

UGANDA ADVANCED CERTIFICATE OF EDUCATION S.5 MATHEMATICS HOLIDAY WORK TERM I 2023

INSTRUCTIONS

- ✓ Attempt all questions in a new book for Mathematics holiday work.
- ✓ A fee of Ugshs 10,000 only will be payable upon reporting

PURE MATHEMATICS

1. Solve the equation $4\cos x - 2\cos 2x = 1$ for $0^\circ \leq x \leq 360^\circ$
2. Solve $10 \sin^2 x + 5\sin 2x - \cos^2 x = 2$, for $0^\circ \leq x \leq 360^\circ$
3. If $\sin A = \frac{15}{17}$ and angle A is acute, find the value of $\cos 2A + 3\tan 3A$
4. If $\frac{2+\sqrt{2}}{2-\sqrt{2}} + \frac{1-\sqrt{2}}{1+\sqrt{2}} = a + b\sqrt{2}$ Find the values of a and b .
5. The ninth term of an arithmetic progression is twice the third term, and the fifteenth term is 27. Evaluate the sum of the first 25 terms of the series.
6. Solve the equation $3 \cos 2\theta - 7 \cos \theta - 2 = 0$ for $0^\circ \leq \theta \leq 360^\circ$
7. The roots of the equation $x^2 + px + 7 = 0$ are α and β . Given that $\alpha^2 + \beta^2 = 22$, find the possible values of p .
8. Prove that $\log_a x = \frac{1}{\log_x a}$. Hence solve the equation $\log_{10} x + \log_x 100 = 3$
9. Prove the identity: $\sin 7A \cos A - \cos 5A \sin 3A = \sin 4A \cos 2A$
10. Solve the equation: $(1 - \sin \theta)^2 + (1 + \cos \theta)^2 = 1$; for $0^\circ \leq \theta \leq 360^\circ$
11. Solve the equation $\sqrt{x-3} = \sqrt{x} - 3$
12. In the expansion of $\left(2x + \frac{y}{x^2}\right)^9$, find;
 - (i) The value of y if the coefficient of the term in x^{-12} is 314928.
 - (ii) The term independent of x .
13. The distance of the point $(2, -1)$ from the line $y = \frac{3}{4}x + c$ is twice its distance from the line $y = -\frac{4}{3}x$. Find the value of c .
14. A quadratic polynomial leaves a remainder 18 when divided by $x + 3$ and $x - 1$ is a repeated factor. Find the polynomial.
15. Show that $\frac{d}{dx} \left[\tan^{-1} x^x \right] = \frac{(1 + \ln x)x^x}{1 + x^{2x}}$.

16. Show that if $y = \cos(2\cos^{-1}x)$, then $y = 2x^2 - 1$
17. Solve: $\sqrt{10 + 3\sqrt{x}} = \sqrt{x}$
18. Given that $y = \frac{\sin x}{x}$, show that $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + xy = 0$.
19. The coefficients of the first three terms in the expansion of $(1 + \frac{x}{2})^n$ are in an arithmetic progression. Find the value of n .
20. Solve: $\cos x + \sin x = 1$ for $0 \leq x \leq 2\pi$
21. Use calculus of small changes to show that $\sqrt[3]{64.96} \approx 4.02$
22. Given that $y = e^{\tan x}$ show that $\frac{d^2y}{dx^2} - (2 \tan x + \sec^2 x) \frac{dy}{dx} = 0$
23. Solve the equation: $4x^4 + 17x^3 + 8x^2 + 17x + 4 = 0$.
24. Prove that in any triangle ABC $\sin\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{bc}}$ where $s = \frac{a+b+c}{2}$
25. Express $7 \cos 2\theta + 6 \sin 2\theta$ in the form $\sqrt{r} \cos(2\theta - \alpha)$ where r is a constant and α is an acute angle hence solve the equation $7 \cos 2\theta + 6 \sin 2\theta = 5$ for $0^\circ \leq \theta \leq 180^\circ$

APPLIED MATHEMATICS

1. The probability that a baby boy is born in a family is α and the probability that girl is born is β where $\alpha + \beta = 1$ Construct a sample space for the possible ways children could have been born in a family of three children. Hence construct the probability distribution table for the number N , of boys in a family of three children.
2. A particle starts from rest at the origin $(0, 0)$. Its acceleration in ms^{-2} is given by $a = 6i - 4j$. Find its speed and position after $t = 2$ seconds.
3. Mary's chance of passing an examination is $\frac{5}{3}$. If she sits for four examinations, find the probability that she passes:
 - (a) Only three examinations.
 - (b) Atleast one examination.
4. A heavy particle is projected from a point O at an angle of elevation α . Prove that the equation of the trajectory is $y = x \left(1 - \frac{r}{x}\right) \tan \alpha$, where r is the horizontal range.

5. Given the table below;

X	0	10	20	30
Y	6.6	2.9	-0.1	-2.9

Use linear interpolation to find;

(a) y when x =16

(b) x when y = -1.

(c) y when x=9

(d) x when y=0

6. A four-man team is to be selected from three women and 4 men. What is the probability that at least 3 men will be on the committee?

7. Forces of 2N, 4N, 3N and 2N act along the sides PQ, QR, RS, SP, of a square PQRS of side 2m. A force of 5N acts along QM where M is the mid-point of PS. Calculate the;

(i) Magnitude of this resultant

(ii) angle this resultant makes with PQ.

8. For a set of 9 numbers $\sum(x - \bar{x})^2 = 60$ and $\sum x^2 = 285$. Find the mean of the numbers.

9. (a)Find the range within which the exact value of z lies, given that

$$Z = \frac{1}{x} + \frac{1}{y} + xy, \quad x = 4.165 + 0.001$$

$$y = 6.72 + 0.01$$

(b) Given that X = 4.8654 and Y = 2.80;

(i) State the maximum possible errors in X and Y.

(ii) Find the limits within which XY lies correct to 4 decimals places.

10 A stone is projected from a point O with initial velocity $5i + 12j$.

(a) Find a vector expression for the velocity and position of the projectile at time t.

(b) A particle is projected horizontally at 168ms^{-1} . Find the magnitude and direction of the velocity of the particle five seconds after projection

11.The senior five mathematics class while studying the topic on price indices carries out a practical lesson of baking a class cake. The table below shows the major items required and their corresponding prices in 2013 and 2014.

ITEM	PRICE		WEIGHT
	2013	2014	
Milk(per litre)	1000	1300	0.5

Eggs (per tray)	3500	4300	1
Sugar (per kg)	3000	3800	2
Blue band (per kg)	7000	9000	1

Taking 2013 as the base year, calculate for 2014 the;

- (a) Price relatives for each item. (b) Simple aggregate price index.
(c) Weighted aggregate price index and comment on your result.

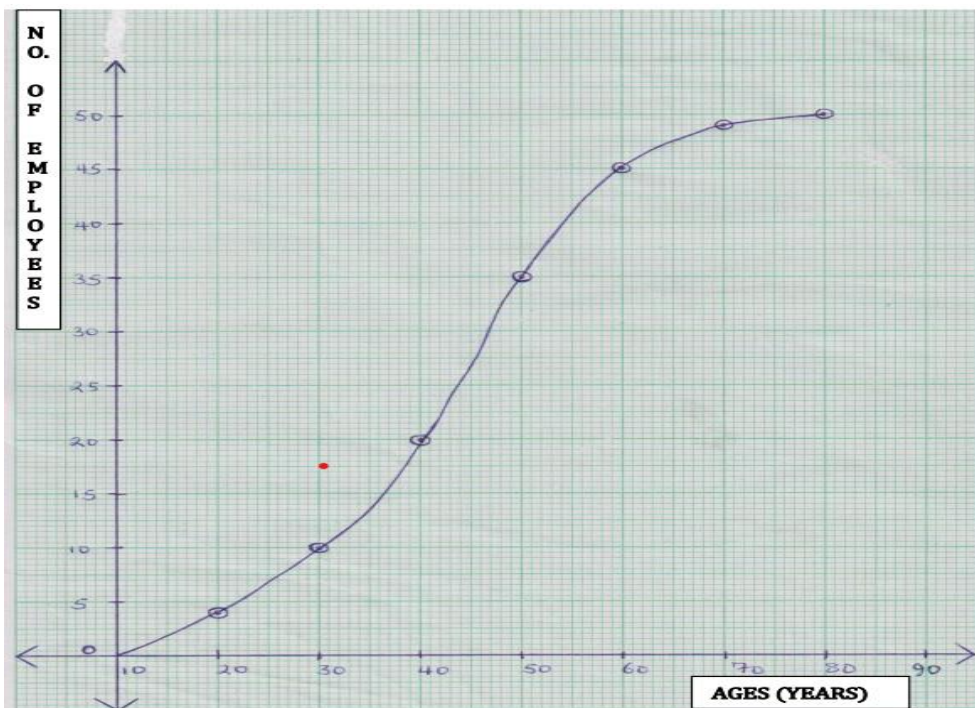
12. The probability mass function of a discrete random variable X is given by

$$f(x) = \begin{cases} k(2)^x, & x = 0, 1, 2, 3, 4. \\ 0, & \text{elsewhere} \end{cases} \quad \text{Where } k \text{ is a constant.}$$

Find the;

- (a) Value of k (b) mean of X.
(c) Standard deviation of X. (d) $P(1 < x \leq 4)$.

13. A cumulative frequency curve for the ages of employees of a certain school



Use the curve below to estimate the;

- (i) variance. (ii) age exceeded by 50% of the school employees.
(iii) age range for the central 70% of the distribution.
(b) Hence or otherwise obtain the;
(i) mean age. (ii) modal age.

END