HANDWRITTEN NOTES

BY~

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SHM

$y = a\sin(\omega t \pm \phi)$ (+) → ACW (−) → CW

$y = a\sin\omega t$

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ ellipse $a \neq b$

$\frac{d^2y}{dt^2} + \omega^2 y = 0$

$A = -\omega^2 y$ $F = -ky$ $k = m\omega^2$

sin (+ve) → θ → I, π − θ → II → +ve (II: S, I: A)

sin (−ve) → π + θ → III, 2π − θ → IV → −ve (III: T, IV: C)

Velo.

$V_{particle}$ → time 't' → $V = a\omega\cos\omega t$ → $V_{max} = a\omega$ at $t = 0$ (mean); $V_{min} = 0$ at $t = T/4$ (extreme)

$V_{particle}$ → displacement 'y' → $V = \omega\sqrt{a^2 - y^2}$ → $V_{max} = \omega a$ ($y = 0$) (mean); $V_{min} = 0$ ($y = \pm a$) (extreme)

$V_{particle} = \frac{dy}{dt}$ → $y = a\sin\omega t$

$t = 0$ → mean; or y or $t = T/4$ → extreme

ENERGY

$K.E = \frac{1}{2}mv^2$

$t \to K.E = \frac{1}{2}Ka^2\cos^2\omega t$

$t = 0 \to (K.E)_{max} = \frac{1}{2}Ka^2$

$t = T/4 \to (K.E)_{min} = 0$

$y \to K.E = \frac{1}{2}K(a^2 - y^2)$

$y = 0 \to (K.E)_{max} = \frac{1}{2}Ka^2$

$y = \pm a \to (K.E)_{min} = 0$

$t \to P.E = \frac{1}{2}Ka^2\sin^2\omega t$

$t = 0 \to (P.E)_{min} = 0$

$t = T/4 \to (P.E)_{max} = \frac{1}{2}Ka^2$

$y \to PE = \frac{1}{2}Ky^2$

$y = 0 \to (PE)_{min} = 0$

$y = \pm a \to (P.E)_{max} = \frac{1}{2}Ka^2$

If $U_0 = 0$

P.E → $U = U_0 + \frac{1}{2}Ky^2$ (due to configuration; due to position)

$T.E = \frac{1}{2}Ka^2$

$U_0 = 0$, $y = 0$ (mean): $(P.E)_{min} = 0$, $(T.E)_{mean} = (K.E)_{max}$

$U_0 \neq 0$, $y = 0$ (mean): $(P.E)_{min} \neq 0$, $(T.E)_{mean} \neq (K.E)_{max}$

$(T.E)_{mean} = (K.E)_{max} + (P.E)_{min.}$

$(T.E)_{extreme} = (P.E)_{max} + (K.E)_{min.}$

$(K.E)_{min} = 0$ (Always)

$(P.E)_{max} = (T.E)_{extreme}$ (Always)

AVG. ENERGY

Time 't' dependent → $T.E = \frac{1}{2}Ka^2$

$(K.E)_{avg} = \frac{1}{4}Ka^2$

$(P.E)_{avg} = \frac{1}{4}Ka^2$

Position dependent (y) → $(K.E)_{avg} = \frac{1}{3}Ka^2$, $(P.E)_{avg} = \frac{1}{6}Ka^2$

Agr ques me mentioned na ho to by default 't' dependent wala case lenge

BASIC MATHS

$1 + \cos\theta = 2\cos^2\theta/2$

$1 - \cos\theta = 2\sin^2\theta/2$

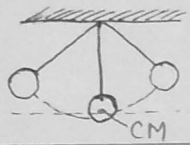
$1 + \cos 2\theta = 2\cos^2\theta$

$1 - \cos 2\theta = 2\sin^2\theta$

SIMPLE PENDULUM

$T = 2\pi\sqrt{\frac{L}{g_{eff}}}$ $g_{eff} = 0$ (Space)

$\theta \leq 5° \to$ it will be SHM



Effect of buoyancy

$g' = \left(1 - \frac{m'}{m}\right)g$ (Displaced mass / Actual mass)

$T' = 2\pi\sqrt{\frac{l}{g\left(1 - \frac{\sigma}{\rho}\right)}}$

$\sigma \to$ Liq. density

$\rho \to$ Body "

SECOND'S PENDULUM

$T = 2\pi\sqrt{\frac{L}{g}} = 2s$

$L \simeq 1m$

Compound pendulum

$I = I_{CM} + ml^2$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{L_{eff}}{g}} = 2\pi\sqrt{\frac{\frac{K^2}{l} + l}{g}}$

$T = 2\pi\sqrt{\frac{1}{\left(\frac{1}{l} + \frac{1}{R}\right)g}}$ → If length to Radius (Earth)

Simple pendulum $l << R$ $\frac{1}{l} >> \frac{1}{R}$

SPRING PENDULUM

$T = \frac{2\pi}{\omega}$ $\omega = \sqrt{\frac{K}{m}}$ $K = m\omega^2$

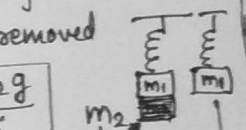
$T = 2\pi\sqrt{\frac{m}{K}}$ $T = \infty$ (Astronauts me)

$K \propto \omega^2 \propto v^2 \propto \frac{1}{T^2}$

Amp. (A) = $y_1 - y_2$

To Remove ya add kia

Suppose m_2 removed → $A = \frac{m_2 g}{K}$



Freq. (ω) = $\sqrt{\frac{K}{m}}$ [Jo Reh gya]

m_1 bach gya → $\omega = \sqrt{\frac{K}{m_1}}$

Parallel → $y_1 = y_2$

$K_p = K_1 + K_2 = nK$

$T_p = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$

Series → $F_1 = F_2$

$\frac{1}{K_s} = \frac{1}{K_1} + \frac{1}{K_2}$

$T_s^2 = T_1^2 + T_2^2$

$K_s = K/n$

SPRING CUTTING

I. Equal parts (n equal parts)

$K' = nK$ $T = T/\sqrt{n}$

Series → $K_s = K$, $T_s = T = 2\pi\sqrt{\frac{m}{K}}$

Parallel → $K_p = n^2K$, $T_p = T/n$

II. Unequal parts $l = l_1 + l_2$

$l_2 = nl_1$ or $l_1 = nl_2$

$K_1 = (n+1)K$ $K_2 = \left(\frac{n+1}{n}\right)K$

$Ky^2 + I\omega^2 + mv^2 = 2E$

$T = 2\pi\sqrt{\frac{mK'^2}{R^2} + m}{K}$

$Kl = K_1l_1 + K_2l_2 + \cdots + K_nl_n$