Soil Mechanics II

Tutor for Exit-Exam

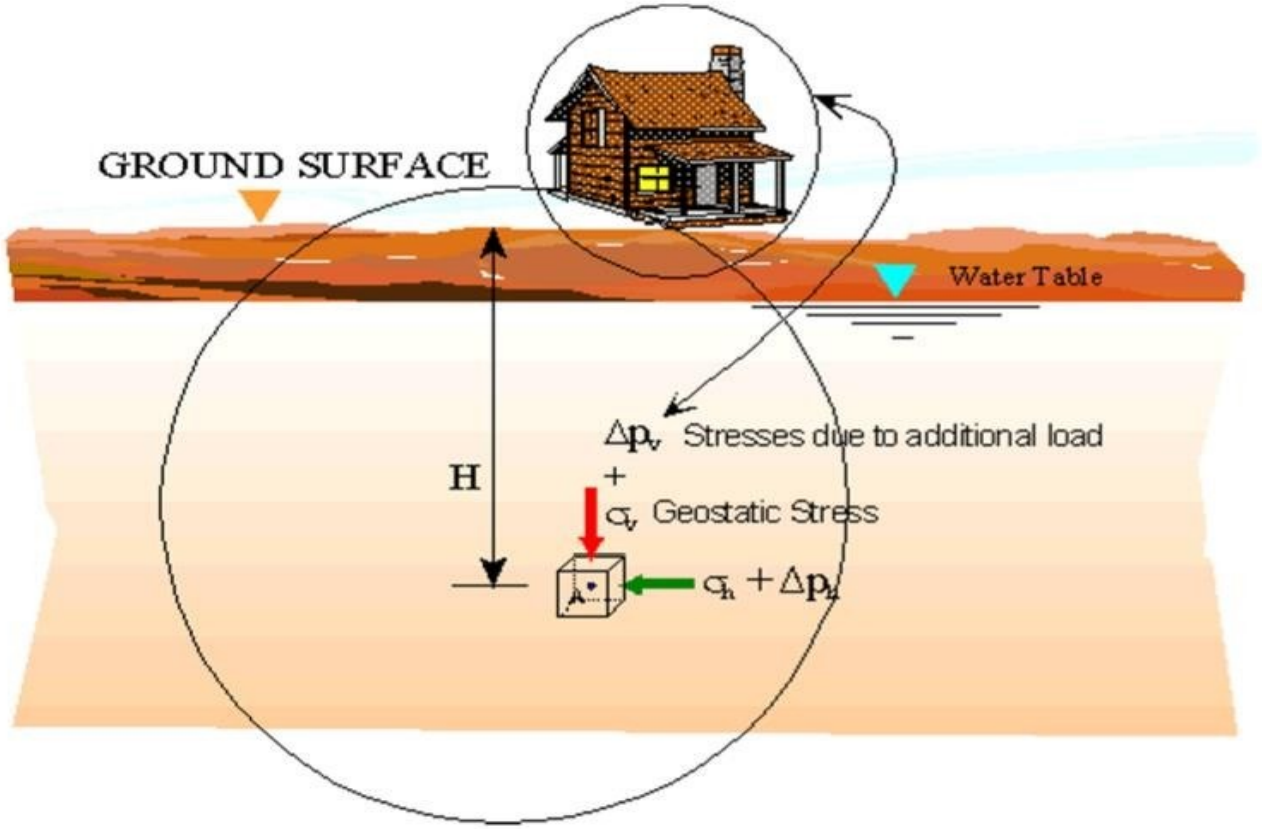
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(MSc. in Geotechnical Engineering)

Objectives of the tutorial

- Determine the stress distribution on soil.
- Conduct shear strength tests for soils and interpret their corresponding results.
- Determine bearing capacity of a soil.
- Determine lateral earth pressure.
- Estimate the stability of a slope.

Stresses in Soil Mass

- ***Geostatic stresses:*** from self weight of soil
- ***Additional stress:*** from imposed load



Stresses in Soil Masses *Cont…*

- ***Geo-static/Overburden Pressure:*** from self weight of soil; increases linearly with depth.
- ***Hydrostatic Pressure:*** from ground water; increases linearly with depth.
- ***Additional stress:*** from an imposed structural load; decreases with depth in a non-linear manner
- ***Surcharge load:*** a load of infinite extent on the surface; does not vary with depth.

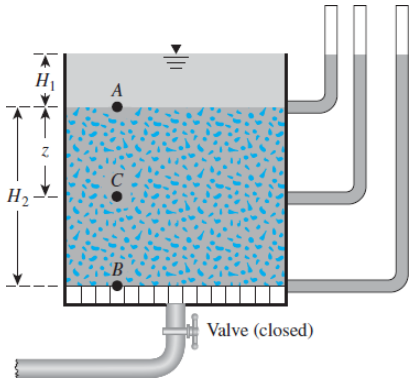
Geostatic Stress

- Total Stress
- Pore Pressure
- Effective Stress
  - ✓ Effects of Capillarity
  - ✓ Effects of Seepage

Geostatic Stress Cntd…

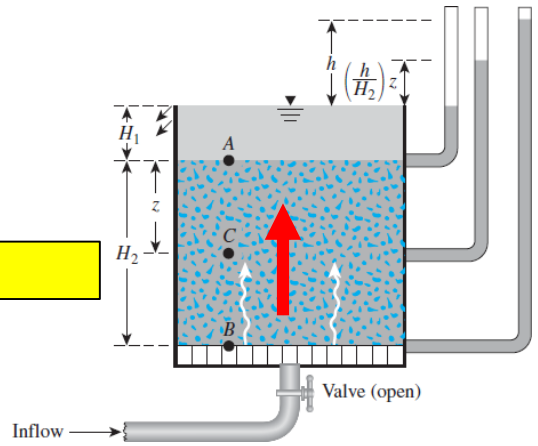
Static Case

$$\sigma'_C = z\,\gamma'$$



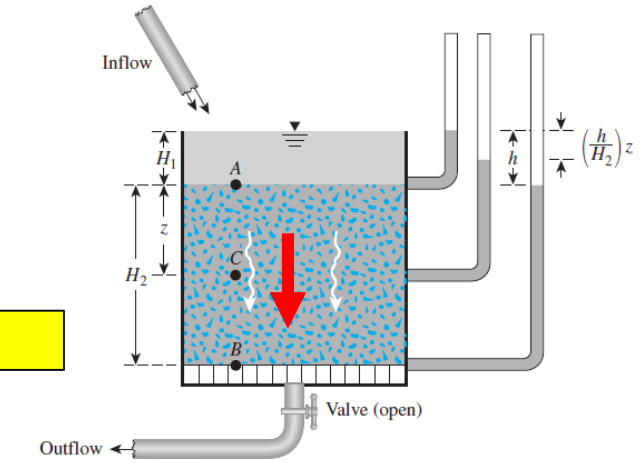
Upward Seepage

$$\sigma'_C = z\gamma' - iz\gamma_w$$



Downward Seepage

$$\sigma'_C = z\gamma' + iz\gamma_w$$



Geostatic Stress Cntd

EFFECT OF SEEPAGE

UPWARD SEEPAGE	DECREASES EFFECTIVE STRESS
DOWNWARD SEEPAGE	INCREASES EFFECTIVE STRESS

EFFECT OF CAPILLARY	INCREASES EFFECTIVE STRESS

Geostatic Stress Cntd…

Quick Sand Condition (Boiling)

$$\sigma'_c = z\gamma' - iz\gamma_w$$

$$z\gamma' - iz\gamma_w = 0 \longrightarrow i_c = \frac{\gamma'}{\gamma_w}$$

- i_c is called the ***Critical Hydraulic Gradient***. It is the value of i when a quick condition occurs.

➢ Recall that $\gamma' = \gamma_{sub} = \frac{G_s - 1}{1+e}\gamma_w$

$$i_c = \frac{G_s - 1}{1+e}$$

Stresses in Soil Mass *Cont…*

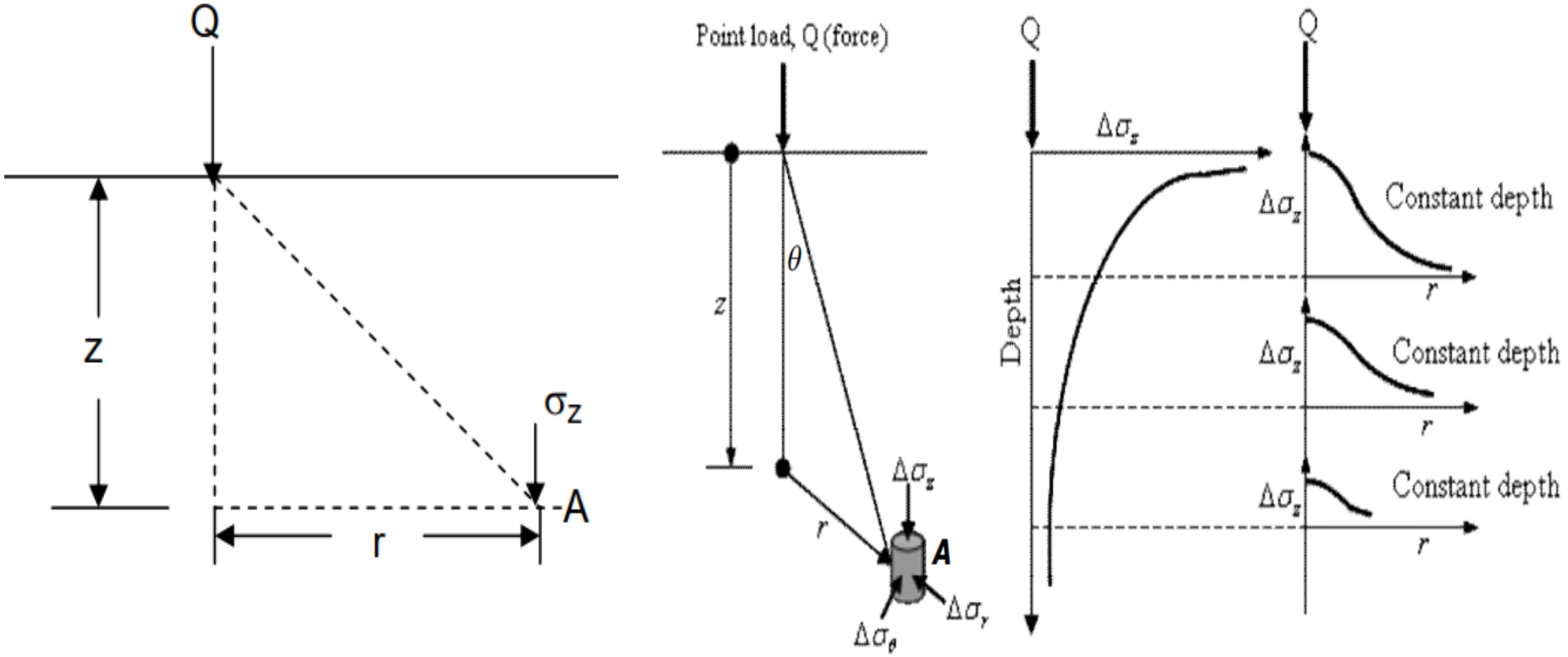
Additional Stress

- Infinite Loads
  - ✓ Surcharge Load
- Finite Loads
  - ✓ Point Load
  - ✓ Strip Load
  - ✓ Circular Load
  - ✓ Rectangular Load
    - Approximate Methods
  - ✓ Irregularly Shaped Areas

Additional Stress *Cont…*

Stress Distribution for a Point Load

- Stresses are induced by the ***application of a point load at the surface of a soil mass***.



Cont…

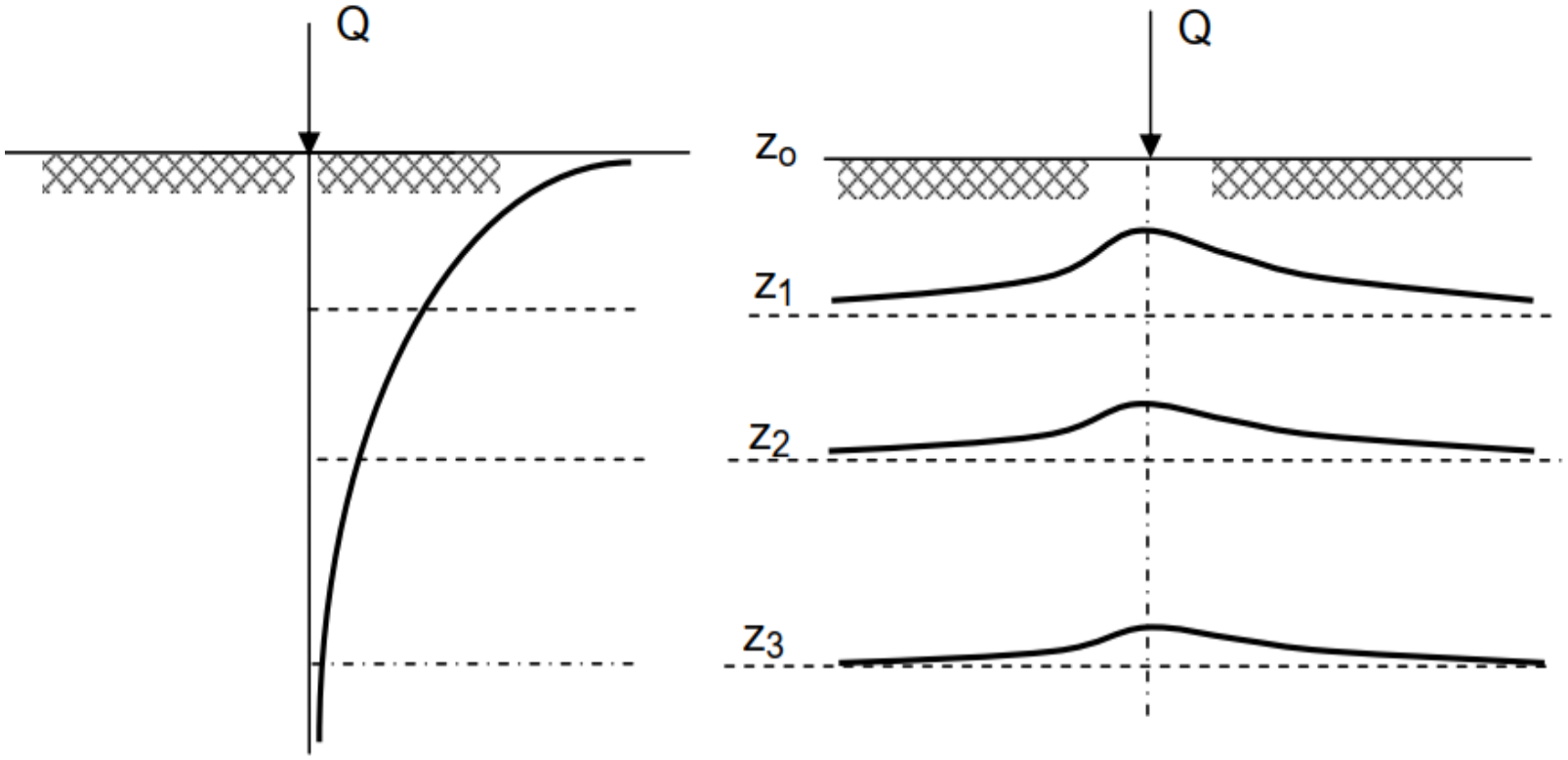
- Q is a point load applied on the ground surface.
- Point A is the point where the stress caused by the load Q.
- Z-vertical distance from point application to point A.
- r - radial distance of ***point "A"*** from the point load

- Thus the vertical normal stress at ***point A*** is given by

$$\sigma_Z = \frac{Q}{2\pi}\frac{3z^3}{(r^2+z^2)^{\frac{5}{2}}} \quad \Rightarrow \quad \sigma_Z = \frac{Q}{z^2}\frac{3/2\pi}{\left[1+\left(r/z\right)^2\right]^{\frac{5}{2}}}$$

$$\sigma_Z = \frac{Q}{z^2} I_B \qquad Where I_B = \frac{3/2\pi}{\left[1+\left(r/z\right)^2\right]^{\frac{5}{2}}}$$

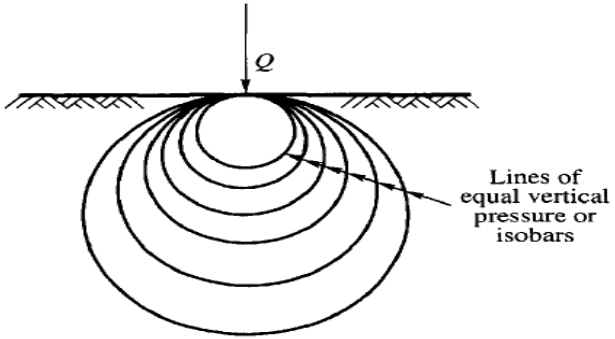
Variation of σ_z with depth z and radial distance r



a) Variation of stress with depth **b) Stress distribution on horizontal planes**

Pressure Isobars-Pressure Bulb

- An *isobar is a line which connects all points of equal stress below the ground surface.*
- Isobar is *a stress contour*.
- They are also termed *bulb of pressure or simply the pressure bulb.*
- *Normally isobars are drawn for vertical, horizontal and shear* stresses. The one that is most important in the ***calculation of settlements of footings*** is the vertical pressure isobar.



Integral form of Bossinesq equation for specific shape

The solution as obtained for obtaining at a corner of rectangular footing of dimension mz and nz at depth z carrying a uniform load intensity Q/unit area.

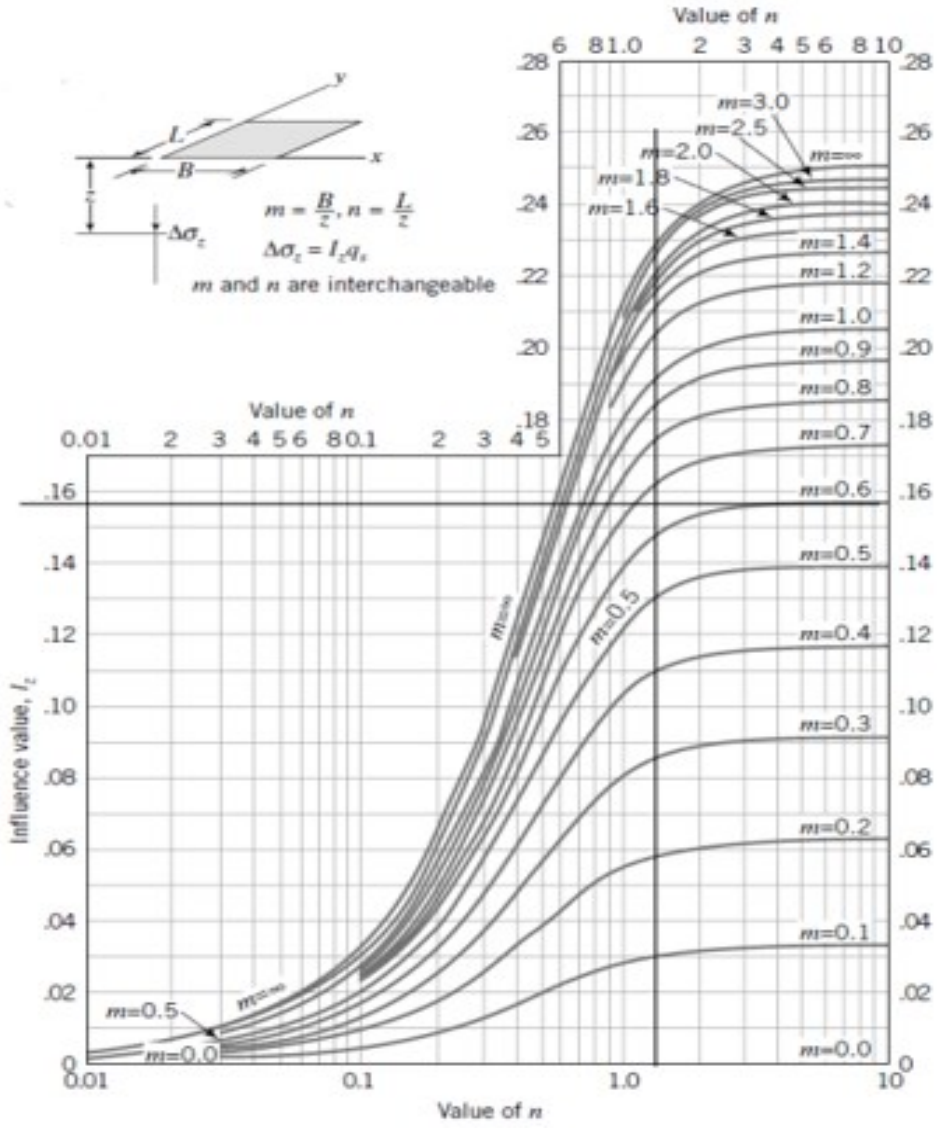
$$\sigma_z \; QI_R \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

Where I_R is another influence factor and is given by

$$I_R = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\right]\times\left[\frac{m^2+n^2+2}{m^2+n^2+1}\right]$$

$$+\frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\right]\ldots\ldots\ldots\ldots(13)$$

- I_R *can also read from the chart*

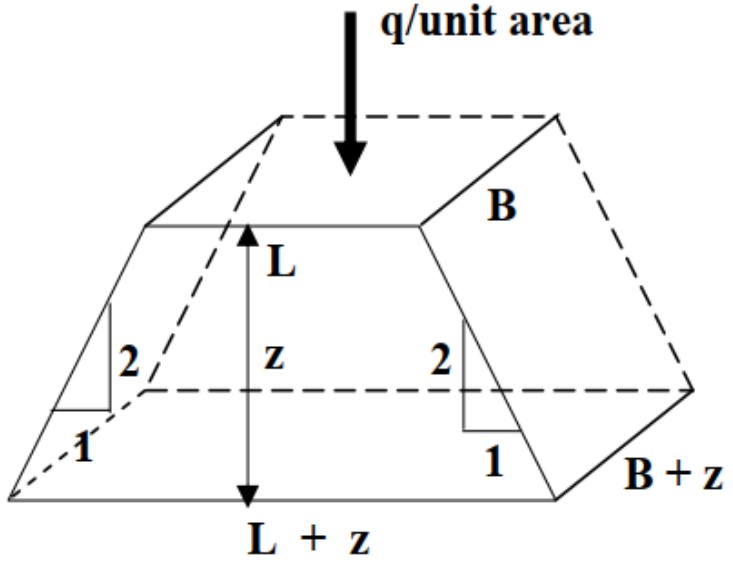


2:1 Approximation Method

- Total load on the surface = q x B x L; and
  Area at depth z = (L + z) (B + z)

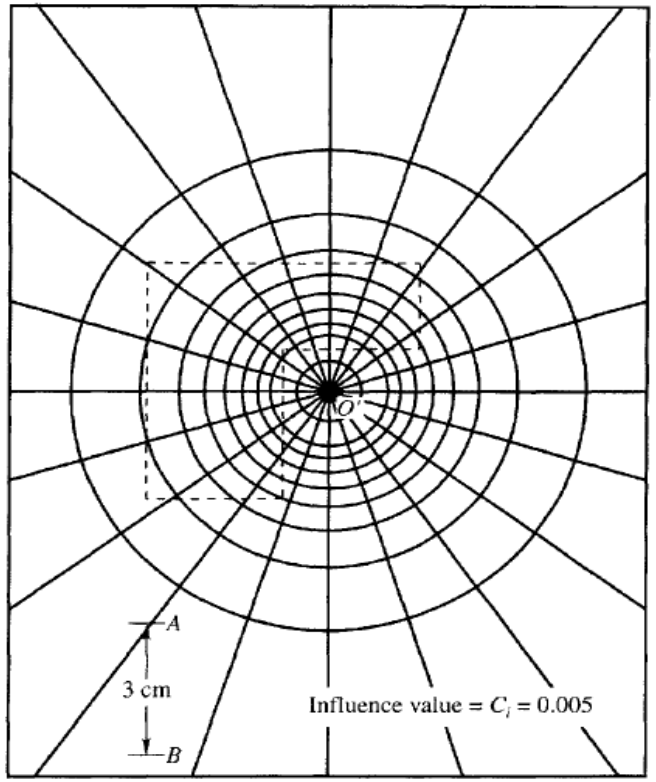
$$\sigma_z = \frac{q.B.L}{(L+z)(B+z)}$$

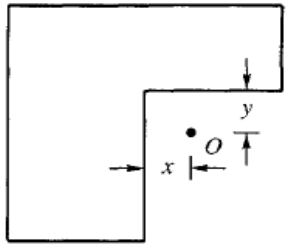
Type of footing	Area, A_z
Square	$(B+z)^2$
Rectangular	$(B+z)(L+z)$
Circular	$\pi(D+z)^2/4$
Strip or wall	$(B+z).1$



Newmarks Influence Chart

- ***To determine the increase in vertical stress due to a uniformly loaded area of any shape.***
- ***Consists of concentric circles divide by radial lines.***
- ***The area of each segment represents an equal proportion of the applied surface stress at depth z below the surface.***
- ***Constructed by 10 concentric circles and 20 radial lines.***
- ***The chart should show a scale and an influence factor.***





$$I_N = \frac{1}{10*20} = 0.005$$

$$\Delta\sigma_z = q_s I_N N_S$$

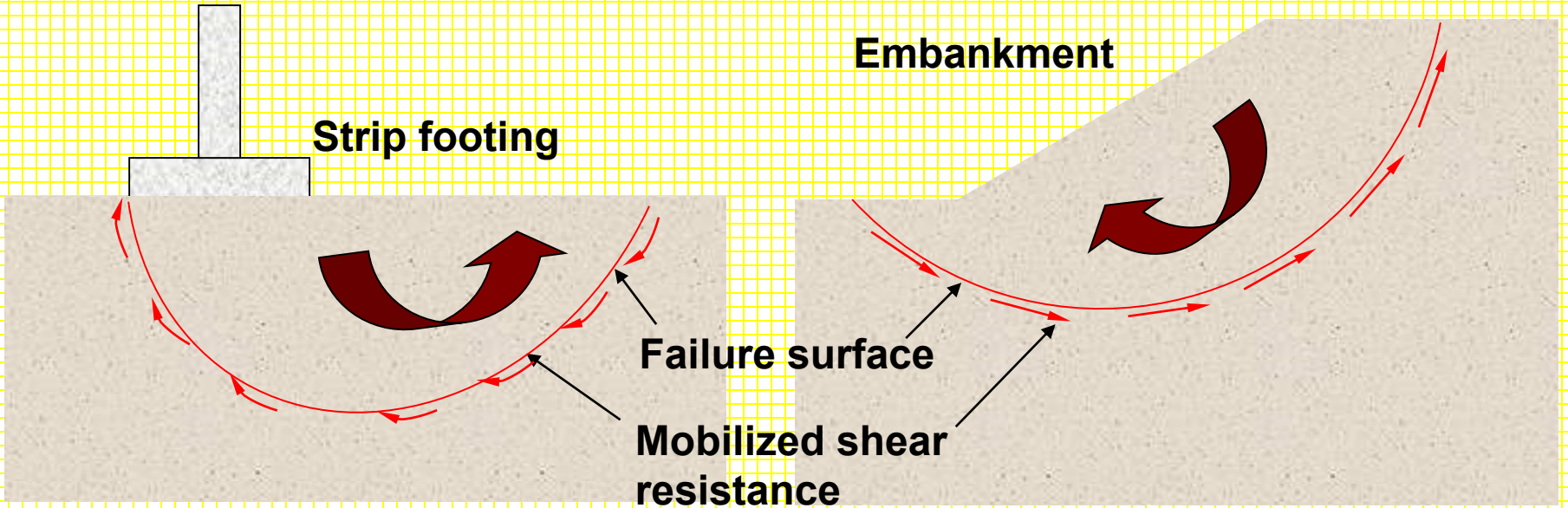
SHEAR STRENGTH OF SOILS

Shear Strength of Soils

- **The safety** of any geotechnical structure is dependent on the strength of the soil.
- Majority of soil mechanics problems (such as bearing capacity, lateral pressure against retaining walls, slope stability, etc.), only failure in **shear.**
- Shear strength along any plane is mobilized by **cohesion** and **frictional resistance** to sliding between soil particles.
- The cohesion ***c* and** ϕ angle of friction of a soil- ***shear strength parameters***.

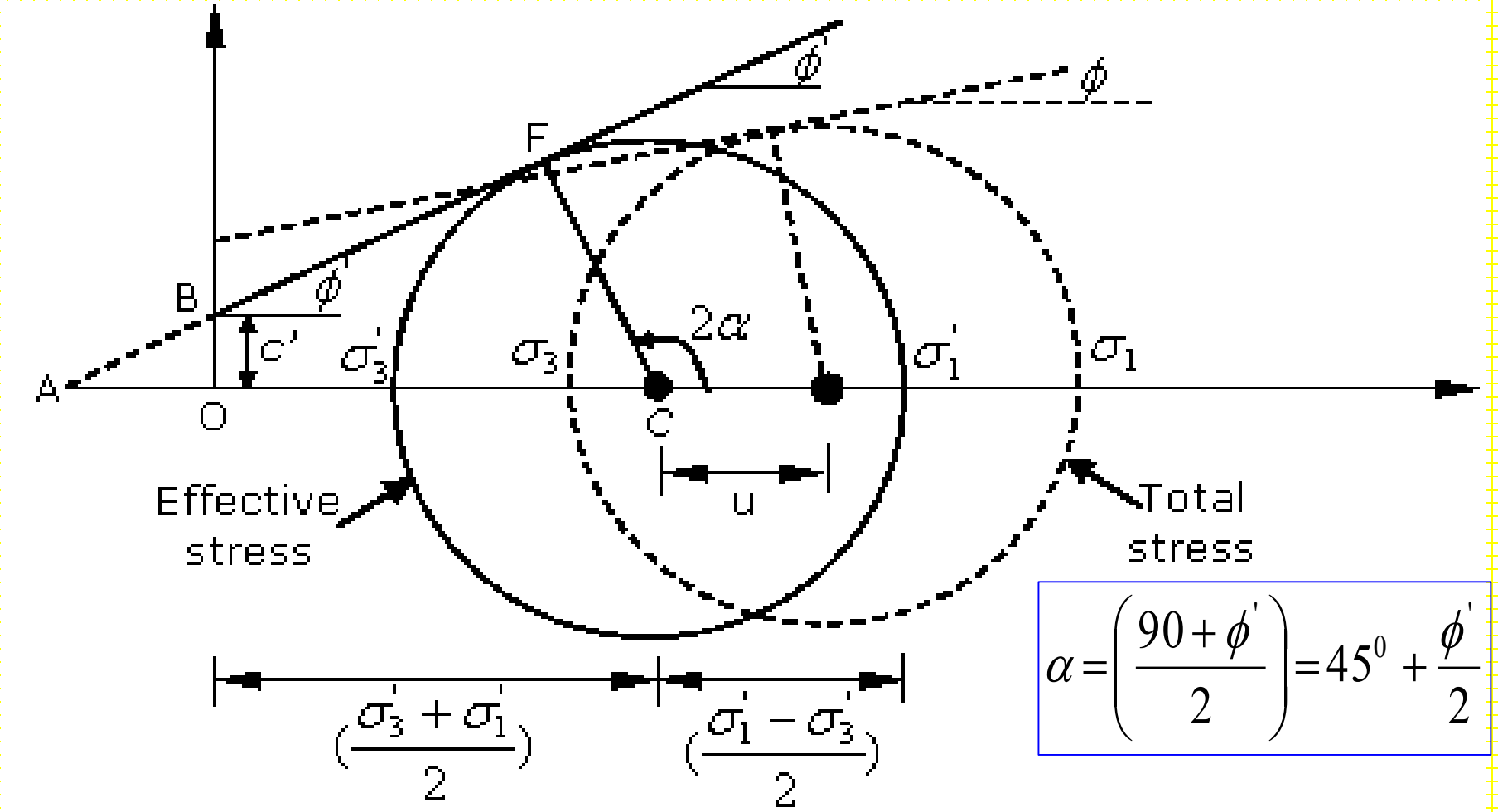
Shear failure of soils

✓ **Shear failure** happens when the shear stress along the failure surface reaches the soil's shear strength.



✓ At failure, shear stress along the failure surface (mobilized shear resistance) reaches the shear strength.

Cont…

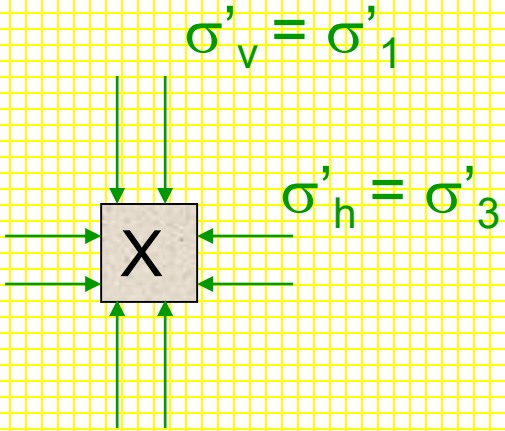


$$\alpha = \left(\frac{90 + \phi'}{2}\right) = 45^0 + \frac{\phi'}{2}$$

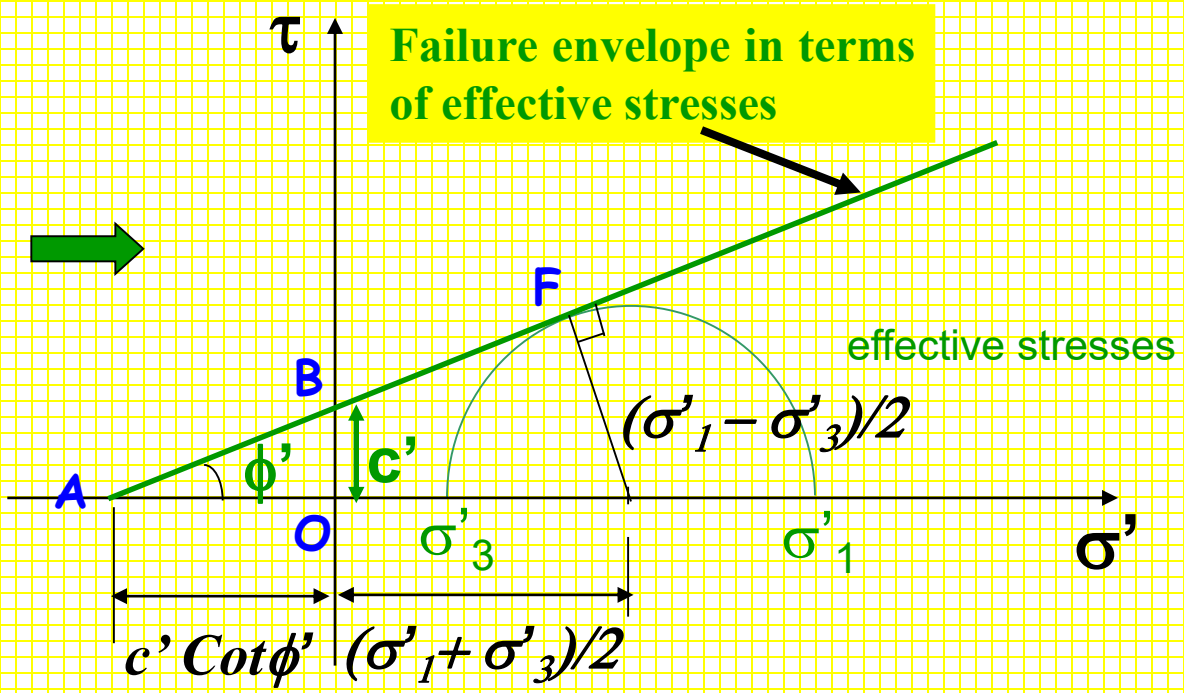
Mohr-Coulomb failure criterion.

$$\sigma_z - \sigma_x = (\sigma_z' + u) - (\sigma_x' + u) = \sigma_z' - \sigma_x'$$

Mohr Coulomb failure criterion with Mohr circle of stress (Relation of stresses can be calculated as)



X is on failure



Therefore,

$$\left[c'Cot\phi' + \left(\frac{\sigma_1' + \sigma_3'}{2}\right)\right] Sin\phi' = \left(\frac{\sigma_1' - \sigma_3'}{2}\right)$$

tanϕ= op/adj= OB/AO
=c'/AO
AO= c'/ tanϕ = c' *cotϕ

Mohr Coulomb failure criterion with Mohr circle of stress

$$\left[c' Cot\phi' + \left(\frac{\sigma_1' + \sigma_3'}{2}\right)\right] Sin\phi' = \left(\frac{\sigma_1' - \sigma_3'}{2}\right)$$

$$\left(\sigma_1' - \sigma_3'\right) = \left(\sigma_1' + \sigma_3'\right) Sin\phi' + 2c' Cos\phi'$$

$$\sigma_1'\left(1 - Sin\phi'\right) = \sigma_3'\left(1 + Sin\phi'\right) + 2c' Cos\phi'$$

$$\sigma_1' = \sigma_3' \frac{\left(1 + Sin\phi'\right)}{\left(1 - Sin\phi'\right)} + 2c' \frac{Cos\phi'}{\left(1 - Sin\phi'\right)}$$

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

Mohr Coulomb failure criterion…

If the cohesion c', is small or zero;

$$\sin\phi' = \left(\frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'}\right)$$

By rearranging $$\frac{\sigma_3'}{\sigma_1'} = \frac{1-\sin\phi'}{1+\sin\phi'}$$

Or $$\frac{\sigma_3'}{\sigma_1'} = \tan^2\left(45 - \frac{\phi'}{2}\right)$$

Mohr-Coulomb Failure Criterion

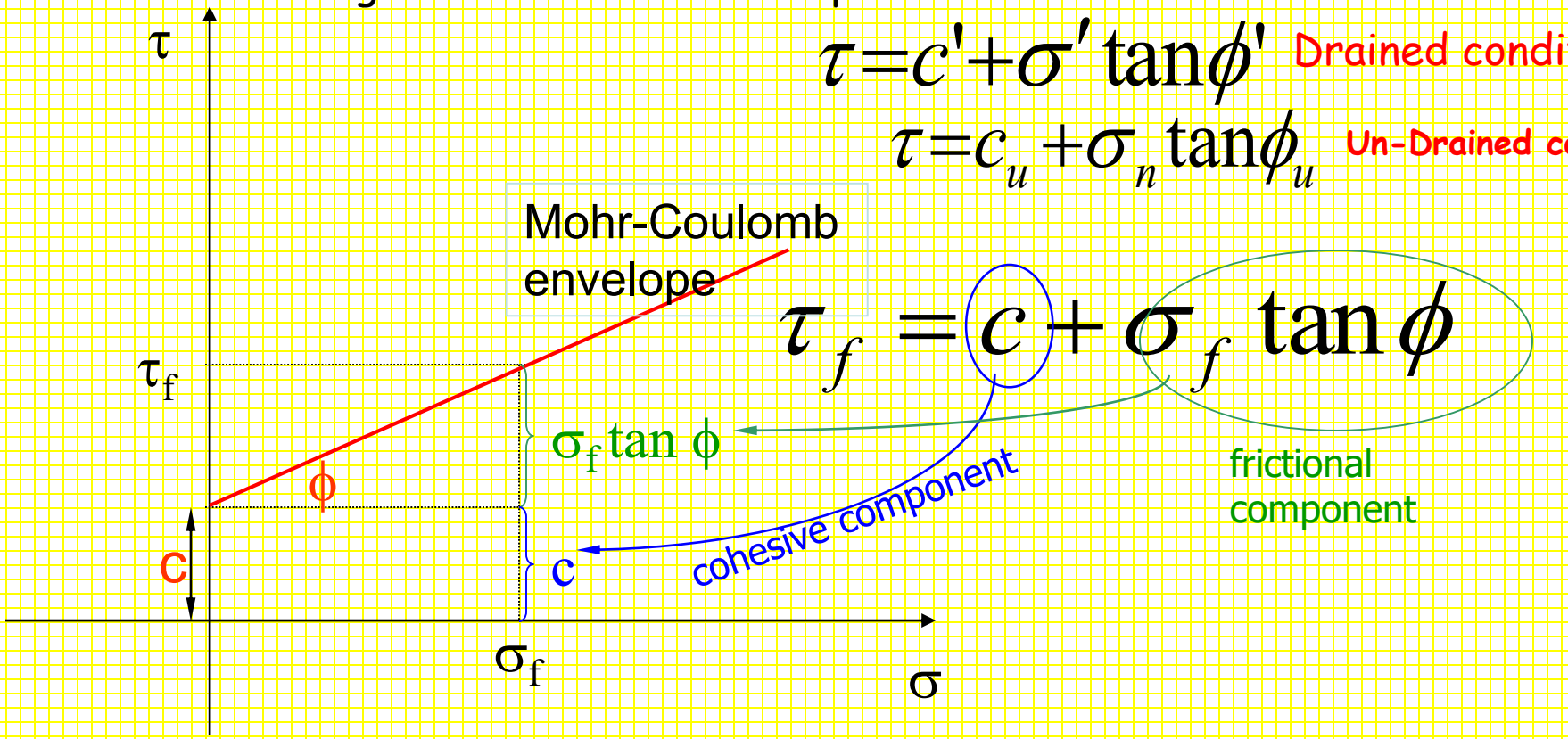
✓ Shear strength of a soil along a failure plane could be described by:

$$\tau_f = c + \sigma_f \tan\phi$$

✓ Shear strength consists of two components: cohesive and frictional.

$$\tau = c' + \sigma' \tan\phi' \quad \text{Drained condition}$$

$$\tau = c_u + \sigma_n \tan\phi_u \quad \text{Un-Drained condition}$$



$$\tau_f = c + \sigma_f \tan\phi$$

Drained and Undrained Shear strength

- **Drained condition-** ***Long-term condition***
- **Undrained condition-*Short-term condition***

- Specimen tends to **expand and swell during shear**, the induced pore water pressure is ***negative***.
- Specimen tends to compress or contract **during shear**, then the **induced pore water pressure** is ***positive***.

Laboratory Shear Strength Tests

- Methods for testing shear strength of soils in a laboratory are:

I. Direct shear test.

II. Triaxial compression test.

III. Unconfined compression test.

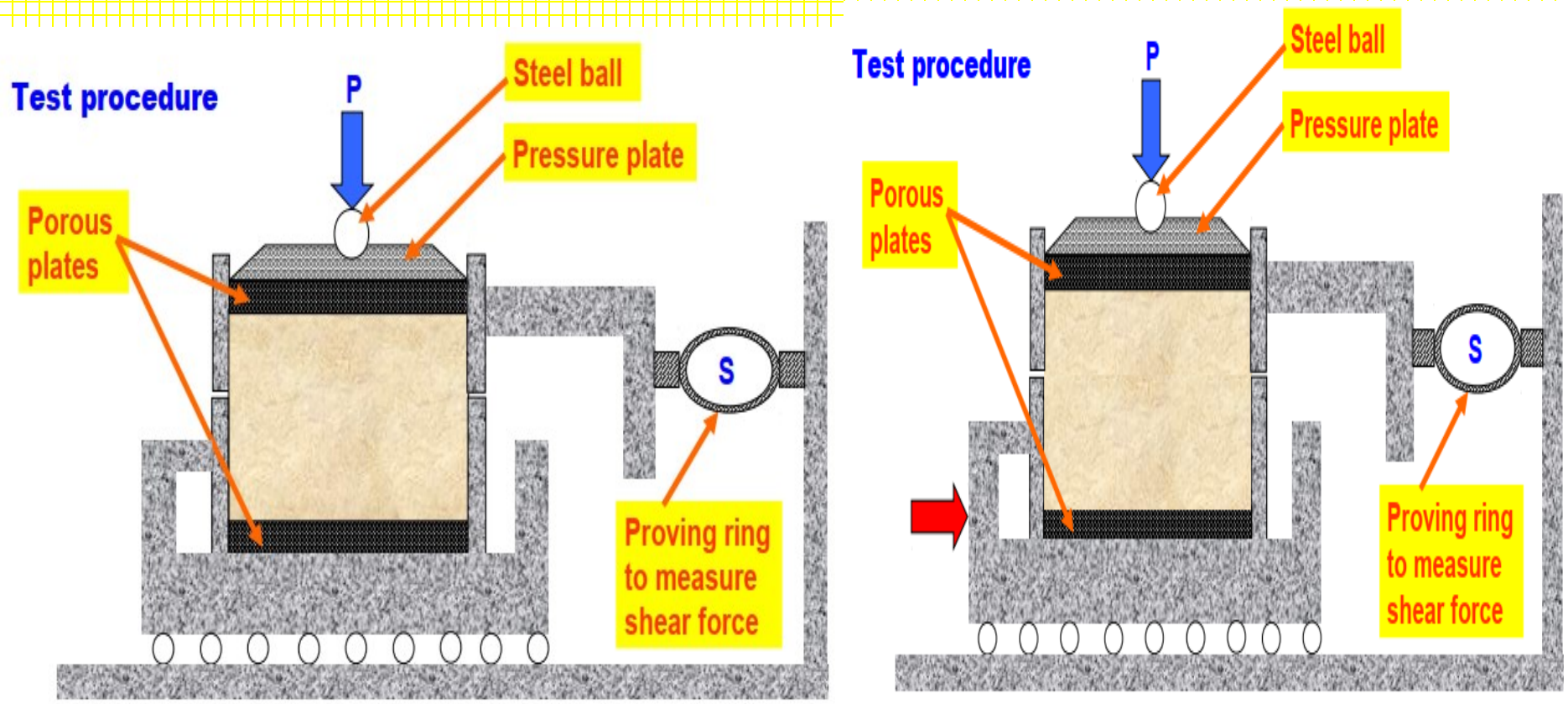
To do what?

➢ **Determination of shear strength parameters of soils (c, Φ or c', Φ')**

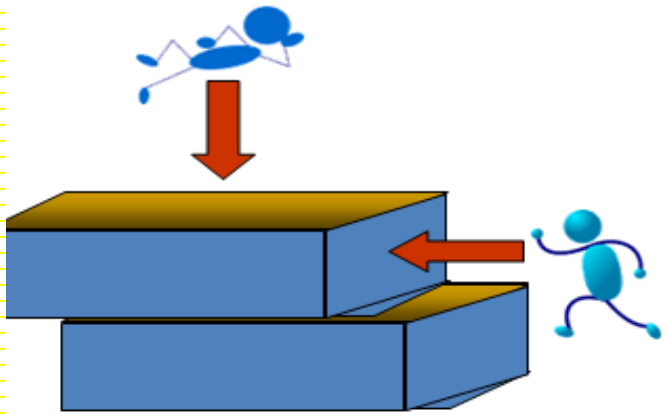
Direct shear test



Cont…



Schematic of direct shear apparatus.



Cont…

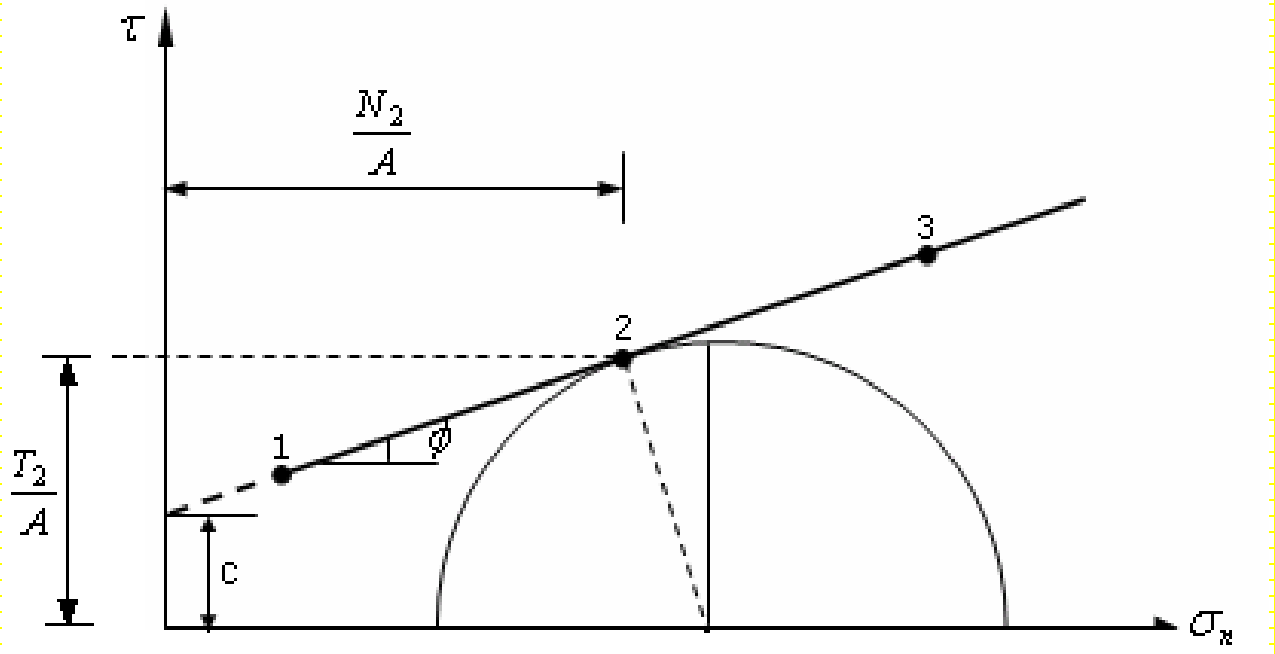
- ✓ By plotting the normal stresses and corresponding shear stresses from the results of such tests, a **failure envelope** is obtained.

$$\sigma = N/A$$

$$\tau = T/A,$$

Cont…

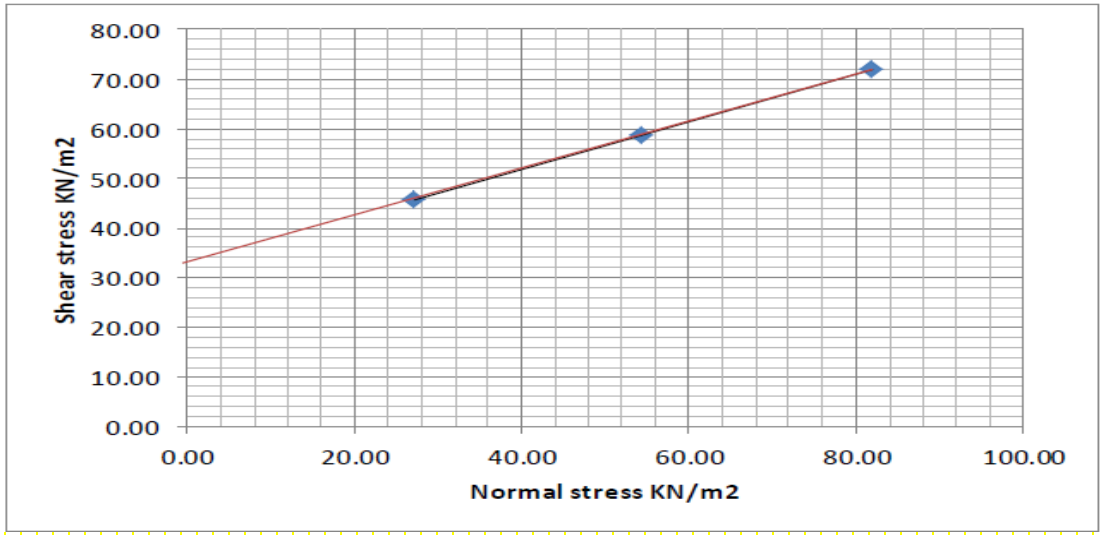
- ✓ The corresponding **Mohr's circle** for this case is shown in below and the **shear strength parameters ϕ and c** could be measured directly.



Direct shear data for above curves!

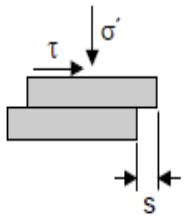
Test no	Normal stress=(KN/m2)	shear stress(KN/m2)
1	27.27	45.82
2	54.56	58.63
3	81.86	72.03

Direct shear curve!



Plotted direct shear test results and a Mohr circle.

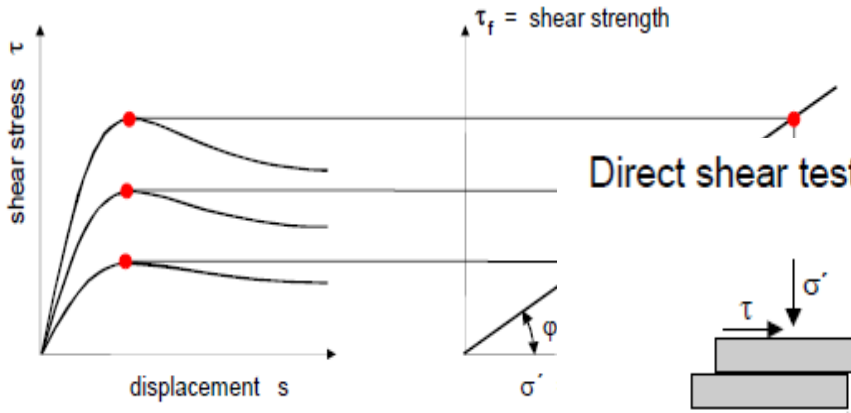
Direct shear tests - coarse grained soil



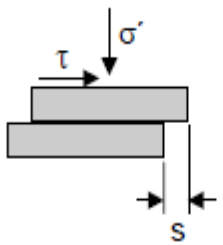
shear strength is caused by friction:

$\tau_f = \sigma' \cdot \tan \varphi'$

φ' = angle of internal friction

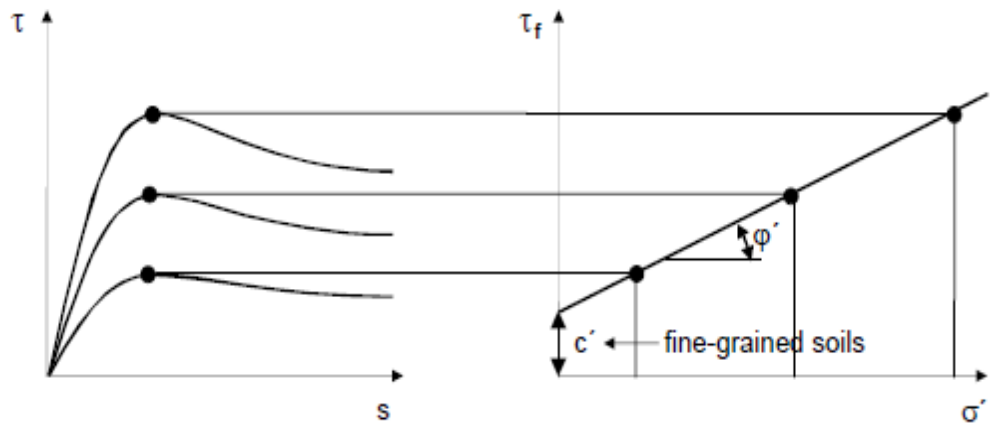


Direct shear tests - fine grained soil



$\tau_f = c' + \sigma' \cdot \tan\varphi'$ c' = effective cohesion

shear strength criterion after Coulomb !



Cont…

Advantages

- ✓ Test in inexpensive
- ✓ Fast and simple
- ✓ Easy to prepare for **cohesion less** samples

Disadvantages

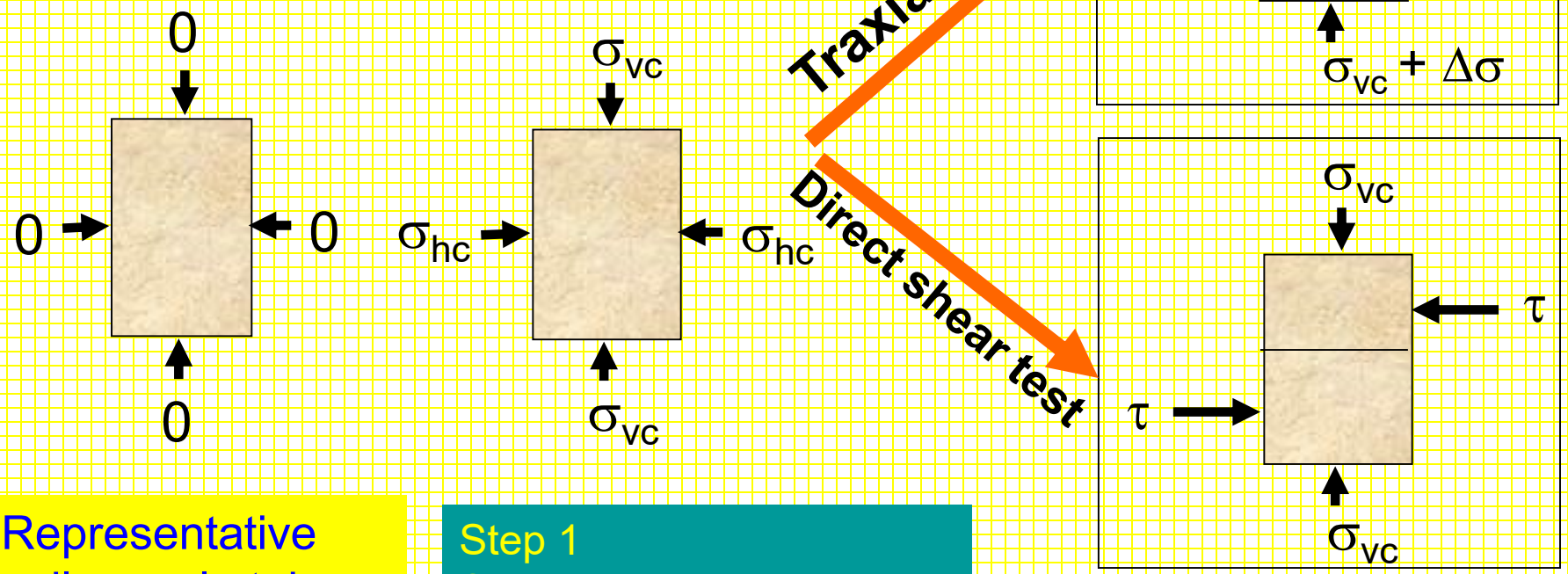
- ✓ Failure plane is forced to be horizontal.
- ✓ Cannot control drainage
- ✓ Stress concentrations at the sample boundaries
- ✓ Uncontrolled rotation of principal planes and stresses

Triaxial Compression Test



Laboratory tests

Simulating field conditions in the laboratory



Representative soil sample taken from the site

Step 1
Set the specimen in the apparatus and apply the initial stress condition

Step 2
Apply the corresponding field stress conditions

Cont…

Advantage over DST

- ✓ More versatile
- ✓ Drainage can be well controlled
- ✓ No rotation of principal plane and stresses
- ✓ Failure plane can occur anywhere

Triaxial tests are run by:

- – **saturating** the soil
- – ***applying*** the confining stress (called σ_3)
- – Then applying the vertical stress until failure

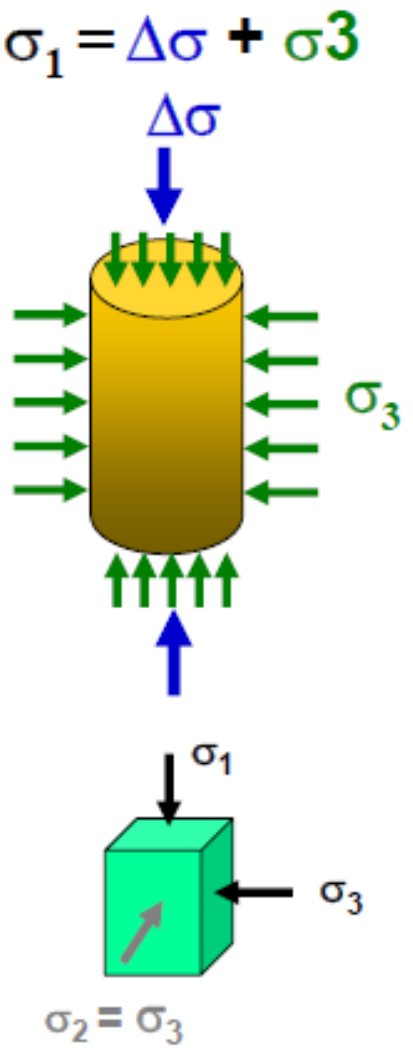
Cont…

Principles of triaxial compression test(TC)

- ✓ Is used to measure shear strength of soil under **controlled drainage condition**
- ✓ Cylindrical soil specimen is enclosed and subjected to ***confining fluid/air pressure*** and then loaded axially to failure.
- ✓ Test is called **triaxail** b/c of three principal stresses are known and controlled.

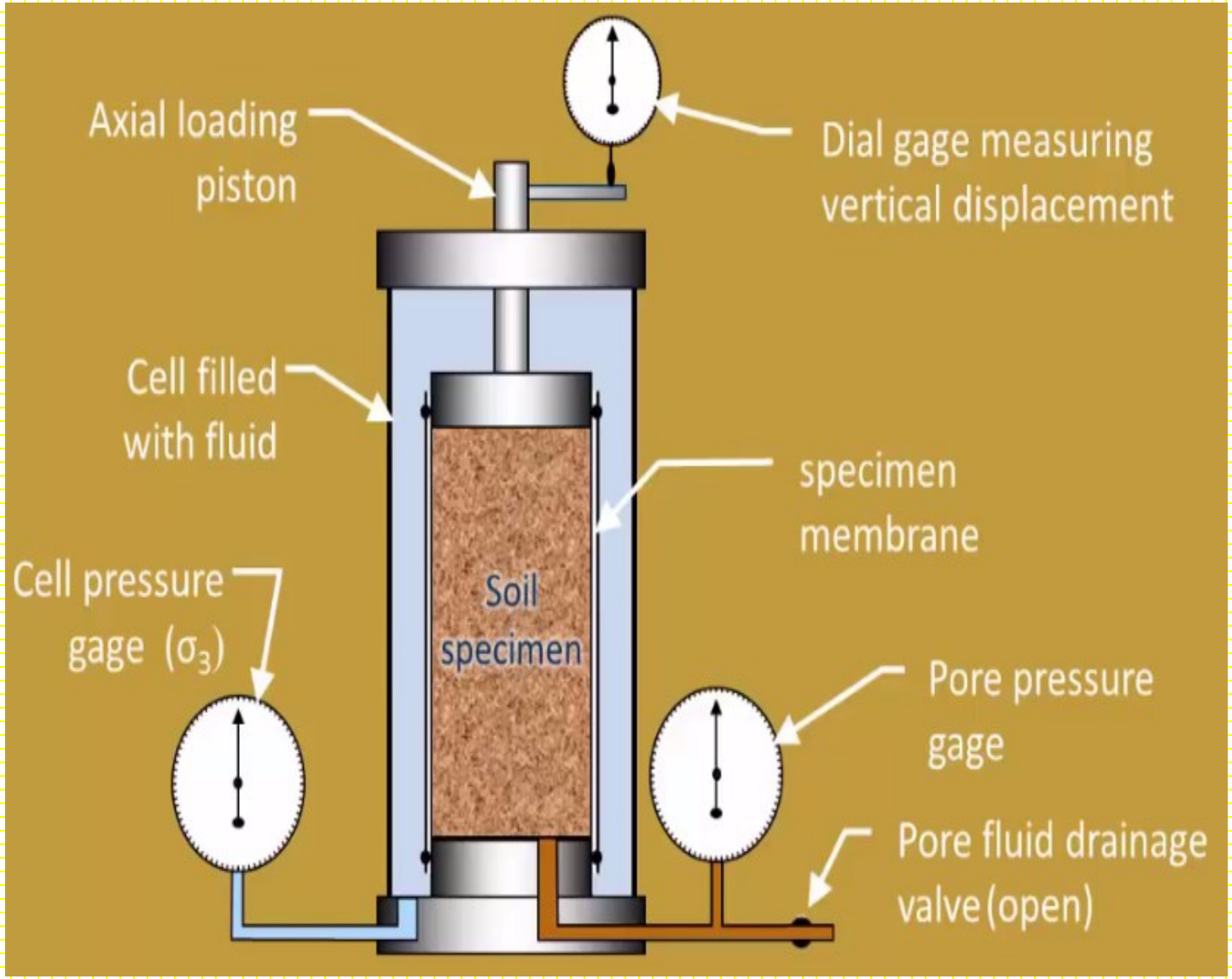
Principles of triaxial compression test(TC)..

- During shear, the major principal stress, σ_1 is equal to the applied axial stress ($\Delta\sigma$ = P/A) plus the chamber (confining) pressure, σ_3
- The applied axial stress, $\sigma_1 - \sigma_3$ is termed the "principal stress difference" or sometimes the "deviatory stress"
- The intermediate principal stress, σ_2 and the minor principal stress, σ_3 are identical in the test, and are equal to the confining or chamber pressure



Cont…

Triaxial cell





Soil sample at failure

Cont…

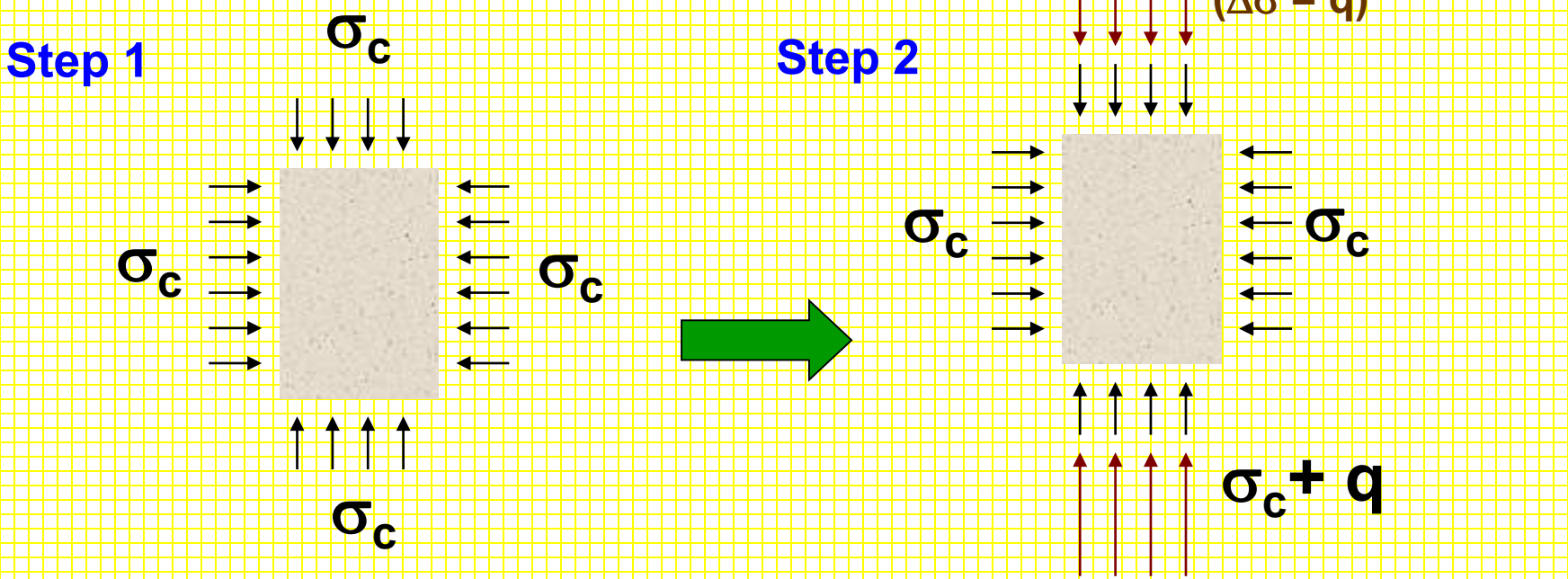
- ✓ The **triaxial apparatus is versatile** because we can ① independently control the applied axial and radial loads, ② conduct tests under drained and undrained conditions, and ③ control the applied displacements or stresses.
- ✓ Recorded measurements include **deviatoric stress** at different stages of the test, **vertical displacement** of the ram, **volume change** and **pore water pressure**.

Types of Triaxial Tests

- ✓ Depending on whether drainage is allowed or not
  - ❖ **initial isotropic** cell pressure application/ **consolidation phase**, and
  - ❖ **shearing**(during loading valve is open or not) ,
- ✓ Having these; there are three special types of triaxial tests that have practical significances.

Consolidated Drained (CD) test
Consolidated Undrained (CU) test
Unconsolidated Undrained (UU) test

Types of Triaxial Tests



Under all-around cell pressure σ_c

Shearing (loading)

Is the drainage valve open?

yes → **Consolidated sample**

no → **Unconsolidated sample**

Is the drainage valve open?

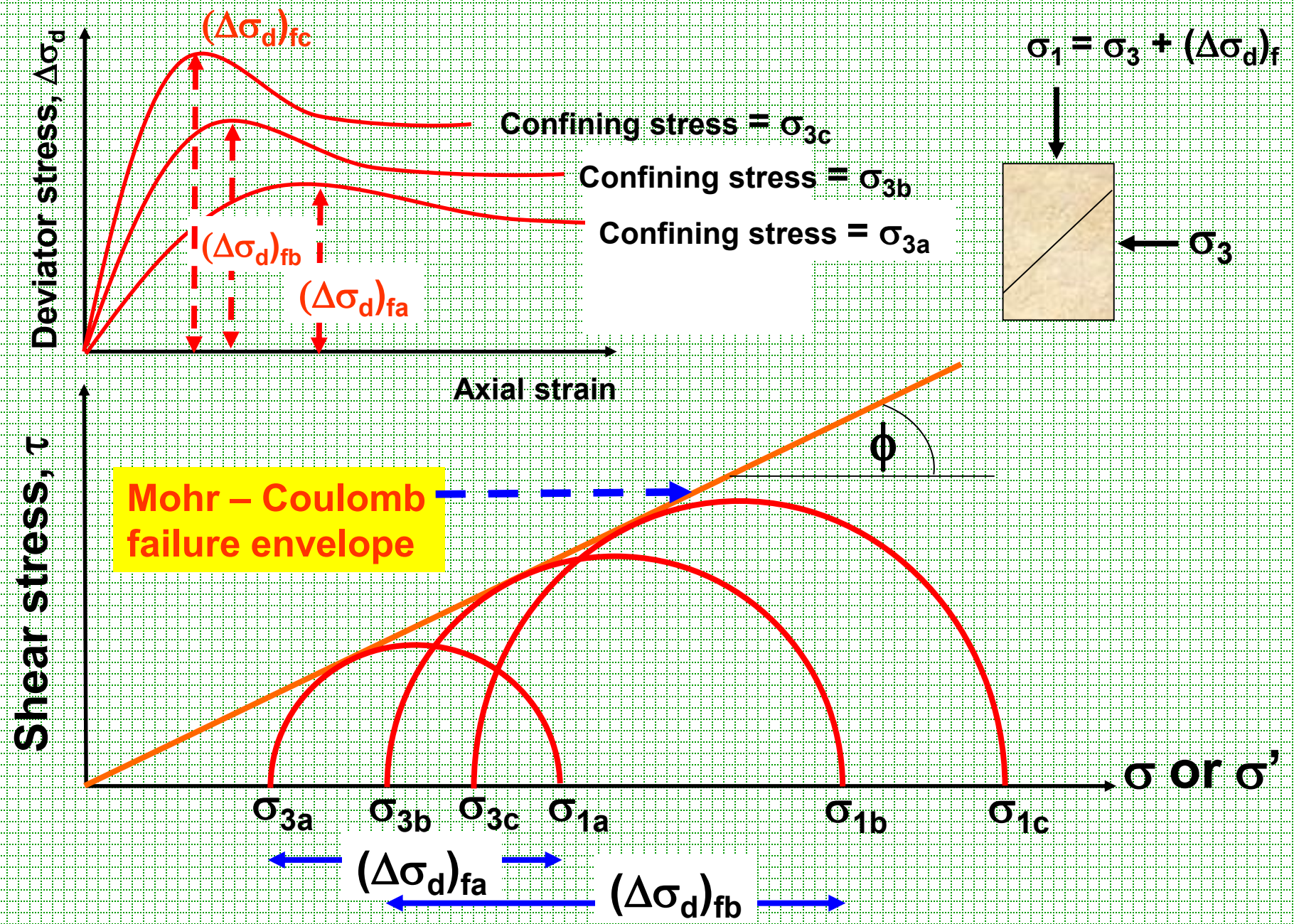
yes → **Drained loading**

no → **Undrained loading**

A. Consolidated – Drained Triaxial Test

- ✓ The purpose of a **CD test** is to determine the **drained shear strength parameters** ϕ' **and** c'.
- ✓ The specimen is saturated
- ✓ Confining stress (σ_3) is applied (stage 1)
  - This squeezes the sample causing **volume decrease**
  - Drain lines kept open and must wait for full consolidation (**u = 0**) to continue with test
- ✓ Once full consolidation is achieved, **normal stress applied** to failure with drain lines **still open** (stage 2)
  - Normal stress applied **very slowly allowing** full drainage and full consolidation of sample during test (u = 0)
- ✓ Test can be run with varying values of σ_3 to create a Mohrs circle and to obtain a plot showing **c' and φ'**
- ✓ Applicable in conditions ***long term stability analysis.***

CD tests How to determine strength parameters c and ϕ



B. Consolidated Undrained (CU) Test

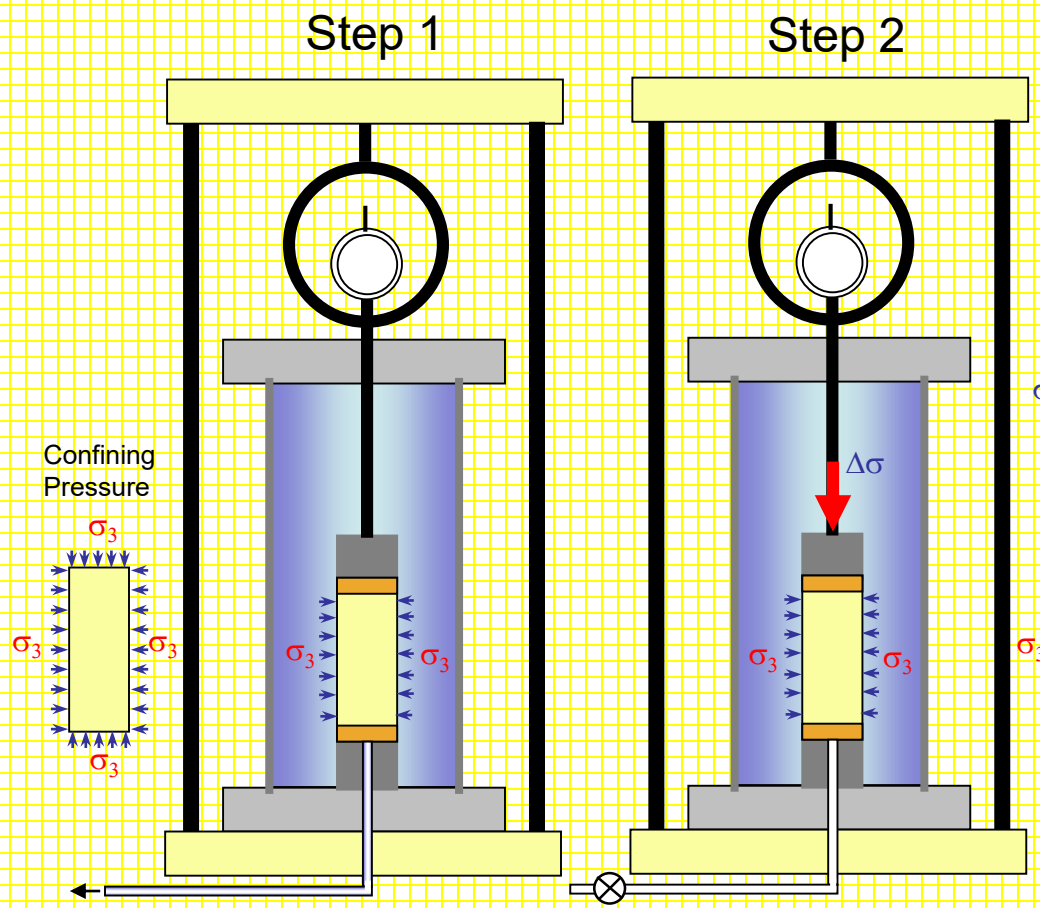
- ✓ The purpose of a CU test is to determine both the undrained (c_u, ϕu) and drained (c', ϕ') shear strength parameters; E_u and E are also obtained from this test.
- ✓ The CU test is conducted in a similar manner to the CD test except that **after isotropic consolidation**, the axial load is increased under **undrained condition** and **the excess pore water pressure is measured**.
- ✓ The excess pore pressure developed during shear can either be **positive or negative**.
- ✓ Positive pore pressures occur in loose sands and normally consolidated clays. Negative pore pressures occur in dense sands and heavily overconsolidated clays.

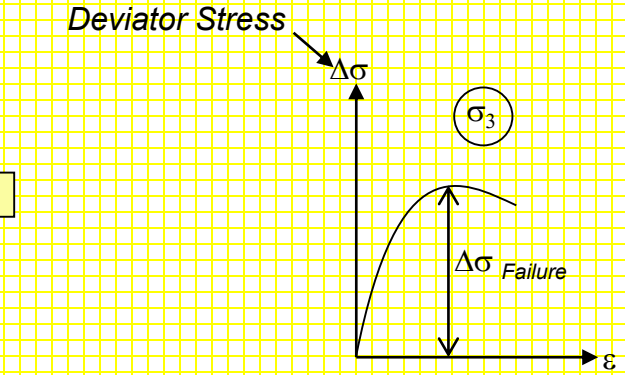
B. Consolidated – Undrained Triaxial Test

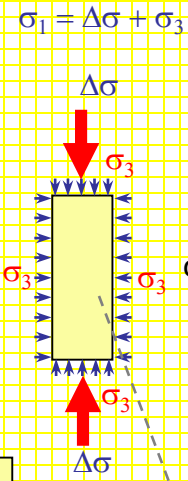
- ✓ The specimen is **saturated**
- ✓ Confining stress (σ_3) is applied
  - This squeezes the sample causing volume decrease
  - Again, must wait for full consolidation (u = 0)
- ✓ Once full consolidation is achieved, **drain lines are closed** (no drainage for the rest of the test), and normal stress applied to failure
  - Normal stress can be **applied faster** since no drainage is necessary (**u not equal to 0**)
- ✓ Test can be run with varying values of σ_3 to create a Mohrs circle and to obtain a plot showing **c & φ /c' & φ'**
- ✓ Applicable in situations where failure may occur **suddenly** such as a rapid drawdown in a dam or levee
- ✓ This test can simulates **long term as** well as **short term shear** strength for cohesive soils if pore water pressure is measured during the shearing phase.

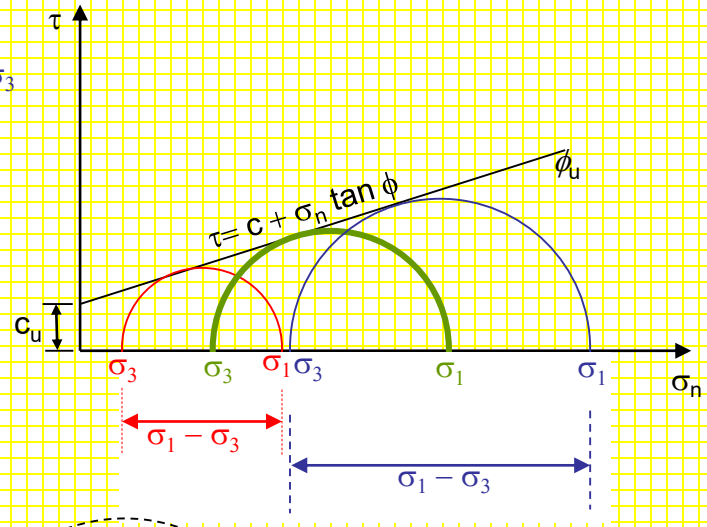
Triaxial Compression Test

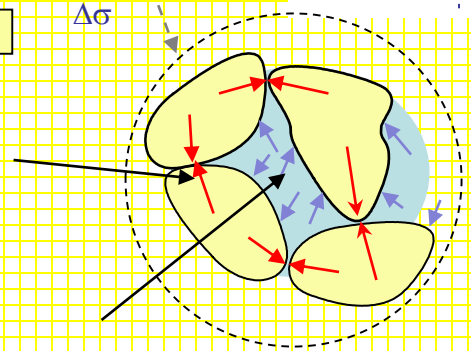
Consolidated Undrained Test (CU)











C. Unconsolidated Undrained (UU) Test

- ✓ The purpose of a **UU test** is to determine the undrained shear strength (S_u) of a **saturated soil.**
- ✓ The UU test consists of applying a cell pressure to the soil sample **without drainage** of pore water followed by increments of **axial stress**.
- ✓ The cell pressure is **kept constant** and the test is completed **very quickly** because in neither of the two stages – **consolidation and shearing** – is the excess pore pressure allowed to drain. In the UU test, pore water pressures are usually **not measured**.

c. Unconsolidated – Undrained Test

- ✓ The specimen is **saturated**
- ✓ Confining stress (σ_3) is applied **without drainage** or consolidation (drains closed the entire time).
- ✓ Normal stress then increased to failure without allowing **drainage or consolidation.**
- ✓ This test can be run **quicker** than the other 2 tests since no consolidation or drainage is needed. Test can be run with varying values of σ_3 to create a Mohrs circle and to obtain a plot showing c and φ
- ✓ **UU test simulates short term shear strength for cohesive soils.**
- ✓ Applicable in most practical situations – **foundations for example.**
- ✓ This test commonly shows a $\varphi = 0$ condition

Failure Conditions

Unconsolidated Undrained (UU)

✓ - Failure that occurs rapidly during or **shortly after construction.**

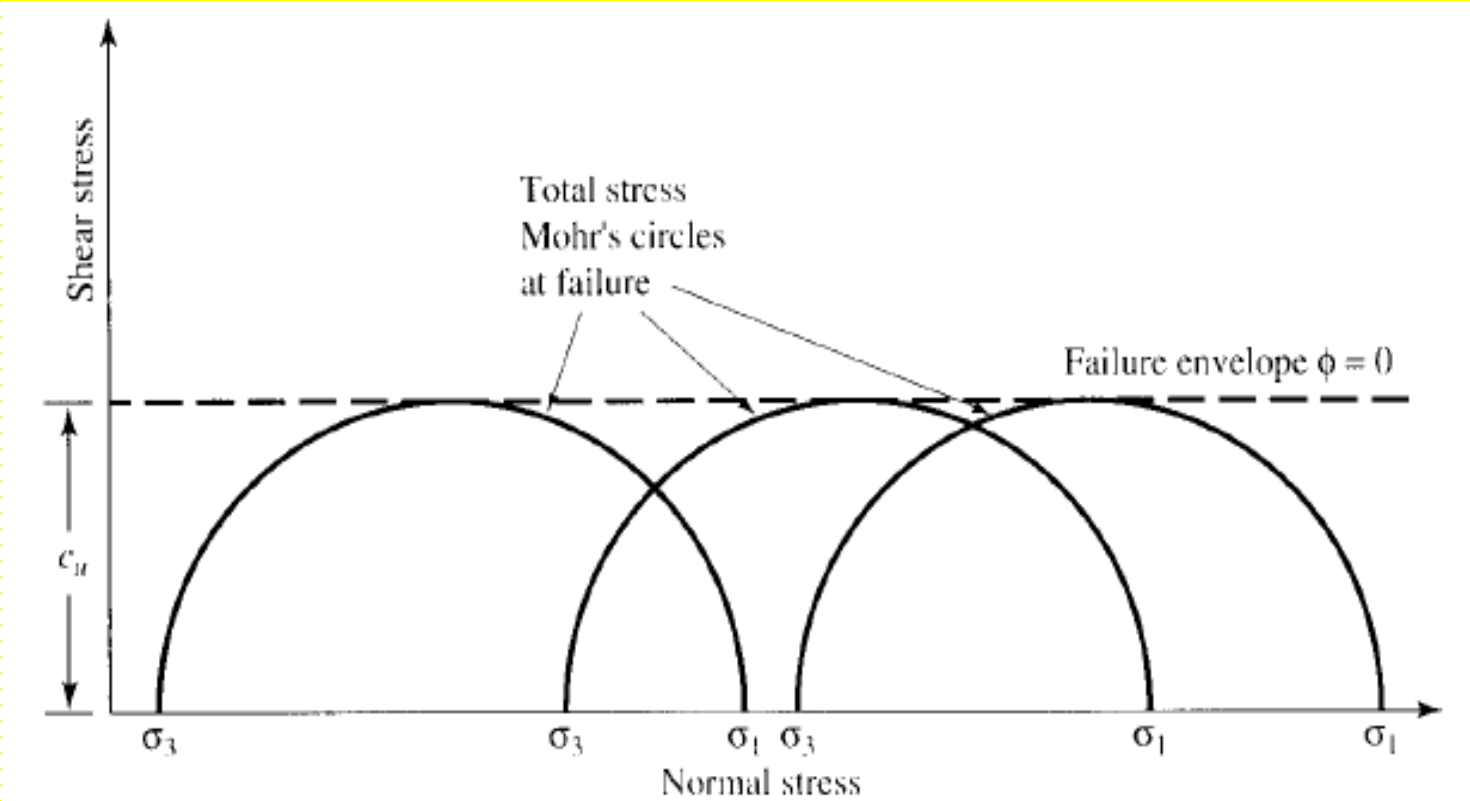
Consolidated Undrained (CU)

✓ - Failure that occurs rapidly after the soil has had time to consolidate.

Consolidated Drained (CD)

✓ - Failure that occurs slowly after the soil has had **time to consolidate.**

Mohr circle for ϕ=0 soil UU test



$$\tau_f = c = c_u$$

Unconfined Compression (UC) Test

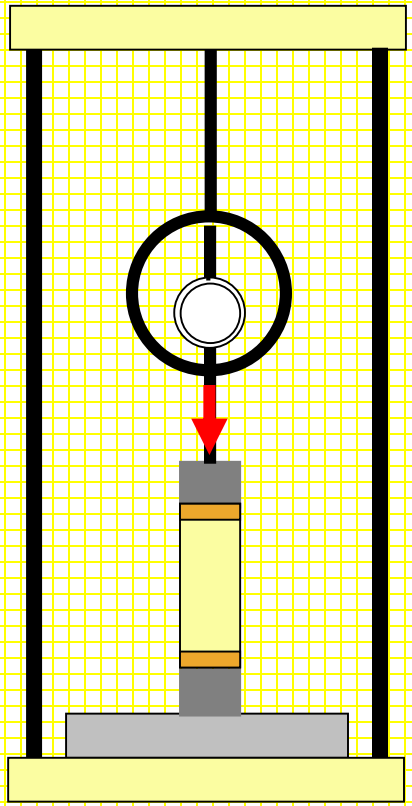
- ✓ The **UC test** is the simplest and quickest test used to determine the shear strength of a **cohesive soil**.
- ✓ An undisturbed or remolded sample of **cylindrical shape**, about 38 mm in diameter and 76 mm in height is subjected to **uniaxial compression** until the soil fails.
- ✓ Since the sample is **laterally unconfined**, only cohesive soils can be tested. The sample is tested quickly and there is **no drainage**.
- ✓ Therefore, it is **a special case of the UU test in which σ_3=0**.

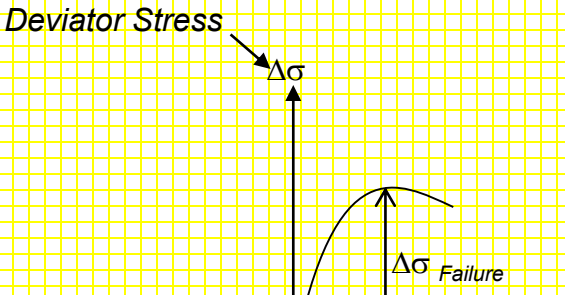


Unconfined Compression Test…

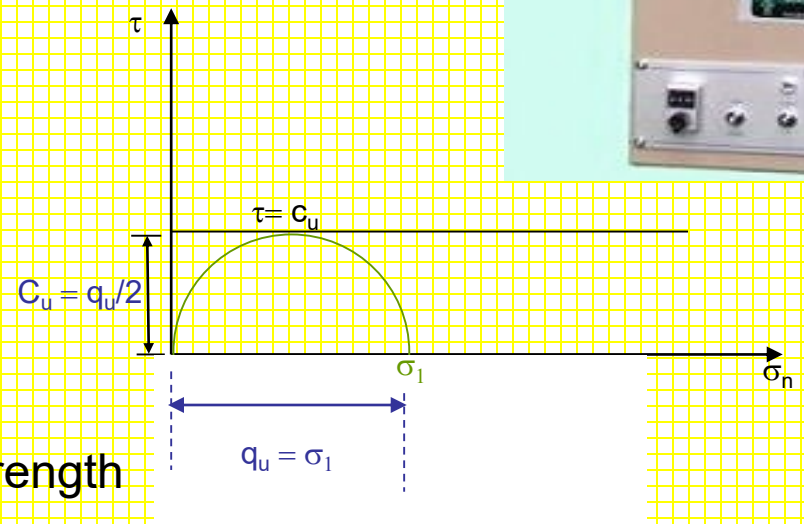
- ✓ The specimen is **not placed in the cell.**
- ✓ Specimen is open to **air with a σ_3 of 0.**
- ✓ Test is similar to **concrete compression test**, except with soil (**cohesive** – why?).
- ✓ Applicable in most practical situations – **foundations**
- ✓ Drawing Mohrs circle with **σ_3 at 0** and the failure (normal) stress σ_3 defining the 2nd point of the circle – often called **q_u** in this special case
- ✓ **c** becomes **$\frac{1}{2}$ q_u** of the failure stress

Unconfined Compression Test





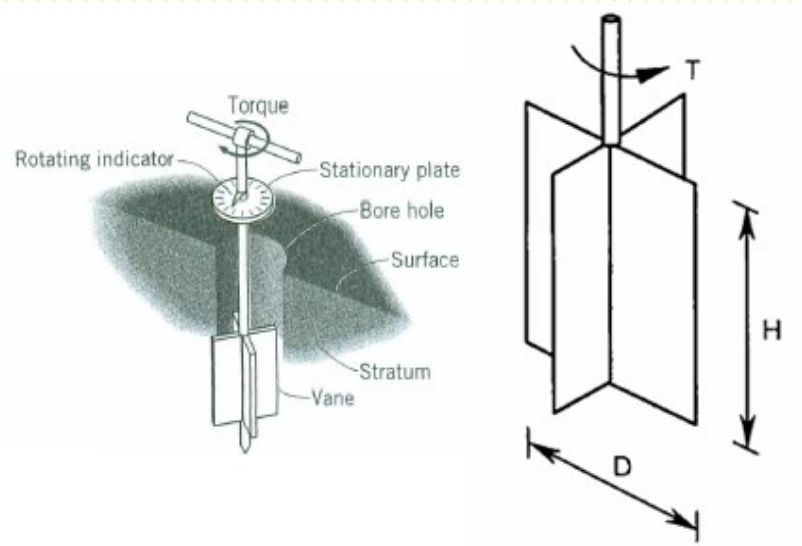




q_u = Unconfined compressive strength
C_u = Unconfined shear strength

Field Tests for shear

- ✓ **Sampling disturbances** and sample preparation for laboratory tests significantly affect the **shear strength parameters**.



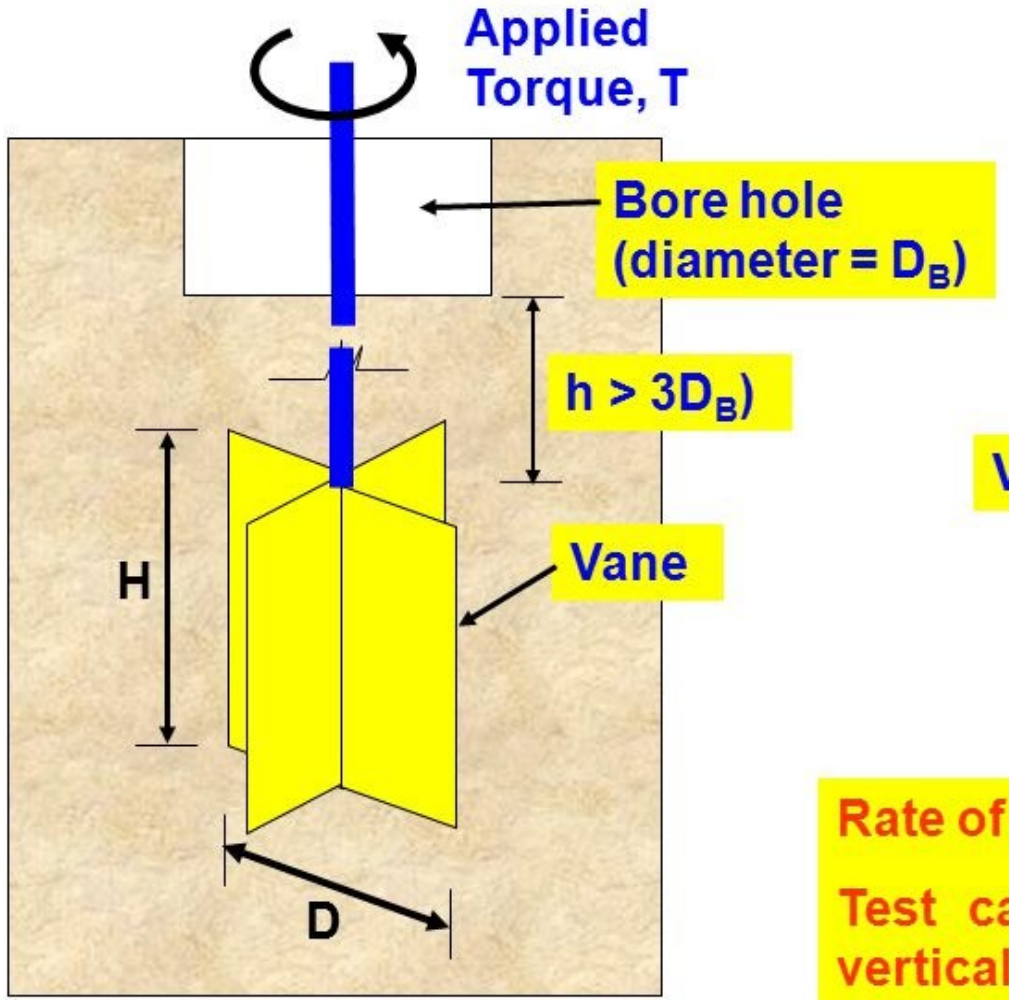
1. Shear Vane

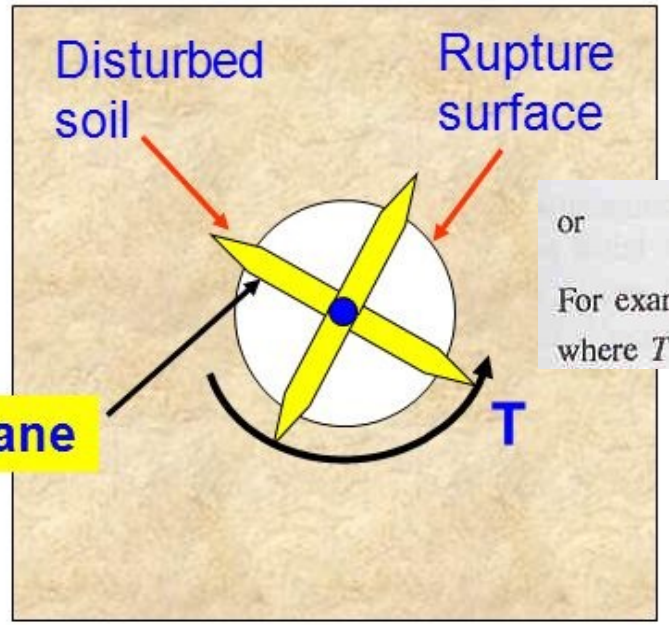
- ✓ In ***soft and saturated clays***, where undisturbed specimen is difficult to obtain.
- ✓ The undrained shear strength is measured using a **shear vane test**. A diagrammatic view of the shear vane apparatus.
- ✓ It consists of four thin metal blades welded orthogonally (90^0) to a rod where the height ***H*** **is twice the diameter** ***D*** .
- ✓ Commonly used diameters are **38, 50 and 75** mm.

Vane shear test

This is one of the most versatile and widely used devices used for investigating undrained shear strength (C_u) and sensitivity of soft clays

$$S_u = \frac{T}{\pi D^2 (H/2 + D/6)}$$





PLAN VIEW

or $$s = \frac{T}{\pi (D^2 H/2 + D^3/6)}$$

For example, if D = 1.2 cm, and H = 2.4 cm, $s = 0.158\ T$

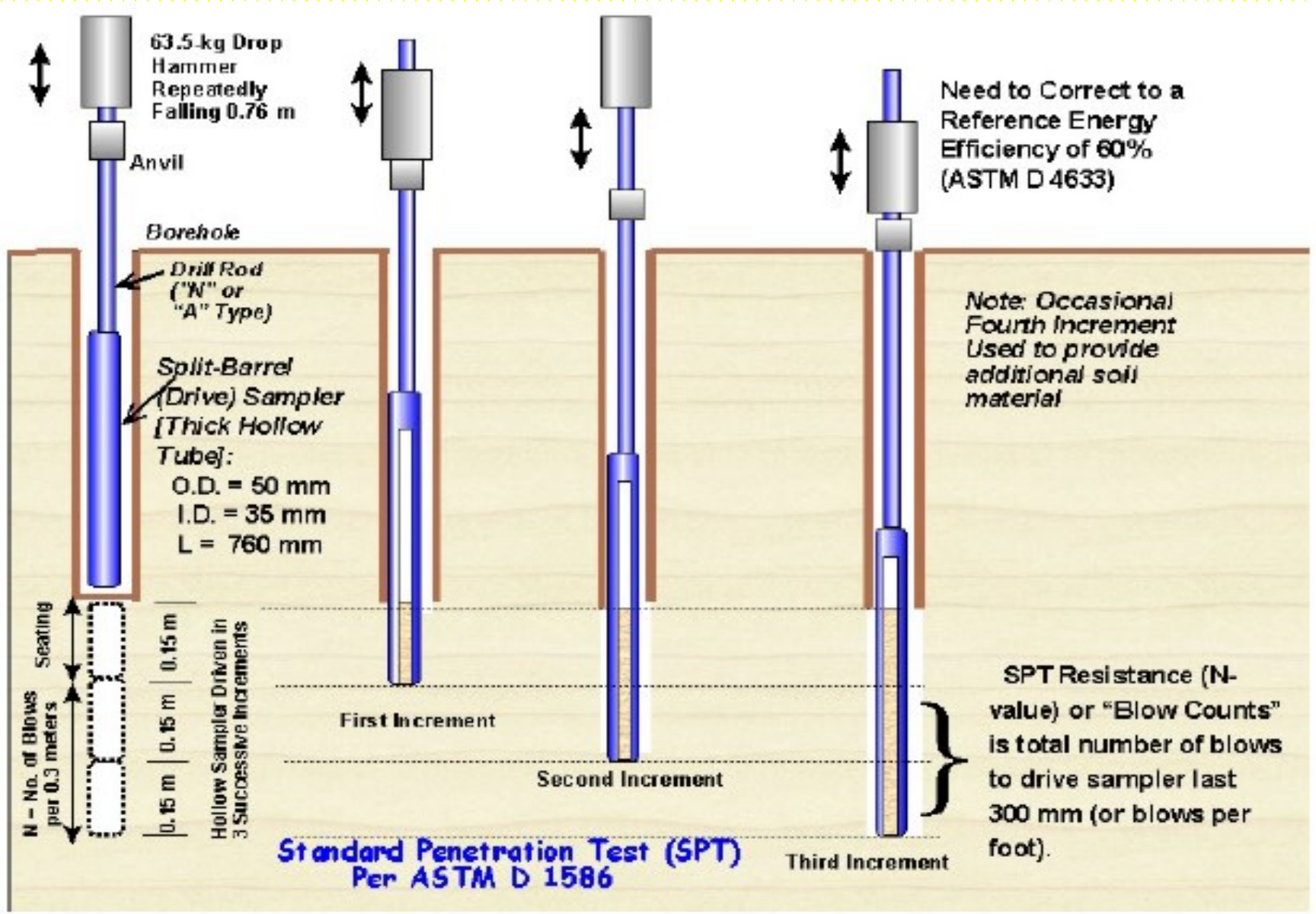
where T is in N-cm and s in N/cm^2.

Rate of rotation : 6^0 – 12^0 per minute

Test can be conducted at 0.5 m vertical intervals

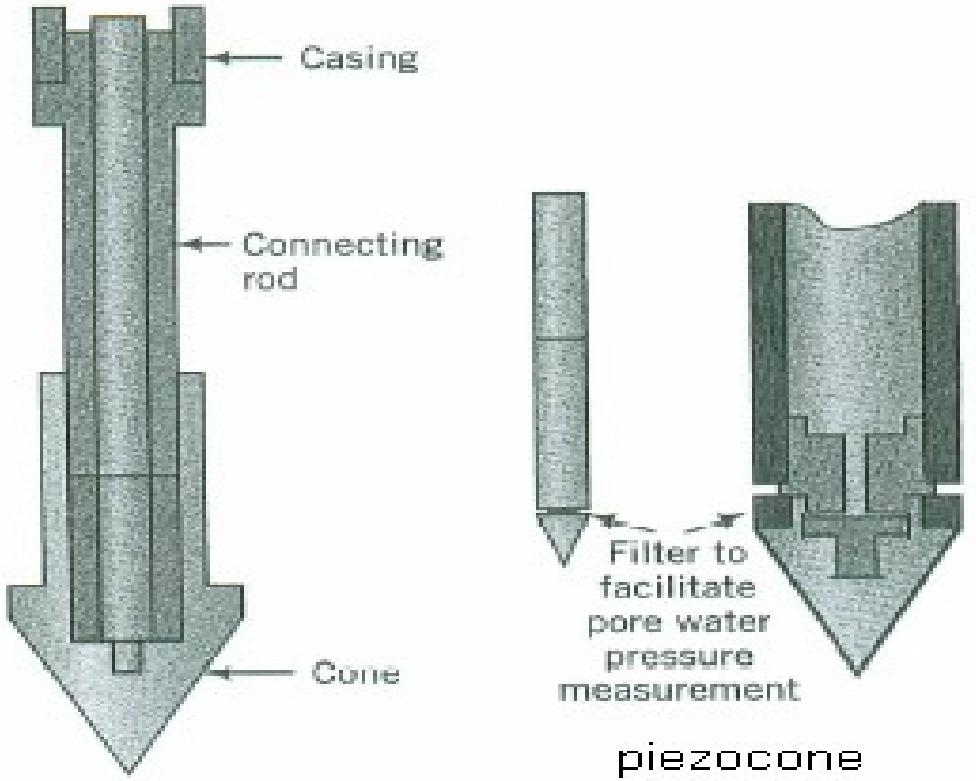
Standard Penetration Test (SPT)

- ✓ Most popular field test performed ***mostly*** ***in coarse grained*** ***(or cohesionless) soils***.
- ✓ The SPT is performed by driving a **standard split spoon sampler** into the ground by blows from a drop hammer of mass **64 kg falling 760 mm.**
- ✓ The sampler is driven **150 mm** into the soil at the bottom of a borehole, and the number of blows (N) required to drive it an additional **300 mm** is counted.
- ✓ The ***number of blows N*** is called the standard penetration number.


63.5-kg Drop Hammer Repeatedly Falling 0.76 m
Anvil
Borehole
Drill Rod ("N" or "A" Type)
Split-Barrel (Drive) Sampler [Thick Hollow Tube]:
O.D. = 50 mm
I.D. = 35 mm
L = 760 mm
Need to Correct to a Reference Energy Efficiency of 60% (ASTM D 4633)
Note: Occasional Fourth Increment Used to provide additional soil material
N = No. of Blows per 0.3 meters
Seating
0.15 m
0.15 m
0.15 m
Hollow Sampler Driven in 3 Successive Increments
First Increment
Second Increment
Third Increment
SPT Resistance (N-value) or "Blow Counts" is total number of blows to drive sampler last 300 mm (or blows per foot).
Standard Penetration Test (SPT)
Per ASTM D 1586

Cone Penetration Test (CPT)

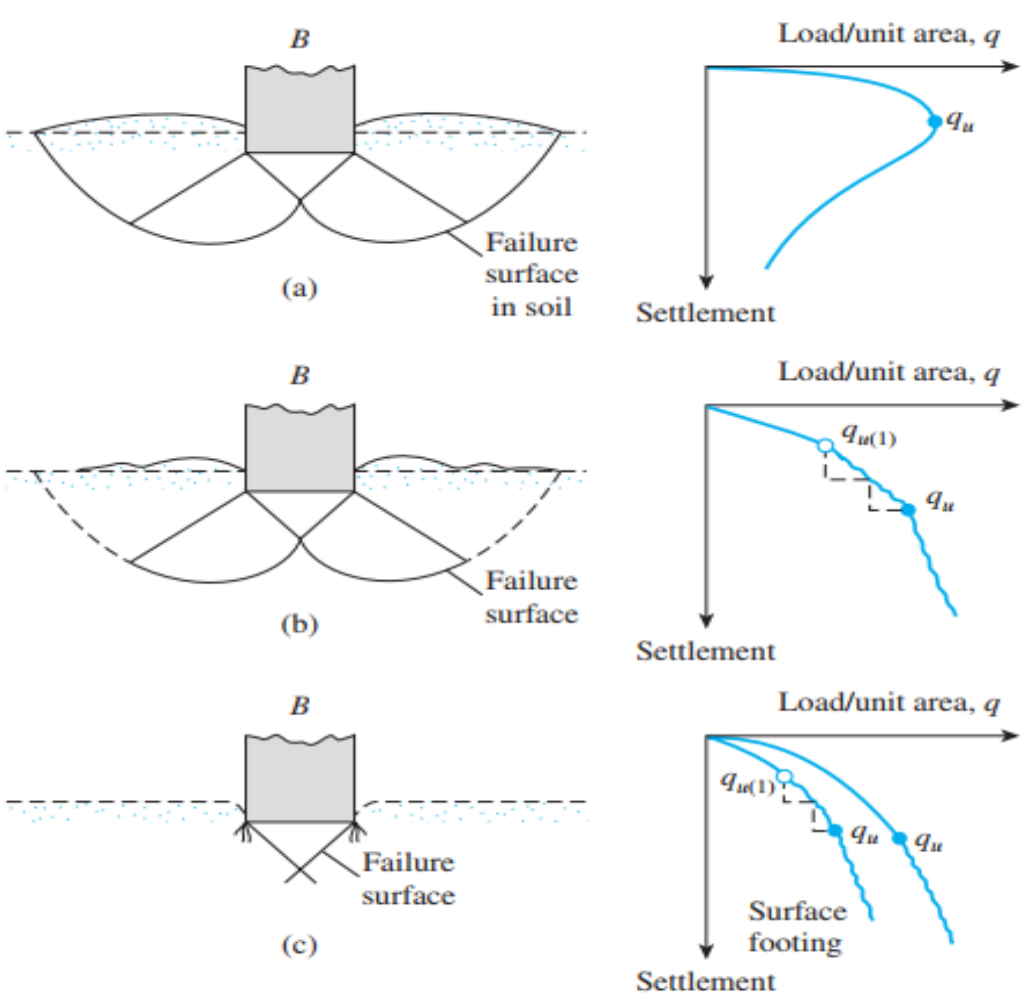
- ✓ **The Cone Penetrometer Test (CPT)** is an in situ test used for subsurface exploration in **fine and medium sands, soft silts and clays.**
- ✓ The apparatus consists of a cone with a 35.7 mm end diameter, projected area of 1000 mm^2 and 60^0 point angle that is attached to a rod. An outer sleeve encloses the rod.



BEARING CAPACITY OF SOILS

Bearing Failure Modes

- According to Vesic (1973),there are three types of shear/bearing failure modes. There are:
  - → General Shear Failure
  - → Local Shear Failure
  - → Punching Shear Failure



General Shear failure

- Usually associated with soils of low compressibility soils such as dense sand and stiff cohesive soils.
- Load is gradually applied to the foundation, settlement will increase.
- At a certain point when the applied load per unit area equals to the ultimate load, q_u a sudden failure in the soil supporting the foundation takes place.
- The failure surface in the soil will extend to the ground surface and full shear resistance of the soil is developed along the failure surface.
- Bulging of the soil near the footing is usually apparent.
- Shear failure is accompanied by both tilting and settlement of footing.

Local shear failure

- Is common in sands and clays of medium compaction.
- The failure surface will gradually extend outward from the foundation but will not reach the ground surface.
- The shear resistance is fully developed over one part of the failure surface.
- There is a certain degree of bulging of soil.
- There is significant compression of soils.
- The failure surface is not clearly defined.

Punching shear failure

- Is common in loose and very compressible soils.
- Occur due to the compression of soil under the footing.
- There is no heaving/bulging of soil.
- Considerable vertical settlement may take place with the failure surface restricted to vertical planes immediately adjacent to the side of the foundation.
- Large settlement is occurred.
- The failure surface not clearly defined.

Ultimate Bearing Capacity Equations

- First proposed by Prandtl, 1920's based on the failure mechanism by assuming:

✓ Smooth footing

✓ Ignoring the weight of the soil in the failure zone.

- Terzaghi (1946), improve the Prandtl assumption by improving:

✓ Roughness of the footing

✓ The weight of the soil above the footing

Terzaghi's bearing capacity equation

- **Assumptions**

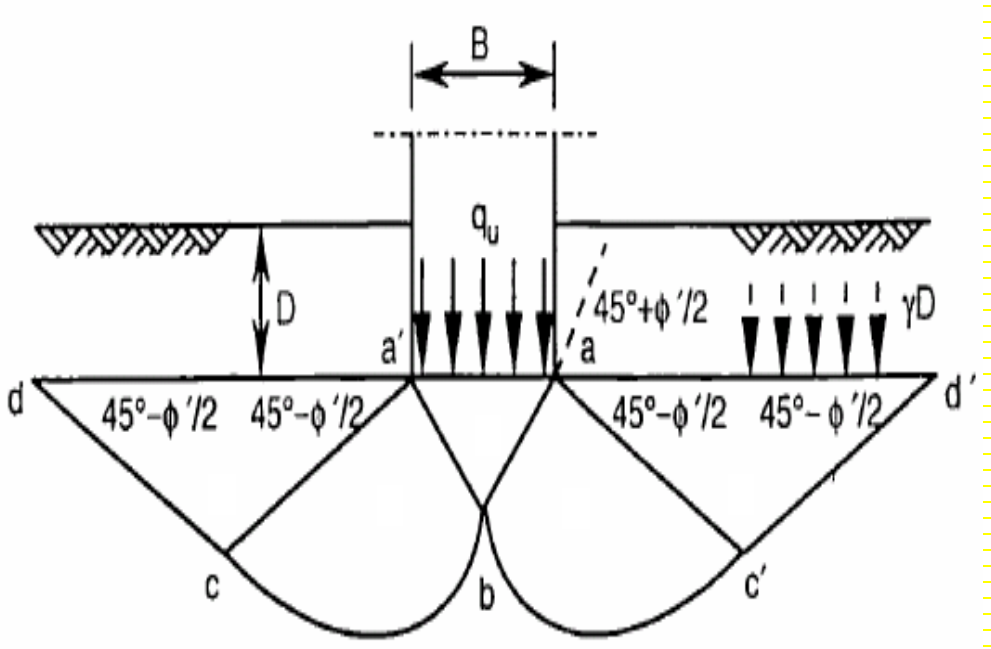
✓ General shear failure.

✓ Base of a footing is rough.

✓ Shallow foundation.

✓ Soil is homogeneous and isotropic.

✓ shear strength of soil is represented by Mohr-Coulombs Criteria.



$$q_u = c'N_c + \gamma D N_q + 0.5B\gamma N_\gamma$$

Terzaghi's BC Equation for Strip (or long) footing

Cont…

- **Square footing:** $q_u = 1.3c'N_c + \gamma D N_q + 0.4B\gamma N_\gamma$
- **Circular footing:** $q_u = 1.3c'N_c + \gamma D N_q + 0.3B\gamma N_\gamma$

Where: Nc, Nq and Nγ are called the bearing capacity factors.

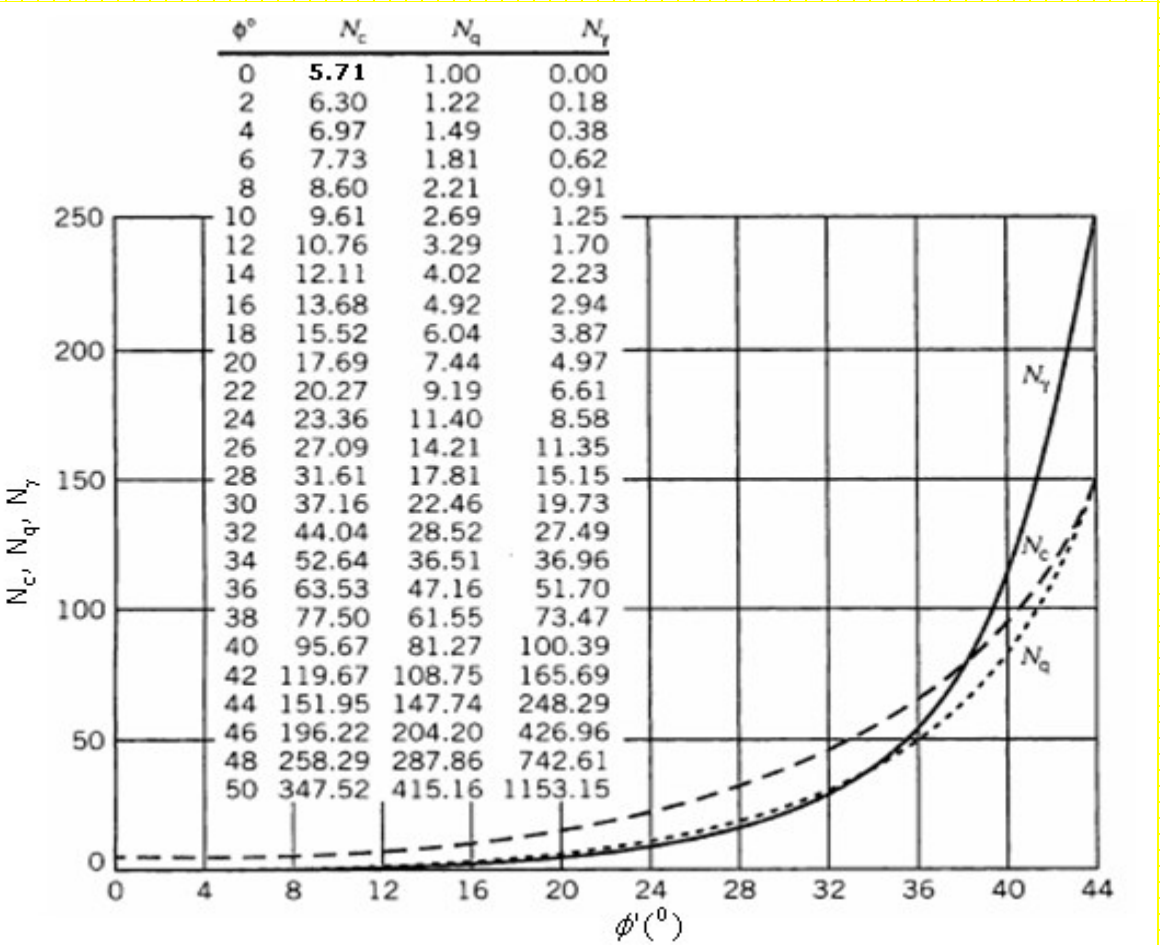
$$N_q = \frac{e^{(3\pi/2-\varphi')\tan\varphi'}}{2\cos^2(45+\varphi'/2)}$$

$$N_\gamma = \frac{1}{2}\tan\phi'\left(\frac{K_{p\gamma}}{\cos^2\phi'} - 1\right)$$

$$N_c = \cot\phi'(N_q - 1)$$

$$K_{p\gamma} = (8\phi'^2 - 4\phi' + 3.8)\tan^2(60^0 + \phi'/2)$$

ϕ^0	N_c	N_q	N_γ
0	5.71	1.00	0.00
2	6.30	1.22	0.18
4	6.97	1.49	0.38
6	7.73	1.81	0.62
8	8.60	2.21	0.91
10	9.61	2.69	1.25
12	10.76	3.29	1.70
14	12.11	4.02	2.23
16	13.68	4.92	2.94
18	15.52	6.04	3.87
20	17.69	7.44	4.97
22	20.27	9.19	6.61
24	23.36	11.40	8.58
26	27.09	14.21	11.35
28	31.61	17.81	15.15
30	37.16	22.46	19.73
32	44.04	28.52	27.49
34	52.64	36.51	36.96
36	63.53	47.16	51.70
38	77.50	61.55	73.47
40	95.67	81.27	100.39
42	119.67	108.75	165.69
44	151.95	147.74	248.29
46	196.22	204.20	426.96
48	258.29	287.86	742.61
50	347.52	415.16	1153.15



Meyerhof's Bearing Capacity equation

- Developed a bearing capacity equation by extending Terzaghi's.
- Taking into account the effects of *footing shape, load inclination and footing depth* by adding the corresponding factors of *s, d, and i*.
- For a rectangular footing of L by B (L > B) and inclined load,

$$q_u = c'N_c s_c i_c d_c + \gamma D N_q s_q i_q d_q + 0.5 B \gamma N_\gamma s_\gamma i_\gamma d_\gamma$$

$$N_q = \exp(\pi \tan\phi') \tan^2(45 + \phi'/2)$$

$$N_c = \cot\phi'(N_q - 1)$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi')$$

In the undrained conditions (c_u and Φ=0)

$$N_q = 1$$

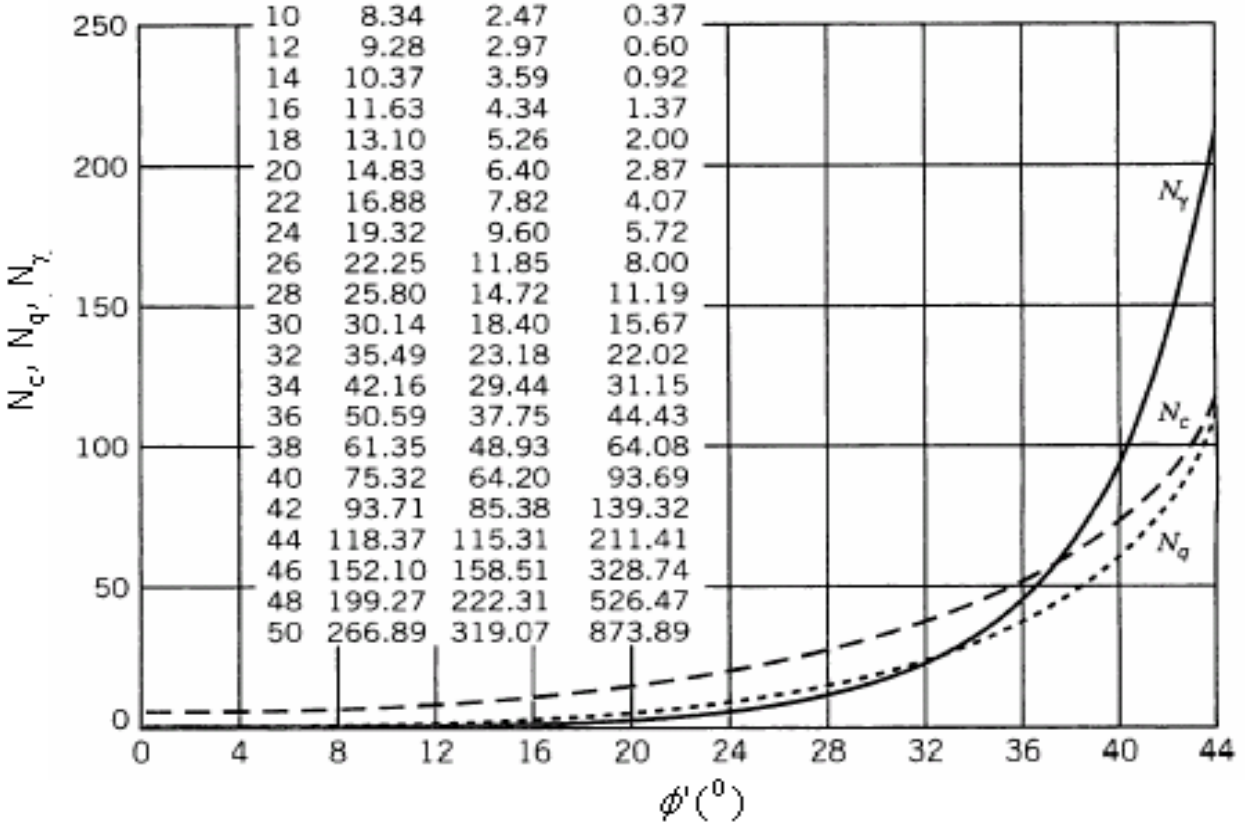
$$N_c = (\pi + 2) = 5.71$$

$$N_\gamma = 0$$

Cont…

Meyerhof's bearing capacity factors

ϕ°	N_c	N_q	N_γ
0	5.14	1.00	0.00
2	5.63	1.20	0.01
4	6.19	1.43	0.04
6	6.81	1.72	0.11
8	7.53	2.06	0.21
10	8.34	2.47	0.37
12	9.28	2.97	0.60
14	10.37	3.59	0.92
16	11.63	4.34	1.37
18	13.10	5.26	2.00
20	14.83	6.40	2.87
22	16.88	7.82	4.07
24	19.32	9.60	5.72
26	22.25	11.85	8.00
28	25.80	14.72	11.19
30	30.14	18.40	15.67
32	35.49	23.18	22.02
34	42.16	29.44	31.15
36	50.59	37.75	44.43
38	61.35	48.93	64.08
40	75.32	64.20	93.69
42	93.71	85.38	139.32
44	118.37	115.31	211.41
46	152.10	158.51	328.74
48	199.27	222.31	526.47
50	266.89	319.07	873.89



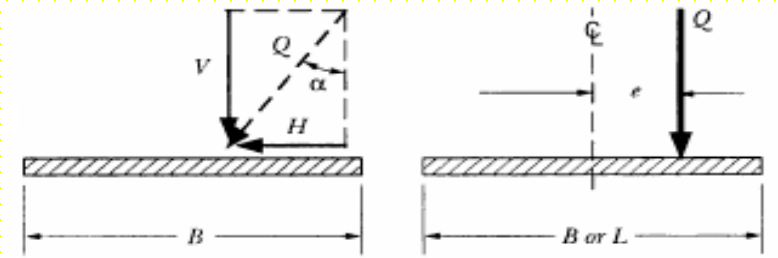
Cont…

Meyerhof's shape, inclination and depth factors

	Shape	Depth	Inclination
Any ϕ'	$s_c = 1 + 0.2K_p \frac{B}{L}$	$d_c = 1 + 0.2\sqrt{K_p}\frac{D}{B}$	$i_c = i_q = \left(1 - \frac{\alpha^0}{90^0}\right)^2$
For $\phi' = 0$	$s_q = s_\gamma = 1$	$d_q = d_\gamma = 1$	$i_\gamma = 0$
For $\phi' \geq 10^0$	$s_q = s_\gamma = 1 + 0.1K_p \frac{B}{L}$	$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B}$	$i_\gamma = \left(1 - \frac{\alpha^0}{\phi'^0}\right)^2$

$K_p = \tan^2\left(45 + \frac{\phi'}{2}\right)$, α =angle of resultant measured from vertical axis.

when triaxial ϕ' is used for plane strain, adjust ϕ' to obtain $\phi' = \left(1.1 - 0.1\frac{B}{L}\right)\phi'_{triaxial}$



For the eccentric load, the length and width of the footing rectangle are modified to: $L' = L - 2e_L$ and $B' = B - 2e_B$ where e_L and e_B represent the eccentricity along the appropriate directions.

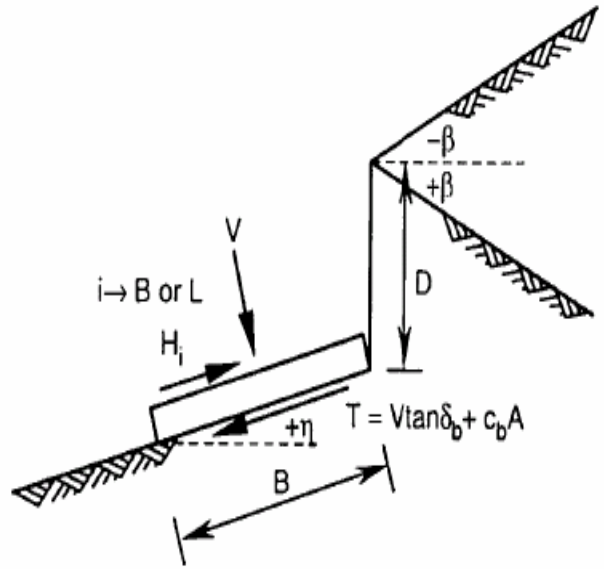
Hansen's Bearing Capacity equation

- Extended Meyerhof's solutions by considering the effects of sloping ground surface and tilted base as well as modification of $N\gamma$ and other factors.
- For a rectangular footing of L by B (L > B) and inclined ground surface, base and load,

$$q_u = c'N_c s_c d_c i_c b_c g_c + \gamma D N_q s_q d_q i_q b_q g_q + 0.5B\gamma N_\gamma s_\gamma d_\gamma i_\gamma b_\gamma g_\gamma$$

- For the case of a horizontal ground surface

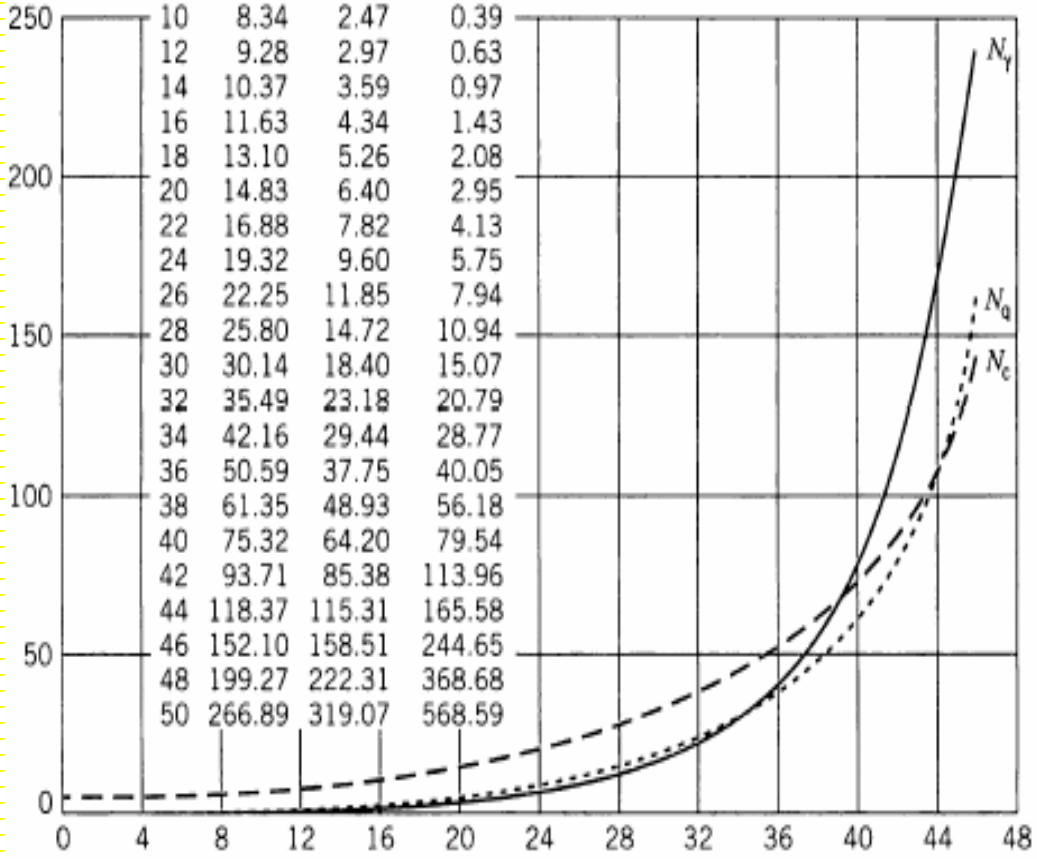
$$q_u = c'N_c s_c d_c i_c b_c + \gamma D N_q s_q d_q i_q b_q + 0.5B\gamma N_\gamma s_\gamma d_\gamma i_\gamma b_\gamma$$



Cont…

- **Meyerhof's bearing capacity factors**

ϕ°	N_c	N_q	N_γ
0	5.14	1.00	0.00
2	5.63	1.20	0.01
4	6.19	1.43	0.05
6	6.81	1.72	0.11
8	7.53	2.06	0.22
10	8.34	2.47	0.39
12	9.28	2.97	0.63
14	10.37	3.59	0.97
16	11.63	4.34	1.43
18	13.10	5.26	2.08
20	14.83	6.40	2.95
22	16.88	7.82	4.13
24	19.32	9.60	5.75
26	22.25	11.85	7.94
28	25.80	14.72	10.94
30	30.14	18.40	15.07
32	35.49	23.18	20.79
34	42.16	29.44	28.77
36	50.59	37.75	40.05
38	61.35	48.93	56.18
40	75.32	64.20	79.54
42	93.71	85.38	113.96
44	118.37	115.31	165.58
46	152.10	158.51	244.65
48	199.27	222.31	368.68
50	266.89	319.07	568.59



- ✓ The bearing capacity factors N_c and N_q are identical with Meyerhof's factors.
- ✓ N_γ is defined by:

$$N_\gamma = 1.5(N_q - 1)\tan\phi$$

Cont…

- Since failure can take place either along the long side or along the short side, Hansen proposed two sets of shape, inclination and depth factors.

✓ **The shape factors**

$$s_{c,B} = 1 + \frac{N_q}{N_c} \cdot \frac{B}{L} i_{c,B} \qquad s_{q,B} = 1 + \frac{B}{L} i_{q,B} \cdot \sin\phi' \qquad s_{\gamma,B} = 1 - 0.4 \frac{B}{L} i_{\gamma,B} \geq 0.6$$

$$s_{c,L} = 1 + \frac{N_q}{N_c} \cdot \frac{L}{B} i_{c,L} \qquad s_{q,L} = 1 + \frac{L}{B} i_{q,L} \cdot \sin\phi' \qquad s_{\gamma,L} = 1 - 0.4 \frac{L}{B} i_{\gamma,L} \geq 0.6$$

For c, ϕ_u=0 soil: $s_{c,B} = 0.2 \frac{B}{L} i_{c,B}$ $\qquad$ $s_{c,L} = 0.2 \frac{L}{B} i_{c,L}$

Cont...

✓ Inclination factors

$$i_{c,i} = i_{q,i} - \frac{1 - i_{q,i}}{N_q - 1} \qquad i_{q,i} = \left(1 - \frac{0.5H_i}{V + Ac_b \cot\phi'}\right)^{\alpha_1} \qquad i_{\gamma,i} = \left(1 - \frac{0.7H_i}{V + Ac_b \cot\phi'}\right)^{\alpha_2}$$

For the tilted base $i_{\gamma,i} = \left[1 - \frac{(0.7 - \eta^0/450^0)H_i}{V + Ac_b \cot\phi'}\right]^{\alpha_2}$

✓ Depth factors

→ ***For D/B≤1 and D/L≤1***

$$d_{c,B} = 1 + 0.4 \cdot D/B \qquad d_{q,B} = 1 + 2\tan\phi'(1 - \sin\phi')^2 \cdot D/B$$

$$d_{c,L} = 1 + 0.4 \cdot D/L \qquad d_{q,L} = 1 + 2\tan\phi'(1 - \sin\phi')^2 \cdot D/L$$

→ ***For D/B>1 and D/L>1***

$$d_{c,B} = 1 + 0.4 \cdot \tan^{-1}(D/B) \qquad d_{q,B} = 1 + 2\tan\phi'(1 - \sin\phi')^2 \cdot \tan^{-1}(D/B)$$

$$d_{c,L} = 1 + 0.4 \cdot \tan^{-1}(D/L) \qquad d_{q,L} = 1 + 2\tan\varphi'(1 - \sin\varphi')^2 \cdot \tan^{-1}(D/L)$$

→ ***For both sets,*** $d_\gamma = 1$

Cont…

- For the sloping ground and tilted base, the ground factors g_i and base factors b_i are proposed by the following equations.
- The angles β and η are at the same plane, either parallel to B or L.

$$g_c = 1 - \beta^0 / 147^0 \qquad g_q = g_\gamma = (1 - 0.5 \tan\beta)^5$$

✓ **For c_u, ϕ_u soil:** $g_c = \beta^0 / 147^0$

$$b_c = 1 - \eta^0 / 147^0 \qquad b_\gamma = e^{-2.7\eta \tan\phi'} \qquad b_q = e^{-2\eta \tan\phi'}$$

✓ **For c_u, ϕ_u soil:** $b_c = \eta^0 / 147^0$

A comparative summary of the three bearing capacity equations

→ Practitioners use Terzaghi's equations for a very cohesive soil and D/B < 1.

→ Terzaghi's equations have the following major drawbacks:

- Shape, depth and inclination factors are not considered.
- Terzaghi's equations are suitable for a concentrically loaded horizontal footing
- The equations are generally conservative than Meyerhof's and Hansen's.

® Meyerhof's and Hansen's equations are more widely used.

® Are less conservative and applicable to more general conditions.

® Hansen's is, however, used when the base is tilted or when the footing is on a slope and for D/B > 1.

Effects of Groundwater Table on Bearing Capacity

Situation 1:

- Groundwater level at a depth B below the base of the footing.
- In this case no modification of the bearing capacity equations is required.

Situation 2:

- Groundwater level within a depth B below the base of the footing.
- In the groundwater level is at a below the depth z below the base, such that z<B, then the term γB is $\gamma z+\gamma'(B-z)$ *or* $\gamma_{sat} z+\gamma'(B-z)$.
- The later equation is used if the soil above the groundwater level is also saturated. γD term remain unchanged.

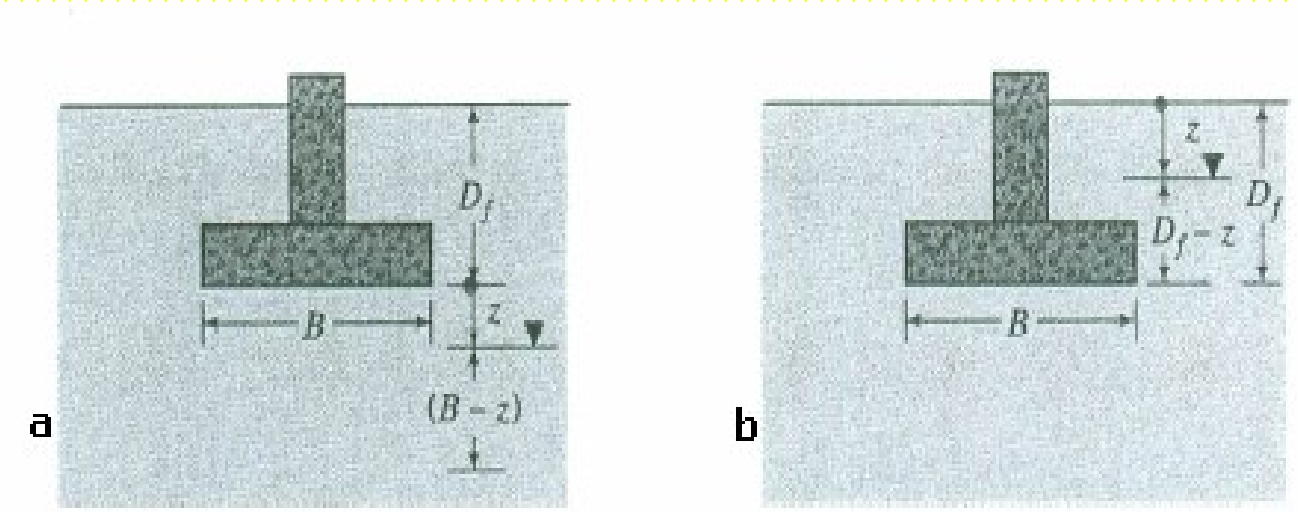
Cont…

Situation 3:

- Groundwater level within the embedment depth.
- If the groundwater is at a depth z within the embedment such that $z<D_f$ then the term γD_f is

$$\gamma z+\gamma'(D_f-z) \text{ or } \gamma_{sat} z+\gamma'(D_f-z).$$

- The later equation is used if the soil above the groundwater level is also saturated. *The term* γB becomes $\gamma' B$



Allowable bearing capacity and factor of safety

- Allowable bearing capacity, q_a is calculated by dividing the ultimate bearing capacity by a factor, called the factor of safety, FS.
- The FS is intended to compensate for assumptions made in developing the bearing capacity equations, soil variability, inaccurate soil data, and uncertainties of loads.
- The magnitude of FS applied to the ultimate bearing capacity may be between 2 and 3.

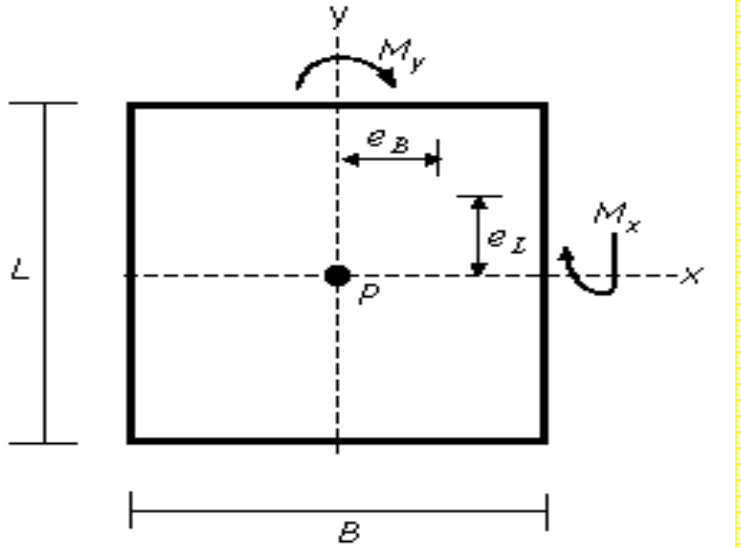
$$q_a = \frac{q_u}{FS}$$

$(\sigma_a)_{\max}$ is known and the dimension of the footing is also known

$$FS = \frac{q_u}{(\sigma_a)_{\max}}$$

Eccentric Loads

- Meyerhof, proposed an approximate method for loads that are located off-centered.



- He modified base area rectangular footing of width B and length L as:

$B' = B - 2e_B$ and $L' = L - 2e_L$

$$e_B = \frac{M_y}{P} \qquad e_L = \frac{M_x}{P}$$

Cont…

- The maximum and minimum vertical stresses along the x axis are:

$$\sigma_{max} = \frac{P}{BL}\left(1 + \frac{6e_B}{B}\right) \text{ and } \sigma_{min} = \frac{P}{BL}\left(1 - \frac{6e_B}{B}\right)$$

- The maximum and minimum vertical stresses along the y axis are:

$$\sigma_{max} = \frac{P}{BL}\left(1 + \frac{6e_L}{B}\right) \text{ and } \sigma_{min} = \frac{P}{BL}\left(1 - \frac{6e_L}{B}\right)$$

- ***σ_{min} should always be greater than zero!!!***

  → e_B & e_L should always be less than B/6 & L/6 respectively.

® ***The bearing capacity equations are modified for eccentric loads by replacing B with B'.***

Field Tests

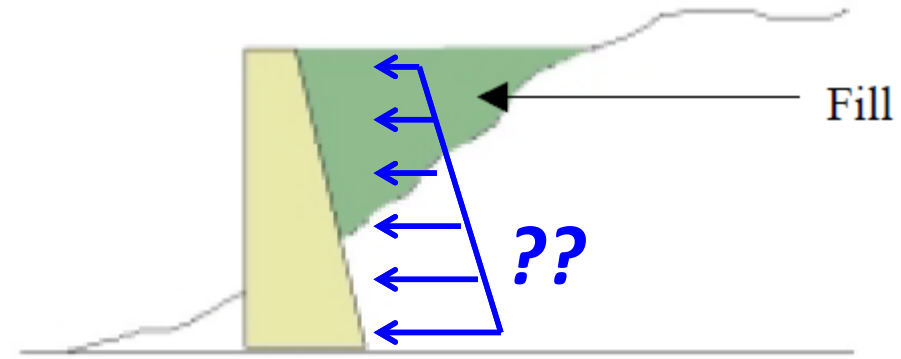
- Some of the most common methods used for field tests for the determination of ultimate bearing capacity of soils are:

→ Plate loading test

→ Standard penetration test(SPT)

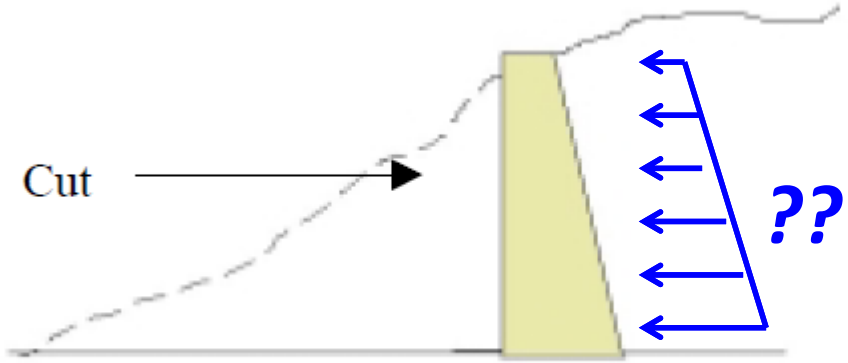
Lateral Earth Pressure

Lateral Earth Pressure



Retaining Wall to Support a Fill.





Retaining Wall to Support a Cut.



Lateral Earth Pressure ...

- LEP is a function of several factors, including the following:
  - The type & amount of wall movement
  - Type of backfill used
  - The effective unit weight of the backfill soil,
  - Ground water position
  - The drainage condition in the backfill
  - Surcharge application

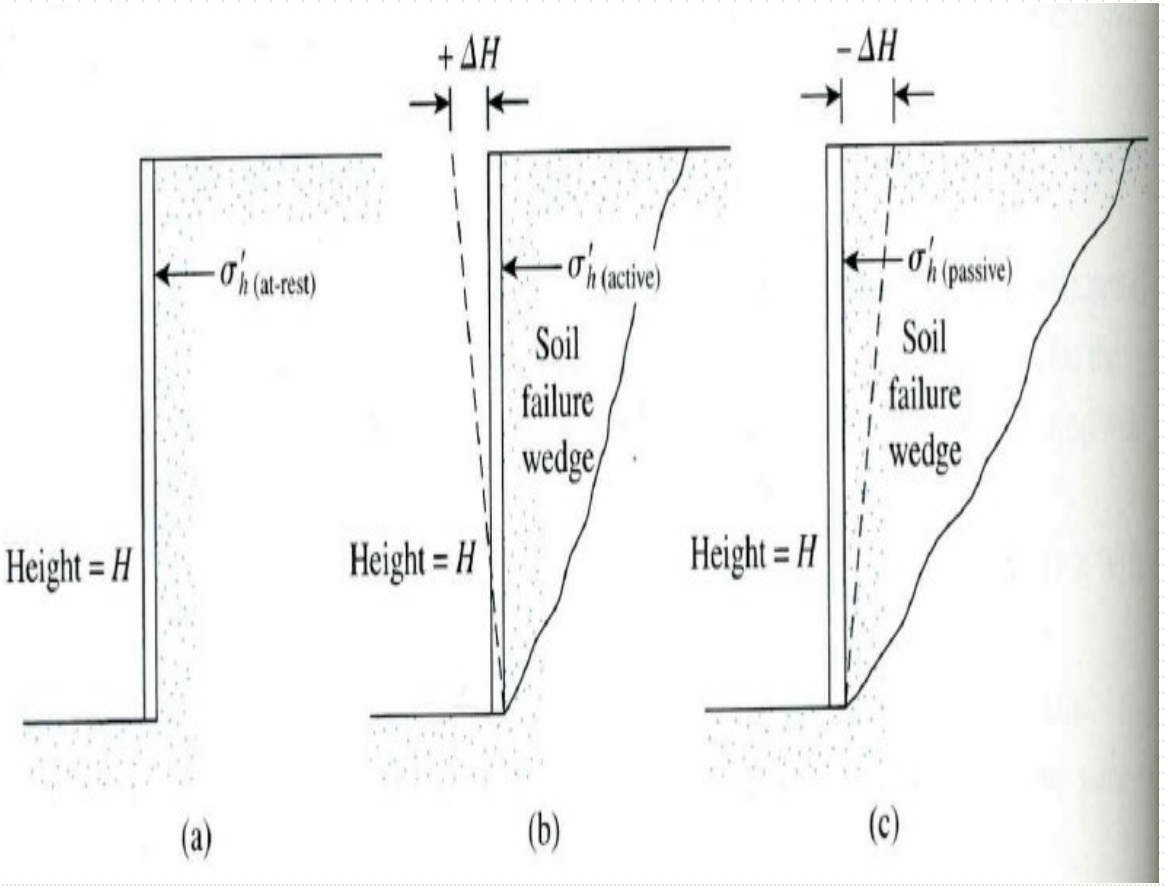
Lateral Earth Pressure…

Three Types of LEP

a. **PRESSURE AT REST**-If the wall **restrained/Fixed** from moving. – the LEP at any depth is referred to as ***earth pressure at-rest***

b. **ACTIVE EARTH PRESSURE**-If the wall tilts **away** from the retained soil – ***Active earth pressure.***

c. **PASSIVE EARTH PRESSURE**- If the wall is **pushed into** the retained soil – ***Passive earth pressure.***



I. LEP at-rest

- ✓ If the wall is at rest and is **not allowed to move** at all, either from the soil mass or into the soil mass (i.e., there is zero horizontal strain), the lateral pressure at a depth **Z** is:

But for ϕ soils;

$$\sigma_h = K_o \sigma_o^{'}$$

Without GWT

$$\sigma_h = K_o \sigma_o^{'} + u$$

If GWT is there

But for C- ϕ soils;

$$\sigma_{ho} = K_o \gamma H - 2\, c\, (K_o)^{1/2}$$

Where

u **= pore water pressure**

K_o **= Coefficient of at-rest earth pressure**

Rankine's Active Condition

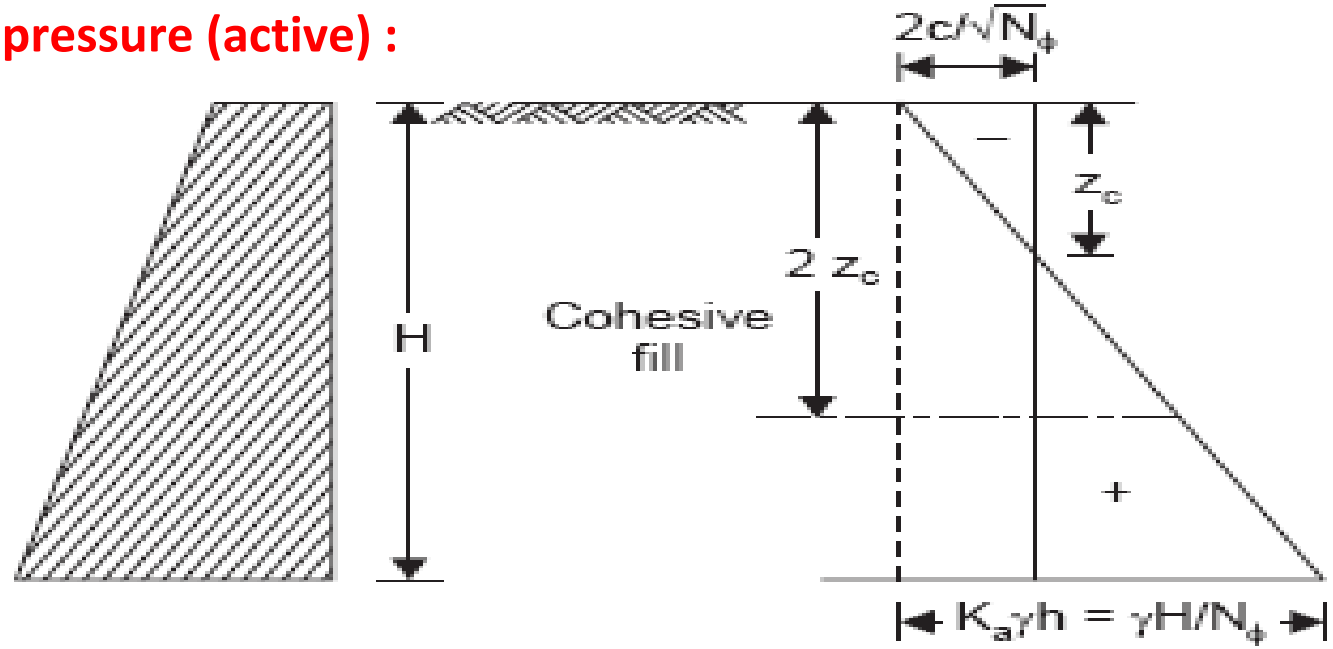
✓Consider a vertical **frictionless** (smooth) wall retaining a soil mass in both front and back of the wall; **vertical walls only.**

$$\sigma_a = K_a \gamma z \quad \text{for non-cohesive soil}$$

$$\sigma_a = \gamma z K_a - 2c\sqrt{K_a}$$

For cohesive backfill

✓ **For lateral earth pressure (active) :**



Rankine Active Earth Pressure

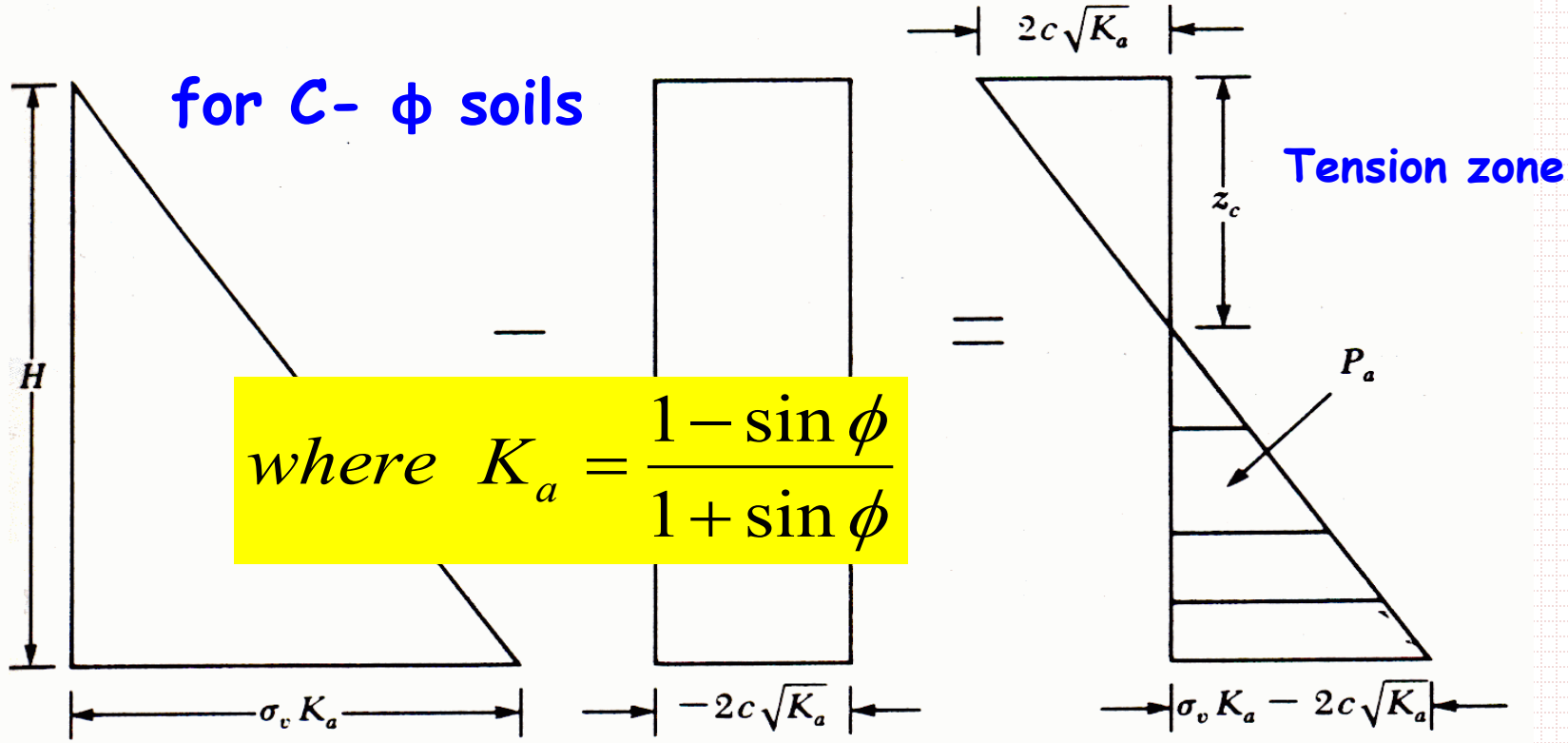
$\sigma_3 = \sigma_1 \cdot \tan^2 (45-\phi/2) - 2c.\tan (45-\phi/2)$ (From Mohr-coulomb theory)

$\sigma_a = \sigma_v \cdot \tan^2 (45-\phi/2) - 2c.\tan (45-\phi/2)$

$\sigma_a = \sigma_v \cdot Ka - 2c\sqrt{Ka}$

$Ka = \tan^2 (45 - \phi/2)$

for C- ϕ soils



$$where \quad K_a = \frac{1-\sin\phi}{1+\sin\phi}$$

Active Earth Pressure

Note :

$z = 0 \rightarrow \sigma_v = 0 \; ; \; \sigma_a = -2c\sqrt{K_a}$

$z = H \rightarrow \sigma_v = \gamma H$

✓ **The tensile stress decreases** with depth and becomes zero at a depth $z = z_c$ or

$$\gamma z_c K_a - 2c\sqrt{K_a} = 0$$

and

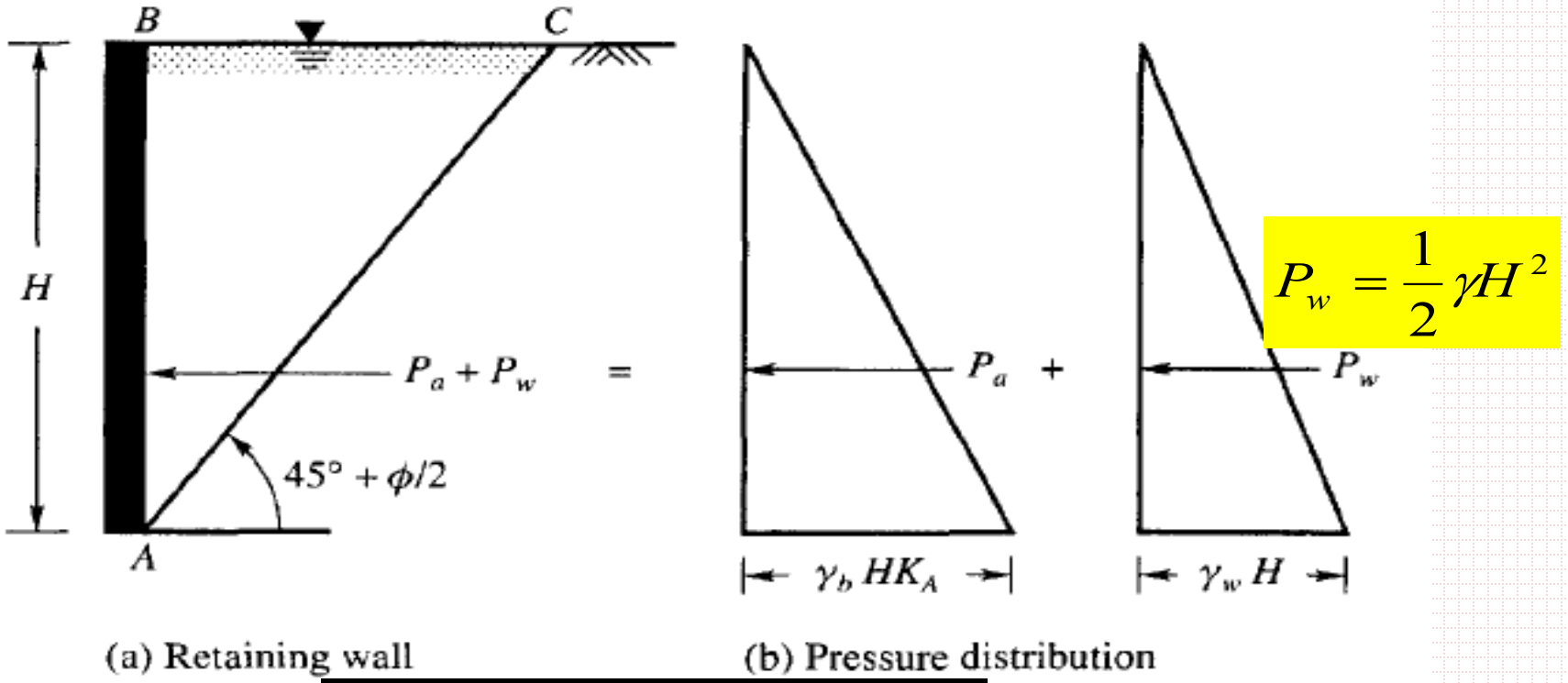
$$z_c = \frac{2c}{\gamma\sqrt{K_a}}$$

z_c = depth of tensile crack

$$z_o = \frac{2c_u}{\gamma}$$ **For undrained condition**

Active Earth Pressure- Backfill Soil Submerged

- ✓ If backfill is affected by GWT, its effect is considered during determination of LEP



(a) Retaining wall (b) Pressure distribution

$$P_w = \frac{1}{2}\gamma H^2$$

$$\sigma_w = \gamma H$$

$$\overline{P_a} = P_a + P_w = \frac{1}{2}\gamma_b H^2 K_A + \frac{1}{2}\gamma_w H^2$$

RANKIN'S PASSIVE CONDITION

✓ **For total passive pressure for non cohesive soils:**

$$\sigma_p = K_p \gamma H$$ for non-cohesive soil

$$P_p = \frac{1}{2} K_p \gamma H^2$$

✓ **For total passive pressure for c- ϕ soils :**

$$\sigma_p' = \sigma_z' \frac{1+\sin\varphi'}{1-\sin\varphi'} + 2c' \sqrt{\frac{1+\sin\varphi'}{1-\sin\varphi'}}$$

$$= \sigma_z' k_p + 2c' \sqrt{k_p}$$

$$k_p = \frac{1+\sin\phi'}{1-\sin\phi'} = \tan^2(45+\frac{\phi'}{2})$$

$K_a < K_0 < K_p$

$$k_p = \frac{1}{k_a}$$

Lateral Earth Pressure due to Surcharge

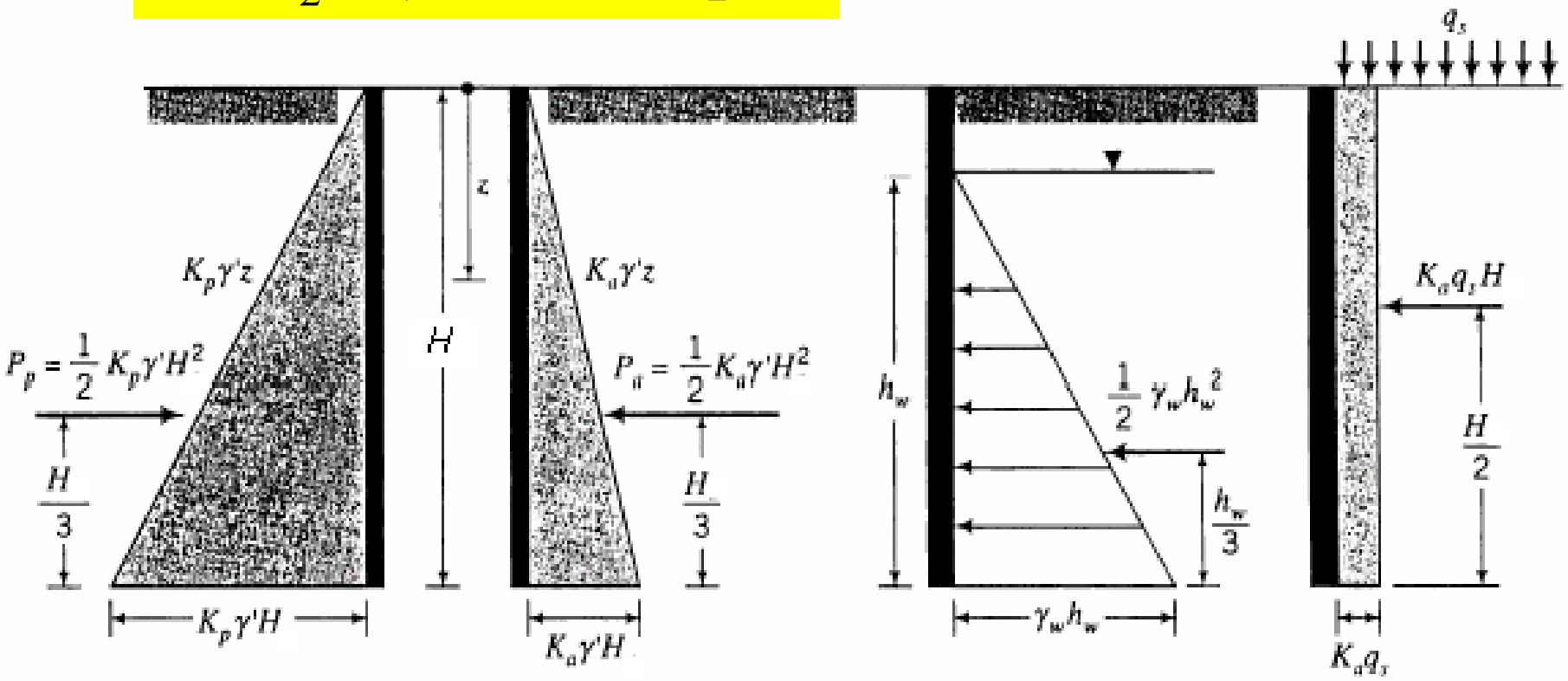
For ϕ soils;

$$\sigma_a' = k_a \gamma' z + k_a q_s$$

$$P_a = \frac{1}{2} k_a \gamma' H^2 + k_a q_s H$$

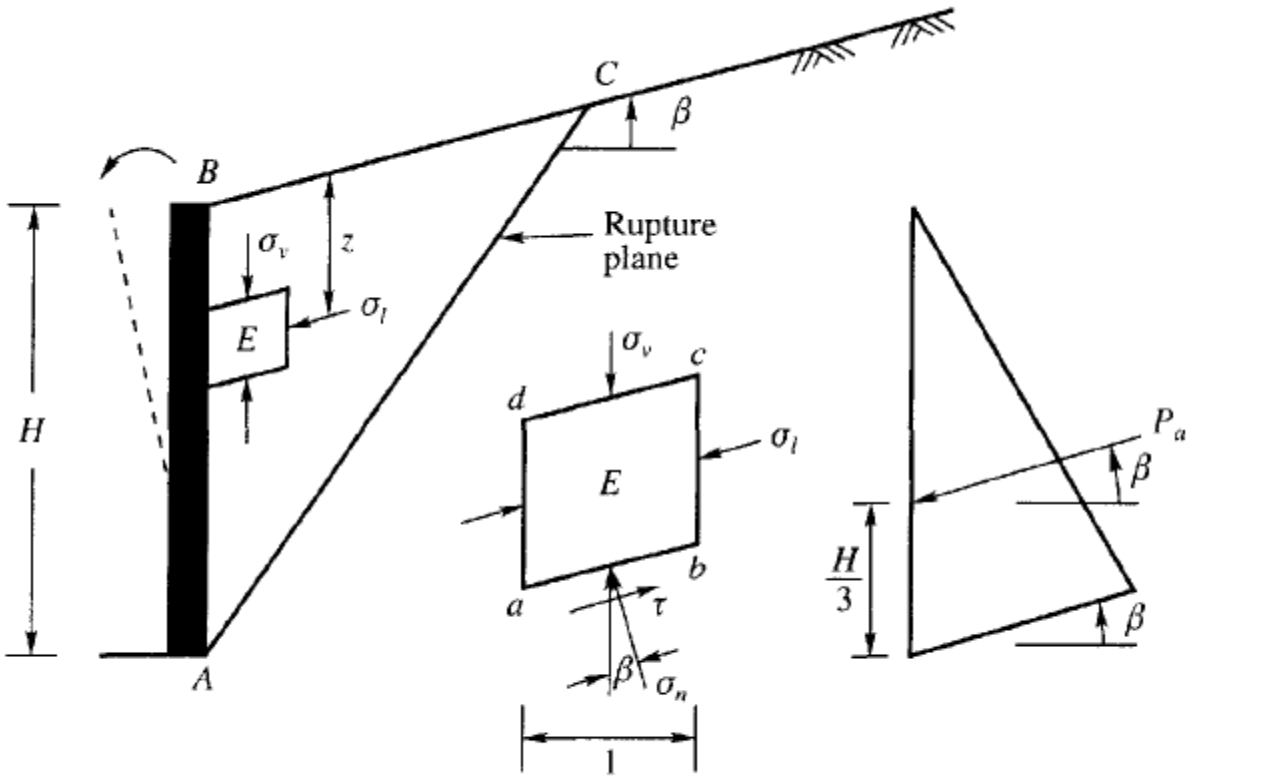
$$\sigma_p' = k_p \gamma' z + k_p q_s$$

$$P_p = \frac{1}{2} k_p \gamma' H^2 + k_p q_s H$$



(a) Passive (b) Active (c) Hydrostatic pressure (d) Surface stress

SLOPING SURFACE-ACTIVE EARTH PRESSURE

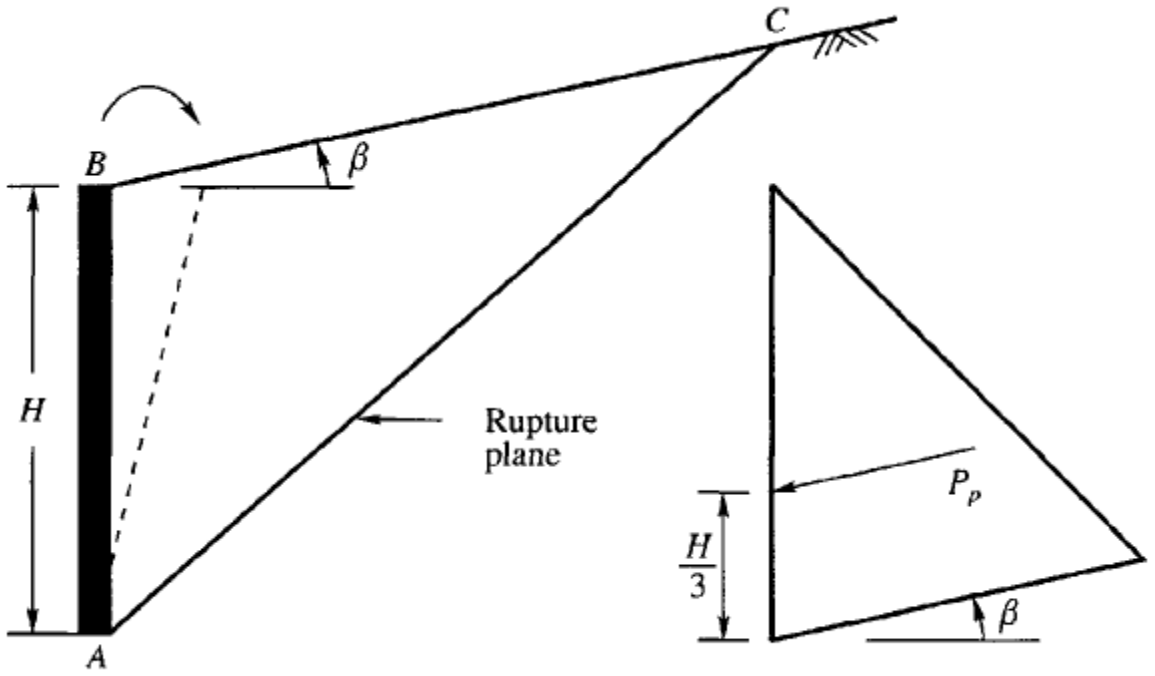


(a) Retaining wall (b) Pressure distribution

$$P_a = \frac{1}{2} K_a \gamma H^2$$

$$k_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi'}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi'}}$$

SLOPING SURFACE-PASSIVE EARTH PRESSURE



(a) Retaining wall (b) Pressure distribution

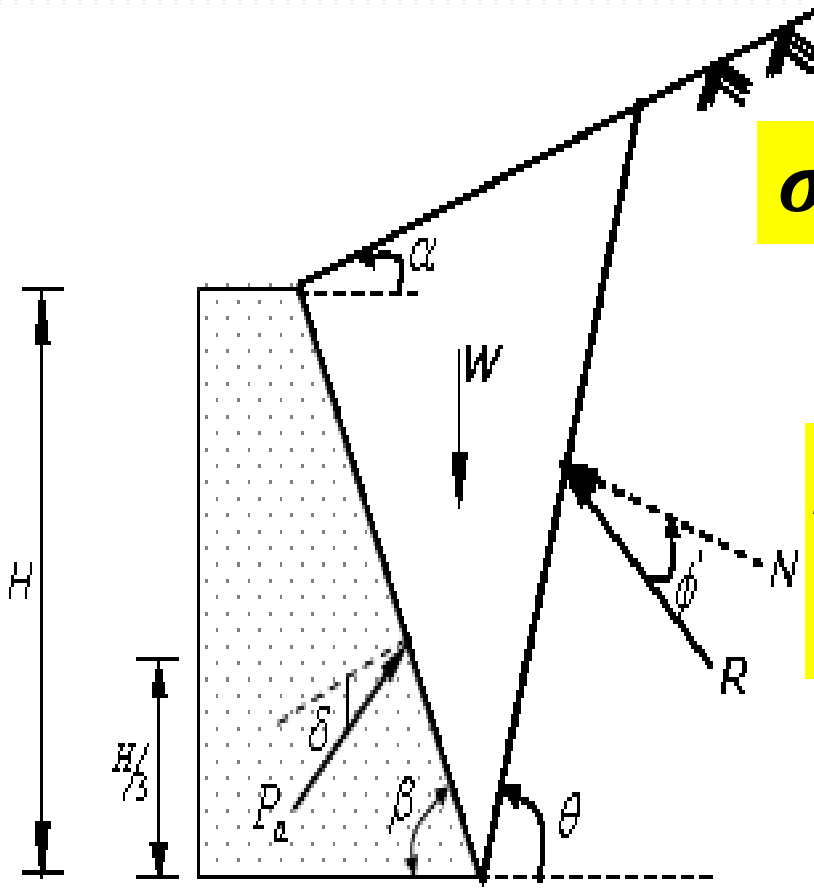
$$P_p = \frac{1}{2} K_p \gamma H^2$$

where, $$K_p = \cos\beta \times \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}$$

COULOMB'S EARTH PRESSURE THEORY

- Coulomb made the following assumptions in the development of his theory:
  - The soil is isotropic and homogeneous
  - The rupture surface is a plane surface
  - The failure wedge is a rigid body
  - The pressure surface is a plane surface
  - There is wall friction on the pressure surface
  - Failure is two-dimensional and
  - The soil is cohesionless

Coulomb's Active earth pressure at sloped fill



$$\sigma_a = K_{ac}\gamma H \text{ for non-cohesive soil}$$

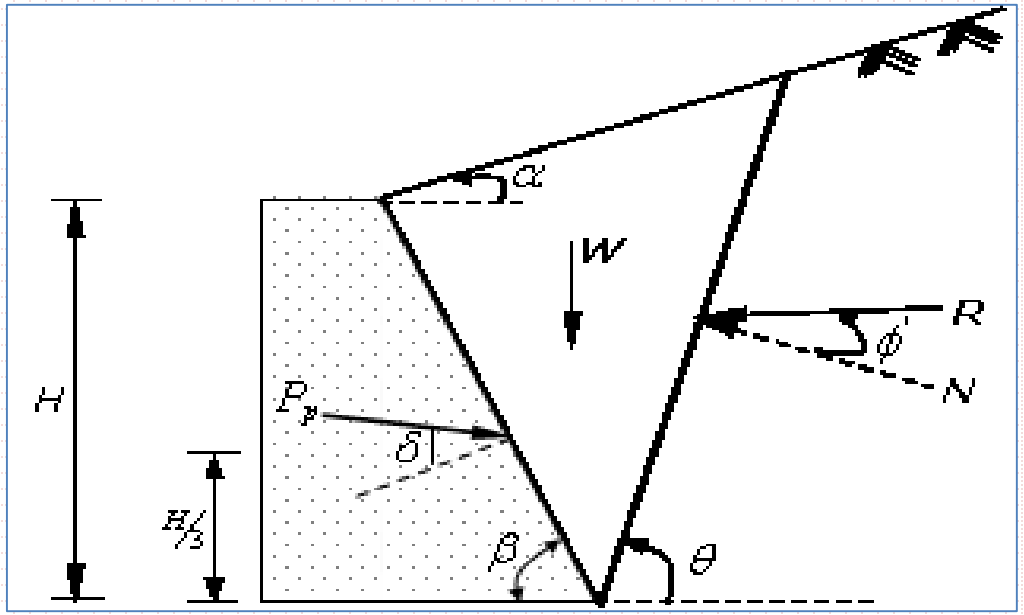
$$P_a = \frac{1}{2}k_{ac}\gamma' H^2$$

$$k_{ac} = \frac{\sin^2(\beta+\phi')}{\sin^2\beta\sin(\beta-\delta)\left[1+\sqrt{\frac{\sin(\phi'+\delta)\sin(\phi'-\alpha)}{\sin(\beta-\delta)\sin(\alpha+\beta)}}\right]^2}$$

k_{ac} is Coulomb's active pressure coefficient

Figure 3.9: Retaining wall with slopping back, wall friction, and sloping soil surface for use with Coulomb's method for active state.

Coulomb's passive earth pressure at sloped fill



$$\sigma_{pc} = K_{pc}\gamma H$$

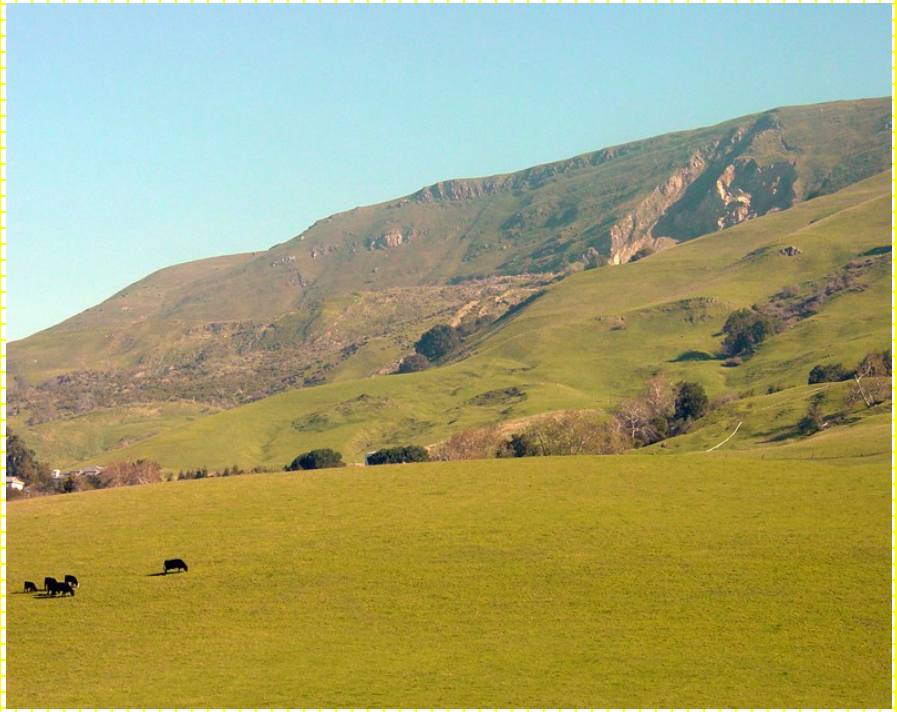
for non-cohesive soil

Figure 3.10: Retaining wall with slopping back, wall friction, and sloping soil surface for use with Coulomb's method for passive state.

$$k_{pc} = \frac{\sin^2(\beta - \phi')}{\sin^2\beta \sin(\beta+\delta)\left[1 - \sqrt{\frac{\sin(\phi'+\delta)\sin(\phi'+\alpha)}{\sin(\beta+\delta)\sin(\alpha+\beta)}}\right]^2}$$

Stability of Slopes

Slope failures



Natural slope





Natural slope failure

Cont…

- ✓ Geotechnical engineers have to pay to **geology**, **drainage**, **groundwater**, and the **shear strength** of the soils for the analysis of slope stability.
- ✓ The most common slope stability analysis methods are based on:
- ➢ **simplifying assumptions.**
- ➢ The design of stable slope relies on experience and careful site investigation.

Types of Slope Failure

- ✓ **Slope failure**, also referred to as mass wasting, is the **downslope movement** of rock debris and soil in response to gravitational stresses.
- ✓ Slope failures depend on **the soil type**, **soil stratification**, **groundwater**, **seepage**, and **the slope geometry**.
- ✓ Failure of a slope along a weak zone of soil is called a *translational slide* .
- ✓ Translational slides are common in **coarse-grained soils**.

Types of Slope Failure

1) Translational slide (weak layer)

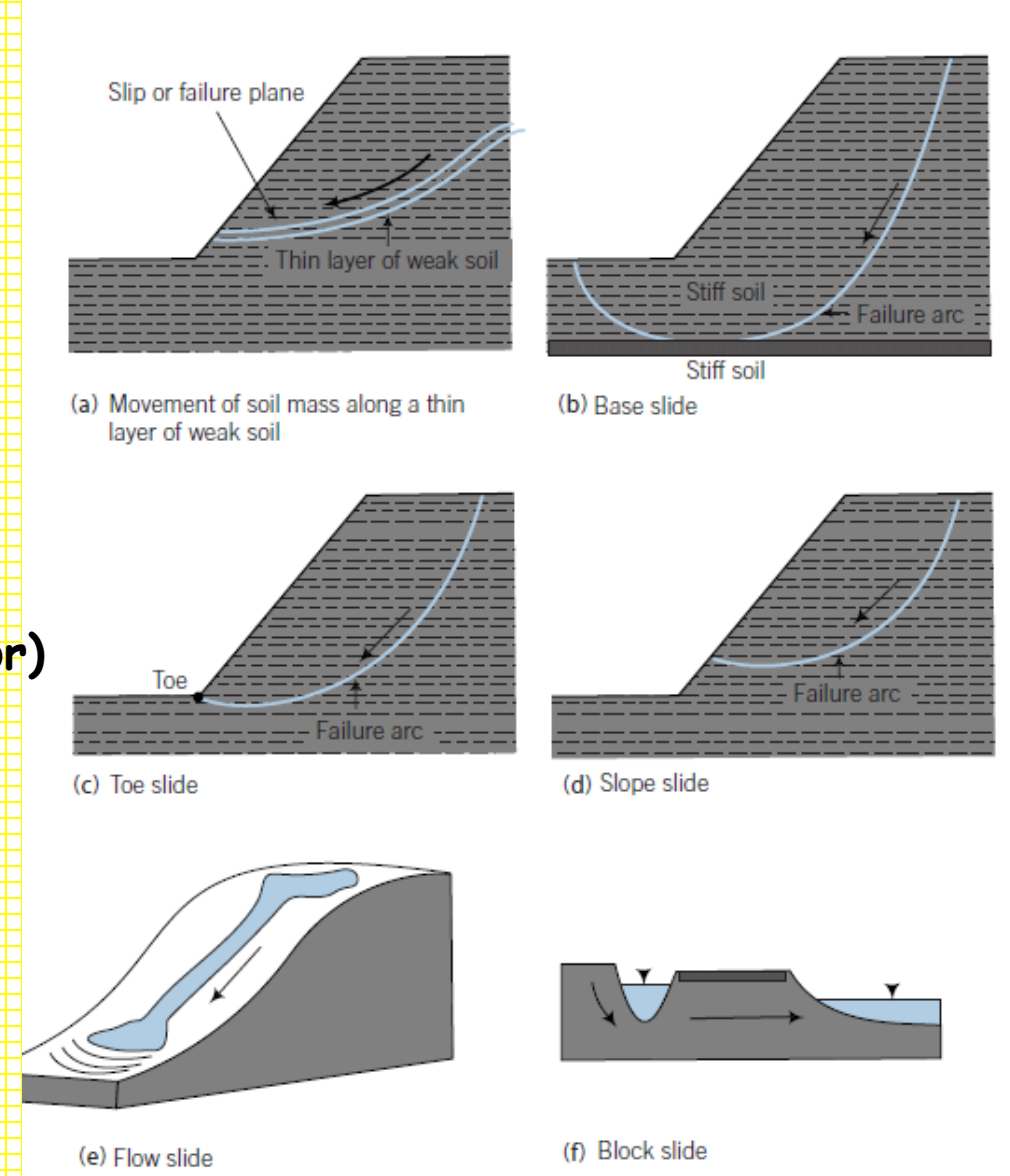
2) Rotational slide (base failure)

3) Rotational slide ("toe slide")

4) Rotational slide ("slope slide")

5) Flow slide (viscous fluid behavior)

6) Block slide

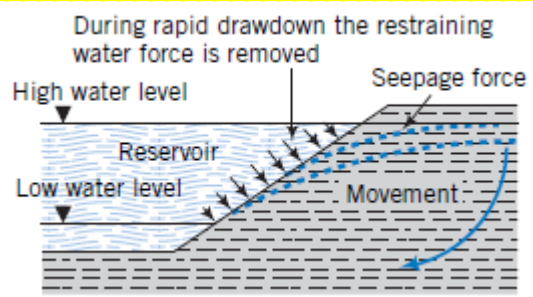


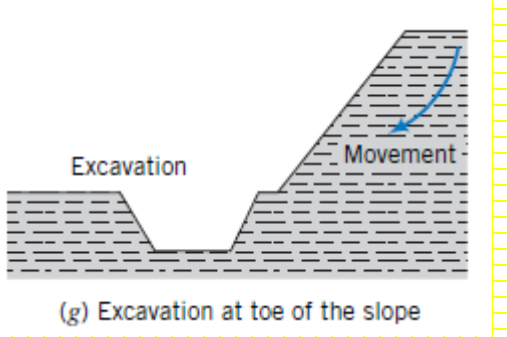
Causes of slope failures/Triggering Mechanisms

✓Slope failures are caused in general by natural forces, human mismanagement and activities.

✓The most common causes of slope failures are:

1) **Intense rainfall** (role of pore pressure)
2) **Erosion** (removal of toe support, increased slope angle)
3) **Earthquake loading** (dynamic forces, liquefaction)
4) **Construction activities** (toe excavation)
5) **Surcharge loading**
6) **Rapid drawdown**



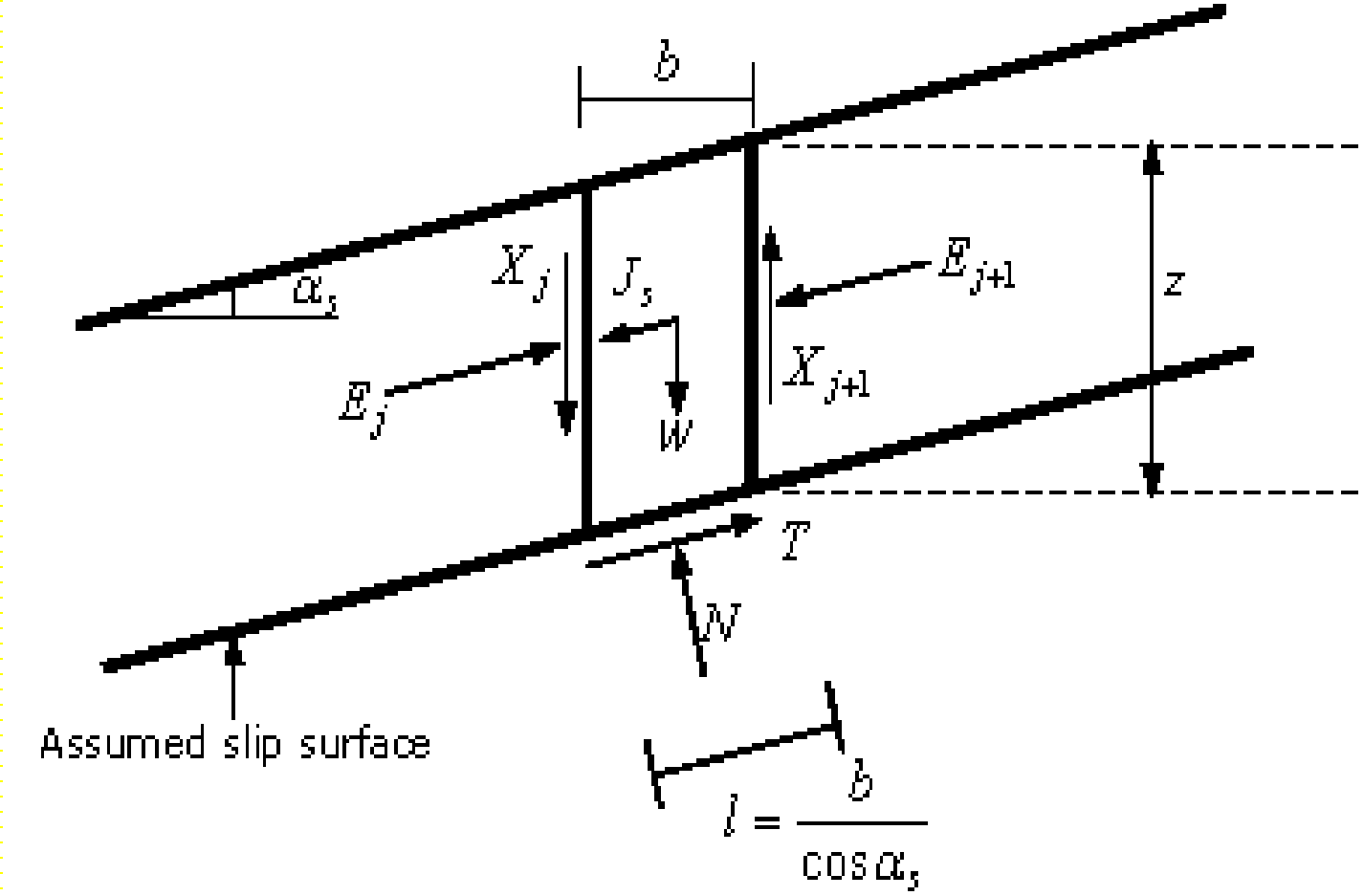


(g) Excavation at toe of the slope

Two-Dimensional Slope Stability Analysis

- ✓Slope stability can be analyzed using one or more of the following: **the limit equilibrium method**, **finite difference method**, and **finite element method**.
- ✓Limit equilibrium is the most widely used method for stability analysis.

Infinite Slopes…



Forces on a slice of soil in an infinite slope.

- **The factor of safety** of a slope is defined as the ratio of the available shear strength, τ_f, to the minimum shear strength required to maintain stability, τ_m.

- The shear strength of the soil is governed by the Mohr-Coulomb failure criterion

$$FS = \frac{\tau_f}{\tau_m}$$

$$F_s = \frac{c' + \sigma' \tan \phi'}{c'_d + \sigma' \tan \phi'_d}$$

i) Stability of Infinite Slopes in φ=0, c_u soil

- ✓ For the φ=0, Cu soil, the Mohr-Coulomb shear strength is given by:

$$\tau_f = c_u$$

- ✓ From statics and using Figure 4.3,

$$N = W\cos\alpha_s \qquad T = W\sin\alpha_s$$

- ✓ The shear stress per unit length (A=L*1) on the slip plane is given by: (shear stress =s.forece/area thus:)

$$\tau_m = \frac{T}{l} = \frac{W\sin\alpha_s\cos\alpha_s}{b} = \frac{\gamma bz}{b}\sin\alpha_s\cos\alpha_s = \gamma z\sin\alpha_s\cos\alpha_s$$

- ✓ The factor of safety is then,

$$FS = \frac{c_u}{\gamma z \sin\alpha_s \cos\alpha_s} = \frac{2c_u}{\gamma z \sin(2\alpha_s)}$$

- ✓ At limit equilibrium, FS = 1. Therefore, the critical slope is($\alpha s = \alpha c$)

$$\alpha_c = \tfrac{1}{2}\sin^{-1}\left(2c_u / \gamma z\right)$$

- ✓ And the critical depth is:

$$z_c = \frac{2c_u}{\gamma \sin(2\alpha_s)}$$

ii) Stability of Infinite Slopes in c', ϕ soil – without seepage.

- ✓ For a c', ϕ soil, the Mohr-Coulomb shear strength is given by:

$$\tau_f = c' + \sigma_n' \tan\phi'$$

- ✓ The factor of safety FS is then:

$$FS = \frac{c' + \sigma_n' \tan\phi'}{\tau_m} = \frac{c'}{\tau_m} + \frac{\sigma_n' \tan\phi'}{\tau_m}$$

- ✓ The normal and shear stresses per unit length at the failure plane in reference to figure 5.3 are given by:

$$\sigma'_n = \frac{N}{l} \qquad \tau_m = \frac{T}{l}$$

- ✓ For a slope without seepage, Js=0.

$$FS = \frac{c'}{\gamma z \sin\alpha_s \cos\alpha_s} + \frac{W' \cos\alpha_s \tan\phi'}{W \sin\alpha_s} = \frac{c'}{\gamma z \sin\alpha_s \cos\alpha_s} + \frac{\tan\phi'}{\tan\alpha_s}$$

- ✓ At limit equilibrium FS = 1. Therefore, the critical depth z_c is given by

$$z_c = \frac{c'}{\gamma}\left(\frac{\sec^2\alpha_s}{\tan\alpha_s - \tan\phi'}\right)$$

- ✓ For the case where $\alpha_s < \phi'$, the factor of safety is always greater than 1 and is computed. This means that there is no limiting value for the depth z, and at an infinite depth, the factor of safety approaches to $\tan\phi/\tan\alpha_s$. For a coarse-grained soil with c' = 0,:

$$FS = \frac{\tan\phi'}{\tan\alpha_s}$$

- ✓ At limit equilibrium FS = 1. Therefore, the critical slope angle is: $\alpha_c = \phi'$

Stability of Infinite Slopes in c', soil – steady state seepage.

- ✓ We will now consider groundwater at the ground surface and assume that seepage is parallel to the slope. The seepage force is:

$$J_s = i\gamma_w bz$$

- ✓ Since seepage is parallel to the slope, $i = \sin\alpha$. From statics,

$$N' = W'\cos\alpha_s = \gamma' bz\cos\alpha_s$$

And

$$\begin{aligned} T &= W'\sin\alpha_s + J_s \\ &= \gamma' bz\sin\alpha_s + \gamma_w bz\sin\alpha_s = (\gamma' + \gamma_w)bz\sin\alpha_s \\ &= \gamma_{sat} bz\sin\alpha_s \end{aligned}$$

✓ Therefore, the shear stress at the slip plane is:

$$\tau_m = \frac{T}{l} = \frac{\gamma_{sat} bz \sin\alpha_s \cos\alpha_s}{b} = \gamma_{sat} z \sin\alpha_s \cos\alpha_s$$

✓ From the definition of factor of safety (Eqn. 5.3), we get:

$$FS = \frac{c'}{\gamma_{sat} z \sin\alpha_s \cos\alpha_s} + \frac{\gamma' bz \cos\alpha_s \tan\phi'}{\gamma_{sat} zb \cos\alpha_s \tan\alpha_s}$$

$$= \frac{c'}{\gamma_{sat} z \sin\alpha_s \cos\alpha_s} + \frac{\gamma'}{\gamma_{sat}} \cdot \frac{\tan\phi'}{\tan\alpha_s}$$

✓ At limit equilibrium, FS=1. Therefore, the critical height is:

$$z_c = \frac{c' c^2 \alpha_s}{\gamma \tan\alpha_s - \gamma' \tan\phi'}$$

✓ At infinite depth the factor of safety:

$$FS = \frac{\gamma'}{\gamma_{sat}} \cdot \frac{\tan\phi'}{\tan\alpha_s}$$

- ✓ The factor of safety for a coarse-grained soil with c′ = 0. At limit equilibrium FS = 1, and hence, the critical slope for a coarse-grained soil with c′ = 0 is given by:

$$\tan\alpha_s = \frac{\gamma'}{\gamma_{sat}}\tan\phi'$$

- ✓ For most soils, $\gamma'/\gamma_{sat} \approx \frac{1}{2}$. Thus, seepage parallel to the slope reduces the limiting slope of a clean, coarse-grained soil by about one-half.
- ✓ If the groundwater level is not at the ground surface, weighted average unit weights have to be used.

Rotational Slope Failure

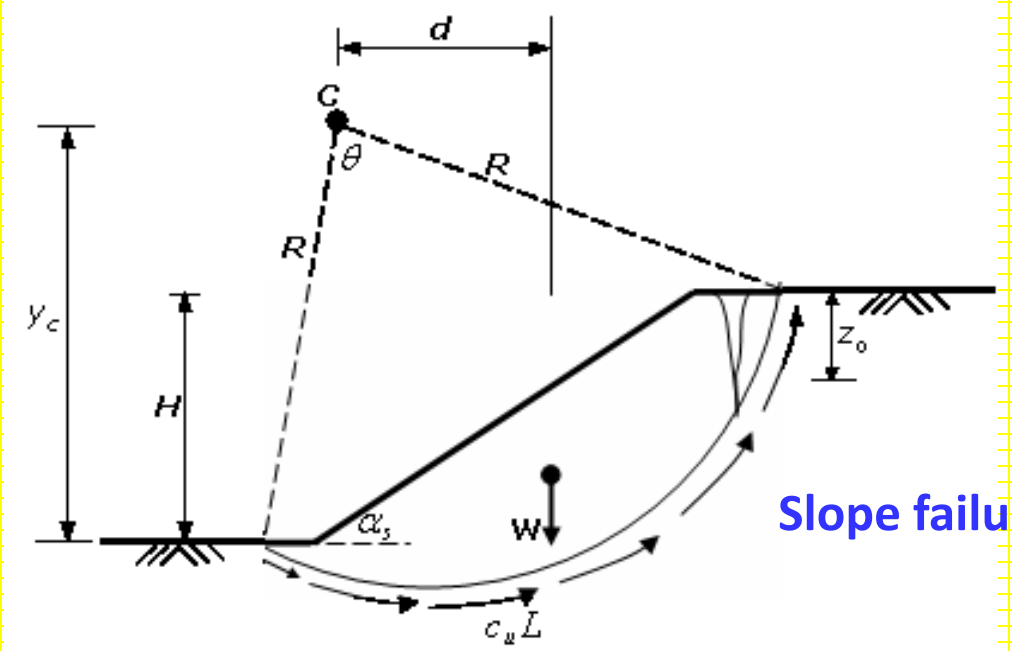
- ✓ Stability of Slopes in c_u, $\phi_u = 0$ soil – circular failure surface.
- ✓ Stability of Slopes in c'– ϕ' soil – Method of Slices.
- ✓ Fellenius or Ordinary or Swedish Method
- ✓ Bishop Simplified Method

Rotational/finite Slope Failure

- ✓ **The infinite slope failure mechanism** is reasonable for infinitely long and homogeneous slopes made of coarse-grained soils, where the failure plane is assumed **to be parallel to the ground surface**.
- ✓ But in many practical problems slopes have been observed to fail through a rotational mechanism of **finite extent**.
- ✓ Rotational failure mechanism involves the failure of a soil mass on a **circular or non-circular failure surface**.
- ✓ Again we will continue to use **the limit equilibrium method** assuming **a circular slip surface**.

i) Stability of Slopes in cu, ϕ =0 soil – circular failure surface.

- ✓ The simplest circular analysis is based on the assumption that a rigid, cylindrical block will fail by rotation about its center and that the shear strength along the failure surface is defined by the undrained strength cu.



Slope failure in cu, ϕ =0.

- ✓ The weight of the sliding block acts at a distance d from the center. Taking moments of the forces about the center of the circular arc, we have:

$$FS = \frac{c_u LR}{Wd} = \frac{c_u R^2 \theta^0}{Wd} \times \frac{\pi}{180^0}$$

- ✓ Where L is the length of the circular arc, W is the weight of the sliding mass and d is the horizontal distance between the circle center, C, and the centroid of the sliding mass. If c_u varies along the failure surface then:

$$FS = \frac{R^2 (c_{u1}\theta_1{}^0 + c_{u2}\theta_2{}^0 + \ldots + c_{un}\theta_n{}^0)}{Wd} \times \frac{\pi}{180^0}$$

- ✓ The centroid of the sliding mass is obtained using a mathematical procedure based on the geometry or the sub-division of the sliding mass into narrow vertical slices.

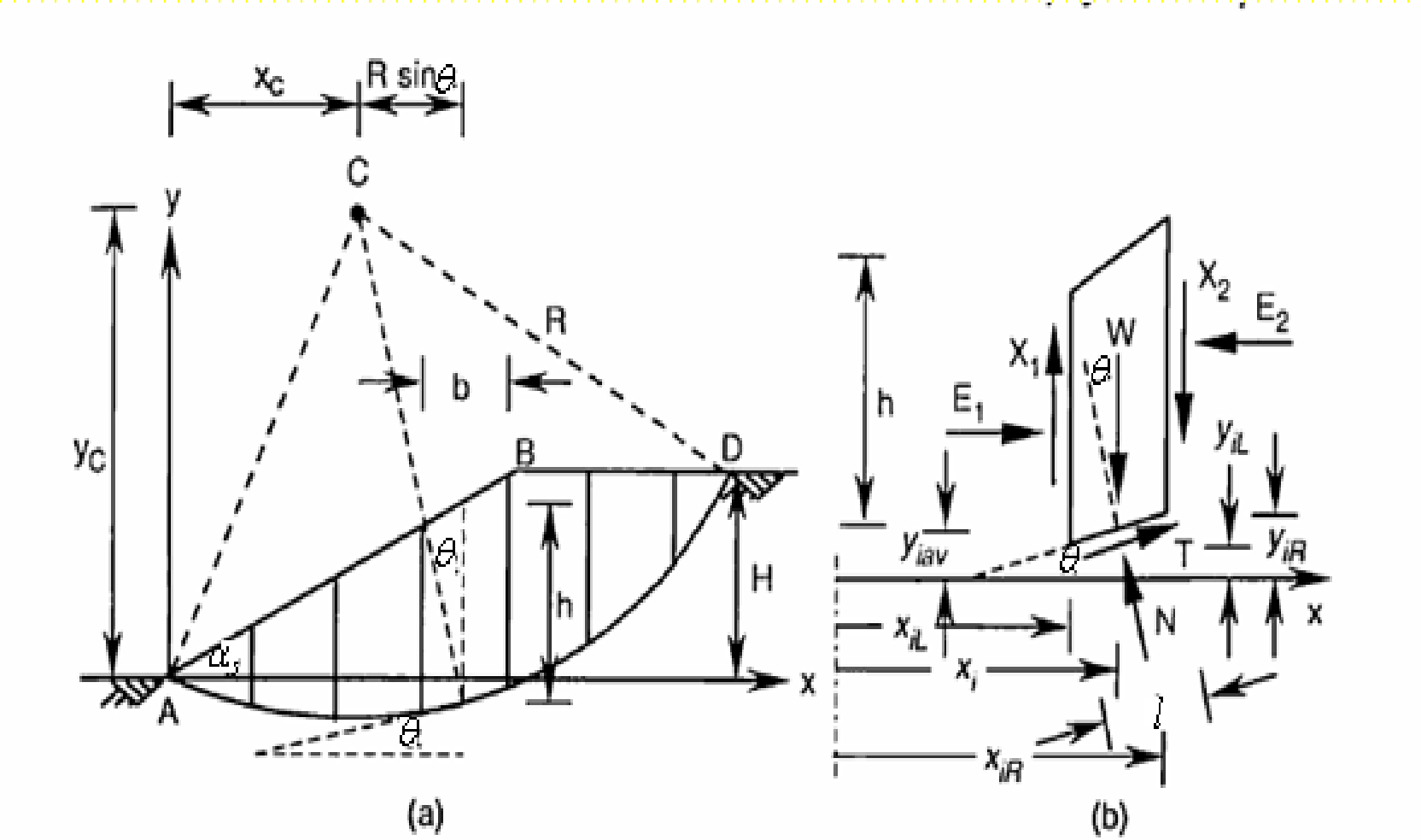
Effect of Tension Cracks

- ✓ Tension cracks may develop from the upper ground surface to a **depth z_0** that can be estimated.
- ✓ The effect of the tension crack can be taken into account by assuming that the trial failure surface terminates at the **depth z_0**, thereby ***reducing the weight W and central angle.***
- ✓ Any ***external water pressure in the crack*** creates a horizontal force that must be included in equilibrium considerations.

Stability of Slopes in c', φ soil – Method of Slices.

- ✓ The stability of a slope in a **c'**, **ϕ** soil is usually analyzed by discretizing the mass of the failure slope.
- ✓ In the method of slices, the soil mass above a trial failure circle is divided into a series of vertical slices of width b.
- ✓ For each slice, ***its base is assumed to be a straight line*** defined by its angle of inclination with the horizontal whilst its height h is measured along the centerline of the slice.

Method of slices



a) Method of slices in c', soil b) Forces acting on a slice.

THANK YOU!