

CHAPTER 1: INTRODUCTION TO REINFORCED DESIGN

1.1. MECHANICAL PROPERTIES OF CONCRETE AND REINFORCING STEEL

Material Properties

Concrete is the most commonly used building material. It has the advantage of being formed into any desired shape most conveniently. It is an artificial stone obtained by mixing aggregates, cement and water and allowing the product to cure for hardening. Its essential ingredients are cement and water, which react with each other chemically, to form another material having useful strength. The strength of concrete depends upon the quality of its ingredients, their relative quantities and the manner in which they are mixed, compacted and cured. It is possible to produce concrete of different specifications for various purposes by suitably adjusting the proportions of cement, aggregate and water.

Reinforced concrete is a composite material made of concrete and steel. Plain concrete possesses high compressive strength but little tensile strength, which makes it weak in bending, shear and torsion. Reinforcing steel possesses high strength both in tension and compression. In reinforced concrete, steel provides the tensile strength and the concrete provides the compressive strength. So, by combining these features of concrete and steel, it attains high utility and versatility.

Important factors contributing to strength of concrete:

- ✚ water cement ratio
- ✚ curing concrete
- ✚ temperature

Advantages of RC (Reinforced concrete)

- ✚ Mouldability to any desired shape in forms while plastic,
- ✚ Economy
- ✚ Suitability of material for architectural & other functions
- ✚ Rigidity
- ✚ Low maintenance
- ✚ It does not deteriorate with time
- ✚ It is fire, weather and corrosion resistant
- ✚ It is monolithic

Disadvantages of RC

- Difficult to dismantle
- Formwork is expensive
- Difficult to supervise after pouring
- Due to crack, large portions of section not effectively used.

Important Features of Concrete

Characteristic Strength of Materials

For both concrete and reinforcement the Code uses the term ‘characteristic strength’ instead of 28-day works cube strength and yield stress, although it is related to these. The characteristic strength for all materials has the notation f_k and is defined as the value of the cube strength of concrete (f_{cu}), the yield or proof stress of reinforcement (f_y), below which 5% of all possible test results would be expected to fall. The value therefore is

$$f_k = f_m - 1.64s$$

Where f_m is the mean strength of actual test results determined in accordance with a standard procedure, s is the standard deviation, and 1.64 is the value of the constant required to comply with 5% of the test results falling below the characteristic strength, as indicated in Fig. 1.

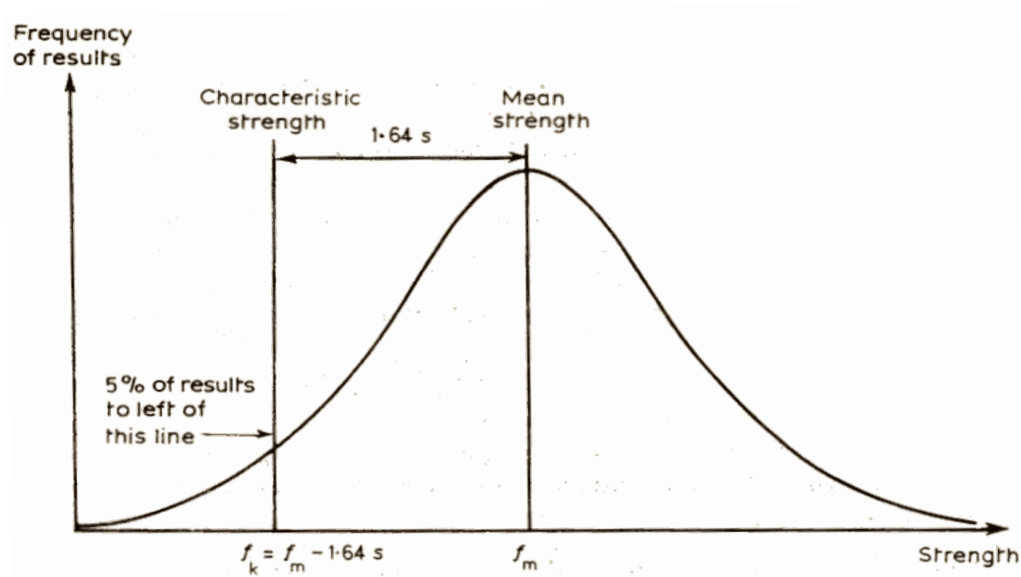


Fig. 1. Characteristic strength

General data on concrete and steel

(As per EBCS 2 – with some explanations)

Classification of concrete works

Concrete works are classified as either Class I or II depending on the quality of workmanship and the competence of the supervisors directing the works.

Works carried out under the direction of appropriately qualified supervisors ensuring the attainment of level of quality control as per EBCS 2 (chapter 9) are classified as Class I works.

Works with a lower level of quality control are classified as Class II works. Class II works is permissible only for single story structures.

Characteristic compressive strength of concrete

The compressive strength of a concrete member at one location may vary from that at another location in the same member, due to many reasons like differences in extent of compaction, time of placement, batch of concrete etc. Likewise, the strength of different specimens can also vary from one another. In order to assess the strength of one particular mix proportion of concrete, EBCS 2 specifies the use of characteristic compressive strength values. Characteristic compressive strength of concrete is defined as that strength below which 5% of all possible strength measurements may be expected to fall. It means that the characteristic strength has 95% reliability or there is only 5% probability of actual strength being less than the characteristic strength. Thus concrete is graded in terms of its characteristic compressive cube strength.

Table 1.1 of EBCS 2 gives the permissible grades of concrete for the two classes of concrete works as shown below. The numbers in the grade designation denote the specified characteristic compressive strength in MPa.

Table 1.1. Permissible grades of concrete

Class	Permissible Grades of Concrete							
I	C5	C15	C20	C25	C30	C40	C50	C60
II	C5	C15	C20					

Grade C5 shall be used only as lean concrete.

As per EBCS 2, compressive strength of concrete is determined from tests on 150 mm cubes at the age of 28 days in accordance with Ethiopian Standards. Cylindrical or cubical specimens of other sizes may also be used with conversion factors determined from a comprehensive series of tests. In the absence of such tests, the conversion factors given in Table 1.2 (of EBCS 2) may be applied to obtain the equivalent characteristic strength on the basis of 150 mm cubes.

Table 1.2 Conversion factors for strength

Size and type of test specimen	Conversion factor
Cube (200 mm)	1.05
Cylinder (150 mm diameter, 300 mm height)	1.25

In table 1.3 the characteristic cylinder compressive strength f_{ck} are given for the different grades of concrete.

Table 1.3 Characteristic cylinder compressive strength of concrete

Grades of concrete	C15	C20	C25	C30	C40	C50	C60
f_{ck}	12	16	20	24	32	40	48

Characteristic tensile strength (f_{ctk})

The characteristic tensile strength refers to the axial tensile strength as determined by tests in accordance with standards issued or approved by Ethiopian Standards.

In the absence of more accurate data, the characteristic tensile strength may also be determined from the characteristic cylinder compressive strength according to the following equations.

$$f_{ctk} = 0.7 f_{ctm}, \text{ where } f_{ctm} \text{ is the mean value given by } f_{ctm} = 0.3 f_{ck}^{2/3}$$

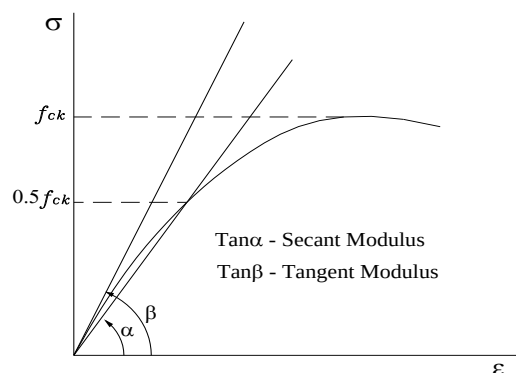
The corresponding values of f_{ctk} and f_{ctm} for different grades of concrete are given in the table 1.4.

Table 1.4 Grades of concrete and values of f_{ctk} and f_{ctm}

Grades of concrete	C15	C20	C25	C30	C40	C50	C60
f_{ctm}	1.6	1.9	2.2	2.5	3.0	3.5	4.0
f_{ctk}	1.1	1.3	1.5	1.7	2.1	2.5	2.8

Modulus of Elasticity

Stress – Strain Curve



As there is no fixed ratio of f_{ck}/ϵ to define the term modulus of elasticity, whenever E is used without further designation, it is usually meant the secant modulus E_c in MPa.

The modulus of elasticity depends not only on the concrete grade but also on the actual properties of the aggregates used.

In the absence of more accurate data, or in cases where great accuracy is not required, an estimate of the mean value of the secant modulus E_{cm} can be obtained from Table 1.5 for a given concrete grade.

Table 1.5 Values of the secant modulus of elasticity E_{cm} in GPa

Grades of concrete	C15	C20	C25	C30	C40	C50	C60
E_{cm}	26	27	29	32	35	37	39

The values given in table 2.5 are based on the following equation:

$$E_{cm} = 9.5 (f_{ck} + 8)^{1/3}$$

Poisson's Ratio

Any value between 0 and 0.2 can be adopted for Poisson's ratio.

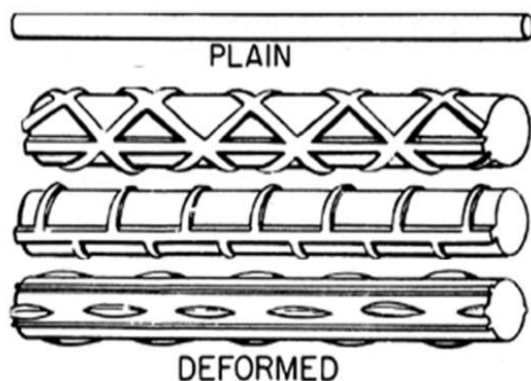
Characteristic strength of reinforcing steel.

Reinforcing steel

It is high strength and high costing material.

- ♦ The two materials – concrete and steel are best used in combination if the concrete is made to resist the compression stresses and the steel, the tensile stresses.
- ♦ When RC elements are used, sufficient bond between the two materials must be developed to ensure that there no relative movements of the steel bars and the surrounding concrete. This bond may be developed by
 - chemical adhesion
 - natural roughness
 - Closely spaced rib-shaped surface deformation of reinforcements.

Reinforcing bars varying in size 6 to 35mm in size are available in which most are surface deformed except $\phi 6$.



(In Ethiopian Iron and Steel Factory, Akaki, deformed bars and round bars of yield strength $f_y = 420$ MPa are being produced. Diameters: 8mm, 10mm, 12mm, 14mm, 16mm, 20mm, 24mm, 30mm, 32mm. Round bars of 6mm diameter are also produced)

- ♦ Some bar size and design areas for design purpose are given below.

Diameter (ϕ mm)	Area (mm ²)	Diameter (ϕ mm)	Area (mm ²)
6	28	16	200
8	50	18	254
10	78.5	20	314
12	113	22	380
14	154	24	450

Characteristic properties of reinforcement bars is expressed using its yield strength f_y (f_{yk}) and modulus of elasticity E_s . f_y is varying between 220 to 550 MPa, with 300MPa common in our country. E_s ranges between 200 to 210 MPa

Simple formula to calculate bar weight.

$$Wt = \frac{\pi D^2}{4} \times \left(\frac{\rho}{1,000,000} \right)$$

Where D=bar diameter & ρ = density of steel (=7850 Kg/m³)

Classification of reinforcing steel

Reinforcing steel shall be classified according to:

- Grade, denoting the value of the specified characteristic yield stress f_{yk} in MPa
- Class, indicating the ductility characteristics
- Size
- Surface characteristics
- Weld ability

In EBCS 2, two classes of ductility are defined. Class A refers to high ductility with $\epsilon_{uk} > 5\%$ and with value of $(f_t / f_y)_k > 1.08$ and Class B refers to normal ductility with $\epsilon_{uk} > 2.5\%$ and with value of $(f_t / f_y)_k > 1.05$, where, ϵ_{uk} denotes the characteristic value of the elongation at maximum load,

f_{tk} = Characteristic tensile strength of reinforcement

f_{yk} = Characteristic yield strength of reinforcement

Two shapes of surface characteristics are defined: (a) Ribbed bars, resulting in high bond action, (b) Plain, smooth bars (round bars), resulting in low bond action.

Physical properties of reinforcing steel

- Density 7850 kg/m³
- Coefficient of thermal expansion 10×10^{-6} per °C

In selecting an appropriate grade of concrete, the designer has to determine the environment and exposure conditions to which the members of the structure will be subjected.

Creep of concrete

Another important factor to be considered in stress-strain of concrete is creep: a property where increase in strain under constant load with time is observed. The final creep strain may be several times as large as the initial elastic strain. Generally creep has little effect on the strength of a structure but it results in increase in service load deflections.

Factors attributing: - loading at an early stage, high water – cement ratio, exposing the concrete to drying condition.

Shrinkage of concrete

The property of diminishing in volume of concrete during the process of hardening is termed shrinkage. The factors affecting shrinkage are:

- (a) Environmental condition (humidity and temperature)
- (b) Water-cement ratio
- (c) Duration of exposure (not duration of loading)
- (d) Percentage of steel

Greater the percentage of steel lesser is the shrinkage, because the reinforcement restrains the shrinkage. The shrinkage is also a **time dependent** process. For computational purpose it may be assumed that 25% of ultimate shrinkage occurs in the first one month and the remaining 75% in six months from the commencement of drying. Creep and shrinkage occur simultaneously, and jointly influence the behavior of reinforced concrete member. In limit state method, it is necessary to assess deformations due to creep and shrinkage from serviceability considerations.

Concrete Placement and Curing

Concrete Placement

When concrete is discharge from the mixer, precaution should be exercised to prevent segregation. Vibration is desirable after pouring the fresh concrete because it eliminates voids and brings particles into close contact. The resulting consolidation also ensures close contact of the concrete with the forms, with reinforcement and other embedded items. For consolidation of structural concrete, immersion vibration are recommended. Oscillation should be at least 7000 vibration per minute when the vibrator head is immersed in concrete. Each yd^3 (0.765m^3) of concrete should be vibrated at least 1 minute.

Formwork retains concrete until it has set and produced the desired shape and sometimes the desired surface finish. Formwork must be supported on false work of adequate strength and rigidity. Forms must also be tight, yet they must be of low cost and often easily demountable to permit reuse. Early striking forms is generally desirable to permit quick reuse, start curing

as soon as possible and allow repairs and surface treatment while the concrete is still green and condition are favorable for good bond.

The time between casting of concrete and removal of the formwork depends mainly on the strength development of the concrete and on the function of the formwork. Provided the concrete strength is confirmed by test on cubes stored under the same condition, formwork can be removed when the cube strength is 50% of the nominal strength or twice the stress to which it will then be subjected whichever is greater, provided such earlier removal will not result in unacceptable deflection such as due to shrinkage and creep [EBCS 2-1995].

In the absences of more accurate data the following minimum periods are recommended by EBCS 2-1995.

1. For non-load bearing parts of formwork like vertical forms for beams, columns and walls..... 18 hours
2. For soffit formwork to slabs 7 days
3. For props to slabs 14 days
4. For soffits formwork to beams 14 days
5. For props to beams 21 days

Curing Concrete

While more than enough water for hydration is incorporated into normal concrete mixes, the loss due to evaporation from the time the concrete is placed is usually so rapid that complete hydration may be delayed or prevented. Rapid drying causes also drying shrinkage surface cracks. Therefore it is important to keep fresh concrete moist for several days after placing either by sprinkling, ponding or by surface sealing. This operation is called curing. If curing is properly done for a sufficiently long period, curing produces stronger and more watertight concrete. The most common field practice is curing by sprinkling. Portland cement concrete should be cured this way for 7-14 days. Curing is especially important in hot climate to replenish water lost due to rapid evaporation.

1.2. Concrete mix design

The art of proportioning the concrete is called concrete mix design. The art of deciding the size, shape and quantity of steel to be provided in reinforced concrete structural component and the fashion how to detail the rebar's to bring out a skeleton of reinforcements is called the reinforced concrete structural design. The major aim of structural design is to produce design for a safe, serviceable and durable structure. The process of structural design requires conceptual thinking, sound knowledge of engineering, imagination, knowledge of relevant design codes and bye-laws backed up by experience and judgment. The process of structural design involves the following stages:

1. Structural planning and configuration
2. Action of loads, their classification and combinations
3. Analysis for the stress resultants
4. Design of components
5. Drawing, detailing and preparation of schedules

Behavior under Load

Loads

Loads that act on structures can be divided into three categories: *dead loads*, *live loads*, and *environmental loads*.

Dead loads are those that are constant in magnitude and fixed in location throughout the lifetime of the structure. Usually the major part of the dead load is the weight of the structure itself. This can be calculated with good accuracy from the design configuration, dimensions of the structure, and density of the material. For buildings, floor fill, finish floors, and plastered ceilings are usually included as dead loads, and an allowance is made for suspended loads such as piping and lighting fixtures. For bridges, dead loads may include wearing surfaces, side walks, and curbing, and an allowance is made for piping and other suspended loads.

Live loads consist chiefly of occupancy loads in buildings and traffic loads on bridges. They may be either fully or partially in place or not present at all, and may also change in location. Their magnitude and distribution at any given time are uncertain, and even their maximum intensities throughout the lifetime of the structure are not known with precision. The minimum live loads for which the floors and roof of a building should be designed are usually specified in the building code that governs at the site of construction. Representative values of minimum live loads to be used in a wide variety of buildings are found in *Minimum Design Loads for Buildings and other structures*.

Live loads in codes are usually approximated by uniformly distributed load. In addition to these uniformly distributed loads, it is recommended that, as an alternative to the uniform load, floors be designed to support safely certain concentrated loads if these produce a greater stress. Certain reductions are often permitted in live loads for members supporting large areas, on the premise that it is not likely that the entire area would be fully loaded at one time. Service live loads for highway bridges are specified by the American Association of State Highway and Transportation Officials (AASHTO) in its Standard Specifications for Highway Bridges. For railway bridges, the American Railway Engineering Association (AREA) has published the Manual of Railway Engineering, which specifies traffic loads.

Environmental loads consist mainly of snow loads, wind pressure and suction, earthquake loads (i.e., inertia forces caused by earthquake motions), soil pressures on subsurface portions of structures, loads from possible ponding of rainwater on flat surfaces, and forces caused by temperature differentials. Like live loads, environmental loads at any given time are uncertain both in magnitude and distribution.

Much progress has been made in recent years in developing rational methods for predicting horizontal forces on structures due to wind and seismic action. Most building codes specify design wind pressure per square foot of vertical wall surface. Depending upon locality, these equivalent static forces vary from about 0.48 KPa up to 2.4KPa. Factors considered in more up to date standards include probable wind velocity exposure (urban vs. open terrain, for example), and height of the structure, the importance of the structure (i.e. consequences of failure), and the gust response factors to account for the fluctuating nature of the wind and its interaction with the structure.

Seismic forces may be found for a particular structure by elastic or inelastic dynamic analysis, considering the expected ground accelerations and the mass, stiffness, and damping characteristic of the construction. However, often the design is based on equivalent static forces calculated from provisions. The base shear is found by considering such factors as location, type of structure and its occupancy, total dead load, and the particular soil condition. The total lateral force is distributed to floors over the entire height of the structure in such a way as to approximate the distribution of forces from a dynamic analysis.

Characteristic loads

For loading we use the ‘characteristic’ load (F_k) as the basis. Ideally this should be determined from the mean load and its standard deviation from the mean, and using the same probability as for the materials we should say that $F_k = F_m + 1.64s$. The characteristic load would be that value of loading such that not more than 5% of the spectrum of loading throughout the life of structure will lie above the value of the characteristic load (Figure 2).

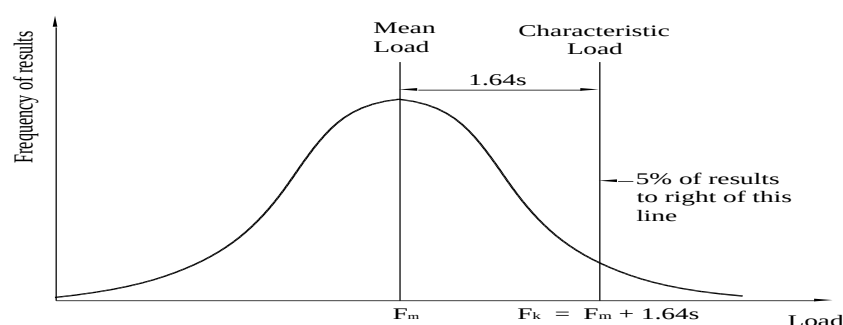


Figure 2. Characteristic load

The characteristic dead, imposed and wind loads have the notation G_k , Q_k , W_k respectively, where the upper-case letters denote the total load on a span. Lower-case letters denote uniform load per square meter, although in design examples for beams the lower-case letters have been used for a uniformly distributed load, so that $G_k = g_k l$.

Behavior

In RC structures such as beams, the tension caused by bending moment is chiefly resisted by the steel reinforcement while the concrete alone is usually capable of resisting the corresponding compression. Such joint action of the two materials is assumed if the relative slip is prevented which is achieved by using deformed bars, with their high bond strength at the steel concrete interface. To illustrate the stress strain development for increased loading consider the following.

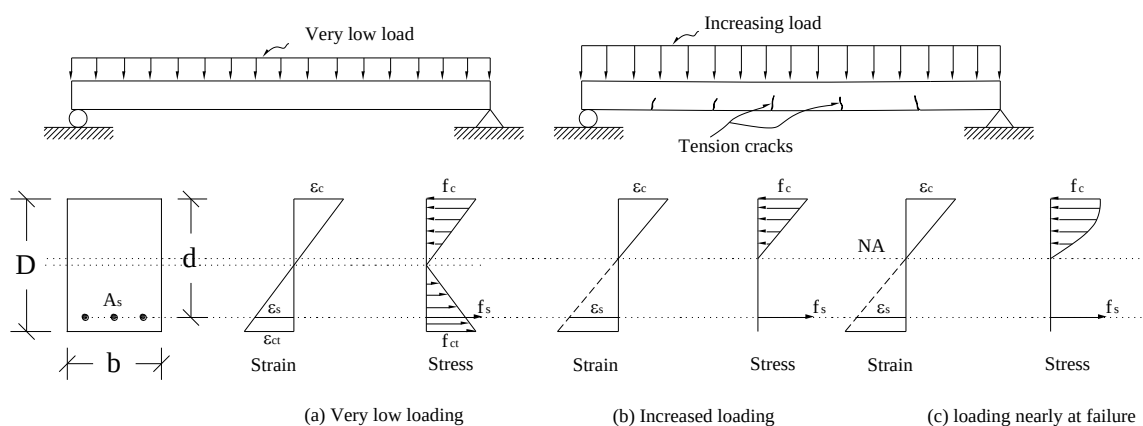


Figure 3. Behavior of RC beam under load.

- At low loads where tensile stress is less than or equals to f_{ctk} stress-strain relation shown in figure 3a results.
- At increased load tensile stress produced is larger than f_{ctk} (figure 3b) $\Rightarrow$ crack develops below neutral axis, the steel alone carries all tensile force and hence the compressive stress at extreme fiber is less than f_c (linear stress distribution).
- For further increment of load, the stress distribution is longer linear as shown in fig. 3c.

If the structure, say the beam, has reached its maximum carrying capacity, one may conclude the following on the cause of failure.

- (i). When the amount of steel is small – at some value of the load, the steel reaches its yield point. In such circumstances, the steel stretches a large amount and tension cracks in the concrete widen visibly resulting in significant deflection of the beam. Compression zone of concrete increases ending up with crushing of concrete (secondary compression failure).

Such failure is gradual and is preceded by visible signs, widening and lengthening of cracks, marked increase in deflection.

- (ii). When a large amount of steel is used, compressive strength of concrete would be exhausted before the steel starts yielding, thus, concrete fails by crushing. Compression failure through crushing of concrete is sudden and occurs without warning.

Thus it is a good practice to dimension sections in such a way that should they be overloaded, yielding of steel rather than crushing of concrete would initiate failure.

CHAPTER Two

Basic Concepts of Design

2.1. Design Philosophy

Structural design should be performed to satisfy three criteria: strength, serviceability, and economy.

Strength pertains to the general integrity and safety of the structure under extreme load conditions. The structure is expected to withstand occasional overloads without severe distress and damage during its lifetime.

Serviceability refers to the proper functioning of the structure as related to its appearance, maintainability, and durability under normal, or service load conditions, deflection, vibration, permanent deformation, cracking, and corrosion are some design considerations associated with serviceability.

Economy concerns the overall material and labor costs required for the design, fabrication, erection, and maintenance processes of the structure.

As a result of the various design criteria, two major design methods have evolved in practice for the design of reinforced concrete structures.

i) Allowable Stress Design (ASD)

In the allowable stress design (ASD), a member is selected such that under expected loads, known as service or working loads, the stress will not exceed one of the previously described limits of usefulness. This design methodology has been in use for decades for reinforced concrete design of buildings and bridges. Allowable stresses are usually expressed as a function of the yield stress (f_y) or tensile stress (f_u) of the material. To account for overload, under-strength, and approximations used in structural analysis, a *factor of safety* is applied to reduce the nominal resistance of the structural member to a fraction of its tangible capacity.

In so far as the method of analysis is concerned, allowable stress design is based on elastic analysis to obtain the structural responses such as moments, shear and axial forces that a member must be designed to carry.

The general formula for an allowable stress design has the form:

$$\frac{R_n}{F_s} \geq \sum_{i=1}^m Q_i$$

Where: R_n = nominal resistance of the structural component expressed in units of stress

Q_i = service or working stress computed from the applied working load type i.

i = load type (dead, live, wind, etc.)

m = number of load types considered in the design.

$\frac{R_n}{F_s}$ = Allowable stress of structural component.

ii) Limit State Design (LSD)

Limit state is a probabilistic design procedure in which a structure being designed will not become unfit for use during its expected life. Therefore the various ways in which the structure could become unfit for use should be examined. The condition of a structure when it becomes unfit for use or unserviceable is called a limit state.

Mathematically it can be expressed as:

$$\frac{R_n}{\gamma_m} \geq \sum_{i=1}^m \gamma_{li} Q_i$$

Where:

$\frac{R_n}{\gamma_m}$ = Design strength

$\sum_{i=1}^m \gamma_{li} Q_i$ = The required strength or load effects for a given load combination.

2.2. Limit State Design (LSD) method

The objective of limit state design is to achieve acceptable probabilities that the structure being designed will not become unfit for its intended purpose during its expected life. When a structure or structural element becomes unfit for its intended use, it is said to have reached a *limit state*. The limit states for reinforced concrete structures can be divided into three basic groups:

1. Ultimate limit states
2. Serviceability limit states
3. Special limit states

Ultimate limit States

These involve a structural collapse of part or all of the structure. Such a limit state should have a very low probability of occurrence since it may lead to loss of life and major financial losses. The major ultimate limit states are:

- a) **Loss of equilibrium** of a part or all of the structure when considered as a rigid body. Such a failure would generally involve tipping or sliding of the entire structure and would occur if the reactions necessary for equilibrium could not be developed.

- b) **Rupture of** critical parts of the structure, leading to partial or complete collapse. It includes flexural failures, shear failures, direct compression or tension failures and so on. The majority of this course deals with this limit state.
- c) **Progressive collapse:** In some cases a minor localized failure may cause adjacent members to be overloaded and fail, until the entire structure has collapsed. Since such failures frequently occur during construction, the designer should be aware of construction loads and procedures.
- d) **For motion of plastic mechanism:** A mechanism is formed when the reinforcement yields to form plastic hinges at enough sections to make the structure unstable.
- e) **Instability due to deformations of the structure:** This type of failure involves buckling due to gravity load effects.
- f) **Fatigue:** Fracture of members due to repeated stress cycles may cause collapse of part or all of a structure. Although, fatigue failure results from repeated application of service loads, this has been listed as an ultimate limit state as it leads to a structural collapse.

Serviceability limit states

These involve disruption of the functional use of the structures but not collapse. Since there is less danger of loss of life, a higher probability of occurrence can generally be tolerated than in the case of an ultimate limit states. The major serviceability limit states include:

- a) **Excessive deflections for normal service:** Excessive deflections may cause machinery to malfunction, may be visually unacceptable, and may lead to damage to nonstructural elements or to changes in the distribution of forces. A minimum depth is recommended for the control of deflection.
- b) **Excessive crack width:** Although reinforced concrete must crack before the reinforcement can act, it is possible to detail the reinforcement to minimize the crack widths. Excessive crack widths lead to leakage through the cracks, corrosion of the reinforcement, and gradual deterioration of the concrete. In addition, wide cracks may be unsightly. Special detailing is required in structures that must be watertight.
- c) **Undesirable vibrations:** Vertical vibrations of floors or bridges and lateral and torsional vibrations of tall buildings may disturb the user.

Special limit states

- a) Damages or collapses in extreme earthquakes
- b) Structural effects of fire, explosions, or vehicular collisions

- c) Structural effects of corrosion or deterioration, particularly when exposed to deicing compounds.

Limit State design Procedure

During design it is necessary to identify whether the ULS or SLS is the conical limit state for a particular structure and base the design on this. Checks are then made to ensure that all other relevant limit states are satisfied by the results produced.

Generally, in the design of reinforced concrete members (except in special cases such as water retaining structures), it is usual to first ensure that the ULS is not exceeded (by applying factor of safeties) and then to check that the relevant SLSs are satisfied.

General Design Equation

Consider the RC simply supported beam shown below. The applied loads cause bending moments, which are directly obtained from the loads using the law of statics. The bending moments, shear force, axial force, torsion, deflection, etc. are referred to as load effects.

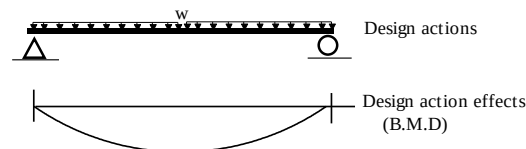


Fig. 2.1 Action and action effects

Fig. 2.2 shows flexural stresses acting on a beam cross-section and the internal couple resulted from their resultants. The compressive and tensile stresses can be replaced by their resultants, C and T , as shown below. The resulting couple is said to be an internal resisting moment. The internal resisting moments when the cross section fails is referred to as the moment capacity or moment resistance. The word resistance can also be used to describe shear resistance or axial load resistance. The beam shown in Fig. will safely support the loads if at every section the resistance of the member exceeds the effects of the loads.

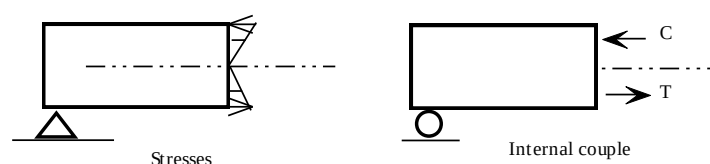


Fig. 2.2 Internal resisting moment

Accordingly, the structure or structural elements shall be verified by the following equations.

Verifications of Static Equilibrium and Strength, Ultimate Limit State.

- 1) When considering a limit state of static equilibrium or of gross displacement of the structure as a rigid body, it shall be verified that:

$$E_{d,dst} \leq E_{d,stab}$$

Where $E_{d,dst}$ is the design value of the effect of destabilizing actions;

$E_{d,stab}$ is the design value of the effect of stabilizing actions.

- 2) When considering a limit state of rupture or excessive deformation of a section, member or connection it shall be verified that:

$$E_d \leq R_d$$

Where E_d is the design value of the effect of actions such as internal force, moment or a vector representing several internal forces or moments;

R_d is the corresponding design resistance, associating all structural properties with the respective design values.

Verification of Serviceability, Serviceability Limit States

It shall be verified that:

$$E_d \leq C_d$$

Where C_d is a nominal value or a function of certain design properties of materials related to the design effects of actions considered; and

E_d is the design value of the action effect (e.g. displacement, acceleration), determined on the basis of one of the combinations defined for serviceability.

Design preliminaries

In the reinforced concrete design, the strength of a given mix of concrete and the imposed load that is likely to act over a specified area cannot be provided with certainty unless actual measurements at an instant are made. Such variables are referred to as random variables in the theory of probability. The behavior of a random variable can be characterized by probability distribution of the random variable. In our case of material strength and imposed load in a structure the uncertainty may be the result of the combined effect of many causes each difficult to isolate and observe. In such cases the distribution can be developed for the variable of interest without knowing the distribution of the cause. The *normal* or *Gaussian probability distribution* is most commonly used for this purpose. Limit state design principle

uses this for deriving the characteristic strength of materials and characteristic loads on the structures.

Design strength

Tests to determine the characteristic strengths of concrete and steel reinforcement are carried out on near perfect specimens, which have been prepared under laboratory conditions. Such conditions will seldom exist in practice. Therefore it is undesirable to use characteristic strengths to size the members. To take account of differences between actual and laboratory values, local weaknesses and inaccuracies in assessment of resistance of section, the characteristic strengths (f_{ck} and f_{yk}) are divided by appropriate partial safety factor for strengths (γ_m) taken from table 3.1 and 3.2 of EBCS 2. The resulting values are termed as design strengths.

Table 3.1 Partial Safety Factor for Materials - Class I Works

Design Situations	Concrete, γ_c	Reinforcing Steel, γ_s
Persistent and Transient	1.50	1.15
Accidental	1.30	1.00

Table 3.2 Partial Safety Factor for Materials - Class II Works

Design Situations	Concrete, γ_c	Reinforcing Steel, γ_s
Persistent and Transient	1.65	1.20
Accidental	1.45	1.10

The difference in values for the two materials is indicative of the comparative lack of control over the production of concrete, the strength of which is affected by such factors as W/C ratio, degree of compaction, rate of drying etc. which frequently cannot be accurately controlled on site to that of steel which is produced under strict control conditions in factory.

The design strength of concrete is defined by:

(a) In compression

$$f_{cd} = \frac{0.85f_{ck}}{\gamma_c}$$

(b) In tension

$$f_{ctd} = \frac{f_{ctk}}{\gamma_c}$$

The design strength of steel in tension and compression is defined by:

$$f_{yd} = \frac{f_{yk}}{\gamma_s}$$

In the formula for design strength of concrete in compression, the value 0.85 accounts for effect of duration of loading.

Design loads

The load in a member may be greater than anticipated because of variability of occupancy and because of unforeseen circumstances which may lead to an increase in the general level of loading, errors in analysis, and errors during construction etc. Therefore, the characteristic loads which are obtained from the standardized values given in EBCS 1, are expected to be increased by some factor to obtain the design loads. Such factors are called partial safety factors for actions. In EBCS 2, table 3.3 gives the partial safety factor values for actions in building structures.

Table 3.3 Partial Safety Factors for Actions in Building Structures

Design Situation	Action	Factor γ	Favorable	Unfavorable
Persistent and Transient	Permanent	γ_G	1.00	1.30
	Variable	γ_Q	0	1.60
Accidental	Permanent	γ_G	1.00	1.00

Having the partial safety factor values and characteristic loads in hand, the designer will have to determine the particular combination of loading which is likely to produce the most adverse effect on the structure in terms of bending moments, shear forces and deflections.

Some of the load combinations for ultimate limit state and serviceability limit states with the corresponding load factors is given below.

Ultimate limit state:

- (a) Permanent action (G_k) and only one variable action (Q_k)

$$\text{Design (Factored) load} = F_d = 1.3G_k + 1.6Q_k$$

- (b) Permanent action (G_k) and two or more variable actions (Q_{ki})

$$\text{Design (Factored) load} = F_d = 1.3G_k + 1.35 \sum_{i=1}^n Q_{ki}$$

- (c) Permanent action (G_k), variable action (Q_k) and accidental (seismic) action (A_E)

$$\text{Design (Factored) load} = F_d = 0.75(1.3G_k + 1.6Q_k) + A_E$$

Serviceability limits state:

- (a) Permanent action (G_k) and only one variable action (Q_k)

$$\text{Design (Factored) load} = F_d = G_k + Q_k$$

- (b) Permanent action (G_k) and two or more variable actions (Q_{ki})

$$\text{Design (Factored) load} = F_d = G_k + 0.9 \sum_{i=1}^n Q_{ki}$$

Specifications and Building Codes.

Then design of concrete structures is generally done within the framework of codes giving specific requirements for materials, structural analysis, member proportioning, etc. Specification serves as a guide for the engineer to arrive at a safe and acceptable design. It is also a guarantee to the owner that the resulting structure will comply with basic standards to ensure safety, utility and economy.

The designer doing concrete structures in various disciplines, such as buildings, bridges, etc., will have to follow closely the relevant design requirements in the appropriate specifications and design codes as *minimum* requirements.

The following are some important specifications for concrete structures.

- EBSC 1 Ethiopian Building Code Standard for Basis of Design and Actions on Structures
- EBCS 2 Ethiopian Building Code Standard for the Design of Concrete Structures

- EBCS 4 Ethiopian Building Code Standard for Design of Composite Steel and Concrete Structures
- EBCS 7 Ethiopian Building Code Standard for Basis Earthquake design of Structures.
- EC 2 European Standards for the Structural Use *of* Concrete
- ACI American Concrete Institute (building code requirements for structural concrete)
- AASHTO American Association *of* State Highway and Transportation Officials
- BS 8100 British Standards for The Structural Use *of* Concrete Works in Buildings
- ASTM American Society for Testing Materials.

CHAPTER 3

DESIGN OF BEAMS FOR FLEXURE

3.1. Singly and doubly reinforced for Rectangular and T-sections.

Introduction

Most reinforced concrete structures can be divided into beams and slabs subjected primarily to flexure (bending) and columns subjected to axial compression accompanied in most cases by flexure. Typical examples of flexural members are the slab and beams shown in Fig. 3.1. The load P applied at point A is carried by the strip of slab shown shaded. The end reactions from this slab strip load the beams at B and C. The beams, in turn, carry the slab reactions to the columns at D, E, F, and G. The beam reactions cause axial loads in the columns. The slab in Fig. 3.1 is assumed to transfer loads in one direction and hence is called one way slab. If there were no beams, the slab would carry the load in two directions. Such a slab is referred to as two-way slab. Two way slab action will be discussed in chapter 4.

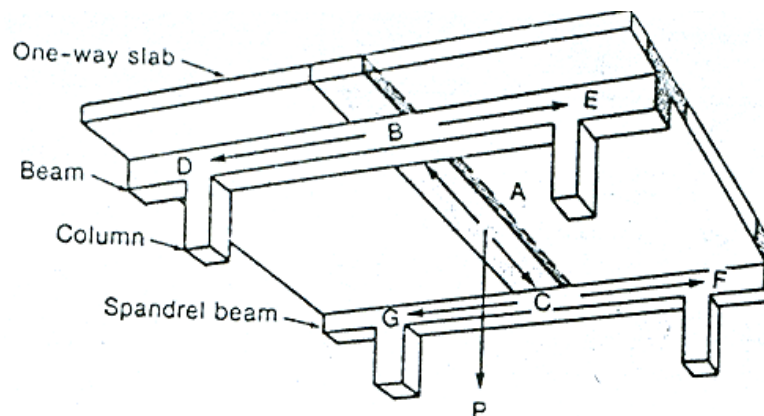


Fig. 3.1 One-way flexure

In this chapter the stress-strain curves for concrete and reinforcement as recommended by EBCS-2 are used to develop flexural theory.

Analysis versus Design

Two different types of problems arise in the study of RC.

1. **Analysis:** Given a cross-section, concrete strength, reinforcement size, location, and yield strength, and compute the resistance or capacity.
2. **Design:** Given a factored load effects such as M_{sd} , and select a suitable cross-section, including dimensions, concrete strength, reinforcement, and so on.

Although both types of problems utilize the same fundamental principles, the procedure followed is different in each case. Analysis is easier as all the decisions concerning reinforcement location, beam size and so on have been made and it is only necessary to apply the strength calculation principles to determine the capacity. Design, on the other hand, involves the choice of the beam sizes, material strengths and reinforcement to produce a cross-section and structural system that can resist the loads and moments which will be imposed on it. **As the analysis problem is easier, most sections in this and other chapters start with analysis to develop the fundamental concept and then move to consider design.**

The fundamental principles involved in the analysis and design of reinforced concrete beams are as follows:

- **At any cross section there exist internal forces which can be resolved in to components normal and tangential to the section.**
- **The normal components are known as the bending stresses (tension on one side of the neutral axis and compression on the other), and their function is to resist the bending moment at the section.**
- **The tangential components are known as the shear stresses, and they resist the transverse or shear forces.**

Behavior of RC beams

In reinforced concrete beams, the tension caused by bending moment is mainly resisted by the steel reinforcement while the concrete alone is usually capable of resisting the corresponding compression. Such joint action of the two materials is assured if relative slip is prevented and it is achieved by using deformed bars with their high bond strength at the steel-concrete interface or by special anchorage of the ends of the bars.

There are three distinct stages of behavior for a reinforced concrete beam when the load is gradually increased from zero to the magnitude that will cause the beam to fail (see Fig. 3.2)

Uncracked concrete Stage: At low loads, as the bending moment in a flexural member is so small that the tension stress in the concrete doesn't exceed the modulus of rupture, no flexural tension cracks will occur and the entire concrete is effective in resisting stress, in compression on one side and in tension on the other side of the neutral axis. In addition, the reinforcement, deforming the same amount as the adjacent concrete, is also subject to tension stresses. At this stage all stresses in the concrete are of small magnitude and are proportional to strains (see Fig.3.2a).

As long as no tension cracks develop the strain and stress distribution is essentially the same as in an elastic, homogeneous beam. The only difference is the presence of another material, the steel reinforcement. In this "transformed section" the actual area of the reinforcement is replaced with an equivalent concrete area equal to nA_s located at the level of the steel (see Fig. 3.2a). Where n is the ratio of the modulus of elasticity of steel to that concrete (modular ratio) and A_s is the area of steel.

Once the transformed section has been obtained, the usual methods of analysis of elastic homogeneous beams apply. That is, the uncracked transformed section (gross cross-section) will be used in the computation of section properties and stresses.

Bending stresses can be obtained from: $\sigma = My/I_{\text{gross}}$

The moment curvature diagram for this stage (segment O-B in Fig. 3.3) is linear.

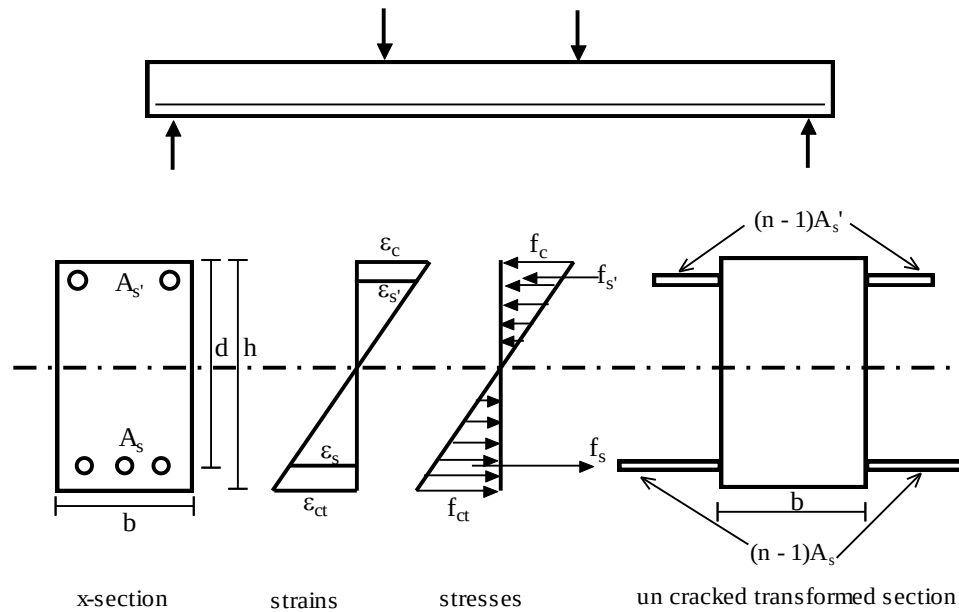
Cracked Concrete Stage: At moderate loads, as the bending moment exceeds the cracking moment of the section, tension cracks start to develop from the bottom extreme fiber and propagate quickly upward to or close to the level of the neutral plane, which in turn shifts upward with progressive cracking (see Fig. 3.2c). In well-designed beams, the width of these cracks is so small (hairline cracks) that they are not objectionable from the viewpoint of either corrosion protection or appearance. Their presence, however, profoundly affects the behavior of the beam under load. Evidently, in a cracked section, the concrete does not transmit any tension stresses and the steel is called upon to resist the entire tension. Therefore, the cracked transformed section will be used in the computation of section properties and stresses. At these loads, stresses and strains continue to be closely proportional (see Fig. 3.2b). At this stage as the stiffness of the beam is reduced due to the reduction in the effective area of concrete, the slope of the moment curvature diagram (shown by B-C-D in Fig. 3.3) is also reduced.

The cracking moment can be obtained using the maximum tensile stress equal to the modulus of rupture of concrete, that is: $M_{\text{cr}} = \sigma_{\text{cr}} I/C$, where $\sigma_{\text{cr}} = 0.7f_{\text{ck}}$

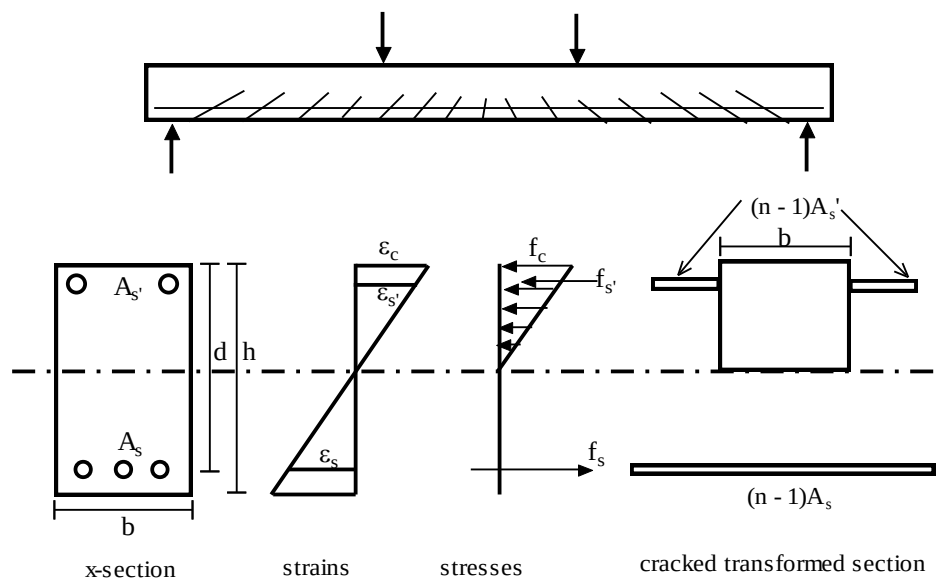
Ultimate Stage: At higher loads (close to the ultimate load), stresses and strains rise correspondingly and are no longer proportional and the distribution of concrete stresses on the compression side of the beam is of the same shape as the stress strain curve (see Fig. 3.2b).

Once yielding has occurred, the curvature increases rapidly with very little increase in moment (see Fig. 3.3). Eventually, the carrying capacity of the beam is reached. And failure

can be caused either due to the attainment of the yield point in steel in moderately reinforced beams or due to crushing of concrete in the compression zone in highly reinforced beams.



(a)



(b)

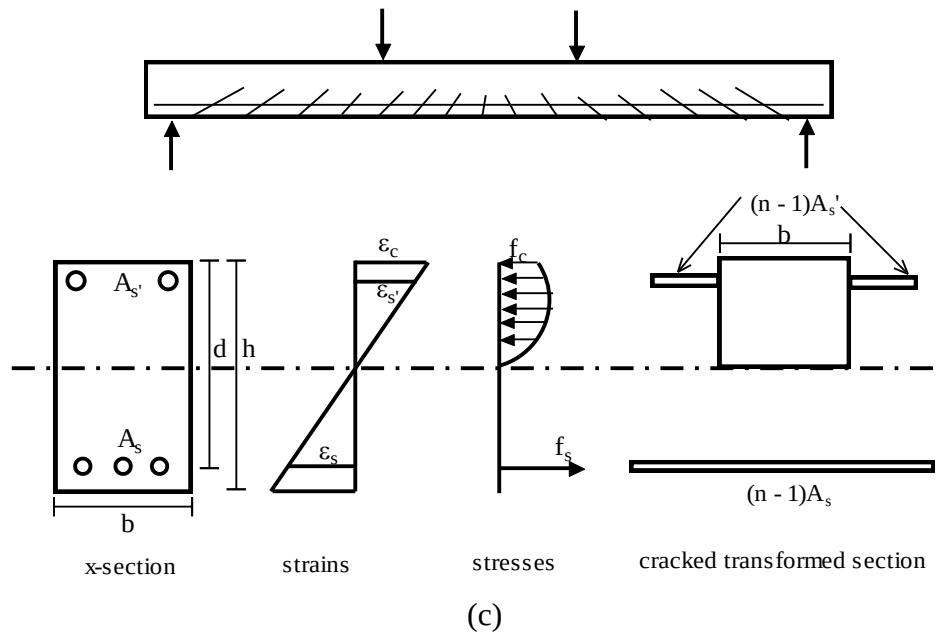


Fig. 3.2 Behavior of reinforced concrete beam under increasing load

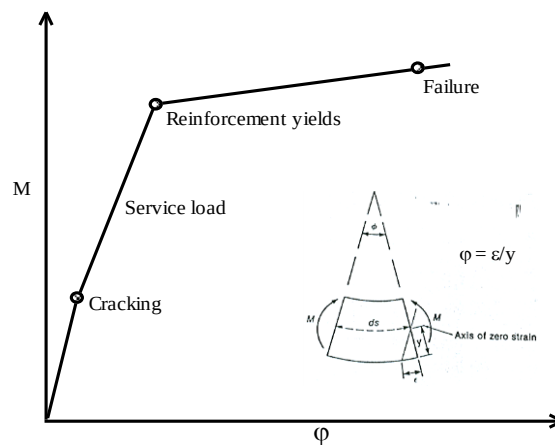


Fig. 3.3 Moment-curvature diagram for a beam under increasing load

Basic principles and assumptions in flexure theory of RC Beams

Although the method used in the analysis of RC beams are different from those used in the design of homogenous beam such as structural steel, the fundamental principles are essentially the same. Accordingly, the basic equations for the flexural design of beams and slabs are derived based on the following basic principles and assumptions at ultimate limit state. The derived equations are then used to develop design Tables and Charts for various grades of concrete and steel.

1. Internal stress resultants such as bending moments, shear forces etc. at any section of a member are in equilibrium with the external action effects.
2. Plane sections before bending remains plane after bending

3. The strain in the reinforcement is equal to the strain in the concrete at the same level
4. The tensile strength of concrete is neglected
5. The stresses in concrete and reinforcement can be computed from the strains using their σ - ϵ curves.
6. The behavior of the concrete under compression is as shown in Fig. 3.3. The equivalent rectangular stress block as recommended by EBCS 2 is shown in Fig. **(Concrete is assumed to fail when the compressive strain reaches its ultimate value. The compressive stress-strain curve for concrete may be assumed to be rectangular trapezoidal, parabolic or any other shape, (which is easier for computation) provided that it adequately predicts the test results).**
7. The stress -strain relationship of the reinforcement is as shown in Fig.3.3
8. The strain diagrams at the ultimate limit state is as shown in Fig. 3.4
 - a) The maximum compressive strain in the concrete is taken to be
 - 0.0035 in bending
 - 0.002 in axial compression
 - b) The maximum tensile strain in the reinforcement is taken to be 0.01

For manual calculation, for the sake of simplicity, the simplified rectangular stress block can be used whereas design Charts and Tables are based on the parabola-rectangle stress distribution diagram.

Strain Distribution at the Ultimate Limit State

The entire range of strain distribution at the ultimate limit state is assumed to pass through one of the three points A, B or C as shown in Fig. 3.4 (reproduce from EBCS-2). This resulted in five possible zones with respect to the limiting values of the ultimate strains in concrete and steel as shown in the same figure.

Each zone is characteristic of the particular type of loading on the section and may be described as follows:

Zone 1 - The section is subjected to a tensile load with a small eccentricity.

Zone 2 - The section is subjected to an axial load combined with bending that will cause the strain in the steel to reach the maximum $\epsilon_{st} = 0.01$ while the strain in the concrete ϵ_c is less or equal to its maximum value of $\epsilon_{cu} = -0.0035$.

Zone 3 - The section is subjected to axial load and large bending moments. The tensile steel strain is in the range $0.01 \epsilon_{st}, \epsilon_{sy}$ while the concrete strain reaches $\epsilon_{cu} = -0.0035$.

Zone 4 - The section is subjected to axial load with moderate eccentricity. The tensile steel strain is less than the yield value ϵ_{sy} while the concrete strain reaches $\epsilon_{cu} = -0.0035$.

Zone 5 - The section is subjected to predominantly compressive load with small eccentricity.

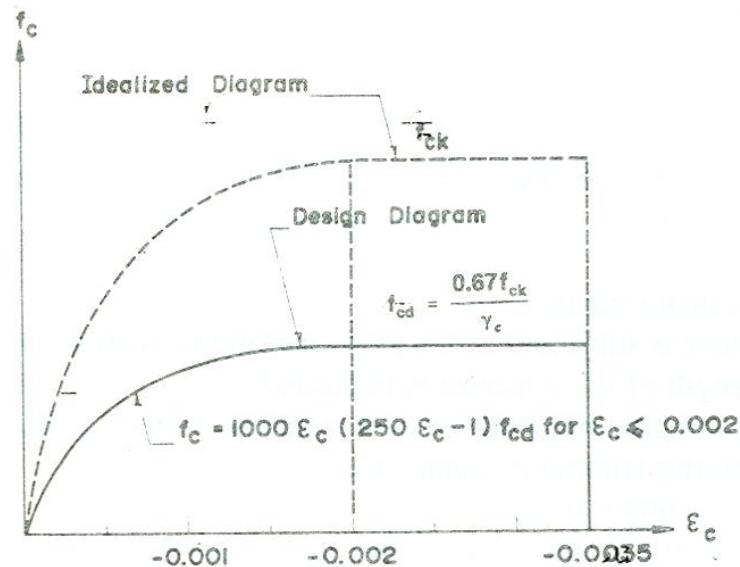


Fig. 3.4 Parabolic-rectangular stress-strain diagram for concrete in compression

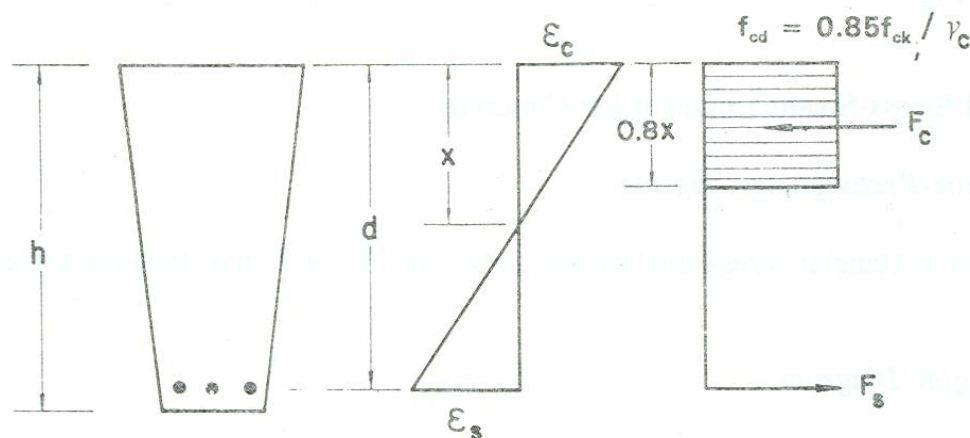


Fig. 3.5 Rectangular stress diagram

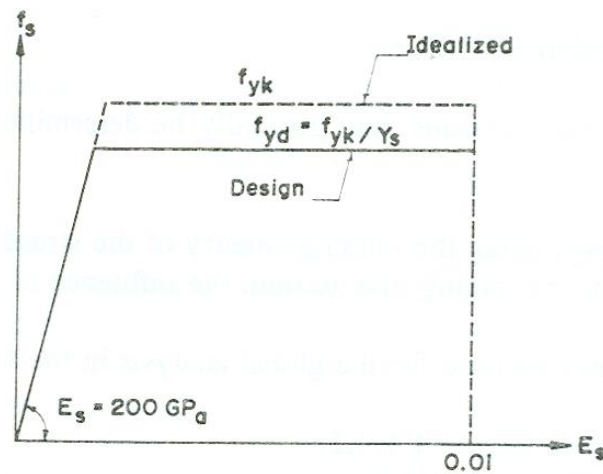


Fig. 3.6 Stress-strain diagram for reinforcing steel

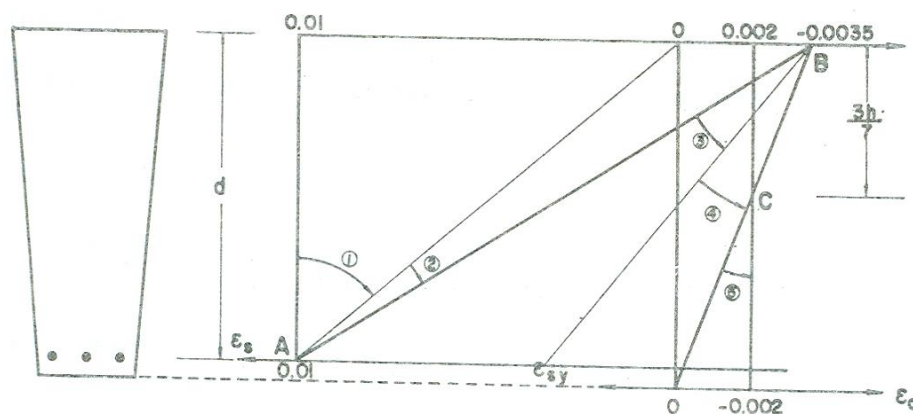


Fig. 3.7 Strain diagram in the Ultimate Limit State

Negative Moment Redistribution in Continuous Beams

In ductile members, plastic hinge regions are formed at the locations of maximum moments and cause a shift in the elastic moment diagram. The result is a reduction in the negative moment and the corresponding increase in the positive moment. Codes, including EBCS 2, permit redistribution of negative moment depending on the rotational capacity of the member. Accordingly, as per EBCS 2:

- Moments obtained from a linear analysis may be reduced by multiplying by the following reduction coefficient δ provided that the moments are increased in other sections in order to maintain equilibrium
- For continuous beams and for beams in rigid jointed braced frames with span /effective depth ratio not greater than 20;

$$\delta = 0.44 + 1.25 \frac{x}{d}$$

The neutral axis depth, x , is calculated at the ultimate limit state and the term x/d refers to the section where the moment is reduced.

- For other continuous beams and rigid jointed braced frames;

$$\delta \geq 0.75$$

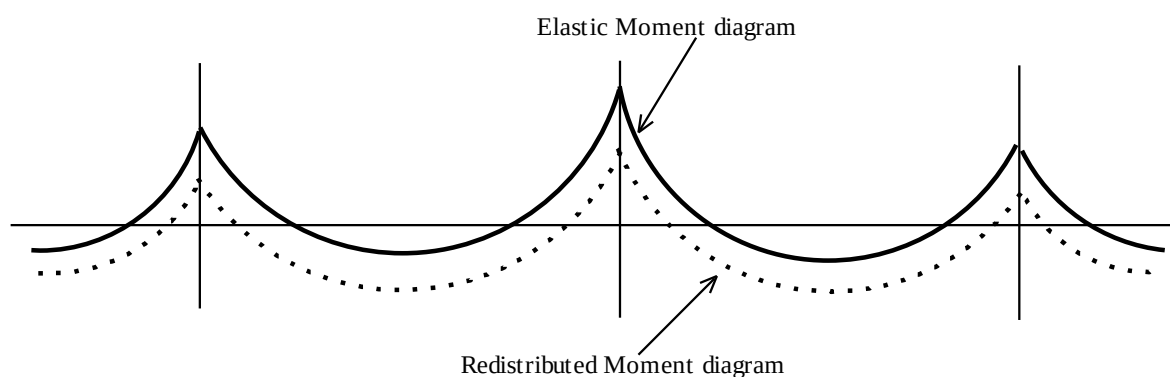


Fig. 3.8 Negative moment redistribution in continuous beam

Analysis of rectangular section for flexure Derivation of design Formula for ULS of flexure

The calculation of the ultimate resistance of members for flexure and axial loads shall be based on the following assumptions.

- ✓ Plane sections remain plane
- ✓ The reinforcement is subjected to the same variations in strain as the adjacent concrete
- ✓ The tensile strength of the concrete is neglected
- ✓ The maximum compressive strain in the concrete is taken to be :
0.0035 bending (simple or compound) & 0.002 in axial compression
- ✓ The maximum tensile strain in the reinforcement is taken to be 0.01

The stress strain relationships to derive expressions for flexural strength of RC members are shown below as given in EBCS 2.

Singly reinforced rectangular sections

Consider the stress & strain distribution for a rectangular cross section of singly reinforced concrete beam shown below.

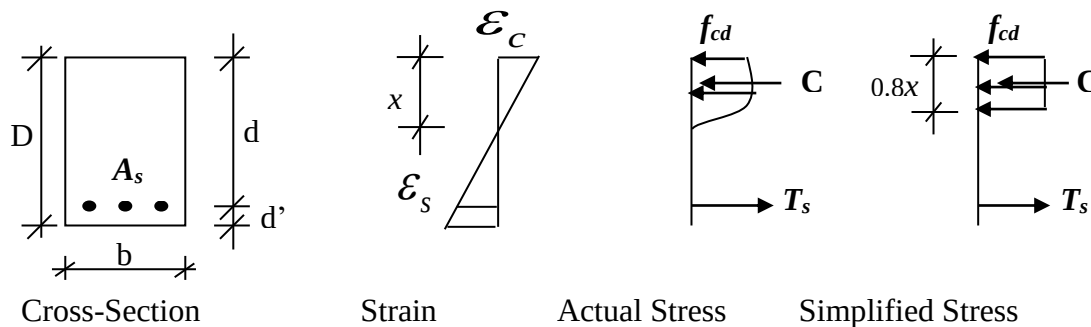
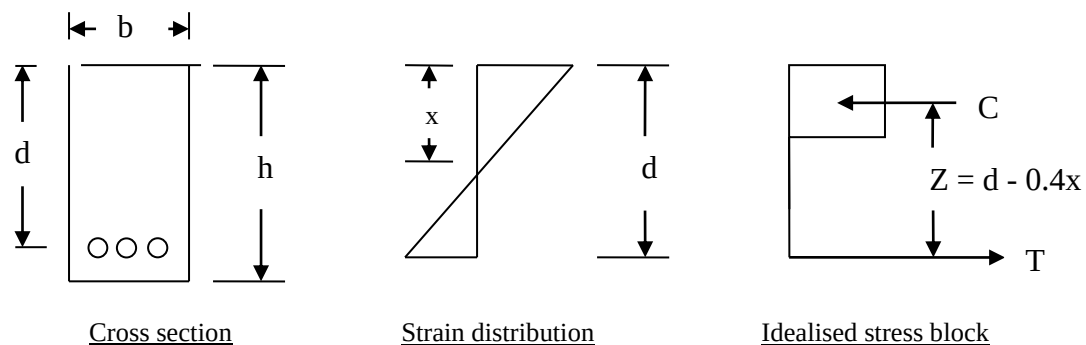


Fig.3.1. Simplified rectangular stress block shown in the figure may be used for the analysis



$$T = A_s f_s$$

$$C = 0.8x b f_{cd}$$

$$M_u = Tz = Cz$$

$$M_u = A_s f_s (d - 0.4x) = 0.8x b f_{cd} (d - 0.4x)$$

Failure Modes of Flexural Members

Depending on the amount of reinforcing steel in a beam, flexural failures may occur in three different ways: **Tension, Compression and balanced Failure**

Three types of sections are possible such as under reinforced section, over reinforced section and balanced section.

Traditional definitions

Tension failure: Under-reinforced section

If the steel content of the section is small, the steel will reach the yield strength f_{yd} before the concrete reaches its maximum capacity. *Such a beam is said to be under reinforced*

If steel's design stress f_{yd} is reaching before the concrete reaches its maximum strain $\epsilon_{cu} = 0.0035$ the failure is tension failure.

From equilibrium, $C = T$, $0.8x b f_{cd} = A_s f_{yd}$

$$x = \frac{A_s f_{yd}}{0.8b f_{cd}}$$

Ultimate moment of resistance

$$M_u = A_s f_{yd} \left[d - 0.4 \frac{A_s f_{yd}}{0.8 b f_{cd}} \right] = \frac{A_s}{b d} f_{yd} b d \left[d - \frac{0.4 A_s f_{yd} d}{0.8 b d f_{cd}} \right]; \quad M_u = \rho f_{yd} b d^2 \left[1 - \frac{0.5 \rho f_{yd}}{f_{cd}} \right]$$

From the above equation for M_u , expression for steel ratio ρ is deduced:

$$\rho = \left[1 - \sqrt{1 - \frac{2 M_u}{b d^2 f_{cd}}} \right] \frac{f_{cd}}{f_{yd}}$$

Compression failure: Over reinforced section

If the steel content of the section is large, the concrete may reach its maximum capacity before the steel yields. Such a beam is said to be *over reinforced*. If A_s is large, concrete fails under compression prior to steel reaching its design yield value. The section is **over-reinforced**. Strength is based on concrete's failure $f_s < f_{yd}$. From similar triangles in strain distribution diagram,

$$\frac{\varepsilon_s}{d-x} = \frac{0.0035}{x}; \quad \varepsilon_s = 0.0035 \frac{d-x}{x}; \quad f_s = \varepsilon_s E_s; \quad f_s = 0.0035 \frac{(d-x)}{x} E_s;$$

From equilibrium, $C = T$, $0.8x b f_{cd} = A_s f_s$; $0.8x b f_{cd} = A_s \times 0.0035 \frac{d-x}{x} E_s$

$$\left[\frac{0.8x b f_{cd}}{0.0035 A_s E_s} \right] x + x - d = 0; \quad \left[\frac{0.8x b f_{cd}}{0.0035 \rho b d E_s} \right] x + x - d = 0$$

$$\frac{0.8 f_{cd}}{0.0035 E_s \rho} x^2 + x d - d^2 = 0 \quad \text{'x' could be obtained from the above equation}$$

The moment capacity can be written as, $M_u = 0.8x b f_{cd} (d - 0.4x)$

Balanced failure: Balanced section

In a balanced failure, steel reaches f_{yd} and concrete reaches a strain of 0.0035 simultaneously.

$$\frac{\varepsilon_y}{\varepsilon_{cu}} = \frac{d-x_b}{x_b} = \frac{f_{yd}/E_s}{0.0035}; \quad x_b f_{yd} = d - x_b (E_s \times 0.0035) = d E_s (0.0035) - x_b E_s (0.0035)$$

$$x_b(0.0035 E_s + f_{yd}) = 0.0035 E_s d; \quad x_b = \frac{0.0035 E_s d}{(0.0035 E_s + f_{yd})}$$

From equilibrium, $C = T$, $0.8x_b b f_{cd} = A_s f_{yd} = \rho b b d f_{yd}$;

$$\rho_b = \frac{0.8x_b b f_{cd}}{b d f_{yd}} = \frac{0.8x_b f_{cd}}{d f_{yd}}$$

$$\text{Substituting for } x_b, \quad \rho_b = \frac{0.8 f_{cd}}{f_{yd}} \left[\frac{0.0035 E_s}{0.0035 E_s + f_{yd}} \right]$$

If $\rho < \rho_b$; $x < x_b$ -----The section is under-reinforced.

If $\rho > \rho_b$; $x > x_b$ -----The section is over-reinforced.

Control over depth of neutral axis

To ensure that all beams have the desirable characteristics of visible warning, if failure is sudden as well as reasonable ductility at failure, it is recommended that depth of neutral axis be limited.

In EBCS 2, article 3.7.9, $\frac{x}{d} \leq 0.8(\delta - 0.44)$, where δ = % moment redistribution,

= Moment after redistribution

Original moment

When no moment is redistributed, $\delta = 1$.

In such a case, $\frac{x}{d} = 0.45$, or, $x = 0.45d$

$$M_u = 0.8x b f_{cd} (d - 0.4x) = 0.8(0.45d) b f_{cd} [d - 0.4(0.45d)]$$

$$M_u = 0.2952 f_{cd} b d^2.$$

In accordance with LSD method, at ULS of collapse:-

$$\bullet \epsilon_c \text{ approaches } \epsilon_{cu} = 0.0035$$

$$\bullet \text{ The reinforcing steel shall yield first } \left(\epsilon_{yd} = \frac{f_{yd}}{E_s} \right)$$

$\Rightarrow$ Ductility is ensured by means of under reinforcement.

- At balanced failure simultaneous failure of the two

Materials (Concrete & Steel) occurs.

Let x_b be the depth to the NA at balanced failure. From the strain relation,

$$\frac{x_b}{\epsilon_{cu}} = \frac{d - x_b}{\epsilon_{yd}} \Rightarrow x_b = \frac{\epsilon_{cu} * d}{\epsilon_{cu} + \epsilon_{yd}}$$

- If $x < x_b \Rightarrow$ Steel yields first
- If $x > x_b \Rightarrow$ Crushing of concrete takes place first.

$$\Sigma F_H = 0 \Rightarrow T_s = C_C \Rightarrow A_s f_{yd} = 0.8 x_b b f_{cd}$$

Substituting for x_b and simplifying,

$$\rho_b = \frac{0.8 * \epsilon_{cu} * f_{cd}}{\epsilon_{cu} + \epsilon_{yd} f_{yd}} \quad (\text{a steel ratio for balanced case})$$

However, for ductility purpose the steel ratio ρ may range b/n $0.75 \rho_b$ to $0.9 \rho_b$, and in some cases as low as $0.5 \rho_b$.

Rewriting the force equilibrium

$$b f_{cd} = A_s f_{yd} \Rightarrow b * 0.8x f_{cd} = \rho b d f_{yd}$$

$$\frac{x}{d} = \frac{\rho * f_{yd}}{0.8 * f_{cd}} = \rho m, \text{ Where } m = \frac{f_{yd}}{0.8 * f_{cd}}$$

$$\Sigma M_c = 0 \Rightarrow M_d = A_s f_{yd} (d - 0.4x)$$

Substituting the value of x and simplifying

$$M_d = 0.8 b d^2 f_{cd} \rho m (1 - 0.4 \rho m)$$

When the above equation is solved for ρ ,
$$\rho = 0.5 \left\{ c_1 - \sqrt{c_1^2 - \frac{4M_d}{b d^2 c_2}} \right\} \leq \rho_{\max}$$

where $c_1 = \frac{2.5}{m}$, $c_2 = 0.32 m^2 f_{cd}$, $m = \frac{f_{yd}}{0.8 * f_{cd}}$

The section capacity for single reinforcement case may be computed from $\Sigma M_t = 0$

$$\Rightarrow M_u = 0.8 b x f_{cd} (d - 0.4x) \quad x = \rho_{\max} m d$$

$$= 0.8 b d^2 f_{cd} \rho_{\max} m (1 - 0.4 \rho_{\max} m)$$

For ductility requirements, $\rho_{\max} = 0.75 * \rho_b = \frac{0.6 * \varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} * \frac{f_{cd}}{f_{yd}}$

Simplified Rectangular Stress Block

The actual distribution of the compressive force in a section has the form of a rising parabola as shown in fig. The compressive stress-strain curve for concrete may be assumed to be rectangular trapezoidal, parabolic or any other shape, which is easier for computation, provided that it adequately predicts the test results. Therefore; as a simplification, EBCS 2 recommends (ACI also) the use of the equivalent rectangular concrete stress distribution for sections which are partly in tension (beams or columns with large eccentricity), as shown below.

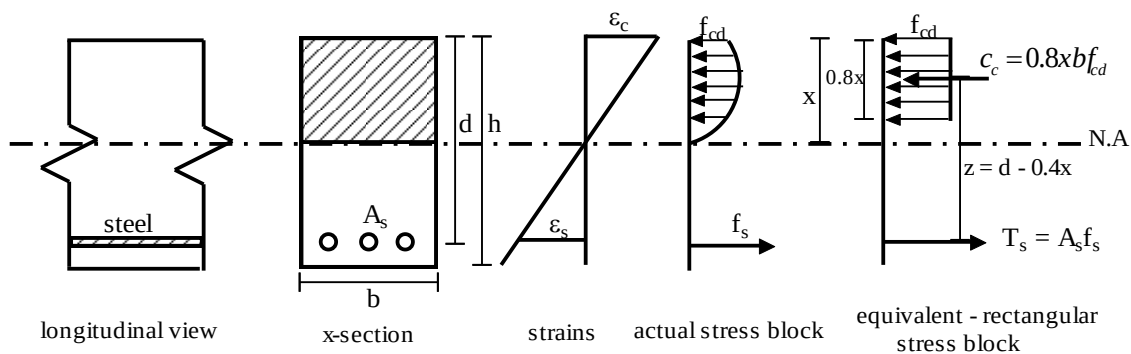


Fig. 3.10 Strain and stress distribution across beam depth/equivalent rectangular stress block

In order to define the effect of concrete compression stresses, it is not really necessary to know the exact shape of the concrete stress distribution. What is necessary is to know for a given distance of neutral axis:

- (1) the total resultant compression force C in the concrete and
- (2) its vertical location i.e. its distance from the outer compression fiber.

Therefore; EBCS 2 recommends (ACI also) the use of the equivalent rectangular concrete stress distribution for sections which are partly in tension (beams or columns with large eccentricity), as shown below.

The equivalent rectangular stress block will be used in all manual calculations.

Note: Two requirements are satisfied though out the analysis and design of reinforced concrete beams and columns (stress and strain compatibility and equilibrium)

- **Stress and strain compatibility:** The stress at any point in a member must correspond to the strain at that point. Except for short deep beams, the distribution of strain over the depth of the member must be linear to satisfy the earlier assumptions 1 and 2.
- **Equilibrium:** The internal forces must balance the external load effects.

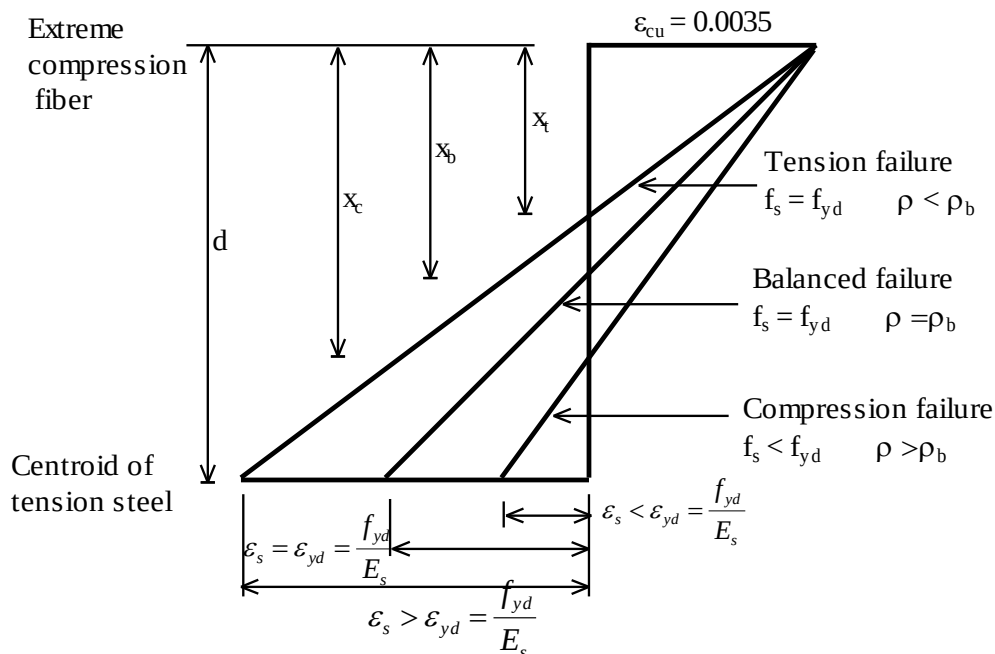


Fig. 3.11 Strain profiles at the flexural strength of a section

Balanced Failure (balanced Reinforcement): $\rho = \rho_b$, $X = X_b$ and $\epsilon_s = \epsilon_{yd} = f_{yd}/E_s$

From the strain line at balanced failure:

$$x_b = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} d \quad (3.3a)$$

Force equation:

$$\Sigma F_H = 0 \Rightarrow A_s f_{yd} = 0.8x_b b f_{cd} \quad (3.3b)$$

Substituting for x_b from eqn (3.3a) in to eqn(3.3b) and simplifying:

$$\begin{aligned} A_s f_{yd} &= 0.8 \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} d b f_{cd} \\ \Rightarrow \frac{A_s}{b d} &= 0.8 \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} \frac{f_{cd}}{f_{yd}} \\ \Rightarrow \rho = \rho_b &= 0.8 \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} \frac{f_{cd}}{f_{yd}} \end{aligned} \quad (3.3c)$$

Moment equation:

The moment equation about Cc results in:

$$M_{rd} = \rho_b d f_{yd} (d - 0.4x_b) \quad (3.4)$$

Tension Failure (under-reinforced Section) : $\rho < \rho_b$, $X < X_b$ and $\varepsilon_s > \varepsilon_{yd} = f_{yd}/E_s$

Force equation:

$$\Sigma F_H = 0 \Rightarrow A_s f_{yd} = 0.8x b f_{cd} \quad (3.5a)$$

$$\begin{aligned} \Rightarrow \rho b d f_{yd} &= 0.8x b f_{cd} \\ \Rightarrow \frac{x}{d} &= \rho \frac{f_{yd}}{0.8 f_{cd}} \\ \Rightarrow x &= \rho d \frac{f_{yd}}{0.8 f_{cd}} \end{aligned} \quad (3.5b)$$

Moment equation:

The moment equation about Cc results in:

$$M_{rd} = A_s f_{yd} (d - 0.4x) \quad (3.6a)$$

Substituting the value of x from eqn (3.5b) and simplifying:

$$M_{rd} = 0.8 b d^2 f_{yd} \rho m (1 - 0.4 \rho m) \quad (3.6c)$$

$$\text{Where } m = \frac{f_{yd}}{0.8 f_{cd}}$$

The singly reinforced section capacity may be obtained from:

$$M_{rd} = 0.8 x b f_{cd} (d - 0.4 x) \quad (3.6d)$$

$$\text{Where } x = x_{\max} = \rho_{\max} \frac{f_{yd}}{0.8 f_{cd}} d = \rho_{\max} m d$$

Compression Failure (over-reinforced Section): $\rho > \rho_b$, $X > X_b$ and $\varepsilon_s < \varepsilon_{yd} = f_{yd}/E_s$

From the strain line at compression failure:

$$\frac{\varepsilon_s}{0.0035_b} = \frac{d - x}{x}$$

$$\Rightarrow \varepsilon_s = 0.0035 \frac{d - x}{x}$$

$$\text{Therefore; } f_s = \varepsilon_s E_s = 0.0035 \frac{d - x}{x} E_s$$

Force equation:

$$\Sigma F_H = 0 \Rightarrow A_s f_s = 0.8 x b f_{cd} \quad (3.7a)$$

$$\Rightarrow \rho b d (0.0035 \frac{d - x}{x} E_s) = 0.8 x b f_{cd}$$

$$\Rightarrow \left(\frac{f_{cd}}{0.0035 E_s \rho} \right) x^2 + d x - 0.8 d^2 = 0 \quad (3.7b)$$

Therefore the above equation may be solved to obtain X.

Moment equation:

The moment equation about T_s results in:

$$M_{rd} = 0.8 x b f_{cd} (d - 0.4 x) \quad (3.8)$$

(Analysis example)

Design of Singly Reinforced Rectangular Beams for Flexure

Generally, ductility is a design requirement in reinforced concrete structures to ensure that a brittle failure will not occur. In EBCS-2, ductility is ensured by limiting the depth of the neutral axis k_x (used to determine the maximum carrying capacity of a singly reinforced beam) to specified values depending on the percent plastic moment redistribution as:

$$K_x \leq 0.8 (\delta - 0.44)$$

Where: δ = reduction coefficient which multiplies the elastic moment (see section 3.4)

Accordingly,

$K_x = 0.448$ for condition of no redistribution and

$K_x = 0.208$ for a recommended max. of 30 %

Other codes of practice such as the ACI ensure ductility by limiting the reinforcement ratio, ρ to a value below some specified value which is a function of the balanced reinforcement ratio, ρ_{bal} .

$$\rho \leq 0.75\rho_{bal}$$

Parabolic-rectangular Stress block

Reconsider the two equilibrium equations for a rectangular section using the expressions developed for a parabolic-rectangular stress block as follow:

$$A_s f_{yd} = \alpha_c f_{cd} b d \quad (3.9a)$$

$$\alpha_c f_{cd} b d^2 k_z = M_{sd} \quad (3.9b)$$

The number of unknowns in equations (3.9a) and (3.9b) are seven which are greater than the number of available equilibrium equations (i.e. two), therefore the designer should make decision on:

- 1) material strengths i.e. f_{cd} , f_{yd}
- 2) Dimensions of cross sections b and d . The minimum thickness for deflection specified in the code can be used as a guide and the ratio b/d varies between 0.3 and 0.6 in usual practice.

So that α_c , A_s and k_z are left as unknowns where α_c , and k_z could both be expressed in terms of K_x or x . Thus the two equations are sufficient to uniquely determine the remaining two unknowns A_s and K_x .

Equivalent-rectangular stress block

Reconsider the two equilibrium equations for a rectangular section using the expressions developed for equivalent-rectangular stress block as follow:

$$A_s f_{yd} = 0.8x b f_{cd} \quad (3.10a)$$

$$0.8x b f_{cd} (d - 0.4x) = M_{sd} \quad (3.10b)$$

The number of unknowns in equations (3.10a) and (3.10b) are six which are greater than the number of available equilibrium equations (i.e. two), therefore the designer should make decision on:

- 3) *material strengths i.e. f_{cd} , f_{yd}*
- 4) *Dimensions of cross sections b and d . The minimum thickness for deflection specified in the code can be used as a guide and the ratio b/d varies between 0.3 and 0.6 in usual practice.*

So that α_c , A_s and k_x are left as unknowns where α_c and k_x could both be expressed in terms of k_x or x . Thus the two equations are sufficient to uniquely determine the remaining two unknowns A_s and K_x .

Equation 3.6c can be re-written and simplified to give the reinforcement ratio as:

$$\rho = \frac{1}{2} \left[c_1 \pm \sqrt{c_1^2 - \frac{4M}{bd^2c_2}} \right] \quad (3.10c)$$

$$\text{Where: } c_1 = \frac{2.5}{m}, c_2 = 0.32m^2 f_{cd}, m = \frac{f_{yd}}{0.8 f_{cd}}$$

Then the area of steel required may be computed from:

$$A_s = \rho b d \quad (3.10d)$$

Congestion of reinforcements at the supports could be reduced by reducing the beam support moments. But redistribution needs more ductile elements. Most design do not use moment redistribution i.e. $\delta = M_p/M_E = 1.0$. Reducing by 20% implies $\delta = 0.8$ which in turn restrict the N.A depth K_x . Limitation in K_x does affect ductility.

Flexural Reinforcement for Beams as per EBCS 2:

- The geometrical main reinforcement ratio ρ at any section of a beam where positive reinforcement is required by analysis shall not be less than that given by: **(to control cracking of concrete)**

$$\rho_{\min} = \frac{0.6}{f_{yk}} \quad \text{where } f_{yk} \text{ is in MPa}$$

- The maximum reinforcement ratio $\rho_{\max}$ for either tensile or compressive reinforcement shall be 0.04. **(the tension reinforcement in a beam shall not exceed**

4% of the gross sectional area of the concrete to ensure proper placing and compacting of concrete around the reinforcement)

Effective Span of Beams, EBCS 2 1995

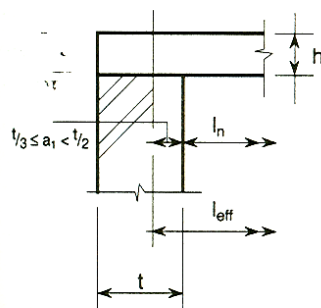
- The effective span or length of a simply supported beam may be taken as the lesser of:
 - a) The distance between the centers of supports
 - b) The clear distance between supports plus the effective depth d .
- The effective length of a continuous element shall normally be taken as the distance between the centerline of the supports.
- For a cantilever the effective span is taken to be its length, measured from:
 - a) The face of the support, for an isolated, fixed-ended cantilever
 - b) The centre line of the support for a cantilever which forms the end of continuous beam.

Effective Span of Beams, EC 2

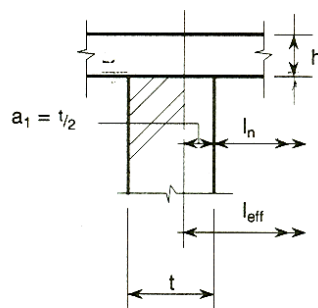
The effective span (l_{eff}) may be calculated as follows:

$$L_{eff} = l_n + a_1 + a_2$$

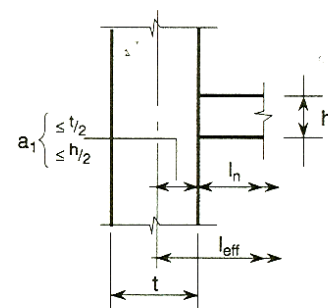
Where l_n is the clear distance between the faces of the supports and a_1 and a_2 are as in the figure below.



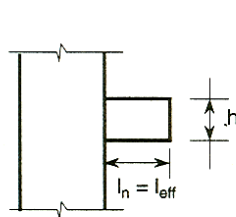
(a) non continuous members



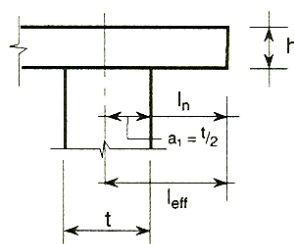
(b) continuous members



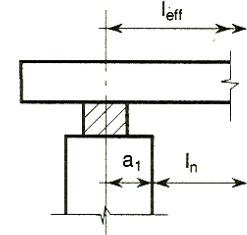
(c) supports fully restrained



(d) isolated cantilever



(e) continuous cantilever



(f) bearing provided

Fig.3.12 Geometrical data for overall analysis

Doubly Reinforced Rectangular Beams

Doubly reinforced sections contain reinforcement both at the tension and at the compression face. Compression steel may be required in design for the following reasons.

- When either architectural limitation restrict the beam web depth at the mid span, or the mid span section dimensions are not adequate to carry the support negative moment even when the tensile steel at the support is sufficiently increased. In such cases about one-third to one-half of the bottom bars at mid span are extended and well anchored at the supports to act as compression reinforcement.
- To increase the ductility of the section at flexural strength. It is evident that if compression steel is in the section, the neutral axis depth will be smaller as the internal compressive force is shared by the concrete and the compression steel.
- To reduce deflection of beams at service loads
- To support the shear reinforcement (stirrups)

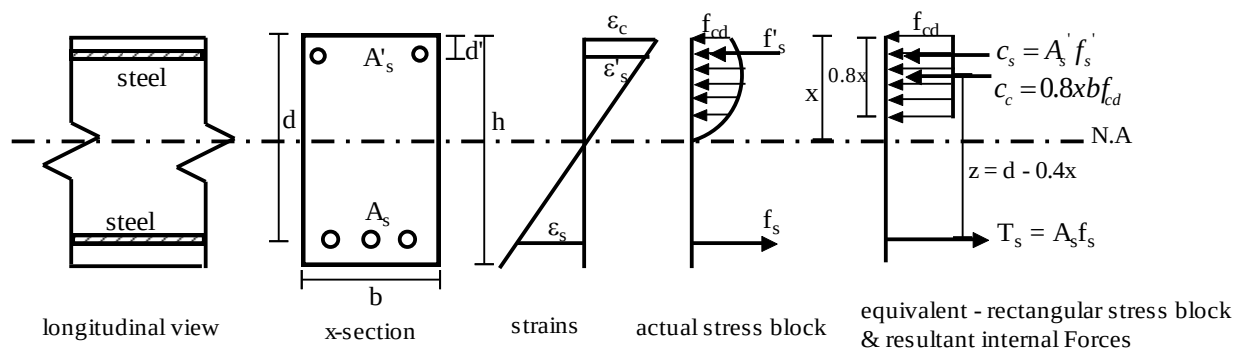


Fig. 3.15 doubly reinforced beam design

In the analysis or design of beams with compression reinforcement A_s' , the section is theoretically split in to two parts, as shown in Fig.

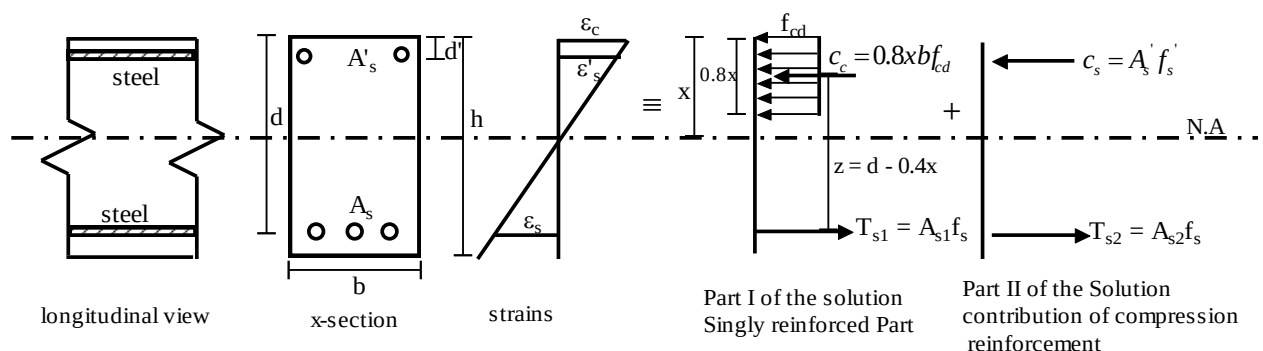


Fig. 3.16 doubly reinforced beam design (singly reinforced part plus contribution of compression reinforcement)

The two parts of the solution comprise:

- (1) The singly reinforced part involving the equivalent rectangular stress block with the area of tension reinforcement being $(A_s - A_s')$; and
- (2) The two areas of equivalent steel A_s' at both the tension and compression side to form the couple T_{s2} and C_s as the second part of the solution. It can be seen from Fig. that the total resistance moment $M_{rd} = M_{d1} + M_{d2}$, that is, the summation of the moments for Parts 1 and 2 of the solution.

The analysis of such section is best carried out by assuming the compression reinforcement bars to be yielded and check for compatibility of strain to verify whether the compression steel yielded or not and use the corresponding stress in the steel for calculating the forces and moments.

Let M_d be the total design bending moment which this section sustains. Then

$$M_d = M_{d1} + M_{d2}$$

$$A_s = A_{s1} + A_{s2}$$

Where M_{d1} = is the bending moment carried by the concrete and the corresponding steel which may be obtained using case of singly reinforced section.

$$M_{d1} = 0.8bd^2f_{cd}p_1m(1-0.4p_1m) \text{ and } A_{s1} = p_1bd$$

If the process involved is design: $p_1 = p_{max} = 0.75p_b$

If the process involved is analysis $p_1 = (A_s - A_s') / bd < p_{max}$

It can be seen from Fig. that the total resisting moment $M_{rd,t} = M_{rd1} + M_{rd2}$, that is, the summation of the moments for parts 1 and 2 of the solution.

From part I:

Force equation

$$T_1 = C_1$$

$$A_{s1}f_{yd} = (A_s - A_s')f_{yd} = 0.8xf_{cd}b$$

$$x = \frac{(A_s - A_s')f_{yd}}{0.8f_{cd}b}$$

Moment equation

Taking the moment about the centroid of the compression zone:

$$M_{d1} = A_{s1}f_{yd}(d - 0.4x) = (A_s - A_s')f_{yd}(d - 0.4x)$$

$$\text{Where } x = \frac{(A_s - A_s')f_{yd}}{0.8f_{cd}b}$$

From part II:

Force equation

$$A_s' = A_{s2} = (A_s - A_{s1})$$

$$T_2 = C_2 = A_{s2}f_{yd}$$

Moment equation

Taking the moment about the tension reinforcement:

$$M_{d2} = A_{s2}f_{yd}(d - d')$$

Adding the moments for parts 1 and 2 yields:

$$\begin{aligned} M_{rd} &= M_{d1} + M_{d2} \\ &= (A_s - A_s')f_{yd}(d - 0.4x) + A_s'f_{yd}(d - d') \end{aligned}$$

This equation is valid if A_s' yields. Otherwise the beam has to be treated as a singly reinforced beam neglecting the compression steel or one has to find the actual stress f_s' in the compression reinforcement A_s' and use the actual force in the moment equilibrium equation.

Beams are built with both tension and compression reinforcement for any of the following reasons:

- **To enhance strength (increase M_{rd})**
- **To increase ductility.**
- **To reduce sustained deflection (creep).**
- **To support the stirrups**

Strain Compatibility Check

It is always necessary to verify that the strain across the depth follow a linear distribution at the strength design levels. We recall that the minimum depth for a singly reinforced section corresponds to $K_x = 0.448$ in order that the cross section possesses the minimum ductility requirement. In this condition $\epsilon_{s1} = 4.3\text{‰}$ and $\epsilon_{cm} = 3.5\text{‰}$. once the strain is verified to be higher than the yield strain, the moment capacity can be computed using the previous equations. If the design action effect is higher than that corresponding to d_{min} , that is , $\mu_{sd,s} > 0.295$, and if it is not possible to increase the depth d , the capacity can be increased by using compression reinforcement, whereby the minimum ductility is maintained.

a) Analysis of doubly reinforced beams

The general procedures are:

- **Divide the x- section in to two hypothetical beams as shown above.**

- Assume the compression reinforcement to be yielded
- Determine the tension steel which balances compressive force due to the compression reinforcement
- Determine the moment capacity of the compression reinforcement and the corresponding tension reinforcement.
- Determine the depth of the stress block by equating the compressive force in the concrete with the tensile force as a result of the remaining tension reinforcement.
- Determine the additional moment capacity.
- Determine the total moment capacity by super imposing the two values.
- Check whether the assumption was correct or not
- If the assumption was not correct, use the force and moment equilibrium equations considering the actual stresses in the compression steels.

b) Design of doubly reinforced beams

Similar to the analysis, the general procedures for the design of doubly reinforced sections are:

- Check whether double reinforcement is required or not
- Determine the moment capacity of the concrete without compression reinforcement
- Determine the tension steel which couple the compressive force
- Determine the extra moment to be resisted by the compression steel and the additional tension steel.
- Determine the additional tension steel to carry this moment
- Calculate the compression reinforcement assuming that it has yielded.
- Check whether the compression reinforcement has yielded or not using compatibility of strain

(Analysis and design Example)

3.1.6. Flanged Beams (T and L-beams)

When concrete roofs or floor slabs are cast monolithically with supporting beams, T or L are created as shown in fig. below. Forms are built for beam soffits and sides and for the under side of slabs, and the entire construction is poured at once, from the bottom of deepest beam to the top of the slab. It is evident, therefore, that a part of the slab will act with the upper part of the beam to resist longitudinal compression. The resulting beam cross-section is T or L-shaped rather than rectangular.

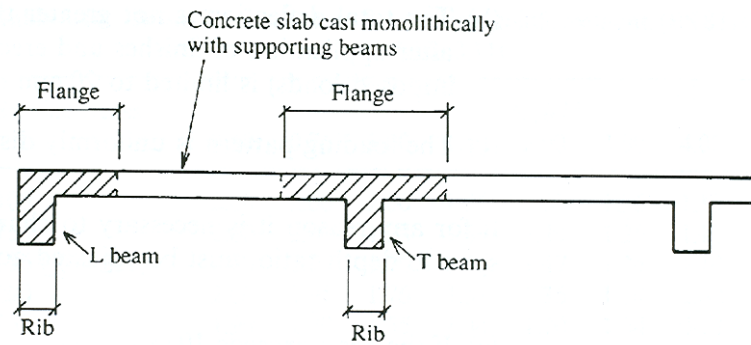


Fig.3.17 Flanged beams

Effective flange width

When the spacing between the beams is large, it is evident that simple bending theory does not strictly apply because the longitudinal compressive stress in the flange will vary with distance from the beam web, the flange being more highly stressed over the web than in the extremities (see Fig.)

This variation in flange compressive stress occurs because of shear deformations in the flange (shear lag), which reduces the longitudinal compressive strain with distance from the web.

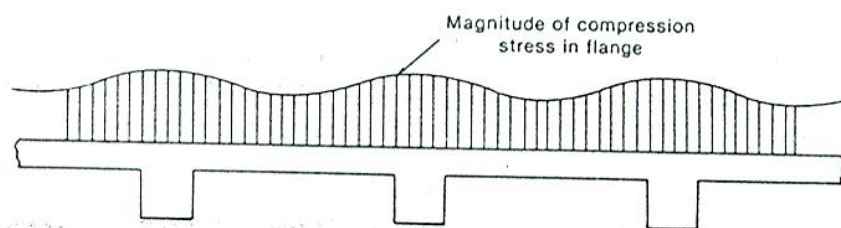


Fig. 3.18 Distribution of maximum flexural compressive strength.

In design, to take the variation of compressive stress across the flange into account, it is convenient to use an effective width of flange that may be smaller than the actual width but is considered to be uniformly stressed (see Fig.)

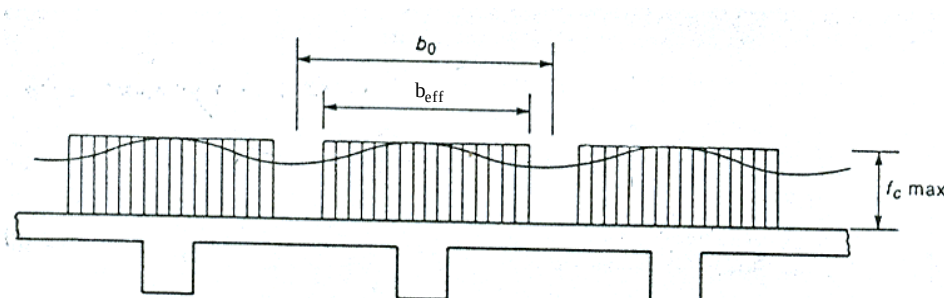


Fig. 3.19 Flexural compressive stress distribution assumed in design

Effective width, EBSC 2, 1995

According to EBCS – 2 (Art. 3.7.8), the effective width b_f shall not exceed the lesser of :

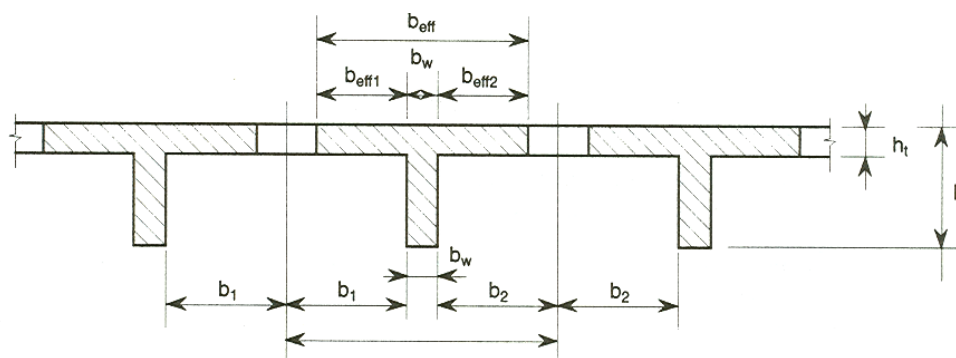
For T beams:

- thickness of the web plus one- fifth of the effective span or
- the actual width of the top slab (extending b/n the centers of the adjacent spans)

For L beams:

- thickness of the web plus one- tenth of the effective span or
- Thickness of the web plus half the clear distance to the adjacent beam.

For analysis when a great accuracy is not required, for example, continuous beams in buildings a constant effective width (b_{eff}) may be assumed over the whole span.



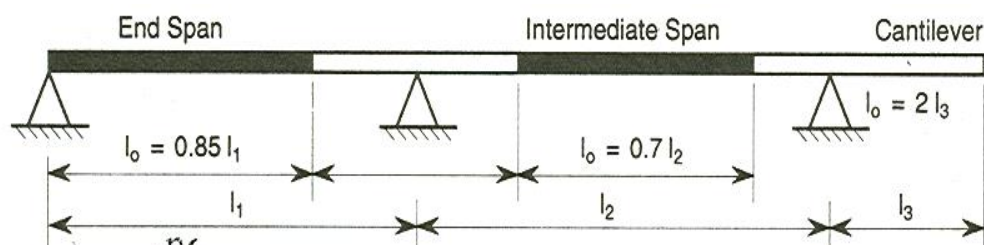
The effective width for a symmetrical T- beam may be taken as :

$$b_{eff} = b_w + 1/5 l_o < b$$

And for an edge beam, that is with floor on one side only

$$b_{eff} = b_w + 1/10 l_o < b_i + b_w \quad (i = 1 \text{ or } 2)$$

The distance l_o between points of zero moment may be obtained from the figure below for typical cases:



The following conditions should be satisfied

- The length of the cantilever should be less than half the adjacent span
- The ratio of adjacent spans should lie between 1 and 1.5

Analysis and Design of Flanged Beams

The basic principle used for analysis and design of rectangular beams are also valid for the flanged beams. The major difference between the rectangular and flanged sections is in the

calculation of compressive force C_c . Depending on the depth of the neutral axis, X , the following cases can be identified.

a) Depth of neutral axis X less than flange thickness h_f , see Fig.

This case can be treated similarly to the standard rectangular section provided that the depth $0.8x$ of the equivalent rectangular block is less than the flange thickness. The flange width b_f of the compression side should be used as the beam width in the analysis or design.

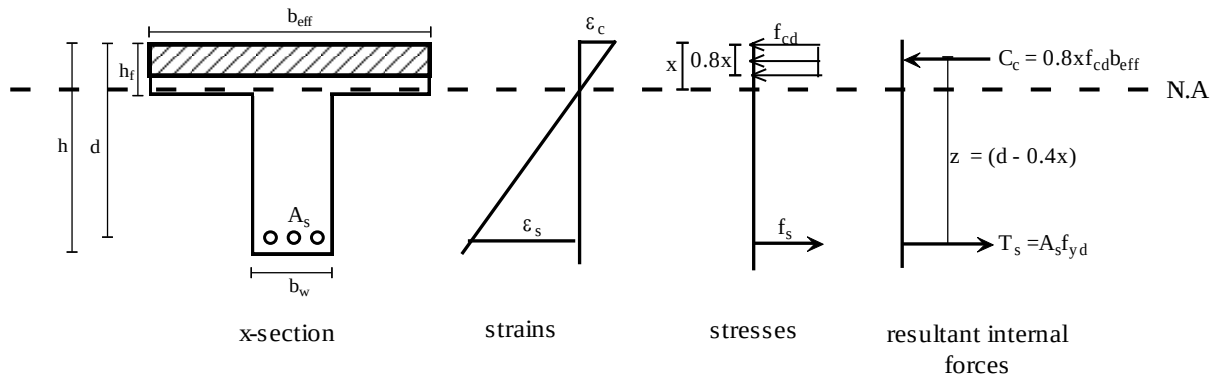


Fig.3.20 T- beam section with neutral axis within the flange

Force equilibrium gives:

$$A_{s1}f_{yd} = 0.8xf_{cd}b_f$$

$$\Rightarrow x = \frac{A_{s1}f_{yd}}{0.8f_{cd}b_f}$$

Moment equilibrium gives:

$$M_{rd} = A_{s1}f_{yd}(d - 0.4x)$$

$$\text{Where } x = \frac{A_{s1}f_{yd}}{0.8f_{cd}b_f}$$

3.2. Design Aids/Design Using Chart

In the general design chart no.1 (EBSC 2, 1995), all values necessary for design are entered as a function of the relative moment about the center of the tension steel. This diagram can be used for any concrete or steel grade. In the zones of negative steel strains (**sections entirely under compression**), however no accurate reading is possible. For that zone the use of interaction diagrams can be used.

The following characteristic values are entered as a function of the relative moment:

$$k_x = \frac{x}{d} \text{ the relative neutral axis depth}$$

$k_z = \frac{z}{d}$ the relative lever arm b/n the internal forces

$\alpha_c = \frac{c_c}{f_{cd}bd}$ the relative compression force in the concrete in the ultimate limit state

ϵ_c = compressive strain in outer most concrete fiber

ϵ_{s1} = strain intension reinforcement

ϵ_{s2} = Strain in compression reinforcement

The upper limits of the design values of the ultimate relative moment capacities (without compression reinforcement) about tension steel, for 0%, 10 %, 20%, and 30% moment redistribution are shown by the broken vertical lines $\mu_{u,s}^* = 0.295, 0.252, 0.205$, and 0.14 respectively. Compliance with these upper limits implies compliance with the upper limit specification for the relative neutral axis depth, K_x , thus ensuring ductile response of the cross section. For the cases that $\mu_{sd,s} > \mu_{u,s}^*$, ductile behavior can be achieved by providing compression reinforcement.

The area of reinforcement required is determined from the following equations:

If $\mu_{sd,s} \leq \mu_{u,s}^*$, compression reinforcement is not required and

$$A_{s1} = \frac{M_{sd,s}}{zf_{yd}} + \frac{N_{sd}}{f_{yd}}$$

If $\mu_{sd,s} > \mu_{u,s}^*$, compression reinforcement is required and

$$A_{s2} = \frac{M_{sd,s} - M_{u,s}^*}{(d - d_2)\sigma_{s2}}$$

$$A_{s1} = \frac{M_{u,s}^*}{zf_{yd}} + \frac{M_{sd,s} - M_{u,s}^*}{(d - d_2)f_{yd}} + \frac{N_{sd}}{f_{yd}}$$

- Starting from a strain profile in ULS: α_c , k_x , k_z , μ_{Rd} etc. are determined.
- In design the chart is entered by equating $\mu_{sd} = \mu_{Rd}$, then k_z is read and A_{s1} is determined from:

$$A_{s1} = M_{sd} / (k_z d f_{yd})$$

- Another advantage is the possibility of handling axial forces in addition to bending. The horizontal relative moment axis is designated as $\mu_{Rd,s}$ for this reason should an axial force be present, then it is shifted to the location of tension reinforcement and the associated moment is added to M_{sd} to give $M_{sd,s}$.

$M_{sd,s} = M_{sd} - N_{sd} y_e$ (N_{sd} is +ve when tension) &

Design Using Tables (K_d - Method)

Procedure of computing design parameters using table involves the following and the table has the following format

K _m					K _s		
C15	C20	C25	C30	C40	S300	S400	S460
15	17	19	21	24	3.95	2.96	2.58
-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-

- Evaluate $k_m = \sqrt{\frac{M}{bd^2}}$

Where: M in KN.m, b & d in m

- Enter the table for appropriate concrete grade used
- Obtain K_s corresponding to steel grade & K_m
- Evaluate the area of steel required as :

$$A_s = K_s \frac{M_d}{d}$$

Discussion

Consider $M_d = 0.8bd^2f_{cd} \rho m$ (1-0.4pm)

Recall $\frac{x}{d} = \rho \frac{f_{yd}}{0.8f_{cd}} \Rightarrow \rho = \frac{x}{d} 0.8 \frac{f_{cd}}{f_{yd}} = \frac{k_x}{m}$ where $k_x = \frac{x}{d}$, $m = \frac{f_{yd}}{0.8f_{cd}}$

When $\rho = \frac{k_x}{m}$ is substituted in equation (3*) and simplified

$$\frac{M_d}{bd^2} = 0.8 f_{cd} k_x (1 - 0.4k_x)$$

Note that (1*) $\Rightarrow k_m = \sqrt{\frac{M_d}{bd^2}} = \sqrt{0.8 f_{cd} k_x (1 - 0.4k_x)}$ which is essentially a function of concrete grade & section property

On the other hand, $A_s = \frac{M_d}{f_{yd}(d - 0.4x)} = \frac{M_d}{d} \frac{1}{f_{yd}(1 - 0.4k_x)} = k_s \frac{M_d}{d}$

$\Rightarrow k_s = \frac{1}{f_{yd}(1 - 0.4k_x)}$ **Which implies that k_s is a function of steel grade and section property.**

Thus the following example can be solved using the table as follows

$$b/h/d = 250/730/675\text{mm}$$

$$M_d = 431.45\text{KN.m}$$

$$k_m = \sqrt{\frac{M}{bd^2}} = \sqrt{\frac{431.45}{0.25 \times 0.675^2}} = 61.55, k_d = \frac{1}{k_m} = 0.0162$$

The value of k_s corresponding to C25, steel S300 is $K_s = 4.68$

$$\text{Then } A_s = K_s \frac{M_d}{d} = 4.68 \frac{432}{0.675} = 2996\text{mm}^2$$

$$\# \phi 20 \text{ bars} = 9.5$$

Use 10 $\phi 20$ bars

(Design examples using parabolic & rectangular)

General design chart can be used.

b) Depth of neutral axis X Larger than flange thickness h_f , see Fig.

In this case, $x > h_f$, the depth of the equivalent rectangular stress block $0.8x$ could be smaller or larger than the flange thickness h_f . If x is greater than h_f and $0.8x$ is less than h_f , the beam could still be considered as a rectangular beam for design purpose. Hence the design procedure explained above is applicable to this case.

If both x and $0.8x$ are greater than h_f , the section has to be considered as a T-section. This type of T-beam can be treated in a manner similar to that for a doubly reinforced rectangular section (see Fig.).

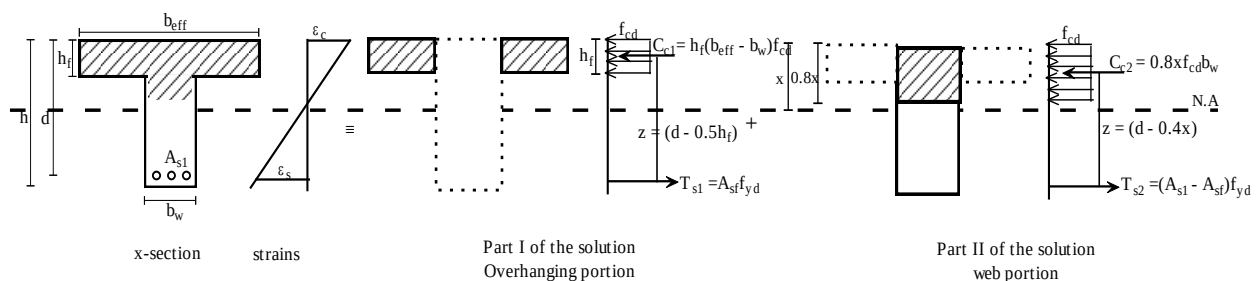


Fig.3.21 T- beam section with neutral axis in the web

As a computational device, it is convenient to divide the total tensile steel into two parts.

The first part, A_{sf} , represents the steel area which, when stressed to f_{yd} is required to balance the compressive force in the overhanging portion of the flange. Thus,

$$A_{sf} * f_{yd} = f_{cd} (b_f - b_w) h_f$$

$$\Rightarrow A_{sf} = f_{cd} * h_f (b_f - b_w) / f_{yd}$$

The partial resisting moment capacity as a result of these forces:

$$M_{ult1} = A_{sf} * f_{yd} (d - h_f/2)$$

The remaining steel area ($A_s - A_{sf}$), at a stress f_{yd} is balanced by the compression in the rectangular portion of the beam.

From force equilibrium:

$$(A_s - A_{sf})f_{yd} = f_{cd} (b_w * 0.8x)$$

From Moment equilibrium:

$$M_{ult2} = (A_s - A_{sf})f_{yd} (d - 0.4x)$$

The total resisting moment, taking moments of the rectangles about the tension steel, gives:

$$M_{rd} = 0.8xf_{cd}b_w(d-0.4x) + f_{cd}(b_f-b_w)h_f(d-0.5h_f)$$

General design chart is not applicable.

The resultant compressive force acts at the centroid of the T-shaped compressed area.

From force equilibrium

$$f_{cd}(0.8xb_w + h_f(b_f-b_w)) = A_s f_{yd}$$

$$\Rightarrow x = \frac{A_s f_{yd} - f_{cd} h_f (b_f - b_w)}{f_{cd} b_w}$$

The total moment

$$M_{rd} = 0.8xf_{cd}b_w (d-0.4x) + f_{cd}(b_f-b_w)h_f(d-0.5h_f)$$

Note:

- When the T-section is subjected to bending moment and tension is produced in the flange portion, the can be considered as a rectangular with $b = b_w$ for design purpose.
- For T-beam sections, when the flexural strength is reached, the depth to the neutral axis is generally small because of the large flange area. Therefore; a tension failure generally occurs and it is usually safe to assume in analysis that $f_s = f_{yd}$; and ϵ_{ck} when the flexural strength is reached check the validity of the assumption when the neutral axis depth is found.

Note: the problem at hand is one of the following

(i) Analysis:

As is given

Determine A_{sf}

Determine $A_s - A_{sf}$

Determine N.A depth from force equilibrium.

(ii) Design:

If the N.A. is within the flange rectangular section(use the general design chart)

If the N.A. lies in the web:

Determine A_{st} and M_{ult1}

Determine $M_{ult2} = M_{sd} - M_{ult1}$

Determine the required amount of reinforcement from the two equations.

(Unknowns are A_s and x .)

Note: For -ve bending moment T- beams are not analyzed. It is rather analyzed (designed) as rectangular beams.

(Analysis and Design Examples)

3.3. DETAILING OF FLEXURAL REINFORCEMENT.

The geometrical ratio of reinforcement ρ at any section of a beam where positive reinforcement is required by analysis shall not be less than that given by:

$$\rho_{\min} = \frac{0.6}{f_{yk}}$$

Where f_{yk} is in MPa.

(2 In T -beams and joists where the web is in tension, the ratio ρ shall be computed for this purpose it using width of web.

(3) The maximum reinforcement ratio $\rho_{\max}$ for either tensile or compressive reinforcement shall be 0.04.

Reinforcement Cover and Spacing

Cover to Reinforcements

- The concrete cover is the distance between the outermost surface of reinforcement (usually stirrups) and the nearest concrete surface.
- The thickness of cover required depends both upon the exposure conditions and on the concrete quality.
- To transmit bond forces safely, and to ensure adequate compaction, the concrete cover should never be less than:

(a) ϕ or ϕ_n ($\leq 40\text{mm}$), or

(b) $(\phi + 5\text{mm})$ or $(\phi_n + 5\text{mm})$ if $d_g > 32\text{mm}$

Where ϕ = the diameter of the bar.

ϕ_n = the equivalent diameter for a bundle.

d_g = the largest nominal aggregate size.

The minimum concrete cover to all reinforcement including links and stirrups should not be less than the appropriate values given in the following table.

Table of Minimum Cover Requirements for Concrete Members

Type of Exposure	Dry environment: Interior buildings of normal habitation or offices (Mild)	Humid environment: Interior components (e.g. laundries); exterior components; components in non-aggressive soil and/or water (Moderate)	Sea water and/or aggressive chemical environment: Components completely or partially submerged in sea water; components in saturated salt air; aggressive industrial atmospheres (Sever)
Minimum cover (mm)	15	25	50

A minimum concrete cover shall be provided in order to ensure:

- the safe transmission of bond forces
- that spalling will not occur
- an adequate fire resistance
- the protection of steel against corrosion

Durability and control of crack width is related with finishing and provision of adequate cover to reinforcement. Nominal cover for structural elements located in the interior of the building with dry environment and mild condition is 15 mm, example slab; humid

environment with moderate exposure is 25 mm, example beam; severe environment is 50 mm, example footing.

Spacing of Reinforcements

- The clear horizontal and vertical distance between bars shall be at least equal to the largest of the following values.
 - (a) 20 mm
 - (b) The diameter of the largest bar or effective diameter of the bundle
 - (c) The maximum size of the aggregate d_g plus 5mm.
- Where bars are positioned in separate horizontal layers, the bars in each layer should be located vertically above each other and the space between the resulting columns of the bars should permit the passage of an internal vibrator.

Deflection limits are assumed to be satisfied when the minimum effective depth for a particular member is

$$d \geq \left(0.4 + \frac{0.6 * f_{yk}}{400} \right) * \frac{L_e}{\beta_a}$$

Where f_{yk} is equal to characteristic strength of reinforcement, L_e is the effective span (the shorter span in case of two way slab), is constant, a function of restraints given below).

Table – values of β_a

Member	Simple	End span	Interior span	cantilever
Beams	20	24	28	10
Slabs:				
Span ratio 2:1	25	30	35	12
Span ratio 1:1	35	40	45	10

* For intermediate values – *interpolation*. The distance l_o between points of zero moment may be obtained from the figure below for typical cases:

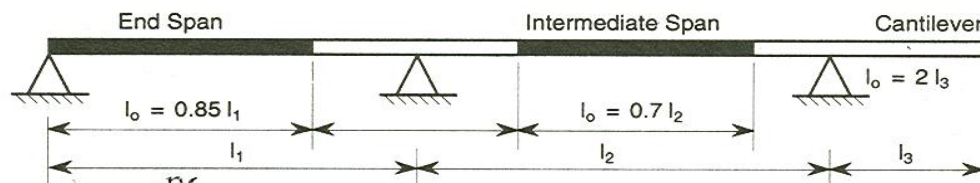


Fig. Locations of contra-flexure

3.4. SHEAR IN BEAMS-TRUSS MODEL. BOND, ANCHORAGE AND DEVELOPMENT LENGTH.

Shear force is present in beams where there is a change in bending moment along the span. Shear force is equal to the rate of change of bending moment.

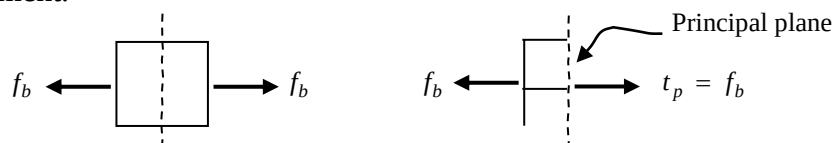
$$\text{i.e.} \quad V = \frac{dM}{dx}$$

Shear force in beam generally develop in combination with bending moment.

Behavior of Reinforced-concrete Beam without Shear Reinforcements.

In reinforced-concrete beam elements, shear force and moment may develop in combination. The combined action of shear force and moment gives rises to principal tensile and compressive stresses on principal planes of the elements of beam. When the principal tensile stress exceeds tensile strength of concrete, formation of cracks takes place along the principal plane.

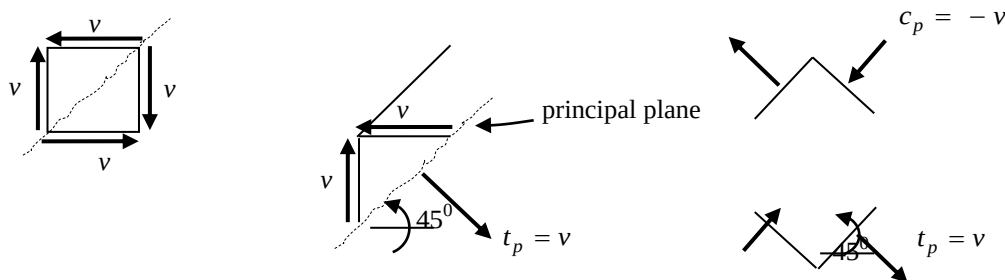
In the vicinity of maximum span moment of simple and continuous beams, where shear force is small and moment is large, the direction of the principal tensile stress is nearly horizontal and is nearly equal to the flexural tensile stress. For such case of a section of the beam subjected to moment only, the state of stresses for an element of beam near to extreme tension fiber is as shown below. When the principal tensile stress in such case exceeds the tensile strength of concrete, flexure tension cracks develop along vertical principal plane. Vertical flexure tension crack can be controlled by the provision of longitudinal tension reinforcement.



Near to the support of simple beams, where the shear force is large and moment is small, the principal tensile stress is nearly equal to the shear stress and is inclined at approximately

45° to the axis of the beam. It is referred to as the diagonal tension and is responsible for the development of inclined cracks. At the level of the neutral axis, longitudinal flexural stress is absent and shear stresses give rise to a diagonal tension which causes inclined cracks. For such case of a section of the beam subjected to maximum shear force and small moment, the state of stresses for an element of beam at the level of the neutral axis is as shown below.

When the principal tensile stress in such case exceeds the tensile strength of concrete, diagonal tension cracks develop along principal plane inclined at angle approximately 45° to the axis of the beam. But for a case of a section of continuous beam subjected to both maximum shear force and maximum moment, diagonal tension cracks develop an extension of vertical flexure crack and it is known as flexure-shear crack. Diagonal tension crack can be controlled by the provision of shear reinforcement.



Shear Strengths of RC beams and Shear Reinforcements

Inclined (diagonal) tension cracks in the web of reinforced concrete beams may develop:

- Either in absence of flexural tension cracks in the vicinity of location of maximum shear force and zero moment. Such crack is known as *web-shear crack*.
- Or as an extension of previously developed flexural tension crack at vicinity of the location the maximum support moment. Such crack is known as *flexure-shear crack*.

i) Web-shear cracks: -It is an inclined (diagonal) tension crack occurs near to supports of deep thin-webbed flanged beams; and it may also occurs near to the inflection points or bar-cutoff points on continuous beam subjected to axial tension.

ii) Flexure-shear cracks: -It is an inclined (diagonal) tension crack develop at the end of the initial vertical tension crack at section where critical combination of flexural and shear stresses develop in the beam.

The transfer of shear in reinforced concrete members occurs by the combination of the following mechanisms.

- 1) Shear resistance of the un-cracked concrete in compression

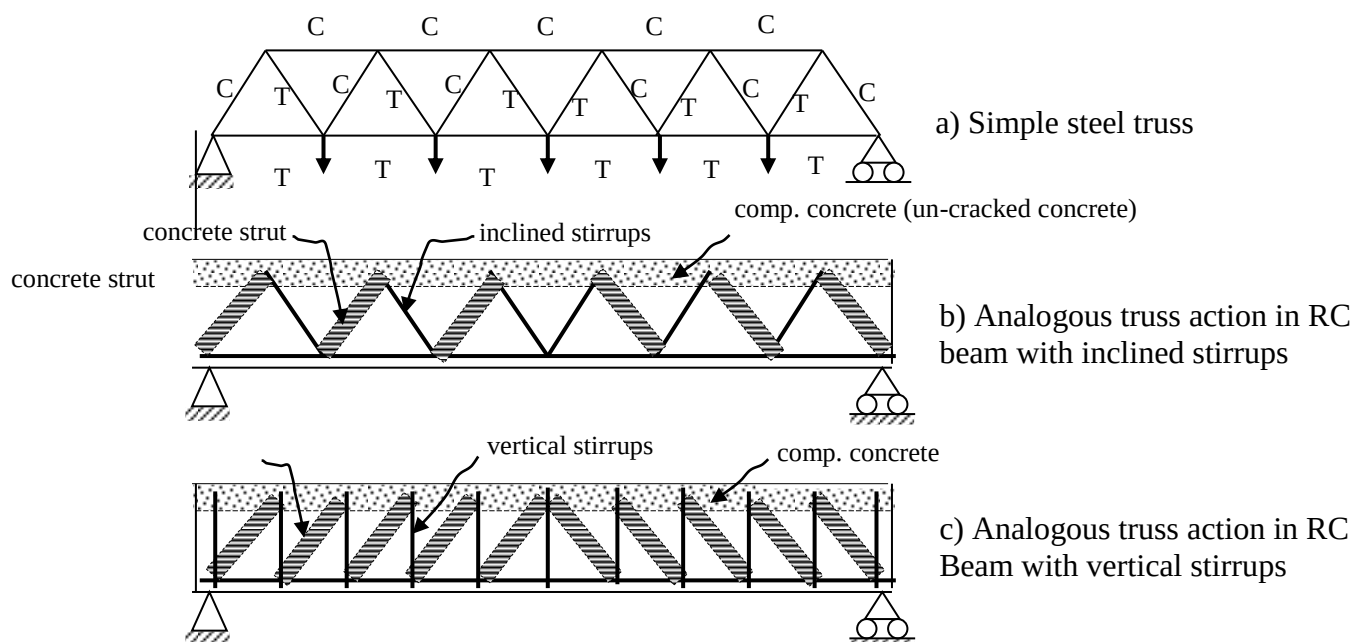
- 2) Aggregate interlock (or interface shear transfer) force: -that develop tangentially along tension crack, and similar to a frictional force due to irregular interlocking of aggregates along the rough concrete surface on each side of the crack.
- 3) Dowel action: - the resistance of the longitudinal reinforcement to a transverse force.
- 4) Arch action on relatively deep beams
- 5) Shear reinforcement resistance from vertical or inclined stirrups (not available in beams without shear reinforcement)

Code's empirical relation for shear strength of concrete preventing diagonal tension failure considers the first four shear transfer mechanisms listed above.

Common types of shear reinforcement are:

- 1) Vertical stirrups
- 2) Inclined stirrups at angle 45° or more from longitudinal axis of beam
- 3) Bent-up bars of longitudinal reinforcement bent at angle of 30° or more from longitudinal axis of beam
- 4) Combination of 1 or 2 with 3

The structural action of shear reinforcement may be described by the analogous steel truss action as shown below.



In a simple steel truss, the upper and lower chords are in compression and tension, respectively; and the diagonal members, usually called web members, are alternatively in compression and tension. The same analogy applied to reinforced concrete beam reinforced with shear reinforcement: vertical and inclined stirrups are similar in action to the tensile web-members, un-cracked compression concrete is similar in action to the upper chord compression member, concrete struts between inclined tension crack are similar in action to the compressive web members, and the longitudinal tension steel is similar in action to the lower chord member of steel truss.

The shear reinforcement must be anchored in the compression zone of the concrete and is usually hopped around the longitudinal tension reinforcement. The provision of shear reinforcement increases the shear strength of a member, but such reinforcement contributes little to the shear resistance prior to the formation of inclined tension cracks. Shear strength of a member increases due to the presence shear reinforcement allowing a redistribution of internal forces across any inclined tension crack forms in the member.

The primary functions of shear reinforcement are:

- 1) to carry part of the shear develop in the member
- 2) to restrict the growth of the inclined crack and reduce their penetration into compression zone, thus helps to maintain aggregate interlock (or interface shear transfer)
- 3) to tie the longitudinal bars in place and thereby increase their dowel capacity.

In addition to these, the provision of stirrups helps to improve member ductility.

Critical Section for Shear-force

The critical section for shear in experimental work was found at the location of the first inclined crack. Since most testing was made on simply supported beams under simple loading arrangements, it was difficult to extend such results to generalized loading on continuous structures. Therefore, the critical inclined crack is expected to develop in the member may be based on shear-span to depth ratio (a_v/d) as:

- 1) If $a_v/d > 2$, critical inclined crack is expected at 'd' distance from the section of maximum shear force.
- 2) If $a_v/d \leq 2$, critical inclined crack is expected at the center of shear-span (at $a_v/2$ from the support).

where a_v --distance from the applied concentrated load to the center of support of beam

Thus, for members subjected to uniform loading, ACI code permits taking the critical section for shear at a distance, 'd' from the face of support, in recognition of the fact that the support reaction being in the direction of the applied shear introduces compression into the end region of the member. This compression in the end region would occur when the beam is gravity loaded and supported by columns or walls.

Otherwise, the critical section for shear must be taken at face of support when one of the following occurs.

- 1) When the member is subjected to uniform loading but the support of member is itself a beam or girder and therefore does not introduce compression into the end region the member.
- 2) When a concentrated load occurs between the face of support and the distance, 'd' from the face of support.
- 3) When any loading may cause a potential inclined crack to occur at the face of the support or extend into instead of away from the support (see figure shown below)

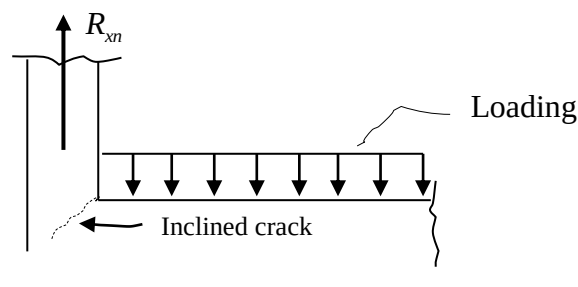


Fig: The reaction at support induces tension in the member

Design Shear Strength of Concrete (EBCS-2/95)-Limit State Method

-In order to prevent diagonal compression failure in concrete, the shear resistance of a section, V_{RD} shall not be less than the design shear force developed in a member due to factored design load. V_{RD} is given by

$$V_{RD} = 0.25 f_{cd} \cdot b_w \cdot d$$

-Design shear strength of concrete flexural member (beams & slabs) without significant axial force preventing diagonal tension failure is given by:

$$V_c = 0.25 f_{cd} \cdot k_1 \cdot k_2 \cdot b_w \cdot d$$

where

$$f_{ctd} = \frac{0.21(0.8 f_{cu})^{2/3}}{\gamma_c} \quad k_1 = (1+50\rho) \leq 2.0$$

$k_2 = (1.6-d) \geq 1.0$ (d in meters). For members where more than 50% of the bottom reinforcement is curtailed, $k_2 = 1.0$

$$\rho = \frac{A_s}{b_w \cdot d}$$

A_s --flexural tension reinforcement at critical section for shear

-For members subjected to axial compression in addition to flexure and shear, section may be designed for the additional shear strength of concrete given by,

$$V_{cn} = \frac{0.1(b_w \cdot d) \cdot P_d}{A_c}$$

where P_d --design axial force

A_c --gross concrete cross section

-For members subjected to axial tension in addition to flexure and shear, shear reinforcement shall be provided to carry total design shear.

Design Requirement for Shear Reinforcement

Analysis for Shear

Design Criteria

i) Only normal reinforcement

If $V_d < V_c$ where

V_d - the design critical shear at 'd' distance from the face of support, then the section is theoretically adequate without shear reinforcement.

However nominal reinforcement is provided as given by

$$\rho_{w,min} = \frac{0.4}{f_{yk}} = \frac{A_v}{b_w * S} \geq \frac{A_v}{b_w * d}$$

$$\Rightarrow S \leq \left\{ \frac{A_v * f_{yk}}{0.4 b_w} \right\}$$

A_v = Pair area of stirrups

S = Spacing in mm

The traverse spacing of legs of stirrups shall not exceed d or 800mm, whichever is the smaller.

When $V_d > V_c$, there exists a requirement for shear reinforcement to withstand the excess shear. Shear reinforcement are provided using vertical stirrups or inclined stirrups or bent-up bars or combination of them. Shear reinforcement required by excess shear should not be less than the minimum shear reinforcement given by code.

Where, $V_s = V_d - V_c$

$$S = \frac{A_{sv} f_{yd} (d)}{V_s}$$

S=spacing

- The maximum spacing, S_{max} between stirrups, in the longitudinal direction, shall be as given below;

$$S_{max} = 0.5d \leq 300\text{mm, if } V_{sd} \leq 2/3 V_{Rd}$$

$$S_{max} = 0.3d \leq 200\text{mm, if } V_{sd} > 2/3 V_{Rd}$$

3.4.1. Bond, Anchorage and Development Length

Bond

In order for reinforced concrete to behave as intended, it is essential that bond forces be developed on the interface between concrete and steel, such as to prevent significant slip from occurring at that interface. If the bar is smooth enough to slip, the assumption that the strain in an embedded reinforcing bar is the same as that in the surrounding concrete, would not be valid. Consequently, the beam would be very little stronger than if it were built of plain concrete, without reinforcement.

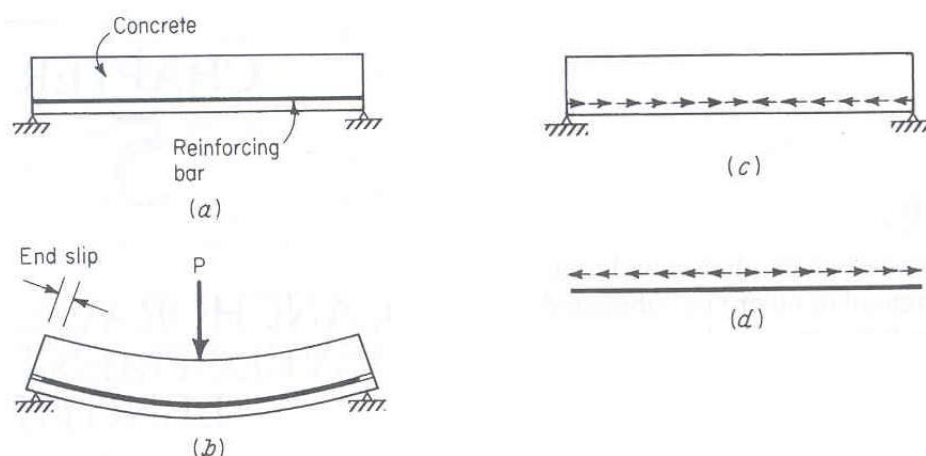


Figure. Bond stresses due to flexure (a) beam before loading; (b) unrestrained slip between concrete and steel; (c) bond forces acting on concrete; (d) bond forces acting on steel.

Formerly plain bars were used with provision of end anchorage in the form of hooks. Such beam forms a broken bond over the entire length between anchorages and acts as a tied arch

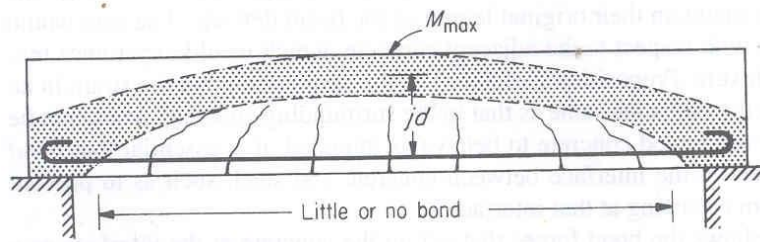


Figure 6.1.2 Tied arch action in a beam with little or no bond

To avoid development of wide cracks and dispense with special anchorage devices, deformed bars are now universally used. With such bars, the shoulders of the projecting ribs bear on the surrounding concrete and result in greatly increased bond strength.

Bond Stress

The force is resisted by the bond at the contact surface between bar and concrete. the unit bond stress is proportional to the shear at a particular section, i.e., to the rate of change of bending moment.

Development Length

Ultimate bond failures for bars in tension are of two types: the first is direct pullout of the bar, which occurs when ample confinement is provided by the surrounding concrete. The second type of failure is splitting of the concrete along the bar when cover, confinement or bar spacing is insufficient to resist the lateral concrete tension resulting from the wedging effect of the bar deformations. The latter is more common than the former.

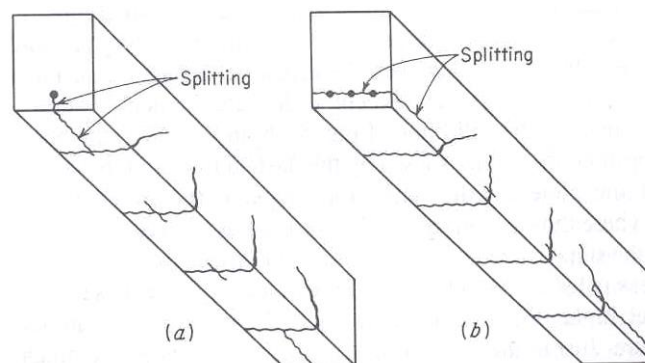


Figure. Splitting of concrete along reinforcement

The development length is defined as that length of embedment necessary to develop the full tensile strength of the bar, controlled by either pullout or splitting. Referring to figure 4.3.3, the moment, and hence the tensile stress, is evidently maximum at point *a* and zero at supports.

The total tension force $A_s f_s$ must be transferred from the bar to the concrete in the distance l by bond stress on the surface.

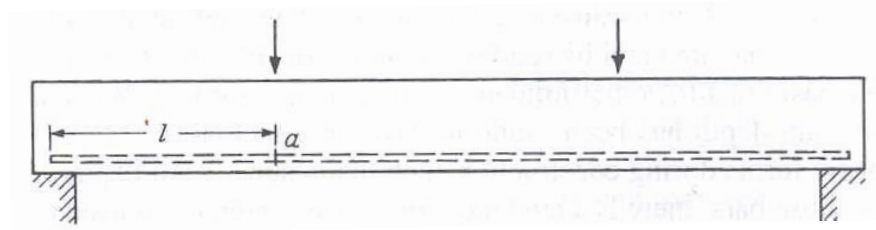


Figure. Development length

The safety against bond failure is that the length of the bar, from any point of given steel stress (f_s or at most f_y) to its nearby free end must be at least equal to its development length.

The basic anchorage length, l_b , is the straight length of bar required to anchor the force $A_s f_{yd}$. For a bar of diameter ϕ , this force must equal the shear force developed between the bar surface and the surrounding concrete:

$$A_s f_{yd} = (\pi \phi l_b) f_{bd}$$

$$\text{But } A_s = \frac{\pi \phi^2}{4}$$

$$\therefore \frac{\pi \phi^2}{4} f_{yd} = \pi \phi l_b f_{bd}$$

$$\Rightarrow l_b = \frac{\phi f_{yd}}{4 f_{bd}}$$

Where, f_{bd} = design bond strength.

Design bond strength

- Depends on the type of reinforcement, concrete strength & Position of bar during concreting.

- For good bond condition the design bond strength f_{bd} is given by

$$f_{bd} = f_{ctd} \quad \text{- Plain bar.}$$

$$f_{bd} = 2 f_{ctd} \quad \text{- Deformed bar.}$$

The required anchorage length $l_{b,net}$ depends on the type of anchorage and on the stress in the reinforcement and can be calculated as:

$$l_{b,net} = a l_b \frac{A_{s,cal}}{A_{s,ef}} \geq l_{b,min}$$

Where, $A_{s,cal}$ = theoretical area of reinforcement required by the design.

$A_{s,ef}$ = area of reinforcement actually provided.

$a = 1.0$,for straight bar anchorage in tension or compression.

0.7 , for anchorage in tension with standard hooks.

For bars in tension, $l_{b,min} = 0.3l_b \geq 10\phi$ or $\geq 200mm$

For bars in compression, $l_{b,min} = 0.6l_b \geq 10\phi$ or $\geq 200mm$

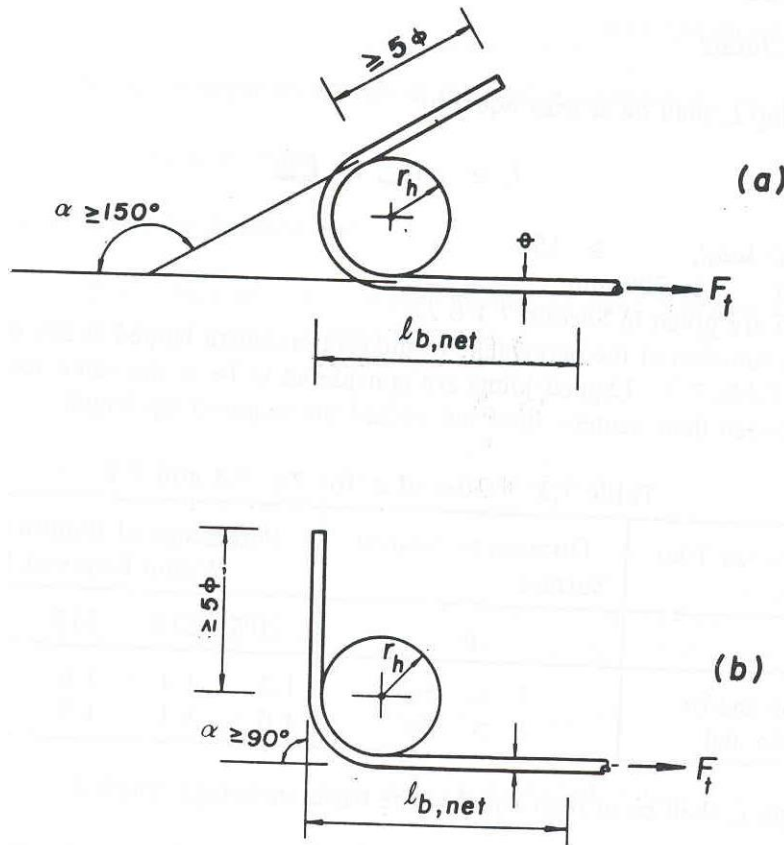


Figure Standard Hooks

Anchorage Length of reinforcement

1- Reinforcement shall extend beyond the point at which it is no longer required to resist tension for a length given by:

a) l_b according to the previous equation

b) $l_{b,net} \geq d$ according to the previous equation provided that in this case, the continuing bars are capable of resisting twice the applied moment at the section

2-The anchorage length of bars that are bent as shear reinforcement shall be at least equal to $1.3l_{b,net}$ in zones subjected to tension and to $0.7l_{b,net}$ in zones subjected to compression

Anchorage Bottom reinforcement at the Supports

At least one-quarter of the positive moment reinforcement in simple beams and one-half of the positive moment reinforcement in slabs shall extend along the same face of the member into the support

- 1) The anchorage of this reinforcement shall be capable of developing the following tensile force F_t .

$$F_t = V_{sd} \frac{a_1}{d} \geq 0.5V_{sd}$$

- 2) The anchorage length is measured from:
 - a) The face of the support, for a direct support
 - b) A plane inside the support located at a distance of $1/3$ the width of the support from the face of the support, for an indirect support.
- 3) The anchorage length of the bottom reinforcement at intermediate supports shall be at least 10ϕ .

Bar Cut off and Bend points

It is a common practice either to cut off bars where they are no longer required to resist stress or in case of continuous beams, to bend up bottom steel so that it provides tensile reinforcement at the top of the beam over the support.

To determine bend points, or bar cutting points, the moment diagram resulting from loading for maximum span moment and maximum support moment is shown bellows

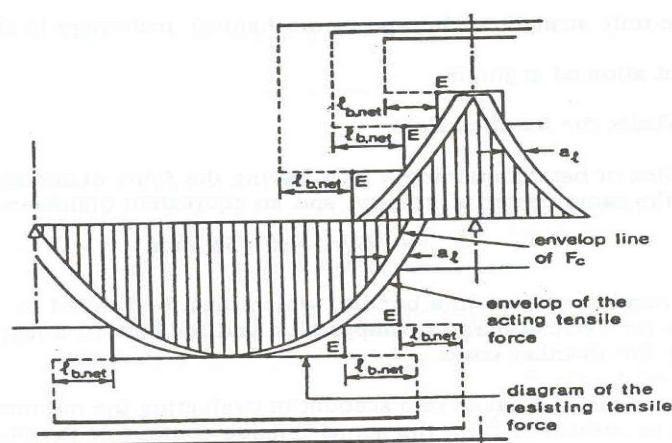


Figure. Tensile force diagram

Recognizing the various uncertainties, for bars with no special end anchorage the full development length $l_{b,net}$ [d or 12ϕ] whichever is larger, must be provided beyond the peak stress location. The critical section may be the point of max moment or a point where adjacent terminated reinforcement is no longer needed to resist bending.

In the absence of explicit calculation, the sketch shown may serve this purpose.

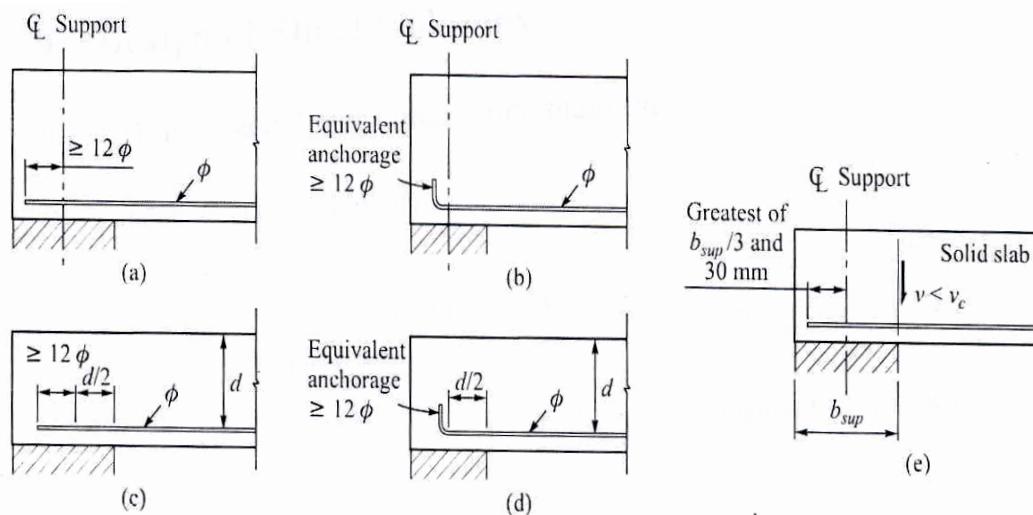
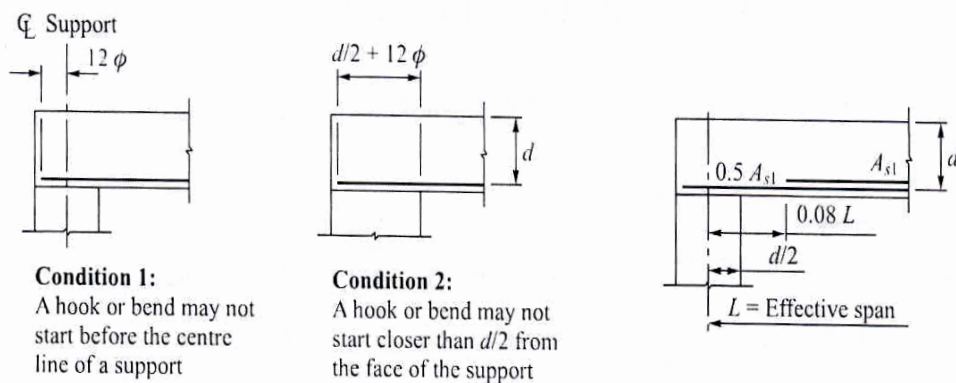
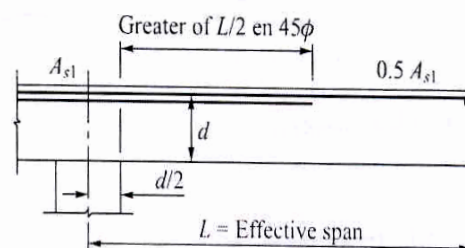


Figure 14: Curtailment of reinforcement—simply supported end.

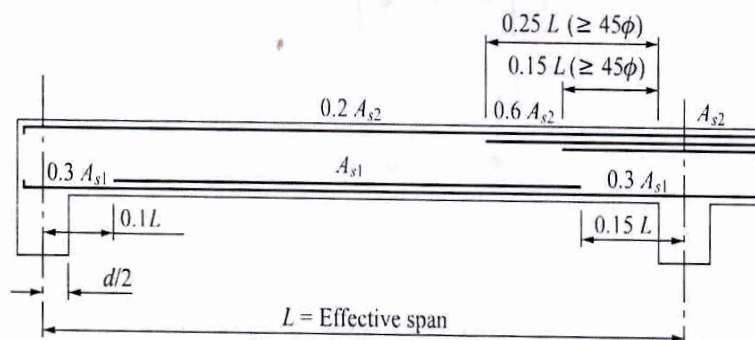


(a) Simple support

(b) Simply supported beam



(c) Cantilever



(d) Continuous beam

Bar splices

Reinforcing bars are as by fabrication limited in length, say 12 m. Thus it is normal to splice bars in the field. To do this, one has to notice the following regarding splicing.

- Splicing of bars must be avoided at points of max-moment.
- Bars which are spliced should be staggered.
- Splices are made simply by lapping the bars a sufficient distance to transfer stress by bond from one bar to the other.
- The required length of lap for tension is approximately $1.3l_b$ and that for compression is l_b .

3.5. DETAILING OF SHEAR REINFORCEMENT**Design of shear reinforcement**

- 1) In beams bent-up bars shall not be used as shear reinforcements except in combination with stirrups. At least 50% of the design shear force V_{SD} shall be resisted by vertical stirrups.
- 2) Where inclined reinforcement is used, the angle between the reinforcement and the longitudinal axis of the beam shall not be less than 45° .
- 3) Where the load is not acting at the top of the beam or when the support is not at the bottom of the beam suspension reinforcement shall be provided to transfer the load to the top of the beam.
- 4) When shear reinforcement perpendicular to the longitudinal axis is used, its shear resistance V_s may be calculated as:

$$V_s = \frac{A_v f_{yd} d}{s} \quad \text{Where } A_v \text{ is the area of shear reinforcement with in distance } s$$

- 5) When inclined stirrups are used, the shear resistance of the stirrups may be calculated as:

$$V_s = \frac{A_v d f_{yd} (\sin \alpha + \cos \alpha)}{s} \quad \text{Where } \alpha \text{ is the angle of inclination from the horizontal.}$$

- 6) When shear reinforcement consists of a single bar or a single group of parallel bars, all bent up at the same distance from the support, the shear resistance of the reinforcement may be calculated as :

$$V_s = A_v f_{yd} \sin \alpha$$

Minimum shear reinforcement

- 1) All beams, except joists of ribbed slabs, shall be provided with at least the minimum web reinforcement given by:

$$\rho_{\min} = \frac{0.4}{f_{yk}} \quad \text{where } f_{yk} \text{ is in MPa}$$

- 2) The maximum spacing $S_{\max}$ between stirrups, in the longitudinal direction, shall be as given below:

$$S_{\max} = 0.5d \leq 300\text{mm} \quad \text{if } V_{sd} \leq \frac{2}{3}V_{rd}$$

$$S_{\max} = 0.3d \leq 200\text{mm} \quad \text{if } V_{sd} > \frac{2}{3}V_{rd}$$

- 3) The transverse spacing of legs of stirrups shall not exceed d , or 800 mm, whichever is smaller.

Design Requirement for Shear Reinforcement

The design of section for shear may be made separately for the following categories.

- 1) When $V_d \leq \frac{V_c}{2}$, no shear reinforcement is required.
- 2) When $\frac{V_c}{2} < V_d \leq V_c$, theoretically no shear reinforcement is required. But code requires the provision of at least minimum shear reinforcement except for thin slab-like flexural members which experience has shown may perform well satisfactorily without shear reinforcement. The thin slab-like member exceptions include:
 - a) floor and roof slabs
 - b) footing slabs
 - c) floor joist construction
 - d) beams where the total depth does not exceed 254mm, 2.5 times the flange thickness of T-shaped section, or one-half of the web width

According to ACI code, minimum shear reinforcement for this category must provide shear resistance at least $= \left(k * \frac{1}{3} \text{MPa} \right) * b_w * d$ and, maximum spacing is limited to,

$S_{v,\max} \leq \frac{d}{2} \leq 600\text{mm}$. Therefore, minimum area of shear reinforcement based on the minimum shear resistance given by ACI code as,

$$A_{sv} = \frac{0.34b_w \cdot s}{f_y}$$

where s —longitudinal spacing of stirrups
 f_y --yield strength of steel used by stirrups

A_{sv} --area of shear reinforcement assumed to be traversed by inclined crack with-
in a distance s

The corresponding minimum area of shear reinforcement given by ESCP-2/83 code as,

$$A_{sv} = \frac{0.4b_w \cdot s}{f_y}$$

3) When $V_d > V_c$, there exists a requirement for shear reinforcement to withstand the excess shear. Shear reinforcement are provided using vertical stirrups or inclined stirrups or bent-up bars or combination of them. Shear reinforcement required by excess shear should not be less than the minimum shear reinforcement given by code.

Flexural Members with Shear Reinforcement

If the design shear force develop in flexural member exceeds the shear strength of concrete, V_c shear reinforcement is required to increase the shear strength of the member. Shear reinforcement may be provided in the forms of vertical-stirrups, or inclined- stirrups at angle 45° or more from longitudinal axis of member, or bent-up longitudinal bars bent an angle 30° or more from longitudinal axis of member, or combination of them.

Expression of strength of shear reinforcement may be developed from truss analogy. Consider analogous trusses as shown below to obtain the shear strength of shear reinforcement. In analogous trusses, it is assume that vertical stirrups are spaced at a distance equal to the effective depth, ' d ' of the beam so that the diagonal compression members are at an angle of 45° , which is more or less agrees with experimental observations of cracking of reinforced concrete beams close to their support. In analogous truss with inclined stirrups, the corresponding spacing of stirrups is equal to ' $d \cdot (1 + \cot \alpha)$ '.

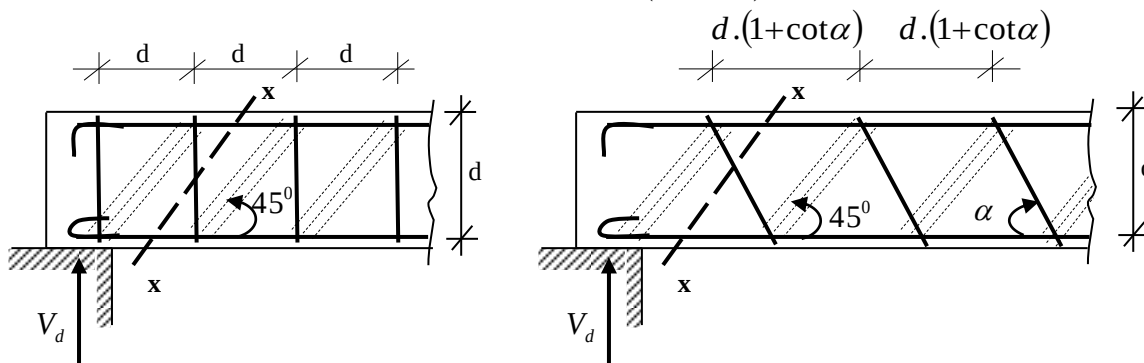


Fig a: Reinforced by vertical stirrups

Fig b: Reinforced by inclined stirrups

In analogous trusses, let

A_{sv} --cross sectional area of two-leg or four-leg stirrups

f_v --tensile stress in steel stirrups

V_d --design shear force due to service or ultimate design load

a) If beams reinforced with vertical Stirrups:-Using the method of sections and applying equilibrium of forces left of section x-x shown in Fig a, that the force in vertical stirrup member must equal to the design shear force, V_d .

i.e $V_d = f_v \cdot A_{sv}$

Since concrete is also capable of resisting a limited amount of shear, the equation of the force in vertical stirrup member is rewritten as,

$$V_s = (V_d - V_c) = f_v \cdot A_{sv}$$

where, $V_c = v_c \cdot b_w \cdot d$ --shear strength of concrete beam for diagonal tension failure

When the stirrup spacing is less than the effective depth, a series of super-imposed equivalent trusses may be considered, so that the force to be resisted by the stirrup is reduced proportionally. Thus, if number of stirrups traversed by diagonal tension crack is,

$$n = \frac{d}{s_v} \quad \text{where, 's_v' is spacing of vertical stirrups}$$

Therefore, equation of strength of shear reinforcement rewritten as,

$$V_s = (V_d - V_c) = n \cdot f_v \cdot A_{sv}$$

$$\Leftrightarrow V_s = (V_d - V_c) = \frac{d}{s_v} \cdot f_v \cdot A_{sv}$$

Rearranging the equation, the spacing of vertical stirrups required by excess shear is obtained as,

$$s_v = \frac{A_{sv} \cdot f_v \cdot d}{(V_d - V_c)}$$

where $f_v = f_{s,allow}$ --allowable stress of steel in WSD method

$f_v = f_{yd}$ --design stress of steel in strength limit state method

b) If beams reinforced with inclined stirrups:-Using the method of sections and applying equilibrium forces left of section x-x shown in Fig b, that the vertical component of force in inclined stirrup member must equal to the design shear force, V_d .

$$\text{i.e } V_d = f_v \cdot A_{sv} \cdot \sin \alpha$$

where α --angle of inclination of inclined stirrups from longitudinal axis of beam

Since concrete is also capable of resisting a limited amount of shear, the equation of the vertical component of force in inclined stirrup member is rewritten as,

$$V_s = (V_d - V_c) = f_v \cdot A_{sv} \cdot \sin \alpha$$

where, $V_c = v_c \cdot b_w \cdot d$ --shear strength of concrete beam for diagonal tension failure

When the stirrup spacing is less than ' $d \cdot (1 + \cot \alpha)$ ', a series of super-imposed equivalent trusses may be considered, so that the force to be resisted by the stirrup is reduced proportionally. Thus, if number of stirrups traversed by diagonal tension crack is,

$$n = \frac{d \cdot (1 + \cot \alpha)}{s_v} \quad \text{where, 's_v' is spacing of vertical stirrups}$$

Therefore, equation of strength of shear reinforcement rewritten as,

$$V_s = (V_d - V_c) = n \cdot f_v \cdot A_{sv} \cdot \sin \alpha$$

$$\Leftrightarrow V_s = (V_d - V_c) = \frac{d \cdot (1 + \cot \alpha)}{s_v} \cdot f_v \cdot A_{sv} \cdot \sin \alpha = \frac{f_v \cdot A_{sv} \cdot d \cdot (\sin \alpha + \cos \alpha)}{s_v}$$

Rearranging the equation, the spacing of inclined stirrups required by excess shear is obtained as,

$$s = \frac{A_{sv} \cdot f_v \cdot d \cdot (\sin \alpha + \cos \alpha)}{(V_d - V_c)}$$

If beam reinforced by with inclined stirrups at angle 45° from the longitudinal axis of beam, the spacing of inclined stirrups required by excess shear is obtained substituting $\alpha = 45^\circ$ as,

$$s = \frac{\sqrt{2} A_{sv} \cdot f_v \cdot d}{(V_d - V_c)} \cong \frac{1.414 A_{sv} \cdot f_v \cdot d}{(V_d - V_c)}$$

where $f_v = f_{s,allow}$ --allowable stress of steel in WSD method

$f_v = f_{yd}$ --design stress of steel in strength limit state method

c) If beams reinforced by bent-up of longitudinal bars with inclination of α from longitudinal axis of member:-If shear reinforcement consist of a single bar or a single group pf parallel bars, all bent up at the same distance from the support, area of shear reinforcement required is given as

$$A_{sv} = \frac{(V_d - V_c)}{f_v \cdot \sin \alpha}$$

where $f_v = f_{s,allow}$ --allowable stress of steel in WSD method

$f_v = f_{yd}$ --design stress of steel in strength limit state method

But, BS:8110 code do not recommend that all the shear be taken by bent-up bars only. If bent-up bars to be used, at least 50% of the shear should be provided by stirrups.