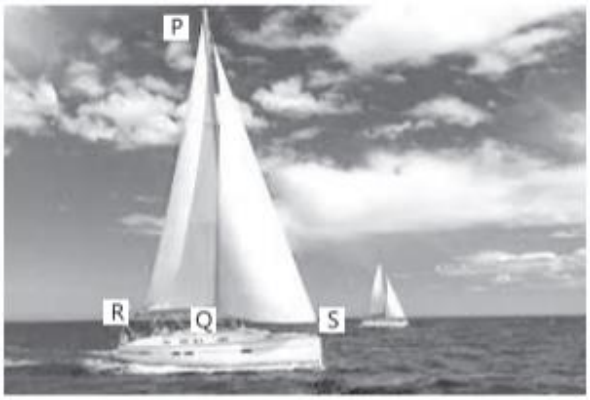
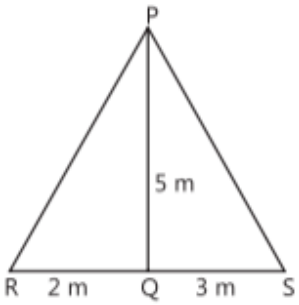
Introduction to Trigonometry

Case Study Based Questions

Case Study 1

A sailing boat with triangular masts is as shown below. Two right triangles can be observed Triangles PQR and PQS, both right-angled at Q. The distance QR = 2 m and QS = 3 m and height PQ = 5 m.





Based on the above information, solve the following questions:

Q 1. The value of sec S is:

a. $\frac{\sqrt{34}}{5}$ b. $\frac{\sqrt{34}}{3}$

c. $\frac{5}{3}$ d. $\frac{3}{\sqrt{34}}$

Q 2. The value of cosec R is:

a. $\frac{\sqrt{29}}{5}$ b. $\frac{\sqrt{29}}{2}$

c. $\frac{2}{5}$ d. $\frac{5}{\sqrt{29}}$

Q 3. The value of tan S + cot R is:

a. $\frac{9}{4}$ b. $\frac{5}{3}$ c. $\frac{31}{15}$ d. $\frac{9}{5}$

Q 4. The value of $\sin^2 R - \cos^2 S$ is:

a. 0 b. 1 c. $\frac{97}{85}$ d. $\frac{589}{986}$

Q 5. The value of $\sin^2 S + \cos^2 R$ is:

a. 0 b. 1

c. $\frac{97}{85}$ d. $\frac{861}{986}$

Solutions

1. In right-angled APQS

$(PS)^2 = (SQ)^2 + (PQ)^2 = (3)^2 + (5)^2 = 9 + 25 = 34$

(by Pythagoras theorem)

$\Rightarrow \quad PS = \sqrt{34}\,m$

$\therefore$ In right-angled ΔPQS,

$$\sec S = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{PS}{SQ} = \frac{\sqrt{34}}{3}$$

So, option (b) is correct.

2. In right-angled ΔPQR

$(PR)^2 = (PQ)^2 + (QR)^2$

(by Pythagoras theorem)

$= (5)^2 + (2)^2 = 25 + 4$

$= 29$

$\Rightarrow \quad PR = \sqrt{29}\,m$

$\therefore$ In right-angled ΔPQR,

$$\text{cosec}\, R = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{PR}{PQ} = \frac{\sqrt{29}}{5}$$

So, option (a) is correct.

3. Use the identity,

$1 + \tan^2 S = \sec^2 S$

$$\Rightarrow \quad \tan S = \sqrt{\sec^2 S - 1} = \sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - 1}$$

(from part 1)

$$= \sqrt{\frac{34}{9} - 1} = \sqrt{\frac{25}{9}} = \frac{5}{3}$$

Use the identity, $1 + \cot^2 R = \operatorname{cosec}^2 R$

$\Rightarrow \quad \cot R = \sqrt{\operatorname{cosec}^2 R - 1} = \sqrt{\left(\frac{\sqrt{29}}{5}\right)^2 - 1}$

(from part 2)

$$= \sqrt{\frac{29}{25} - 1} = \sqrt{\frac{4}{25}} = \frac{2}{5}$$

$\therefore \quad \tan S + \cot R = \frac{5}{3} + \frac{2}{5} = \frac{25+6}{15} = \frac{31}{15}$

So, option (c) is correct.

4. From part (1), $\sec S = \frac{\sqrt{34}}{3}$

$\Rightarrow \quad \cos S = \frac{3}{\sqrt{34}}$

From part (2), $\operatorname{cosec} R = \frac{\sqrt{29}}{5}$

$\Rightarrow \quad \sin R = \frac{5}{\sqrt{29}}$

$\therefore \sin^2 R - \cos^2 S = \left(\frac{5}{\sqrt{29}}\right)^2 - \left(\frac{3}{\sqrt{34}}\right)^2 = \frac{25}{29} - \frac{9}{34}$

$$= \frac{850 - 261}{986} = \frac{589}{986}$$

So, option (d) is correct.

5. From part (1), $\sec S = \frac{\sqrt{34}}{3}$

$\Rightarrow \quad \cos S = \frac{3}{\sqrt{34}}$

$\therefore \quad \sin S = \sqrt{1 - \cos^2 S} = \sqrt{1 - \frac{9}{34}} = \sqrt{\frac{25}{34}} = \frac{5}{\sqrt{34}}$

From part (2), $\operatorname{cosec} R = \frac{\sqrt{29}}{5}$

$$\Rightarrow \quad \sin R = \frac{5}{\sqrt{29}}$$

$$\therefore \quad \cos R = \sqrt{1-\sin^2 R} = \sqrt{1-\frac{25}{29}} = \sqrt{\frac{4}{29}} = \frac{2}{\sqrt{29}}$$

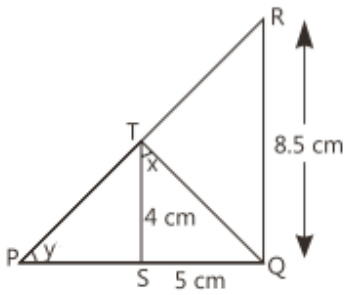
$$\therefore \quad \sin^2 S + \cos^2 R = \left(\frac{5}{\sqrt{34}}\right)^2 + \left(\frac{2}{\sqrt{29}}\right)^2 = \frac{25}{34} + \frac{4}{29}$$

$$= \frac{725+136}{986} = \frac{861}{986}$$

So, option (d) is correct.

Case Study 2

Anika is studying in X standard. She is making a figure to understand trigonometric ratio shown as below.



In ΔPQR, <Q is a right angle, ΔQTR is right- angled at T and ΔQST is right-angled at S, PQ = 12 cm, QR = 8.5 cm, ST=4 cm, SQ = 5 cm, <QTSx and <TPQ = y.

Based on the given information, solve the following questions:

Q1. The length of PT is:

a. 8 cm
b. √65 cm
c. 7.5 cm
d. √69 cm

Q2. The value of tan x is:

a. $\frac{7.5}{13}$
b. $\frac{5}{4}$
c. $\frac{4}{5}$
d. $\frac{13}{7.5}$

Q3. The value of sec x is:

a. $\frac{\sqrt{91}}{6}$ b. $\frac{\sqrt{71}}{6}$

c. $\frac{\sqrt{41}}{4}$ d. $\frac{\sqrt{31}}{5}$

Q4. The value of sin y is:

a. $\frac{4}{\sqrt{65}}$ b. $\frac{4}{7}$

c. $\frac{7}{4}$ d. $\frac{\sqrt{65}}{7}$

Q5. The value of cot y is:

a. $\frac{7}{4}$ b. $\frac{4}{7}$

c. $\frac{\sqrt{65}}{4}$ d. $\frac{\sqrt{65}}{7}$

Solutions

1. We have, PSPQ-SQ=12-5=7cm

In right-angled ΔPST,

$(PT)^2 = (PS)^2 + (ST)^2$ (By Pythagoras theorem)

$= (7)^2 + (4)^2 = 49 + 16 = 65$

$\Rightarrow PT = \sqrt{65}$ cm

So, option (b) is correct.

2. In right-angled ΔTSQ,

$$\tan x = \frac{\text{Perpendicular}}{\text{Base}} = \frac{SQ}{TS} = \frac{5}{4}$$

So, option (b) is correct.

3. We know the identity.

$$\sec^2 x = 1+\tan^2 x = 1+\left(\frac{5}{4}\right)^2 = 1+\frac{25}{16} = \frac{41}{16}$$

$$\Rightarrow \quad \sec x = \sqrt{\frac{41}{16}} = \frac{\sqrt{41}}{4}$$

So, option (c) is correct.

4. In right-angled ΔTSP,

$$\sin y = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{TS}{PT} = \frac{4}{\sqrt{65}}$$

So, option (a) is correct.

5. In right angled ΔTSP

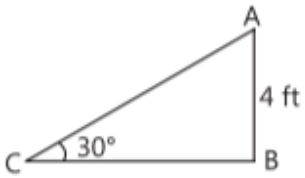
$$\cot y = \frac{\text{Base}}{\text{Perpendicular}} = \frac{PS}{TS} = \frac{7}{4}$$

So, option (a) is correct.

Case Study 3

In structural design, a structure is composed of triangles that are interconnecting. A truss a series of triangle in same plane end is one of the major types of engineering structures and is especially used in the design of bridges and buildings. Trusses are designed to support loads, such as the weight of people. A truss is exclusively made of long, straight members connected by joints at the end of each member.





This is a single repeating triangle in a truss system.

Based on the above information, solve the following questions:

Q1. In the above triangle, what is the length of AC?

Q2. In the above triangle, what is the length of BC?

Q3. If sin A = sin C, what will be the length of BC?

Or

If the length of AB doubles, what will happen the Length of AC?

Solutions

1. In right angled ΔABC,

$$\sin 30^\circ = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{4}{AC}$$

$$\Rightarrow \quad AC = 8 \text{ ft}$$

2. In right-angled ΔABC,

$$\tan 30^\circ = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{4}{BC} \Rightarrow BC = 4\sqrt{3} \text{ ft}$$

3. Given, $\sin A = \sin C$

In right-angled Δ ABC,

$$\frac{BC}{AC} = \frac{AB}{AC} \Rightarrow BC = AB = 4 \text{ ft}$$

Or

Given, $AB = 2 \times 4 = 8$ ft

$\therefore$ In right ΔABC, $\sin 30^\circ = \frac{AB}{AC}$

$$\Rightarrow \quad \frac{1}{2} = \frac{8}{AC} \Rightarrow AC = 16 \text{ ft}$$

So, AC doubles the original length.

Case Study 4

Soniya and her father went to her friend Ruhi to enjoy party. When they reached Ruhi's place, Soniya saw the roof of the house, which was triangular in shape. She imagined the dimensions of the roof which is as given in the figure.



Based on the above information, solve the following questions:

Q1. If D is the mid-point of AC, then find BD.

Q2. Find the measure of <A and <C.

Q3. Find the value of sin A + cos C.

Or

Find the value of $\tan^2 C + \tan^2 A$.

Solutions

1. We have, AB = BC = $6\sqrt{2}$ m and AC = 12 m
D is the mid-point of AC.

$$\therefore \quad AD = DC = \frac{12}{2} = 6 \text{ m}$$

In right-angled ΔADB, use Pythagoras theorem

$$AB^2 = BD^2 + AD^2$$

$$\Rightarrow \quad BD^2 = (6\sqrt{2})^2 - 6^2$$

$$BD^2 = 72 - 36 = 36$$

$$\Rightarrow \quad BD = 6 \text{ m}$$

2. In right ΔADB, $\sin A = \frac{BD}{AB} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$ [from part (1)]

$$\Rightarrow \quad \sin A = \sin 45^\circ \quad \Rightarrow \angle A = 45^\circ$$

In right ΔBDC, $\tan C = \frac{BD}{DC} = \frac{6}{6}$

$\Rightarrow \quad \tan C = 1 = \tan 45^\circ \Rightarrow \angle C = 45^\circ$

3. Here, $\sin A = \frac{1}{\sqrt{2}}$ and $\cos C = \cos 45^\circ = \frac{1}{\sqrt{2}}$

$\therefore \sin A + \cos C = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

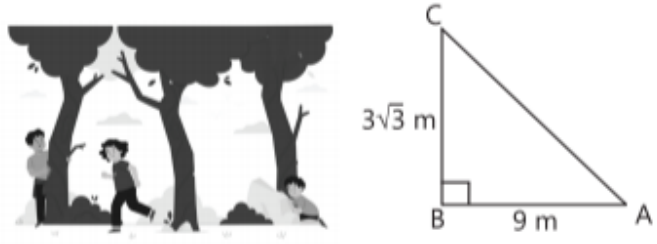
Or

Here, $\tan C = 1$ and $\tan A = \tan 45^\circ = 1$

$\Rightarrow \quad \tan^2 C + \tan^2 A = 1 + 1 = 2$

Case Study 5

Three friends-Sanjeev, Amit and Digvijay are playing hide and seek in a park. Sanjeev and Amit were supposed to hide and Digvijay had to find both of them. If the positions of three friends are at A, B and C respectively as shown in the figure and forms a right-angled triangle such that AB = 9 m,
BC= 3√3 m and B = 90°.



Based on the above information, solve the following questions:

Q1. Find the measure of <A by using trigonometric ratio.

Q2. Find the measure of <C by using trigonometric ratio.

Q3. Find the length of AC.

Q4. Find the value of cos 2A.

Or

Find the value of $\sin\left(\frac{C}{2}\right)$.

Solutions

1. We have, AB = 9 m, BC = 3√3 m

In right AABC, we have

$$\tan A = \frac{BC}{AB} = \frac{3\sqrt{3}}{9} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan A = \tan 30^\circ \Rightarrow \angle A = 30^\circ$$

2. In right ΔABC,

We have, $\tan C = \frac{AB}{BC} = \frac{9}{3\sqrt{3}} = \sqrt{3}$

$$\Rightarrow \quad \tan C = \tan 60^\circ \quad \Rightarrow \angle C = 60^\circ$$

3. In right ΔABC, $\sin A = \frac{BC}{AC}$

$$\Rightarrow \quad \sin 30^\circ = \frac{BC}{AC} \qquad \text{[from part (1)]}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{3\sqrt{3}}{AC} \Rightarrow AC = 6\sqrt{3}\ \text{m}$$

4. $\because \quad \angle A = 30^\circ \qquad$ [from part (1)]

$$\therefore \quad \cos 2A = \cos(2 \times 30^\circ) = \cos 60^\circ = \frac{1}{2}$$

Or

$$\because \quad \angle C = 60^\circ$$

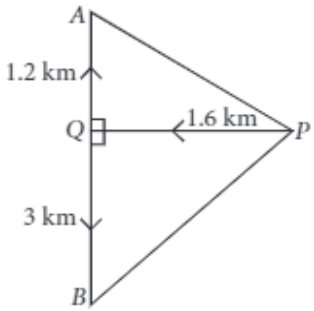
$$\therefore \quad \sin\left(\frac{C}{2}\right) = \sin\left(\frac{60^\circ}{2}\right) = \sin 30^\circ = \frac{1}{2}$$

Two Flights

Case Study 7

Two aeroplanes leave an airport, one after the other. After moving on runway, one flies due North and other flies due South. The speed of two aeroplanes is 400 km/hr and 500 km/hr respectively. Considering PQ as runway and A and B are any two points in the path followed by two planes, then answer the following questions.





(i) Find tanθ; if $\angle APQ = \theta$.

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{3}{4}$

(ii) Find cot$B$.

(a) $\frac{3}{4}$ (b) $\frac{15}{4}$ (c) $\frac{3}{8}$ (d) $\frac{15}{8}$

(iii) Find tan$A$.

(a) 2 (b) $\sqrt{2}$ (c) $\frac{4}{3}$ (d) $\frac{2}{\sqrt{3}}$

(iv) Find sec$A$.

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

(v) Find cosec$B$.

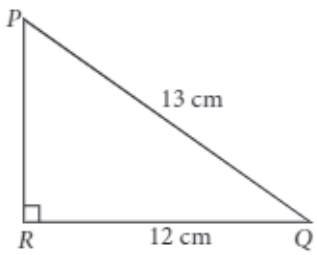
(a) $\frac{17}{8}$ (b) $\frac{12}{5}$ (c) $\frac{5}{12}$ (d) $\frac{8}{17}$

Bird House

Case Study 8

Anita, a student of class 10th, has to made a project on 'Introduction to Trigonometry'. She decides to make a bird house which is triangular in shape. She uses cardboard to make the bird house as shown in the figure. Considering the front side of bird house as right angled triangle PQR, right angled at R, answer the following questions.





(ii) The value of $\sec\theta$ =

(a) $\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{13}{12}$ (d) $\frac{12}{13}$

(iii) The value of $\frac{\tan\theta}{1+\tan^2\theta}$ =

(a) $\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{60}{169}$ (d) $\frac{169}{60}$

(iv) The value of $\cot^2\theta - \text{cosec}^2\theta$ =

(a) –1 (b) 0 (c) 1 (d) 2

(v) The value of $\sin^2\theta + \cos^2\theta$ =

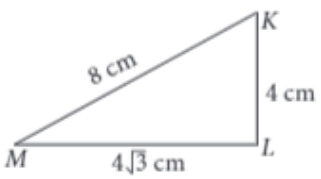
(a) 0 (b) 1 (c) –1 (d) 2

Sandwich Making

Case Study 9

Ritu's daughter is feeling so hungry and so thought to eat something. She looked into the fridge and found some bread pieces. She decided to make a sandwich. She cut the piece of bread diagonally and found that it forms a right angled triangle, with sides 4 cm, $4\sqrt{3}$ cm and 8 cm.





On the basis of above information, answer the following questions.

(i) The value of $\angle M$ =

(a) 30° (b) 60° (c) 45° (d) None of these

(ii) The value of $\angle K$ =

(a) 45° (b) 30° (c) 60° (d) None of these

(iii) Find the value of tan$M$.

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) None of these

(iv) $\sec^2 M - 1$ =

(a) tan$M$ (b) tan2M (c) $\tan^2 M$ (d) None of these

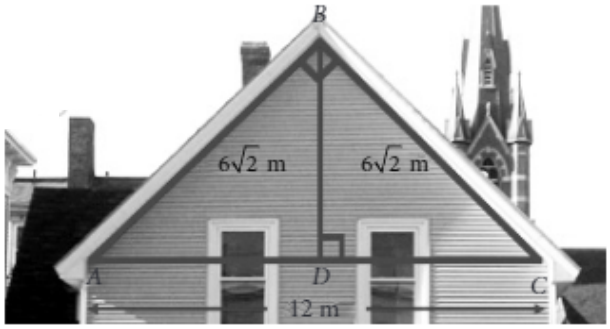
(v) The value of $\frac{\tan^2 45° - 1}{\tan^2 45° + 1}$ is

(a) 0 (b) 1 (c) 2 (d) –1

Roof Top of House

Case Study 10

Aanya and her father go to meet her friend Juhi for a party. When they reached to Juhi's place, Aanya saw the roof of the house, which is triangular in shape. If she imagined the dimensions of the roof as given in the figure, then answer the following questions.



(i) If D is the mid point of AC, then BD =

(a) 2 m (b) 3 m (c) 4 m (d) 6 m

(ii) Measure of $\angle A$ =

(a) 30° (b) 60° (c) 45° (d) None of these

(iii) Measure of $\angle C$ =

(a) 30° (b) 60° (c) 45° (d) None of these

(iv) Find the value of $\sin A + \cos C$.

(a) 0 (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

(v) Find the value of $\tan^2 C + \tan^2 A$.

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

HINTS & EXPLANATIONS

6. **(i)** **(a)**: We have, $AB = 9$ m, $BC = 3\sqrt{3}$ m

In ΔABC, we have

$$\tan A = \frac{BC}{AB} = \frac{3\sqrt{3}}{9} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $\tan A = \tan 30° \Rightarrow \angle A = 30°$ ...(1)

(ii) **(c)**: Similarly, $\tan C = \frac{AB}{BC} = \frac{9}{3\sqrt{3}} = \sqrt{3}$

$\Rightarrow$ $\tan C = \tan 60° \Rightarrow \angle C = 60°$ ...(2)

(iii) **(d)**: Since, $\sin A = \frac{BC}{AC} \Rightarrow \sin 30° = \frac{BC}{AC}$ [Using (1)]

$\Rightarrow$ $\frac{1}{2} = \frac{3\sqrt{3}}{AC} \Rightarrow AC = 6\sqrt{3}$ m

(iv) **(b)**: $\because \angle A = 30°$ [From (1)]

$\therefore$ $\cos 2A = \cos(2 \times 30°) = \cos 60° = \frac{1}{2}$

(v) **(b)**: $\because \angle C = 60°$ [Using (2)]

$\therefore$ $\sin\left(\frac{C}{2}\right) = \sin\left(\frac{60°}{2}\right) = \sin 30° = \frac{1}{2}$

7. **(i)** **(d)**: In ΔAPQ, $\tan\theta = \frac{AQ}{PQ} = \frac{1.2}{1.6} = \frac{3}{4}$

(ii) **(d)**: In ΔPBQ, $\cot B = \frac{QB}{PQ} = \frac{3}{1.6} = \frac{15}{8}$ ...(1)

(iii) **(c)**: In ΔAPQ, $\tan A = \frac{PQ}{AQ} = \frac{1.6}{1.2} = \frac{4}{3}$ ...(2)

(iv) **(d)**: We have, $\tan^2 A + 1 = \sec^2 A$

$$= \sqrt{\frac{16}{9}+1} = \sqrt{\frac{25}{9}} = \frac{5}{3}$$

(v) (a) : Since, $\text{cosec}B = \sqrt{\cot^2 B + 1}$

$$= \sqrt{\left(\frac{15}{8}\right)^2 + 1} \qquad \text{[Using (1)]}$$

$$= \frac{17}{8}$$

$\because$ ΔPQR is a right angled triangle.

$\therefore$ $PR^2 + RQ^2 = PQ^2$

$\Rightarrow$ $PR^2 = (13)^2 - (12)^2 = 25 \Rightarrow PR = 5$ cm

8.

(i) (c) : $\cos\theta = \frac{QR}{PQ} = \frac{12}{13}$

(ii) (c) : $\sec\theta = \frac{1}{\cos\theta} = \frac{13}{12}$

(iii) (c) : $\tan\theta = \frac{PR}{RQ} = \frac{5}{12}$ ...(1)

$$\therefore \quad \frac{\tan\theta}{1+\tan^2\theta} = \frac{\frac{5}{12}}{1+\frac{25}{144}} = \frac{\frac{5}{12}}{\frac{169}{144}} = \frac{60}{169}$$

(iv) (a) : $\cot\theta = \frac{1}{\tan\theta} = \frac{12}{5}$ [Using (1)]

$$\text{cosec}\theta = \frac{PQ}{PR} = \frac{13}{5}$$

$$\therefore \quad \cot^2\theta - \text{cosec}^2\theta = \frac{144}{25} - \frac{169}{25} = -1$$

(v) (b) : $\sin^2\theta + \cos^2\theta = 1$ (Using identity)

$$\frac{LM}{4\sqrt{3}} = \frac{\sqrt{3}}{1}$$

9. $\Rightarrow \tan M = \tan 30^\circ \Rightarrow \angle M = 30^\circ$

(ii) (c) : $\tan K = \frac{ML}{KL} = \frac{4\sqrt{3}}{4} = \sqrt{3} = \tan 60^\circ$

$\Rightarrow$ $\angle K = 60^\circ$

(iii) (b) (iv) (c)

(v) (a) : $\frac{\tan^2 45^\circ - 1}{\tan^2 45^\circ + 1} = \frac{(1)^2 - 1}{1^2 + 1} = \frac{0}{2} = 0$

We have, $AB = BC = 6\sqrt{2}$ m

and $AC = 12$ m.

(i) (d) : $\because$ D is mid point of AC.

10. $\therefore AD = DC = 6$ m

Now, $AB^2 = BD^2 + AD^2$ ($\because$ ΔABD is a right triangle)

$\Rightarrow$ $BD^2 = (6\sqrt{2})^2 - 6^2 = 72 - 36 = 36$

$\Rightarrow$ $BD = 6$ m ...(1)

(ii) (c) : In ΔABD, $\sin A = \frac{BD}{AB} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$ [Using (1)]

$\Rightarrow$ $\sin A = \sin 45^\circ \Rightarrow \angle A = 45^\circ$

(iii) (c) : In ΔBDC, $\tan C = \frac{BD}{DC} = \frac{6}{6}$ [Using (1)]

$\Rightarrow$ $\tan C = 1 = \tan 45^\circ \Rightarrow \angle C = 45^\circ$

(iv) (d) : $\sin A = \frac{1}{\sqrt{2}}$, $\cos C = \cos 45^\circ = \frac{1}{\sqrt{2}}$

$$\therefore \quad \sin A + \cos C = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

(v) (c) : $\tan C = 1$, $\tan A = \tan 45^\circ = 1$

$\Rightarrow$ $\tan^2 C + \tan^2 A = 1 + 1 = 2$