



The Fourth Rwanda Mathematics Competition

Stage 2 - regional elimination

Date: April 29th, 2022

Time limit: 3.5 hours

Please enter the following information in **PRINT**.

NAME:

GENDER: M ☐
F ☐

SCHOOL:

CLASS:

INSTRUCTIONS AND REGULATIONS (please read it):

- This stage of the RMC consists of two parts: 11 Multiple-choice questions and 5 Open questions.
- For each of the 11 multiple-choice questions mark with cross (×) the answer which you find is true. If you, at some point, want to change the marking add a clear note. Only one of the alternatives A, B, C, D and E is true.
- For each of the 5 open questions full, well-justified solutions are requested.
- No calculators, cellphones nor other electronic devices are allowed. Only paper, pen, eraser, straightedge and compass.
- For multiple-choice questions, please do not make any calculation on the problem pages. Those are only for answers. For your own calculations use scratch papers.
- If you need additional paper or go to the toilet, please raise your hand and wait until one of the invigilators will approach you and address your request.
- Students communicating with each other during the test will be asked to leave the classroom.
- When the invigilators announces "TIME'S UP". please collect your papers (with this page on top) and hand them over immediately. Do NOT include scratch papers.
- Note that the total time allowed is 3.5 hours.

Please, **DO NOT** write in the boxes below.

Problems	3 points	4 points	5 points	Open	Total Score
Score	/12	/12	/20	/35	/79

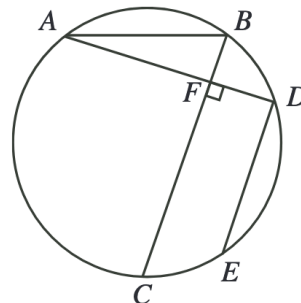
NAME:

PART 1

Each correct answer to the first set of four questions will be awarded **3 points**.

1. In the adjacent figure, AB and BC are chords of a circle with $AB < BC$. Let D be the point on the circle such that AD is perpendicular to BC and let E be the point on the circle such that DE is parallel to BC . What is the measure of $\angle ABE$?

- ☐ (A) 45° ☐ (B) 75° ☐ (C) 105° ☐ (D) 60°
☐ (E) 90°



2. In the picture below we have 11 fields. In the first field we have number 7 and in the ninth field we have number 6. What number has to be placed in the second field if the following condition has to be true: sums of each three successive fields are equal to 21?

7								6		
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- ☐ (A) 7 ☐ (B) 8 ☐ (C) 6 ☐ (D) 10 ☐ (E) 5

3. There are three identical cubes in the picture below, with numbers 0, 1, 3, 4, 5, 9 written identically on each cube. What is the total sum of numbers on the bottom side of the cubes?



- ☐ (A) 6 ☐ (B) 8 ☐ (C) 9 ☐ (D) 10 ☐ (E) 12

4. The result of a school-test was as follows: the average mark for the class was 9.4, while the average mark for the boys was 8.6, and for the girls – 9.8. What ratio of the whole class are boys?

- ☐ (A) $\frac{3}{4}$ ☐ (B) $\frac{2}{3}$ ☐ (C) $\frac{1}{2}$ ☐ (D) $\frac{1}{3}$ ☐ (E) $\frac{1}{4}$

Each correct answer to the next set of four questions will be awarded **4 points**.

5. If a, b, c, d and e are different integers and $(4-a)(4-b)(4-c)(4-d)(4-e) = 12$, then $a+b+c+d+e$ equals

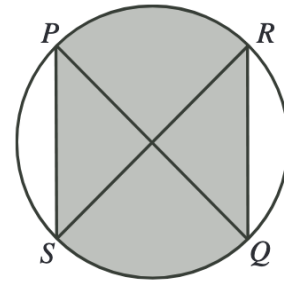
☐ (A) 17 ☐ (B) 20 ☐ (C) 24 ☐ (D) 32 ☐ (E) 40

6. Using the digits 1, 4 and 8 (each digit at most one time) it is possible to compose four perfect squares: $1 = 1^2$, $4 = 2^2$, $81 = 9^2$, $841 = 29^2$. How many different perfect squares is possible to compose using the digits 1, 6 and 9?

☐ (A) 2 ☐ (B) 3 ☐ (C) 4 ☐ (D) 5 ☐ (E) 6

7. In the adjacent figure, PQ and RS are diameters of a circle with radius 4. If PQ and RS are perpendicular, what is the area of the shaded region?

☐ (A) $8 + 4\pi$ ☐ (B) $8 + 8\pi$ ☐ (C) $16 + 4\pi$ ☐ (D) $16 + 8\pi$
☐ (E) $16 + 16\pi$



Each correct answer to the last set of four questions will be awarded **5 points**.

9. A family drives from a city to the beach, at 60 km/h. On the return trip the speed is 20 km/h. What is the average speed for the whole trip?

☐ (A) 24 km/h ☐ (B) 30 km/h ☐ (C) 40 km/h ☐ (D) 45 km/h ☐ (E) 50 km/h

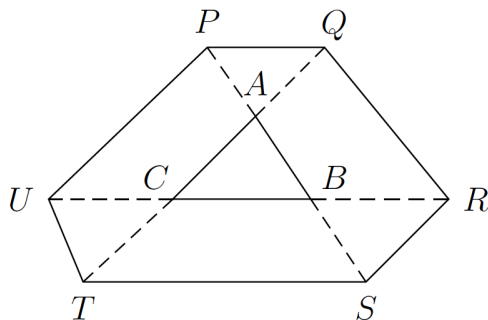
10. Sides of the triangle ABC are extended on both sides to points P, Q, R, S, T and U (see the figure below) so that

$$PA = AB = BS,$$

$$TC = CA = AQ \text{ and}$$

$$UC = CB = BR.$$

Moreover, the area of triangle ABC is 1. What is the area of the hexagon $PQRSTU$?



☐ (A) 9 cm^2 ☐ (B) 10 cm^2 ☐ (C) 12 cm^2 ☐ (D) 13 cm^2 ☐ (E) 15 cm^2

11. M and N are two natural numbers, such that $51 \leq N \leq 101$ and $19 \leq M \leq 49$. What is the greatest integer value (whole number) of the expression $\frac{N+M}{N-M}$?

☐ (A) 20 ☐ (B) 30 ☐ (C) 40 ☐ (D) 50 ☐ (E) 60

12. How many ten-digit numbers composed only of digits 1, 2 and 3 exist, in which any two neighboring digits differ by 1 (it is not necessary that all three digits must occur in such a number).

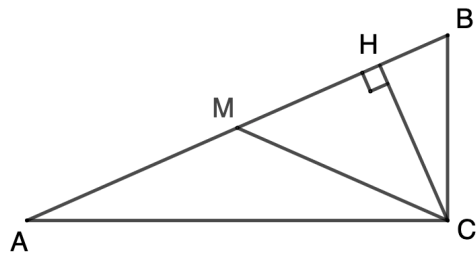
☐ (A) 16 ☐ (B) 32 ☐ (C) 64 ☐ (D) 80 ☐ (E) 100

NAME:

PART 2

Each **solution** will be awarded up to **7 points**.

1. In the right triangle ABC , with the right angle at C , the length of the height $CH = 1$ cm and the length of the median $CM = 2$ cm. Find the angle BAC .



2. A positive integer n has 2022 digits, all of them being 9. What are the digits of n^2 .

3. 100 participants of The Rwanda Mathematics Competition had to solve 4 problems. 90 students solved the first problem, 80 solved the second problem. 70 students solved the third problem, and 60 solved the fourth. None managed to solve all 4 problems. Only those who succeeded with the third and the fourth problems were announced the winners. How many winners were there?

4. Find all prime numbers p such that $p^3 + p^2$ is a perfect square.

5. For which real numbers a the equation $3^{2x} - 3^x + 1 = a$ has two different real positive roots.