frac{|\mathbf{a}_K^H(\theta_s) \mathbf{a}_K(\theta)|^2}{(M_T - K + 1)^2} \cdot \frac{|\mathbf{d}^H(\theta_s) \mathbf{d}(\theta)|^2}{K^2} \cdot \frac{|\mathbf{b}^H(\theta_s) \mathbf{b}(\theta)|^2}{M_R^2}, \\ &= T_O(\theta) \cdot D_O(\theta) \cdot R(\theta) \end{aligned} \quad (4.26)$$

where the waveform diversity beampattern is  $D_O(\theta) \triangleq \frac{|\mathbf{d}^H(\theta_s)\mathbf{d}(\theta)|^2}{K^2}$ , the transmit beampattern is  $T_O(\theta) \triangleq \frac{|\mathbf{a}_K^H(\theta_s)\mathbf{a}_K(\theta)|^2}{(M_T - K + 1)^2}$  and the receive beampattern is  $R(\theta) \triangleq \frac{|\mathbf{b}^H(\theta_s)\mathbf{b}(\theta)|^2}{M_R^2}$ . So, we can see that the overall beampattern of the Overlapped-MIMO radar can be expressed in terms of three distinct and independent beampattern.

For MIMO radar, the subarray number is  $K = M_T$  and the transmitter beampattern  $T_M(\theta) = 1$ . Hence, the overall beampattern for MIMO radar can be expressed as

$$G_{MIMO}(\theta) = D_M(\theta) \cdot R(\theta) \quad (4.27)$$

where the waveform diversity beampattern is  $D_M(\theta) = \frac{|\mathbf{a}^H(\theta_s)\mathbf{a}(\theta)|^2}{M_T^2}$ . Notice that the overall beampattern of the MIMO radar has only the waveform diversity and receive beampattern.

For phased-array radar, the subarray number is  $K = 1$  and the waveform diversity beampattern  $D_P(\theta) = 1$ . Hence, the overall beampattern for phased-array radar can be expressed as

$$G_{PH}(\theta) = T_P(\theta) \cdot R(\theta) \quad (4.28)$$

where the transmit beampattern is  $T_P(\theta) = \frac{|\mathbf{a}^H(\theta_s)\mathbf{a}(\theta)|^2}{M_T^2}$ . Notice that the overall beampattern of the phased-array radar has only transmit and receive beampattern.

#### 4.4.2 SNR Gain Improvement

According to [5], the output SNR of the Overlapped-MIMO radar with nonadaptive transmit/receive beamforming can be expressed as

$$SNR_{OMIMO} = M_R M_T (M_T - K + 1) \frac{\sigma_s^2}{\sigma_n^2} \quad (4.29)$$

where  $\sigma_s^2$  is the variance of the target/source reflection coefficient, thus  $\sigma_s^2 = E\{|\beta|^2\}$  and  $\sigma_n^2$  is the noise variance.

For MIMO radar, the output SNR can be found by substituting  $K = M_T$  at (4.29)

$$SNR_{MIMO} = M_R M_T \frac{\sigma_s^2}{\sigma_n^2} \quad (4.30)$$

For phased-array radar, the output SNR can be found by substituting  $K = 1$  at (4.29)

$$SNR_{PH} = M_R M_T^2 \frac{\sigma_s^2}{\sigma_n^2} = M_T \cdot SNR_{MIMO} \quad (4.31)$$

Finally, from (4.29)–(4.31), we can express the output SNR of Overlapped-MIMO radar as

$$SNR_{OMIMO} = \eta \cdot SNR_{PH} = \eta \cdot M_T \cdot SNR_{MIMO} \quad (4.32)$$

where  $\frac{1}{M_T} \leq \eta \triangleq \frac{(M_T - K + 1)}{M_T} \leq 1$  is the ratio between the overlapped-MIMO radar SNR gain and that of the phased-array radar.

We can see that the SNR gain of the MIMO radar is equal to  $M_R M_T$  and the SNR gain of the phased-array radar is equal to  $M_R M_T^2$  and the SNR gain of the Overlapped-MIMO radar is equal to  $M_R M_T (M_T K + 1)$ . The SNR gain of the phased-array radar is  $M_T$  times greater than that of the MIMO radar. The SNR gain of the Overlapped-MIMO radar is  $(M_T K + 1)$  times greater than that of the MIMO radar and  $\frac{(M_T K + 1)}{M_T}$  times greater than that of the phased-array radar. Hence, we can see an overall SNR gain improvement for the Overlapped-MIMO architecture.

## 4.5 Optimum Subarray Size for Overlapped-MIMO Radar

In order to maximize the impact of the overlapping subarray architecture presented, we have to select a value for the number of subarrays  $K$  that maximizes the virtual array size,  $M_\epsilon$ . Hence,

$$K = \arg \max_K (M_\epsilon) \quad (4.33)$$

where  $M_\epsilon = (M_T - K + 1) K$ .

The number of subarrays  $K$  in the overlapped array can be optimized by

$$\begin{aligned} \frac{\partial}{\partial K} (M_\epsilon) &= 0 \\ \frac{\partial}{\partial K} ((M_T - K + 1) K) &= 0 \\ M_T - 2K + 1 &= 0 \\ K &= \left\lfloor \frac{M_T + 1}{2} \right\rfloor \end{aligned} \quad (4.34)$$

where  $\lfloor \cdot \rfloor$  stands for the floor operation as  $K$  should be an integer. Note that the radar has most significant impact when the number of virtual arrays,  $M_\epsilon$ , on transmitter side is maximized (see Eq.(4.34)).

## 4.6 Radar-Centric Spectrum Sharing Algorithm

In this section, we describe the details of a radar-centric projection algorithm, which projects the Overlapped-MIMO radar signal onto the null space of the communication interference channel via null space projection (NSP) technique proposed in [3]. We start with a generic description of the spectrum sharing algorithm and move on to present the mathematical details of the projection matrix.

### 4.6.1 Null Space Projection (NSP)

This section describes the null space projection (NSP) algorithm, which projects radar signal onto the null space of the interference channel  $\mathbf{H}_I$ . The NSP algorithm requires the radar to have the interference channel's CSI available in advance, which can be obtained in a number of ways and conveyed to the radar via mutual cooperation between the radar and the communication systems [1–3].

The proposed algorithm works as follows. At the beginning, the radar receives  $\mathbf{H}_I$ , which is the CSI between radar and communication node's interference channel. It then calculates the number of null spaces available to project the radar signal by performing singular value decomposition (SVD) on  $\mathbf{H}_I$ . The dimension of the null space is  $M_T - N_R$ . It then calculates the projection channel matrix  $\mathbf{P}$  and constructs a new radar waveform  $\hat{\mathbf{x}}_{\text{Radar}}$ . If  $\mathbf{H}_I$  is the channel matrix and  $\mathbf{P}$  is the projection matrix onto the null space of  $\mathbf{H}_I$ , then the overlapped-MIMO radar waveform projected onto the null space of  $\mathbf{H}_I$  to avoid interference from radar can be written as

$$\hat{\mathbf{x}}_{\text{Radar}}(t) = \mathbf{P}\mathbf{x}_{\text{Radar}}(t). \quad (4.35)$$

Notice that the spectrum sharing algorithm via null space projection (NSP) described above is shown in Algorithm 9.

### 4.6.2 Projection Matrix

In this section, we introduce the formation of the projection matrix  $\mathbf{P}$  and analyze the properties of this projection matrix. Let  $\mathbf{H}_I$  be the interference channel between the radar and communications node. We assume that  $\mathbf{H}_I \in \mathbf{F}^{N_R \times M_T}$  for  $\mathbf{F} = \mathbb{R}$  or  $\mathbf{F} = \mathbb{C}$ . We want a projection matrix  $\mathbf{P} \in \mathbf{F}^{M_T \times M_T}$  of a maximum rank such that it satisfies following properties:

- $\mathbf{H}_I\mathbf{P} = 0$
- $\mathbf{P}^2 = \mathbf{P}$

**Algorithm 9** Spectrum Sharing Algorithm via Null Space Projection (NSP)**loop**Get CSI of  $\mathbf{H}_I$  through feedback from ‘Communications Node’.Send  $\mathbf{H}_I$  to inner loop (i.e., NSP Algorithm) for projection matrix  $\mathbf{P}$  formation.**if**  $\mathbf{H}_I$  received from the outer loop, **then**Perform SVD on  $\mathbf{H}_I$  (i.e.,  $\mathbf{H}_I = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$ ).Construct  $\tilde{\mathbf{\Sigma}} = \text{diag}(\tilde{\sigma}_1, \tilde{\sigma}_2, \dots, \tilde{\sigma}_k)$ .Construct  $\tilde{\mathbf{\Sigma}}' = \text{diag}(\tilde{\sigma}'_1, \tilde{\sigma}'_2, \dots, \tilde{\sigma}'_{M_T})$ .Setup projection matrix  $\mathbf{P} = \mathbf{V}\tilde{\mathbf{\Sigma}}'\mathbf{V}^H$ .Send  $\mathbf{P}$  to the outer loop.**end if**Receive the projection matrix  $\mathbf{P}$  from inner loop.Perform null space projection, i.e.,  $\hat{\mathbf{x}}_{\text{Radar}}(t) = \mathbf{P}\mathbf{x}_{\text{Radar}}(t)$ .**end loop**

The projection matrix  $\mathbf{P}$ , which satisfies above properties and projects onto the null space of interference channel  $\mathbf{H}_I$ , can be found by taking the SVD of  $\mathbf{H}_I$ . The SVD of  $\mathbf{H}_I$  is

$$\mathbf{H}_I = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \quad (4.36)$$

where  $\mathbf{U}$  and  $\mathbf{V}$  are unitary or orthogonal, depending on  $\mathbf{F}$ , of order  $N_R$  and  $M_T$ , respectively, and  $\mathbf{\Sigma} \in \mathbb{R}^{N_R \times M_T}$  is an  $N_R \times M_T$  rectangular diagonal matrix with nonnegative real numbers on the diagonal. Let us define

$$\tilde{\mathbf{\Sigma}} = \text{diag}(\tilde{\sigma}_1, \tilde{\sigma}_2, \dots, \tilde{\sigma}_k) \quad (4.37)$$

where  $k = \min(N_R, M_T)$  and  $\tilde{\sigma}_1 \geq \tilde{\sigma}_2 \geq \dots \geq \tilde{\sigma}_p \geq \tilde{\sigma}_{p+1} = \tilde{\sigma}_{p+2} = \dots = \tilde{\sigma}_k = 0$  are the singular values of  $\mathbf{H}_I$ . Let us define

$$\tilde{\mathbf{\Sigma}}' = \text{diag}(\tilde{\sigma}'_1, \tilde{\sigma}'_2, \dots, \tilde{\sigma}'_{M_T}) \quad (4.38)$$

where  $\tilde{\mathbf{\Sigma}}' \in \mathbb{R}^{M_T \times M_T}$  and

$$\tilde{\sigma}'_i = \begin{cases} 0, & \text{if } i \leq p, \\ 1, & \text{if } i > p. \end{cases}$$

Note that  $\tilde{\mathbf{\Sigma}}\tilde{\mathbf{\Sigma}}' = 0$  and  $(\tilde{\mathbf{\Sigma}}')^2 = \tilde{\mathbf{\Sigma}}'$ . Now, one can define the projection matrix as

$$\mathbf{P} = \mathbf{V}\tilde{\mathbf{\Sigma}}'\mathbf{V}^H. \quad (4.39)$$

We can verify that this matrix  $\mathbf{P}$  is valid projection matrix by computing the properties mentioned above. The details of this proofs are given below.

**Property 4.1**  $\mathbf{P} \in \mathbb{R}^{M_T \times M_T}$  is an orthogonal projection matrix onto the null space of  $\mathbf{H}_I \in \mathbb{R}^{N_R \times M_T}$ , if and only if  $\mathbf{H}_I \mathbf{P} = \mathbf{H}_I \mathbf{P}^H = \mathbf{0}$ .

*Proof* Since  $\mathbf{P} = \mathbf{P}^H$  (see property 2), it can be written

$$\mathbf{H}_I \mathbf{P} = \mathbf{H}_I \mathbf{P}^H = \mathbf{U} \tilde{\Sigma} \mathbf{V}^H \times \mathbf{V} \tilde{\Sigma}' \mathbf{V}^H = \mathbf{0}. \quad (4.40)$$

The result mentioned above follow from the fact that  $\tilde{\Sigma} \tilde{\Sigma}' = \mathbf{0}$  by construction.

**Property 4.2**  $\mathbf{P} \in \mathbb{R}^{M_T \times M_T}$  is a projection matrix, if and only if  $\mathbf{P} = \mathbf{P}^H = \mathbf{P}^2$ .

*Proof* At first, let us prove the ‘only if’ part, where we will have to show  $\mathbf{P} = \mathbf{P}^H$ . By taking the Hermitian of Eq. (4.39), we will get

$$\mathbf{P}^H = \left( \mathbf{V} \tilde{\Sigma}' \mathbf{V}^H \right)^H = \mathbf{P}. \quad (4.41)$$

Then, by squaring the Eq. (4.39) we will get

$$\mathbf{P}^2 = \mathbf{V} \tilde{\Sigma} \mathbf{V}^H \times \mathbf{V} \tilde{\Sigma} \mathbf{V}^H = \mathbf{P} \quad (4.42)$$

where the Eq. (4.42) follows from  $\mathbf{V}^H \mathbf{V} = \mathbf{I}$  (both of them are orthonormal matrices) and  $\left( \tilde{\Sigma}' \right)^2 = \tilde{\Sigma}'$  (by construction). If we follow the Eqs. (4.41) and (4.42), we will find that  $\mathbf{P} = \mathbf{P}^H = \mathbf{P}^2$ .

Next, we will show that  $\mathbf{P}$  is a projector matrix by proving that if  $\mathbf{v} \in \text{range}(\mathbf{P})$ , then  $\mathbf{P}\mathbf{v} = \mathbf{v}$ , i.e., for some  $\mathbf{w}$ ,  $\mathbf{v} = \mathbf{P}\mathbf{w}$ , then

$$\mathbf{P}\mathbf{v} = \mathbf{P}(\mathbf{P}\mathbf{w}) = \mathbf{P}^2\mathbf{w} = \mathbf{P}\mathbf{w} = \mathbf{v}. \quad (4.43)$$

On top of that,  $\mathbf{P}\mathbf{v} - \mathbf{v} \in \text{null}(\mathbf{P})$ , i.e.,

$$\mathbf{P}(\mathbf{P}\mathbf{v} - \mathbf{v}) = \mathbf{P}^2\mathbf{v} - \mathbf{P}\mathbf{v} = \mathbf{P}\mathbf{v} - \mathbf{P}\mathbf{v} = \mathbf{0}. \quad (4.44)$$

This concludes the proof.

## 4.7 Assumptions and Limiting Factors of NSP

In this section, we discuss the assumptions and limiting factors of the NSP algorithm implementation. We consider two spectrum sharing scenario depending upon the number of antenna elements in the radar and communications system.

The key assumption of NSP algorithm implementation is ‘cooperation’. There must be some kind of cooperation between the radar and the communication node to effectively project radar signal onto the null space. They must be exchanging

the CSI of the inference channel though feedback/feedforward or any other kind of mechanism. It will work only when the channel is static or quasi-static, meaning the CSI will not be changed before the projection takes place. A number mechanisms to exchange CSI between radar and communications system is presented in [13].

Notice that we run into two possible scenarios: (1) the number of antenna elements in radar transmit array is less than equal to that of communications system,  $M_T \leq N_R$  and (2) the number of antenna elements in the radar transmit array is greater than that of communications system,  $M_T > N_R$ . For first scenario where we have  $M_T \leq N_R$ , we cannot use the NSP method. However, a possible solution to this problem is using Overlapped-MIMO as it increases the *effective* number of transmit arrays, thus making NSP possible. In this case, the *effective* transmit array aperture,  $M_e$  is equal to  $(M_T - K + 1)K$ , which is greater than  $N_R$ . Note that  $M_e$  is essentially the number of the virtual arrays in the transmitter of the radar. Hence, the Overlapped-MIMO radar results in a total virtual array of size  $((M_T - K + 1)K) M_R$ . On the other hand, if  $M_T > N_R$ , then we will have sufficient degrees of freedom (DoF) to make NSP possible for  $M_T - N_R$  dimensions. However, even in this case the performance can be increased using Overlapped-MIMO since it increases the effective number of transmit arrays.

## 4.8 Simulation and Results

In this section, we simulate an Overlapped-MIMO radar. We assume a ULA with  $M_T = 20$  antenna elements at the transmitter. At the receiver, we also assume  $M_R = 20$  antennas. In both cases the space between elements is  $d_T = 0.5$ , meaning adjacent antenna elements are half a wavelength apart. The signal passes through a Rayleigh distributed channel and is subject to AWGN. Each antenna element is omnidirectional. We assume the target of interest is at  $\theta_s = 15^\circ$  and For the sake of simplicity, we ignored the presence of any interfering signals here. Output SNRs are computed using 10, 000 independent simulations.

Figure 4.3 shows the overall beampattern for four different MIMO radar formulations: (1) Overlapped-MIMO radar with  $K = 1$  (single subarray or phased-array), (2) Overlapped-MIMO radar with  $K = 5$ , (3) Overlapped-MIMO radar with  $K = 10$ , and (4) MIMO radar with  $K = 20$  (MIMO). Here the Overlapped-MIMO radars have two different orientations of 5 and 10 overlapped subarrays and each subarray has 11 and 16 antenna elements, respectively. We can observe that the Overlapped-MIMO with  $K = 1$  (phased-array) and MIMO radars have exactly the same overall transmit/receive beampatterns. However, the Overlapped-MIMO radar (for  $K = 5$  and  $K = 10$ ) has significantly improved sidelobe suppression compared to the beampattern of the MIMO and the phased-array radar.

Figure 4.4 shows the overall beampattern for same radar formulations with NSP algorithm: (1) Overlapped-MIMO radar with  $K = 1$  plus NSP (single subarray or phased-array), (2) Overlapped-MIMO radar with  $K = 5$  plus NSP, (3) Overlapped-MIMO radar with  $K = 10$  plus NSP, and (4) MIMO radar with  $K = 20$  plus NSP

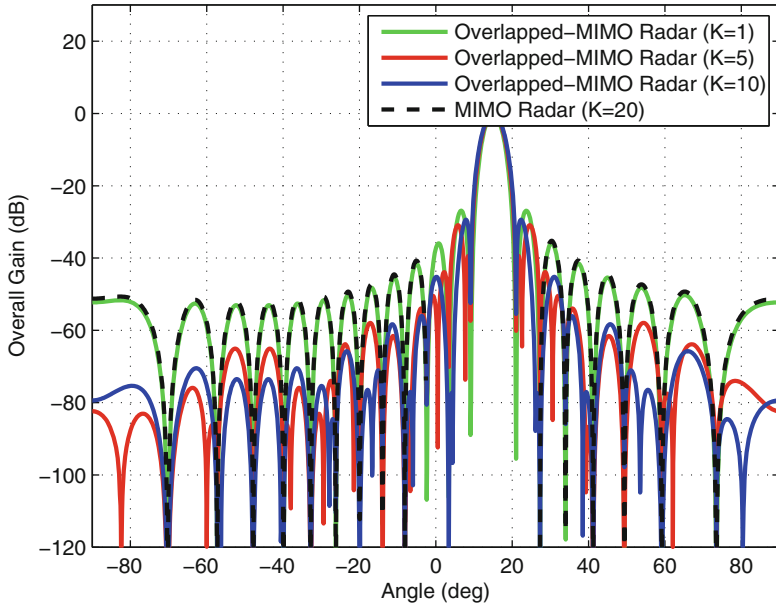


**Fig. 4.3** Overall beampattern using conventional transmit-receive beamformer where the total number of elements is  $M_T = 20$ , the number of overlapped subarrays is  $K = 5$  and  $K = 10$ , respectively; the number of elements in each subarray is  $(M_T - K + 1) = 16$  and  $(M_T - K + 1) = 11$ , respectively, and  $d_T = 0.5$  wavelength

(MIMO). We observe that the projection algorithm has reduced sidelobe suppression as expected. Note that it is still providing encouraging suppression in compared to pure MIMO radar. However, the primary benefits are at the communications side since this NSP algorithm minimizes interference from the radar to the communications system and thus, enables the two to coexist.

The final experiment considers the number of subarrays,  $K$ , in the transmitter of the Overlapped-MIMO radar that maximizes the benefit for the radar in terms of sidelobe suppression. Note that the radar has most significant impact when the number of virtual arrays,  $M_e$ , on transmitter side is maximized (see Eq. 4.34). Figure 4.5 shows the impact of varying the number of subarrays  $K$  from 1 to  $M_T$  on  $M_e$ . For  $M_T = 20$ ,  $K = 11$ , or  $K = 12$  results in the highest impact. This knowledge enables determining the structure of overlapping subarrays. The plot of  $K$  for  $M_T = 10$  and  $M_T = 15$  is shown in the same figure to provide a comparative view. This graph enables picking a value for  $K$  (the number of subarrays in the Overlapped-MIMO structure) that maximizes the virtual antenna array size, thus enhancing the amount of sidelobe suppression in radar beampattern, while retaining the dimension needed for NSP.

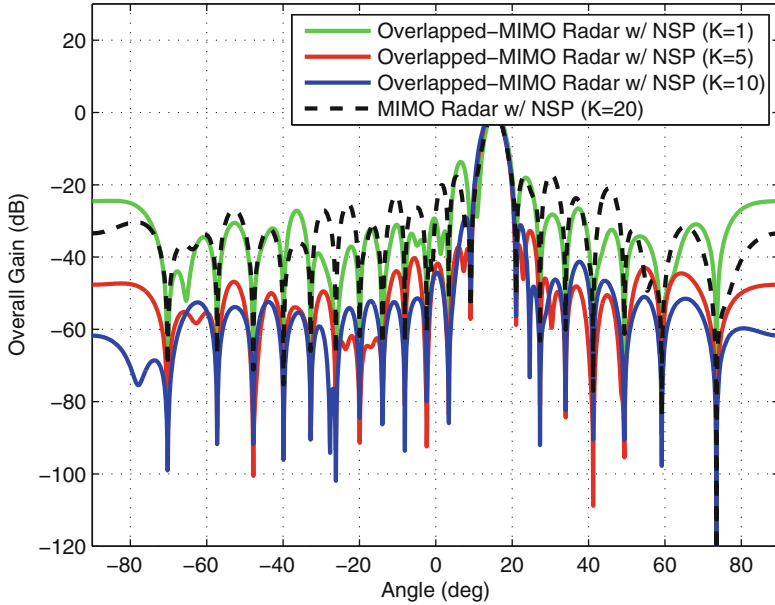


**Fig. 4.4** Overall beam pattern using conventional transmit-receive beamformer and NSP where the total number of elements is  $M_T = 20$ , the number of overlapped subarrays is  $K = 5$  and  $K = 10$ , respectively; the number of elements in each subarray is  $(M_T - K + 1) = 16$  and  $(M_T - K + 1) = 11$ , respectively, and  $d_T = 0.5$  wavelength

## 4.9 Conclusion

In this chapter, we presented a MIMO radar architecture that we named ‘Overlapped-MIMO’ radar and combined a spectrum sharing algorithm called ‘null space projection’ (NSP) for radar-communications coexistence. In the Overlapped-MIMO radar architecture, the transmit array of the radar is partitioned into a number of subarrays that are permitted to overlap. Each antenna subarray transmit waveforms that are orthogonal to other subarrays, but coherent among all the antenna elements of each individual subarrays. The benefit of this architecture is to have a larger *effective* transmit array with augmented *diversity gain*. On top of that, we achieved *coherent processing gain* by designing a weight vector for each subarray to form a beam toward certain direction in space [5]. This formulation also expands the overall sidelobe suppression compared to conventional MIMO radar, making it suitable for coexisting with communication systems.

Further, we hosted a radar-centric spectrum sharing algorithm that projects the radar signal onto the null space of the communications system’s interference channel, which reliefs to evade interference from radar. Note that such conventional null space projection is only possible when the physical/virtual number of transmit antennas

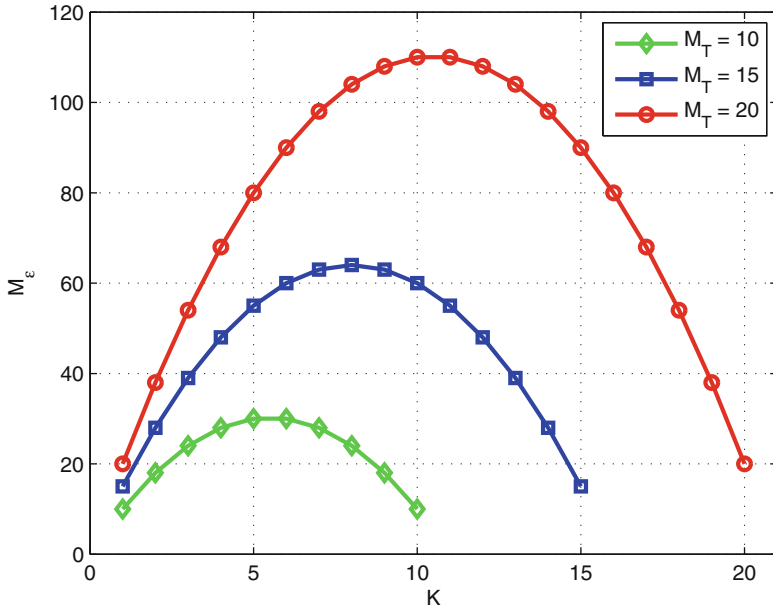


**Fig. 4.5** The number of subarrays,  $K$ , in a Overlapped-MIMO radar is varied from 1 to  $M_T$  and the resulting effective virtual transmitter array number,  $M_\epsilon$  is observed for three different transmit antenna sizes, i.e.,  $M_T = 10$ ,  $M_T = 15$  and  $M_T = 20$ . This graph enables picking a value for  $K$  (the number of subarrays in the overlapped-MIMO structure) that maximizes the virtual antenna array size, thus enhancing the amount of sidelobe suppression in radar beampattern, while retaining the dimension needed for NSP

of the radar is greater than the number of receive antennas of the communications system.

Analytical models for the waveform of the Overlapped-MIMO radar and the NSP algorithm are derived in this chapter. Simulations of the coexistence scenario are presented too. Through analytical derivation and as well, simulation results, we showed that the proposed overlapped-MIMO and NSP algorithms outperform conventional schemes and enable radar-communications system coexistence in the same band. We found that the Overlapped-MIMO architecture achieves more than 20 dB sidelobe suppression above conventional MIMO radar when there are 20 physical antenna elements in the radar system. We also observed that, in a similar setup, even though NSP degrades suppression, it still retains more than 10 dB additional sidelobe suppression compared to conventional MIMO radar while reducing interference to the communication systems.

## 4.10 MATLAB Code

In this section, we provide MATLAB code to generate Overlapped-MIMO radar waveforms subject to spectrum sharing constraints.

### 4.10.1 Overlapped-MIMO Main Module

This MATLAB script is the main module for Overlapped-MIMO.

```

1  % Total number of transmitting antennas
2  M = 20;
3  % Total number of receiving antennas
4  M_r = 20;
5  % transmitter spacings in wavelength
6  d_t = 0.5;
7  % Direction of target
8  theta_tar = 15*pi/180;
9  % Uplink steering vector
10 a_tar = exp(-j*d_t*2*pi*(0:M-1)*sin(theta_tar));
11 % Downlink steering vector
12 b_tar = exp(-j*pi*(0:M_r-1)*sin(theta_tar));
13 % K is the number of subarrays in the MIMO
14 no_subarrays = [1 5 10 20];
15 % Overall beampattern
16 Rx_pattern_conv = [];
17 % Overall beampattern w/ projection
18 Rx_pattern_conv_proj = [];
19 for ksub = 1:length(no_subarrays);
20     % Number of antennas in each subarray
21     K_sub = no_subarrays(ksub);
22     M_sub = M - K_sub + 1;
23     W_u_conv = uplinkbeamform(a_tar, K_sub, M_sub);
24     % Computing Transmit and diversity beampatterns
25     Theta_grid = [linspace(-pi/2,pi/2,1801)];
26     % Compute virtual steering vectors of target
27     [v_tar] = virtual_sv(theta_tar, M, d_t, ...
28         M_r, M_sub, K_sub, W_u_conv);
29     [v_tar_proj] = virtual_sv_proj(theta_tar, M, ...
30         d_t, M_r, M_sub, K_sub, W_u_conv);
31     % conventional downlink beamformer
32     w_d_conv = v_tar/(norm(v_tar));
33     % conventional downlink beamformer
34     w_d_conv_proj = v_tar_proj/(norm(v_tar_proj));
35     % Compute and plot overall Tx/Rx beampattern
36     % Use this to plot overall beampattern
37     w_d = w_d_conv;
38     % Use this to plot overall beampattern
39     w_d_proj = w_d_conv_proj;
40     size_w_d = size(w_d)
41     [V_grid] = virtual_sv(Theta_grid, M, d_t, ...
42         M_r, M_sub, K_sub, W_u_conv);

```

```

43     [V_grid_proj] = virtual_sv_proj(Theta_grid, ...
44     M, d_t, M_r, M_sub, K_sub, W_u_conv);
45     Rx_pattern = [10*log10(abs(w_d'*V_grid).^2)];
46     Rx_pattern_proj = [10*log10(abs(w_d'*V_grid_proj).^2)];
47     Rx_pattern = Rx_pattern - max(Rx_pattern);
48     Rx_pattern_conv = [Rx_pattern_conv; Rx_pattern];
49     Rx_pattern_proj = Rx_pattern_proj - max(Rx_pattern_proj);
50     Rx_pattern_conv_proj = [Rx_pattern_conv_proj;
51     Rx_pattern_proj];
52 end
53
54 % --- Plots --- %
55 Theta = Theta_grid;
56 plot(Theta*180/pi, 1.02*Rx_pattern_conv(1,:), 'g',
57 Theta*180/pi, 1.0*Rx_pattern_conv(2,:), 'r',
58 Theta*180/pi, Rx_pattern_conv(3,:), 'b', Theta*180/pi,
59 Rx_pattern_conv(4,:), 'k--', 'linewidth', 2)
60 grid
61 axis([-90 90 -120 30])
62 xlabel('Angle (deg)')
63 ylabel('Overall Gain (dB)')
64 legend('Overlapped-MIMO Radar (K=1)',
65 'Overlapped-MIMO Radar (K=5)',
66 'Overlapped-MIMO Radar (K=10)', 'MIMO Radar (K=20)')
67 figure;
68 Theta = Theta_grid;
69 plot(Theta*180/pi, 1.02*Rx_pattern_conv_proj(1,:), 'g',
70 Theta*180/pi, 1.0*Rx_pattern_conv_proj(2,:), 'r',
71 Theta*180pi, Rx_pattern_conv_proj(3,:), 'b', Theta*180pi,
72 Rx_pattern_conv_proj(4,:), 'k--', 'linewidth', 2), grid
73 axis([-90 90 -120 30])
74 xlabel('Angle (deg)')
75 ylabel('Overall Gain (dB)')
76 legend('Overlapped-MIMO Radar w/ NSP (K=1)', ...
77 'Overlapped-MIMO Radar w/ NSP (K=5)', 'Overlapped-MIMO Radar
78 ...
79 w/ NSP (K=10)', 'MIMO Radar w/ NSP (K=20)')

```

### 4.10.2 Uplink Beamforming Matrix

This MATLAB function computes the uplink beamforming matrix. The  $k$ th column represents the beamforming weight vector of the  $k$ th subarray.

```

1  %-----
2  % This function computes the the uplink beamforming matrix.
3  % The kth column represents the beamforming weight vector
4  % of the kth subarray
5  %-----
6  function [W_u] = uplinkbeamform(a_tar, K_sub, M_sub);
7  % Uplink weight vector (equal for all subarrays)
8  w_u = a_tar(1:M_sub);

```

```

9 size_w_u = size(w_u)
10 % Uplink weight vector normalized
11 w_u = w_u/(norm(w_u));
12 % Weights that are equal for all subarrays
13 W_u = kron(ones(1, K_sub), w_u);

```

### 4.10.3 Virtual Steering Vector

This MATLAB function computes the virtual steering vectors with projection.

```

1 %-----
2 % This function computes the virtual steering vectors
3 % with projection
4 %-----
5 function [v_sv] = virtual_sv_proj(Theta, Mt, d_t, Mr, M_sub,
6     ...
7     no_subarrays, W_u);
8 % No of receive Antennas
9 N_R = 15;
10 %---TX and RX Array ---%
11 Tx_sv = exp(-j*d_t*2*pi * (0:Mt-1) '*sin(Theta));
12 % Receiving steering vectors
13 Rx_sv = exp(-j*pi * (0:Mr-1) '*sin(Theta));
14
15 %--- VIRTUAL ARRAY W/O A_R ---%
16 P = [];
17 H = randn(N_R, no_subarrays * M_sub) + j * randn(N_R, ...
18     no_subarrays * M_sub);
19 P = null(H) * ctranspose(null(H));
20 v_sv2 = [];
21 for kk = 1:no_subarrays
22     for mm = 1:M_sub
23         v_temp2 = [];
24         w_u = W_u(mm, kk);
25         for jj = 1:length(Theta)
26             v_temp2 = [v_temp2, (w_u' * Tx_sv(kk + mm -1, jj)
27                 )];
28             v_sv2 = [v_sv2; v_temp2];
29         end
30     end
31 v_sv2_P = P * v_sv2;
32
33 v_sv3 = [];
34 for vv = 1:no_subarrays * M_sub
35     v_temp3 = [];
36     for jj = 1:length(Theta)
37         v_temp3 = [v_temp3, (v_sv2_P(vv, jj)) * Rx_sv(:,
38             jj)];
39     end
40 v_sv3 = [v_sv3; v_temp3];

```

```

40 end
41
42 v_sv = v_sv3;

```

#### 4.10.4 Number of Subarray

This MATLAB script enable us to determine the number of subarray.

```

1  % Determine the number of subarray
2
3  K1 = 1:1:10;
4  K2 = 1:1:15;
5  K3 = 1:1:20;
6  M1 = 10;
7  M2 = 15;
8  M3 = 20;
9
10 for ii = 1:1:10
11     ME1(ii) = (M1-ii+1)*ii;
12 end
13
14 for jj = 1:1:15
15     ME2(jj) = (M2-jj+1)*jj;
16 end
17
18 for kk = 1:1:20
19     ME3(kk) = (M3-kk+1)*kk;
20 end
21
22 figure;
23 plot(K1,ME1,'g-d','linewidth',2), grid
24 hold on
25 plot(K2,ME2,'b-s','linewidth',2)
26 plot(K3,ME3,'r-o','linewidth',2)
27 xlabel('K')
28 ylabel('M_{\epsilon}')
29 legend('M_T = 10','M_T = 15','M_T = 20')
30 axis([0 21 0 120])

```

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