SUBJECT : MATHEMATICS

Topic Name: Relation and Function

1. The function f:R→ [–1, 1] defined by f(x) = cos x is **[2020, 1M]**

(a) Bothe one-one and onto
(b) Not one-one, but onto
(c) One-one, but not onto
(d) Neither one-one, nor onto

2. The relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1), (1, 1)} is **[2020, 1M]**

(a) Symmetric and transitive, but not reflexive
(b) Reflexive and symmetric, but not transitive
(c) Symmetric, but neither reflexive nor transitive
(d) An equivalence relation

3. Check whether the relation R in the set N of natural numbers given by R={(a,b):a is a divisor of b} is reflexive, symmetric or transitive. Also determine whether R is an equivalence relation. **[2020, 4M]**

4. If * is defined on the set R of all real number by*: $a*b=\sqrt{a^2+b^2}$ find the identity element if exist in R with respect to * **[2019, 2M]**

5. If a*b denotes the larger of 'a' and 'b' and if $a\circ b$ =(a*b)+3, then write the value of $(5)\circ(10)$, where * and ∘ are binary operations. **[2018, 1M]**

6. Let $A=\{x\in Z:0\le x\le 12\}$ show that $R=\{(a,b):a,b\in A,|a-b|\}$ is divisible by 4} is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2]. **[2018, 6M]**

OR

7. Show that the function f:R→R defined by f(x) $=\frac{x}{x^2+1},\forall x\in R$ is neither one – one nor onto. **[2018, 6M]**

Also, if g: R → R is defined as g(x) = 2x –1 find fog (x)

8. Consider $f:R-\left\{-\frac{4}{3}\right\}\to R-\left\{\frac{4}{3}\right\}$ given by f(x) $=\frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence (0) $f^{-1}(0)$ and x such that $f^{-1}(x)=2$. **[2017, 6M]**

OR

Let A = Q × Q and let * be a binary operation on A defined by (a, b) * (c, d) = (ac, b + ad) for (a, b), (c, d) ∈ A Determine, whether * is commutative and associative. Then with respect to * on A

(i) Find the identity element in A.

(ii) Find the invertible elements of A.

9. Let f : N → N be a function defined as $f(x)=9x^2+6x-5$. Show that f : N → S, where S is the range of f, is invertible. Find the inverse of f and hence find $f^{-1}(43)$ and $f^{-1}(163)$. **[2016, 6M]**

10. Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if ad (b + c) = bc (a + d). show that R is an equivalence relation. **[2015, 6M]**

11. If R ={(x, y): x + 2y = 8} is a relation on N, write the range of R. **[2014, 1M]**

12. If the function f : R → R be given by f(x) = x^2 + 2 and g : R → R be given by $g(x)=\frac{x}{x-1}, x \neq 1$, find fog and gof and hence find fog (2) and gof (–3). **[2014, 4M]**

13. Consider f : R_+ → [4, ∞) given by f(x) = x^2 + 4. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y)=\sqrt{y-4}$, where R + is the set of all non-negative real numbers. **[2013, 4M]**

14. A relation R is defined on N. which of the following is the reflexive relation? **[2021, 1M]**

(a) R = {(x, y) : x > y; x, y ∈ N}

(b) R = {(x, y) : x + y = 10; x, y ∈ N}

(c) R = {(x, y) : xy is the square number; x, y ∈ N}

(d) R = {(x, y) : x + 4y = 10; x, y ∈ N}

15. The function f : R → R defined by f(x) = 4 + 3 cos x is: **[2021, 1M]**

(a) Bijective

(b) One-one but not onto

(c) Onto but not one-one

(d) Neither one-one nor onto

16. The number of functions defined from {1, 2, 3, 4, 5} → {a, b} which are one-one is: **[2021, 1M]**

(a) 5

(b) 3

(c) 2

(d) 0

17. The number of equivalence relations in the set {1, 2, 3} containing the elements (1, 2) and (2, 1) is: **[2021, 1M]**

(a) 0

(b) 1

(c) 2

(d) 3

18. Let f : R → R be defined by f(x) = $\frac{1}{x}$, for all x ∈ R. Then, f is: **[2021, 1M]**

(a) One-one

(b) Onto

(c) Bijective

(d) Not defined

19. The function $f: N \to N$ is defined by $f(n) = \begin{cases} \frac{n+1}{2}, \text{if n is odd} \\ \frac{n}{2}, \text{if n is even} \end{cases}$. **[2021, 1M]**

The function f is:

(a) Bijective

(b) One-one but not onto

(c) Onto but not one-one

(d) Neither one-one nor onto

20. A relation R is defined on Z as: **[2021, 1M]**

aRb if and only if $a^2 - 7ab + 6b^2 = 0$.

Then, R is:

(a) Reflexive and symmetric

(b) Symmetric but not reflexive

(c) Transitive but not reflexive

(d) Reflexive but not symmetric

Answer Key

1. **(d)**

2. **(a)**

3. **(The relation is not an equivalence relation)**

4. **(There for 0 is the edentity element)**

5. **(13)**

6. **(R is an equivalence relation, Hence the set of all element of A which are related to 1 is {1,5,9}i.e. [1] = [1, 5, 9] & [2] = [2, 6, 16])**

7. $(fog)(x) = \dfrac{2x-1}{4x^2-4x+2}$

8. $f^{-1}(0) = -\dfrac{3}{4}, x = \dfrac{11}{10}$

9. $\therefore \quad f^{-1}(y) = \dfrac{\sqrt{y+6}-1}{3}, f^{-1}(43) = 2, f^{-1}(163) = 4$

10. **(?)**

11. **{1, 2, 3}**

12. $\text{fog} = \dfrac{3x^2-4x+2}{(x-1)^2}$, $\text{gof} = 1+\dfrac{1}{x^2}$, $\therefore fog(2) = 6$, $gof(-3) = 1\dfrac{1}{10}$

13. $\text{Or } f^{-1}(y) = \sqrt{y-4}$

14. **(c)**

15. **(d)**

16. **(d)**

17. **(c)**

18. **(d)**

19. **(c)**

20. **(d)**

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