

# Questions With detailed Answers and illustrative Arithmetic

#1, let  $x$  be the first no and  $y$  be the second no then:-

$$P = xy = 288$$

$$S = y + 2x$$

then

$$xy = 288$$

$$y = \frac{288}{x}$$

$$S = y + 2x$$

$$= \frac{288}{x} + 2x$$

$$S = \frac{2x^2 + 288}{x}$$

By using quotient rule

$$S'(x) = \frac{(2x^2 + 288)'(x) - (x)'(2x^2 + 288)}{x^2}$$

$$= \frac{4x^2 - 2x^2 - 288}{x^2}$$

$$= \frac{2x^2 - 288}{x^2} = 0$$

$$= \frac{2x^2 - 288}{x^2} = 0$$

$$= 2x^2 = 288$$

$$= x^2 = 144$$

$$= \underline{x = \pm 12}$$

$$- xy = 288$$

$$- 12y = 288$$

$$- y = 24$$

$\Rightarrow$  So, the numbers that minimize the sum of the 2<sup>nd</sup> no and twice the first is  $\frac{12}{x}$  and  $\frac{24}{y}$  respectively for  $x$  and  $y$ .

#2, let the points be  $(x, y)$

a)  $y = f(x) = x^2 - 4$

the point given =  $(0, 2) \Rightarrow (x, y) = (x, x^2 - 4)$

then

the distance b/n  $(x, x^2 - 4)$  and the closest point given  $(0, 2)$  can be determined by distance formula.

$$d = \sqrt{(x-0)^2 + (y-2)^2}$$

$$= \sqrt{x^2 + (x^2 - 4 - 2)^2}$$

$$= \sqrt{x^2 + x^4 - 12x^2 + 36}$$

$$= \sqrt{x^4 - 11x^2 + 36}$$

$$f(x) = x^4 - 11x^2 + 36$$

$$f'(x) = 4x^3 - 22x$$

$$f'(x) = 2x(2x^2 - 11)$$

$$2x(2x^2 - 11) = 0$$

$$2x = 0, \quad 2x^2 - 11 = 0$$

$$x = 0, \quad 2x^2 = 11$$

$$x^2 = \frac{11}{2}$$

$$x = \pm \sqrt{\frac{11}{2}}$$

$\Rightarrow$  let us substitute the value of "x" that we have got previously.

$$f(x) = x^2 - 4$$

$$= \left(\pm \sqrt{\frac{11}{2}}\right)^2 - 4$$

$$= \frac{11}{2} - 4 = \frac{3}{2}$$

then the value of "x" and "y" respectively are  $\pm \sqrt{\frac{11}{2}}$  and  $\frac{3}{2}$

$\left(-\sqrt{\frac{11}{2}}, \frac{3}{2}\right)$  ;  $\left(\sqrt{\frac{11}{2}}, \frac{3}{2}\right)$  are the coordinates of points.

$$b) y = f(x) = x^2 + 1$$

$$\text{the point given} = (0, 4) = \Delta (x, y) = (x, x^2 + 1)$$

$$d = \sqrt{(x-0)^2 + (x^2+1-4)^2}$$

$$= \sqrt{x^2 + (x^2-3)^2}$$

$$= \sqrt{x^2 + x^4 - 6x^2 + 9}$$

$$= \sqrt{x^4 - 5x^2 + 9}$$

$$f(x) = x^4 - 5x^2 + 9$$

$$f'(x) = 4x^3 - 10x$$

$$4x^3 - 10x = 0$$

$$x(4x^2 - 10) = 0$$

$$x=0, 4x^2 = 10$$

$$x^2 = \frac{10}{4}$$

$$x = \pm \sqrt{\frac{5}{2}}$$

So, substituting  $\pm \sqrt{\frac{5}{2}}$  in the  $f(x)$

$$f(x) = \left(\pm \sqrt{\frac{5}{2}}\right)^2 + 1$$

$$y = \frac{5}{2} + 1$$

$$y = \frac{7}{2}$$

$$\left(-\sqrt{\frac{5}{2}}, \frac{7}{2}\right) \text{ ; } \left(\sqrt{\frac{5}{2}}, \frac{7}{2}\right)$$

$$C, f(x) = y = x^2$$

The given point = (2, 1)

$$(x, y) = (x, x^2)$$

$$d = \sqrt{(x-2)^2 + (x^2-1)^2}$$

$$d = \sqrt{x^2 - 4x + 4 + x^4 - 2x^2 + 1}$$

$$d = \sqrt{x^4 - x^2 - 4x + 5}$$

$$g(x) = x^4 - x^2 - 4x + 5$$

$$g'(x) = 4x^3 - 2x - 4$$

∵ Since it can't be factorized we can't find the critical number

#3, Let "x" be positive no the reciprocal of "x" is " $\frac{1}{x}$ ".

$$f(x) = x + \frac{1}{x}$$

$$f(x) = \frac{x^2 + 1}{x}$$

Using quotient rule to determine the above function.

$$f'(x) = \frac{(x^2+1)'(x) - (x)'(x^2+1)}{x^2}$$

$$= \frac{2x - (x^2+1)}{x^2}$$

$$0 = \frac{x^2 - 1}{x^2}$$

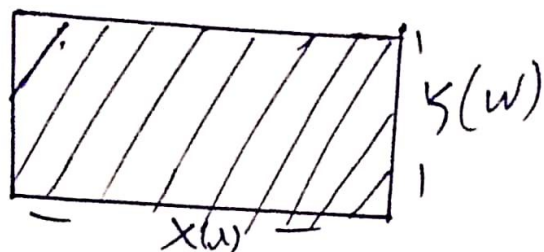
$$\hookrightarrow x^2 - 1 = 0$$

$\hookrightarrow x = \pm 1$ , hence it is said to be "1" is the answer.

$$f(x) = \frac{x^2 + 1}{x}$$

$$f(1) = \frac{1^2 + 1}{1} = 2 \rightarrow \text{is the final minimum value.}$$

#4,



$$P_{\text{perimeter}} = 2x + 2y = 100$$

$$= \boxed{x + y = 50} \rightarrow \boxed{y = 50 - x}$$

$$\text{Area} = xy$$

solving for  $x$  and  $y$  substituting must be the criteria.

$$A = x(50 - x)$$

$$A = -x^2 + 50x$$

$$A'(x) = -2x + 50$$

$$-2x + 50 = 0$$

$$-2x = -50$$

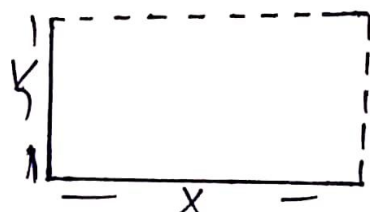
$$\underline{\underline{x = 25}}$$

then substituting into  $y = 50 - x$

$$y = 50 - 25$$

$\underline{\underline{y = 25}}$ , dimensions! -  $\underline{\underline{x = 25}}$  and  $\underline{\underline{y = 25}}$  maximize the area.

#5,



$\Rightarrow$  b/c the farmer needs to fence the two adjacent sides of the rectangular field.

$$P = x + y = 200m \Rightarrow y = 200 - x$$

$$A = xy \Rightarrow A = x(200 - x)$$

$$A = 200x - x^2$$

$$-A'(x) = 200 - 2x = 0$$

$$= 200 = 2x$$

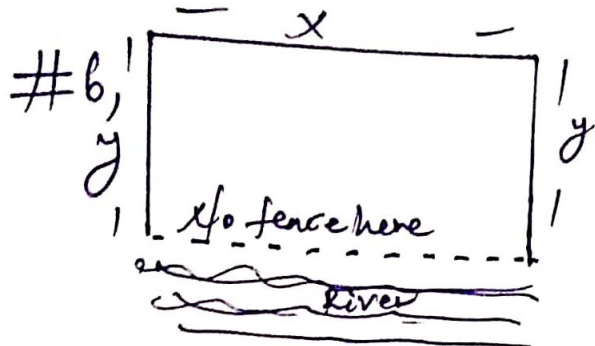
$$= \underline{\underline{x = 100}}$$

$$\Rightarrow x + y = 200$$

$$\Rightarrow 100 + y = 200$$

$$\underline{\underline{y = 100}}$$

then  $x = \underline{\underline{100}}$  and  $y = \underline{\underline{100}}$  yield the maximum area



$$P = 2y + x = \text{sum}$$

$$A = x\zeta = 180,000 \text{ m}^2$$

$$\zeta = \frac{180,000}{x}$$

$$S = 2y + x$$

$$S(x) = 2\left(\frac{180,000}{x}\right) + x$$

$$S(x) = \frac{360,000}{x} + x$$

$$= 360,000x^{-1} + x$$

$$S'(x) = -360,000x^{-2} + 1$$

$$-360,000x^{-2} + 1 = 0$$

$$+360,000x^{-2} = +1$$

$$360,000x^{-2} = 1$$

$$\Rightarrow \frac{360,000}{x^2} = 1$$

$$\Rightarrow 360,000 = x^2$$

$$\Rightarrow \underline{x = 600}$$

$$\zeta = \frac{180,000}{x}$$

$$= \frac{180,000}{600}$$

$$\underline{\underline{\zeta = 300}}$$

⊗  $\underline{x=600}$ ;  $\underline{\zeta=300}$  yield the smallest amount of fencing.

#7,  $2x + 2y = 5$

$$A = x\zeta = 64 \text{ m}^2 \rightarrow \boxed{\zeta = \frac{64}{x}}$$

$$\Rightarrow 2x + 2\left(\frac{64}{x}\right) = 5$$

$$\Rightarrow 2x + \frac{128}{x} = 5$$

$$S(x) = 2x + \frac{128}{x}$$

$$S'(x) = 2 - \frac{128}{x^2}$$

$$S'(x) = \frac{2x^2 - 128}{x^2}$$

$$\frac{2x^2 - 128}{x^2} = 0$$

$$2x^2 - 128 = 0$$

$$2x^2 = 128$$

$$x = \pm 8$$

$$A = 8\zeta = 64 \text{ m}^2$$

$$\Rightarrow \underline{\underline{\zeta = 8}}$$

⊗  $\underline{x=8}$  and  $\underline{\zeta=8}$  yields minimum value

#8,

$$P_{\text{cimeter}} = P_{\text{circle}} + P_{\text{square}}$$

$$\Rightarrow 2\pi r + 4l = 16 \rightarrow \pi r + 2l = 8$$

$$\Rightarrow A = (\pi r^2) l^2$$

$$\boxed{l = \frac{8 - \pi r}{2}}$$

$$\Rightarrow A = (\pi r^2) \left( \frac{8 - \pi r}{2} \right)^2$$

$$\Rightarrow A = (\pi r^2) \left( 4 - \frac{\pi r}{2} \right)^2$$

$$\Rightarrow A = \pi r^2 + l^2$$

$$A = \pi r^2 + \left( 4 - \frac{\pi r}{2} \right)^2$$

$$A'(r) = 2\pi r + 2 \left( 4 - \frac{\pi r}{2} \right) \cdot \left( -\frac{\pi}{2} \right)$$

$$A'(r) = 2\pi r + (8 - \pi r) \left( -\frac{\pi}{2} \right) = 0$$

$$-2\pi r - \frac{8\pi}{2} + \frac{\pi^2 r}{2} = 0$$

$$-2\pi r - 4\pi + \frac{\pi^2 r}{2} = 0$$

$$2\pi r + \frac{\pi^2 r}{2} = 4\pi$$

$$2 \left( 2\pi r + \frac{\pi^2 r}{2} \right) = (4\pi) 2$$

$$4\pi r + \pi^2 r = 8\pi$$

$$\frac{r(4\pi + \pi^2)}{4\pi + \pi^2} = \frac{8\pi}{4\pi + \pi^2}$$

$$r = \frac{8\pi}{4\pi + \pi^2}$$

$$r = \frac{\pi(8)}{\pi(4 + \pi)}$$

$$\boxed{r = \frac{8}{4 + \pi}}$$

$$-2\pi r + 4l = 16$$

$$-2\pi\left(\frac{8}{4+\pi}\right) + 4l = 16$$

$$\frac{16\pi}{4+\pi} + 4l = 16$$

$$\Rightarrow 4l = 16 - \left(\frac{16\pi}{4+\pi}\right)$$

$$\Rightarrow 4l = 16 - \frac{16\pi}{4+\pi}$$

$$\Rightarrow 4l = \frac{64 + 16\pi - 16\pi}{4+\pi}$$

$$\Rightarrow 4l = \frac{64}{4+\pi}$$

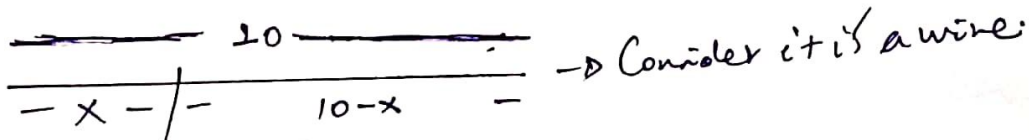
$$\Rightarrow l = \left(\frac{64}{4+\pi}\right) \cdot \frac{1}{4}$$

$$\boxed{l = \frac{16}{4+\pi}}$$

So,  $r = \frac{8}{4+\pi}$  and  $l = \frac{16}{4+\pi}$  are possible answers.

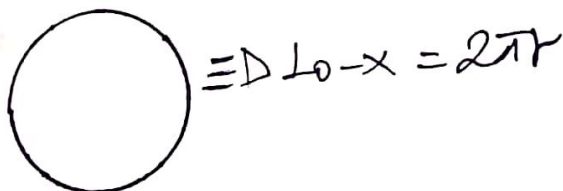
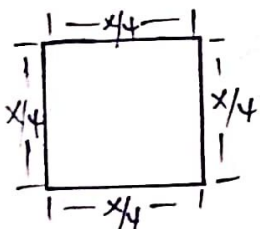
#9,

(a)



$x$  → Used to form a square

$10-x$  → Used to form a circle.



$$\frac{10-x}{2\pi} = \frac{2\pi r}{2\pi}$$

$$r = \frac{10-x}{2\pi}$$

$$A = \pi r^2$$

$$A_{\text{circle}} = \pi \left(\frac{10-x}{2\pi}\right)^2$$

$$\text{Area of square} = \left(\frac{x}{4}\right)^2$$

$$A(x) = \text{Area of circle} + \text{Area of square}$$

$$\boxed{A(x) = \pi \left(\frac{10-x}{2\pi}\right)^2 + \left(\frac{x}{4}\right)^2}$$

(b) The domain of  $f(x)$  is

$$10 - x > 0$$

$$-x > -10$$

$$\underline{x < 10}$$

$$(c) A(x) = \pi \left( \frac{10-x}{2\pi} \right)^2 + \left( \frac{x}{4} \right)^2$$

$$A'(x) = \pi' \left( \frac{10-x}{2\pi} \right)^2 + \pi \left( \left( \frac{10-x}{2\pi} \right)^2 \right)' + \frac{x}{8}$$

$$A'(x) = 2\pi \left( \frac{10-x}{2\pi} \right) \left( -\frac{1}{2\pi} \right) + \frac{x}{8}$$

$$= -\left( \frac{10-x}{2\pi} \right) + \frac{x}{8}$$

$$= -\frac{10+x}{2\pi} + \frac{x}{8}$$

$$\Rightarrow -\frac{10+x}{2\pi} + \frac{x}{8} = 0$$

$$\Rightarrow -\frac{5}{\pi} + \frac{x}{2\pi} + \frac{x}{8} = 0$$

$$2\pi \left( \frac{5}{\pi} \right) = 2\pi \left( \frac{x}{2\pi} + \frac{x}{8} \right)$$

$$10 = x + \frac{1}{4}\pi x$$

$$10 = x \left( 1 + \frac{1}{4}\pi \right)$$

$$\frac{4}{4+\pi} (10) = x \left( \frac{4+\pi}{4} \right) \times \frac{4}{4+\pi}$$

$$\boxed{x = \frac{40}{4+\pi}} \approx \underline{\underline{5.6m}}$$

$$(x) = 10 - x$$

$$= 10 - \left( \frac{40}{4+\pi} \right)$$

$$= 10 - 5.6 = \underline{4.4m} \text{ is the wire should be used for a circle}$$

$$\Rightarrow \underline{5.6m} \text{ is a wire should be used for square.}$$

$$\textcircled{10} R(x) = -x^3 + 450x^2 + 52,500x$$

$$R'(x) = \frac{-3x^2}{-3} + \frac{900x}{-3} + \frac{52,500}{-3}$$

$$R'(x) = x^2 - 300x - 17,500$$

Using quadratic formula

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$X = \frac{300 \pm \sqrt{90,000 + 70,000}}{2(1)}$$

$$X = \frac{300 \pm 400}{2}$$

$$X = 350 \text{ and } X = -50$$

$X = 350 \rightarrow$  b/c it is given as maximum

$$R(350) = -(350)^3 + 450(350)^2 + 52,500(350)$$

$$= \underline{30,625,000} \text{ yield the maximum revenue.}$$

$$\textcircled{11} C(x) = 0.008x^2 + 2x + 304$$

$$C'(x) = 0$$

$$C'(x) = 0.016x + 2$$

$$0.016x + 2 = 0$$

$$0.016x = -2$$

$$x = \frac{-2}{0.016}$$

$$x = -125$$

$$C(x) = 0.008x^2 + 2x + 304$$

$$C(-125) = 0.008(-125)^2 + 2(-125) + 304$$

$$C(-125) = 0.008(15,625) + -250 + 304$$

$$C(-125) = 125 + 54$$

$$C(-125) = \underline{179} \rightarrow \text{the minimum cost.}$$

#12,

$$X(t) = \sin 2t + \sqrt{3} \cos 2t$$

$$X'(t) = 2\cos 2t - 2\sqrt{3} \sin 2t$$

$$X'(t) = 0$$

$$2\cos 2t - 2\sqrt{3} \sin 2t = 0$$

$$\frac{2\cos 2t}{\cos 2t} = \frac{2\sqrt{3} \sin 2t}{\cos 2t}$$

$$2 = \frac{2\sqrt{3} \sin 2t}{\cos 2t}$$

$$1 = \sqrt{3} \frac{\sin 2t}{\cos 2t}$$

$$1 = \sqrt{3} \tan 2t$$

$$\tan 2t = \frac{1}{\sqrt{3}}$$

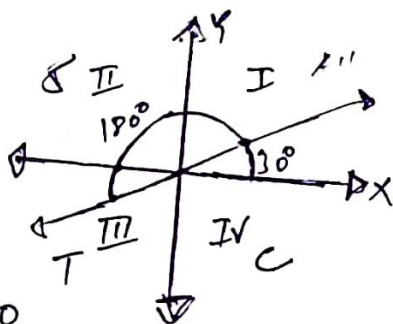
$$\tan 2t = \tan \theta = \frac{1}{\sqrt{3}}$$

$$= \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \underline{\underline{\theta = 30^\circ}}$$

$$2t = \theta = 30^\circ$$

$\Rightarrow$  let's find the Co-terminal angle  $30^\circ$  by referring the reference Angle.



$$\theta_{III} = 180^\circ + 30^\circ$$

$$\theta_{III} = \underline{\underline{210^\circ}}$$

$$2t = 30^\circ$$

$$t = 15^\circ$$

$$2t = 210^\circ$$

$$t = 105^\circ$$

$\Rightarrow$   $15^\circ$  and  $105^\circ$  are critical number.

$\Rightarrow$  Let's check the 2<sup>nd</sup> derivative test whether  $15^\circ$  and  $105^\circ$  is Valid.

$$X'(t) = 2 \cos 2t - 2\sqrt{3} \sin 2t$$

$$X''(t) = -4 \sin 2t - 4\sqrt{3} \cos 2t$$

$$X''(105^\circ) = -4 \sin 2 \times 105^\circ - 4\sqrt{3} \cos 2 \times 105^\circ$$

$$= -4 \times -\frac{1}{2} - 4\sqrt{3} \times \frac{\sqrt{3}}{2}$$

$$= 2 + \frac{12}{2} = 2 + 6 = 8 > 0 \rightarrow \text{hence it is minimum, so it is invalid b/c the question mainly aimed at maximum distance.}$$

$$X''(15^\circ) = -4 \times \frac{1}{2} - 4\sqrt{3} \cos(2 \times 15^\circ)$$

$$= -2 - 4\sqrt{3} \times \frac{\sqrt{3}}{2}$$

$$= -2 - 6 = \underline{\underline{-8}} < 0 \rightarrow \text{it is maximum, so } 15^\circ \text{ is Valid for this function } X(*) \text{ in order to maximize.}$$

$$\text{So, } X(t) = 5 \sin 2t + \sqrt{3} \cos 2t$$

$$X(15^\circ) = 5 \sin 2(15^\circ) + \sqrt{3} \cos 2(15^\circ)$$

$$= 5 \sin 30^\circ + \sqrt{3} \cos 30^\circ$$

$$= \frac{1}{2} + \frac{\sqrt{3} \times \sqrt{3}}{2}$$

$$= \frac{1}{2} + \frac{3}{2}$$

$$= \frac{4}{2} = \underline{\underline{2}}$$

$\Rightarrow \underline{\underline{2}} \rightarrow$  is the maximum distance starting from the origin of  $X(t)$ .

#13,

$$C(t) = 37 + \frac{4}{\sqrt{t+1}}$$

$$C(3) = 37 + \frac{4}{\sqrt{3+1}}$$

$$C(3) = 37 + \frac{4}{\sqrt{4}}$$

$$C(3) = 37 + \frac{4}{2}$$

$$C(3) = 37 + 2$$

$$C(3) = \underline{\underline{39^\circ\text{C}}}$$

$$C'(3) = 37 + \frac{4}{\sqrt{t+1}}$$
$$= 0 - (4) \left( \frac{1}{2\sqrt{t+1}} \right)$$
$$\frac{1}{(\sqrt{t+1})^2}$$

$$= 0 - 4 \left( \frac{1}{2 \times 2} \right)$$
$$\frac{1}{(\sqrt{4})^2}$$

$$= \frac{0 - 1}{4} = \underline{\underline{-\frac{1}{4}}}$$

⇒ from the derivation above of the function  $C(t) = 37 + \frac{4}{\sqrt{t+1}}$ ,

We found  $39^\circ\text{C}$  and  $\frac{1}{4}$ , from  $C(3)$  and  $C'(3)$  respectively,  
it indicates that after 3 hrs the body temperature is decreased to  
 $39^\circ\text{C}$  and it is decreasing at the rate of  $-\frac{1}{4}^\circ\text{C/hr}$ .



scholastic cluster