

SIMILARITY AND CONGRUENCE

1. SIMILARITY

Two objects are said to be similar if:

- The corresponding angles are equal
- The ratio of the corresponding sides is the same or equal.

1.1 SIMILARITY IN TRIANGLES

For two triangles to be similar, they need to satisfy any of the three cases:

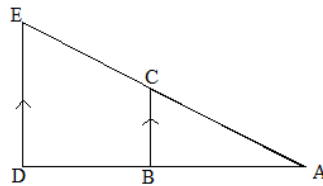
- Three pairs of corresponding angles are equal (AAA)
- The ratio of corresponding sides is the same (SSS)
- Two pairs of corresponding sides are proportional and the included angles are equal (SAS).

2. CONGRUENCY

Two objects are congruent if they have the same shape and size.

QUESTIONS

- 1) State the two triangles in the diagram below which are similar. Give the reason why.

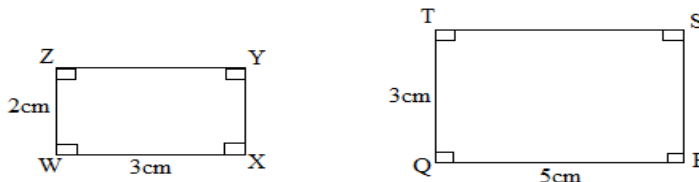


Answer: $\triangle ABC$ and $\triangle ADE$ are similar.

Since DE and BC are parallel, $\angle ABC = \angle ADE$ and $\angle ACB = \angle AED$

(Corresponding Angles). $\angle A$ is common to both triangles, hence satisfying **AAA**.

- 2) Determine whether or not the two rectangles below are similar

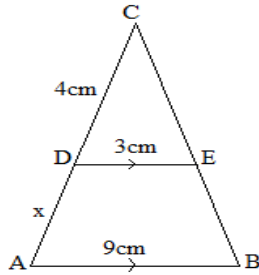


Solution

$$2:3 \neq 3:5$$

Therefore XYZW and TQRS are not similar.

- a) In the diagram, DE is parallel to AB, $DE = 3\text{cm}$, $AB = 9\text{cm}$ and $CD = 4\text{cm}$



b) Find the ratio of corresponding sides.

c) Find the value of x

Solution

a) $3: 9 = 1: 3$

b) $\frac{3}{9} = \frac{4}{4+x}$

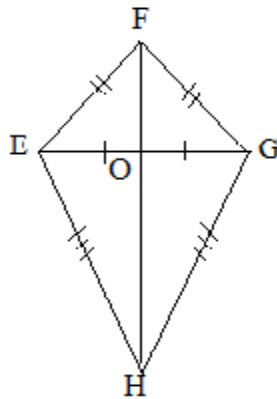
$$3(4+x) = 9 \times 4$$

$$12 + 3x = 36$$

$$3x = 24$$

$$\mathbf{X = 8cm}$$

3) Name all the pairs of congruent triangles in the figure below:



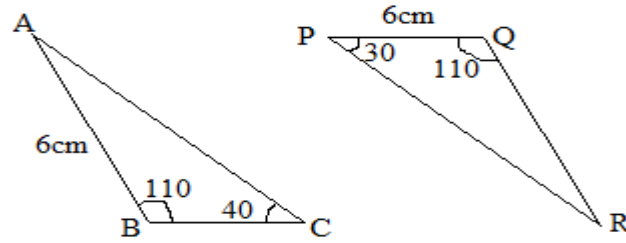
Solution

$$\mathbf{\Delta EOF = \Delta GOF}$$

$$\mathbf{\Delta EOH = \Delta GOH}$$

$$\mathbf{\Delta FEH = \Delta FGH}$$

4) Show that ΔABC and ΔPQR below are congruent



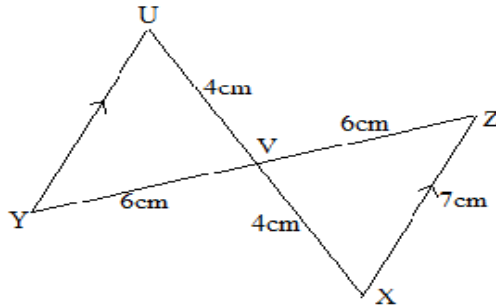
Answer:

$\angle ABC = \angle PQR$,

$\angle BAC = \angle RPQ = 30^\circ$ and

$\angle BCA = \angle PRQ = 40^\circ$

5) Find the length of YU in the diagram below



Solution:

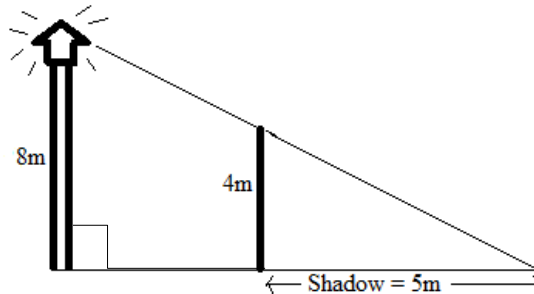
$\angle UYV = \angle XVZ$ (Vertically opposite angles)

$UV = XV$

$YV = VZ$

Therefore $UY = XZ = 7\text{cm}$.

6) A wall, which is 4m high, is built next to a street light that is 8m high. The shadow of the wall is 5m long. How far is the wall from the street light?



Solution

$$\frac{4}{8} = \frac{5}{5+x}$$

$$4(5+x) = 8 \times 5$$

$$20 + 4x = 40$$

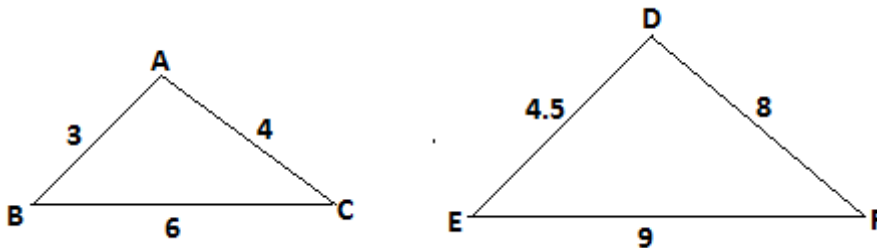
$$X = 5\text{m}$$

NOTE: If two figures are similar and the lengths of their corresponding sides are in the ratio $p:q$, then the ratio of their area is $p^2:q^2$, and the ratio of their volumes is $p^3:q^3$.

PRACTICE QUESTIONS:

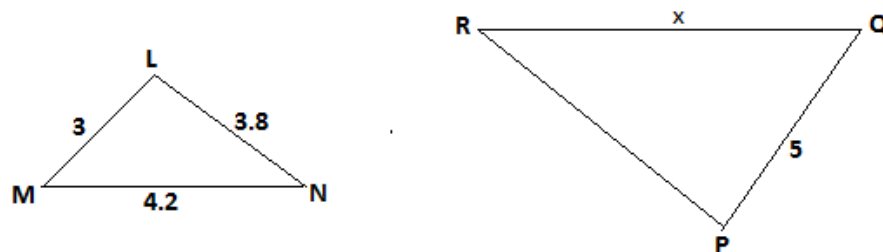
- (1) A model of a tanker is made using a scale of 1: 20.
- (a) The length of the tanker is 15 m. Calculate the length, in centimetres, of the model.
- (b) The model holds 12 litres of liquid. Calculate the number of litres the tanker will hold.

(2)(a)



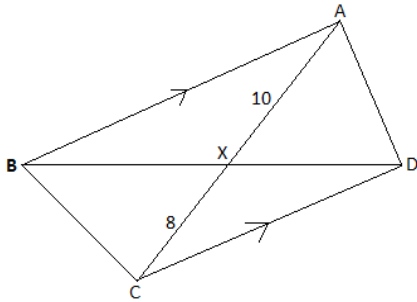
Are triangles ABC and DEF similar? Explain your answer clearly.

(b)



Triangle LMN and PQR are similar. Calculate the value of x.

- (3) In the diagram, ABCD is a quadrilateral with BA parallel to CD. AC and BD meet at X where CX = 8 cm and XA = 10 cm.

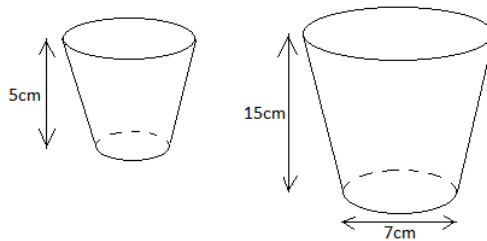


(a) Given that $BD = 27 \text{ cm}$, find the length BX .

(b) Find the ratio **area of triangle BXC : area of triangle AXD**.

(4) Two pots are geometrically similar. The height of the smaller pot is 5 cm.

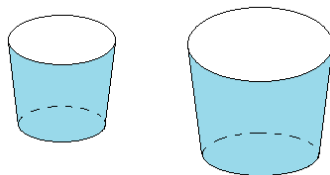
The height of the bigger pot is 15 cm.



(a) The diameter of the base of the larger pot is 7 cm. Find the diameter of the base of the smaller pot.

(b) Find the ratio of the volume of the smaller pot to that of the larger. Give your answer in the form **1 : n**.

(5) The ratio of the **areas** of the bases of two geometrically similar buckets is 4 : 9.



(a) The **area** of the top of the smaller bucket is 480 cm^2 .

What is the **area** of the top of the larger bucket?

(b) **Write down** the ratio of the **heights** of the two buckets.

(c) Both buckets are filled with sand. The **mass** of sand in the larger bucket is 36 kg.

Find the **mass** of sand in the smaller bucket.

ANSWERS:

- (1) (a) 75cm , (b) 96000 litres
- (2) (a) **not similar, because not all** the 3 corresponding sides are proportional.
(b) 7
- (3) (a) 15 cm , (b) 1 : 1
- (4) (a) $2\frac{1}{3}$ cm, (b) 1 : 27
- (5) (a) 1080 cm² , (b) 2 : 3 , (c) $10\frac{2}{3}$ kg .