

SETS

1. Given that Set A has 5 elements and B has 128 subsets.
 - (a) Find the number of subsets of A
 - (b) Find the number of elements of Set B.

Solutions

- a) Number of subsets is given by 2^n where n is the number of elements in a given set.

$$\text{Number of subset} = 2^n$$

A has 5 elements

$$= 2^5$$

$$= 32, \text{A has 32 subsets}$$

- b) No. of subset = 2^n

$$128 = 2^n$$

$$2^7 = 2^n$$

$$n = 7, \text{B has 7 elements}$$

2. If $E = \{\text{Natural numbers less than 13}\}$
 $P = \{x: x \text{ is a prime number}\}$
 $O = \{x: x \text{ is an odd number}\}$
 $S = \{x: x \text{ is a square number}\}$

List Sets E, P, O and S and hence find the following:

(a) P'

(b) $(P \cap O)'$

(c) $(P \cup S)'$

(d) $(P \cup S \cup O)'$

Solutions

$$E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$P = \{2, 3, 5, 7, 11\}$$

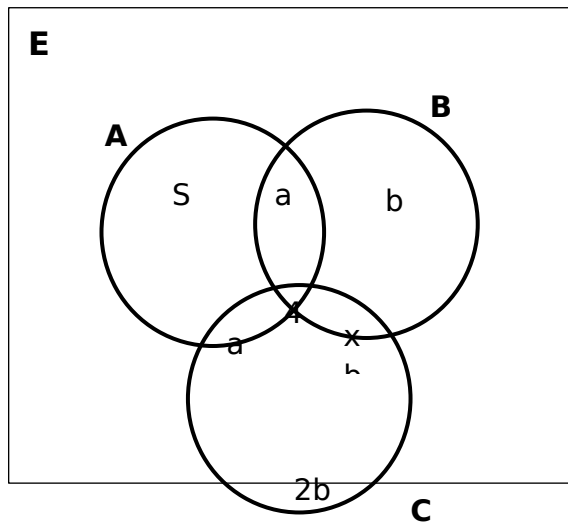
$$O = \{1, 3, 5, 7, 9, 11\}$$

$$S = \{1, 4, 9\}$$

- a) P' elements in the universal set that are not in P: $P' = \{1, 4, 6, 8, 9, 10, 12\}$
- b) $(P \cap O)'$ elements in the universal set that are not in $P \cap O = \{3, 5, 7, 11\}$
 $\therefore (P \cap O)' = \{1, 2, 4, 6, 8, 9, 10, 12\}$
- c) $P \cup S$ elements of both Sets $P \cup S = \{1, 2, 4, 5, 7, 9, 11\}$ and common elements should be listed once.

d) First list $P \cup S \cup O = \{1, 2, 3, 4, 5, 7, 9, 11\}$ then list elements of universal that are not $P \cup S \cup O \therefore (P \cup S \cup O)' = \{6, 8, 10, 12\}$

3. The diagram below shows three intersecting sets A, B and C. It is given that $n(A) = 50$, $n(B) = 42$ and $n(C) = 62$



a) By considering Sets B and C, show that

$$a + b = 38 - x \quad (i)$$

$$a + 2b = 58 - x \quad (ii)$$

b) Hence or otherwise, find the value of b

c) Given that

$S = b + 10$ find the values of S, a and x.

Solutions

a) From Sets B, $a + b + 4 + x = 42$ and from C, $a + 2b + 4 + 4 + x = 62$

$$a + b = 42 - 4 - x$$

$$a + 2b = 62 - 4 - x$$

$$a + b = 38 - x$$

$$a + 2b = 58 - x$$

$$\therefore a + b = 38 - x \quad (i)$$

$$a + 2b = 58 - x \quad (ii) \text{ Hence shown}$$

b) Value of b can be solved by showing the two equations above simultaneously.

$$\begin{array}{r} -(a+b=38-x) \\ a+2b=58-x \\ \hline -b=-20 \\ \therefore b=20 \end{array}$$

Value a from Set A

Value of x from Set

B

c) $S = b + 10$

$$S + 2a + 4 = 50$$

$$a + b + x + 4 = 42$$

$$S = 20 + 10$$

$$S = 30$$

$$30 + 4 + 2a = 50$$

$$34 + 2a = 50$$

$$2a = 50 - 34$$

$$\frac{2a}{2} = \frac{16}{2}$$

$$a = 8$$

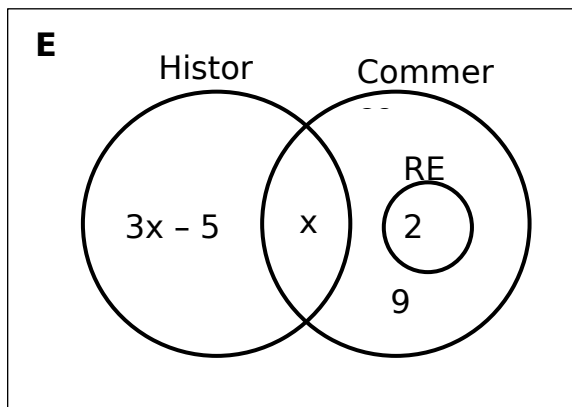
$$8 + 20 + x + 4 = 42$$

$$x = 42 - 8 - 20 - 4$$

$$x = 10$$

$$\therefore a = 8, s = 30 \text{ and } x = 10$$

4. At Hillcrest Technical Secondary School, a group of 70 take optional subjects as illustrated in the Venn diagram below.



- (i) Calculate the value of x
- (ii) Find the value of Girls who take
 - a) History only
 - b) Commerce

Solutions

$$\begin{aligned}
 \text{(i)} \quad & 3x - 5 + x + 2x + 9 = 70 \\
 & 3x - 5 \\
 & 3x + x + 2x - 5 + 9 = 70 \\
 & 6x + 4 = 70 \\
 & 6x = 70 - 4 \\
 & \frac{6x}{6} = \frac{66}{6}
 \end{aligned}$$

Value of $x = 11$

9

$$\text{(ii) (a)} \quad \text{History only} =$$

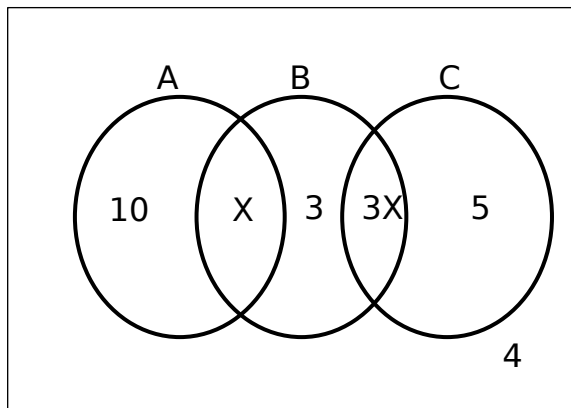
$$\begin{aligned}
 & 3(11) - 5 \\
 & 33 - 5
 \end{aligned}$$

28 History only

$$\text{(b) Commerce} = x + 2x +$$

$$\begin{aligned}
 & = 11 + 2(11) + 9 \\
 & = 11 + 22 + 9 \\
 & = \mathbf{42}
 \end{aligned}$$

5. In the diagram below shows three Sets A, B and C



Give that $n(A \cup B \cup C) = 50$ find

- (i) The value of x
- (ii) $n(A \cup B)$
- (iii) $n(B \cup C)$
- (iv) $n(A' \cap C')$

Solution

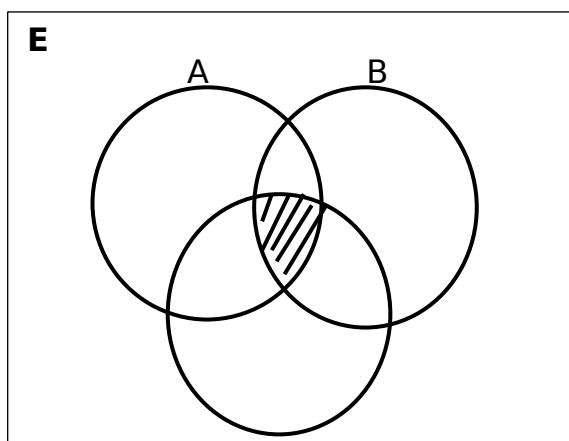
$$\begin{aligned}
 \text{(i)} \quad & 10 + x + 3 + 3x + 5 + 4 = 50 \\
 & 10 + 3 + 5 + 4 + x + 3x = 50 \\
 & 22 + 4x = 50 \\
 & 4x = 50 - 22 \\
 & \frac{4x}{4} = \frac{28}{4} \\
 & \underline{x = 7}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & n(A \cup B) = 10 + x + 3 + 3x \\
 & = 10 + 7 + 3 + 3(7) \\
 & = 10 + 7 + 3 + 21 \\
 & = \mathbf{41}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & n(B \cup C) = 10 + 4 \\
 & = 10 + 4 \\
 & = \mathbf{14}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & n(A' \cap C') = 3 + 4 \\
 & = \mathbf{7}
 \end{aligned}$$

6. On the Venn diagram below, shade the Set $(A' \cap B) \cap C$



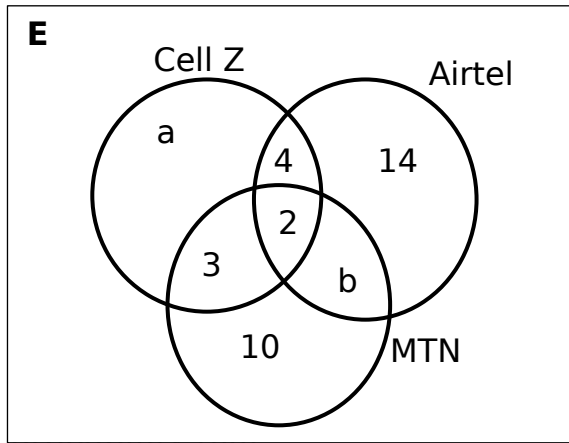
Solution

Hint

Shade A' and B to give the region common to $(A'$ and B)
 - Shade C to give the region common to $(A' \cap B) \cap C$

C

7. A survey was conducted on 60 women connecting the types of Sim cards used in their cell phones for the past 2 years. Their responses are given in the diagram below.



- a) Given that 23 women have used Cell Z Sim cards, find the values of a and b
- b) How many women have used only two different Sim cards.
- c) From the group, what is the probability that
- She has no cell phone
 - She used only type of a Sim card
- d) How many women did not use MTN and Cell Z Sim Cards?
- e) How many women used either Airtel or MTN Sim Cards but not Cell Z.?

Solutions

a) $a + 4 + 2 + 3 = 23$

$a + 9 = 23$

$a = 23 - 9$

$a = 14$

b) $a + b + 4 + 2 + 3 + 10 + 14 + 8 = 60$

$14 + b + 4 + 2 + 3 + 10 + 14 + 8 = 60$

$b + 55 = 60$

$b = 60 - 55$

$b = 5$

$\therefore a = 14 \wedge b = 5$

c) (i) No cell phone $\frac{8}{60} = \frac{2}{15}$

(iii) Only one type of Sim card = $a + 14 + 10$
 $= 14 + 14 + 10$
 $= 38$

$$\therefore \text{probability} = \frac{38}{60} = \frac{19}{30}$$

d) $(\text{Cell Z} \cup \text{MTN})' = 14 + 8$
 $= \underline{\underline{22}}$

e) $10 + 5 + 2 + 3 + 14 + 4 = \underline{\underline{38}}$ used either Airtel or MTN but not Cell Z

8. If $P = \{-2, -1, 0, 1, 2, 3, 4, \dots\}$, express

a) P in set builder notation

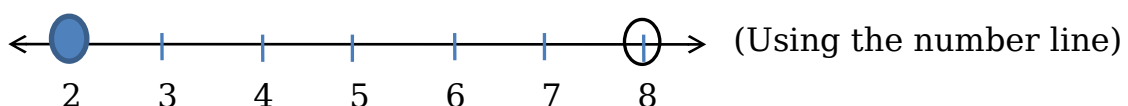
$P = \{x : x \geq -2; x \in \mathbb{Z}\}$ (The inequality $\geq$ is used since elements are building

from 2 (inclusive) to the positive side).

b) Write Set A in listed form,

$$A = \{x : 2 \leq x < 8$$

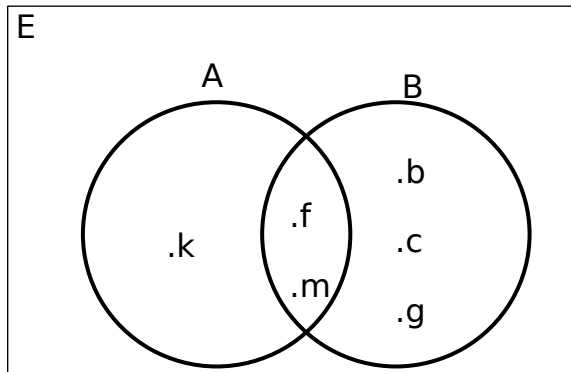
$$x < 8, x \geq 2 \quad (\text{By splitting the set builder notation})$$



$$\therefore A = \{2, 3, 4, 5, 6, 7\}$$

(2 is included in the solution set when 8 is **not** included)

9. Using the information given in the Venn diagram below



a) List the Set $(A \cap B)$
 $(A \cap B) = \{f, m\}$

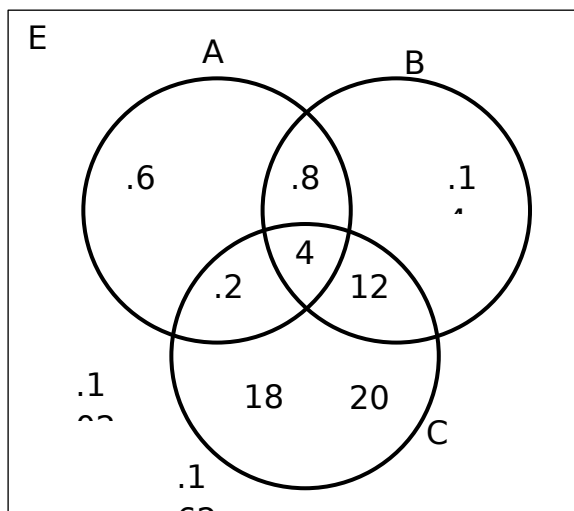
b) List $(A \cap B)'$
 $(A \cap B)' = \{k, b, c, g\}$ (Complement of a set are elements outside the given sets $(A \cap B)$)

10. E (Universal Set) = $\{2, 4, 6, 8, 10, 12, 14, 18, 20\}$

$$A = \{2, 4, 6, 8\}$$

$$B = \{4, 8, 12, 14\} \quad C = \{2, 4, 12, 18, 20\}$$

i. Illustrate the above information on a Venn diagram.



- ii. List the Set
 $(A \cap B \cap C) = \{4\}$ (The intersection of three sets)

- iii. $(A \cap B') \cup C$
 $A \cap B' = \{2, 4, 6, 8\} \cap \{2, 6, 10, 16, 18, 20\}$ (Dealing with what is inside the brackets by listing Set A and Set B')

$$(A \cap B') = \{2, 6\}$$

$$\text{Set } C = \{2, 4, 12, 18, 20\}$$

$$\therefore (A \cap B') \cup C = \{2, 6\} \cup \{2, 4, 12, 18, 20\}$$

$$(A \cap B') \cup C = \{2, 4, 6, 12, 18, 20\} \quad (\text{Do not repeat elements in the union of sets})$$

- iv. Find the value of $n(A \cup B \cup C)$

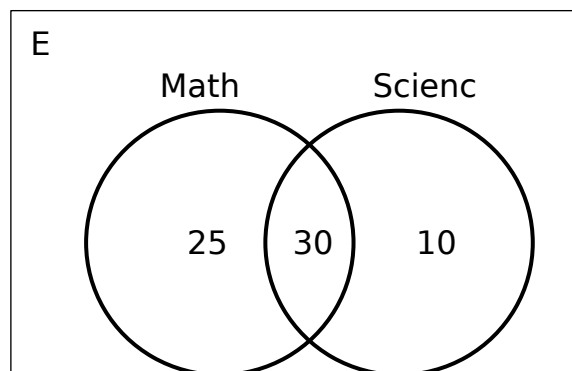
$$(A \cup B \cup C) = \{10, 16\} \quad \text{By listing}$$

$$\therefore n(A \cup B \cup C) = 2 \quad (\text{There are two elements in that Set})$$

- v. In the Venn diagram, shade the region $(A' \cup B) \cap C$
 $(A' \cup B) = \{12, 14, 18, 20, 10, 16\} \cup \{4, 8, 12, 14\}$
 $(A' \cup B) = \{4, 8, 10, 12, 14, 16, 18, 20\}$
 $(A' \cup B) \cap C = \{4, 8, 10, 12, 14, 16, 18, 20\} \cap \{2, 4, 12, 18, 20\}$
 $(A' \cup B) \cap C = \{4, 12, 18, 20\}$ Shade the region where these elements are lying.

11. 70 learners at Mansa Secondary School were asked to mention their favourite subjects between Maths and Science.

The results are shown in a Venn diagram below



- i. How many learners like Maths only? **25 learners**
- ii. How many learners do not like Maths nor Science?

$$70 - (25 + 30 + 10)$$

$$70 - (65)$$

$$70 - 65$$

= 5 learners do not like Maths nor Science