

UNIVERSITY OF GONDAR

INSTITUTE OF TECHNOLOGY

DEPARTMENT OF CIVIL ENGINEERING

Reinforced Concrete Structure II

(Tutorial NOTE for Exit Exam)

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Contents which Covered in RC II

- ⇒ Introduction
- ⇒ Inelastic Moment Redistribution
- ⇒ Analysis and Design of Columns
- ⇒ Analysis and Design of beams for Torsion
- ⇒ Flat Slabs
- ⇒ Yield Line Theory for Slabs

Introduction

- ⚙ Limit State Design
- ⚙ Material Properties (RC)
- ⚙ Inelastic Moment Redistribution
- ⚙ Analysis of Continuous Floor Beams
 - ✓ Transferring Floor loads to beams
 - ✓ Load cases: arrangement of live loads
 - ✓ Moment envelopes
 - ✓ Inelastic moment redistribution
- ⚙ Design and Detailing of Continuous Beams

1: INELASTIC MOMENT REDISTRIBUTION

General Introduction

- The objective of limit state design is to achieve acceptable probabilities that the structure being designed will not become unfit for its intended purpose during its expected life.
- When a structure or structural element becomes unfit for its intended use, it is said to have reached a Limit State.
- In RC structures Limit State has three basic groups:
 - ✓ Ultimate limit states
 - ✓ Serviceability limit states
 - ✓ Special limit states

Ultimate Limit States

- Structural Collapse of part or all of the structure
- Very Low Probability of occurrence
- Loss of life and major financial losses
- The major ultimate limit states are:
 - ✓ Loss of equilibrium of a part or all of the structure
 - ✓ Rupture of critical parts of the structure
 - ✓ Formation of Plastic Mechanism
 - ✓ Instability
 - ✓ Fatigue

Serviceability Limit States

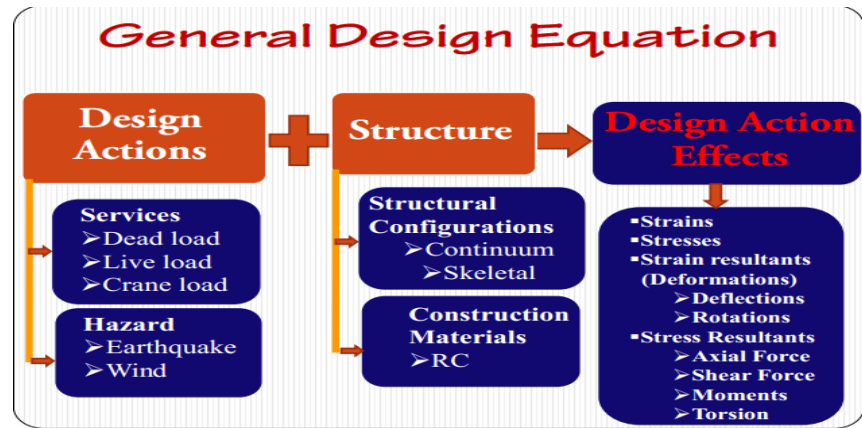
- Disruption of the functional use of the structures but not collapse.
- There is less danger of loss of life; a higher probability of occurrence can generally be tolerated than in the case of an ULS.
- The major serviceability limit states include:
 - ✓ Excessive Deflections
 - ✓ Excessive Crack Width
 - ✓ Undesirable Vibrations

Special Limit States

- Damages or collapses in Extreme Earthquakes
- Structural effects of Fire, Explosions, or Vehicular Collisions
- Structural effects of corrosion or deterioration, particularly when exposed to deicing compounds

Limit State Design Procedure

- Identify whether the ULS or SLS is the conical LS for a particular structure.
- Mostly ULS governs except for water tight structures.



Material Properties

Concrete

- Compressive strength: class and strength
- Tensile strength
- Stress Strain behavior:

Reinforcement

- Strength
- Deformability
- Durability (concrete cover)

Class of Concrete

- Class-indicates workmanship
 - ✓ Class I concrete: used for all elements of structures
 - ✓ Class II concrete: used in footings, pedestals, massive pier shafts, and gravity walls

Class	Permissible Grades of Concrete					
I	C20	C25	C30	C40	C50	C60
II	C20					

Grade of Concrete

- Grade indicates characteristic (5%) cubic compressive strength at 28 days, f_{cuk}

Grades of Concrete	C25	C30	C40	C50	C60
f_{ck} (cylindrical strength in MPa)	20	24	32	40	48
f_{cuk} (cubic strength in MPa)	25	30	40	50	60
$f_{ck} = 0.8f_{cuk}$					
□ Design compressive strength: $f_{cd} = \alpha_{cc} f_{ck} / \gamma_c$					

Where:

- γ_c is the partial safety factor for concrete, which is equal to 1.5 for persistent and transient design situation, and
- α_{cc} is the coefficient taking account of long term effects on the compressive strength and of unfavorable effects resulting from the way the load is applied.
- The value of α_{cc} should lie between 0.8 and 1.0, is taken to be equal to 0.85

Tensile Strength of Concrete

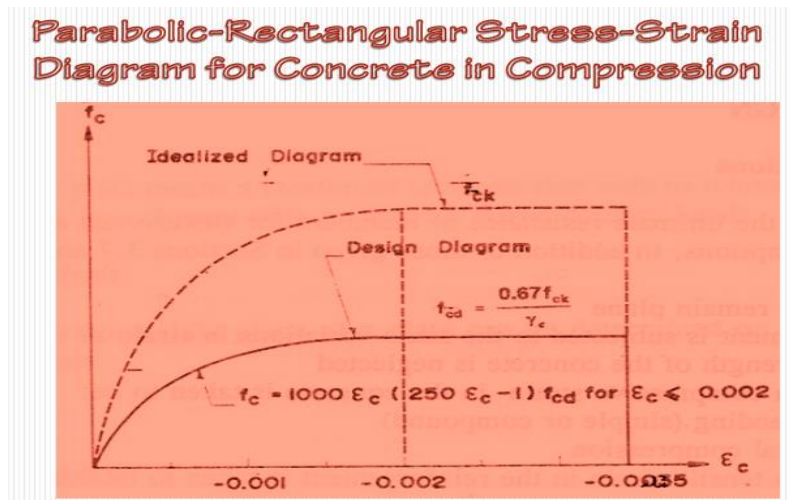
- The characteristic tensile strength

$$f_{ctk} = 0.21 f_{ck}^{2/3}$$

- The tensile strength of concrete is neglected for ULS

Stress-Strain of Concrete

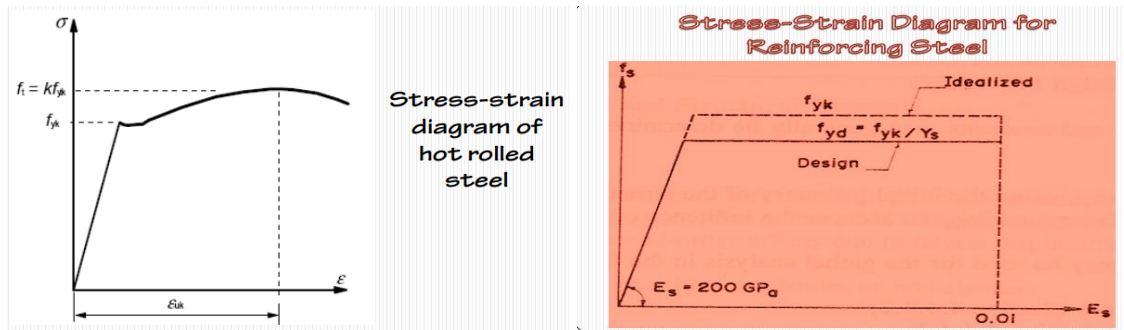
The concrete compressive stress-strain distribution may be assumed to be rectangular, parabolic, or any other shape that results in a prediction of strength in substantial agreement with the test results.



Steel reinforcement bars

Tensile Strength:

- ✓ Yield strength => grade
- ✓ Ultimate strength = tensile strength



Inelastic moment redistribution

When designing a structure and its components, the designer must decide on the appropriate structural model. The choice of the model effects:

- the analysis of the structure, which is aimed at the determination of the stress (internal forces and moments), and
- the calculation of cross section resistance

- ✚ **Redistribution of moments:** shifting of moment after the formation of **first plastic hinge** at sections having the **highest bending moment** till the collapse of the structure.
- ✚ **Redistribution of moment** is possible if the section forming the plastic hinge has the ability to **rotate at constant moment**, which depends on the **amount of reinforcement** provided at that section.
- ✚ The section must be **under – reinforced** and should have sufficient **ductility**.

A limited amount of redistribution is permitted by ESEN depending up on the degree of ductility without explicit calculation of rotation requirements and capacities.

The amount of redistribution allowed is dependent on the grade of the concrete and on the ductility characteristics of the reinforcement as well as the neutral axis.

The limiting maximum neutral axis depth ratio used for proportioning of sections of continuous beams and rigid jointed braced frames are obtained as follow:

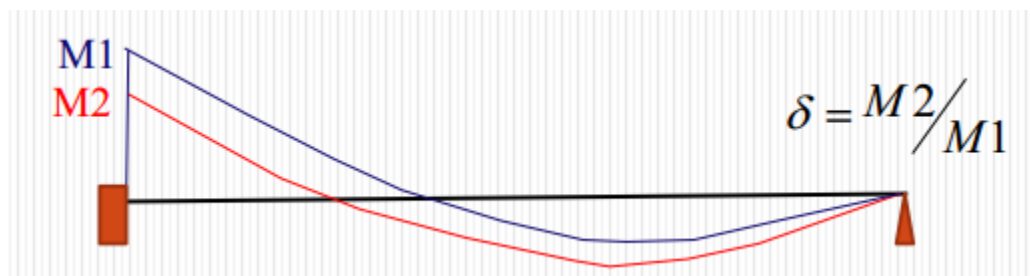
- For 30% redistribution of elastic moment, $x/d = 0.208$ ($\epsilon_c = 0.00263$), $\omega_{\max} = 0.1552$ & $\mu_{\max} = 0.142$
- For 20% redistribution of elastic moment, $x/d = 0.288$ ($\epsilon_c = \epsilon_{cu}$), $\omega_{\max} = 0.2304$ & $\mu_{\max} = 0.204$
- For 10% redistribution of elastic moment, $x/d = 0.368$ ($\epsilon_c = \epsilon_{cu}$), $\omega_{\max} = 0.2944$ & $\mu_{\max} = 0.251$

For no reduction of elastic moment, $x/d = 0.448$ ($\epsilon_c = \epsilon_{cu}$), $\omega_{\max} = 0.3584$ & $\mu_{\max} = 0.294$

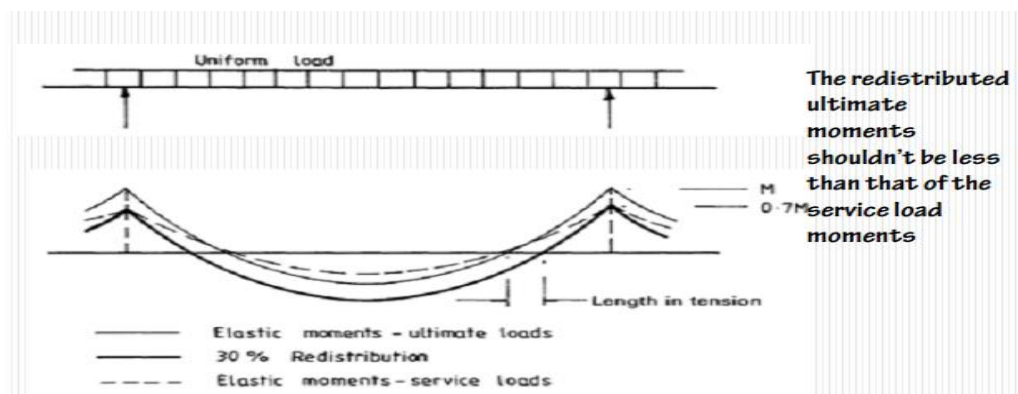
Both continuous beams and building frames are statically indeterminate structures. Internal forces developed and the resulting deformations of statically indeterminate structures depend not only on the loads but also on relative stiffness of connected members. The determination of internal forces (moments, shears and normal forces) in continuous concrete structures is usually based on elastic analysis of structures. Therefore, we can use any of the following elastic methods of analysis of indeterminate structures:

- Moment distribution method (both Cross- and Kani-moment distribution)
- Slope deflection method
- Methods of consistent-deformation
- Matrix method of analysis of structures (both Flexibility and Stiffness methods)
- Available commercial soft-wares such as SAP-2000, STAD-Pro, ANSY and etc. All commercial soft-wares are based on Stiffness matrix-method.

Inelastic Moment redistribution allows designing critical sections (support sections) for a reduced elastic moment.



Moments obtained from a linear elastic analysis may be reduced at critical sections provided that the moments are increased in other sections in order to maintain equilibrium.



A limited amount of redistribution is permitted by ESEN.

- For other continuous beams and rigid jointed braced frames

$$\delta \geq 0.75$$

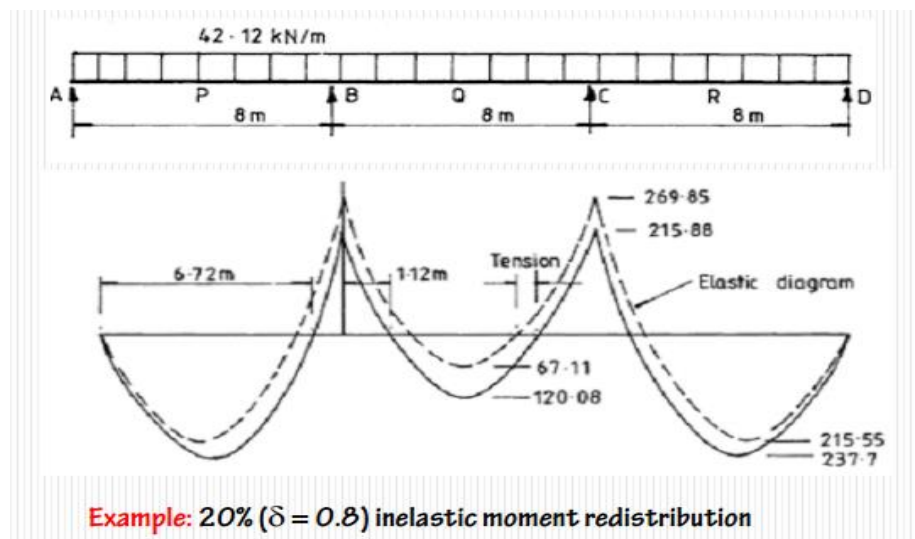
- For sway frames with slenderness ratio of column less than 25

$$\delta \geq 0.9$$

The above restrictions for inelastic moment redistribution coefficient are based on the fact that the reduced ultimate moments should be greater than the corresponding service moments. [$\delta > 2/(\gamma G + \gamma Q)$, inverse of partial load factors average].

Advantages of moment distribution:

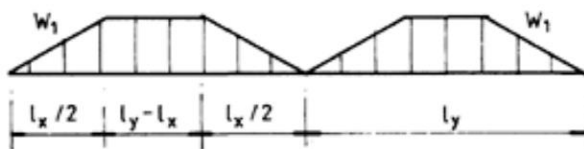
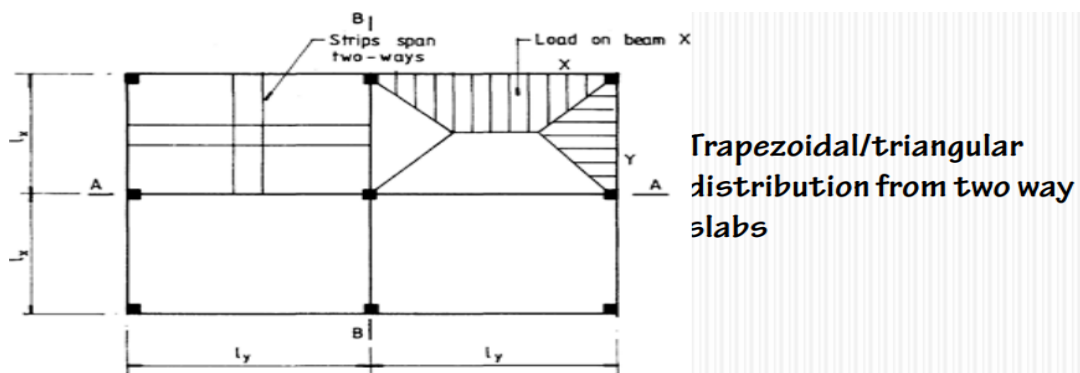
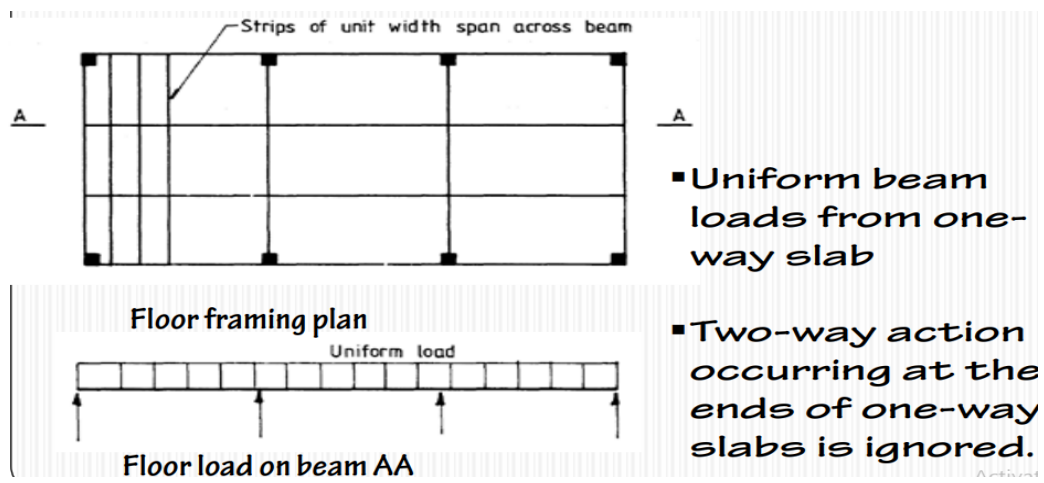
- Results in even arrangement of rebar
- Relieves rebar congestion at supports
- Saves the required amount of reinforcement



Analysis of continuous floor beams:

- ✓ Floor Load Distribution on Beams
- ✓ Load Cases: arrangement of live loads
- ✓ Moment Envelopes & Inelastic Moment Redistribution

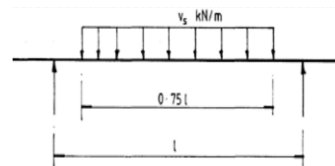
Floor Load Distribution on Beams



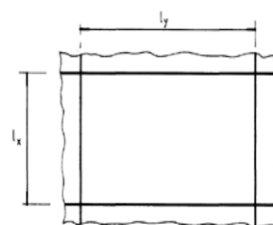
Floor load on beam AA



Floor load on beam BB



Rectangular distribution from two way slabs



$$\begin{cases} v_{sx} = \beta_{vx} q l_x & \text{for short span} \\ v_{sy} = \beta_{vy} q l_y & \text{for short span} \end{cases}$$

Activate
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Load cases: Arrangement Imposed Loads

The gravity loading should be arranged to give the most adverse conditions at any section along the beam.

Responses of a continuous beam may be obtained from analysis of the structure under a single load case of maximum design load if:

- $Q_k / G_k < 1.25$, and
- $Q_k < 5 \text{ kN/m}^2$ (excluding partitions)

Response envelopes for continuous beams should be obtained considering the following possible arrangements.

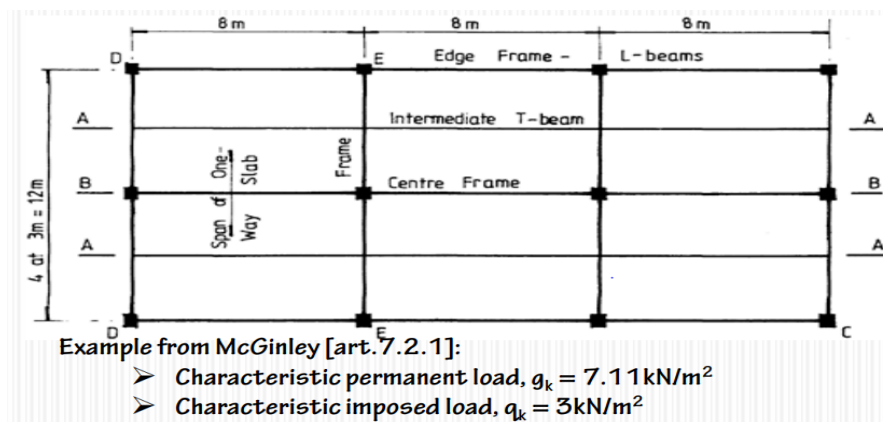
- All spans loaded with the maximum design ultimate load ($1.35G_k + 1.5Q_k$)
- Alternate spans are loaded with the maximum design ultimate ($1.35G_k + 1.5Q_k$) and all other spans are loaded with the minimum design ultimate load ($1.0G_k$)

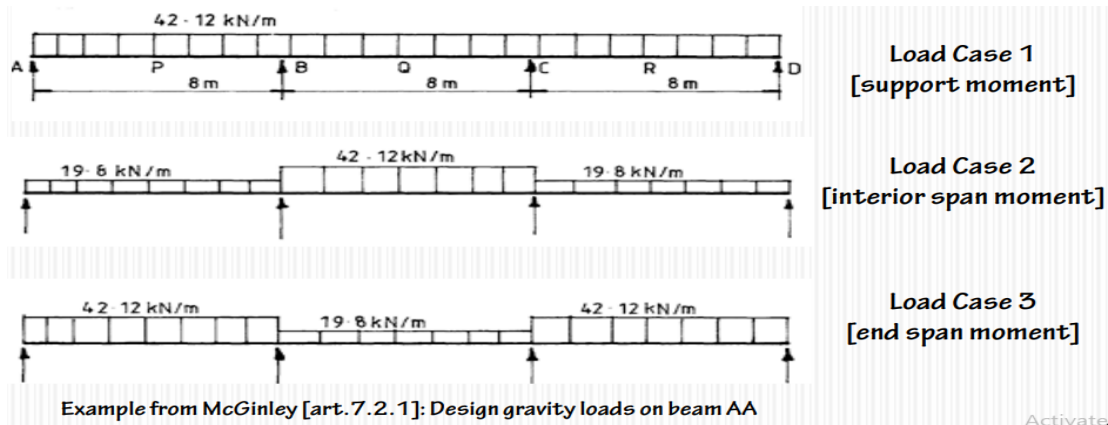
Example: Analysis of Continuous Beams

Design the flexural reinforcement required for beam on axis AA, and sketch the rebar detailing on the longitudinal and x-section considering:

- 1) Elastic moments without inelastic Moment Redistribution
- 2) Moments with 20% inelastic moment redistribution.

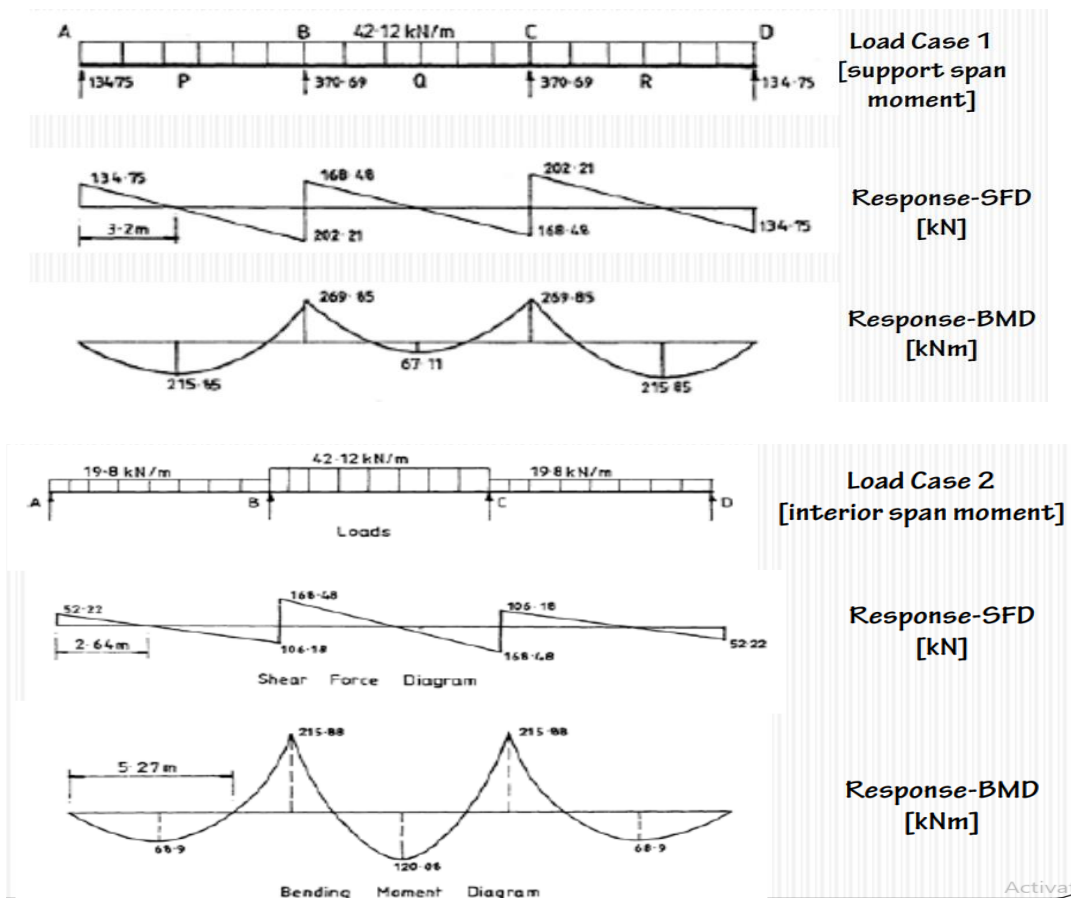
- Assume concrete grade C-25, Rebar Grade S-400, and section $b_w/h = 250/450$, slab thickness = 150mm

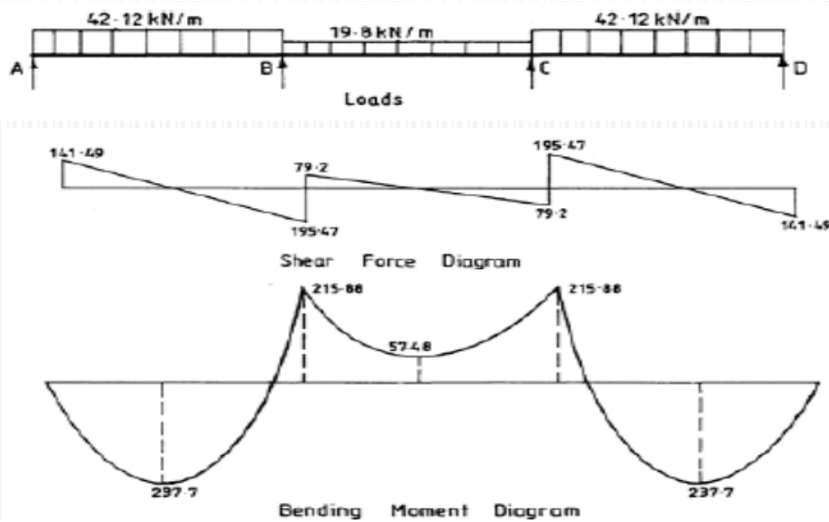




Analysis of Continuous Beams

- ⇒ Approximate: Codal coefficients
- ⇒ Elastic:
 - ✓ **Manual:** Moment Distribution Method
 - ✓ **Computer:** Finite Element Method



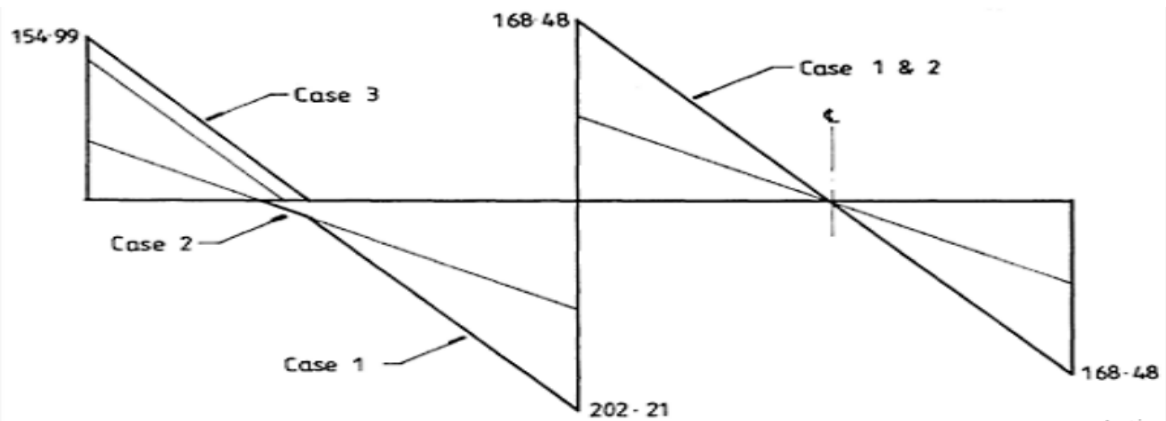


Load Case 3
[End span moment]

Response-SFD
[kN]

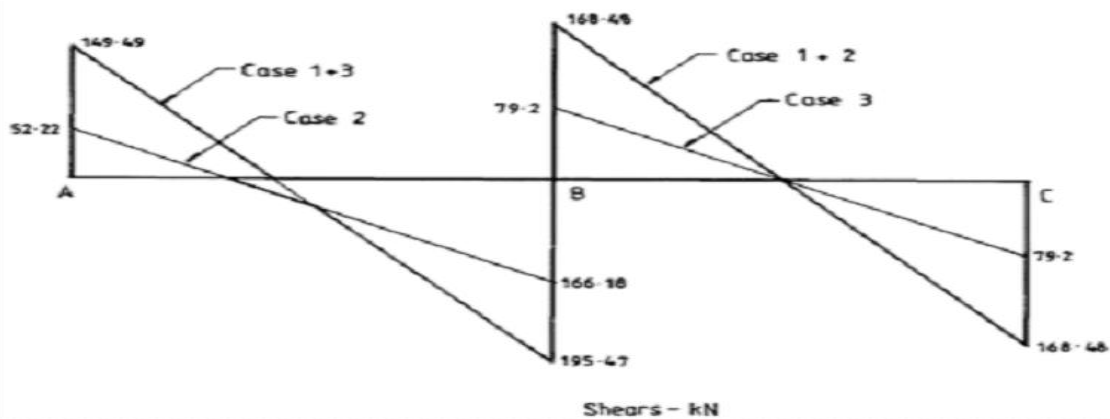
Response-BMD
[kNm]

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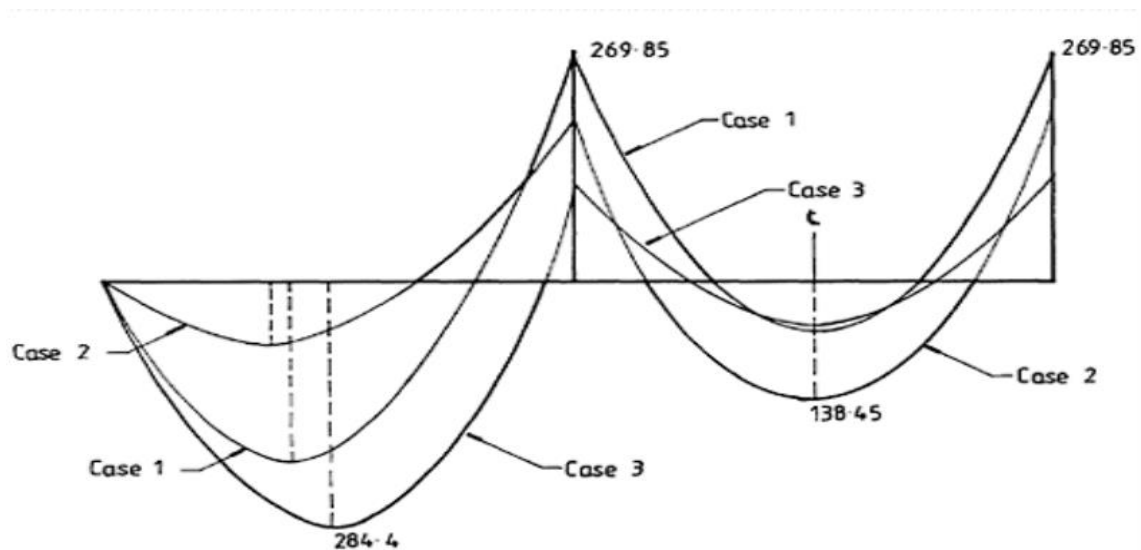


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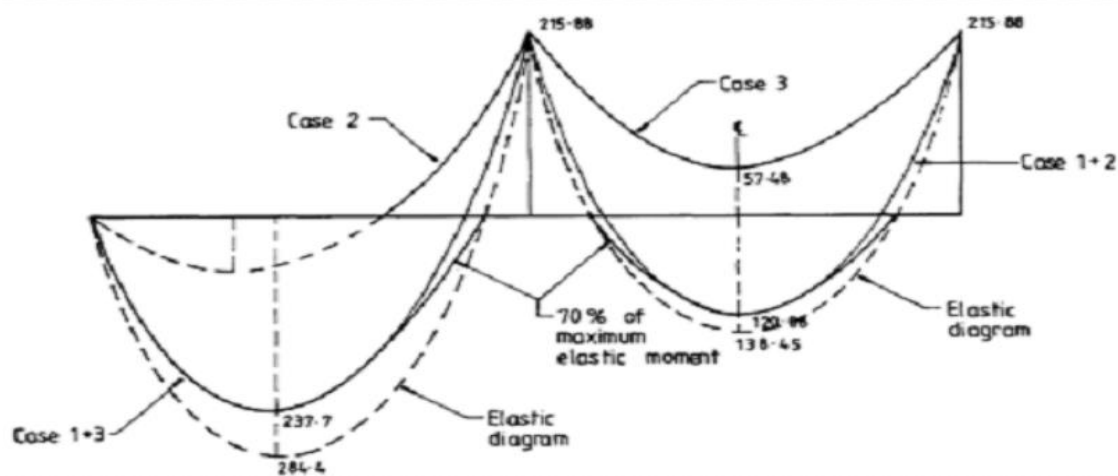
Shear force envelope (kNm) without inelastic moment redistribution



Shear force Envelope (kNm), after 20% inelastic moment redistribution



Elastic bending moment envelope (kNm) without inelastic moment redistribution



Bending moment Envelope (kNm), after 20% inelastic moment redistribution

* Flexural Design

- ✓ Specify the grades of materials
- ✓ Specify preliminary section size
 - Deflection requirement
 - Rigidity requirement
- ✓ Estimate designed ultimate and service action effects
 - Δ_s
 - V_{sd} , M_{sd} , with inelastic moment redistribution
- ✓ Design for flexural strength: Calculate reinforcement

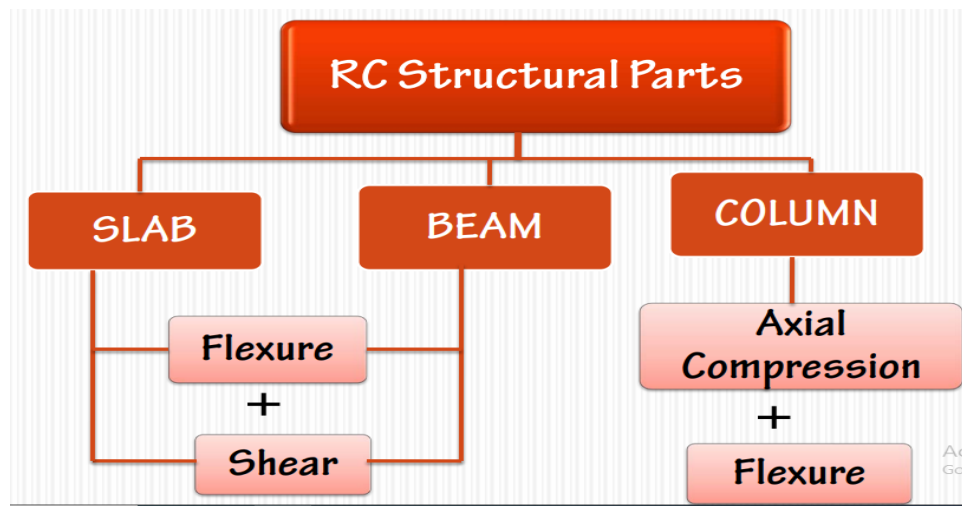
- using the design chart (rectangular parabolic)
- using formula (rectangular stress distribution)
- ✓ Check flexural ductility [max. reinforcement]

$$x/d \leq 0.8(\delta - 0.44)$$

- ✓ If it doesn't satisfy the ductility compliance criteria, either $\neg$
 - increase the depth or
 - provide compression reinforcement
- ✓ Check serviceability requirements
 - Deflection
 - Cracking

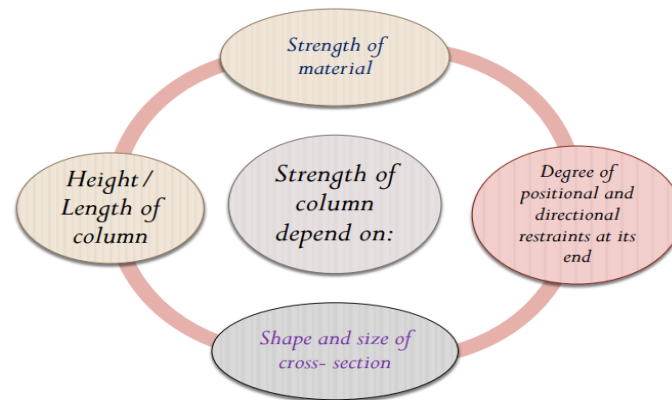
2. Analysis and Design of Columns

General Information



Column:

- Vertical Structural members
- Transmits axial compressive loads with or without moment
- Transmit loads from the floor & roof to the foundation
- The x-sectional dimensions of a column are less than its height.



CLASSIFICATION OF COLUMNS

Columns may be classified based on the following criteria:

- ✚ Depending on level of loading or construction requirements; concrete columns may have different x-sectional detailing.

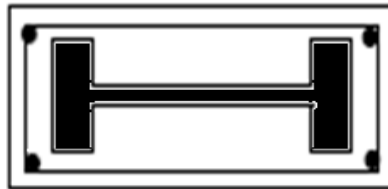
➤ **Reinforced Concrete :**

- Concrete + Rebars



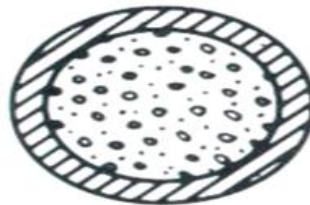
➤ **Composite columns:-** consists of steel or cast iron structural member encased in concrete.

- Concrete + Structural steel



➤ **Infilled columns:-** are those having steel pipe filled with plain or lightly reinforced concrete.

- Steel case + Plain/slightly reinforced concrete



✚ Depending on the structural or architectural requirements; columns may have different xsectional shape.

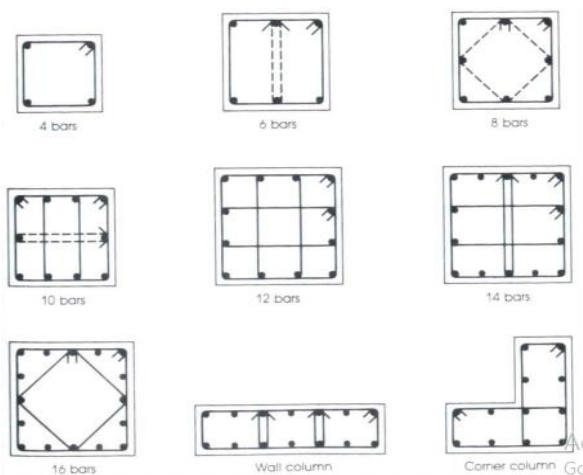
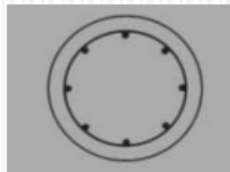
➤ **Rectangular,**

➤ **Square,**

➤ **Circular,**

➤ **L-shaped**

➤ **T-shaped, etc..**



- ✚ Depending on the shape of columns and workmanship; longitudinal rebars of columns may be held in position by either ties or spirals.

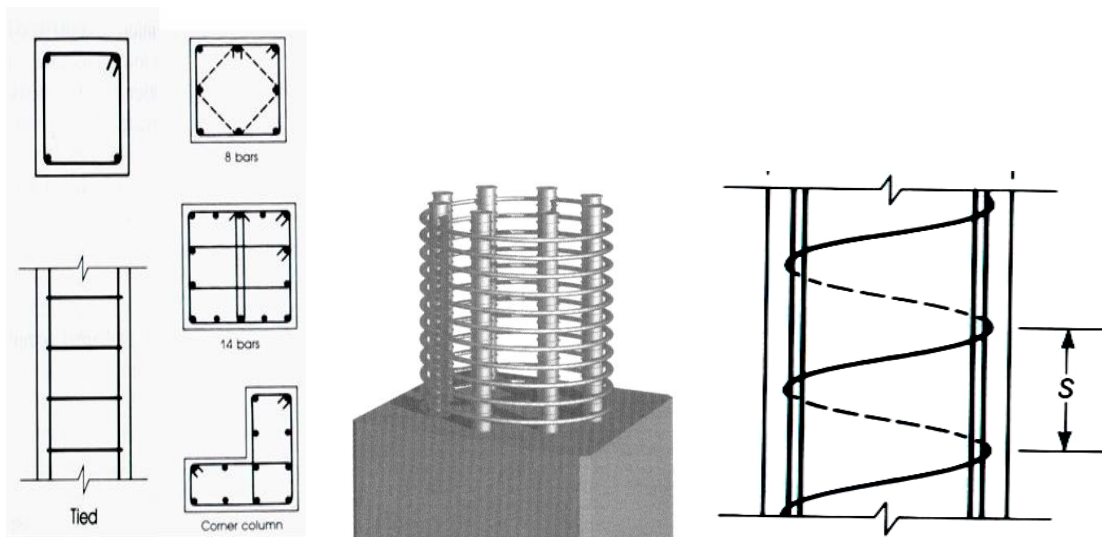
Tied/Spiral Columns

◆ Tie Columns

- Longitudinal reinforcement held in position by separate ties
- Main (longitudinal) reinforcements are held in position by separate ties spaced at equal intervals
- 95% of all columns in buildings are tied
- Over 95% of all columns in buildings in non seismic regions are tied columns
- Tie support long bars (reduce buckling)
- Ties provide negligible restraint to lateral expose of core

◆ Spiral Columns

- Longitudinal bars are wrapped by a closely spaced spiral
- Restrains Poisson's expansion of concrete
- Axial load delays failure (ductile)



- Ductility in columns can be ensured by providing spirals or closely spaced ties.



On the basis of loading:

- Axially loaded column
- Columns under uni-axial bending
- Columns under biaxial bending

Structural Frames

Frame: rigidly connected beam-columns that can transmit both vertical and lateral loads to the foundation.

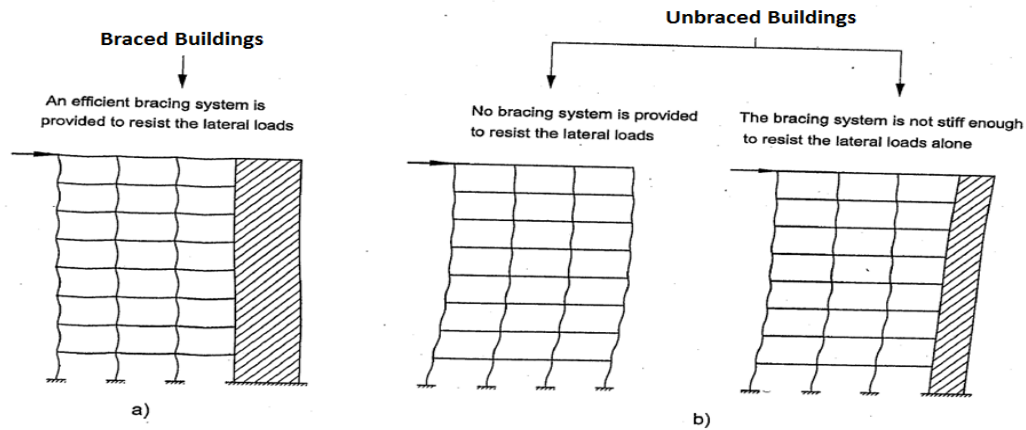
Un-braced frames:

- ⇒ Resist horizontal loads by frames action.
- ⇒ Transmit the horizontal loads to the foundations through bending action in the beams and columns.
- ⇒ Are flexible and allow horizontal displacement
 - Laterally flexible
 - The lateral drift-sway may be significant

Braced frames:

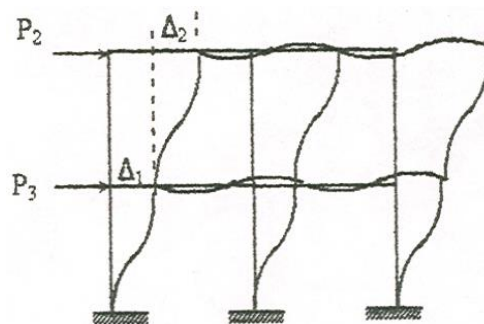
- ⇒ Resist lateral loads substantially by bracing members
- ⇒ Relatively Laterally RIGID — Braces may resist up to 90% of the lateral loads

- ⇒ Where the lateral loads are resisted by diagonal bracing ,shear walls, lift shafts, or other forms of bracing capable of transmitting all horizontal loading to the foundation.



Sway/Non-Sway Columns

- *Sway Frame:*
 - Where effects of horizontal displacements of its nodes ($p-\Delta$) are significant to be considered in design
 - If the lateral inter-storey drift is sufficiently large to influence significantly the column moments, the structure is termed a SWAY FRAME



- *Non-sway frame:*
 - Where effects of horizontal displacements of its nodes ($p-\Delta$) can be neglected in design.
 - A frame may be classified as non-sway if it satisfies:-

$$\frac{N_{sd}}{N_{cr}} \leq 0.1$$

N_{sd} is the design value of the total vertical load

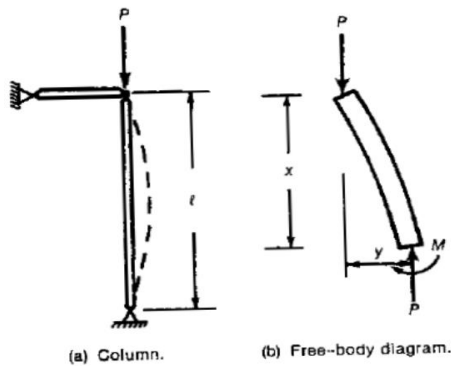
N_{cr} is its critical value for failure in a sway mode

$$N_{cr} = \frac{\pi^2 EI_e}{L_e^2}$$

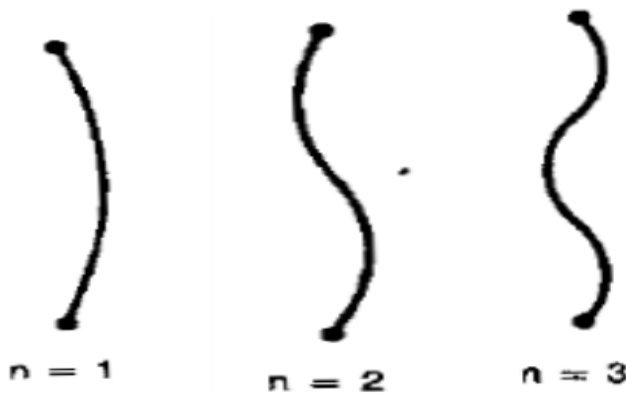
EI_e is the effective stiffness of the substitute column

$$EI_e = 0.2E_c I_c + E_s I_s$$

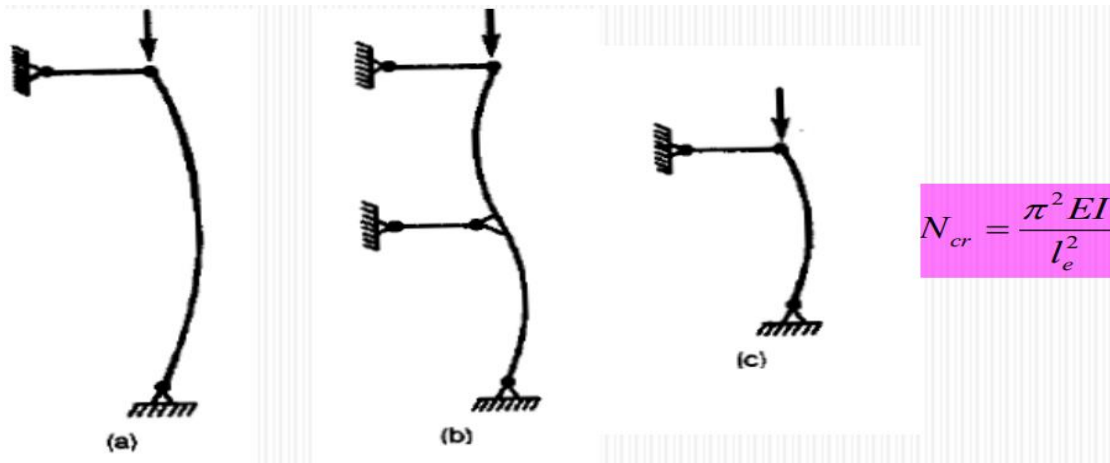
Buckling of Columns



$$\Rightarrow EI \frac{d^2 y}{dx^2} = -Ny$$



$$\Rightarrow N_{cr} = \frac{\pi^2 EI}{(Le/n)^2}$$



The cross sectional dimensions of a column are generally considerably less than its height.

- Short column
- Slender column

Buckling: Effective height of a column

- The effective height of a pin-ended column is the actual height.
- The effective height of a general column is the height of an equivalent pin-ended column of the same strength as the actual member.
- Theoretically the effective height is the distance between the points of inflexion along the member length.

Slenderness Ratio

$$\lambda = \frac{L_e}{i} \quad i = \sqrt{I/A}$$

Storey Frame

Slenderness Ratio

$$\lambda = \sqrt{\frac{12A}{K_1 L}}$$

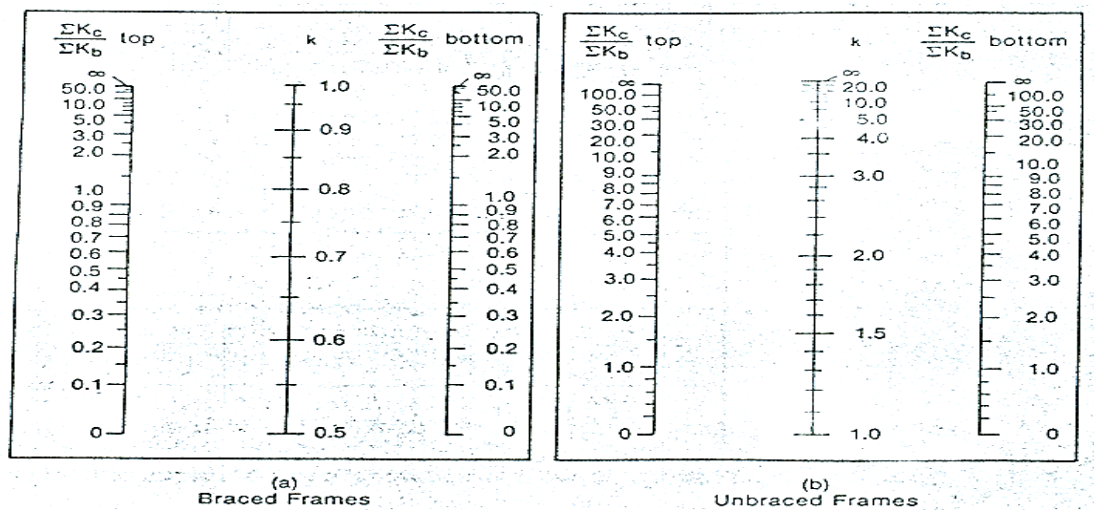
Effective Length of Columns

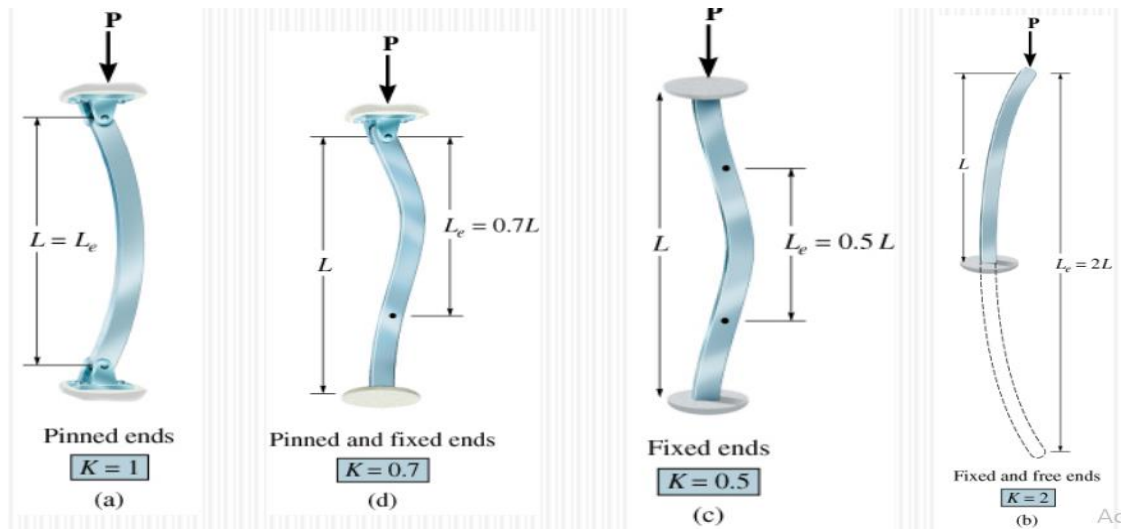
Effective length of column

- The effective length of the column can be determined using
- ❖ Theoretical K-factors for columns with some idealized end connections

Alignment chart

$$\alpha_1 (\text{or } \alpha_2) = \frac{\sum E_{cm} I_{col} / L_{col}}{\sum E_{cm} \beta I_b / L_b}$$





Effective Length of Columns on actual Frame System

$$\alpha_1 (or \alpha_2) = \frac{\sum E_{cm} I_{col} / L_{col}}{\sum E_{cm} \beta I_b / L_b}$$

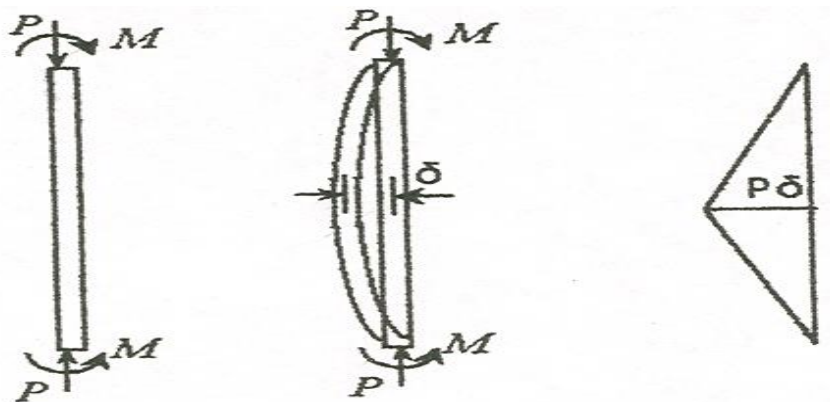
$\beta = 1.0$ opposite end elastically or rigidly restrained

$\beta = 0.5$ opposite end free to rotate

$\beta = 0$ for cantilever beam

Short/Slender Columns

- At the center total moment is $= M + P\delta$



This classification is based on slenderness ratio:

$$\lambda = \frac{l_e}{r} \qquad r = \sqrt{\frac{I}{A}}$$

r is the minimum radius of gyration

l_e is the effective length of column $l_e = k l_{col}$

Limits of Slenderness

- The slenderness ratio of concrete columns shall not exceed 140
- Second order moment in a column can be ignored if

$$\text{a) For sway frames, the greater of } \lambda \leq 25 \text{ or } \lambda \leq \frac{15}{\sqrt{\nu_d}}$$

Where

$$\nu_d = N_{sd} / f_{cd} A_c$$

a) For non-sway frames

$$\lambda \leq 50 - 25 \frac{M_1}{M_2} \quad M_2 (+ve) \text{ and } > M_1$$

❖ Approximate equations

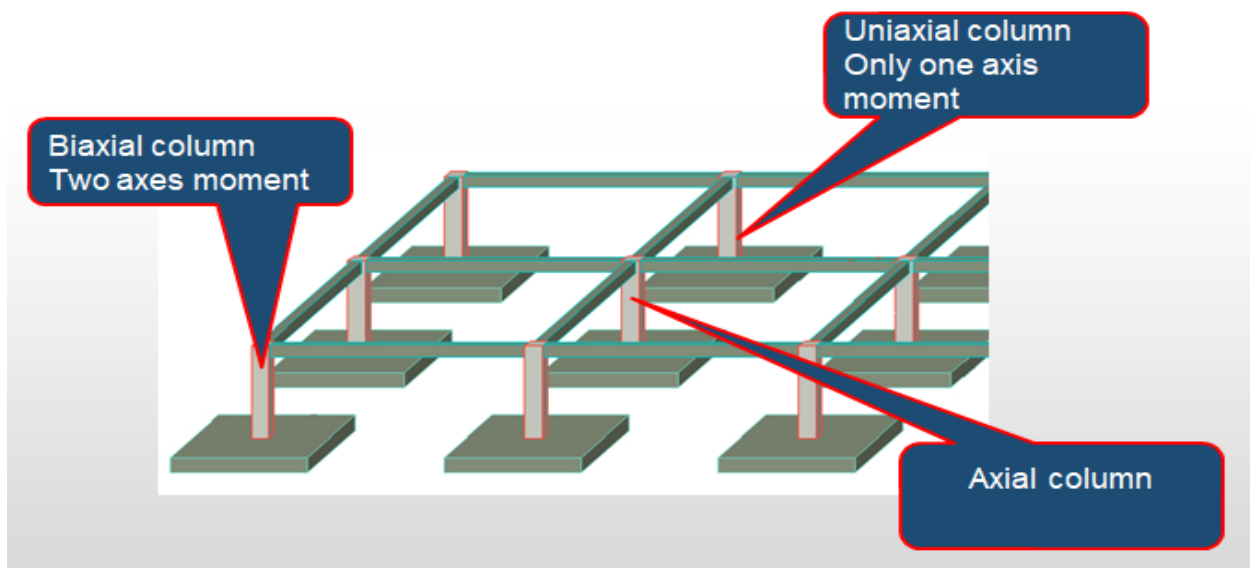
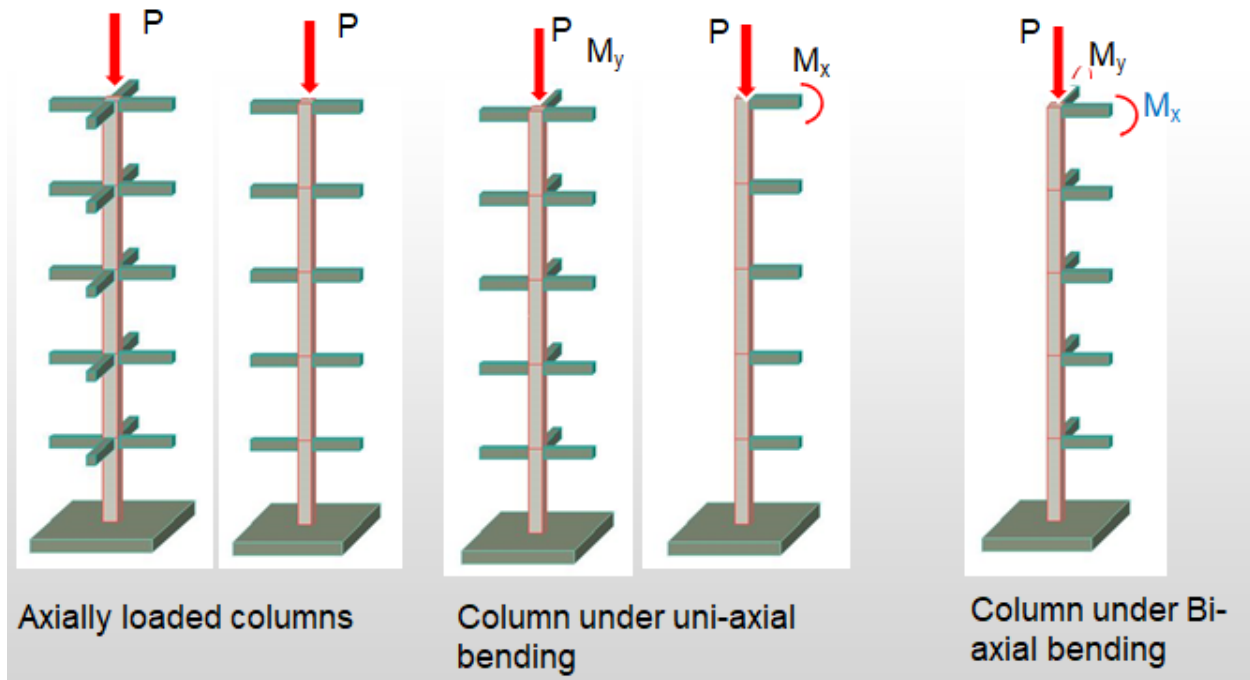
- Non-sway mode $\frac{L_e}{L} = \frac{\alpha_m + 0.4}{\alpha_m + 0.8} \geq 0.7$

- Sway mode $\frac{L_e}{L} = \sqrt{\frac{7.5 + 4(\alpha_1 + \alpha_2) + 1.6\alpha_1\alpha_2}{7.5 + \alpha_1 + \alpha_2}} \geq 1.15$

Or Conservatively $\frac{L_e}{L} = \sqrt{1 + 0.8\alpha_m} \geq 1.15$

$$\alpha_m = \frac{\alpha_1 + \alpha_2}{2}$$

Classification of Columns on the Basis of Loading



Axially loaded columns capacity

- When concentric axial load acts on a short column

$$P_{do} = f_{cd} (A_g - A_{st}) + A_{st} f_{yd}$$

A_g is gross concrete area

A_{st} is total reinforcement area

- When concentric axial load acts on a long column

$$P_{dul} = Cr P_{do} \quad \text{Where} \quad Cr = 1.25 - \frac{Le}{48b}$$

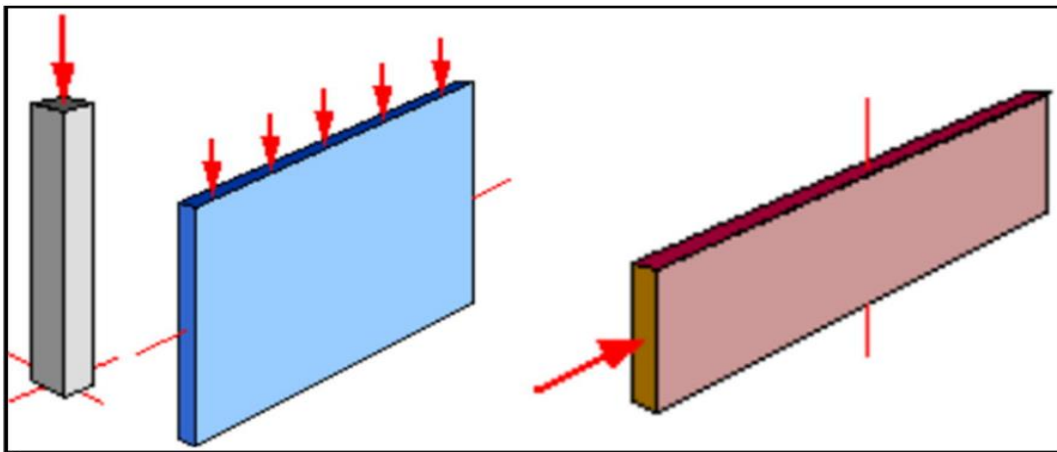
$$\frac{Le}{b} > 12$$

Generally

SLENDERNESS EFFECTS IN STRUCTURES

In structures, slenderness effects occur mainly in ...

- Columns, about one or both axes,
- Walls, about the minor axis, and
- Sometimes in beams which are narrow compared to either the span or the depth.



Methods for assessing slenderness effects in structures involve three basic steps:

1. Determine the slenderness ratio of column
2. Establish whether slenderness effects are significant:
3. Take account of significant effects in the design. This is achieved, essentially, by designing for additional bending moments.

Based on ES EN 1992-2015; column can be considered as short and ignore 2nd order effects if one of the following conditions are met.

If 2nd order effects are less than 10 % of the corresponding 1st order effects (for all)

If the slenderness ratio, λ_{col} is less than λ_{lim}

Active

$$\lambda_{lim} = 20 A.B.C / \sqrt{\eta} \text{ (for isolated members)}$$

Where

$$A = \frac{1}{1 + 0.2\phi_{ef}} \quad [\text{ If } \phi_{ef} \text{ is not known take } A = 0.7]$$

$$B = \sqrt{1 + 2\omega} \quad [\text{ If } \omega \text{ is not known take } B = 1.1]$$

$$C = 1.7 - r_m \quad [\text{ If } r_m \text{ is not known take } C = 0.7]$$

$$\eta = \frac{N_{ED}}{A_c f_{cd}}$$

- ω = is the mechanical reinforcement ratio $= \frac{A_s f_{yd}}{A_c f_{cd}}$
- r_m = is the moment ratio $= \frac{M_{01}}{M_{02}}$
- M_{01}, M_{02} = are the first order end moment $Abs(M_{02}) \geq Abs(M_{01})$
- ϕ_{ef} = is the effective creep ratio $= \phi(\infty, t_o) \frac{M_{oEqp}}{M_{oEd}}$
- M_{oEqp} = is the 1st order moment due to quasi- permanent load combination (SLS)
- M_{oEd} = is the 1st order moment due to design load combination (ULS)

If the end moments give tension on the same side then r_m is taken as positive, otherwise negative

In the following cases r_m should be taken as 1

For braced member where 1st order moment predominantly due to geometric imperfections or transverse loading

For unbraced member in general



Columns Based on Slenderness Ratio

SHORT COLUMN

- are columns with low slenderness ratio $\lambda_{cal} \leq \lambda_{lim}$
- strengths are governed by strength of materials & geometry of the cross section.

LONG (SLENDER) COLUMN

- are columns with high slenderness ratio $\lambda_{cal} > \lambda_{lim}$
- strength may be significantly reduced by lateral deflection.

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STRENGTH OF AXIALLY LOADED COLUMNS

- When a symmetrical column is subjected to a concentric axial load, P , longitudinal strains ϵ , develop uniformly across the section as shown.
- Because the steel & concrete are bonded together, the strains in the concrete & steel are equal.
- Failure occurs when P_o reaches a maximum

Concentric compressive axial load capacity:

$$P_o = f_{cd} (A_g - A_{stot}) + f_{yd} A_{stot} \text{ (in compression) in short column}$$

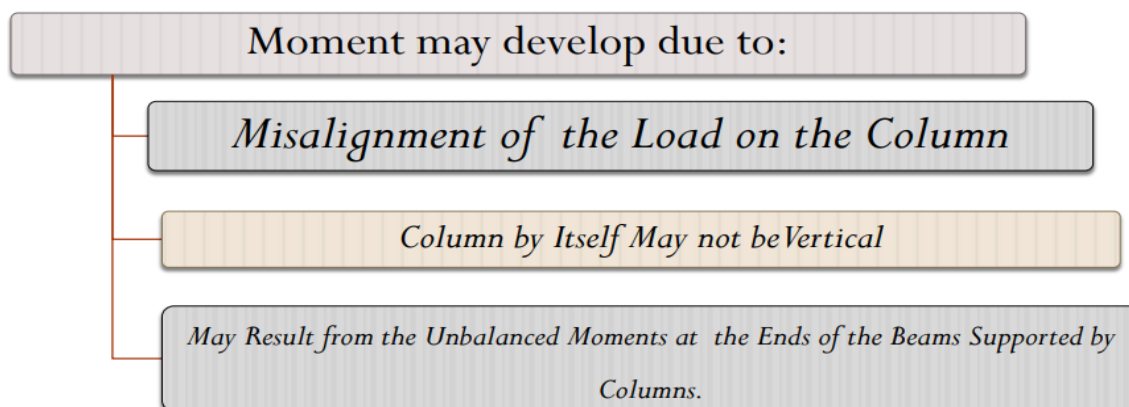
- When concentric axial load acts on a **long column**, its ultimate capacity may be obtained from:

$$P_{dul} = C_r P_o \text{ where } C_r = 1.25 - \frac{L_e}{48 b}$$

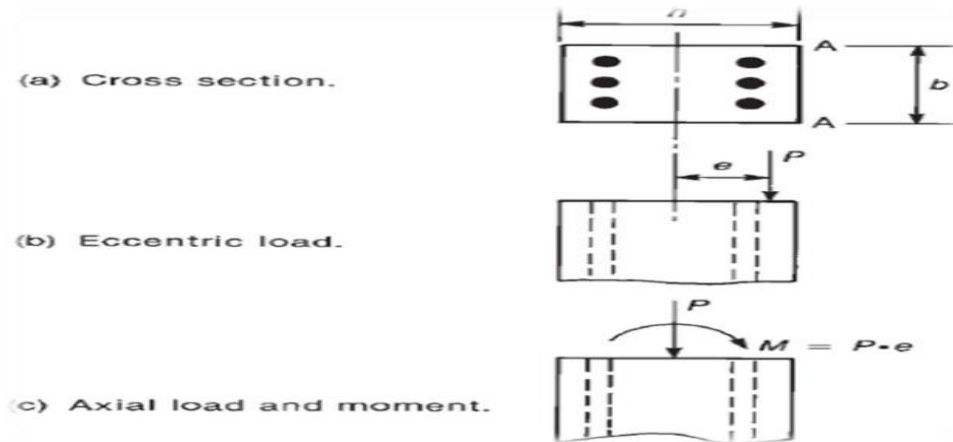
$$P_o = - f_{yd} A_{stot} \text{ (in tension) in short column}$$

COLUMN UNDER UNI-AXIAL BENDING

- Almost all compression members in concrete structures are subjected to moments in addition to axial loads.



- Due to the above and other reasons there is a distance b/n the **geometric center of the column** and the **line of action of the load**, which is called **eccentricity** (e).



- Case on b and c are the same, because the eccentric load can be replaced by an axial load P plus a moment $M = P \cdot e$ about the centroid of the cross-section.

The presence of bending in axially loaded members can reduce axial load capacity of member.

To Illustrate the Interaction b/n Moment & Axial Load in A Column,
the Following Assumption are Considered

Column is made from **homogenous materials**

Column is made from **elastic material**

compressive strength of concrete (f_{cu}) = tensile strength of concrete (f_{tu})

- For **uniaxial column** failure would occurs in a compression when the maximum stresses reached f_{cu} as given by:

$$f_{cu} = \frac{P}{A} + \frac{M \cdot y}{I}$$

DESIGN ECCENTRICITY S PER ES EN 1-1: 1992-2015

Eccentricity (e)- is the distance b/n the geometric center of the section and the line of action eccentric load of column.

In column design we may include the following eccentricity

- Initial eccentricity for geometric imperfection (e_i)
- Equivalent first order eccentricity (e_e)
- Second order eccentricity (e_2)

- The eccentricity to be used for design of columns of constant cross section at the critical section is given by:

$$e_{tot} = e_i + e_e + e_2$$

Initial Eccentricity for Geometric Imperfection (e_i)

$$e_i = \theta_i \frac{l_o}{2} \text{ OR } e_i = l_o / 400, \quad \theta_i = \theta_o * a_h * a_m; \quad a_h = 2 / \sqrt{L} \quad 2/3 \leq a_h \leq 1,$$

$$a_m = \sqrt{0.5(1 + 1/m)}; \quad \theta_o = 1/200$$

m- is the number of vertical members contributing to the total effect

- **Initial eccentricity (e_i)**

Accounts:-Imperfection in vertical alignment

Imperfection in horizontal alignment

Improper connection b/n slabs, beams, etc

Equivalent First Order Eccentricity (e_e)

$$e_e = \max \left\{ \begin{array}{l} 0.6e_{o2} + 0.4e_{o1} \\ 0.4e_{o2} \end{array} \right.,$$

$$e_{o1} = \frac{M_{o1}}{P} \quad \text{and} \quad e_{o2} = \frac{M_{o2}}{P}$$

Where:

- e_{o1} and e_{o2} - are the 1st order eccentricity at the ends.
- e_{o2} - being positive and greater in magnitude than e_{o1}
- e_{o1} - is negative if the member is bent in double curvature other ways it is positive

Minimum Reinforcement Code Requirement

Transverse reinforcement (lateral)

- Min. diameter of ties or spirals 6mm or $\Phi/4$ longitudinal
- c/c spacing $\leq \begin{cases} 12\phi \text{ main longitudinal} \\ b \text{ (least diemnsion of column)} \\ 300\text{mm} \end{cases}$

Longitudinal Reinforcement According to ES EN 1-1 1992-2015

- The total amount of longitudinal reinforcement should not be less than $A_{s,min}$.

$$A_{s,min} = \frac{0.10 * N_{Ed}}{f_{yd}} \text{ or } 0.002 A_c \text{ which ever is the greater one}$$

Where:

- f_{yd} – is the design yield strength of the reinforcement
- N_{Ed} – is the design axial compression force
- The area of longitudinal reinforcement should not exceed $A_{s,max}$.
- The recommended value is $0.04 A_c$ outside lap locations. This limit should be increased to $0.08 A_c$ at laps.
 - Minimum diameter of main reinforcement, $\Phi_{min} = 12\text{mm}$
 - Minimum cover to reinforcement $\Phi_{longitudinal}$
 - Minimum number of bars shall be 6 for circular arrangement and 4 for rectangular

Lateral Reinforcement According to ES EN 1-1 1992-2015

- The diameter of the transverse reinforcement (links, loops or helical spiral reinforcement) **should not be less than 6 mm or 1/4 of the maximum** diameter of longitudinal bars, whichever is the greater.
- The transverse reinforcement should be anchored adequately.
- The spacing of the transverse reinforcement along the column should not exceed $s_{cl,tmax}$.
- The recommended value is the least of the following three distances:
 - 20 times the minimum diameter of the longitudinal bars
 - the lesser dimension of the column
 - 400 mm
- **Spirals or circular ties** may be used for longitudinal bars located around the perimeter of a circle. The pitch of spirals shall **not exceed 100 mm**.
- Every longitudinal bar or **bundle of bars** placed in a corner should be held by **transverse reinforcement**. No bar within a compression zone should be further than 150 mm from a restrained bar.

DESIGN PROCEDURE FOR UNIAXIAL COLUMN

In the actual design, interaction charts prepared for uniaxial bending can be used. The procedure involves:

1. Assume a cross section, d' and evaluate d'/h to choose appropriate chart
2. Compute:

$$\text{Normal force ratio: } \nu = \frac{Nu}{b \cdot h \cdot f_{cd}}; \quad \text{Moment ratios: } \mu = \frac{Mu}{b \cdot h^2 \cdot f_{cd}}$$

3. Enter the chart and pick ω (the mechanical steel ratio), if the coordinate (ν, μ) lies within the families of curves. If the coordinate (ν, μ) lies outside the chart, the cross section is small and a new trial need to be made.
4. Compute $A_{stot} = \frac{\omega \cdot A_c \cdot f_{cd}}{f_{yd}}$
5. Check A_{stot} satisfies the maximum and minimum provisions
6. Determine the distribution of bars in accordance with the charts requirement

BIAXIAL LOADED COLUMNS

♣ Up to this point in the chapter we have dealt with columns subjected to axial loads accompanied by bending about one axis.

♣ Axial compression is accompanied by simultaneous bending about both principal axes of the section

♣ Axial force P acting with eccentricities e_z and e_y , such that $e_z = My/p$, $e_y = Mz/p$ from centroidial axes

♣ This is the case in corner columns, interior or edge columns with irregular column layout

→ One approach is, we can convert biaxial bending into uniaxial bending by finding the resultant moment vector thus:

There are different methods for the design of Biaxial loaded columns:

- Strain compatibility method
- The equivalent eccentricity method
- Load contour method
- Bresler reciprocal load method

All the methods used for uniaxial column is also used to design biaxial column, the only difference is that in biaxial column design moment is considered in each principal directions.

→ **Separate design** in each principal direction, disregarding biaxial bending

→ Separate design in each principal direction is **possible** if the following conditions are satisfied:

$$\hookrightarrow \frac{\lambda_y}{\lambda_z} \leq 2 \quad \text{and} \quad \frac{\lambda_z}{\lambda_y} \leq 2 \quad \text{slenderness ratios}$$

$$\hookrightarrow \frac{e_y/h_{eq}}{e_z/b_{eq}} \leq 0.2 \quad \text{or} \quad \frac{e_z/b_{eq}}{e_y/h_{eq}} \leq 0.2 \quad \text{relative eccentricities}$$

DESIGN OF BI-AXIAL COLUMN, AS PER ES EN 1992 1-1-2015

Where:

b, h are the width and depth of the section

$$b_{eq} = i_y \cdot \sqrt{12} \quad \text{and} \quad h_{eq} = i_z \cdot \sqrt{12}$$

λ_y, λ_z are the slenderness ratios with respect to y- and z- axis respectively

i_y, i_z are the radii of gyration with respect to y- and z-axis respectively

$e_y = M_{Edz}/N_{Ed}$ eccentricity along y-axis

$e_z = M_{Edy}/N_{Ed}$ eccentricity along z-axis

M_{Edy} is the design moment about y-axis, including second order moment

M_{Edz} is the design moment about z-axis, including second order moment

N_{Ed} is the design value of axial load in the respective load combination

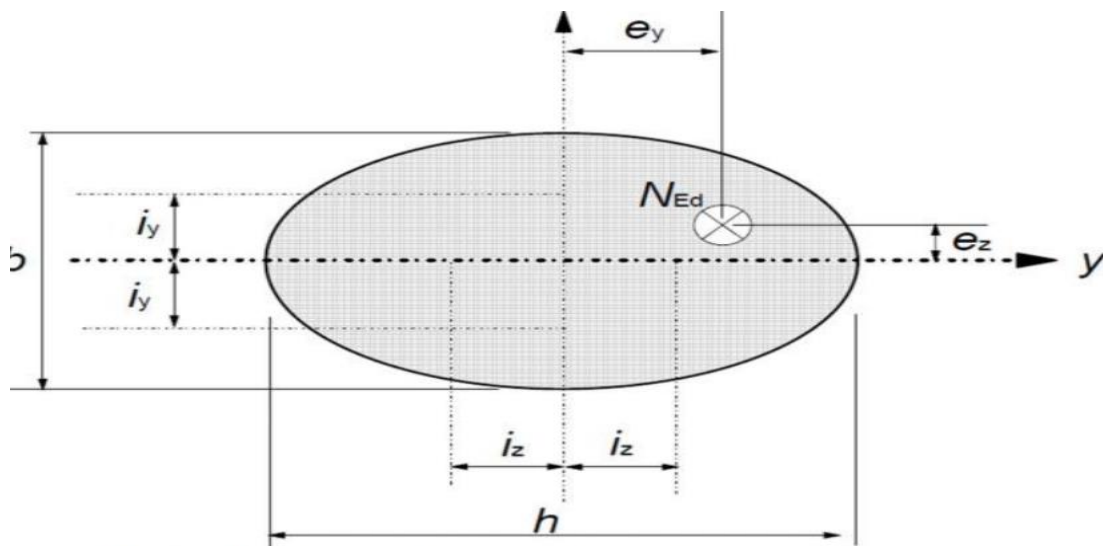


Fig. definition of eccentricities e_y and e_z

- If the conditions of above expression are not fulfilled, biaxial bending should be taken into account including the 2nd order effects in each direction (unless they may be ignored). In the absence of an accurate cross section design for biaxial bending, the following simplified criterion may be used

$$\left(\frac{M_{Edz}}{M_{Rdz}}\right)^a + \left(\frac{M_{Edy}}{M_{Rdy}}\right)^a \leq 1$$

Where:

$M_{Edz/y}$ is the design moment around the respective axis, including a 2nd order moment.

$M_{Rdz/y}$ is the moment resistance in the respective direction

a is the exponent;

for circular and elliptical cross sections: $a = 2$

For rectangular cross section

N_{Ed}/N_{Rd}	0.1	0.7	1.0
$a =$	1.0	1.5	2.0

N_{Ed} is the design value of axial force

$N_{Rd} = A_c f_{cd} + A_s f_{yd}$, design axial resistance of section.

where:

A_c is the gross area of the concrete section

A_s is the area of longitudinal reinforcement

DESIGN OF BIAXIAL COLUMN USING INTERACTION DIAGRAM, AS PER ES EN

Interaction chart in Design

In the actual design, interaction charts prepared for **biaxial bending** can be used.

The procedure involves:

1. Calculate **first order moment** and **axial force**
2. Determine **design moments** including required **eccentricities**
3. Select cross section dimensions **h** and **b** and also **h'** and **b'**
4. Calculate **h' / h** and **b' / b** (range of values of 0.05, 0.1, 0.15, ...etc are available)

5. Compute Normal force ratio: $\nu = \frac{Nu}{b \cdot h \cdot f_{cd}}$

Moment ratios: $\mu_h = \frac{M_h}{h \cdot A_c \cdot f_{cd}}$ and $\mu_b = \frac{M_b}{b \cdot A_c \cdot f_{cd}}$

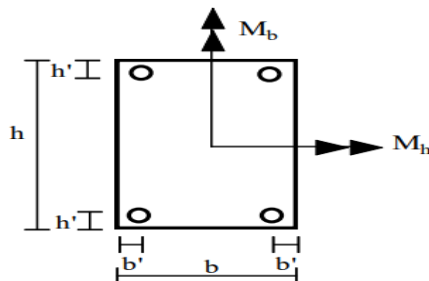
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5. Enter the chart and pick ω (the mechanical steel ratio)

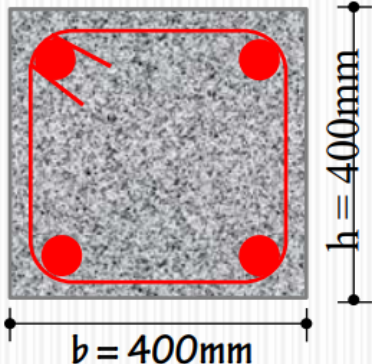
6. Compute $A_{stot} = \frac{\omega \cdot A_c \cdot f_{cd}}{f_{yd}}$

7. Check A_{stot} satisfies the maximum and minimum provisions

8. Determine the distribution of bars in accordance with the charts requirement



Problem: Draw the dimensionless interaction diagram of the following section. Use C-25 concrete and S-400 rebar.



Use:

- 25mm Clear cover
- $\Phi 24$ main longitudinal rebar
- $\Phi 8$ ties

3. Analysis and Design of beams for Torsion

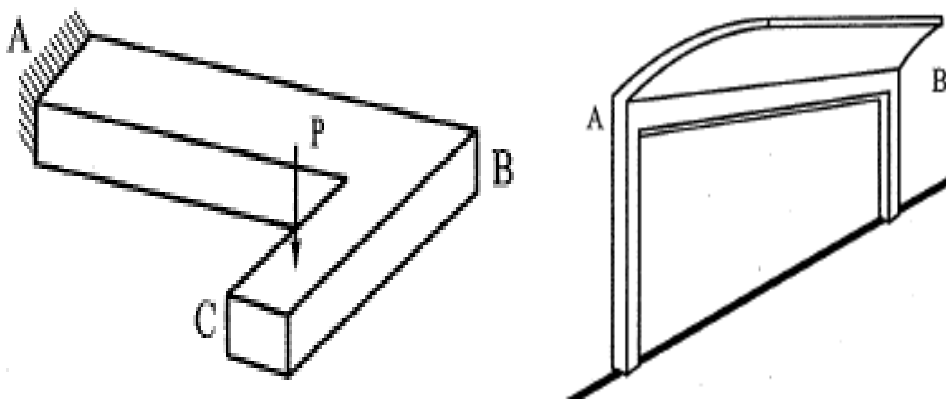
Introduction

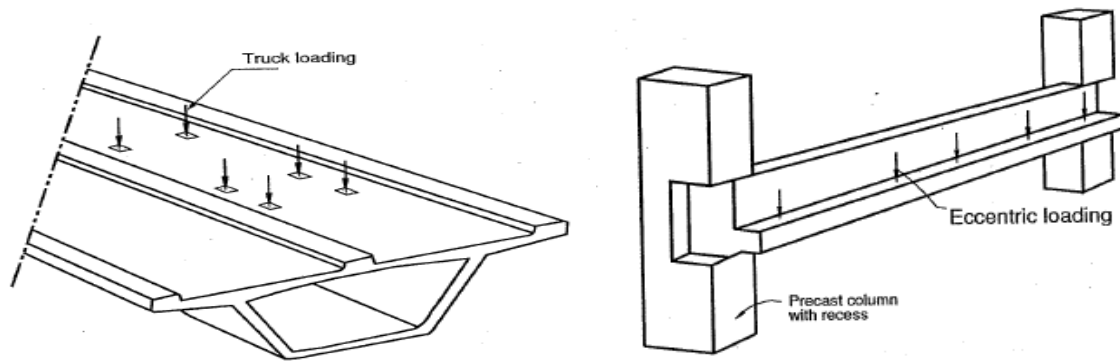
Members subjected to a torsional moment, commonly known as a **torque**, develop shear stresses. In general, these tend to increase in magnitude from the longitudinal axis of the member to its surface. If the **shear stresses** are sufficiently large, cracks will propagate through the member and, if torsion reinforcement is not provided, the member will **collapse** suddenly.

Two types of torsion are commonly identified:

- a) **Equilibrium torsion**: where the torsional moment is required to maintain equilibrium of the structure. In such situation the external load has no other option but to be carried by torsion.
Equilibrium torsion is of primary interest in design because failure of the member is inevitable if it has insufficient torsional strength.
- b) **Compatibility Torsion**: torsion caused due to the resultant of angular rotation induced by rigidly connected members. It arises from the requirement of continuity, i.e., compatibility of deformation between adjacent parts of a structure

Curved bridge girders, edge beams of slabs and shells, spiral stair-cases, and eccentrically loaded box beams constitute examples for members subjected to high **twisting moments** accompanied by bending moments and shear forces.





Failure of concrete members with torsion

Consider the rectangular member of Fig. below subjected to a torque T . Since there are no other external forces (and ignoring self-weight) the member is considered to be in pure torsion. The torque causes the member to twist and to develop shear stresses.

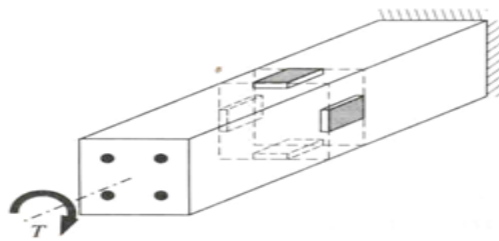


Figure Elements in member subjected to torsion

The member illustrated in Fig. below is subjected to a vertical force, V , in addition to the applied torque. This results in a combination of **bending**, **shear** and **torsion** and alters the orientation of the inclined cracks.

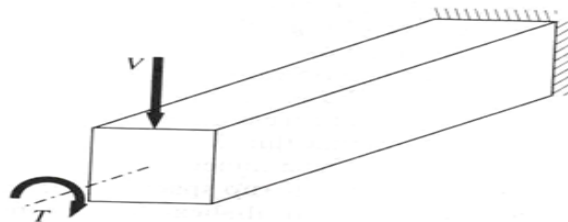


Figure Combined shear, moment and torsion

For members with **no form of reinforcement** to prevent the opening of Torsional cracks, failure of the member will occur almost as soon as the cracking begins.

Therefore, **torsional failure of a member** without reinforcement is prevented only if the shear strength of the concrete exceeds the shear stress due to applied torsion. In practice, the **shear strength** is increased slightly through dowel action by the longitudinal reinforcing bars which cross the cracks.

The torsional strength of a concrete member can be significantly increased by providing **suitable torsion reinforcement** across the cracks. This is usually provided in the form of 'closed' **four-sided stirrups**, as illustrated in Fig. below, in combination with longitudinal bars distributed around the periphery of the section. **This reinforcement** controls the propagation of cracks and ensures that when failure occurs due to yielding of the reinforcement, it is not sudden.

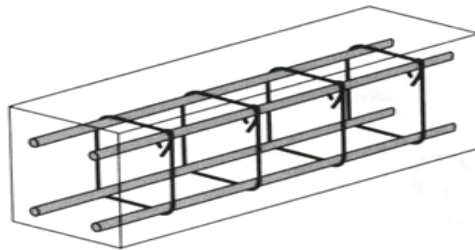


Figure Torsion reinforcement

Torsion in Beams

Two types of torsion

- Primary or equilibrium torsion

Which is required to maintain basic static equilibrium?

- Secondary or compatibility torsion

Is as a result of the restraints, which required maintaining only compatibility condition b/n members

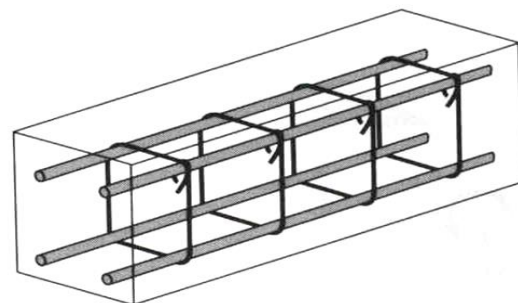
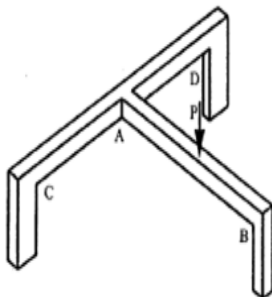
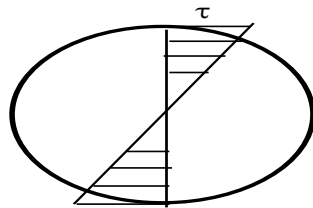


Figure Torsion reinforcement

Behaviours of Members with torsion

- Torsional moment causes shear stresses on cross sectional planes and on radial planes extending from the axis of the member to the surface.

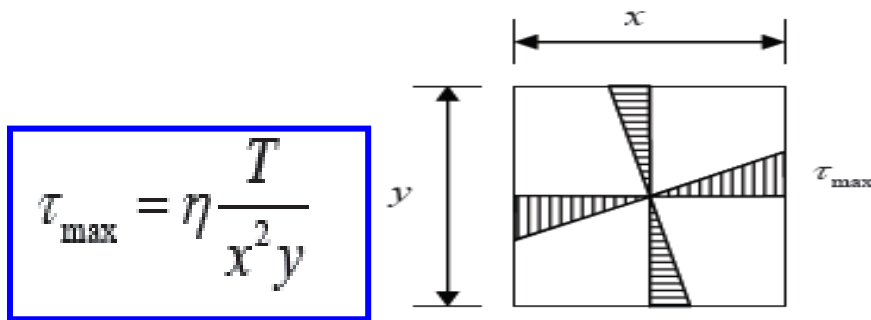
1. For circular member



$$\tau_{\max} = \frac{Tr}{j}$$

2. For non-circular member:- Two approaches

- Elastic approach : St. Venant Principle



where η = shape factor, T = torque,

x, y = narrow and wide side dimensions of the cross section

- **Plastic Approach**
 - Yielding of sections (for ductile materials shear occurs over the entire areas)

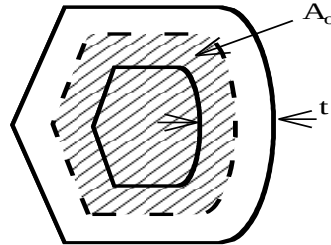
Nada “**Sand Heap Analogy**”

- Finally, similar expression is obtained using both approaches

3. For thin-walled hollow section of any shape

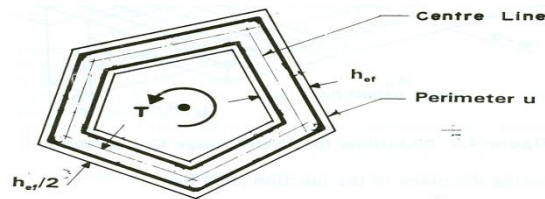
1. The shear stress in the walls is reasonably constant and is:

$$\tau = \frac{T}{2A_o t}$$



Equivalent hollow section

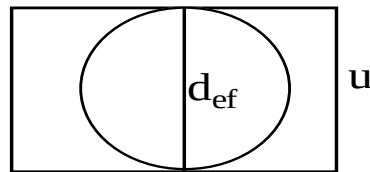
- Extensive tests indicate that the presence of concrete at the centre of the member doesn't have a very significant effect on its torsional resistance



- The equivalent thickness h_{ef} obtained as

$$h_{ef} = \frac{d_{ef}}{5} \leq \frac{A}{u}$$

Where d_{ef} , denotes the diameter of the largest circle which can be inscribed within u



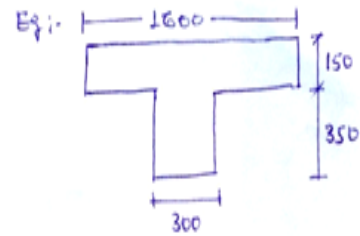
- u is the outer perimeter and A is the area of the cross section enclosed by the outer perimeter
- For T, L, or I shaped sections the torsional resistance is the sum of the capacity of individual rectangular sections
- The subdivision of shape into its rectangles should be done in order to maximize the calculated resistance

$$T_i = \frac{k_i (h_{\min}^3 h_{\max})_i}{\sum k_i (h_{\min}^3 h_{\max})_i} * T$$

Table 1: St. Venant's Torsional constant k .

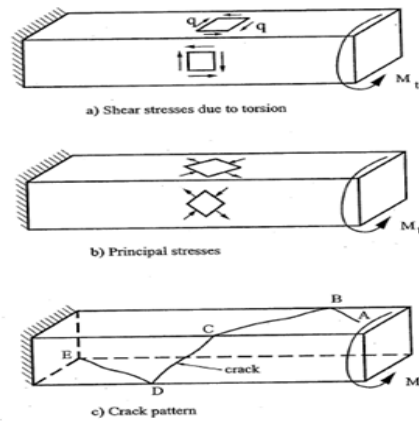
h_{max}/h_{min}	k
1.0	0.14
1.2	0.17
1.5	0.20
2.0	0.23
2.5	0.25

h_{max}/h_{min}	k
3.0	0.26
4.0	0.28
5.0	0.29
10.0	0.31
>10.0	0.33



Principal Stresses due to Torsion

- Longitudinal bars in the corners and closed stirrups are required to control such spiral cracks



Design consideration

I. Limiting value of ultimate torque

- In order to prevent diagonal compression failure in the concrete, the torsional resistance of a section T_{rd} shall not be less than the applied torque T_d .

$$T_{rd} = 0.8f_{cd} A_{ef} h_{ef}$$

Where A_{ef} is the area enclosed with in the centre line of the thin wall cross section

- The critical section for torsion at the face of support

II. Torsional resistance of concrete

$$T_c = 1.2 f_{ctd} A_{ef} h_{ef}$$

- Torsional effects may be disregarded whenever $T_d < T_c$. However, minimum web reinforcement may be provided in such a way that $\rho_{min} = 0.4/f_{yk}$; f_{yk} (Mpa) or the spacing of the stirrups shall not exceed $U_{ef}/8$.

III. Design of torsional reinforcement

- Torsional reinforcement in the form of closed links and longitudinal reinforcement is required to carry the excess torque whenever the applied torque exceeds the concrete resistance, T_c .
- The volume of the longitudinal torsional reinforcement shall be chosen to be equal to the volume of the links (closed stirrups).

- The torsional resistance of the reinforcement T_{ef} is given by:

$$T_{ef} = \frac{2A_{ef} f_{yd} A_s}{S} \qquad T_{ef} = \frac{2A_{ef} f_{yd} A_l}{u_{ef}}$$

Or

Where A_s is the cross sectional area of the stirrups in the effective wall

A_l is the cross sectional area of the longitudinal reinforcement

u_{ef} is the mean perimeter enclosing the area A_{ef}

IV. Combined action effects

Torsion and bending and/or axial stresses

- The longitudinal reinforcement shall be determined separately for torsion, flexure and axial loads.
- The area of reinforcement furnished shall be the sum of the areas thus determined.

Torsion and shear

- The limiting values of torsional and shear resistance are:

$$\begin{aligned} \text{Torsion:} \\ T_{rd, (com)} &= \beta_t T_{rd} \quad \text{where} \quad \beta_t = \frac{1}{\sqrt{1 + \left(\frac{V_{sd} / V_{rd}}{T_{sd} / T_{rd}} \right)^2}} \\ \\ \text{Shear:} \\ V_{rd, (com)} &= \beta_v V_{rd} \quad \text{where} \quad \beta_v = \frac{1}{\sqrt{1 + \left(\frac{T_{sd} / T_{rd}}{V_{sd} / V_{rd}} \right)^2}} \end{aligned}$$

Torsion and shear

The torsional and shear resistance of the concrete are

$$\begin{aligned} \bullet \text{ Torsion:} \\ T_{c,(com)} &= \beta_{tc} T_c \quad \text{where} \quad \beta_{tc} = \frac{1}{\sqrt{1 + \left(\frac{V_{sd} / V_c}{T_{sd} / T_c} \right)^2}} \\ \bullet \text{ Shear:} \\ V_{c,(com)} &= \beta_{vc} V_c \quad \text{where} \quad \beta_{vc} = \frac{1}{\sqrt{1 + \left(\frac{T_{sd} / T_c}{V_{sd} / V_c} \right)^2}} \end{aligned}$$

Exercise:- Member with pure torsion

Problem Determine the maximum torque which can be applied to the member of **Fig. E5.1** given that $f_{ck} = 30 \text{ N/mm}^2$, the yield strength for the longitudinal reinforcement is $f_y = 460 \text{ N/mm}^2$ and the yield strength for the stirrup reinforcement is $f_y = 250 \text{ N/mm}^2$.

4. FLAT SLAB

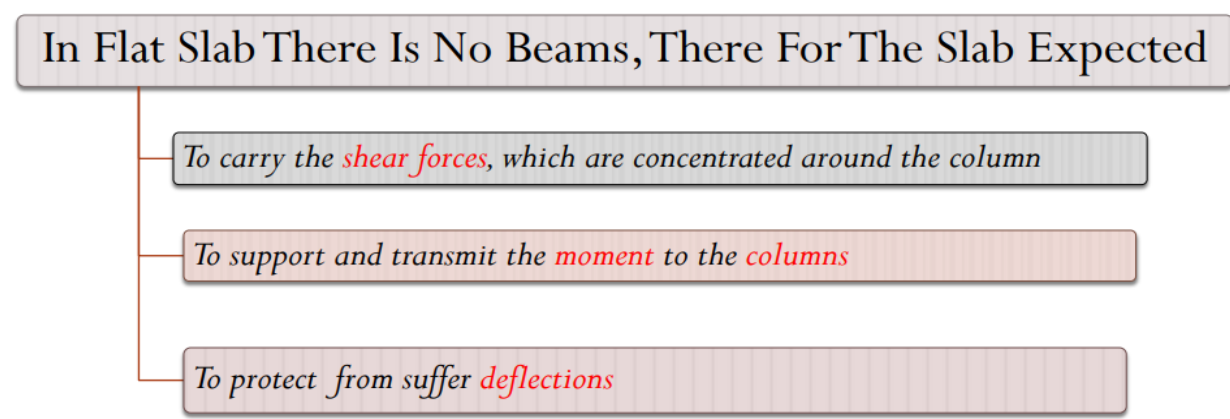
Introduction

Slabs which are not supported by beams or walls along the edges but are supported directly by columns are known as flat slabs.

Beams may also be used where the slab is interrupted around stair, walls or at discontinuous edges.

Shortly: RC slabs supported directly by concrete columns without the use of columns are known as Flat slabs.

The required slab depth for such a system is generally less than for a one-way spanning slab system but greater than for a two-way spanning slab system.



❖ **Column supported** two-way slabs take the following forms:

- Flat Plate /slab
- Flat slab with drop panels/capitals

Flat Plate:

- Flat plate is flat slabs with flat soffit.
- Have uniform thickness supported on columns
- Used for relatively light loads, as experienced in apartments or similar buildings.
- Most economical for spans from 4.5m to 6m

Flat slab:

- Load transfer to the column is accomplished by thickening the slab near the column, using drop panels and/or by flaring the top of the column to form a column capital.
- Are suitable for spans up to 9m and $LL \geq 5\text{kN/m}^2$
- Weak for lateral forces (like wind and earthquake load)
- In flat slab design 100% lateral load is give to wall/ core

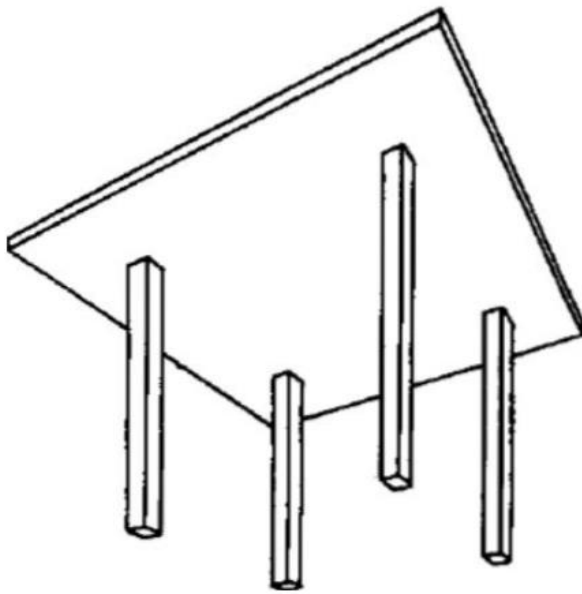
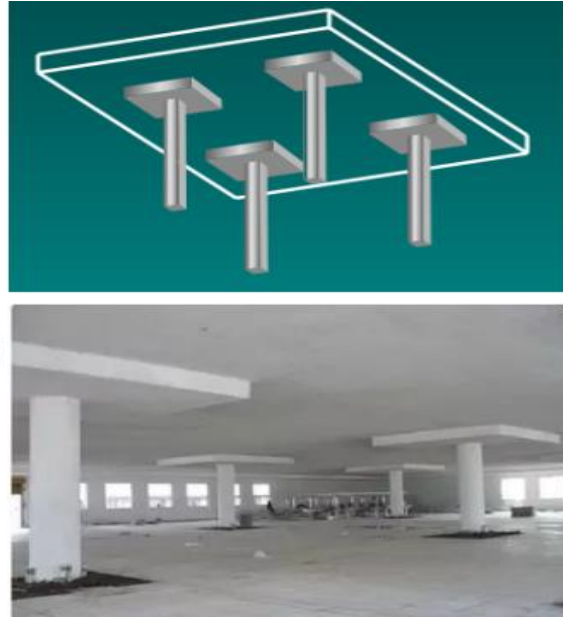


Fig. a) Flat plate



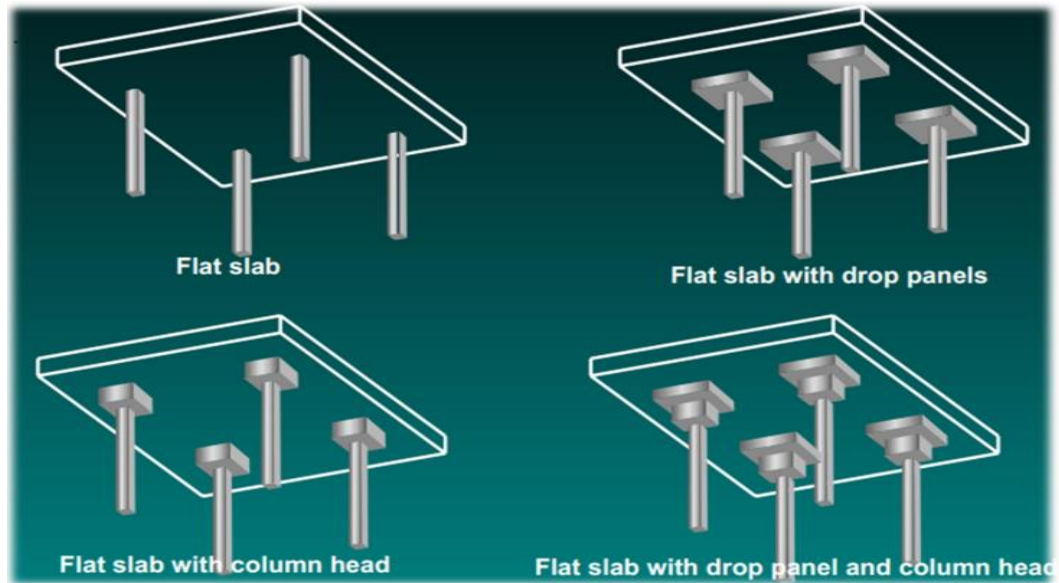
b) Flat slab

Significance of Column Heads:

- Increase shear strength of slab
- Reduce the moment in the slab by reducing the clear or effective span

Significance of Drop Panels

- Increase shear strength of slab
- Increase negative moment capacity of slab
- Stiffen the slab and hence reduce deflection



Advantage of flat slabs

- reduced depth of structure overall as beams
- only requires simple shuttering – there are no beams for which formwork must be prepared
- Economical for ordinary span (up to 9m) and heavy loads.
- Formwork requirement is reduced.
- More light can be admitted.
- Flexibility in room layout
- Saving in building height
- Shorter construction time
- Ease of installation of Mechanical & Electrical services

Disadvantage of flat slab

- arrangement of reinforcement can be very complex
- often provided with ‘drops’ or enlarged column heads to increase the shear strength of the slab around the column supports

The slabs may be of constant thickness throughout or, in the area of column it may be thickened as a drop panel.

The column may also be of constant section or it may be flared to form a column head or capital.

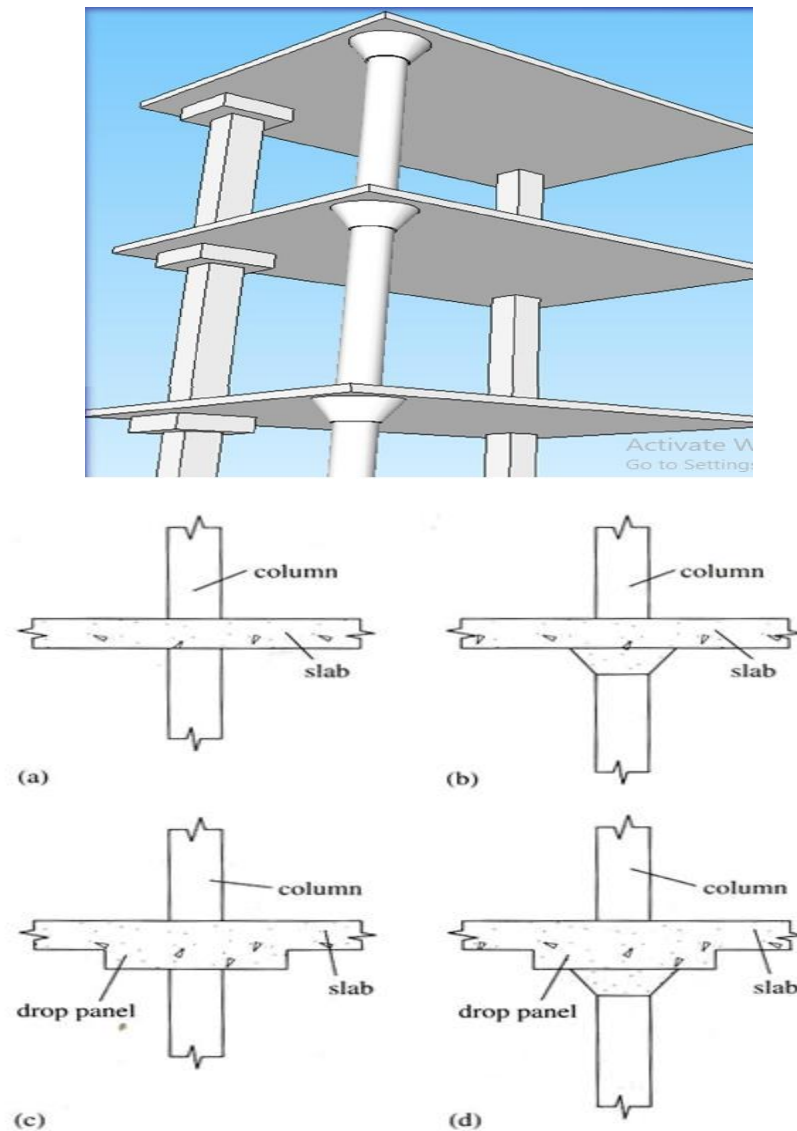
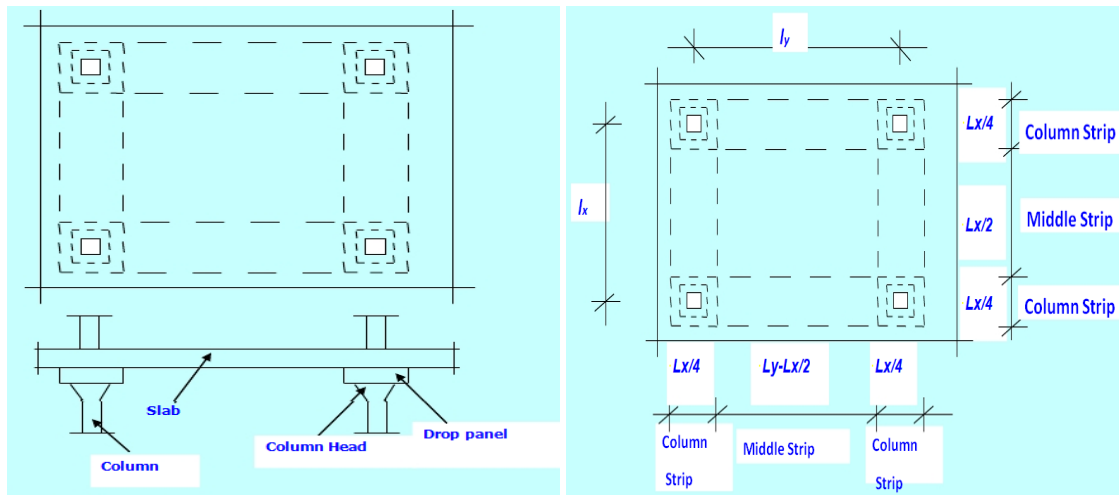


Figure Flat slab construction details: (a) no column head; (b) flared column head; (c) slab with drop panel; (d) flared column head and drop panel.

- ✚ Flat slabs are suitable for spans up to 9 m and $LL \geq 5.0 \text{ kN/m}^2$ such as in storage, warehouses, parking lots, and garages where structures carry heavy loads. For lighter loads flat plates are suitable.
- ✚ The simplified formwork and the reduced story heights make it more economical. Windows can extend up to the underside of the slab, and there are no beams to obstruct the light and the circulation of air.

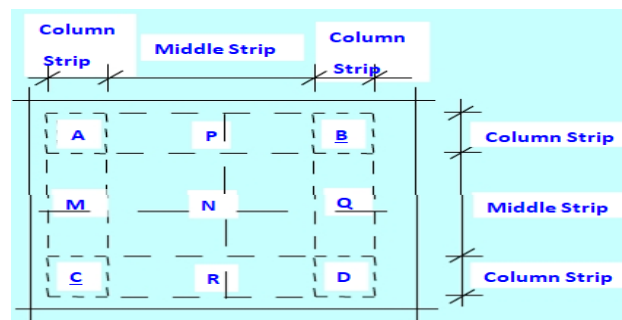
✚ For analysis purpose, the panel in flat slab is divided into column strip and middle strip



- A column strip is a design strip with a width on each side of a column centerline equal to $0.25 l_x$ or if drops with dimensions not less than $l_x/3$ are used, width equal to the drop dimension.
- Middle strip is a design strip bounded by two column strips.
- The drop panels are rectangular or square and influence the distribution of moments with in the slabs.
- The smaller dimension of the drop is $l_x/3$ otherwise can be ignored.
- The drop may be 25 to 50% thicker than the rest of the slab.
- Effective diameter of a column on a column head is defined as the diameter of a circle whose area equals the cross- sectional area of the column or, if column heads are used the area of column head based on effective dimensions.

Behavior of flat slabs under uniform load

Consider a typical interior panel under uniform gravity load.



- Under the uniform load, the points M, P, Q and R are higher than point N, but lower than the support points (A, B, C, D).
- Positive moments at N and Negative moments at M & Q along MQ (middle strip)
- Negative moments at P & R along PR (middle strip) are introduced.

However, considering the column strips,

- -ve moments at A & B along AB while +ve moment at P.
- -ve moments at C & D along CD while +ve moment at R.
- -ve moments at A & C along AC while +ve moment at M.
- -ve moments at B & D along BD while +ve moment at Q.

Based on static principle, the total BM (positive and negative) for a typical interior panel under uniform design loads F_d in X and Y direction may be given as.

$$M_{ox} = \frac{F_d L_y L_x^2}{8} \left(1 - \frac{2}{3} \frac{h_c}{L_x}\right)^2 \dots\dots\dots (*)$$

$$M_{oy} = \frac{F_d L_x L_y^2}{8} \left(1 - \frac{2}{3} \frac{h_c}{L_y}\right)^2 \dots\dots\dots (**)$$

Where:

L_x – distance C/C of column strip under constant elevation

h_c – the effective diameter of a column head.

Flat Slab Design Method As Per ESEN 1992-1-1_2015

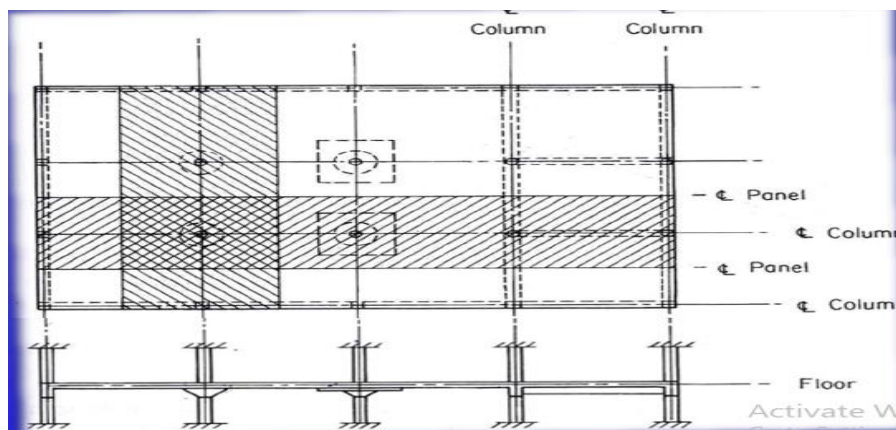
- Finite Element Method (FEM)
- Equivalent Frame Method (EFM)
- Yield Line Method (YLM)
- Grillage Analysis (GA)

1. Finite element method

- Based upon the division of complicated structures into smaller and simpler pieces (elements) whose behavior can be formulated.
- Allows for design of irregular column layouts and can provide the design of reinforcement details.
- Where the appropriate software is available, it is possible to obtain reasonable assessment of deflections.
- Adopted Finite element software SAFE, ADAPT, Lisa, Abacus, etc
- Analysis results includes moment and shear envelopes, contour of structural deformation

2. Equivalent frame method (Approximate method)

- Suitable for regular column layouts, but requires engineering judgment for irregular layouts.
- Flat slab structure is divided longitudinally and transversely into frames consisting of Interior strips and edge strips of slabs.
- The structure is divided longitudinally and transversely into frames considering of columns and strips of slab.
- Entire slab width is considered in establishing the slab load and slab stiffness.
- Stiffness is based on moment of inertia of the concrete section alone.
- During analysis maximum design load on all spans may be only considered.
- if $QK/GK < 1.25$ and $QK \leq 5.0 \text{ KN/m}^2$, excluding partitions.
- Columns are fixed at their far ends. (i.e. at the floor above and below the floor considered.)
- Spans are (measured) from centerlines of columns



- If single load case of maximum design load is not appropriate, alternative spans are loaded with maximum design load and other spans with minimum design load (say $1.0 \cdot G_k$).
- Each frame is analyzed using any elastic method.
 - Hardy Cross method (moment distribution)
 - Kani's method
 - Matrix method
 - Etc...

3. Yield line method

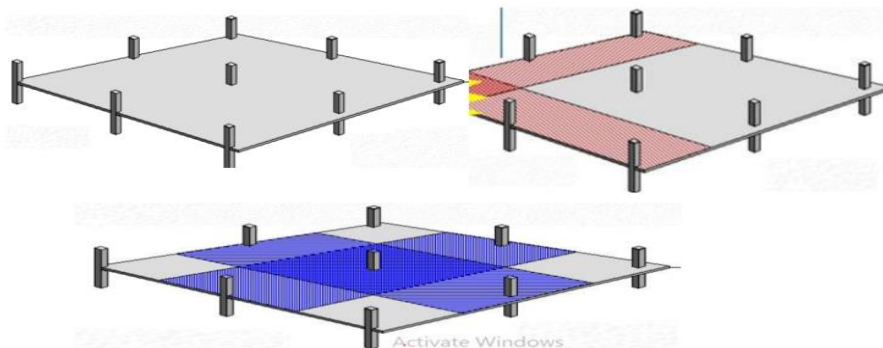
- These can provide suitable designs for ULS but do not give adequate information for serviceability design.

4. Grillage analysis

- This method has similar facilities to finite element models and can also be used for irregular layouts of columns.

ES EN 1992-1-1_2015 RECOMMENDATION FOR (EFM)

- The slab can be analyzed by considering as Equivalent frame system.
- This provides an acceptable representation of the behavior of the floor by a system of Interior strip and Edge strip analyzed separately in each direction.
- Rather than a full height frame, a series of sub frames comprising a single floor with columns above and below is more commonly used, subject to the most unfavorable arrangement of load
- The final moments can be redistributed.
- The structure should be divided longitudinally and transversely into frames consisting of columns and sections of slabs contained between the centre lines of adjacent panels or the edge of the slab as appropriate.
- In the slab shown NS, the edge strips and the interior strips are illustrated as follows:



- The stiffness of members may be calculated from their gross cross-sections.
- For vertical loading the stiffness may be based on the full width of the panels.
- For horizontal loading 40% of this value should be used to reflect the increased flexibility of the column/slab joints in flat slab structures compared to that of column/beam joints.
- Total load on the panel should be used for the analysis in each direction.

The panels should be assumed to be divided into column and middle strips (see Figure below).

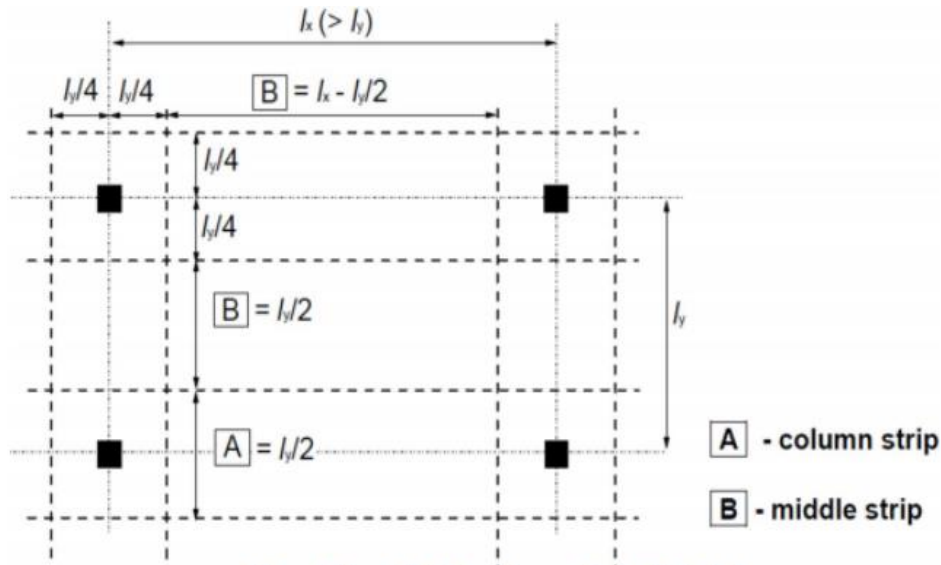


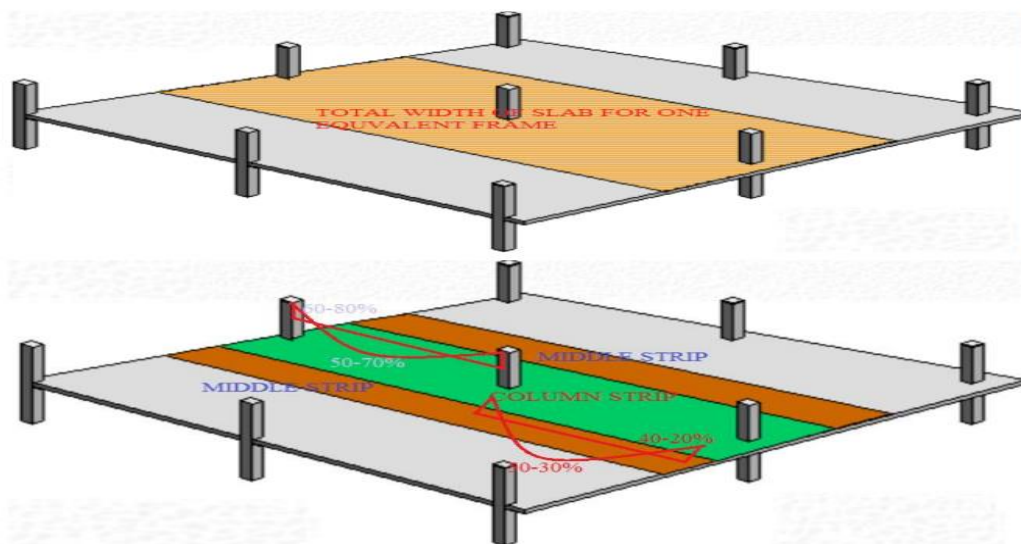
Figure I.1: Division of panels in flat slabs

- When drops of width $> l_y/3$ are used the column strips may be taken to be the width of drops. The width of middle strips should then be adjusted accordingly.
- The total bending moments obtained from analysis should be distributed across the width of the slab.
- In elastic analysis negative moments tend to concentrate towards the center lines of the columns.
- The moment obtained from the analysis of equivalent frame is total moment on the slab strip. However, the distribution of moment across the width of the strip is quite obviously uniform, since the slab is only supported in the center of the strip

Table I.1 Simplified apportionment of bending moment for a flat slab

	Negative moments	Positive moments
Column Strip	60 – 80%	50 – 70%
Middle Strip	40 – 20%	50 – 30%
Note: Total negative and positive moments to be resisted by the column and middle strips together should always add up to 100%.		

Where the width of the column strip is different from $0.5l_x$ as shown in Figure I.1 (e.g.) and made equal to width of drop the width of middle strip should be adjusted accordingly.



FLEXURAL REINFORCEMENT IN FLAT SLAB CONSTRUCTION

Slab at Internal Columns

At internal columns, unless rigorous serviceability calculations are carried out,

- Top reinforcement of area $0.5 A_t$ should be placed in a width equal to the sum of 0.125 times the panel width on either side of the column.
 - A_t = area of reinforcement required to resist the full negative moment from the sum of the two half panels each side of the column.
- Bottom reinforcement (≥ 2 bars) in each orthogonal direction should be provided at internal columns and this reinforcement should pass through the column.

Slab at Edge and Corner Columns

- Reinforcement perpendicular to a free edge required to transmit bending moments from the slab to an edge or corner column should be placed within the effective width be shown in Figure below

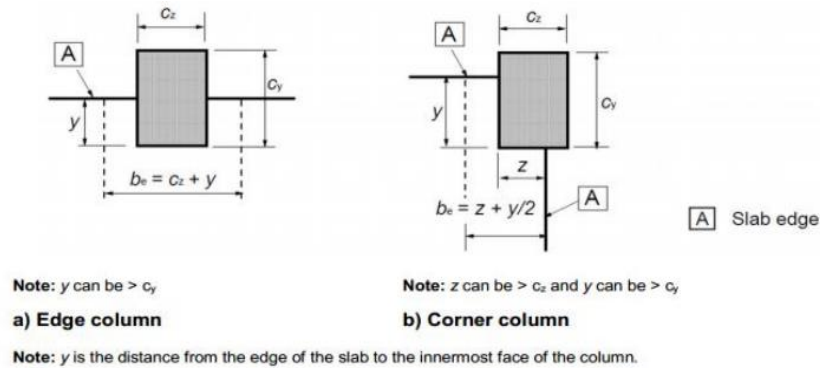


Fig. Effective width b_e , of flat slab

GENERAL DESIGN PROCEDURE

1. Determine effective depth and overall for flat slab considering deflection requirement

$$L_y / d = K * N * F1 * F2 * F3$$

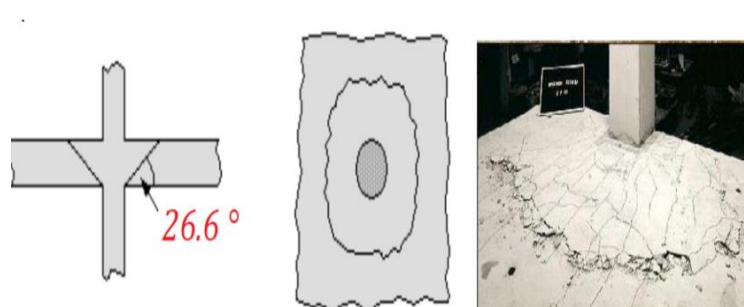
L_y – shorter span length

$K = 1.2$ – for Slab supported on columns without beams
2. Determine design loads
3. Divide slabs in to edge and interior strip
4. Arrange edge strip and interior strip into separate frame
5. For analysis use Hardy cross moment distribution and consider the advantage of symmetry.
 - Moment of inertia for slab and columns
 - Stiffness Calculation (k)
 - Distribution factor (DF)
 - Fixed end moment (FEM)
 - Final moment calculation
6. Distribute support and span moment to column strip (CS) and middle strip (MS)
7. Reinforcement provision for flexure
8. Check for punching shear

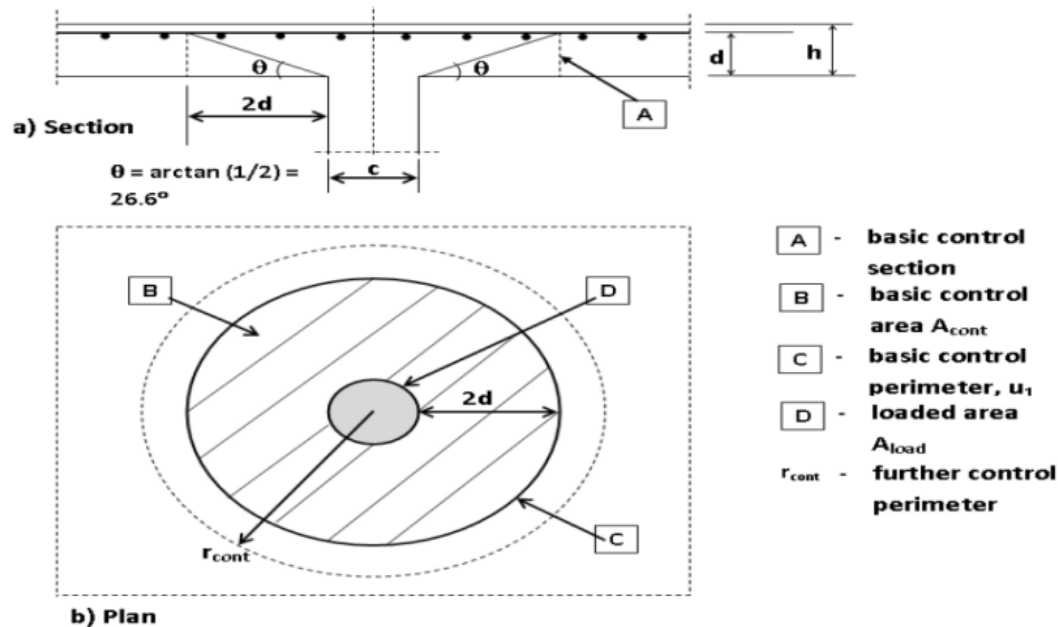
Punching shear:

- ✓ is perimeter shear which occurs in flat slabs around columns.
 - ✓ Results from a concentrated load or reaction acting on small area (loaded area).
 - ✓ Significant on Flat Slab and foundation.
- Punching Shear is the most common, and is a major design consideration, in flat slab construction.

- Tests show that cracks develop radially from the column position, in a sudden and brittle failure on inclined faces of truncated cones or pyramids at an angle of about 26.6° to the horizontal.



- An appropriate verification model for checking punching failure at the ULS is shown in Figure below.
- The shear resistance should be checked along defined control perimeters.



⇒ LOAD DISTRIBUTION AND BASIC CONTROL PERIMETER

- ✓ The basic control perimeter (U_1) may normally be taken to be at a distance $2d$ from the loaded area (column face).
- ✓ The effective depth of the slab is assumed constant and may normally be taken as: $deff = dy + dz$; Where:
 - dy and dz - are the effective depths of the reinforcement in two orthogonal directions

Load Distribution and Basic Control Perimeter for:

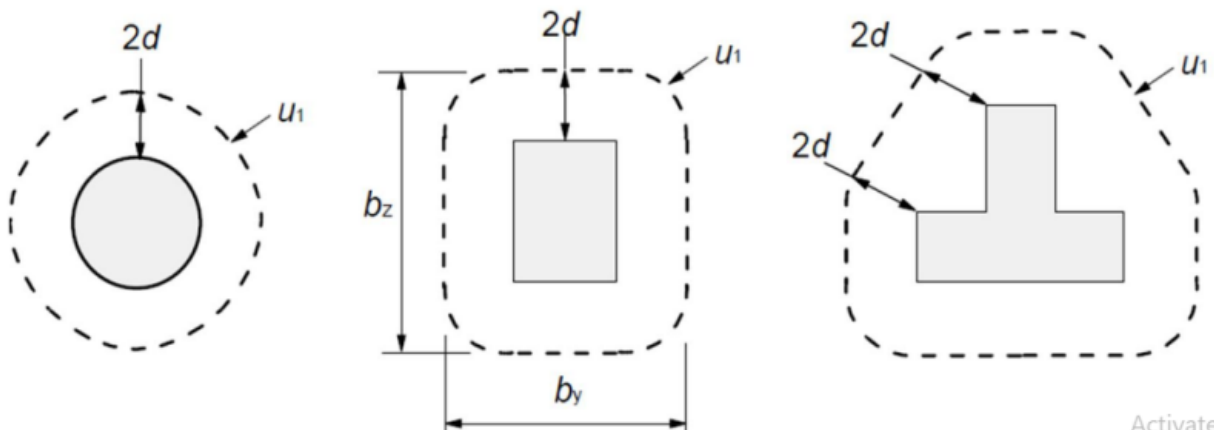
Circular Column

Rectangular Column

Edge Column

Corner Column

T or I—Shaped Column

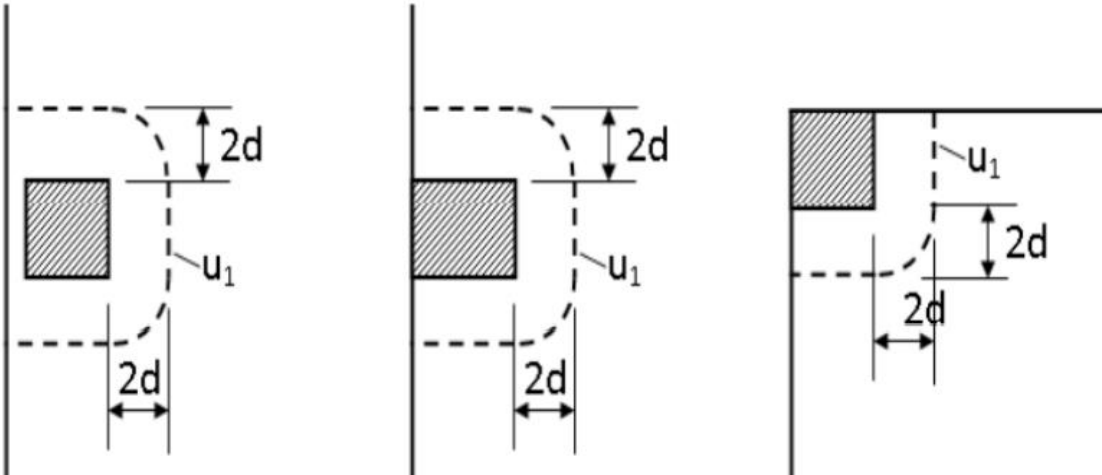


Activate V

Control perimeters at a distance less than $2d$ should be considered where the concentrated force is opposed by a high distributed pressure (e.g. soil pressure in a base), or

By the effects of a load or reaction within a distance $2d$ of the periphery of area of application of the force.

If the loaded area is at edge or a corner, the control perimeter (U_1) should be taken as shown in Figure below.



Basic Control Perimeter & Types of Column Heads

Basic control perimeter for flat slab with circular column heads ($l_H < 2h_H$)

Basic control perimeter for flat slab with rectangular column with a rectangular column head ($l_H < 2h_H$)

Basic control perimeter for flat slabs with enlarged column heads ($l_H \geq 2h_H$)

A
Gc

1. Flat Slabs with Circular Column Heads ($l_H < 2h_H$)

- Check the punching shear stresses is required on the control section outside the column head. The distance of this section from the centroid of the column (r_{cont}) may be taken as:

$$r_{cont} = 2d + l_H + 0.5C$$

Where:

- l_H - is the distance from the column face to the edge of the column head
- C - is the diameter of a circular column

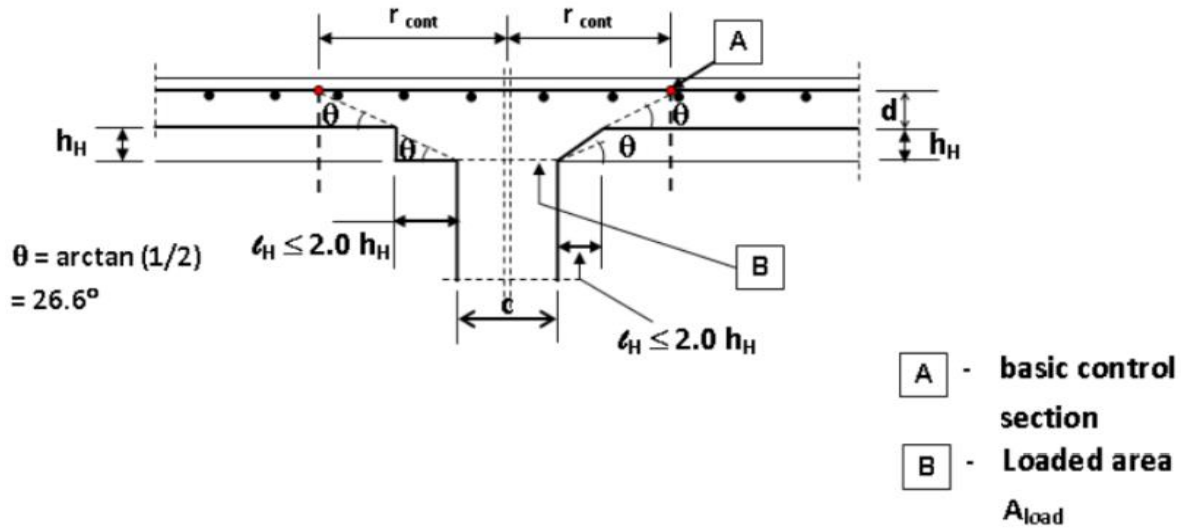


Fig: Slab with enlarged column head where $l_H < 2.0 h_H$

2. For flat slab with rectangular column with a rectangular column head ($l_H < 2h_H$) and overall dimensions

$$r_{cont} = \min \text{ of } 2d + 0.56 (l_2 l_1)^{1/2} \text{ or } 2d + 0.69 l_1$$

$$l_1 = c_1 + 2/l_H, l_2 = c_2 + 2/l_H, l_1 \leq l_2$$

3. For flat slabs with Enlarged Column Heads ($l_H \geq 2h_H$)

- The stress should be checked at critical sections (both within the head and in the) slab.

a. Check at column head with d taken as d_H .

b. For circular columns the distances from the centroid of the column to the control sections may be taken as:

$$r_{cont,ext} = l_H + 2d + 0.5c$$

$$r_{cont,int} = 2d + h_H + 0.5c$$

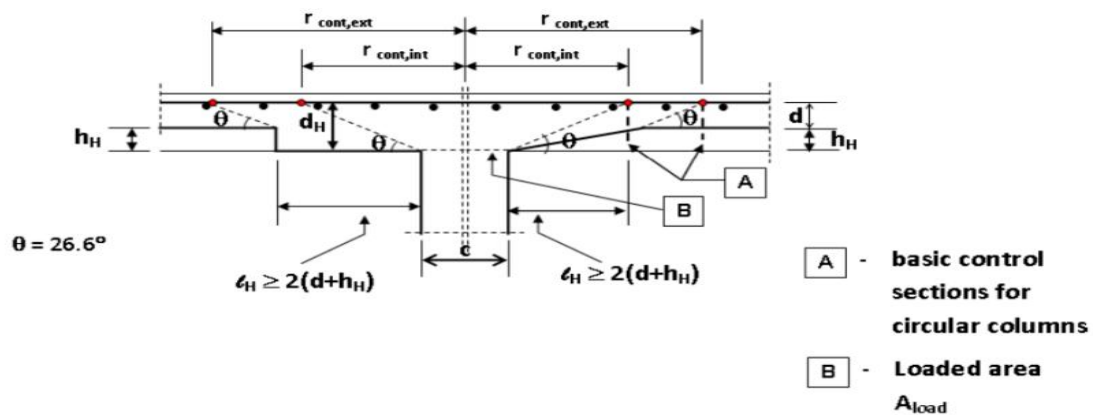


Fig: Slab with enlarged column head where $l_H > 2(d+h_H)$

PUNCHING SHEAR CALCULATION

The design procedure for punching shear is based on checks at the face of the column and at the basic control perimeter (u_1).

The following checks should be carried out:

- At the column perimeter, or the perimeter of the loaded area,

$$V_{Ed} \leq V_{Rd,max}$$

*If $V_{Ed} \leq V_{Rd,c}$, punching shear reinforcement is not necessary.

*If $V_{Ed} \geq V_{Rd,c}$ at the control section considered, punching shear reinforcement should be provided

If shear reinforcement is required a further perimeter $u_{out,ef}$ should be found where shear reinforcement is no longer required.

Where:

- $V_{Rd,max}$ - is the design value of the maximum punching shear resistance along the control section considered
- $V_{Rd,c}$ - is the design value of the punching shear resistance of a slab without punching shear reinforcement along the control section considered
- $V_{Rd,cs}$ - is the design value of the punching shear resistance of a slab with punching shear reinforcement along the control section considered

If support reaction is eccentric with regard to the control perimeter, the maximum shear stress (V_{Ed}) should be taken as:

$$V_{Ed} = \beta^{V_{Ed}} / u_i d$$

where:

- d - is **mean effective depth** of slab, taken as $= \frac{(d_y + d_z)}{2}$
- u_i - is length of control perimeter being considered
- $\beta = 1 + k \frac{M_{Ed}}{V_{Ed}} \cdot \frac{u_1}{W_1}$;
- k - is a coefficient dependent on the ratios b/n the column dimensions c_1 and c_2 :

Table 6.1: Values of k for rectangular loaded areas

c_1/c_2	≤ 0.5	1.0	2.0	≥ 3.0
k	0.45	0.60	0.70	0.80

W_1 - corresponds to a **distribution of shear** and is a function of the basic control perimeter u_1 :

a. For a rectangular column:

$$W_1 = \frac{c_1^2}{2} + c_1 c_2 + 4c_2 d + 16d^2 + 2\pi d c_1;$$

where:

- c_1 - is the column dimension parallel to the eccentricity of the load; and
- c_2 - is the column dimension $\perp$ to the eccentricity of the load.

b. For internal circular columns β is given by:

$$\beta = 1 + 0.6\pi \frac{e}{D+4d},$$

Where:

- D - is the diameter of the circular column and
- e - is the eccentricity of the applied load $e = M_{Ed} / V_{Ed}$

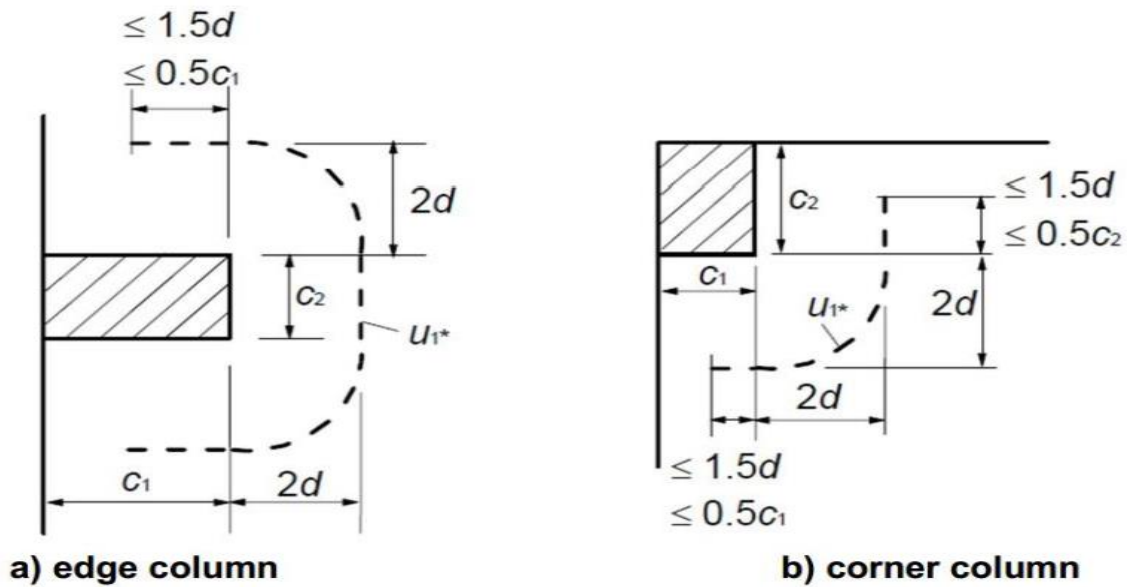
c. For an internal rectangular column : where the loading is eccentric to both axes, the following approximate expression for β may be used:

$$\beta = 1 + 1.8 \sqrt{\left(\frac{e_y}{b_z}\right)^2 + \left(\frac{e_z}{b_y}\right)^2};$$

Where:

- e_y and e_z are the eccentricities M_{Ed} V_{Ed} along y and z axes respectively
- b_y and b_z is the dimensions of the control perimeter

d. For edge column connections,



If there are eccentricities in both orthogonal directions,

$$\beta = \frac{u_1}{u_1^*} + k \frac{u_1}{W_1} e_{par}$$

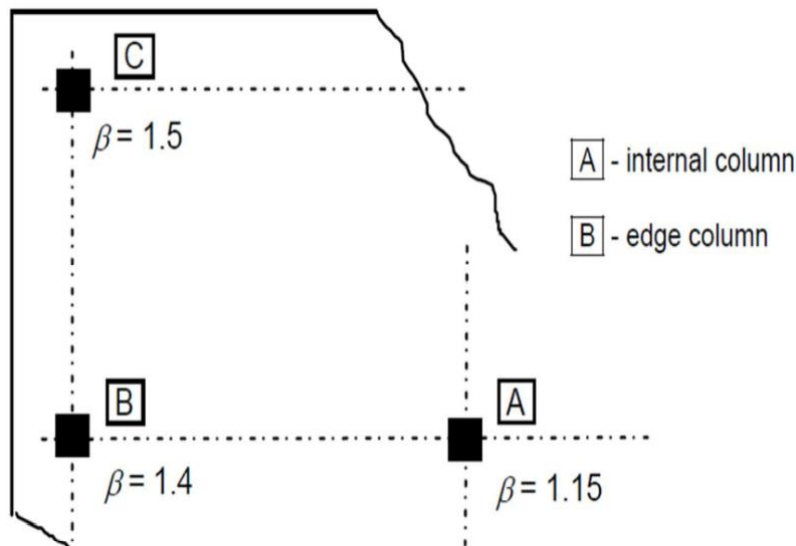
Where:

- u_1 - is the basic control perimeter
- u_1^* - is the reduced basic control perimeter
- e_{par} - is the eccentricity parallel to the slab edge resulting from a moment about an axis perpendicular to the slab edge.
- k - is a coefficient dependent on the ratios b/n the column dimensions c_1 and c_2
- W_1 - is calculated for the basic control perimeter u_1

- For a rectangular column:

$$W_1 = \frac{c_2^2}{4} + c_1 c_2 + 4c_1 d + 8d^2 + 2\pi d c_2$$

- For corner column connections,
 - If the eccentricity is toward the interior of the slab, it is assumed that the punching force is uniformly distributed along the reduced control perimeter u_1 * as defined above.
 - The β value may then be considered as: $\beta = u_1 / u_1^*$
- For structures
 - If the lateral stability does not depend on frame action b/n the slabs and the columns, and
 - If the adjacent spans do not differ in length by more than 25%, then approximate values for β may be used
 - ✓ $\beta = 1.15$ - for internal columns
 - ✓ $\beta = 1.4$ - for edge columns
 - ✓ $\beta = 1.5$ - for corner columns



PUNCHING SHEAR RESISTANCE FOR SLABS OR COLUMN BASES W/O SHEAR REINFORCEMENT

- The design punching stress (resistance) is given by:

$$V_{Rd,c} = C_{Rd,c} k ((100 \rho_l f_{ck}))^{1/3} + k_1 \sigma_{cp} \geq (V_{min} + k_1 \sigma_{cp})$$

where

- f_{ck} - is in MPa;
- $k = 1 + \sqrt{\frac{200}{d}} \leq 2, d \text{ in mm};$

$$\rho_l = \sqrt{\rho_{ly} \cdot \rho_{lz}} \leq 0.02;$$

ρ_{ly} , ρ_{lz} - relate to the bonded tension steel in the y- and z- directions respectively. The values ρ_{ly} , ρ_{lz} should be calculated as mean values taking into account a slab width equal to the column width plus 3d each side. The recommended value for:

$$\circ C_{Rd,c} = 0.18 / \gamma_c$$

$$\circ V_{min} = 0.035 k^{3/2} \cdot f_{ck}^{1/2}$$

$$\circ k_1 = 0.1$$

$$\sigma_{cp} = (\sigma_{cy} + \sigma_{cz}) / 2$$

Where:

- ✓ σ_{cy} , σ_{cz} - are the normal concrete stresses in the critical section in y- and z directions (MPa, positive if compression): $\sigma_{cy} = N_{Ed,y} / A_{c,y}$ and $\sigma_{cz} = N_{Ed,z} / A_{c,z}$
 - $N_{Ed,y}$, $N_{Ed,z}$ are the longitudinal forces across the full bay for internal columns and the longitudinal forces across the control section for edge columns. The force may be from a load or prestressing action;
- ✓ A_c is the area of concrete according to the definition of N_{Ed}

If shear reinforcement is required, it can be calculated as shown below:

$$V_{Rd,cs} = 0.75V_{Rd,c} + 1.5 \left(\frac{d}{S_r} \right) A_{Sw} f_{ywd,ef} \left(\frac{1}{u_1 d} \right) \sin \alpha$$

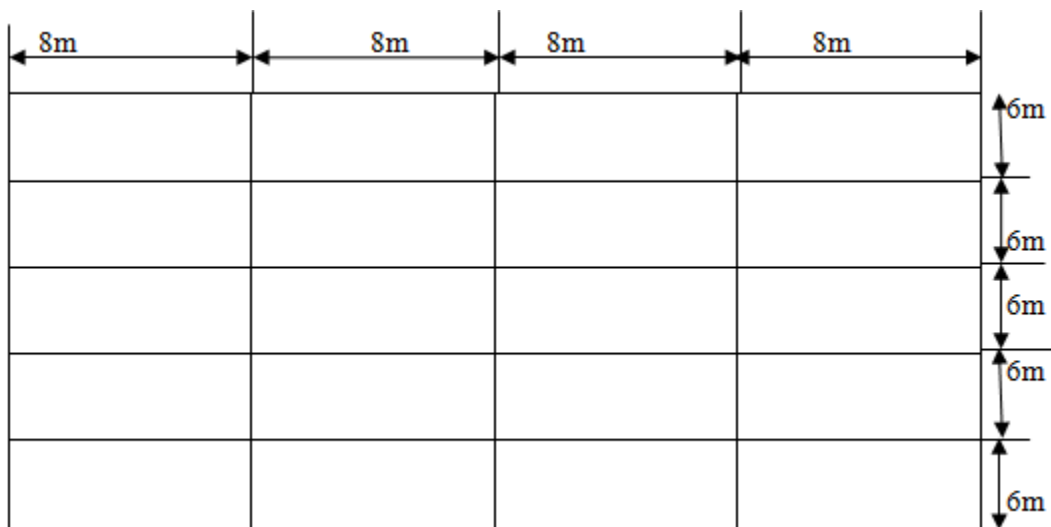
Where:

- A_{Sw} - is the area of one perimeter of shear reinforcement around the column in mm^2
- S_r - is the radial spacing of perimeters of shear reinforcement
- $f_{ywd,ef}$ - is the effective design strength of the punching shear reinforcement, according to $f_{ywd,ef} = 250 + 0.25d \leq f_{ywd} [MPa]$
- d - is the mean of the effective depths in the orthogonal directions [mm]
- α - is the angle between the shear reinforcement and the plane of the slab

$\frac{a}{S_r} = 0.67$ - for a single line of bent-down bars provided,

Design of flat slab

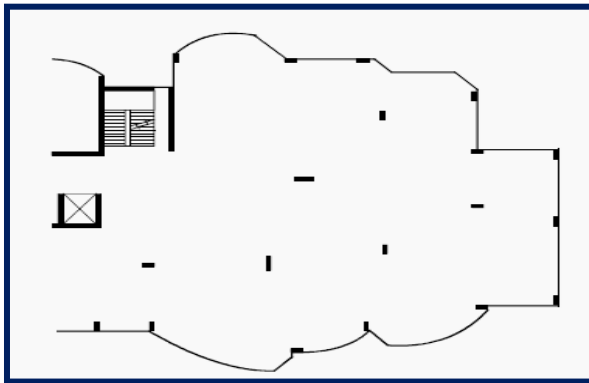
Exercise: a reinforced concrete floor system consists of flat slabs, framed on a system of columns with four and five panels in the long and short directions respectively. The panels are 6m and 8m on center lines of columns. The floor is intended to give service to characteristic live load of 5 kN/m^2 and load from partition 2 kN/m^2 . Determine the cross sectional dimension of various elements of the floor system and the reinforcement details for typical interior panel. Materials include , concrete C-25, S-300, class-I works. Assume column size of $550 \text{ mm} \times 550 \text{ mm}$.



5. Yield line Analysis of Slabs

Introduction

- Rectangular one way or two way slabs can be analyzed and designed using coefficients obtained from Tables published for this purpose
- For irregular shapes, varied support conditions, presence of openings, varied loading and for more complex conditions, the yield line theory is found useful.

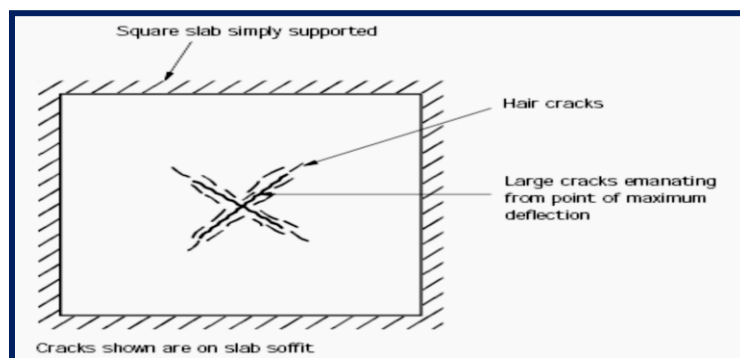


An irregular flat slab....

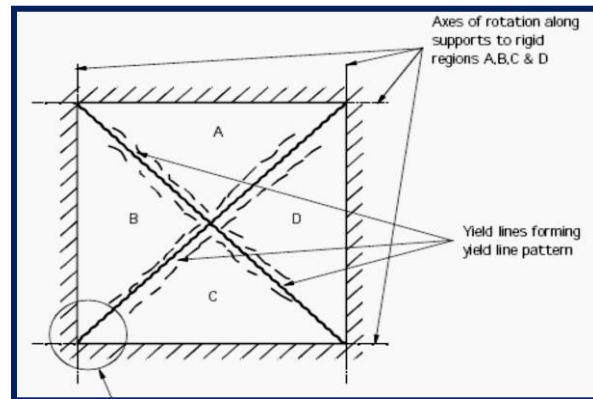
What is Yield Line Design?

- A **yield line** is a crack in a reinforced concrete slab across which the reinforcing bars have yielded and along which plastic rotation occurs
- The **yield line theory** is an ultimate load method of analysis of slab, i.e. the BM at the verge of collapse is used as the basis for design
- **Yield Line Design** uses Yield Line Theory in designing reinforced concrete slabs, and similar types of elements

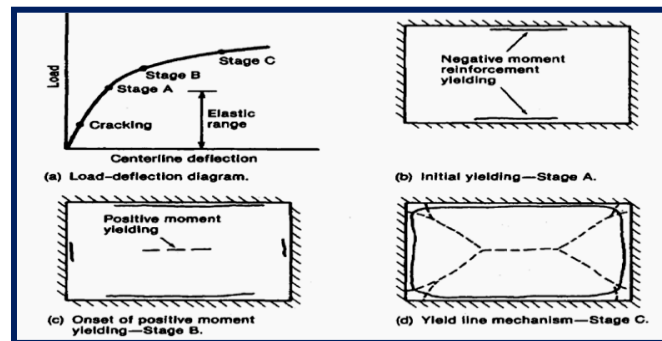
Initially, at service load, the response of the slab is elastic with the maximum steel stress and, it is possible that some hairline cracking will occur on the soffit



- Increasing the load hastens the formation of these hairline cracks
- Increasing the load further will increase the size of the cracks further and induce yielding of the reinforcement, initiating the formation of large cracks emanating from the point of maximum deflection
- On increasing the load yet further, these cracks migrate to the free edges of the slab at which time all the tensile reinforcement passing through a yield line yields

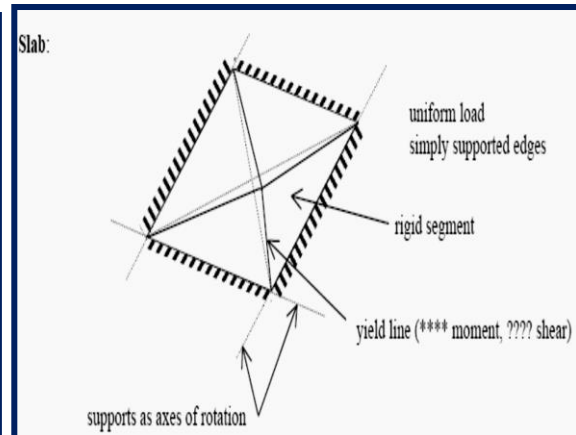
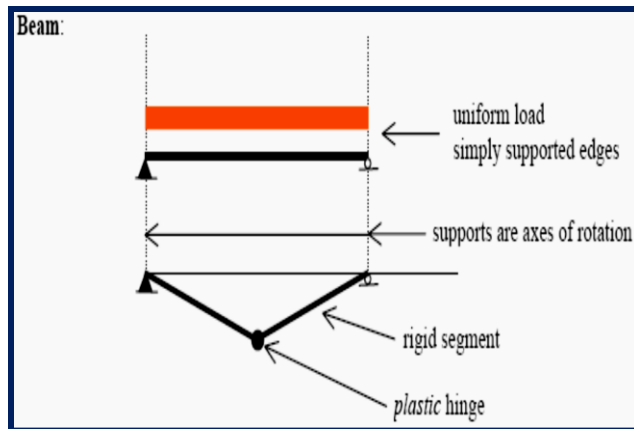


The formation of a mechanism in a simply supported two-way slab with the bottom steel having yielded along the yield lines



The yield line phenomenon involves:

- a slab under increasing loads where cracking and reinforcement yielding occur in the most highly stressed zone (i.e. around maximum moment)
- the highly stressed zone normally acts as a plastic hinge where the subsequent loads are distributed to other region of the slab
- collapse is then indicated by increasing deflection under constant load



Advantages of Yield Line Design

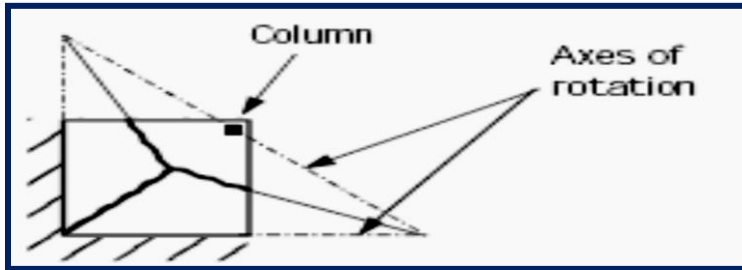
- Yield Line Design has the advantages of:
 - Economy
 - Simplicity and
 - Versatility
- The resulting slabs are thin and have very low amounts of reinforcement in very regular arrangements.
- The reinforcement is therefore easy to detail and easy to fix and the slabs are very quick to construct.
- Above all, Yield Line Design generates very economic concrete slabs, because it considers features at the ultimate limit state.

Drawing notation

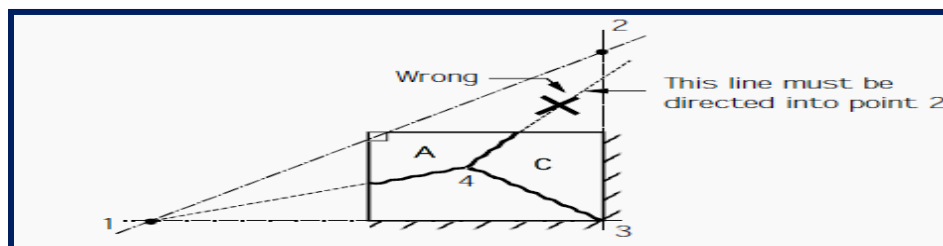
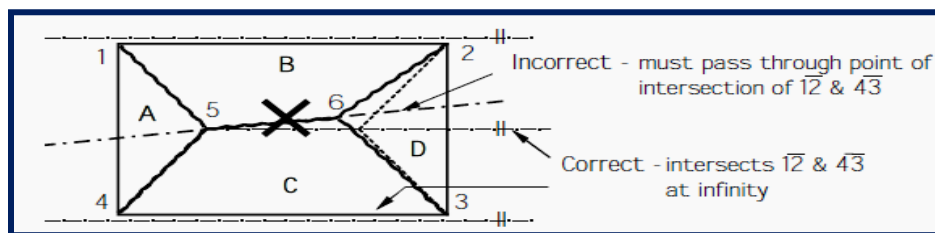
Supports			
	Free edge		Continuous support
	Simple support		Column support
Yield lines			Axis of rotation
	Positive (sagging) yield line, kNm/m		Plastic hinge (in sectional elevation or in plan)
	Negative (hogging) yield line, kNm/m		
Loads			Centre of gravity of load kN
	Line load, kN/m		
	Point load, kN		

Rules for yield line patterns

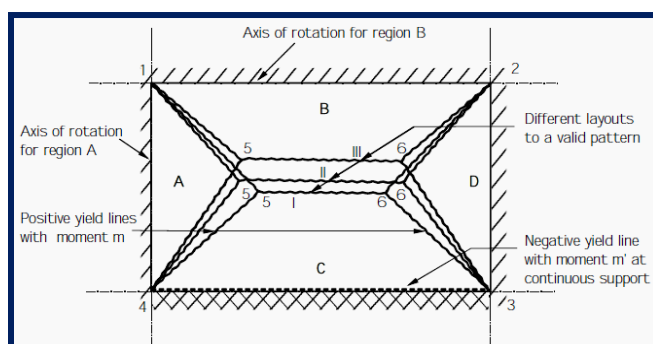
1. Yield lines are straight
2. Axes of rotation generally lie along lines of support and pass along side any columns
3. Yield lines must end at a slab boundary



4. Yield lines between adjacent rigid regions must pass through the point of intersection of the axes of rotation of those regions
5. Continuous supports repel and a simple supports attract yield lines

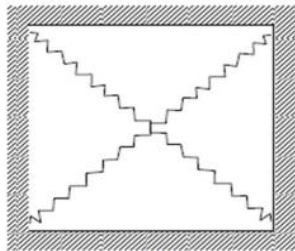


Invalid pattern

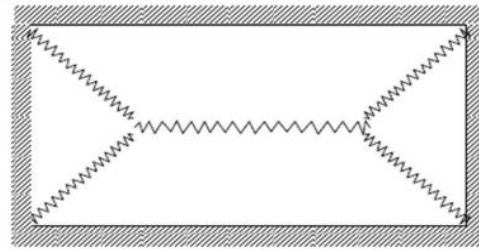


GUIDELINES FOR ESTABLISHING AXES OF ROTATION & YL

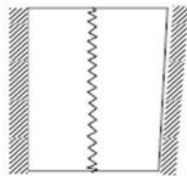
1. Yield lines are generally straight and end at a slab boundary or the intersection of other yield lines.
2. Axes of rotation generally lie along lines of support
3. The supported edges of the slab are axes of rotation.
 - If the edge is fixed negative YL may form, providing constant resistance to rotation.
 - If the edge is simply supported, the axis of rotation provides zero restraining moment
4. An axis of rotation will pass over any column support. The slab segments are considered to rotate as rigid bodies in space about these axes of rotation.
5. YL form under concentrated loads, radiating outward from the point of application.
6. A YL between two slab segments must pass through the point of intersection of the axes of rotation of the adjacent slab segments.



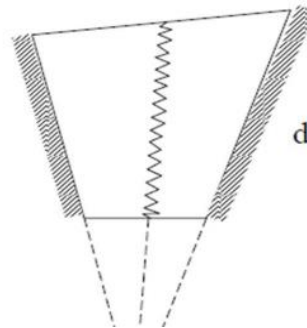
a) Simply Supported Square Slab



b) Simply Supported Rectangular Slab



c) One-Way Rectangular Slab



d) One-Way Skew Slab

Active

Yield Line is an upper bound theory

Ultimate load theories for slabs fall into two categories:

- upper bound (unsafe or correct) or
- lower bound (safe or correct).

Plastic analysis is either based on

- upper bound (kinematic) methods, or on
- lower bound (static) methods.

Upper bound (kinematic) methods include:

- plastic or yield hinges method for beams, frames and one-way slabs;
- **Yield Line Theory for slabs.**

Lower bound (static) methods include:

- the strip method for slabs,
- the strut and tie approach for deep beams, corbels, anchorages, walls and plates loaded in their plane.

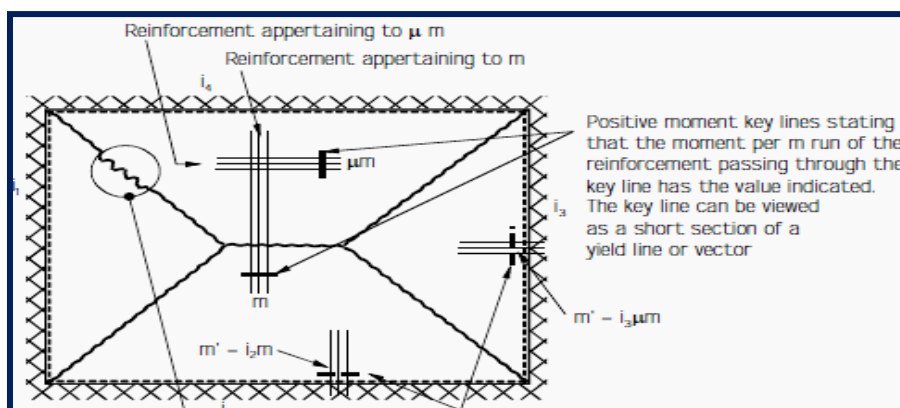
Definition

Isotropy

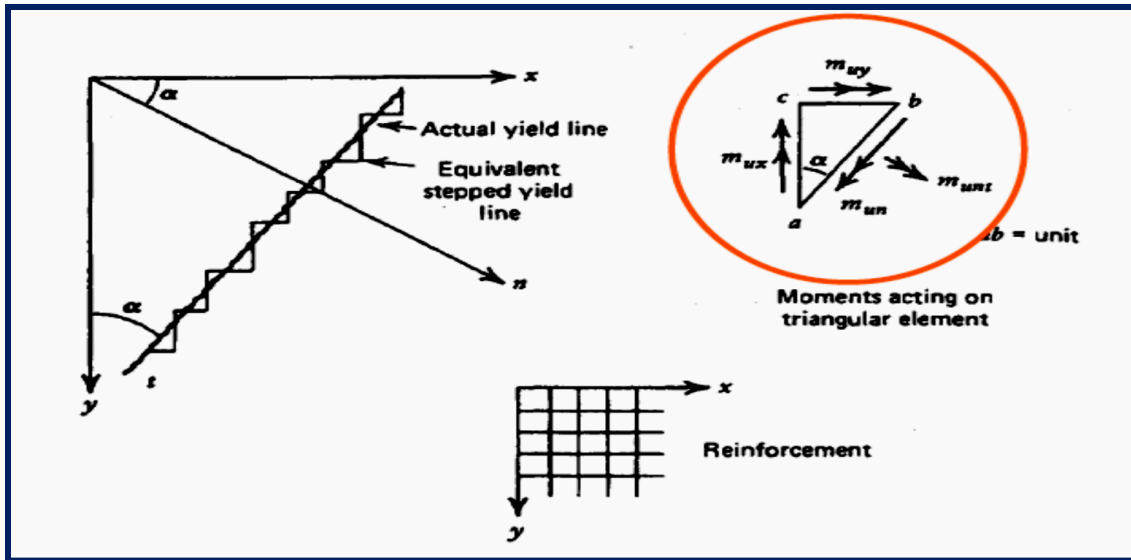
- An isotropic slab is one with the same amount of bottom reinforcement both ways
- Moment capacities in the two directions are equal, i.e. $M_x = M_y$ by assuming effective depths are equal at the interface of the two layers

Orthotropy

- Orthotropic slabs have different amounts of reinforcement in the two directions. i.e. the ultimate moment per unit length of the slab is different in the two orthogonal directions.
- Moment capacity in stronger and weak direction is assumed to have the, m , and μm . The value of μ is usually based on the relative amounts of reinforcement the designer wishes to use in the two directions.



Moments along Yield Lines



Moments along Yield Lines

- From FBD of figure (b)

$$\Sigma M = 0,$$

$$m_{un} [ab] = m_{ux} [ca] \cos \alpha + m_{uy} [cb] \sin \alpha$$

$$m_{un} = m_{ux} \cos^2 \alpha + m_{uy} \sin^2 \alpha$$

- If a slab is isotropic ally reinforced, $m_{ux} = m_{uy} = m_u$

$$m_{un} = m_u (\cos^2 \alpha + \sin^2 \alpha) = m_u$$

- Expression for toraional moment m_{ut} .

$$\Sigma M_{ab} = 0$$

$$m_{ut} [ab] - m_{ux} [cb] \sin \alpha + m_{uy} [cb] \cos \alpha = 0$$

- For isotropically reinforced slab $m_{ut} = 0$

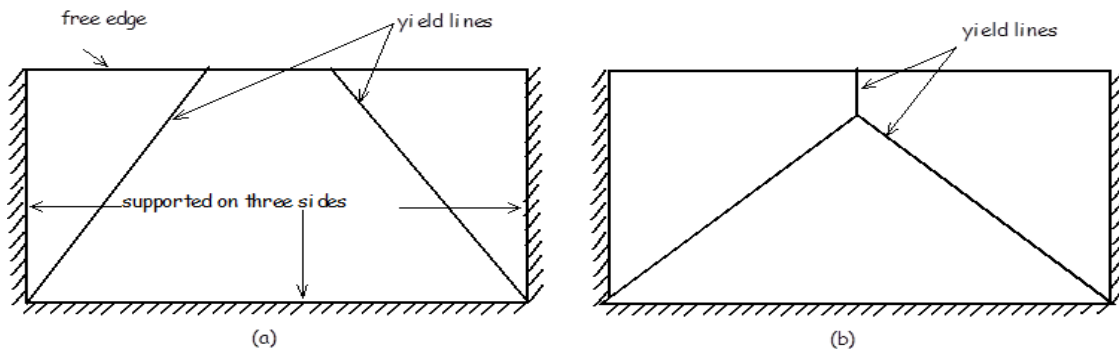
Methods of Yield Line Analysis

- Two methods of analysis to predict the ultimate load capacity of reinforced concrete slabs:

1) **Equilibrium method and**

2) **Virtual work method**

- In either method, a YL pattern is assumed so that a collapse mechanism is produced



Equilibrium method

- It makes use of the equilibrium equations for individual segment to obtain the collapse load
- The FBD represented by each collapsing segment is in equilibrium under
 - applied loads
 - yield moments
 - Reactions or shear along support lines.
 - Essentially, the yield lines form at lines of maximum moment where neither shear nor torsion is typically present at positive yield lines.
 - **NB:** Unlike elastic methods, Yield Line Theory is non-linear and the principle of superimposition of loads is strictly inapplicable.

Equilibrium method

- Demonstration Example

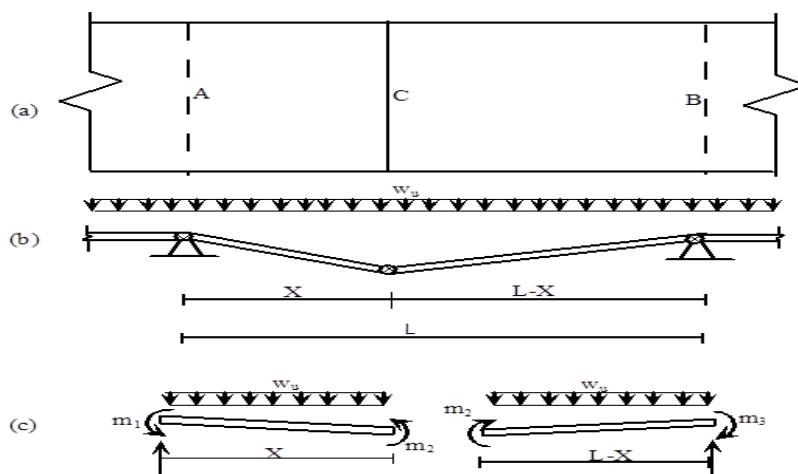


Fig. C

- For the problem posed, consider the FBD in Figure (c);
- From left segment: $\sum M_A = 0; \quad M_1 + M_2 - \frac{w_u x^2}{2} = 0$
- From right segment: $\sum M_B = 0; \quad M_2 + M_3 - \frac{w_u (L-x)^2}{2} = 0$
- Solving for w_u from the two expressions and equating, one may obtain a practical

solution for x as:

$$x = \frac{M_1 + M_2}{M_3 - M_1} L \left\{ -1 + \sqrt{1 + \frac{M_3 - M_1}{M_1 + M_2}} \right\}$$

- For instance, when $L = 3\text{m}$, $m_1 = m_2 = 7\text{kN-m/m}$
and $m_3 = 10\text{kN-m/m}$

$$x = 1.427\text{m and } w_u = 13.75\text{kN/m}^2$$

- For $m_1 = m_3 = 10\text{kN-m/m}$ and $m_2 = 7\text{kN-m/m}$, $L = 3\text{m}$

$$x = 1.5\text{m and } w_u = 15.11\text{kN/m}^2$$

- With the equilibrium method considerable care must be taken to show all forces acting on each element, including twisting moments, especially when several YLs intersect or when YLs intersect free edges.
 - ✓ At such locations, off-setting vertical nodal forces are required at the intersection of YLS.
 - ✓ Because of the possibility of that the nodal forces will be given the wrong sign or location, some building codes don't recommend the methods

Virtual Work Method (Energy Method)

- The basis of the Work Method is simply that at failure the potential energy expended by loads moving must equal the energy dissipated (or work done) in yield lines rotating

External energy expended by the displacement of loads	=	Internal energy dissipated by the yield lines rotating
Expend	=	Dissipated
E	=	D
$\sum (N \times \delta) \text{ for all regions}$	=	$\sum (m \times l \times \theta) \text{ for all regions}$

Virtual Work Method (Energy Method)

$$E = D$$

$$\Sigma (N \times \delta) \text{ for all regions} = \Sigma (m \times l \times \theta) \text{ for all regions}$$

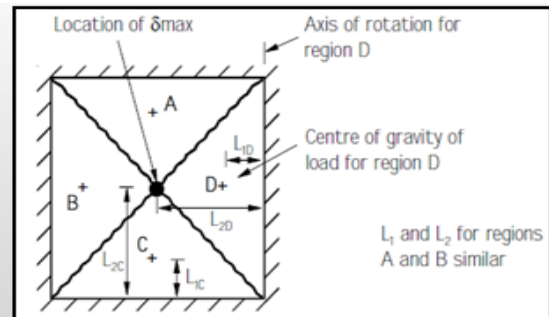
where

- N is the Load(s) acting within a particular region [kN]
- δ is the vertical displacement of the load(s) N on each region expressed as a fraction of unity^C [m]
- m is the moment or moment of resistance of the slab per metre run represented by the reinforcement crossing the yield line [kNm/m^D]
- l is the length of yield line or its projected length onto the axis of rotation for that region [m]
- θ is the rotation of the region about its axis of rotation [m/m]

General simplification

- Maximum deflection $\delta_{\max}$, is assigned the value of unity i.e. $\delta_{\max} = 1$.
- When Calculating the Expended external energy, E, the Values of L_1/L_2 when dealing with uniformly distributed loads:

- 1/2 for all rectangular regions
- 1/3 for all triangular regions with apex at point of max. deflection
- 2/3 for all triangular regions with apex on the axis of rotation



General simplification

- When calculating the Expended external energy, E, a factor of $1/L_2$ is used to determine θ_i ,

θ_i is the rotation of the region about its axis of rotation

L_2 as before, is the distance normal from the axis of rotation (or supported edge) to the location of $\delta_{\max}$ of that region. This distance can vary for each region.

Design procedure

- When applying the Work Method the calculations for the expenditure of external loads and the dissipation of energy within the yield lines are carried out independently

- The results are then made equal to each other and from the resulting equation the unknown, be it the ultimate moment ' m_u ' generated in the yield lines or the ultimate failure load ' w_u ' of the slab, evaluated.

Common failure patterns

