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Engineering Mechanics I (CENG-1001)

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Lecture Overview

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- Scalars and Vectors

- Definition
- Representation of Vectors

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- Classification of Vectors

- Fundamental Principles

- Newton’s Laws
- Law of Gravitation, Units

- Vector operations

- Addition
- Subtraction
- Multiplication

Objectives of the Present lecture

- *To define Mechanics/ Statics*
- *To define scalar and vector quantities*
- *To provide an overview of most common vector operations*

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Definition

Mechanics …

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Definition

Mechanics is a science that describes and predicts the conditions of rest or motion under the action of forces.

Mechanics

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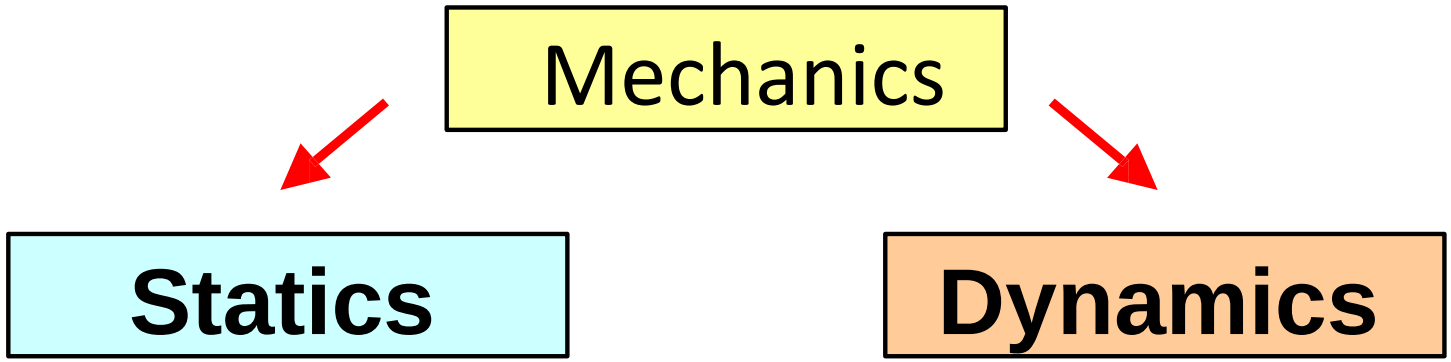
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Mechanics



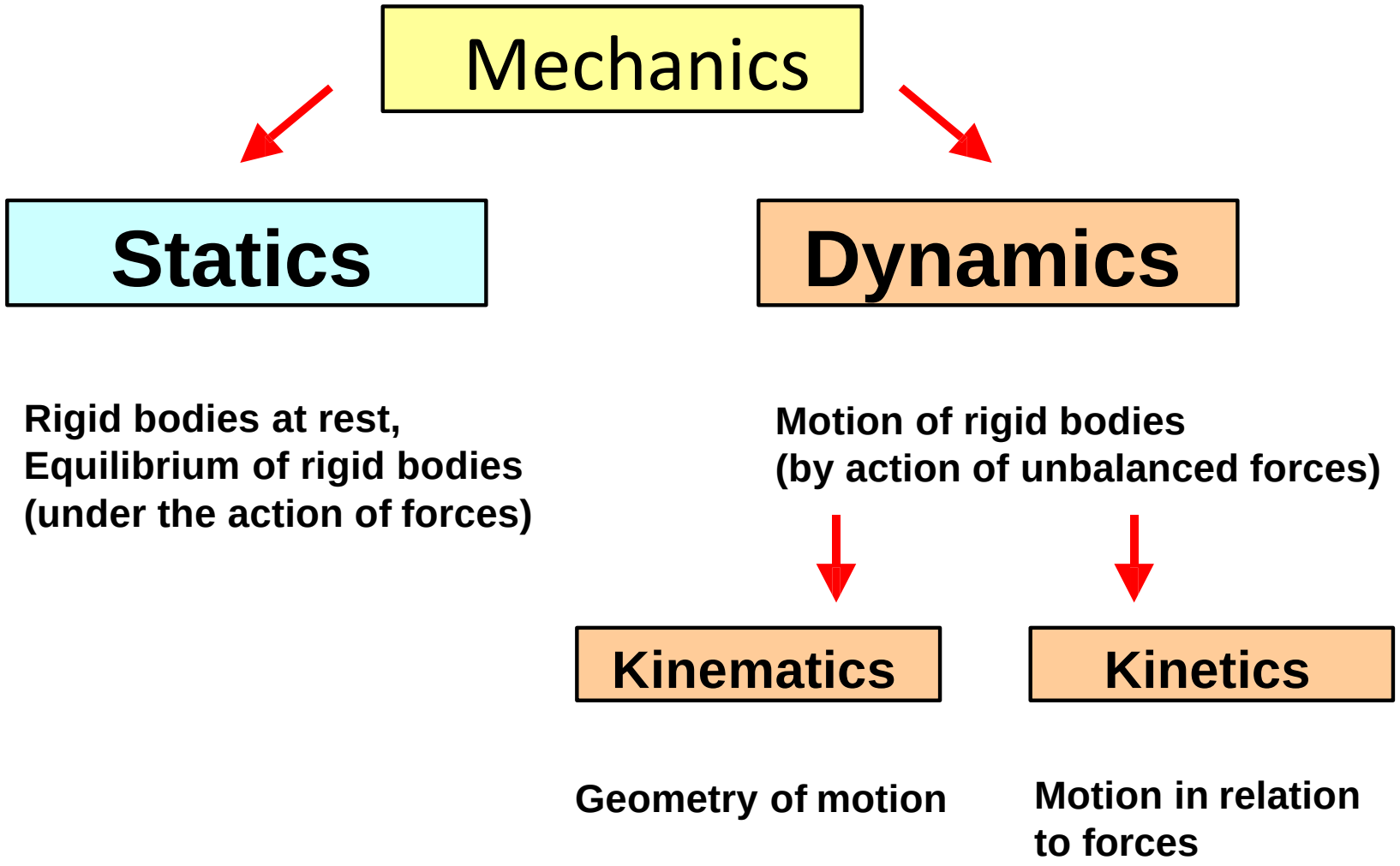
Statics

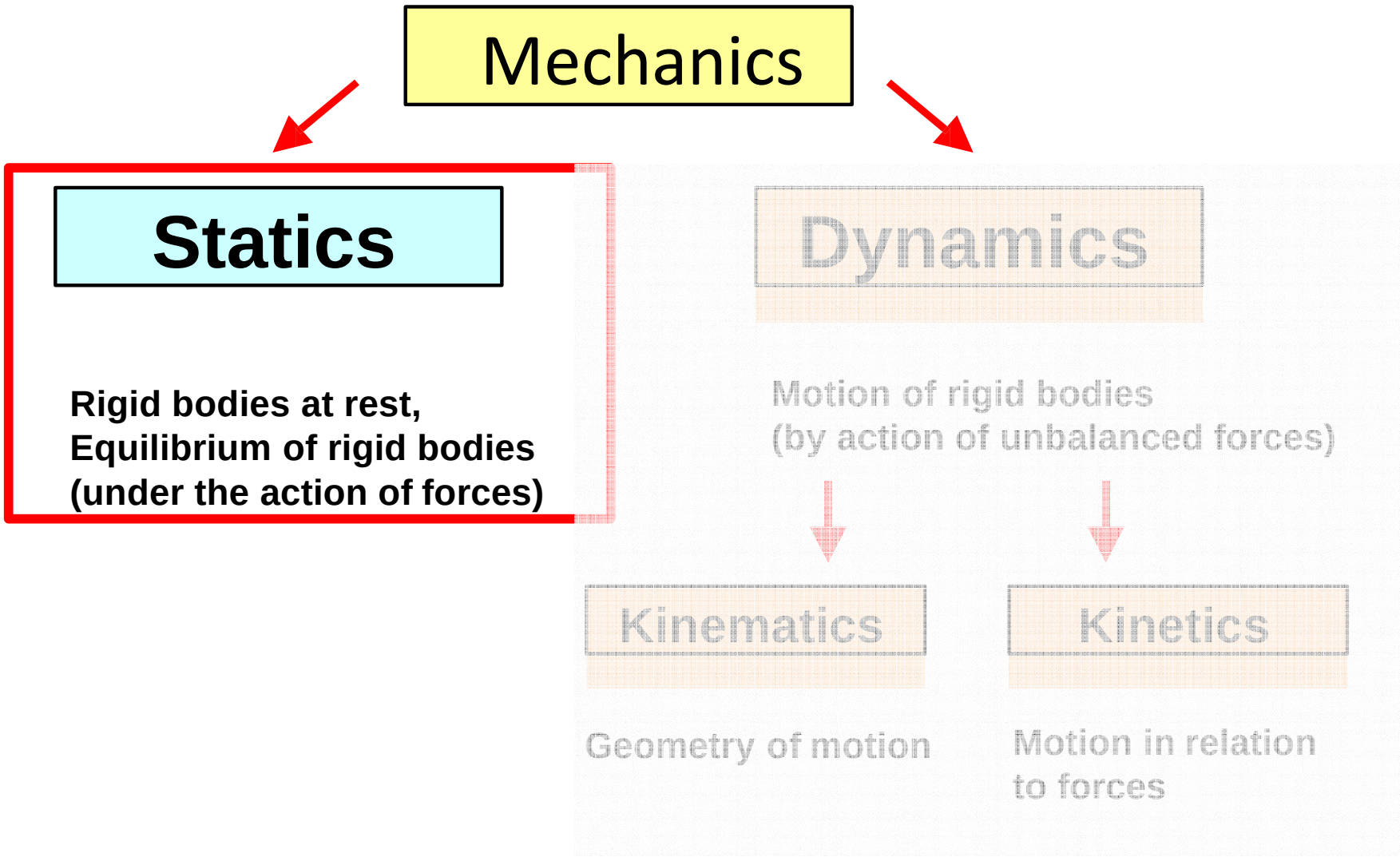
Rigid bodies at rest,
Equilibrium of rigid bodies
(under the action of forces)

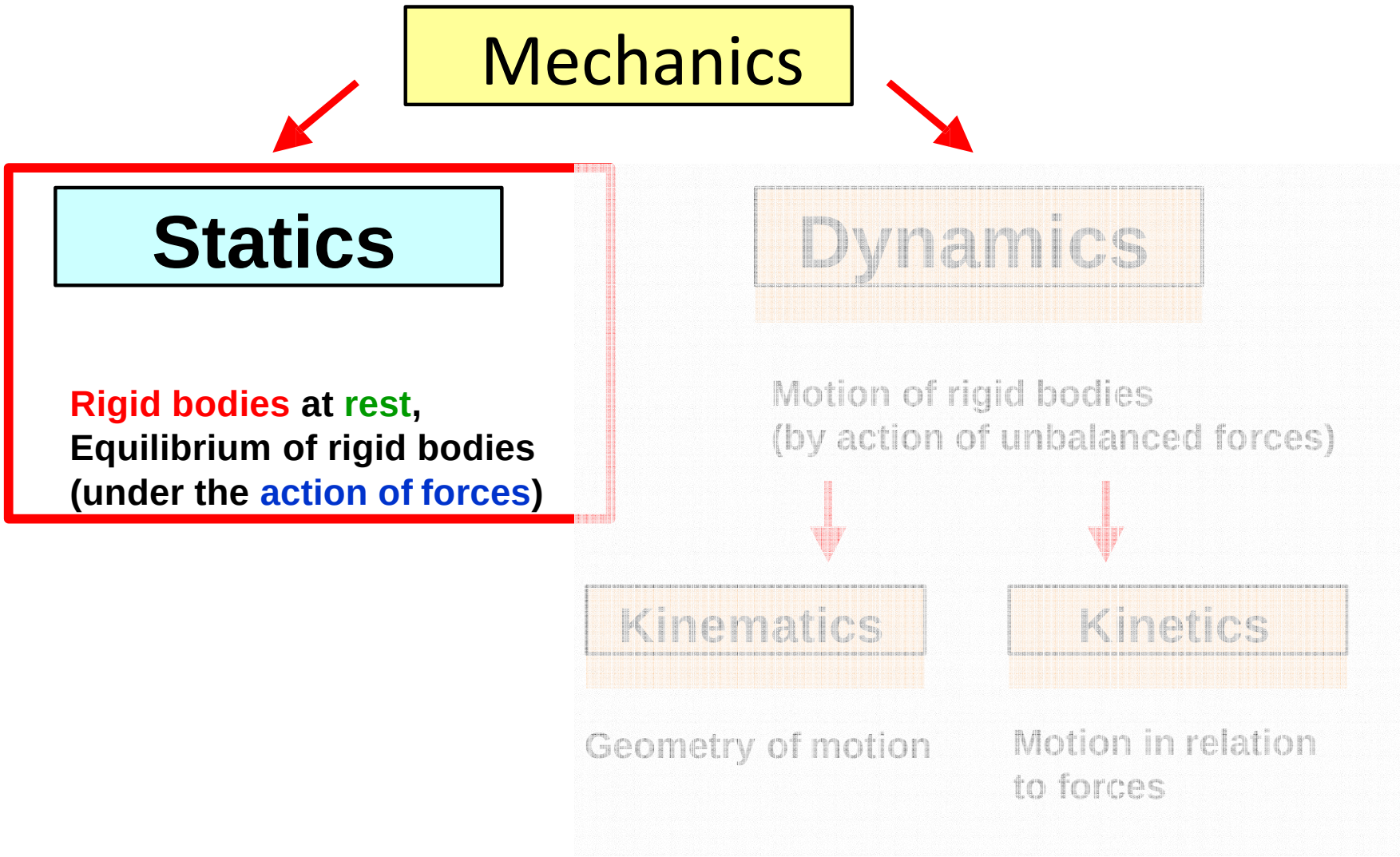


Rigid bodies at rest,
Equilibrium of rigid bodies
(under the action of forces)

Motion of rigid bodies
(by action of unbalanced forces)


Mechanics
Statics
Dynamics
Rigid bodies at rest,
Equilibrium of rigid bodies
(under the action of forces)
Motion of rigid bodies
(by action of unbalanced forces)
Kinematics
Kinetics
Geometry of motion
Motion in relation
to forces


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SCALAR QUANTITIES

A scalar is a quantity that has only magnitude, either positive or negative.

For example; mass, length, area, volume and speed are the scalar quantities frequently used in Statics.

Scalars are indicated by letters in *Itallic* type, such as the scalar '*V*'.

VECTOR QUANTITIES

A vector is a quantity that has both a magnitude and a direction.

For example; weight, force, moment, position, velocity and acceleration are the vector quantities frequently used in Statics.

Vectors are indicated by bold letters, such as '**V**' or $\vec{\mathbf{V}}$

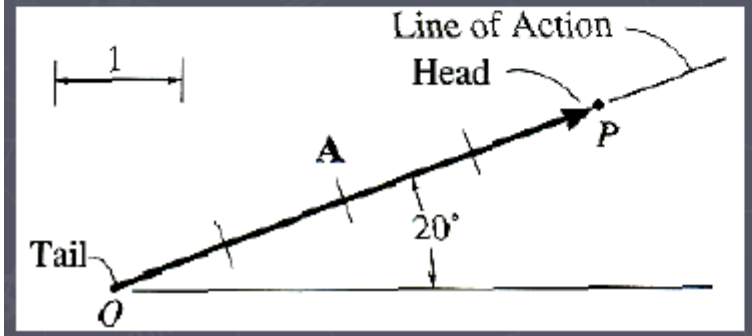
The magnitude of a vector is always a positive quantity and is symbolized in *Itallic* type, written as *V* or |V|

Scalars and Vectors

SCALARS AND VECTORS
Vector Quantities----contd.

- A vector is represented graphically by an arrow, which is used to define its
  - magnitude (by the length of the arrow)
  - direction (by the angle between a reference axis and the arrow's line of action)
  - sense (by the arrowhead)

For example, the vector shown below has



- A magnitude = 4 units
- A direction = 20°, measured counterclockwise from the horizontal axis
- A sense which is upward and to the right
- Point O is called *tail* of the vector
- Point P is called *tip or head* of the vector

Orientation of Vectors

collinear - same line of action

coplanar - located in the same plane

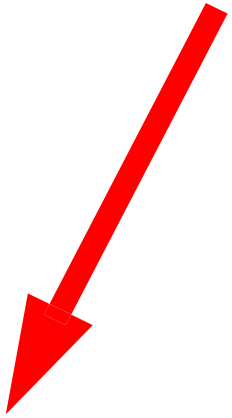
concurrent - passing through a common point

Classification of Vectors

1. Free Vector

2. Sliding Vector

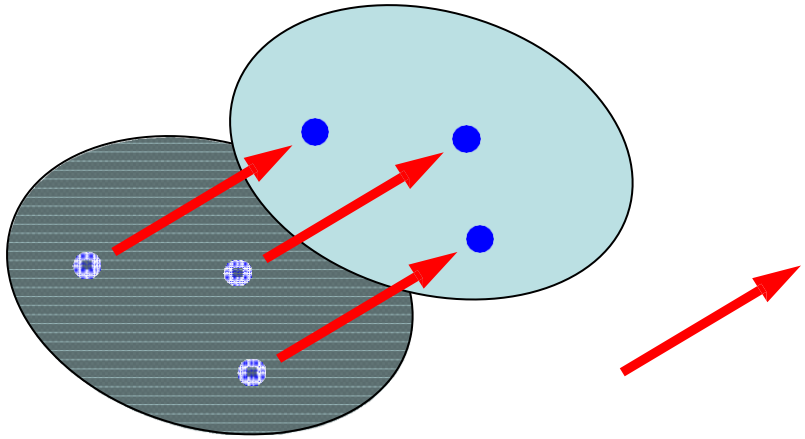
3. Fixed Vector



Vectors Classifications Cont'd

1. Free Vector: **action in space not associated with a unique line**

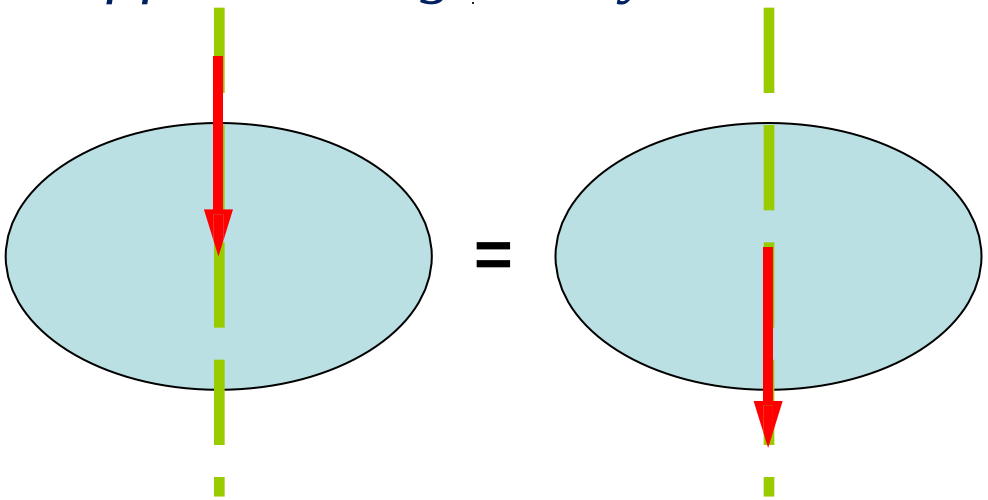
e.g. uniform displacement of a body



Vectors Classifications Cont'd

2. Sliding Vector: **Action in space described by a unique line**

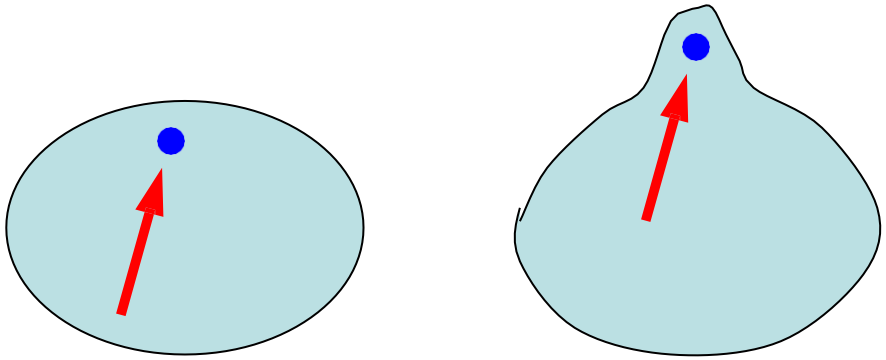
e.g. action of force on rigid body



Vectors Classifications Cont'd

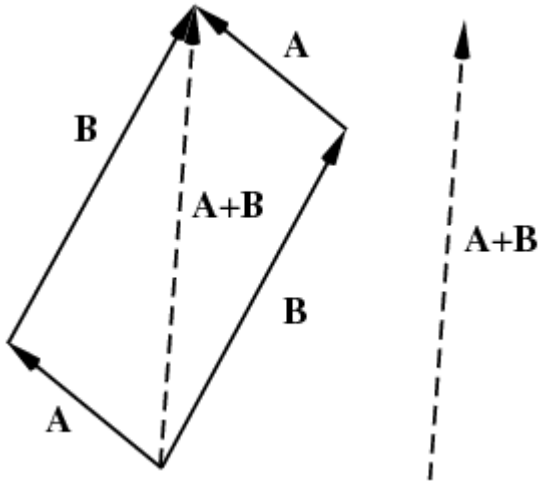
3. Fixed Vector: **Action in space described by a unique point**

e.g. action of force on non rigid body



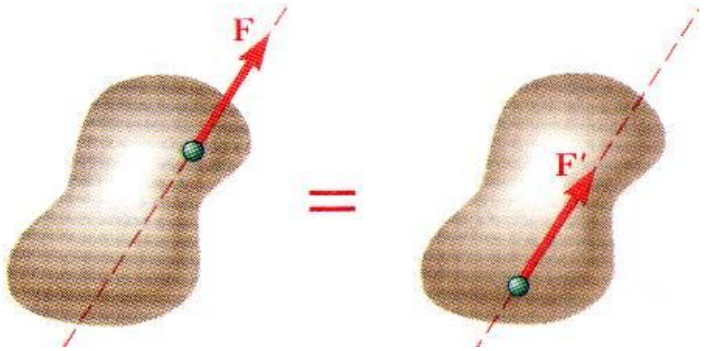
Fundamental Principles – Paralellogram Law & Transmissibility

- ***Parallelogram Law***



Vectors must obey the parallelogram law of combination. This law states that two vectors A and B, treated as free vectors, may be replaced by their equivalent vector (A+B), which is the diagonal of the parelellogram formed by A and B as its two sides, as shown in the figure.

- ***Principle of Transmissibility***
  Conditions of equilibrium or motion are not affected by *transmitting* a force along its line of action.
  NOTE: **F** and **F**' are equivalent forces.



VECTOR OPERATIONS

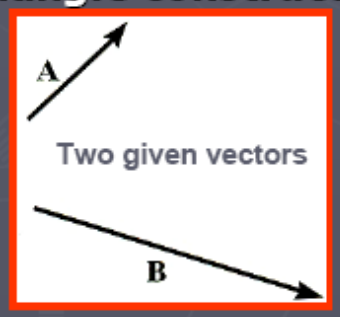
Vector Addition

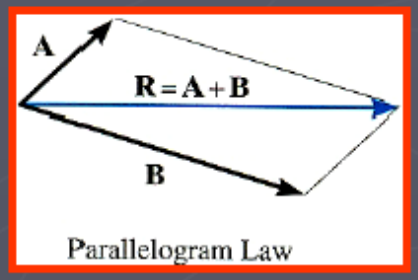
Two vectors A and B may be added to form a resultant vector

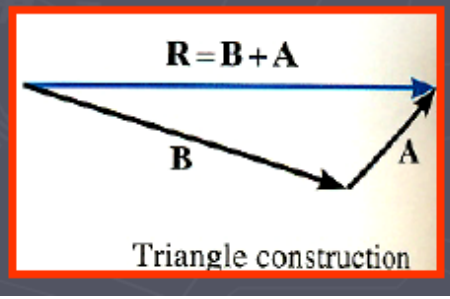
$$R = A + B$$

using the following methods:

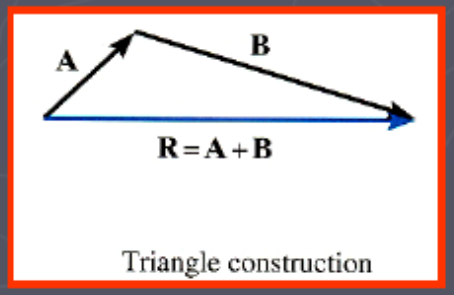
- **Parallelogram law**
- **Triangle construction**





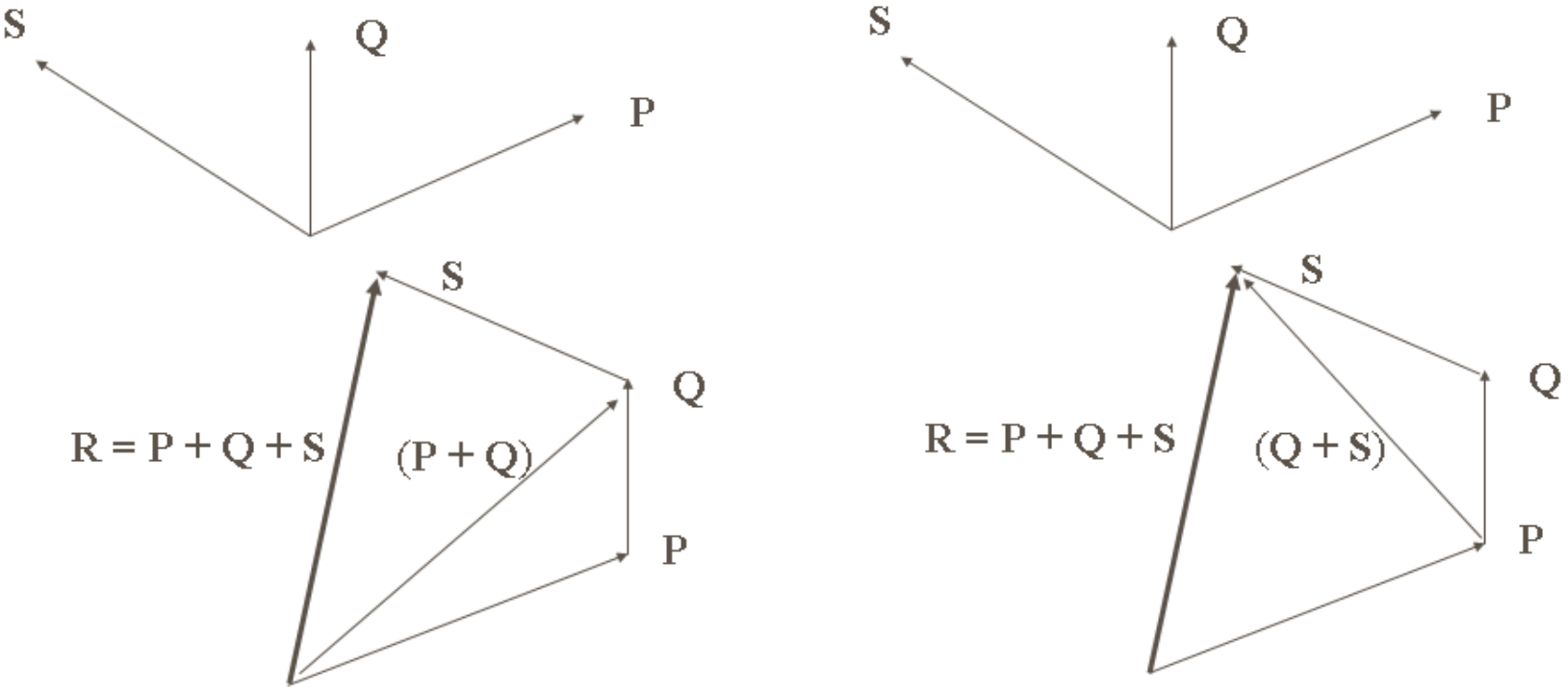


or



Polygon Rule:

can be used for the addition of more than two vectors. Two vectors are actually summed and added to the third and so on...



P + Q + S = (P + Q) + S = P + (Q + S)

This is the Aassociative Law of Vector Addition

VECTOR OPERATIONS

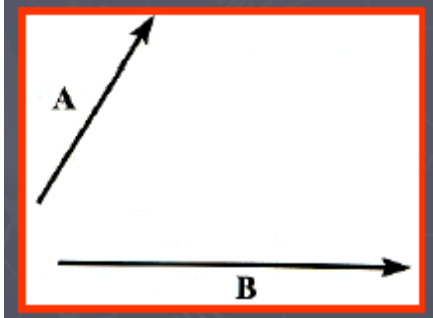
Vector Subtraction

- Vectors **A** and **B** may be subtracted to form a resultant vector

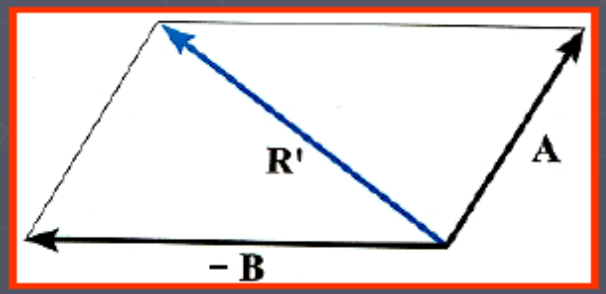
$$R' = A - B = A + (-B)$$

using the following methods:

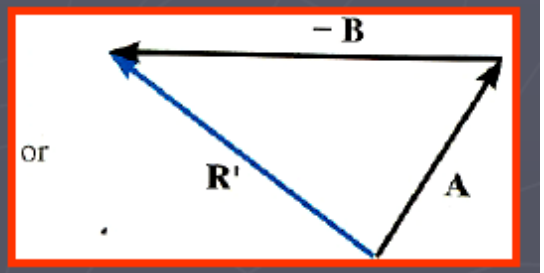
- Parallelogram law
- Triangle construction



Two given vectors



Parallelogram law

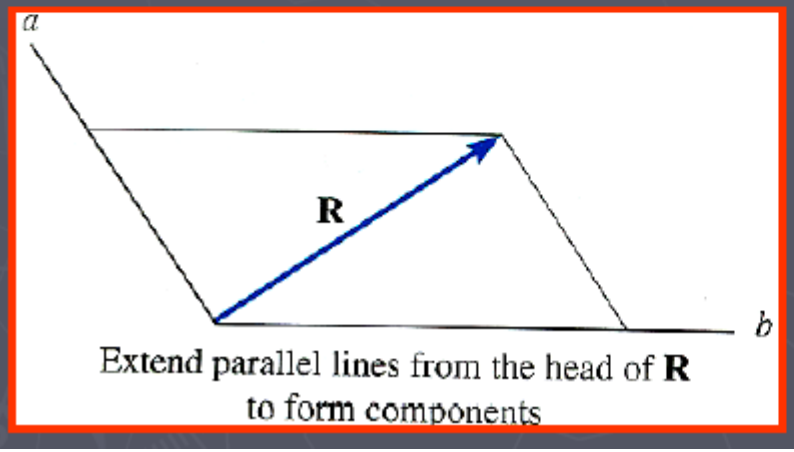


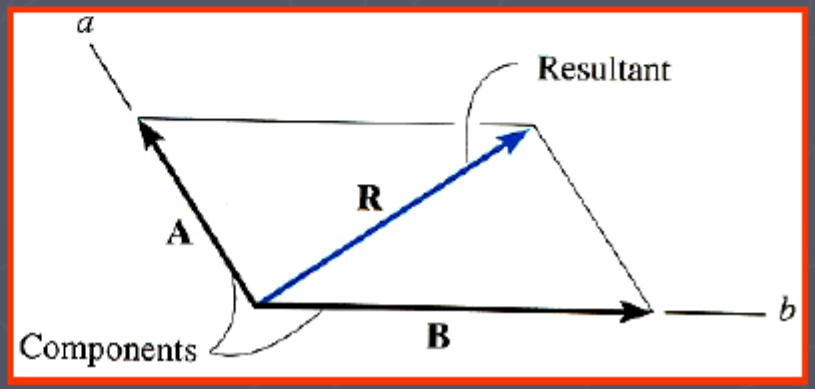
Triangular construction

VECTOR OPERATIONS

Resolution of Vector

- A vector may be resolved into two components having known lines of action by using the parallelogram law.
- For example, a vector R may be resolved into two vectors A and B along the lines a and b, using the parallelogram law, as shown below:



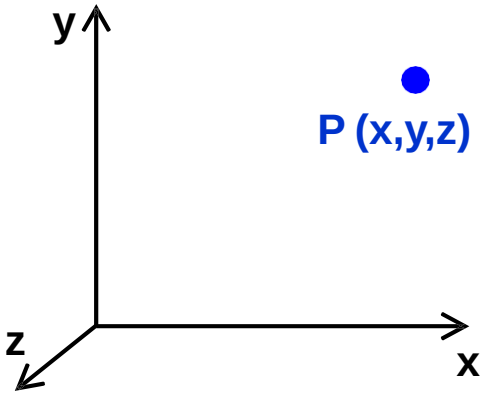


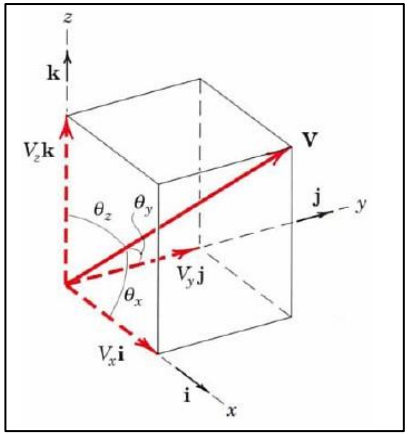
Fundamental Principles

Basic concepts used in mechanics:

space, time, mass, force, particle, rigid body

coordinates - position of a point P (x, y, z)
measured from a certain point of reference



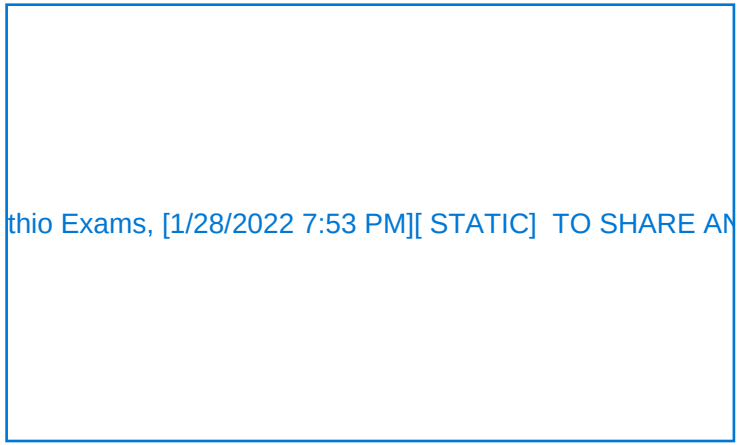


Fundamental Principles

Basic concepts used in mechanics:

space, time, mass, force, particle, rigid body time

of an event taking place,
determination of velocity and acceleration



Fundamental Principles

Basic concepts used in mechanics:

space, time, mass, force, particle, rigid body

mass of a body [kg, to]

action of weight, behaviour under the action of an external force

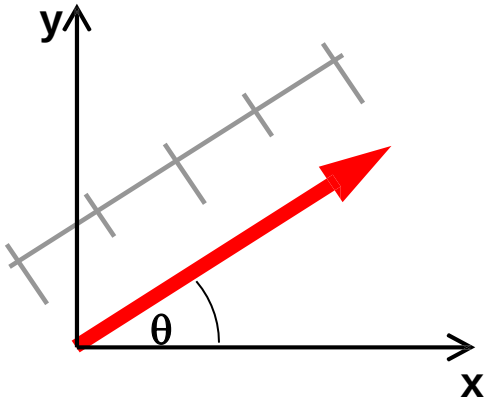
Fundamental Principles

Basic concepts used in mechanics:

space, time, mass, force, particle, rigid body

magnitude, direction, point of application

e.g. action on a rigid body, action of one body onto another

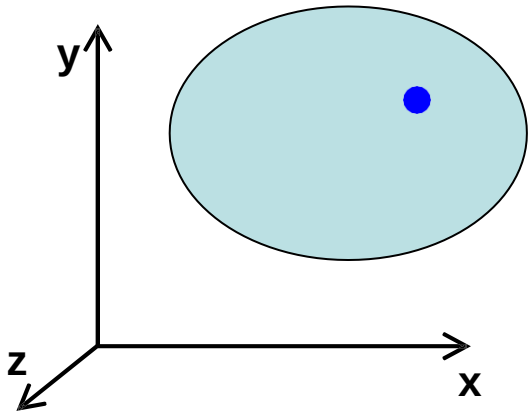


Fundamental Principles

Basic concepts used in mechanics:

space, time, mass, force, particle, rigid body

infinitesimal small piece of a body, single point in space



Fundamental Principles

Basic concepts used in mechanics:

space, time, mass, force, particle, rigid body body

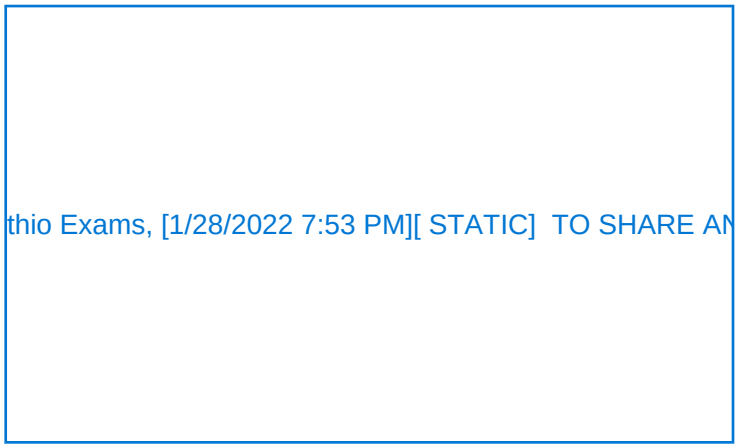
consisting of a non-deformable material
(no displacement under the action of forces)

Fundamental Principles –

Newton's Laws

Sir Isaac Newton (1642-1727)

1st Law: **A particle remains at rest or continues to move with constant velocity if the resultant force acting on it is zero.**



Fundamental Principles –

Newton's Laws

Sir Isaac Newton (1642-1727)

2nd Law: **The acceleration of a particle is proportional to the resultant force acting on it (magnitude and direction).**

F = ma

m = mass of particle a

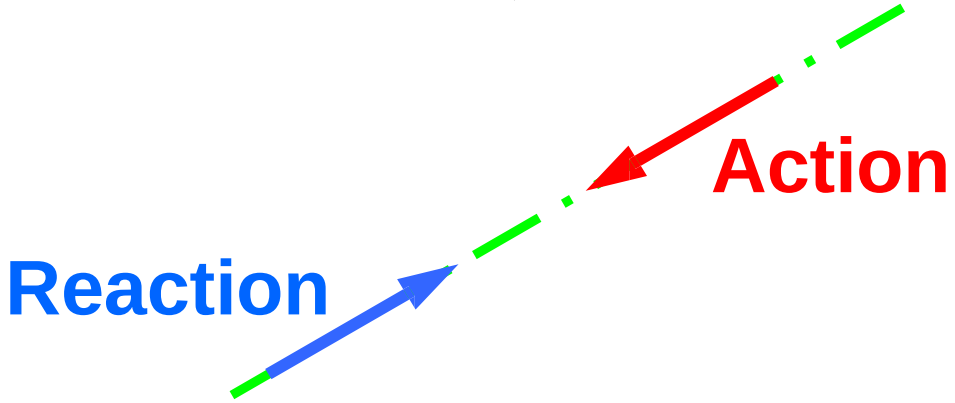
= acceleration

Fundamental Principles –

Newton's Laws

Sir Isaac Newton (1642-1727)

3rd Law: **The forces of action and reaction between bodies in contact are equal in magnitude, opposite in direction and collinear (same line of action).**



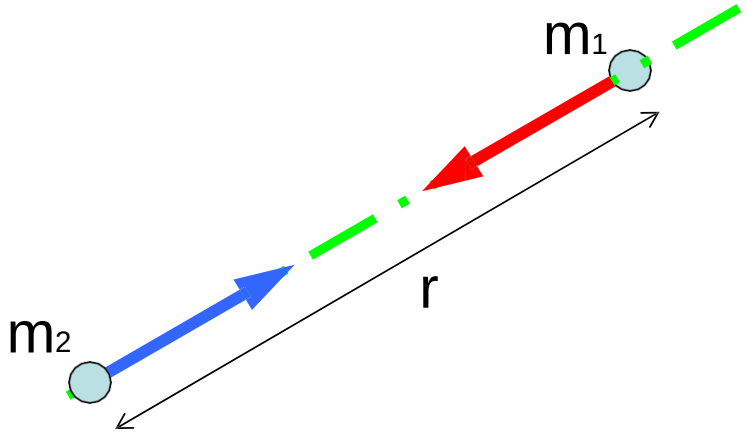
Fundamental Principles – Newton's Laws

Law of Gravitation

Two particles of mass m_1 and m_2 are mutually attracted with equal and opposite forces F and F' of magnitude F.

$$F = G\frac{m_1 m_2}{r^2}$$

G = constant of gravitation



Fundamental Principles – Newton's Laws

Law of Gravitation

Weight = **Gravitational Force acting on a body** (attraction between earth and body)

$W = m \cdot g$ g = acceleration of gravity = 9.81 m/s^2

Fundamental Principles – Newton's Laws

Law of Gravitation

Weight = **Gravitational Force acting on a body** (attraction between earth and body)

$$W[N] = m[kg] \cdot g[m/s^2]$$

$g = 9.81\ m/s^2$

Units

International System of Units (SI units)

Mass	m	[gm, kg]
Force	F	[kN, N]
Time	t	[s]
Length	L	[m, cm, mm]

Units

International System of Units (SI units)

example

Express a body mass of 80 kg in the unit of Newton.

$W = 80\ \text{kg} \cdot 9.81\ \text{m/s}^2 = 784.8\ \text{N} = 0.785\ \text{kN}$

approximation: e.g. 100 kg = 1.0 kN

commonly used in the field of civil engineering

Vector Operations

- **Vector addition**

- parallelogram law, triangle rule
- composition and decomposition of forces

- **Vector multiplication**

- dot and cross products

Vector Addition – graphical method

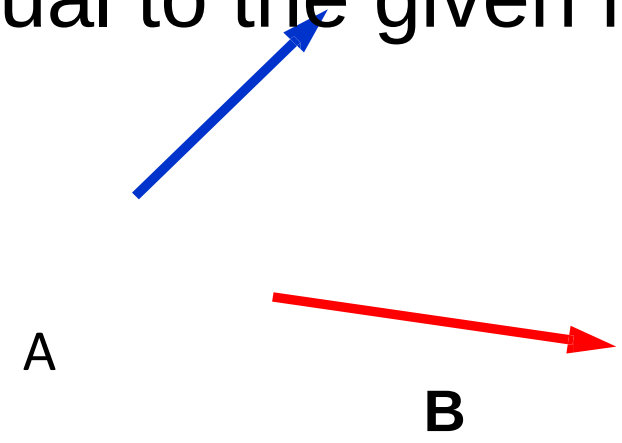
- The parallelogram law – Resultant force

- Two forces maybe replaced by a single force (resultant) obtained by drawing the diagonal of the parallelogram having sides equal to the given forces.

Vector Addition – graphical method

- The parallelogram law – resultant force

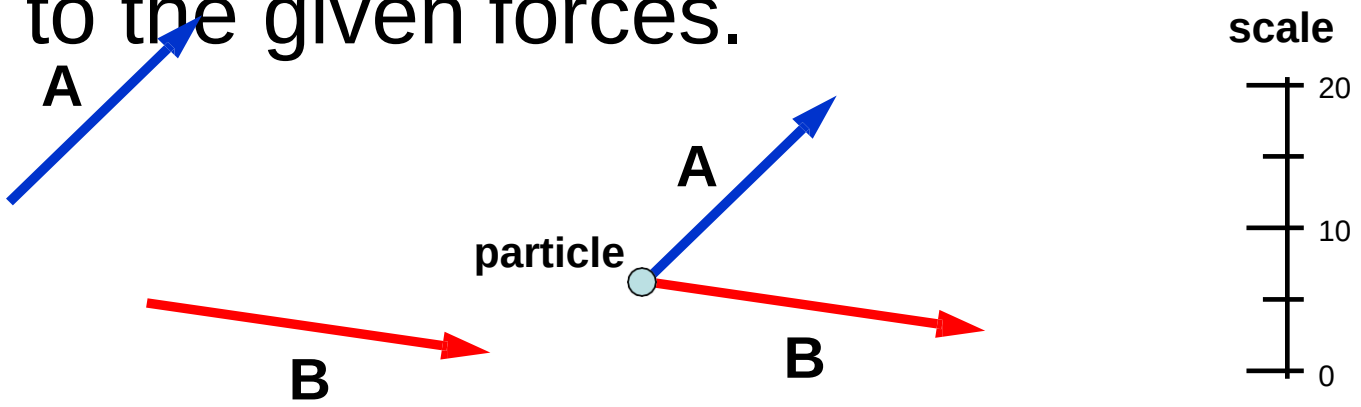
- Two forces maybe replaced by a single force (resultant) obtained by drawing the diagonal of the parallelogram having sides equal to the given forces.



Vector Addition – graphical method

- The parallelogram law – resultant force

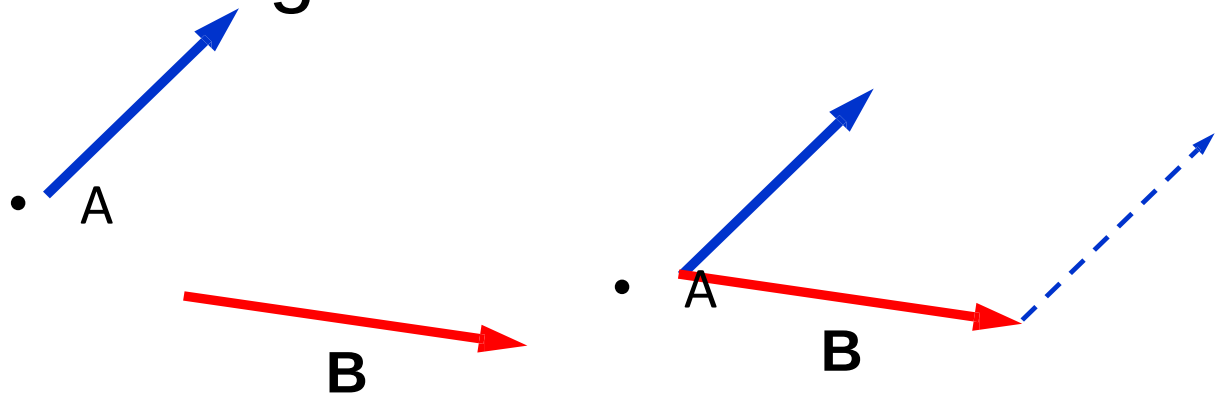
- Two forces maybe replaced by a single force (resultant) obtained by drawing the diagonal of the parallelogram having sides equal to the given forces.



Vector Addition – graphical method

- The parallelogram law – resultant force

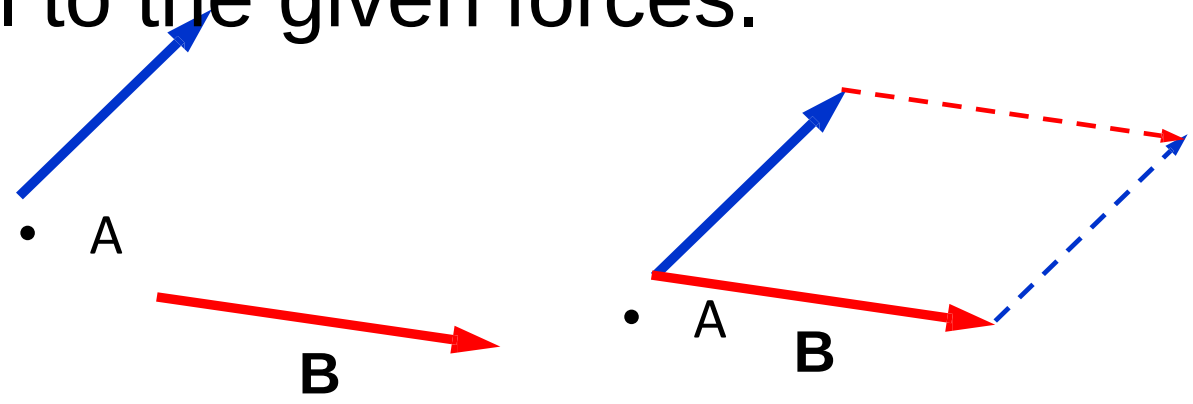
- Two forces maybe replaced by a single force (resultant) obtained by drawing the diagonal of the parallelogram having sides equal to the given forces.



Vector Addition – graphical method

- The parallelogram law – resultant force

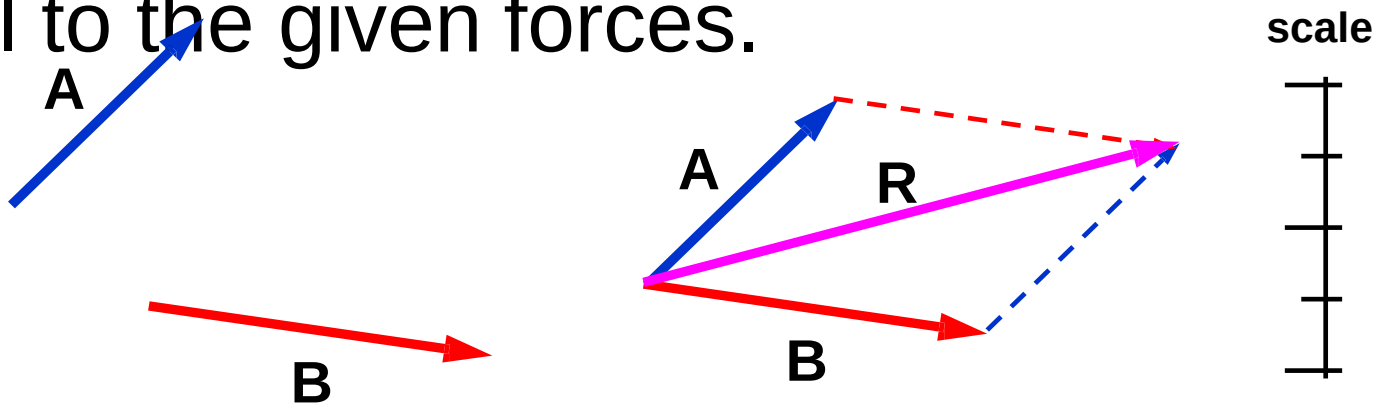
- Two forces maybe replaced by a single force (resultant) obtained by drawing the diagonal of the parallelogram having sides equal to the given forces.



Vector Addition – graphical method

- The parallelogram law – resultant force

- Two forces maybe replaced by a single force (resultant) obtained by drawing the diagonal of the parallelogram having sides equal to the given forces.



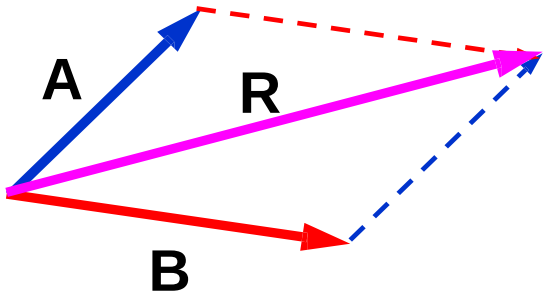
Vector Addition – graphical method

- The parallelogram law – resultant force

- Two forces maybe replaced by a single force (resultant) obtained by drawing the diagonal of the parallelogram having sides equal to the given forces.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Vector addition is commutative!



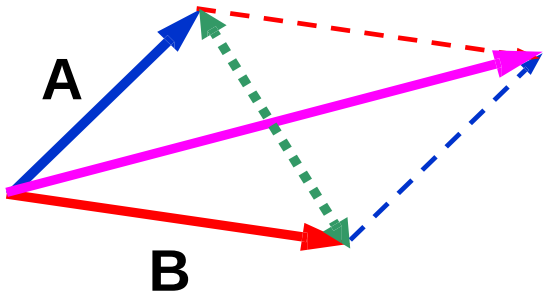
Vector Addition – graphical method

- The parallelogram law – resultant force

- Two forces maybe replaced by a single force (resultant) obtained by drawing the diagonal of the parallelogram having sides equal to the given forces.

$$\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$$

Vector subtraction is **not** commutative!

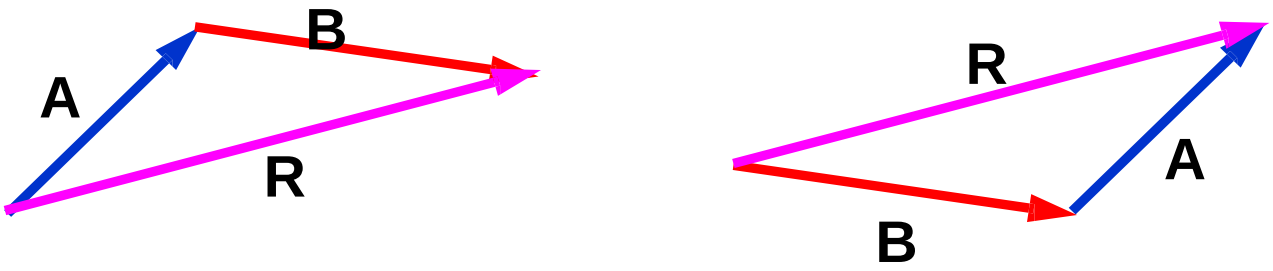


Vector Addition – graphical method

The triangle rule (from parallelogram law)

for two forces

$$R = A + B = B + A$$

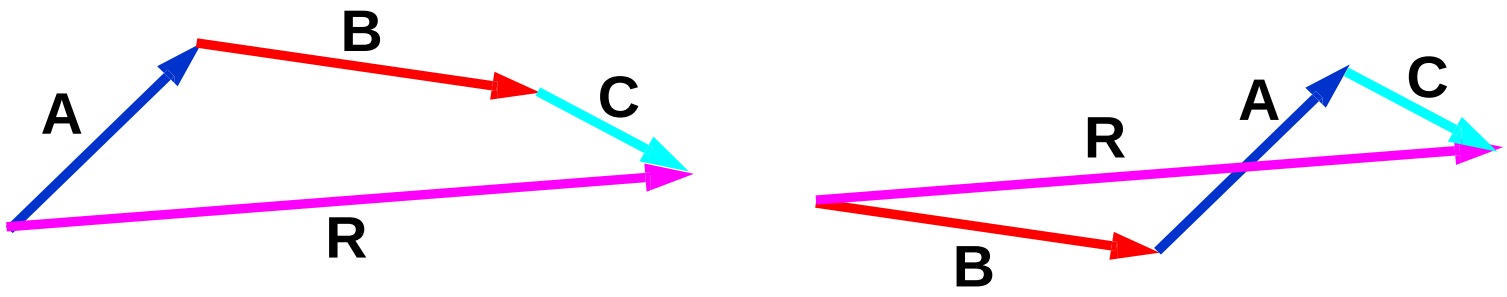


Vector Addition – graphical method

The triangle rule (from parallelogram law)

for three forces

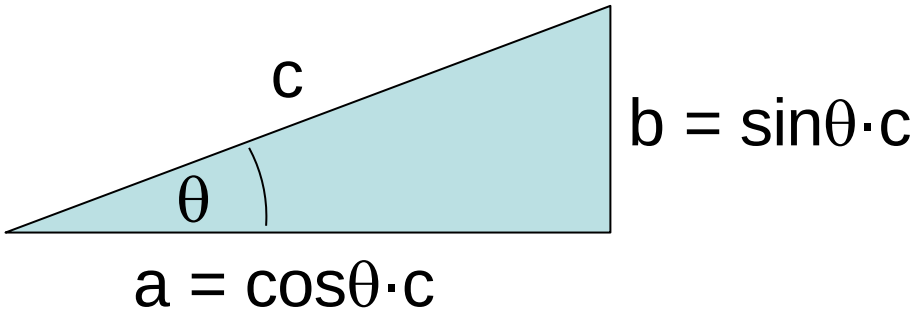
$$\vec{R} = \vec{A} + \vec{B} + \vec{C} = \vec{B} + \vec{A} + \vec{C} = \ldots$$



Vector Addition – analytic method

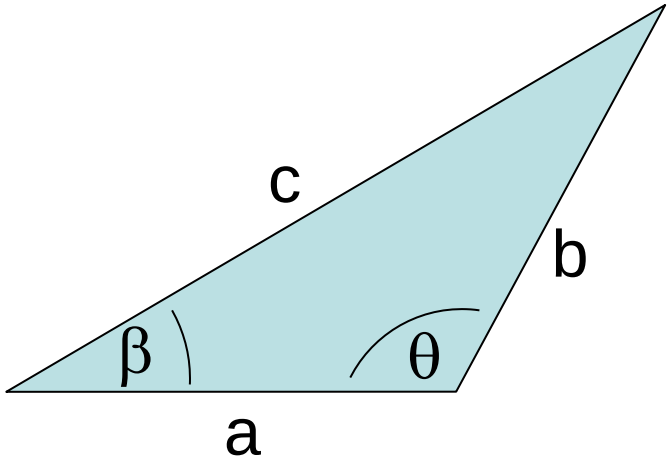
trigonometric rules

applying sine and cosine rules



Vector Addition – analytic method

trigonometric rules

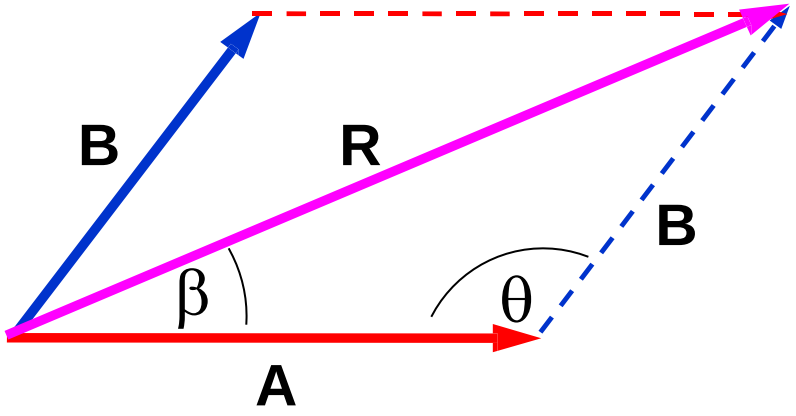


$$c^2 = a^2 + b^2 - 2ab \cdot \cos\theta$$

$$\frac{\sin\beta}{b} = \frac{\sin\theta}{c}$$

Vector Addition – analytic method

trigonometric rules

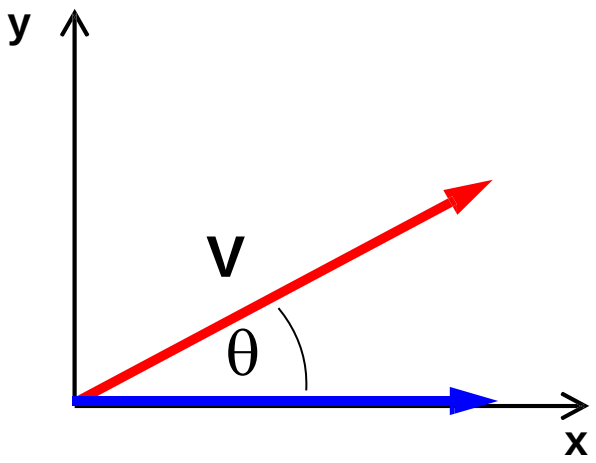


$$\|R\|^2 = \|A\|^2 + \|B\|^2 - 2\|A\|\|B\|\cdot\cos\theta$$ magnitude

$$\frac{\sin\beta}{\|B\|} = \frac{\sin\theta}{\|R\|}$$ Inclination of R from A

Decomposition of Vectors

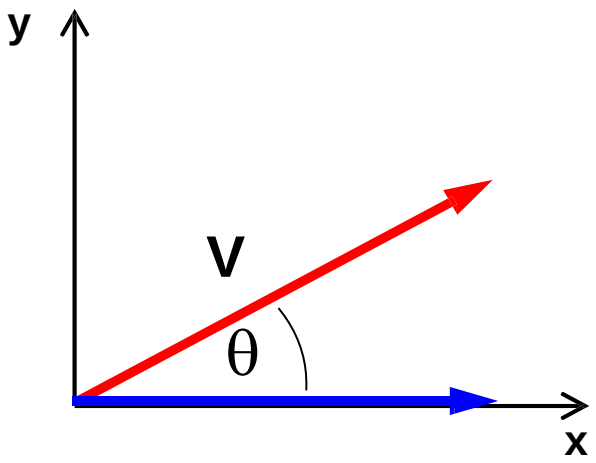
Components, perpendicular



horizontal component of V:

Decomposition of Vectors

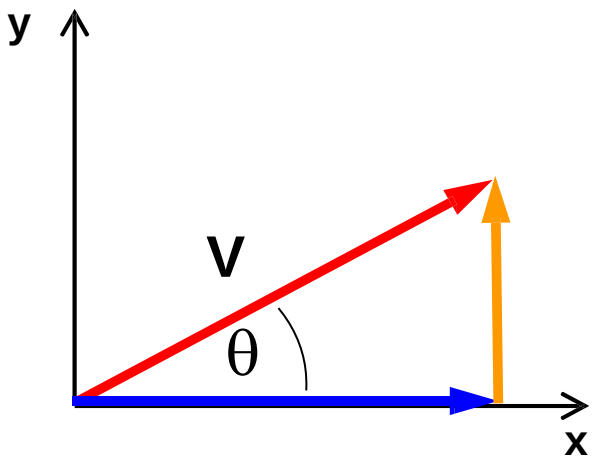
Components



horizontal component of V: $V_x = V \cdot \cos\theta$

Decomposition of Vectors

Components

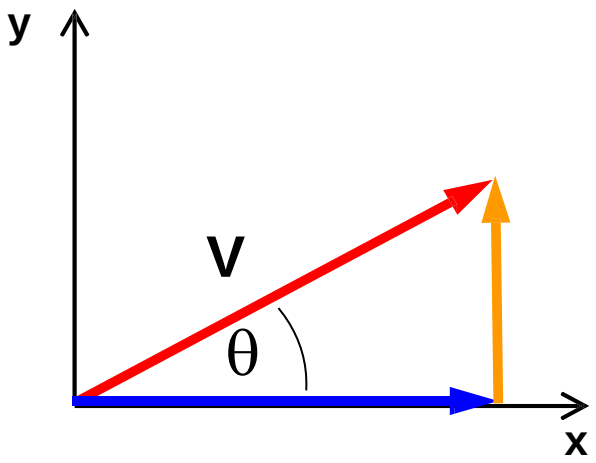


horizontal component of V: $V_x = V \cdot \cos\theta$

vertical component of V:

Decomposition of Vectors

Components

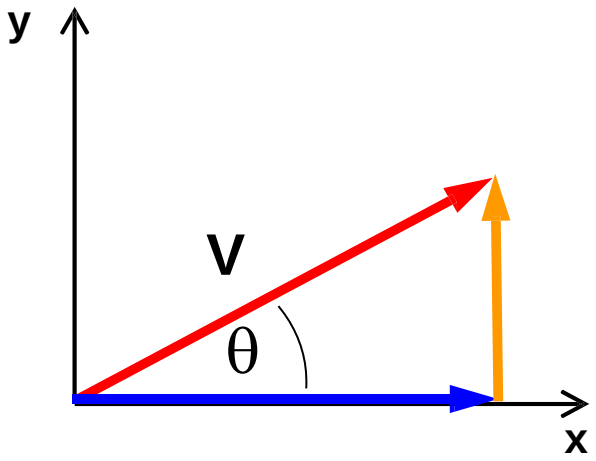


horizontal component of V: $V_x = V \cdot \cos\theta$

vertical component of V: $V_y = V \cdot \sin\theta$

Decomposition of Vectors

Components



horizontal component of V: $V_x = V \cdot \cos\theta$

vertical component of V: $V_y = V \cdot \sin\theta$

Multiplication

Multiplication of vectors by scalars

$$n \cdot \vec{A}$$

$$n \cdot \left(\vec{A} + \vec{B} \right) = nA + nB$$

$$(n + m) \cdot \vec{A} = nA + mA$$

Multiplication

Multiplication of vectors by vectors

- dot product (scalar product)

- cross product (vector product)

Dot Product (scalar product)

Vectors A and B are θ inclined from each other

$$A.B = \|A\| \cdot \|B\| \cdot \cos\theta$$

result = scalar

$$A.B = a_x\, b_x + a_y b_y + a_z b_z$$

Cross Product (vector product)

Vectors A and B are θ inclined from each other.

Result : Vector of determined magnitude and direction perpendicular to the plane formed by A and B

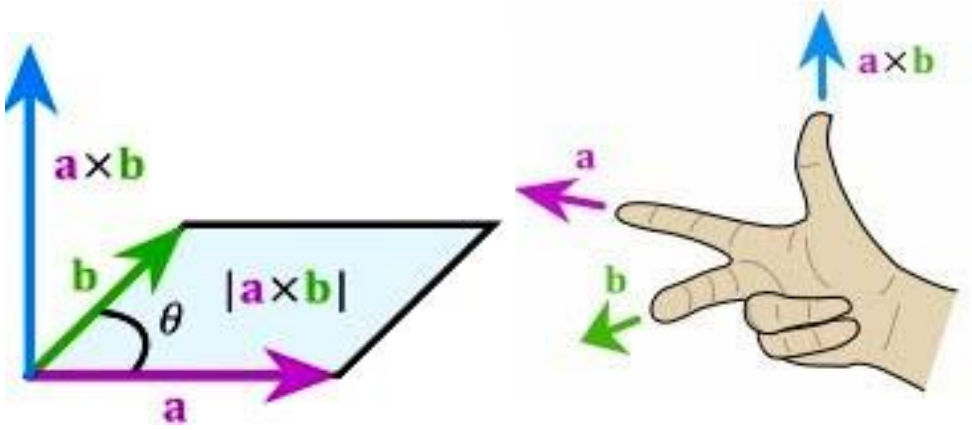
$$\mathrm{A} \times \mathrm{B} = \left\|\mathrm{A}\right\| \cdot \left\|\mathrm{B}\right\| \cdot \sin\theta$$

Cross Product (vector product)

Vectors A and B are θ inclined from each other.

Result : Vector of determined magnitude and direction perpendicular to the plane formed by A and B

$$A \times B = \|A\| \cdot \|B\| \cdot \sin\theta$$



Cross Product (vector product)

Vectors A and B are θ inclined from each other.

Result : Vector of determined magnitude and direction perpendicular to the plane formed by A and B

$$A \times B = \|A\| \cdot \|B\| \cdot \sin\theta$$

$$A \times B = \left(a_y b_z - a_z b_y\right)i + \left(a_z b_x - a_x b_z\right)j + \left(a_x b_y - a_y b_x\right)k$$

Cross Product (vector product)

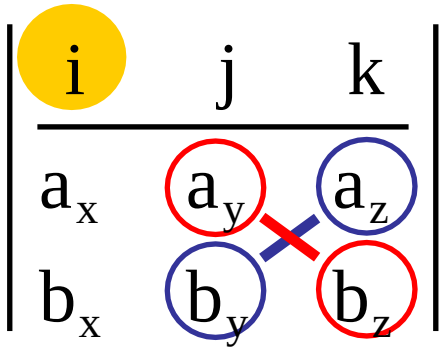
Determination of resulting vector by three by three matrix

$$\begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$A \times B = \left(a_y b_z - a_z b_y\right)i + \left(a_z b_x - a_x b_z\right)j + \left(a_x b_y - a_y b_x\right)k$$

Cross Product (vector product)

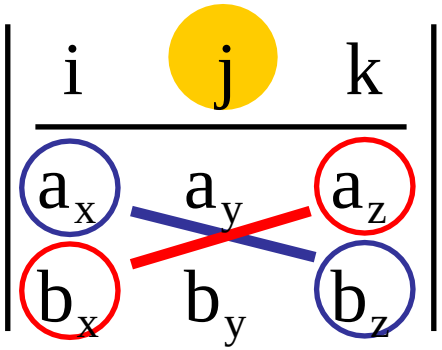
Determination of resulting vector by three by three matrix



$$A \times B = (a_y b_z - a_z b_y)i + (a_z b_x - a_x b_z)j + (a_x b_y - a_y b_x)k$$

Cross Product (vector product)

Determination of resulting vector by three by three matrix



$$A \times B = \left(a_y b_z - a_z b_y\right)i + \left(a_z b_x - a_x b_z\right)j + \left(a_x b_y - a_y b_x\right)k$$

Cross Product (vector product)

Determination of resulting vector by three by three matrix

$$\begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$A \times B = \left(a_y b_z - a_z b_y\right)i + \left(a_z b_x - a_x b_z\right)j + \left(a_x b_y - a_y b_x\right)k$$

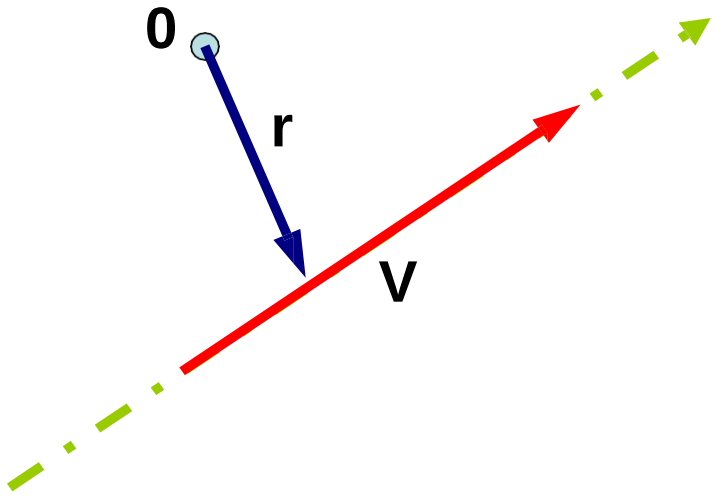
Cross Product (vector product)

Moment of a vector V about any point 0

r is a position vector from point 0 to any point on the line of action of vector **V**.

$$\mathbf{M}_0 = \mathbf{r} \times \mathbf{V}$$

$$\mathbf{M}_0 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ V_x & V_y & V_z \end{vmatrix}$$



Thank you …

thio Exams, [1/28/2022 7:53 PM]
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