

Pre-university Physics Remedial Program

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Chapter 6. Oscillations and Waves

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Simple Harmonic Motion

Oscillation ,wave and Periodic motions

- **Oscillation:** is a disturbance in a physical system that is repetitive in time. Oscillation involves a continuous back and forth flow of energy between two different energy types: e.g., kinetic and potential energy, in the case of a pendulum.
- **Wave:** is a disturbance in an extended physical system that is both repetitive in time and periodic in space.
- A wave involves similar repetitive energy flows to an oscillation, but, in addition, is capable of transmitting energy and information from place to place.
- **Periodic motions:** motion of back and forth about the equilibrium or mean position.
- **Oscillatory motion:** periodic motion in which there is existence of a restoring force and the body moves along the same path to and fro about a definite point, equilibrium position.

Simple Harmonic Motion

Types of oscillatory motion and Oscillatory system

- Types of oscillatory motion:

- ① **linear oscillation:** Example

- ① Oscillation of mass spring system.
 - ② Oscillation of fluid column in a U-tube.
 - ③ Oscillation of floating cylinder.
 - ④ Oscillation of body dropped in a tunnel along earth diameter.
 - ⑤ Oscillation of strings of musical instruments.

- ② **circular oscillation:** Example

- ① Oscillation of simple pendulum.
 - ② Oscillation of solid sphere in a cylinder (If solid sphere rolls without slipping).
 - ③ Oscillation of a circular ring suspended on a nail.
 - ④ Oscillation of balance wheel of a clock.
 - ⑤ Rotation of the earth around the sun.

- Oscillatory system: Oscillators are the basic building blocks of waves. Oscillatory systems are of two types, mechanical and non- mechanical systems.

Simple Harmonic Motion

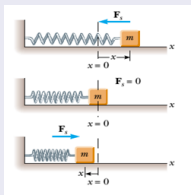
Mechanical and None Mechanical Oscillatory system

- **Mechanical oscillatory system:** In this type of system a body itself changes its position.
- For mechanical oscillation two things are especially responsible, inertia and restoring force.
- Examples: The vibrations of violin strings, the oscillations of quartz crystals used in clocks, and the vibrations of atoms in a molecule.
- **Non-mechanical oscillatory system:** In this type of system, the body itself doesn't change its position but its physical property varies periodically.
- Examples: The electric current in an oscillatory circuit, the lamp of a body which is heated and cooled periodically, the pressure in a gas through a medium in which sound propagates, the electric and magnetic waves propagates undergoes oscillatory change.

Simple Harmonic Motion

Simple Harmonic Motion

- Consider a block of mass ' m ' attached to the end of a spring, with the block free to move on a horizontal, frictionless surface as shown in the figure.



- When the block is displaced to the right of equilibrium , ($x > 0$), the force exerted by the spring acts to the left.
- When the block is at its equilibrium position ($x=0$), the force exerted by the spring is zero.
- When the block is displaced to the left of equilibrium ,($x < 0$), the force exerted by the spring acts to the right.

Simple Harmonic Motion

Simple Harmonic Motion

- Applying Newton's second law, $\sum F_x = ma_x$
 $-kx = ma_x$
 $a_x = -\frac{k}{m}x$
 $\frac{d^2x}{dt^2} = -\omega^2x, \quad \omega^2 = \frac{k}{m}$
- Where K is the constant of proportionality called the spring constant or stiffness factor and ω is the angular frequency of the oscillator.
- The following cosine function is a solution to the differential equation.
 $x(t) = A \cos(\omega t + \phi), \quad A, \omega, \phi$ are constants.
- Explicitly, $\frac{dx}{dt} = A \frac{d}{dt}(\cos(\omega t + \phi)) = -\omega A \sin(\omega t + \phi)$
and $\frac{d^2x}{dt^2} = -\omega A \frac{d}{dt}(\sin(\omega t + \phi)) = -\omega^2 A \cos(\omega t + \phi)$
- $\frac{d^2x}{dt^2} = -\omega^2x$
- Let us investigate further the mathematical description of simple harmonic motion. The values of x and v for the particle at time t equal the values of x and v at time $t+T$.

Simple Harmonic Motion

Simple Harmonic Motion

- We can relate the period to the angular frequency by using the fact that the phase increases by 2π radians in a time interval of T :

$$[\omega(t + T) + \phi] - (\omega t + \phi) = 2\pi \Rightarrow \omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}} \quad , f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

- Not all periodic motions over the same path can be classified as simple harmonic motion.
- Characteristics of SHM:
 - 1 The amplitude A is constant.
 - 2 The frequency and period are independent of the amplitude.
 - 3 The fluctuating quantity can be expressed in terms of sinusoidal function of a single frequency.
- For SHM to occur,
 - 1 there must be a stable equilibrium position
 - 2 there must be no dissipation of energy and the acceleration is proportional to the displacement and opposite in direction.

Simple Harmonic Motion

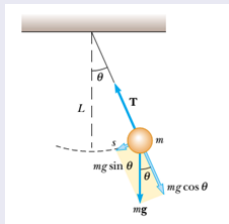
Displacement, Velocity and Acceleration in a SHM

- Displacement: $x = A \sin(\omega t)$
- Velocity: $V = \frac{dx}{dt} = \omega A \cos(\omega t)$
- Acceleration: $a = \frac{dv}{dt} = -\omega^2 A \sin(\omega t) = -\omega^2 x$
- Maximum velocity occurs at equilibrium position with, $v_{max} = \omega A$
- Maximum acceleration is observed at $x = \pm A$, $a_{max} = \omega^2 A$
- Example: An object oscillates with simple harmonic motion along the x axis. Its position varies with time according to the equation $x = (4.00)m \cos(\pi t + \frac{\pi}{4})$. Where t is in seconds and the angles in the parentheses are in radians.
 - 1 Determine the amplitude, frequency, and period of the motion.
 - 2 Calculate the velocity and acceleration of the object at any time t.
 - 3 Using the results of part (b), determine the position, velocity, and acceleration of the object at $t = 1.00$ s.
 - 4 Determine the maximum speed and maximum acceleration of the object.

The Simple Pendulum

The Simple Pendulum

- A simple pendulum is another mechanical system that exhibits periodic motion. It consists of a small bob of mass m suspended by a light string of length L fixed at its upper end, as in Fig below.



- By a light string, we mean that the string's mass is assumed to be very small compared with the mass of the bob and hence can be ignored.
- We shall show that, provided the angle θ is small (less than about 10°), the motion is very close to that of a simple harmonic oscillator.
- The forces acting on the bob are the T exerted by the string and mg .

The Simple Pendulum

The Simple Pendulum

- The tangential component $mg \sin \theta$ of the gravitational force always acts toward $\theta = 0$, opposite the displacement of the bob from the lowest position.
- Therefore, the tangential component is a restoring force, and we can apply Newton's second law for motion in the tangential direction:

$$F_t = -mg \sin \theta = m \frac{d^2 s}{dt^2}$$

$$\frac{d^2 \theta}{dt^2} = -\frac{g}{L} \sin \theta \quad \text{where } s = L\theta$$

$$\frac{d^2 \theta}{dt^2} = -\frac{g}{L} \theta \quad \text{where for small } \theta, \sin \theta \equiv \theta$$

- The period of the motion is $T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}}$ where $\omega^2 = \frac{g}{L}$
- Period of a simple pendulum doesn't depend on the mass, but only on the pendulum's length and on the free-fall acceleration.

The Simple Pendulum

Energy of the Simple Harmonic Oscillator

- We assume a mass less spring, so the kinetic energy of the system corresponds only to that of the block and expressed as
$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$
- The elastic potential energy stored in the spring for any elongation x is given by $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$
- We see that K and U are always positive quantities. Because $\omega^2 = k/m$ we can express the total mechanical energy of the simple harmonic oscillator as

$$E = K + U = \frac{1}{2}kA^2[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

$$E = \frac{1}{2}kA^2 \quad \text{Where } \sin^2(\omega t + \phi) + \cos^2(\omega t + \phi) = 1$$

- The total mechanical energy of a simple harmonic oscillator is proportional to the square of the amplitude.
- Velocity for an arbitrary position can be obtained from conservation of energy as $\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \Rightarrow v = \pm \sqrt{\frac{k}{m}(A^2 - x^2)} = \pm \omega \sqrt{A^2 - x^2}$

Free, Damped and Forced Oscillations

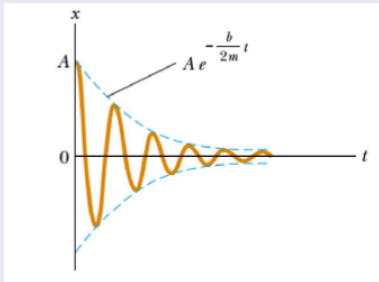
Free, Damped Oscillations

- The oscillatory motions we have considered so far have been for ideal systems that is, systems that oscillate indefinitely under the action of only one force a linear restoring force.
- Such Oscillations are called **Free oscillation**. The oscillator then oscillates at its natural frequency, $\omega_0 = \sqrt{\frac{k}{m}}$.
- In many real systems, non conservative forces, such as friction, retard the motion.
- Consequently, the mechanical energy of the system diminishes in time, and the motion is said to be **damped**.
- Since the retarding force can be expressed as $R = -bv$ (where b is a constant and called damped coefficient) and the restoring force of the system is $-kx$.
- we can write Newton's second law as $\sum F_x = -kx - bv_x = ma_x$
 $-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$

Damped and Forced Oscillations

Damped Oscillations

- As a result the amplitude of the oscillation will decay exponentially, as shown in Fig. below.
- Note that the period of the oscillation does not change as the amplitude gets smaller.

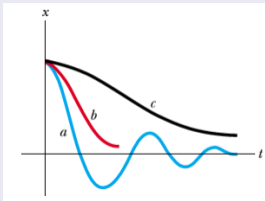


- When the retarding force is small, the oscillatory character of the motion is preserved but the amplitude decreases in time, with the result that the motion ultimately ceases.

Damped and Forced Oscillations

Damped Oscillations

- When the magnitude of the max. retarding force $R_{max} = bv_{max} < kx$, the system is said to be **under damped** (shown by a).



- As the value of b increases, the amplitude of the oscillations decreases more and more rapidly.
- When b reaches a critical value b_c such that $\frac{b_c}{2m} = \omega_0$, the system does not oscillate and is said to be **critically damped**.
- In this case the system, once released from rest at some non equilibrium position, approaches but does not pass through the equilibrium position (shown by b).

Damped and Forced Oscillations

Damped Oscillations

- The damping in a car suspension is always a compromise somewhere near to the critical damping and it is not normally so heavy, as this would produce a very 'hard' and uncomfortable ride for the passengers.
- If the medium is so viscous that the retarding force is greater than the restoring force that is, if $R_{max} = bv_{max} > kA$ and $\frac{b}{2m} > \omega_0$ the system is **over damped** (shown by c).
- Whenever friction is present in a system, whether the system is over damped or under damped, the energy of the oscillator eventually falls to zero.
- The lost mechanical energy is transformed into internal energy in the object and the retarding medium.

Damped and Forced Oscillations

Forced Oscillations

- We have seen that the mechanical energy of a damped oscillator decreases in time as a result of the resistive force. It is possible to compensate for this energy decrease by applying an external force, that does positive work on the system.
- At any instant, energy can be transferred into the system by an applied force that acts in the direction of motion of the oscillator. For example, a child on a swing can be kept in motion by appropriately timed pushes.
- The amplitude of motion remains constant if the energy input per cycle of motion exactly equals the decrease in mechanical energy in each cycle that results from resistive forces.
- A common example of a forced oscillator is a damped oscillator driven by an external force that varies periodically, such as $F(t) = F_o \sin \omega t$
- Where ω is the angular frequency of the driving force and F_o is a constant

Damped and Forced Oscillations

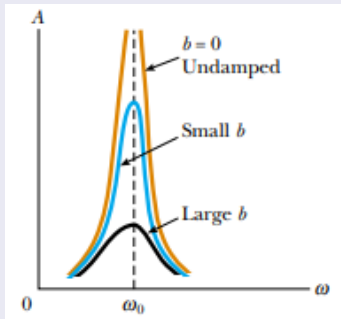
Forced Oscillations

- In general, the frequency ω of the driving force is variable while the natural frequency of undamped oscillator ($b=0$), $\omega_o = \sqrt{\frac{k}{m}}$ of the oscillator is fixed by the values of k and m .
- Newton's second law in this situation gives $\sum F = ma \Rightarrow F_o \sin \omega t - b \frac{dx}{dt} - kx = m \frac{d^2x}{dt^2}$ and
- Has solution of $x = A \cos(\omega t + \phi)$
- Where $A = \frac{\frac{F_o}{m}}{\sqrt{(\omega^2 - \omega_o^2)^2 + (\frac{b\omega}{m})^2}}$ is amplitude of a driven oscillator
- The forced oscillator vibrates at the frequency of the driving force and that the amplitude of the oscillator is constant for a given driving force because it is being driven in steady-state by an external force.
- For small damping, the amplitude is large when the frequency of the driving force is near the natural frequency of oscillation, or when $\omega \approx \omega_o$.

Damped and Forced Oscillations

Forced Oscillations

- The dramatic increase in amplitude near the natural frequency is called **Resonance**, and the natural frequency ω_o is also called the resonance frequency of the system.



- The reason for large-amplitude oscillations at the resonance frequency is that energy is being transferred to the system under the most favorable conditions.

Wave and Its Characteristics

Wave and Its Characteristics

- Wave: is a disturbance from normal or equilibrium condition that travels, or propagates, carrying energy and momentum through space without the transport of matter.
- Pulse: is a single disturbance traveling into a medium. Wave supplies energy to the particles in a medium to set them in to motion.
- Terminologies in Wave
 - ① Crests/Troughs: are positions in a wave with maximum displacements above/below the equilibrium position.
 - ② Amplitude (A): is the maximum displacement from the equilibrium position.
 - ③ Wave length (λ): distance between any two consecutive points which are in phase.
 - ④ Period (T): is the time taken by a wave to move one wave length.
 - ⑤ Frequency (f): number of oscillations performed per unit time.
 - ⑥ Speed (v): is constant in a medium provided the medium is homogeneous. $V = \lambda f$

Types of waves

Transverse and longitudinal

- Waves can also be categorized as transverse and longitudinal waves based on the way they are propagating.
 - 1 Transverse Wave- is a wave where particles of the disturbed medium oscillate perpendicular to the direction of wave motion. Sinusoidal graphs can represent this motion.
Examples are:
 - water waves,
 - waves on strings, and
 - all EM waves.
 - 2 Longitudinal Wave - is a wave where particles of the disturbed medium oscillate parallel to the direction of wave motion. Example: sound wave

traveling and standing waves

- 1 Waves can either move in space (e.g water waves), the so called traveling waves
- 2 or be stationary in an enclosure, the so called standing waves.

Types of waves

Mechanical and Electromagnetic wave

- Waves can be categorized as Mechanical and Electromagnetic waves based on the need of material medium for its propagation.
 - 1 Mechanical Waves: are waves produced by the oscillation of particles of a mechanical medium and need a medium for propagation. Examples are water waves, sound wave, waves in strings etc.

All mechanical waves require:

 - some source of disturbance
 - a medium that can be disturbed and
 - physical medium through which elements of the medium can influence each other.
 - 2 Electromagnetic (EM) waves: are produced by accelerated charged particles and can propagate through both material medium and vacuum.
 - Examples: Light, radio and television waves, micro waves, x-rays, etc.
 - All EM waves in vacuum propagate with speed $c = 3.0 \times 10^8 \text{ m/s}$.

Characteristics of Waves

Common Behaviors of Waves

- **Reflection:** rebounding of wave from a surface from boundary.
- **Refraction:** is the change in direction of a wave passing from one medium to another caused by its change in speed.
- **Diffraction:** is the spreading of waves around obstacles.
- **Interference :** is the net effect of the combination of two or more wave trains moving on intersecting or coincident paths. It can be
 - ① Constructive interference
 - ② Destructive interference
- Interference also occurs between two wave trains moving in the same direction but having different wavelengths or frequencies.
- The resultant effect is a complex wave. A pulsating frequency, called a beat, results when the wavelengths are slightly different.
- Exercise: Draw the diagrams illustrating the different characteristics of wave.

Wave Equation

Wave Equation

- If the wavelength is λ , and f wavelength pass a point per second, then the speed of the wave, v must be given by the wave equation: $v = \lambda f$
- The speed of any travelling wave depends on the media it is travelling.
- For a mechanical wave travelling along a string the speed of the wave depends on the tension of the string and the mass per unit length (Which is called linear density). $v = \sqrt{\frac{T}{\mu}}$
- μ mass per unit length and T is tension in the string in N.
- It shows us that the 'tighter' the string the faster the waves will travel down its length.
- Additionally the 'lighter' the string, (the smaller its mass/length ratio), the faster the waves will travel down its length.
- The phase speed of a wave is the rate at which the phase of the wave travels through space and given in term of λ and period T as $v_{phase} = \frac{\lambda}{T}$