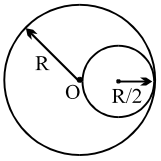
EXERCISE # (S-1)

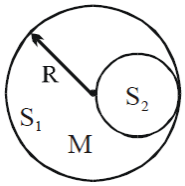
HCV Worked out Examples(Chapter No.10 - 18,19,20)

1. A hole of radius R/2 is cut from a solid sphere of radius R. If the mass of the remaining part is M, then find moment of inertia of the body about an axis through O perpendicular to plane.



Ans. $57/140MR^2$

Sol.



$S_1 + S_2 = S$

$I_{S_1} + I_{S_2} = I_S$

$$M_2 = \rho\frac{4}{3}\pi\frac{R^3}{8} = \frac{M}{7}$$

where $\rho = \dfrac{M}{\frac{4}{3}\pi R^3 - \frac{4}{3}\pi\left(\frac{R}{2}\right)^3}$

$$\rho = \frac{6M}{7\pi R^3}$$

Now

$$I_{S_2} = \frac{2}{5}\frac{M}{7}\left(\frac{R}{2}\right)^2 + \frac{M}{7}\left(\frac{R}{2}\right)^2$$

$$= \frac{7}{5}\frac{M}{7}\frac{R^2}{4} = \frac{MR^2}{20}$$

$$I_S = \frac{2}{5}\left(M + \frac{M}{7}\right)R^2 = \frac{16M}{35}R^2$$

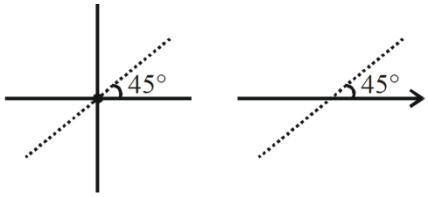
$$\therefore\ I_{S_1} = I_S - I_{S_2} = \frac{16}{35}MR^2 - \frac{MR^2}{20}$$

$$\frac{(64-7)MR^2}{140} = \frac{57}{140}MR^2$$

2. Two rods of equal mass m and length l lie along the x axis and y axis with their centres origin. What is the moment of inertia of both about the line x=y :

Ans. $\frac{ml^2}{12}$

Sol.



$$I_1 = \frac{M\ell^2}{12}\cos^2 45$$

$$= \frac{M\ell^2}{12}\times\frac{1}{2}$$

$$I_1 = I_2 \qquad \therefore\ I_1 + I_2 = \frac{M\ell^2}{12}$$

HCV Exercises (Chapter No.10 - 1,7 to 13,16)

HCV Worked out Examples(Chapter No.10 - 3,4)

3. A force of $\vec{F} = 2\hat{i} + 3\hat{j}\,N$ is applied to an object that is pivoted about a fixed axle aligned along the z coordinate axis. If the force is applied at the point $\vec{r} = 4\hat{i} + 5\hat{j}\,m$, find (a) the magnitude of the net torque about the z axis and (b) the direction of the torque vector $(\vec{\tau})$.

Ans. (a) 2.00 N·m, (b) $\hat{k}$

Sol. $\vec{F} = 2\hat{i} + 3\hat{j};\ \ \vec{r} = 4i + 5\hat{j}$

$$\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} i & j & \hat{k} \\ 4 & 5 & 0 \\ 2 & 3 & 0 \end{vmatrix} = \hat{k}(12-10) = 2\hat{k}$$

HCV Exercises (Chapter No.10 - 17,19,20,21)

HCV Worked out Examples(Chapter No.10 - 7,8,9)

4. In an experiment with a beam balance on unknown mass m is balanced by two known mass m is balanced by two known masses of 16 kg and 4 kg as shown in figure.

Find the value of the unknown mass m.

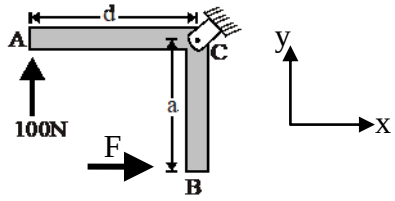
Ans. 8 kg

Sol. $16\ell_1 = m\ell_2$

$m\ell_1 = 4\ell_2$

$\frac{16}{m} = \frac{m}{4}$ $\quad\therefore$ m = 8kg

5. Find force F required to keep the system in equilibrium. The dimensions of the system are d = 0.3 m and a = 0.2 m. Assume the rods to be massless.



Ans. **150(+ $\hat{i}$)**

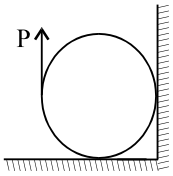
Sol. –d(i) × 100(j) + (–aj) × F(i) = 0

$-100\ d\hat{k} + aF\hat{k} = 0$

$\therefore F = \frac{100d}{a} = \frac{300}{2} = 150$

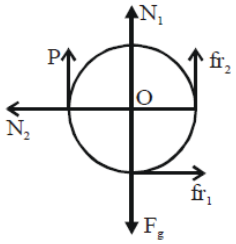
$\therefore F = 150\hat{i}$

6. Figure shows a vertical force applied tangentially to a uniform cylinder of weight F_g. The coefficient of static friction between the cylinder and both surfaces is 0.500. In terms of F_g, find the maximum force **P** that can be applied that does not cause the cylinder to rotate.



Ans. (3/8) F_g

Sol.



$N_1 + P + fr_2 = F_g$ ....(i)

$N_2 = fr_1 = \mu N_1$

$fr_1 = \mu N_1$ & $fr_2 = \mu N_2$

τ about O = 0

$-Pr + fr_1 r + fr_2 r = 0$

$\therefore P = fr_1 + fr_2 = \mu (N_1 + N_2)$ ....(3)

from (1) $N_1 + P + \mu N_2 = Fg$ ....(4)

$\therefore N_1 (1 + \mu^2) = F_g - P$

$$\therefore N_1 = \frac{F_g - P}{(1+\mu^2)}$$

$\therefore$ Putting in $P = \mu(N_1 + \mu N_1)$

$= \mu N1(1 + \mu)$

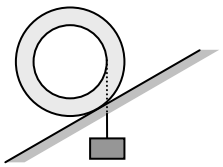
$$P = \mu \frac{(F_g - P)(1+\mu)}{1+\mu^2}$$

$$\therefore P = \frac{\frac{1}{2}\left(1+\frac{1}{2}\right)}{1+\frac{1}{4}}(F_g - P)$$

$\Rightarrow 5P = 3F_g - 3P$

$$\therefore P = \frac{3F_g}{8}$$

7. A 3.0 kg bobbin consists of a central cylinder of radius 5.0 cm and two end plates each of radius 6.0 cm. It is placed on a slotted incline, where friction is sufficient to prevent sliding. A block of mass 4.5 kg is suspended from a cord wound around the bobbin and passing through the slot under the incline. If the bobbin is in static equilibrium, what is the angle of tilt of the incline?



Ans. 30

Sol.

$fr = 75 \sin \theta$

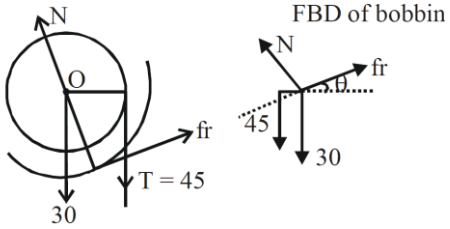
τ about 0 = 0

$\therefore T \times 5 = fr \times 6$

$45 \times 5 = 75 \times 6 \times \sin \theta$

$\Rightarrow \theta = 30°$

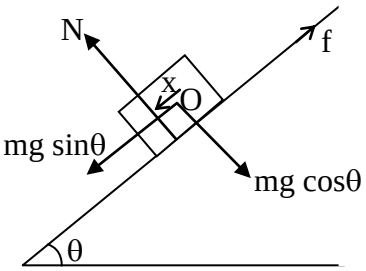
$\tau = I\alpha$ and its calculation



8. A solid cubical block of mass m and side a slides down a rough inclined plane of inclination θ with a constant speed. Calculate the torque of the normal force acting on the block about its centre and the perpendicular distance 'x' from centre of mass at which it is acting.

Ans. $\frac{1}{2}mg\ a\ \sin\theta$, $x = \frac{a\tan\theta}{2}$

Sol.



$f = mg\sin\theta$

$N = mg\cos\theta$

$\vec{\tau}$ = about o should be zero

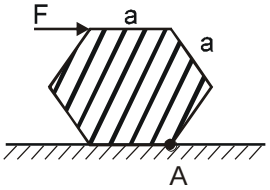
$$Nx = f\frac{a}{2}$$

$$(mg\cos\theta)\ x = (mg\sin\theta)\frac{a}{2}$$

$$\boxed{x = \frac{a\tan\theta}{2}}$$

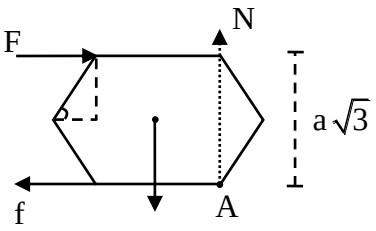
$\vec{\tau}$ of normal $= Nx = (mg\sin\theta)\frac{a}{2}$

9. A regular hexagonal uniform block of mass m rests on a rough horizontal surface with coefficient of friction μ as shown in figure. A constant horizontal force is applied on the block as shown. If the coefficient of friction is sufficient to prevent slipping before toppling, then the minimum force required to topple the block about its corner A.



Ans. $\frac{mg}{2\sqrt{3}}$

Sol.



maximum possible shift for normal is upto A.

$\vec{\tau}_A = 0$ for just topple

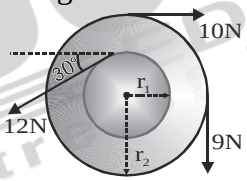
$$mg\frac{a}{2} - F(a\sqrt{3}) = 0$$

$$\boxed{F = \frac{mg}{2\sqrt{3}}}$$

HCV Exercises (Chapter No.10 - 40 to 45)

HCV Worked out Examples(Chapter No.10 - 5,6)

10. In the following figure r_1 and r_2 are 5 cm and 30 cm respectively. If the moment of inertia of the wheel is 5100 kg–m^2 about the axis passing through its center and perpendicular to the plane of wheel, then what will be its angular acceleration?



Ans. $\alpha = 10^{-3}$ rad/s^2

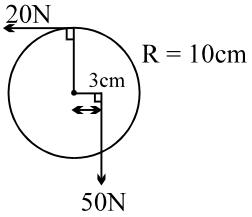
Sol. $\tau = I\alpha$

$\tau = (-9 \times 30 - 10 \times 30 + 12 \times 5) \times 10^{-2}$

$= (-570 + 60) \times 10^{-2} = -510 \times 10^{-2}$

$\therefore |\alpha| = \frac{510 \times 10^{-2}}{5100} = 10^{-3}$ rad/s^2

11. A uniform cylinder of mass 2 kg can rotate about its fixed axis. Forces as shown in figure are applied on it to produce angular acceleration. The angular acceleration is _____ in _____ (clockwise/anticlockwise) sense.



Ans. 50 rad/s^2, anticlockwise

Sol. Taking outward as positive

$$\tau = 20 \times \left(\frac{10}{100}\right) - 50\left(\frac{3}{100}\right) = I\alpha$$

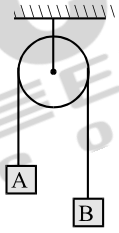
$$\Rightarrow I\alpha = \frac{1}{2}$$

$$I = \frac{MR^2}{2} = \frac{2\left(\frac{1}{10}\right)^2}{2} = \frac{1}{100}$$

$$I\alpha = \frac{1}{100}\alpha = \frac{1}{2}$$

$$\alpha = 50 \text{ rad/s}^2.$$

12. In the figure A & B are two blocks of mass 4 kg & 2 kg respectively attached to the two ends of a light string passing over a disc C of mass 40 kg and radius 0.1 m. The disc is free to rotate about a fixed horizontal axes, coinciding with its own axis. The system is released from rest and the string does not slip over the disc. Find:



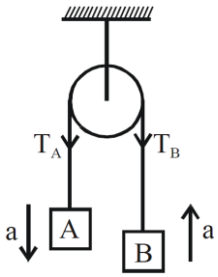
(i) the linear acceleration of mass B.

(ii) the number of revolutions made by the disc at the end of 10 sec. from the start.

(iii) the tension in the string segment supporting the block A.

Ans. (i) 10/13m/s^2, (ii) 5000/26π,(iii) 480/13N

Sol.



$40 - TA = 4a$ …(i)

$TB - 20 = 2a$ …(ii)

$(T_A - T_B) \times 0.1 = \frac{M(0.1)^2}{2}\alpha$ …(iii)

$T_A - T_B = 2\alpha$ …(vi)

$a = \alpha R \Rightarrow a = \frac{\alpha}{10}$ …(iv)

Now (1) + (2) $\Rightarrow T_B - T_A + 20 = 6a$

$\therefore -2\alpha + 20 = 6a$

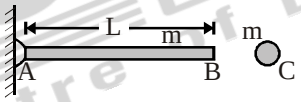
$\therefore -2\alpha + 20 = \frac{6\alpha}{10} \Rightarrow 20 = \frac{26\alpha}{10} \quad \therefore \alpha = \frac{200}{26}$

$\therefore \alpha = \frac{100}{13} \qquad \therefore a = \frac{\alpha}{10} = \frac{10}{13}\ m/s^2$

(ii) $\frac{1}{2}\alpha t^2 = \frac{1}{2} \times \frac{100}{13} \times \frac{100}{2\pi} = \frac{5000}{26\pi} r\ell v$

(iii) $40 - T_A = 4 \times \frac{10}{13} \Rightarrow T_A = 40\left(1 - \frac{1}{13}\right) = \frac{480}{12}\ N$

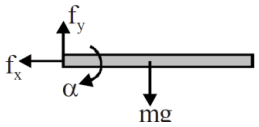
13. A uniform bar AB of mass m and a ball of the same mass are released from rest from the same horizontal position. The bar is hinged at end A. There is gravity downwards. What is the distance of the point from point B that has the same acceleration as that of ball, immediately after release?



Ans. $\frac{L}{3}$

Sol. $mg\frac{L}{2} = \frac{1}{3}mL^2\alpha$

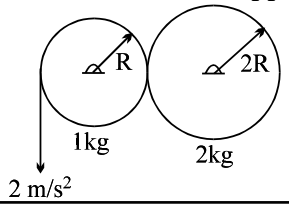
$\therefore \alpha = \frac{3g}{2L}$



and $ax = g \Rightarrow \frac{3g}{2L}x = g \therefore x = \frac{2L}{3}$

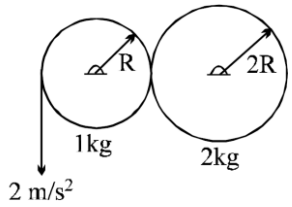
$\therefore$ distance from B = $\frac{L}{3}$

14. Two discs A and B touch each other as in figure. A rope tightly wound on A is pulled down at 2 m/s^2. Find the friction force between A and B if slipping is absent



Ans. 2N

Sol.



Now for (2) $f \times 2R = I_2\alpha_2$

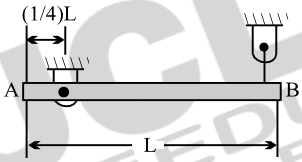
$I_2 = \dfrac{2\times 4R^2}{2} = 4R^2$

Now $\alpha_2 2R = 2$ (a is same)

$f2R = 4R^2\dfrac{1}{R}$

$\therefore\ f = 2$

15. A uniform beam of length L and mass m is supported as shown. If the cable at B suddenly breaks, determine ;



(a) the acceleration of end B.
(b) the reaction at the pin support.

Ans. (a) $\dfrac{9g}{7}\downarrow$ (b) $\dfrac{4mg}{7}\uparrow$

Sol. about hinge $mg\ \dfrac{L}{4} = I\alpha$

$I = \dfrac{M_2}{12} + \dfrac{ML^2}{16} = \dfrac{7ML^2}{48}$

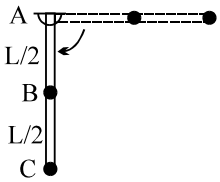
$\therefore\quad \dfrac{MgL}{4} = \dfrac{7ML^2\alpha}{48}$

$\therefore\quad \alpha = \dfrac{12g}{7L}$

Now $\alpha\ \dfrac{3L}{4} = \dfrac{12g}{7L}\dfrac{3L}{4} = \dfrac{9g}{7}\downarrow$

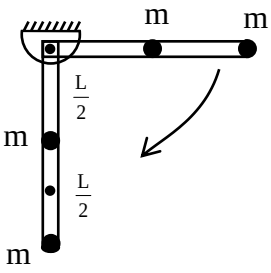
(a) $a_{cm} = \dfrac{\alpha L}{4} = \dfrac{3g}{7}$ and $mg - F_{hinge} = ma_{cm}$

16. Two masses each of m are attached at mid point B & end point C of massless rod AC which is hinged at A. It is released from horizontal position as shown. Find the force at hinge A when rod becomes vertical



Ans. $\frac{28mg}{5}$

Sol.



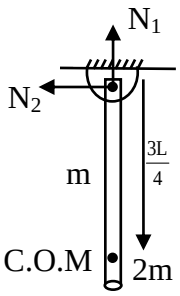
$K_i + U_i = k_f + U_f$

$$0 + 0 = \frac{1}{2} I\omega^2 + \left(-mg\frac{L}{2} - mgL\right)$$

$$\frac{3mgL}{2} = \frac{1}{2}\left(m\left(\frac{L}{2}\right)^2 + m(L)^2\right)\omega^2$$

$$3\,mg\,L = \frac{5mL^2}{4}\omega^2$$

$$\omega = \sqrt{\frac{12g}{5L}}$$



$N_1 - 2mg = 2m\,\omega^2\,\frac{3L}{4}$ …(1)

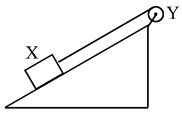
$N_2 = 0$ …(2)

$N_1 = 2mg + 2m\frac{12g}{5L}\frac{3L}{4}$

$= 2\ mg + \frac{18}{5}mg$

$= \frac{28mg}{5}$

17. A block X of mass 0.5kg is held by a long massless string on a frictionless inclined plane of inclination 30° to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and of radius 0.2 m as shown in the fig. The drum is given an initial angular velocity such that the block X starts moving up the plane. ($g = 9.8\ m/s^2$) **[JEE 94]**

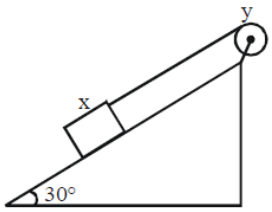


(i) Find the tension in the string during the motion

(ii) At a certain instant of time the magnitude of the angular velocity of Y is 10 rad/sec. Calculate the distance travelled by X from that instant of time until it comes to rest.

Ans. 1.63 N, 1.224 m

Sol. M = 2kg



m = 0.5 kg)

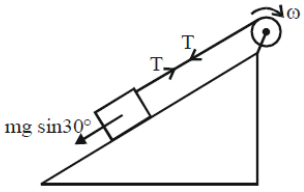
$I = \frac{1}{2} \times Mr^2$

mg sin 30 – T = ma

$Tr = I\alpha$

$\alpha r = a$

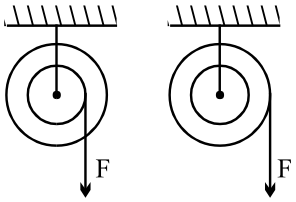
get T and a.



(b) $\omega = 10 \Rightarrow v = \omega r = 10 \times 0.2 = 2\ m/s$

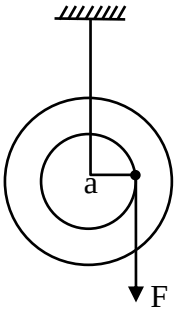
$\frac{v^2}{2a} = s$

18. A person pulls along a rope wound up around a pulley with a constant force F for a time interval of t seconds. If a and b are the radii of the inner and the outer circumference (a < b), then find the ratio of work done W_1/W_2 by the person in the two cases shown in the figure.



Ans. a^2/b^2

Sol.



Case-I

$\tau = Fa$

$I\alpha_1 = Fa$

$\alpha_1 = \dfrac{Fa}{I}$

$\omega_1 = \alpha t$

$= \dfrac{Fa}{I}t$

$K\cdot E_1 = \dfrac{1}{2}I\omega_1^2 = \dfrac{1}{2}I\dfrac{F^2a^2}{I^2}t^2$

$K\cdot E_1 = \dfrac{1}{2}\dfrac{F^2a^2t^2}{I}$

Case-II

Similarly $K\cdot E_2 = \dfrac{1}{2}\dfrac{F^2b^2t^2}{I}$

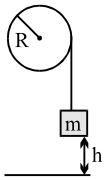
$\dfrac{\omega_1}{\omega_2} = \dfrac{K\cdot E_1}{K\cdot E_2} = \dfrac{a^2}{b^2}$

$\Rightarrow$

HCV Exercises (Chapter No.10 - 22,25,28 to 36,38,39)

HCV Worked out Examples(Chapter No.10 - 13,17)

19. A mass m is attached to a pulley through a cord as shown in the figure. The pulley is a solid disk with radius R. The cord does not slip on the disk. The mass is released from rest at a height h from the ground and at the instant the mass reaches the ground, the disk is rotating with angular velocity ω. Find the mass of the disk.



Ans. $M = 2m\left(\frac{2gh}{R^2\omega^2} - 1\right)$

Sol.

$v = \omega R$

$-T + mg = ma$

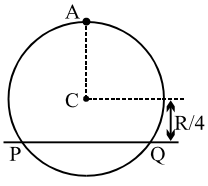
$TR = \frac{MR^2}{2}\alpha$

$\alpha R = a$

$v = \sqrt{2ah}$

get M.

20. A uniform circular disc has radius R and mass m. A particle also of mass m is fixed at a point A on the edge of the disc as in fig. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance R/4 from the centre C of the disc. The line AC is perpendicular to PQ. Initially the disc is held vertical with the point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. Find the linear speed of the particle at it reaches its lowest position. **[JEE 98]**



Ans. $v = \sqrt{5gR}$

Sol.

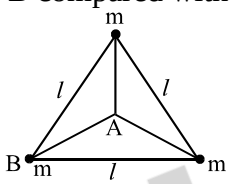
$KE_f = \frac{I}{2}I\omega^2$; $KE_i = 0$; $W_g = mg\left(\frac{2R}{4}\right) + mg2\left(\frac{5R}{4}\right)$

$I = m\left(\frac{5R}{4}\right)^2 + \left(\frac{MR^2}{4} + \frac{MR^2}{16}\right) = MR^2\left(\frac{25}{16} + \frac{1}{4} + \frac{1}{16}\right)$

$W_g = KE_f - KE_i$

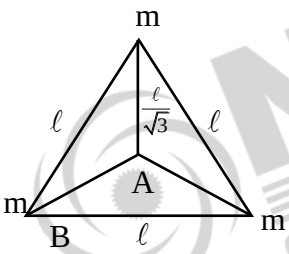
get $v = \omega\frac{5R}{4}$

21. Three equal masses m are rigidly connected to each other by massless rods of length *l* forming an equilateral triangle, as shown above. The assembly is to be given an angular velocity ω about an axis perpendicular to the triangle. For fixed ω, what is the ratio of the kinetic energy of the assembly for an axis through B compared with that for an axis through A(centroid).



Ans. 2

Sol.



$K\cdot E = \frac{1}{2}I\omega^2$

I about $A = \left[m\left(\frac{\ell}{\sqrt{3}}\right)^2\right]\times 3$

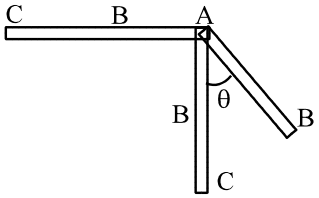
$I_A = m\ell^2$

I about $B = m\ell^2 + m\ell^2$

$I_B = 2m\ell^2$

$\frac{K\cdot E_B}{K\cdot E_A} = \frac{I_B}{I_A} = 2$

22. A rod hinged at one end is released from the horizontal position as shown in the figure. When it becomes vertical its lower half separates without exerting any reaction at the breaking point. Then find the maximum angle 'θ' made by the hinged upper half with the vertical.



Ans. 60°

Sol. $mg\frac{\ell}{2}=\frac{1}{2}\frac{m\ell^2}{3}\omega^2$ $\quad\therefore\ \omega^2=\frac{3g}{\ell}$ at bottom

$W_g = \Delta KE$

$$\Rightarrow -\frac{m}{2}g\frac{\ell}{4}(1-\cos\theta)=0-\frac{1}{2}\left(\frac{m}{2}\right)\frac{\ell^2}{4}\times\frac{1}{3}\omega^2$$

$$\therefore\ \frac{mg\ell}{8}(1-\cos\theta)=\frac{mg\ell}{16}$$

$\therefore \cos\theta = \frac{1}{2}$ $\quad\therefore\ \theta = 60°$

HCV Exercises (Chapter No.10 - 30,65,66,67,69)

HCV Worked out Examples(Chapter No.10 - 11,12,15,16)

23. Two men, each of mass 75 kg , stand on the rim of a horizontal large disc, diametrically opposite to each other. The disc has a mass 450 kg and is free to rotate about its axis. Each man simultaneously start along the rim clockwise with the same speed and reaches their original starting points on the disc. Find the angle turned through by the disc with respect to the ground in this duration.

Ans. $\frac{4\pi}{5}$

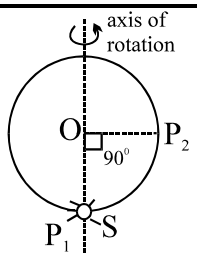
Sol. $2mvR = \frac{M_1R^2}{2}\omega_d \Rightarrow \frac{v}{R}=\frac{3}{2}\omega_d$

$$\therefore\ \frac{\omega_m}{\omega_D}=\frac{3}{2} \qquad \therefore\ \omega_{m/D}=\frac{5\omega_D}{2}$$

$\omega_{m/D}t = 2\pi$

$\omega_D t = 4\pi/5$

24. A uniform ring is rotating about vertical axis with angular velocity ω initially. A point insect (S) having the same mass as that of the ring starts walking from the lowest point P_1 and finally reaches the point P_2 (as shown in figure). What is the final angular velocity of the ring?



Ans. $\frac{\omega}{3}$

Sol. $I_1\omega_1 = I_2\omega_2$

$$\left(\frac{mR^2}{2}+0\right)\omega = \left(\frac{mR^2}{2}+mR^2\right)\omega_2$$

$$\therefore \omega_2 = \frac{\omega}{3}$$

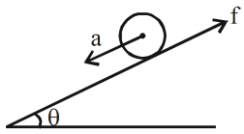
HCV Exercises (Chapter No.10 - 46 to 49,51,53,55 to 58,62)

HCV Worked out Examples(Chapter No.10 - 22,24,26,27,28)

25. A solid uniform disk of mass m rolls without slipping down a fixed inclined plane with an acceleration a. Find the frictional force on the disk due to surface of the plane :]

Ans. 1/2 ma

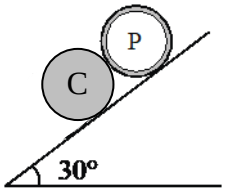
Sol.



$\alpha = ar$

$$fr = \frac{1}{2}mr^2\alpha = \frac{1}{2}ma$$

26. A solid cylinder C and a hollow pipe P of same diameter are in contact when they are released from rest as shown in the figure on a long incline plane. Cylinder C and pipe P roll without slipping. Determine the clear gap (in m) between them after 6 seconds.



Ans. 0015

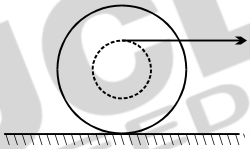
Sol. $a_{SC} = \dfrac{g\sin 30}{1+\dfrac{mR^2}{2mR^2}} = \dfrac{g}{2\times\dfrac{3}{2}} = \dfrac{g}{3}$

$a_{HP} = \dfrac{g\sin 30}{1+\dfrac{mR^2}{mR^2}} = \dfrac{g}{4}$

$\therefore\ a_{rel} = \dfrac{g}{3} - \dfrac{g}{4} = \dfrac{g}{12}$

$\therefore \dfrac{1}{2}\dfrac{g}{12}\times 6^2 = 15\text{m}$

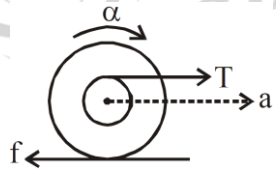
27. A spool of inner radius R and outer radius 3R has a moment of inertia = MR^2 about an axis passing through its geometric centre, where M is the mass of the spool. A thread wound on the inner surface of the spool is pulled horizontally with a constant force = Mg. Find the acceleration of the point on the thread which is being pulled assuming that the spool rolls purely on the floor.



Ans. 16m/s^2

Sol. $T - f = ma$

$TR + f3R = mR^2\alpha$; $a = \alpha 3R$

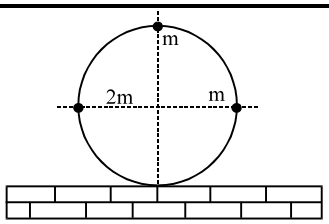


Answer is $a + \alpha R$

HCV Exercises (Chapter No.10 - 59,72,81,83 to 86)

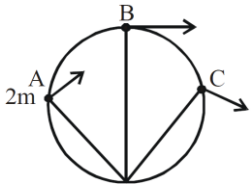
HCV Worked out Examples(Chapter No.10 - 23)

28. A ring of mass m and radius R has three particles attached to the ring as shown in the figure. The centre of the ring has a speed v_0. Find the kinetic energy of the system. (Slipping is absent).



Ans. $6\,mv_0^2$

Sol.



$\omega = \dfrac{v_0}{R}$

$\therefore KE_{ring} = \dfrac{1}{2}I\omega^2 + \dfrac{1}{2}MV_{CM}^2$

$= \dfrac{1}{2}MR^2\omega^2 + \dfrac{1}{2}mv_0^2 = mv_0^2$

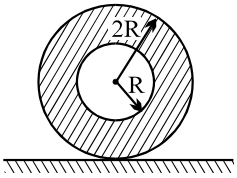
$\dfrac{1}{2}2m\omega^2\left(\sqrt{2}R\right)^2 = m2v^2 \rightarrow KE_A$

$KE_{sys} = KE_{ring} + KE_A + KE_B + KE_C$

$\dfrac{1}{2}m\omega^2(2R)^2 = 2mv^2 \rightarrow KE_B$

$\dfrac{1}{2}m\omega\left(\sqrt{2}R\right) = mv^2 = 6mv_0^2 \rightarrow KE_C$

29. A hollow cylinder with inner radius R, outer radius 2R mass M is rolling with speed of its axis v. Its kinetic energy is

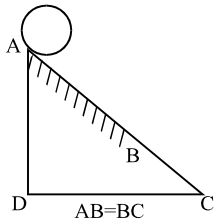


Ans. $\dfrac{13}{16}Mv^2$

Sol. $KE = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2$; get I by subtraction. ($I_{outer} - I_{inner}$)

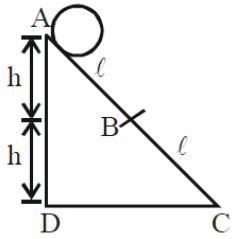
$$\omega = \frac{v}{2R}$$

30. Portion AB of the wedge shown in figure is rough and BC is smooth. A solid cylinder rolls without slipping from A to B. Find the ratio of translational kinetic energy to rotational kinetic energy, when the cylinder reaches point C.



Ans. 5

Sol.



$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2$$

$$mgh = \frac{3}{4}mv^2$$

$$2mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2$$

$$2mgh = \frac{1}{2}mv_f^2 + mgh - \frac{1}{2}mv^2$$

$$mgh = \frac{1}{2}mv_f^2 - \frac{1}{2}mv^2$$

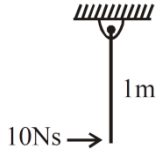
$$\frac{1}{2}mv_f^2 = \frac{5}{3}mgh$$

$$\therefore \quad \text{Ratio} = \frac{\frac{1}{2}mv_f^2}{\frac{1}{2}\frac{mv^2}{2}} = \frac{\frac{5}{3}mgh}{\frac{1}{4}\frac{4}{3}mgh} = 5$$

HCV Exercises (Chapter No.10 - 71,73,75 to 79)

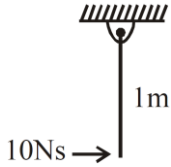
HCV Worked out Examples(Chapter No.10 - 25)

31. A thin uniform straight rod of mass 2 kg and length 1 m is free to rotate about its upper end when at rest. It receives an impulsive blow of 10 Ns at its lowest point, normal to its length as shown in figure. Find the kinetic energy of rod just after impact.



Ans. 75 J

Sol.



$J = 10 \times 1 = I\omega - 0$

$I = \frac{1}{3}m\ell^2 \quad \therefore \omega = 15$

$\therefore KE = \frac{1}{2}I\omega^2 = \frac{1}{2}\times 2 \times \frac{1^2}{3} \times 15^2 = 75J$

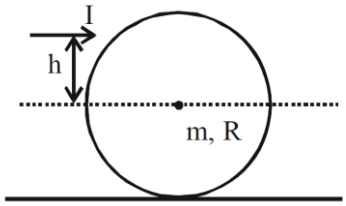
32. A solid sphere of mass m and radius R is placed on a smooth horizontal surface. A sudden blow is given horizontally to the sphere at a height h = 4R/5 above the centre line. If I is the impulse of the blow then find

(a) the minimum time after which the highest point B will touch the ground

(b) the displacement of the centre of mass during this internal.

Ans. (a) $t = \frac{\pi Rm}{2I}$; (b) $s = \frac{\pi R}{2}$

Sol.



$I\frac{4R}{5} = \frac{2}{5}MR^2\omega$

$\frac{2I}{mR} = \omega$

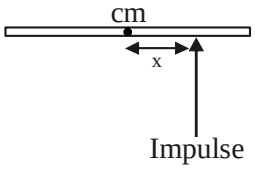
$\omega t = \pi$

$t = \frac{\pi}{\omega} = \frac{\pi MR}{2I}$

(b) $v_{CM} = \frac{I}{m}$

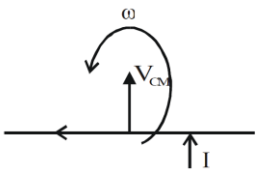
$t \times v_{cm} = \frac{\pi R}{2}$

33. A uniform rod of length *l* is given an impulse at right angles to its length as shown. Find the distance of instantaneous centre of rotation from the centre of the rod.



Ans. $\frac{l^2}{12x}$

Sol.



$V_{cm} = \frac{I}{m}$

$Ix = \frac{m\ell^2}{12}\omega$

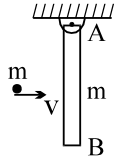
$v_{cm} - wy = 0$

$\Rightarrow y = \frac{v_{cm}}{\omega}$

HCV Exercises (Chapter No.10 - 50,80,82)

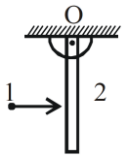
HCV Worked out Examples(Chapter No.10 - 21,29)

34. A uniform rod AB of length L and mass m is suspended freely at A and hangs vertically at rest when a particle of same mass m is fired horizontally with speed v to strike the rod at its mid point. If the particle is comes to rest after the impact. Then find the impulsive reaction at A.



Ans. $\frac{l^2}{12x}$

Sol.



Angular momentum conservation about O.

$\therefore\ mV\frac{\ell}{2} = 0 + \frac{m\ell^2}{3}\omega$

$\therefore\ \omega\ell = V\frac{3}{2}$

Now V_{CM} of rod = $\frac{\omega}{2} = \frac{3V}{4}$

$\therefore\ I_1$ on (1) = mv

$I_1 - N_{imp} = m\frac{3v}{4}$

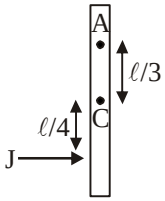
$\therefore\ mv - N_{Im} = \frac{3mv}{4}$

$\therefore\ N_{Im} = \frac{mv}{4}$

35. A uniform rod of mass m and length ℓ is placed in gravity free space and linear impulse J is given to the rod at a distance x = ℓ/4 from centre 'C' and perpendicular to the rod. Point A is at a distance ℓ/3 from centre as shown in the figure. Then find

(i) Speed of centre of rod (ii) Speed of point A

(iii) Speed of upper end of rod (iv) Speed of lower end of rod



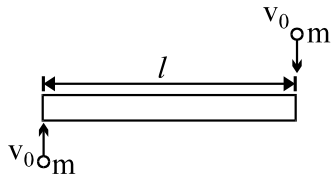
Ans. (i) $\frac{J}{m}$; (ii) zero; (iii) $\frac{J}{2m}$; (iv) $\frac{5}{2}\frac{J}{m}$

Sol. $J\frac{\ell}{4} = \frac{m\ell^2}{12}\omega \Rightarrow \omega = \frac{3J}{m\ell}$, $v_{cm} = \frac{J}{m}$; $v_A = v_{cm} - \frac{\ell}{3}\omega = 0$, $v_{upper} = \left|v_{cm} - \frac{\ell}{2}\omega\right| = \frac{J}{2m}$

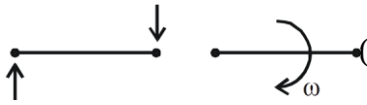
$v_{lower} = \left|v_{cm} + \frac{\ell}{2}\omega\right| = \frac{5}{2}\frac{J}{m}$

36. On a smooth table two particles of mass m each, travelling with a velocity v_0 in opposite directions, strike the ends of a rigid massless rod of length *l*, kept perpendicular to their

velocity. The particles stick to the rod after the collision. Find the tension in rod during subsequent motion.



Ans. $\frac{2mv_0^2}{l}$

Sol.  (V_{cm} of rod = 0 as $P_i = P_f$)

$L_i = L_f$

$\therefore\ \omega = \frac{2v_0}{\ell}$

$T = m\omega^2 \frac{\ell}{2} = \frac{m4v_0^2}{\ell^2} \times \frac{\ell}{2} = \frac{2mv_0^2}{\ell}$

37. A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end 'A' of the rod with a velocity v_0 in the direction perpendicular to AB. The collision is completely elastic. After the collision the particle comes to rest.

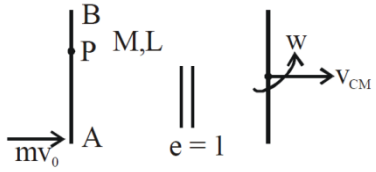
(a) Find the ratio m/M .

(b) A point P on the rod is at rest immediately after the collision. Find the distance AP.

(c) Find the linear speed of the point P at a time $\pi L/(3v_0)$ after the collision.

Ans. (a) $\frac{m}{M} = \frac{1}{4}$; (b) $x = \frac{2L}{3}$; (c) $\frac{v_0}{2\sqrt{2}}$

Sol.



$e = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{\frac{\omega L}{2} + v_{cm}}{v_0}$

$v_{CM} = \frac{mv_0}{M}$ & $mv_0 \frac{L}{2} = \frac{m_1 L^2}{12}\omega$

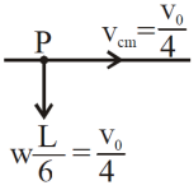
$\omega = \frac{6mv_0}{ML}$

$$\frac{\frac{3mv_0}{M}+\frac{mv_0}{M}}{v_0}=1\Rightarrow\frac{m}{M}=\frac{1}{4}$$

(b) $v_{cm} - \omega x = 0$

$\therefore x = \frac{L}{6}$ $\qquad\qquad$ $\therefore \frac{L}{2}+\frac{L}{6}=\frac{2L}{3}$

(c) $\omega\ell = \frac{3}{2}v_0$



$\omega t = \frac{\omega\pi\ell}{3v_0}$

$\frac{\pi\ell}{3v_0}\times\frac{3v_0}{2\ell}=\frac{\pi}{2}$

$\therefore$ after $\frac{\pi}{2}$

HCV Exercises (Chapter No.10 - 60,61,63,64,68)

EXERCISE # (S-2)

1. Find the MI of a rod about an axis through its centre of mass and perpendicular to the length whose linear density varies as $\lambda=ax$ where a is a constant and x is the position of an element of the rod relative to its left end. The length of the rod is ℓ.

Ans. $\frac{a\ell^4}{36}$

Sol. $\int dmx^2 = \int_0^{\ell} \lambda dx x^2 = \frac{a\ell^4}{4}$

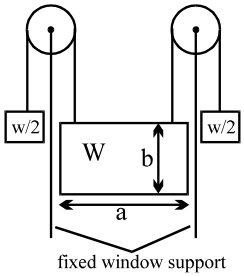
Now CM = d = $\frac{\int_r^{\ell} \lambda dx x}{\int_0^{\ell} \lambda dx} = \frac{2\ell}{3}$

$\therefore\ I_{CM} = I_0 - md^2 = \frac{a\ell^4}{4} - md^2$

$m = \int \lambda dx = \frac{a\ell^2}{2}$

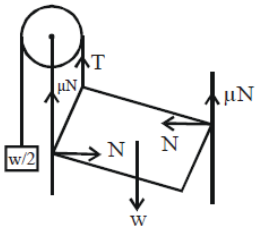
$\therefore\ \frac{a\ell^4}{4} - \left(\frac{a\ell^2}{2}\right)\frac{4}{9}\ell^2 = \frac{a\ell^4}{36}$

2. A slightly loosely fit window is balanced by two strings which are connected to weights w/2 each. The strings pass over the frictionless pulleys as shown in the figure. The strings are tied almost at the corner of the window. The string on the right is cut and then the window accelerates downwards. If the coefficients of friction between the window and the side supports is μ then calculate the acceleration of the window in terms of μ, a, b and g, where a is width and b is the length of the window.



Ans. $a = \left[\frac{b-\mu a}{3b+\mu a}\right] g$

Sol.



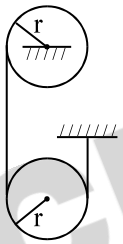
$w - 2\mu N - T = m \times acc.$

$$2N\frac{b}{2} = T\frac{a}{2} \Rightarrow N = \frac{Ta}{2b}$$

$$w - 2\mu\frac{Ta}{2b} - T = m \times acc$$

$T - w/2 = m/2 \times acc.$

3. Two uniform cylinders, each of mass m = 10 kg and radius r = 150 mm, are connected by a rough belt as shown. If the system is released from rest, determine

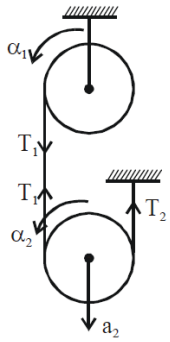


(a) the velocity of the centre of cylinder A after it has moved through 1.2 m &
(b) the tension in the portion of the belt connecting the two cylinders.

Ans. (a) $4\sqrt{\frac{3}{7}}$ m/s, (b) $\frac{200}{7}$ N

Sol. $$T_1 r = \frac{mr^2}{2}\alpha_1$$

$$T_2 r - T_1 r = \frac{mr^2}{2}\alpha_2 = \frac{ma_2}{2}$$



$a_1 r = a_2 + \alpha_2 r = 2a_2$

$mg - T_1 - T_2 = ma_2$

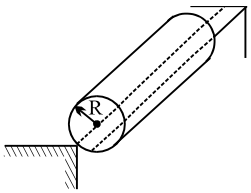
$a = \alpha 2r$

$\therefore\ T_1 = ma$

$T_2 - T_1 = \frac{ma}{2}$

$mg - T_1 - T_2 = ma$ $\quad\therefore\ mgT_1 = \frac{5}{2}ma$

4. A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius R is placed horizontally at rest with its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane as shown in figure. there is sufficient friction present at the edge so that a very small displacement cause the cylinder to roll of the edge without slipping. Determine **[JEE 95]**

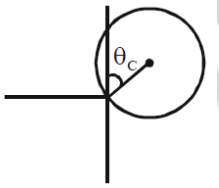


(a) the angle θ_C through which the cylinder rotates before it leaves contact with the edge,
(b) the speed of the centre of mass of the cylinder before leaving contact with the edge, and
(c) the ratio of the translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge.

Ans. (a) $\theta_C = \cos^{-1}(4/7)$, (b) $v = \sqrt{4/7\ gR}$, (c) $K_T/K_R = 6$

Sol. $mg\cos\theta_C = \frac{mv^2}{R}$

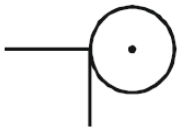
$mg\,R(1-\cos\theta_C) = \frac{1}{2}mV_C^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2$



$mgR(1-\cos\theta) = \frac{3}{4}mV_1^2$

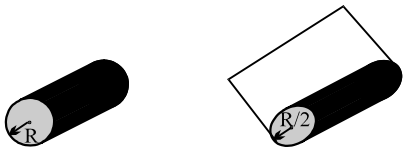
$\therefore\ \frac{1-\cos\theta}{\cos\theta} = \frac{3}{4}$ $\qquad\therefore\ \frac{4r}{7} = \cos\theta_C$

(b) $V_C = \sqrt{\frac{4}{7}gR}$ $\qquad\therefore\ \omega_C = \sqrt{\frac{4}{7}\frac{g}{R}}$

(c)  $\qquad mgR = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2\omega^2}{2}$

5. A carpet of mass 'M' made of inextensible material is rolled along its length in the form of a cylinder of radius 'R' and is kept on a rough floor. The carpet starts unrolling without sliding on

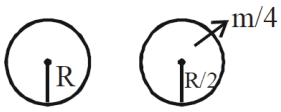
the floor when a negligibly small push is given to it. The horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2 will be: **[JEE 90]**



Ans. $v = \sqrt{\frac{14gR}{3}}$

Sol. $w_{fr} = 0$

$w_g = KE_f$

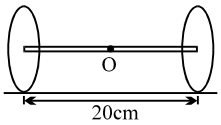


$$mgR - \frac{m}{4}\frac{gR}{2} = \frac{1}{2}\frac{m}{4}v^2 + \frac{1}{2}\frac{\frac{m}{4}\left(\frac{R}{2}\right)^2}{2}w^2$$

$$\frac{7}{8}mgR = mv^2\left(\frac{1}{8}+\frac{1}{16}\right) = mv^2\frac{3}{16}$$

$$\therefore\ v = \sqrt{\frac{14}{3}gR}$$

6. Two thin circular disks of mass 2kg and radius 10 cm each are joined by a rigid massless rod of length 20cm. The axis of the rod is along the perpendicular to the planes of the disks through their centre. The object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of motion of the truck. Its friction with the floor of the truck is large enough so that the object can roll on the truck without slipping. Take x-axis as the direction of motion of the truck and z -axis as the vertically upwards direction. If the truck has an acceleration of $9m/s^2$ calculate : **[JEE 97]**

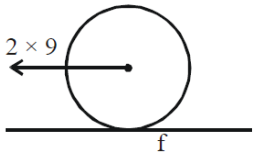


(a) the force of friction on each disk

(b) The magnitude and the direction of the frictional torque acting on each disk about the centre of mass O of the object. Express the torque in the vector form of unit vectors in the x-y and z directions.

Ans. *6N*, $-0.6\hat{j} \pm 0.6\hat{k}$

Sol.



(a) $18 - f = 2a$

$f \times .1 = \frac{2(.1)^2}{2}\alpha$

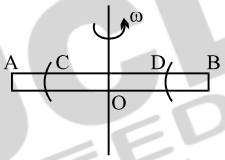
$\therefore\ f = 6N$

(b) $\frac{2(.1)^2}{2}\alpha$

$\therefore\ -.6\left(\hat{j}\times\hat{i}\right)-.6\left(\hat{k}\times\hat{j}\right)$

$= .6\hat{k} - .6\hat{j}$

7. A rigid horizontal smooth rod AB of mass 0.75 kg and length 40 cm can rotate freely about a fixed vertical axis through its mid point O. Two rings each of mass 1 kg are initially at rest at a distance of 10 cm from O on either side of the rod. The rod is set in rotation with an angular velocity of 30 radians per second. Find the velocity of each ring along the length of the rod in m/s when they reach the ends of the rod



Ans. 3

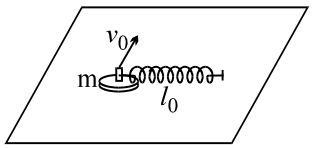
Sol. $\left(\frac{M_r\ell^2}{12}+2m_p(.1)^2\right)w=\left(\frac{M\ell^2}{12}+2M_p(.2)^2\right)w'$

$\Rightarrow \left(\frac{.75\times.4\times.4}{12}+2\times1\times.4\right)30=\left(\frac{70\times.4\times.4}{12}+2\times1\times(.2)^2\right)w$

$KE_i = KE_f$

$v = 3m/s$

8. One side of a spring of initial, unstretched length l_0 = 1 m, lying on a frictionless table, is fixed, the other one is fastened to a small puck of mass m = 0.1 kg. The puck is given velocity in a direction perpendicular to the spring, at an initial speed v_0 = 11 m/s. In the course of the motion, the maximum elongation of the spring is $l=l_0/10$. What is the force constant of the spring (in SI units)?



Ans. 210

Sol. At maximum elongation velocity is again perpendicular to the spring.

$$\vec{L}_i = \vec{L}_f$$

$$mv_0\ell_0 = mv\left(\ell_0 + \frac{\ell_0}{10}\right)$$

$$mv_0\ell_0 = mv\ \frac{11\ell_0}{10}$$

$$\Rightarrow \qquad v = 10 m/s \qquad (\because v_0 = 11\ m/s)$$

Energy conservation

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}K\left(\frac{\ell_0}{10}\right)^2$$

On soling

K = 210 N/m.

9. A uniform disk of mass m and radius R is projected horizontally with velocity v_0 on a rough horizontal floor so that it starts off with a purely sliding motion at t=0. After t_0 seconds it acquires a purely rolling motion as shown in figure. **[JEE 97]**

(i) Calculate the velocity of the centre of mass of the disk at t_0.

(ii) Assuming the coefficient of friction to be μ calculate t_0. Also calculate the work done by the frictional force as a function of time and the total work done by it over a time t much longer than t_0.

Ans. (i) $2v_0/3$, (ii) $t = v_0/3\mu g$, $W = \frac{1}{2}[3\mu^2 m g^2 t^2 - 2\mu m g t v_0]$ $(t < t_0)$, $W = -\frac{1}{6}mv_0^2$ $(t > t_0)$

Sol. $mv_0R = mv'R + \frac{mR^2}{2}\omega = \frac{3}{2}mv'$

$$\therefore\ v' = \frac{2v_0}{3}$$

$a = \mu g \qquad \therefore\ \frac{v_0 - v'}{\mu g} = t = \frac{v_0}{3\mu g}$

$v = v_0 - \mu g t$

$$\mu_0 mgR = \frac{mR^2}{2}\alpha$$

$$\therefore\ \alpha = \frac{2\mu_0 g}{R} \qquad \therefore\ \omega = \frac{2\mu_0 g}{R}t$$

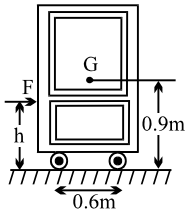
$KE_f - KE_i = W_{fr}$

$$\frac{1}{2}m(v_0 - \mu g t)^2 + \frac{1}{2}\frac{\mu R^2}{2}w^2$$

$$= \frac{1}{2}m\left(v_0^2 + \mu^2 g^2 t^2 - 2\mu v_0 g t\right) + \frac{1}{2}\frac{\mu R^2}{2}\frac{4\mu_0^2 g^2 t^2}{R^2}$$

after $t = t_0$, w_{fr} = constant = $-\frac{1}{6}mv_0^2$

10. A 20 kg cabinet is mounted on small casters that allow it to move freely ($\mu = 0$) on the floor. If a 100 N force is applied as shown, determine



(a) the acceleration of the cabinet,

(b) the range of values of h for which the cabinet will not tip.

Ans. (a) 5 m/s^2 $\rightarrow$, (b) $0.3 < h < 1.5$ m

Sol. $F = ma \Rightarrow a = 5$ m/s^2

(ii) $F(h - .9) = mg \times .3$

$\therefore\ 100\ (h - .9) = 200 \times .3$

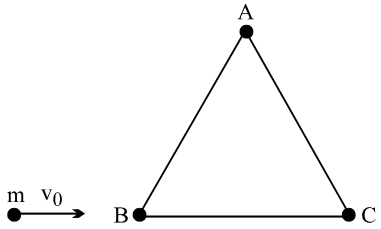
$\therefore\ h = 1.5$

& $100\ (.9 - h) = 200 \times 3$

$\Rightarrow h = .3$

$\therefore\ .3 < h < 1.5$

11. Three particles A, B, C of mass m each are joined to each other by massless rigid rods to form an equilateral triangle of side a. Another particle of mass m hits B with a velocity v_0 directed along BC as shown. The colliding particle stops immediately after impact.

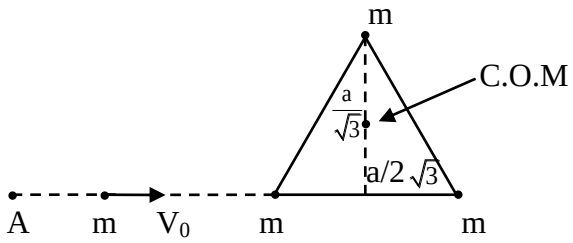


(i) Calculate the time required by the triangle ABC to complete half-revolution in its subsequent motion.

(ii) What is the net displacement of point B during this interval ?

Ans. (i) $t = \frac{6a\pi}{\sqrt{3}v_0}$; (ii) $s = \frac{a}{\sqrt{3}}\sqrt{1 + \left(2\pi + \sqrt{3}\right)^2}$

Sol.



Angular momentum about A is conserved

$$\vec{L}_i = \vec{L}_f$$

$$O = (3m)V_{cm}\frac{a}{2\sqrt{3}} - \left[\left(m\left(\frac{a}{\sqrt{3}}\right)^2\right)\times 3\right]\omega \qquad \ldots(1)$$

Conserving linear momentum.

$$\vec{P}_i = \vec{P}_f$$

$$mv_0 = 3mV_{cm}$$

$$V_{cm} = \frac{V_0}{3} \qquad \ldots(2)$$

from (1) & (2)

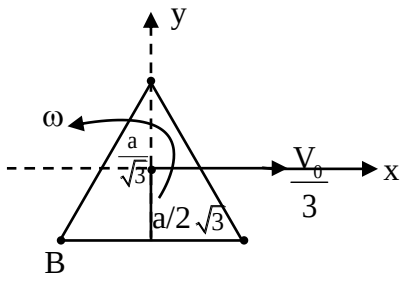
$$\omega = \frac{V_0}{2\sqrt{3}a}$$

(i) $\theta = \omega t = \pi$

$$t = \frac{\pi}{\omega} = \frac{\pi}{V_0}2\sqrt{3}a$$

$$\boxed{\omega = \frac{6\pi a}{\sqrt{3}V_0}}$$

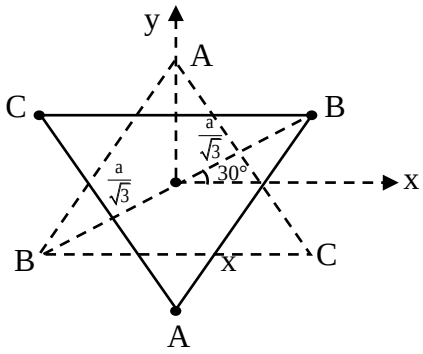
(ii)



$$\left(-\frac{a}{2}, -\frac{a}{2\sqrt{3}}\right)$$

$$\vec{S}_B = \vec{S}_{B/cm} + \vec{S}_{cm}$$

In half revolution, B is rotated angle π about C.O.M



displacement of B w.r.t. C.O.M

$$\vec{S}_{B/cm} = \frac{2a}{\sqrt{3}}\cos 30°\hat{i} + \frac{2a}{\sqrt{3}}\sin 30°\hat{j}$$

$$\vec{S}_{B/cm} = a\hat{i} + \frac{a}{\sqrt{3}}\hat{j}$$

Displacement of C.O.M

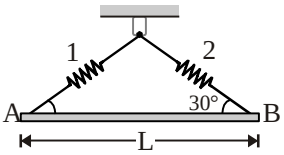
$$\vec{S}_{cm} = \frac{V_0}{3}t = \frac{V_0}{3}\left(\frac{6\pi a}{\sqrt{3}V_0}\right) = \frac{2\pi a}{\sqrt{3}}\hat{i}$$

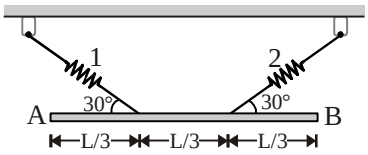
$$\vec{S}_B = \vec{S}_{B/cm} + \vec{S}_{cm}$$

$$= \left(a + \frac{2\pi a}{\sqrt{3}}\right)\hat{i} + \frac{a}{\sqrt{3}}\hat{j}$$

$$|\vec{S}| = \frac{a}{\sqrt{3}}\sqrt{(\sqrt{3}+2\pi)^2 + 1}$$

12. A uniform slender bar AB of mass m is suspended from two springs as shown. If spring 2 breaks, determine at that instant;





(a) the angular acceleration of the bar.

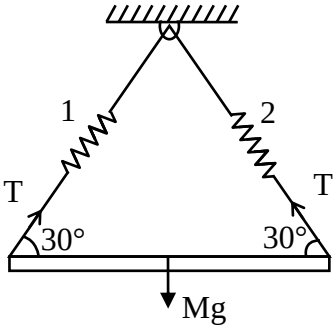
(b) the acceleration of point A.

(c) the acceleration of point B.

Ans. (i) (a) 3 g/L (cw) (b) $\left(\frac{\sqrt{3}}{2}\hat{i}+\hat{j}\right)g = 1.323g \angle 49.1°$ (c) $\left(\frac{\sqrt{3}}{2}\hat{i}-2\hat{j}\right)g = 2.18g \angle -66.6°$

(ii) (a) g/L (cw) (b) $-\frac{\sqrt{3}}{2}g\hat{i}$ (c) $-\left(\frac{\sqrt{3}}{2}\hat{i}+\hat{j}\right)g = 1.323g \angle -130.9°$

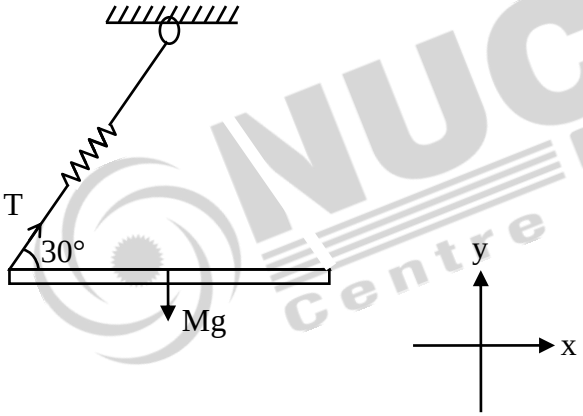
Sol.



$2T \sin 30° = Mg$

$T = Mg$

After break of spring 2



$\Sigma F = Ma_{cm}$ $\quad T\cos 30° = M(a_{cm})_x$

$Mg - T\sin 30° = M(a_{cm})_y$ $\quad \boxed{(a_{cm})_x = \frac{\sqrt{3}}{2}g}$

$\boxed{(a_{cm})_y = -\frac{g}{2}}$

(a) about C.O.M.

$\tau = I\alpha$

$(T\sin 30°)\frac{L}{2} = \left(\frac{ML^2}{12}\right)\alpha$

$\boxed{\alpha = \frac{3g}{L}}$ Clock wise

(b) $\vec{a}_A = \vec{a}_{A/cm} + \vec{a}_{cm}$

$= \left(\alpha\frac{L}{2}\right)\hat{j} + \left(\frac{\sqrt{3}}{2}g\hat{i} - \frac{g}{2}\hat{j}\right)$

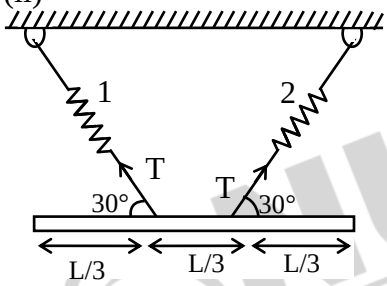
$= \left(\frac{\sqrt{3}}{2}\hat{i} + \hat{j}\right)g.$

(c) $\vec{a}_B = \vec{a}_{B/cm} + \vec{a}_{CM}$

$= \alpha\frac{L}{2}(-\hat{j}) + \left(\frac{\sqrt{3}}{2}g\hat{i} - \frac{g}{2}\hat{j}\right)$

$= \left(\frac{\sqrt{3}}{2}\hat{i} - 2\hat{j}\right)g.$

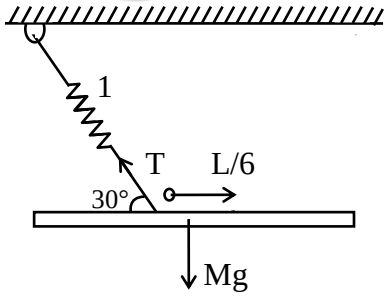
(ii)



$2T\sin30° = Mg$

$T = Mg$

after cutting



$\boxed{(a_{cm})_y = -\frac{g}{2}}$ $\boxed{(a_{cm})_x = -\frac{\sqrt{3}}{2}g}$

(a) $\tau = I\alpha$ about C.O.M

$(T\sin30°)\ \frac{L}{6} = \left(\frac{ML^2}{12}\right)\alpha.$

$\boxed{\alpha = \frac{g}{L}}$ Clock rise.

(b) $\vec{a}_{A/cm} + \vec{a}_{cm} = \vec{a}_A$

$= \left(\alpha \frac{L}{2}\right)\hat{j} + \left(-\frac{\sqrt{3}}{2} g\hat{i} - \frac{g}{2}\hat{j}\right)$

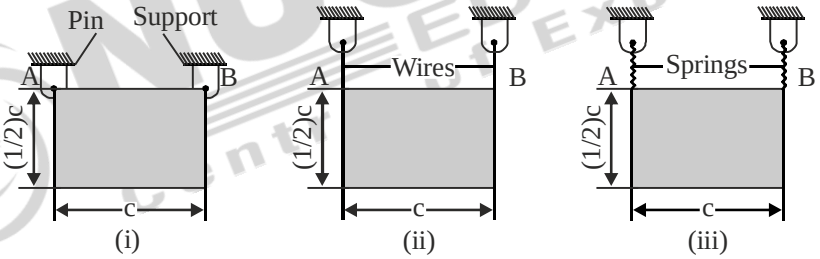
$= -\frac{\sqrt{3}}{2} g\hat{i}$

(c) $\vec{a}_B = \vec{a}_{B/cm} + \vec{a}_{cm}$

$= \alpha \frac{L}{2}\left(-\hat{j}\right) + \left(-\frac{\sqrt{3}}{2} g\hat{i} - \frac{g}{2}\hat{j}\right)$

$= \left(-\frac{\sqrt{3}}{2}\hat{i} - \hat{j}\right) g.$

13. A uniform plate of mass m is suspended in each of the ways shown. For each case determine immediately after the connection at B has been released;



(a) the angular acceleration of the plate.
(b) the acceleration of its mass center.

Ans. (i) (a) $\frac{1.2g}{c}$ (cw) (b) –03. $(\hat{i} + 2\hat{j})g$ (ii) (a) 24g/17c (cw) (b) 12 g/17↓
(iii) (a) 2.4 g/c (cw) (b) 0.5 g↓

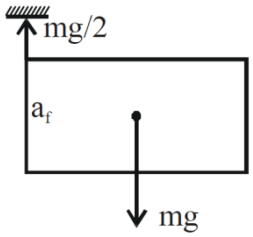
Sol. $mg\frac{C}{2} = I\alpha$

$= \left(m\frac{C^2}{3} + \frac{m\left(\frac{C}{2}\right)^2}{3}\right)\alpha$

$\Rightarrow \alpha = 1.2\ \frac{g}{c}$

$$-\alpha\frac{C}{2}\hat{j}-\alpha\frac{C}{4}\hat{i}=.6g\hat{j}-.3g\hat{i}$$

(ii)



$$\frac{TC}{2}=\left(\frac{mC^2}{12}+\frac{m(C/2)^2}{12}\right)\alpha$$

$$mg-T=ma_y=\frac{m\alpha C}{2}$$

$$\frac{\alpha C}{2}=a_y$$

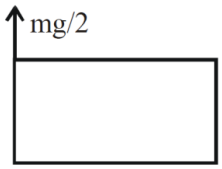
$$\therefore\ T\frac{C}{2}=\frac{5}{4}\frac{mL^2}{12}\alpha$$

$$\therefore\ mg=\left(\frac{5}{24}+\frac{1}{2}\right)mC\alpha$$

$$\therefore\ \alpha=\frac{24}{17}gC$$

(b) $a_y=\frac{\alpha C}{2}=\frac{12g}{17}$

(iii)

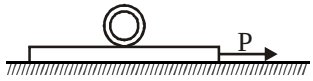


$$\frac{mg}{2}\frac{C}{2}=\frac{5}{4}\frac{mC^2}{12}\alpha$$

$$\therefore\ \alpha=\frac{2.4g}{C}$$

$$mg-\frac{mg}{2}=ma_{cm}\Rightarrow\frac{g}{2}=a_{cm}$$

14. A 160 mm diameter pipe of mass 6 kg rests on a 1.5 kg plate. The pipe and plate are initially at rest when a force P of magnitude 25 N is applied for 0.75s. Knowing that μ_s = 0.25 & μ_k = 0.20 between the plate and both the pipe and the floor, determine;

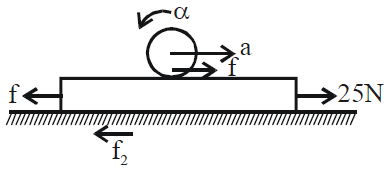


(a) whether the pipe slides with respect to the plate .

(b) the resulting velocities of the pipe and of the plate .

Ans. **(a)** pipe rolls without sliding **(b)** pipe : 5/6 m/s →, 125/24 rad/s (ccw) ; plate : 5/3 m/s →

Sol.

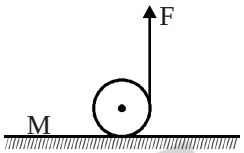


$25 - f - f_2 = 1.5(a + \alpha R)$

$f = 6a$

$fR = 6R^2 a$

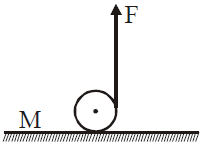
15. The free end of the string wound on the surface of a solid cylinder of mass M & radius r is pulled up by a force F as shown. If there is sufficient friction between cylinder & floor so that the cylinder is able to roll without slipping, find the maximum angular acceleration that the cylinder can have .



Ans. $\alpha_{max} = \dfrac{2g}{3r}$

Sol. $Fr - fr = \dfrac{mr^2}{2}\alpha = \dfrac{ma}{2}$

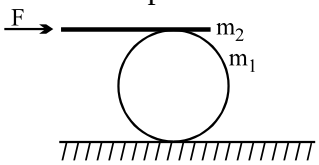
$f = ma$



$\therefore\ F = \dfrac{3}{2}ma$ $\qquad$ $\therefore\ \dfrac{2F}{3mr} = \dfrac{a}{r}$

Now $F_{max} = mg$ $\qquad$ $\therefore\ \dfrac{2g}{3r} = \alpha_{max}$

16. A man pushes a cylinder of mass m_1 with the help of a plank of mass m_2 as shown. There is no slipping at any contact. The horizontal component of the force applied by the man is F. Find



(a) the accelerations of the plank and the center of mass of the cylinder, and
(b) the magnitudes and directions of frictional forces at contact points.]

Ans. **(a)** $a_c = \dfrac{4F}{(3m_1 + 8m_2)}$, $a_p = \dfrac{8F}{(3m_1 + 8m_2)}$; $f_1 = \dfrac{3m_1F}{(3m_1 + 8m_2)}$, $f_2 = \dfrac{m_1F}{(3m_1 + 8m_2)}$

(b) friction at the top of the cylinder = $3m_1F(3m_1+8m_2)$ towards right;
friction at the bottom = $m_1F/(3m_1+8m_2)$ towards right.

Sol. $F - f_1 = M_2(a + \alpha_r) = m_2(2a)$

$f_1 - f_2 = m_1a$

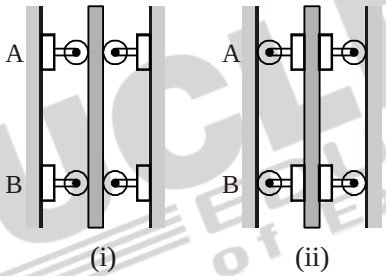
$$(f_1 + f_2)r = \frac{m_1r^2}{2}\alpha = \frac{m_1a}{2}$$

$$f_1 = \frac{3}{4}m_1a$$

$$\therefore F = 2m_2a + \frac{3}{4}m_1a$$

$$\therefore a = \frac{4F}{8m_2 + 3m_1}$$

17. A bar of mass m is held as shown between 4 disks, each of mass m & radius r = 75 mm. Determine the acceleration of the bar immediately after it has been released from rest, knowing that the normal forces exerted on the disks are sufficient to prevent any slipping and assuming that;

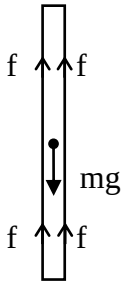


(i) (ii)

(a) m = 5 kg and m' = 2 kg .
(b) the mass of m' of the disks is negligible.
(c) the mass of m of the bar is negligible.

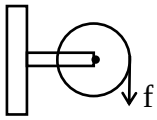
Ans. **(i)** (a) $5g/9\downarrow$ (b) $g\downarrow$ (c) 0; **(ii)** (a) $\frac{13g}{17}\downarrow$ (b) $g\downarrow$ (c) $\frac{2g}{3}\downarrow$

Sol.



(a)

$mg - 4f = ma$ …(1)



$fr = I\alpha$ ...(2)

$\alpha = \frac{a}{r}$ …(3)

Solving (2) & (3)

$f = \frac{m'a}{2}$ …(4) $\left(\because I = \frac{m'r^2}{2}\right)$

solving (1) & (4)

$mg - 4a = ma$

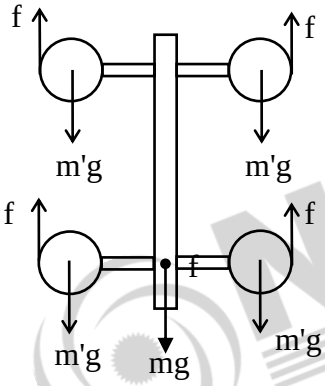
$\boxed{a = \frac{5g}{9}}$

(b) If m' is negligifble then I = 0

$a = g$

(c) a = 0 if m = 0.

(ii) (a)



$(m + 4m')g - 4f = (m + 4m')a$ …(1)

$fr = I\alpha$ …(2)

$\alpha = a/r$ …(3)

$I = \frac{m'r^2}{2}$ …(4)

On solving $\boxed{a = \frac{13g}{17}}$

(b) I of disc ≈ 0

$a = g$

(c) $4m'g - 4f = (4m')a$ …(1)

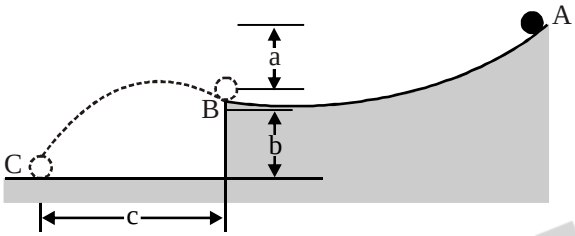
$fr = I\alpha$ …(2)

$\alpha = a/R$ …(3)

$$I = \frac{m'r^2}{2} \qquad \ldots(4)$$

On solving $\boxed{a = \frac{2g}{3}}$

18. A small sphere of mass m and radius r is released from rest at A and rolls without sliding on the curved surface to point B where it leaves the surface with a horizontal velocity. Knowing that a = 1.5 m & b = 1.2 m, determine;



(a) the speed of the sphere as it strikes the ground at C .

(b) the corresponding distance c .

Ans. **(a)** 6.68 m/s **(b)** 2.27 m

Sol. $mg(1.5) = \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}m\omega^2r^2$

$\therefore\ v_A^2 = \frac{10}{7}\times 10\times 1.5$

Now $mg(1.2) = \frac{1}{2}mv_C^2 + \frac{1}{2}mr^2\omega^2 - \left(\frac{1}{2}mv_A^2 + \frac{1}{2}mr^2\omega^2\right)$

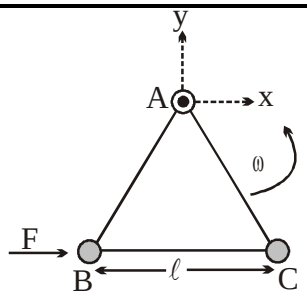
$\therefore\ 24 = v_C^2 - v_A^2$

$\Rightarrow\ v_c^2 = \left(24 + \frac{150}{7}\right)(.98)$

$V_C = (6.67)$m/s

(b) $\sqrt{\frac{2b}{g}}v_A = \sqrt{\frac{2\times 1.2}{9.8}}\times\sqrt{\frac{10\times 1.5\times 9.8}{7}} = 2.27$m

19. Three particles A,B, and C each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side ℓ. This body is placed on a horizontal frictionless table (x-y plane) and is hinged to it at the point A, so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about A with a constant angular velocity ω. **[JEE 2002]**



(a) Find the magnitude of the horizontal force exerted by the hinge on the body.
(b) At time T, when the side BC is parallel to the x-axis, a force F is applied on B along BC (as shown).Obtain the x-component and the y = component of the force exerted by the hinge on the body, immediately after time T.

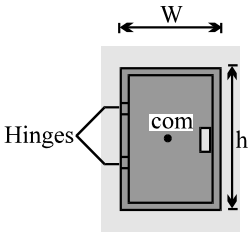
Ans. (a) $\sqrt{3}\,m l\omega^2$ (b) $(F_{net})_x = -\frac{F}{4}$, $(F_{net})_y = \sqrt{3}\,m l\omega^2$

Sol. (a) $3mw^2\left(\frac{\sqrt{3}}{2}\ell\frac{2}{3}\right) = \sqrt{3}\,mw^2\ell$

(b) $F\frac{\sqrt{3}}{2}\ell = 2m\ell\alpha$

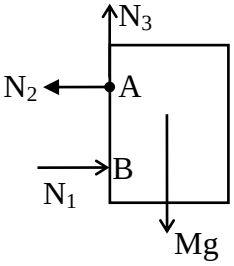
& $F - F_x = \frac{F}{4m}3m$

20. A uniform wood door has mass m, height h, and width w. It is hanging from two hinges attached to one side; the hinges are located h/3 and 2h/3 from the bottom of the door.
Suppose that m = 20.0 kg, h = 2.20 m, and w = 1.00 m and the bottom smooth hinge is not screwed into the door frame. Find the forces acting on the door.



Ans. $F_{dlx} = \frac{3mgw}{2h}$ and $F_{duy} = mg$

Sol.



$N_3 = mg$ …(1)

$N_1 = N_2$ …(2)

About B $\vec{\tau} = 0$

$N_2 \frac{h}{3} = Mg \frac{\omega}{2}.$

$$\boxed{N_2 = \frac{3mg\omega}{2h}}$$

from (2) $N_1 = N_2 = \frac{3mg\omega}{2h}$

& $N_3 = mg$

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EXERCISE # (O-1)

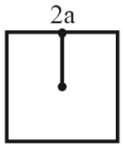
1. For the same total mass which of the following will have the largest moment of inertia about an axis passing through its centre of mass and perpendicular to the plane of the body

(A) a disc of radius a (B) a ring of radius a

(C) a square lamina of side 2a (D) four rods forming a square of side 2a

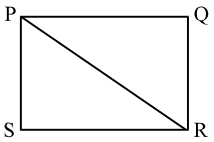
Ans. (D)

Sol. (a) $\frac{ma^2}{2}$; (b) ma^2; (c) $\frac{m(4a^2)}{6}$ (d)



$4 \times \left(\frac{m}{4}\frac{4a^2}{12} + \frac{ma^2}{4}\right) \Rightarrow \frac{ma^2}{3} + ma^2$

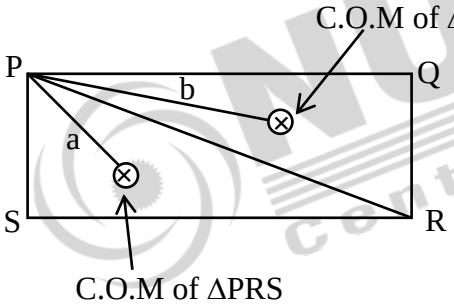
2. Moment of inertia of a rectangular plate about an axis passing through P and perpendicular to the plate is I. Then moment of inertia of PQR about an axis perpendicular to the plane of the plate:



(A) about P = I/2 (B) about R = I/2 (C) about P > I/2 (D) about R > I/2

Ans. (C)

Sol.



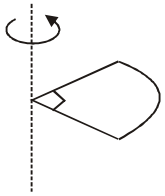
Let mass of each Δ is M.

Then moment of inertia of rectangle about P is

$I_{rectangle} = I_{\Delta PQR} + I\Delta PRS$

$I = \underbrace{\left(I_{cm} + Mb^2\right)}_{\text{this is greater than } \frac{I}{2} \text{ as } b > a} + (I_{cm} + Ma^2)$

3. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the center of the original disc. Its moment of inertia about the axis of rotation is

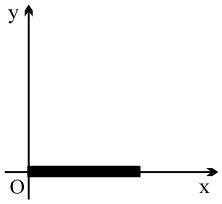


(A) $\frac{1}{2}MR^2$ (B) $\frac{1}{4}MR^2$ (C) $\frac{1}{8}MR^2$ (D) $\sqrt{2}\,MR^2$

Ans. (A)

Sol. $4I = \frac{4MR^2}{2}$; $I = \frac{MR^2}{2}$

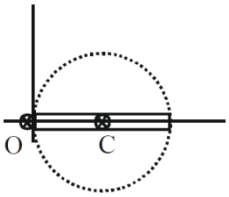
4. The figure shows a uniform rod lying along the x-axis. The locus of all the points lying on the xy-plane, about which the moment of inertia of the rod is same as that about O is



(A) an ellipse (B) a circle (C) a parabola (D) a straight line

Ans. (B)

Sol.



By parallel axis theorem

$I_0 = I_C + m\left(\frac{\ell}{2}\right)^2$

at points at distance of $\frac{\ell}{2}$ from C

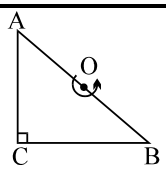
5. Three bodies have equal masses m. Body A is solid cylinder of radius R, body B is a square lamina of side R, and body C is a solid sphere of radius R. Which body has the smallest moment of inertia about an axis passing through their centre of mass and perpendicular to the plane (in case of lamina)

(A) A (B) B (C) C (D) A and C both

Ans. (B)

Sol. $A = \frac{MR^2}{2}; B = \frac{MR^2}{6}; C = \frac{2}{5}MR^2$

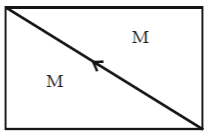
6. Find the moment of inertia of a plate cut in shape of a right angled triangle of mass M, side AC = BC = a about an axis perpendicular to the plane of the plate and passing through the mid point of side AB



(A) $\frac{Ma^2}{12}$ (B) $\frac{Ma^2}{6}$ (C) $\frac{Ma^2}{3}$ (D) $\frac{2Ma^2}{3}$

Ans. (B)

Sol.



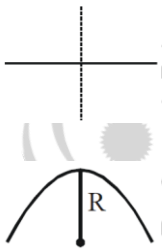
$2I = \frac{2Ma^2}{6}$

7. A thin uniform rod of mass M and length L has its moment of inertia I_1 about its perpendicular bisector. The rod is bend in the form of a semicircular arc. Now its moment of inertia through the centre of the semi circular arc and perpendicular to its plane is I_2. The ratio of $I_1 : I_2$ will be

(A) < 1 (B) > 1 (C) = 1 (D) can't be said

Ans. (A)

Sol.



$I_1 = \frac{ML^2}{12}$

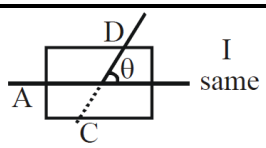
$MR^2 = \frac{ML^2}{\pi^2} = \frac{ML^2}{10} = I_2$

8. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

(A) I (B) $I\sin^2\theta$ (C) $I\cos^2\theta$ (D) $I\cos^2(\theta/2)$

Ans. (A)

Sol.



9. A rigid body can be hinged about any point on the x-axis. When it is hinged such that the hinge is at x, the moment of inertia is given by

$$I = 2x^2 - 12x + 27$$

The x-coordinate of centre of mass is

(A) x = 2 (B) x = 0 (C) x = 1 (D) x = 3

Ans. (D)

Sol. I_C = min

$$\therefore \frac{dI}{dx} = 0 \Rightarrow 4x - 12 = 0 \qquad \therefore x = 3$$

10. If a rigid body is subjected to two forces $\vec{F}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$ acting at (3,3,4) and $\vec{F}_2 = -2\hat{i} - 3\hat{j} - 4\hat{k}$ acting at (1, 0, 0) then which of the following is (are) true? **[REE-94]**

(A) The body is in equilibrium.

(B) The body is under the influence of a torque only.

(C) The body is under the influence of a single force.

(D) the body is under the influence of a force together with a torque .

Ans. (A)

Sol. $\vec{F}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$ at (3,3,4,)

$\vec{F}_2 = -2\hat{i} - 3\hat{j} - 4\hat{k}$ at (1,0,0)

$\Sigma\vec{F} = 0$, hence for equilibrium $\vec{\tau}$ about any point should be zero.

$\vec{\tau}$ about origin.

$$\vec{\tau} = \vec{r}_2 \times \vec{f}_2 + \vec{r}_1 \times \vec{f}_1$$

$$= (-3\hat{k} + 4\hat{j}) + (9\hat{k} - 12\hat{j} - 6\hat{k} + 12\hat{i} + 8\hat{j} - 12\hat{i})$$

$$\vec{\tau} = 0$$

$\Rightarrow$ equilibrium.

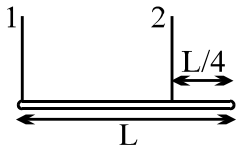
11. In case of torque of a couple, if the axis is changed by displacing it parallel to itself, torque will

(A) increase (B) decrease (C) remain constant (D) None of these

Ans. (C)

Sol. if $\Sigma\vec{F} = 0$, then torque about every point is same.

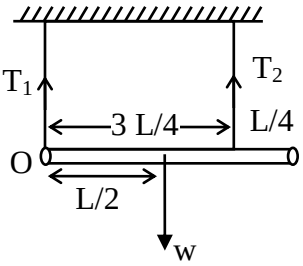
12. A heavy rod of length L and weight W is suspended horizontally by two vertical ropes as shown. The first rope is attached to the left end of the rod while the second rope is attached a distance L/4 from right end. The tension in the second rope is:



(A) (1/2)W (B) (1/4)W (C) (1/3)W (D) (2/3)W

Ans. (D)

Sol.

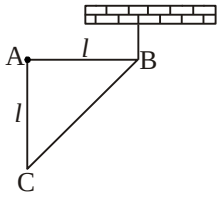


$\vec{\tau}$ about O is zero

$$\frac{wL}{2} - T_2 \frac{3L}{4} = 0$$

$$\Rightarrow T_2 = \frac{2w}{3}$$

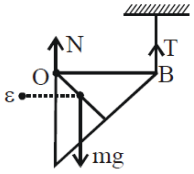
13. A right triangular plate ABC of mass m is free to rotate in the vertical plane about a fixed horizontal axis through A. It is supported by a string such that the side AB is horizontal. The reaction at the support A in equilibrium is:



(A) $\frac{mg}{3}$ (B) $\frac{2mg}{3}$ (C) $\frac{mg}{2}$ (D) mg

Ans. (B)

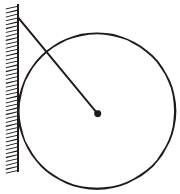
Sol.



N only ↑ because no other Horizontal component

$\tau_B = 0 \Rightarrow N\ell = \frac{mg2\ell}{3}$ $\quad\quad$ $\therefore N = \frac{2mg}{3}$

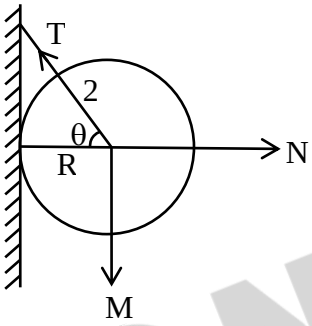
14. A uniform disk of radius R and mass m is connected to a wall by string of length 2R. The normal reaction of wall is



(A) mg (B) $\frac{mg}{2}$ (C) $\frac{mg}{\sqrt{3}}$ (D) 2 mg

Ans. (C)

Sol.



$\theta = 60°$

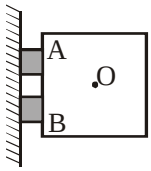
$T\sin\theta = Mg$

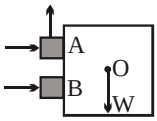
$T\cos\theta = N$

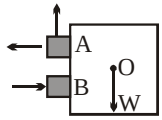
$\tan\theta = \frac{mg}{N}$.

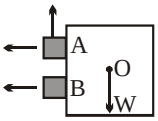
$$\boxed{N = \frac{mg}{\sqrt{3}}}$$

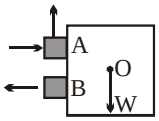
15. A vertical rectangular door with its centre of gravity at O (see figure) is fixed on two hinges A and B along one vertical length side of the door. The entire weight of the door is supported by the hinge A. Then the free body force diagram for the door (the arrows indicate the direction of the forces) is:



(A) 

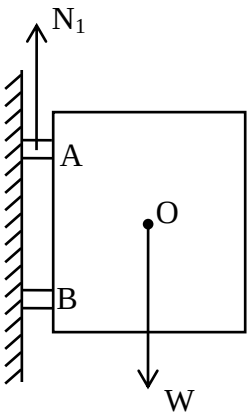
(B) 

(C) 

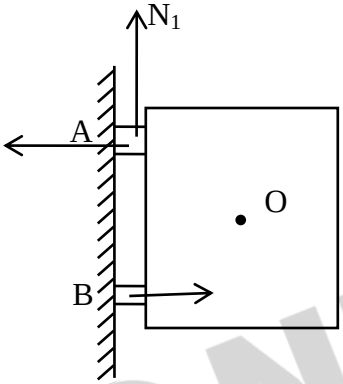
(D) 

Ans. (B)

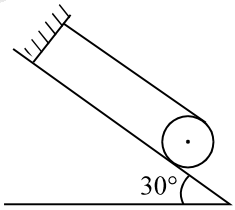
Sol.



Tongue about each point A, B and O should be zero.



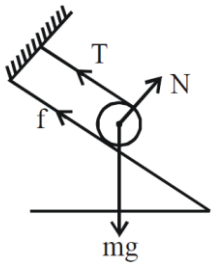
16. A thin hoop of weight 500 N and radius 1 m rests on a rough inclined plane as shown in the figure. The minimum coefficient of friction needed for this configuration to be in equilibrium



(A) $\frac{1}{3\sqrt{3}}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2}$ (D) $\frac{1}{2\sqrt{3}}$

Ans. (D)

Sol.



$T + f = mg \sin 30$

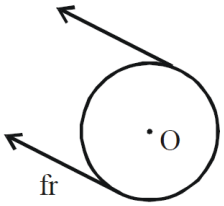
$N = mg \cos 30$

$\tau_0 = 0 \Rightarrow TR = fR$

$\therefore\ 2f = mg \sin 30 = 2\mu N$

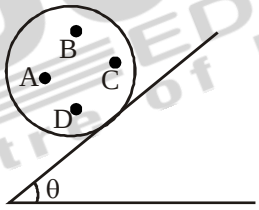
$N = mg \cos 30$

$\Rightarrow 2\mu\ mg \cos 30 = mg \sin 30$



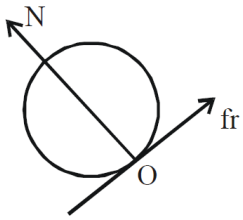
$\therefore\ \mu = \dfrac{1}{2\sqrt{3}}$

17. A non uniform sphere can be kept on a rough inclined plane so that it is in equilibrium. In the figure below the dots represents location of centre of mass. In which one of the positions can sphere be in equilibrium?

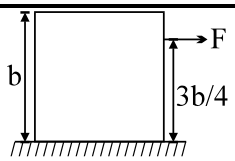


Ans. (C)

Sol. $\tau_0 = 0$, mg should pass through O



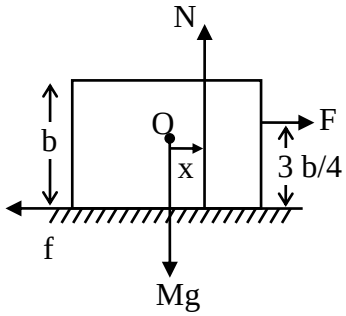
18. A uniform cube of side 'b' and mass M rest on a rough horizontal table. A horizontal force F is applied normal to one of the face at a point, at a height 3b/4 above the base. What should be the coefficient of friction (μ) between cube and table so that is will tip about an edge before it starts slipping?



(A) $\mu > \frac{2}{3}$ (B) $\mu > \frac{1}{3}$ (C) $\mu > \frac{3}{2}$ (D) none

Ans. (A)

Sol.



$N = mg$ …(1)

$F = f$ …(2)

$\vec{\tau}$ about O = 0

$$Nx - f\frac{b}{2} - F\frac{b}{4} = 0 \quad \ldots(3)$$

from (1) & (2)

$$mg\,x - f\frac{b}{2} - f\frac{b}{4} = 0$$

$$f = \frac{4mgx}{3b}$$

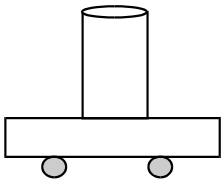
$$x \le \frac{b}{2}$$

$$f_{max} = \frac{4mg}{3b}\left(\frac{b}{2}\right)$$

$$= \frac{2mg}{3} < \mu N \qquad \boxed{\mu > \frac{2}{3}}$$

19. A uniform 2 kg cylinder rests on a laboratory cart as shown. The coefficient of static friction between the cylinder and the cart is 0.5. If the cylinder is 4 cm in diameter and 10 cm in height,

which of the following is closest to the maximum acceleration of the cart such that cylinder neither slips nor tips over ?



(A) 2 m/s^2 (B) 4 m/s^2 (C) 5 m/s^2 (D) 6 m/s^2

Ans. (B)

Sol. N = 20

$f_{max} = .5 \times 20 = 10$

Now for toppling

$2g \times 2 = f \times 5$

$\therefore f = 8$

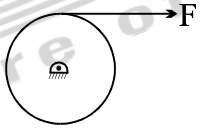
which is < 10

$\therefore$ no slipping

before toppling

$\therefore f = 2a \Rightarrow a = 4$ m/s^2

20. A pulley is hinged at the centre and a massless thread is wrapped around it. The thread is pulled with a constant force F starting from rest. As the time increases,



(A) its angular velocity increases, but force on hinge remains constant

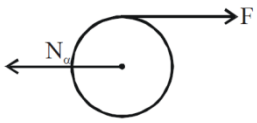
(B) its angular velocity remains same, but force on hinge increases

(C) its angular velocity increases and force on hinge increases

(D) its angular velocity remains same and force on hinge is constant

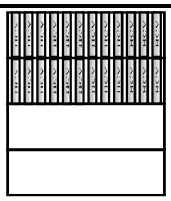
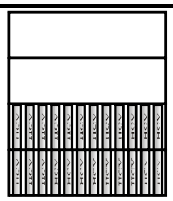
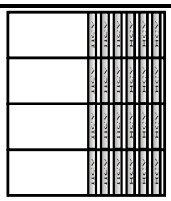
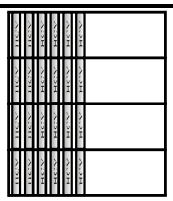
Ans. (A)

Sol.



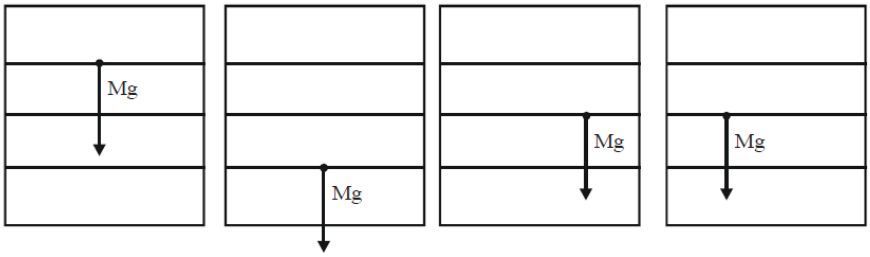
$F = N_\alpha$

21. Same number of books are placed in four book cases as shown. Which bookcase is most likely to topple forward if pulled a little at the top towards right :

(A)  (B)  (C)  (D) 

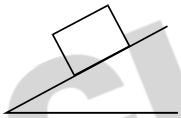
Ans. (C)

Sol.



In C, mg will also help in toppling

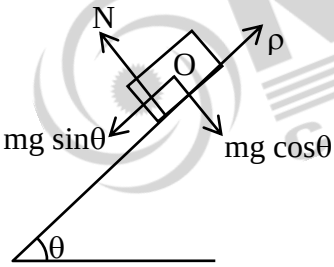
22. A homogeneous cubical brick lies motionless on a rough inclined surface. The half of the brick which applies greater pressure on the plane is :



(A) left half (B) right half
(C) both applies equal pressure (D) the answer depends upon coefficient of friction

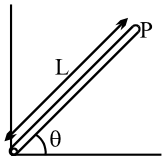
Ans. (A)

Sol.



For torque to be zero about O normal should be shift towards left.
Hence more pressure in on left side.

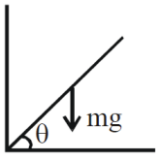
23. A uniform flag pole of length L and mass M is pivoted on the ground with a frictionless hinge. The flag pole makes an angle θ with the horizontal. The moment of inertia of the flag pole about one end is $(1/3)ML^2$. If it starts falling from the position shown in the accompanying figure, the linear acceleration of the free end of the flag pole (labeled P) immediately after that start falling off would be:



(A) (2/3) gcosθ (B) (2/3) g (C) g (D) (3/2) gcosθ

Ans. (D)

Sol.



$$mg\frac{\ell}{2}\cos = \frac{m\ell^2\alpha}{3}$$

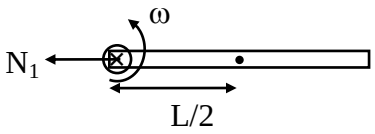
$$\therefore\ \alpha\ell = \frac{3g\cos\theta}{2}$$

24. One end of a uniform rod having mass m and length ℓ is hinged. The rod is placed on a smooth horizontal surface and rotates on it about the hinged end at a uniform angular velocity ω. The force exerted by the hinge on the rod has a horizontal component

(A) $m\omega^2\ell$ (B) zero (C) mg (D) $\frac{1}{2}\ m\omega^2\ell$

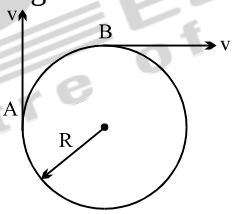
Ans. (D)

Sol.



$N_1 = m\omega^2\ L/2$

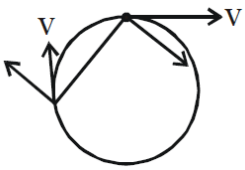
25. Two points of a rigid body are moving as shown. The angular velocity of the body is:



(A) $\frac{\upsilon}{2R}$ (B) $\frac{\upsilon}{R}$ (C) $\frac{2\upsilon}{R}$ (D) $\frac{2\upsilon}{3R}$

Ans. (B)

Sol.



$$\omega = \frac{\frac{v}{\sqrt{2}} + \frac{v}{\sqrt{2}}}{\sqrt{2}R} = \frac{v}{R}$$

26. A man, sitting firmly over a rotating stool has his arms stretched. If he folds his arms, the work done by the man is

(A) zero (B) positive

(C) negative (D) may be positive or negative.

Ans. (B)

Sol. $\vec{L} = I\omega$ conserve

$I_1\omega_1 = I_2\omega_2$

as arm folds I decreases

$I_2 < I_1 \Rightarrow \omega_1 < \omega_2$

$K{\cdot}E_i = \frac{1}{2} I_1\omega_1^2 \qquad K{\cdot}E_f = \frac{1}{2} I_1\omega_2^2$

$= \frac{1}{2}\frac{(I_1\omega_1)^2}{I_1} \qquad = \frac{1}{2}\frac{(I_2\omega_2)^2}{I_2}$

$\frac{K{\cdot}E_i}{K{\cdot}E_f} = \frac{I_2}{I_1} \quad \Rightarrow \quad K{\cdot}E$ increases

$\Rightarrow$ +ve work

27. A particle of mass 2 kg located at the position $(\hat{i} + \hat{j})$ m has a velocity $2(+\hat{i} - \hat{j} + \hat{k})$ m/s. Its angular momentum about z-axis in kg-m^2/s is:

(A) zero (B) +8 (C) 12 (D) –8

Ans. (D)

Sol. $AM = m\vec{r} \times \vec{v} = 2(+i + j) \times (2)(+i - j + k)$

$= -8\hat{k} - 4j + 4i$

$\therefore$ about z axis = –8

28. A spinning ice skater can increase his rate of rotation by bringing his arms and free leg closer to his body. How does this procedure affect the skater's angular momentum and kinetic energy?

(A) angular momentum remains the same while kinetic energy increases

(B) angular momentum remains the same while kinetic energy decreases

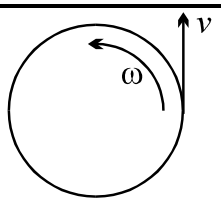
(C) both angular momentum and kinetic energy remain the same

(D) angular momentum increases while kinetic energy remains the same

Ans. (A)

Sol. Moment of inertia decreases therefore angular velocity increases. KE also increases and therefore work done is positive.

29. A child with mass *m* is standing at the edge of a disc with moment of inertia I, radius R, and initial angular velocity ω. See figure given below. The child jumps off the edge of the disc with tangential velocity *v* with respect to the ground. The new angular velocity of the disc is



(A) $\sqrt{\frac{I\omega^2 - mv^2}{I}}$

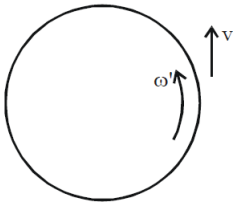
(B) $\sqrt{\frac{(I + mR^2)\omega^2 - mv^2}{I}}$

(C) $\frac{I\omega - mvR}{I}$

(D) $\frac{(I + mR^2)\omega - mvR}{I}$

Ans. (D)

Sol.

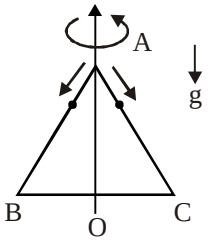


$L_i = L_f$

$\Rightarrow m\omega R^2 + I\omega = mvR + I\omega'$

$\therefore \omega' = \frac{(mR^2 + I)\omega - mvR}{I}$

30. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are



(A) Angular velocity and total energy (kinetic and potential)

(B) Total angular momentum and total energy

(C) Angular velocity and moment of inertia about the axis of rotation.

(D) Total angular momentum and moment of inertia about the axis of rotation.

Ans. (B)

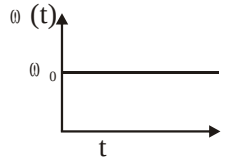
Sol. As external torque about axis is zero hence angular momentum is conserved about axis.
Moment of Inertia about axis is increases as bead moves downward.

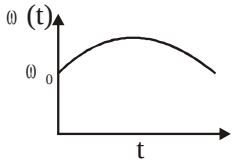
$L_i = L_\rho$

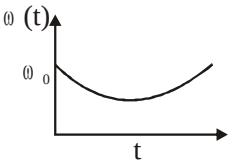
$I_1\omega_1 = I_1\omega_2$

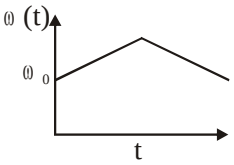
$I_2 > I_1 \Rightarrow \omega_2 < \omega_1$

31. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0 . When the tortoise move along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform $\omega(t)$. will vary with time t as

(A) 

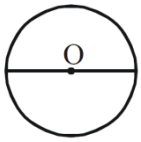
(B) 

(C) 

(D) 

Ans. (B)

Sol.



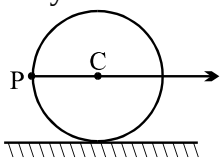
Angular momentum will be conserved about O

$I\omega$ = constant

$I \downarrow\uparrow \quad \therefore \omega \uparrow\downarrow$

$\therefore I = m|d - vt]^2 + I_{disk}$

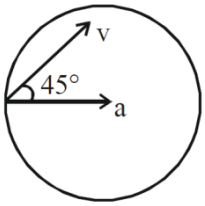
32. A disc of radius R is rolling purely on a flat horizontal surface, with a constant angular velocity. The angle between the velocity and acceleration vectors of point P is



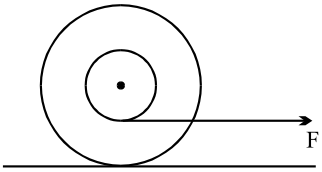
(A) zero (B) 45° (C) 135° (D) $\tan^{-1}(1/2)$

Ans. (B)

Sol.



33. Inner and outer radii of a spool are r and R respectively. A thread is wound over its inner surface and placed over a rough horizontal surface. Thread is pulled by a force F as shown in fig. then in case of pure rolling



(A) Thread unwinds, spool rotates anticlockwise and friction act leftwards

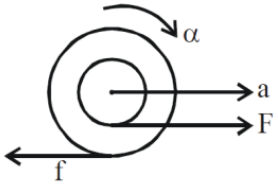
(B) Thread winds, spool rotates clockwise and friction acts leftwards

(C) Thread winds, spool moves to the right and friction act rightwards

(D) Thread winds, spool moves to the right and friction does not come into existence.

Ans. (B)

Sol.



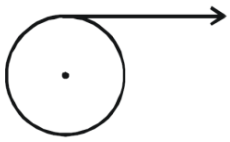
34. A body kept on a smooth horizontal surface is pulled by a constant horizontal force applied at the top point of the body. If the body rolls purely on the surface, its shape can be :

(A) thin pipe (B) uniform cylinder

(C) uniform sphere (D) thin spherical shell

Ans. (A)

Sol.



$x = \frac{I}{MR}$ $\therefore\ R = \frac{I}{MR}$ $\therefore\ I = MR_2$

35. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are

(A) Up the incline while ascending and down the incline while descending

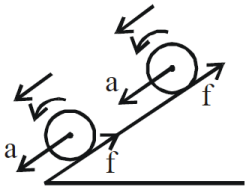
(B) Up the incline while ascending as well as descending

(C) down the incline while ascending and up the incline while descending

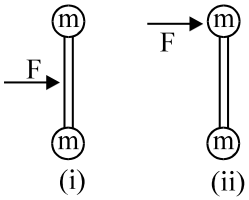
(D) down the incline while ascending as well as descending

Ans. (B)

Sol.



36. A force F is applied to a dumbbell for a time interval, t, first as in (i) and then as in (ii). In which case does the dumbbell acquire the greater centre-of-mass speed?



(A) (i)
(B) (ii)
(C) there is no difference
(D) the answer depends on the rotational inertia of the dumbbell

Ans. (C)

Sol. $V_{cm} = \frac{I}{M} = \frac{Ft}{M}$ same

37. A solid sphere with a velocity (of centre of mass) v and angular velocity ω is gently placed on a rough horizontal surface. The frictional force on the sphere:

(A) must be forward (in direction of v) (B) must be backward (opposite to v)
(C) cannot be zero (D) none of the above

Ans. (D)

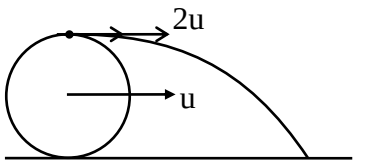
Sol. Depends on v & ω.

38. A bit of mud stuck to a bicycle's front wheel of radius R detaches and is flung horizontally forward when it is at the top of the wheel. The bicycle is moving forward at a speed v and it is rolling without slipping. The horizontal distance travelled by the mud after detaching from the wheel is:

(A) $\sqrt{2rv^2/g}$ (B) $\sqrt{8rv^2/g}$ (C) $\sqrt{4rv^2/g}$ (D) $\sqrt{16Rv^2/g}$

Ans. (D)

Sol.

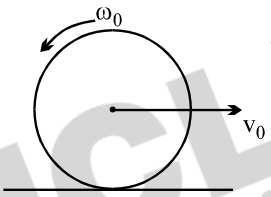


$$t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2(2R)}{g}} = 2\sqrt{\frac{R}{g}}$$

Horizontal Range = $2ut = 4u\sqrt{\frac{R}{g}}$

$$= \sqrt{\frac{16RV^2}{g}}$$

39. A uniform circular disc placed on a rough horizontal surface has initially a velocity v_0 and an angular velocity ω_0 as shown in the figure. The disc comes to rest after moving some distance in the direction of motion. Then is



(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) 2

Ans. (A)

Sol. $L_f = 0 \therefore L_i = 0$

$$mv_0R - \frac{mR^2}{2}\omega_0 = 0$$

$$\therefore \frac{v_0}{r\omega_0} = \frac{1}{2}$$

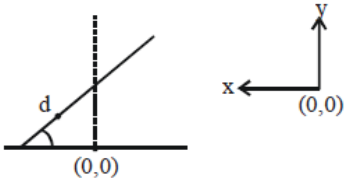
40. A straight rod of length L is released on a frictionless horizontal floor in a vertical position. As it falls + slips, the distance of a point on the rod from the lower end, which follows a quarter circular locus is

(A) L/2 (B) L/4 (C) L/8 (D) None

Ans. (B)

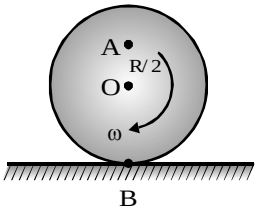
Sol. $x = \left(\frac{L}{2} - d\right)\cos\theta$ & $y = d\sin\theta$

$$\therefore \frac{x^2}{\left(\frac{L}{2} - d\right)} + \frac{y^2}{d^2} = 1$$



for circle $\frac{L}{2} - d = d \Rightarrow d = \frac{L}{4}$

41. A disc is rotating with angular velocity (ω) about its axis (without any translation push) on a smooth surface :
The directions and magnitudes of velocity at points B and A are :



(A) $V_A = +\frac{\omega R}{2}$ (Towards right), $V_B = -\omega R$ (Towards left)

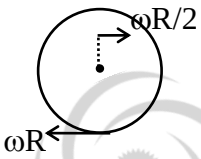
(B) $V_A = -\omega R$ (Towards right), $V_B = \frac{\omega R}{2}$ (Towards left)

(C) $V_A = +\frac{\omega R}{4}$ (Towards right), $V_B = -\omega R$ (Towards left)

(D) $V_A = +\frac{\omega R}{2}$ (Towards right), $V_B = \frac{\omega R}{2}$ (Towards left)

Ans. (A)

Sol.

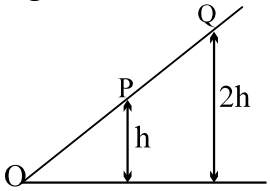


42. A hoop and a solid cylinder have the same mass and radius. They both roll, without slipping, on a horizontal surface. If their kinetic energies are equal
(A) the hoop has a greater translational speed than the cylinder
(B) the cylinder has a greater translational speed than the hoop
(C) the hoop and the cylinder have the same translational speed
(D) the hoop has a greater rotational speed than the cylinder

Ans. (B)

Sol. $\frac{1}{2}MV_{cm}^2 + \frac{1}{2}MR^2\omega^2 = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}\frac{MR^2}{2}\omega'^2$

43. A ball rolls down an inclined plane, figure. The ball is first released from rest from P and then later from Q. Which of the following statement is/are correct?



(i) The ball takes twice as much time to roll from Q to O as it does to roll from P to O.

(ii) The acceleration of the ball at Q is twice as large as the acceleration at P.
(iii) The ball has twice as much K.E. at O when rolling from Q
as it does when rolling from P.
(A) i, ii only (B) ii, iii only (C) i only (D) iii only

Ans. (D)

Sol. $KE_{QO} = mg2h$ & $KQ_{PO} = mgh$

44. The moment of inertia of a solid cylinder about its axis is given by $(1/2)MR^2$. If this cylinder rolls without slipping, the ratio of its rotational kinetic energy to its translational kinetic energy is
(A) 1 : 1 (B) 2 : 2 (C) 1 : 2 (D) 1 : 3

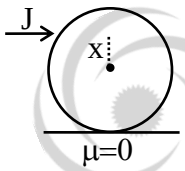
Ans. (C)

Sol. $$\frac{\frac{1}{2}\frac{MR^2}{2}\omega^2}{\frac{1}{2}Mv^2} = \frac{1}{2}$$

45. A non uniform ball of radius R and radius of gyration about geometric center = R/2, is kept on a frictionless surface. The geometric center coincides with the centre of mass. The ball is struck horizontally with a sharp impulse = J. The point of application of the impulse is at a height 'h' above the surface. Then
(A) The ball will slip on surface for all cases. (B) The ball will roll purely if h = 5R/4.
(C) The ball will roll purely if h = 3R/2. (D) None of these

Ans. (B)

Sol.

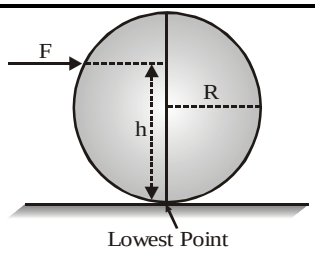


$$x = \frac{I}{MR} \text{ (for pure rolling)}$$

$$I = M\left(\frac{R}{2}\right)^2 \qquad x = \frac{\frac{MR^2}{4}}{MR} = \frac{R}{4}$$

$$h = R + x = \frac{5R}{4}.$$

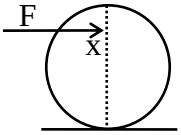
46. A solid sphere of radius R is placed on smooth horizontal surface. A horizontal force 'F' is applied at height 'h' from the lowest point. For the maximum acceleration of centre of mass, which is correct-



(A) h = R (B) h = 2R (C) h = 0 (D) Independent on h

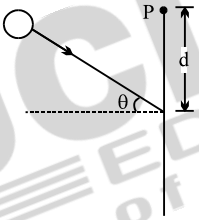
Ans. (D)

Sol.



$a_{cm} = \frac{F}{M}$ (always)

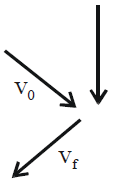
47. A ball of mass m moving with velocity v, collide with the wall elastically as shown in the figure. After impact the change in angular momentum about P is:



(A) 2 *mvd* (B) 2 *mvd cosθ* (C) 2 *mvd* sin θ (D) zero

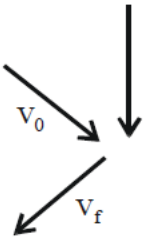
Ans. (B)

Sol.



$Am_i = mrv \sin(90 - \theta)$

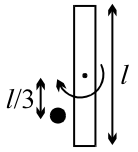
$= mdv \cos\theta$



$Am_f = -mdv \cos\theta$

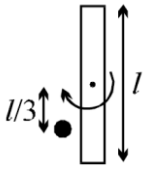
$\therefore \Delta Am = 2mdv \cos\theta$

48. A uniform rod of length l and mass M rotating about a fixed vertical axis on a smooth horizontal table. It elastically strikes a particle placed at a distance $l/3$ from its axis and stops. Mass of the particle is



(A) 3M (B) $\frac{3M}{4}$ (C) $\frac{3M}{2}$ (D) $\frac{4M}{3}$

Ans. (B)



Sol. m_1 be mass of particle

$$\frac{1}{2}I\omega^2 = \frac{1}{2}m_1v^2$$

$L_i = L_f$

$$I\omega = m_1v\frac{\ell}{3}$$

get m_1

49. A mass m is moving at speed v perpendicular to a rod of length d and mass $M = 6m$ which pivots around a frictionless axle running through its centre. It strikes and sticks to the end of the rod. The moment of inertia of the rod about its centre is $Md^2/12$. Then the angular speed of the system right after the collision is

(A) $2v/d$ (B) $2v/(3d)$ (C) v/d (D) $3v/(2d)$

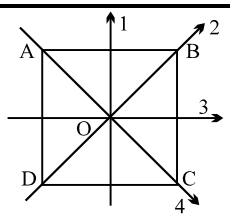
Ans. (B)

Sol. $$mv\frac{d}{2} = m\frac{\omega d}{2}\frac{d}{2} + \frac{6md^2}{12}\omega$$

$$\therefore\ \omega = \frac{2v}{3d}$$

MULTIPLE CORRECT TYPE QUESTIONS

50. ABCD is a square plate with centre O. The moments of inertia of the plate about the perpendicular axis through O is I and about the axes 1, 2, 3 & 4 are I_1, I_2, I_3 & I_4 respectively. It follows that:



(A) $I_2 = I_3$ (B) $I = I_1 + I_4$ (C) $I = I_2 + I_4$ (D) $I_1 = I_3$

Ans. (ABCD)

Sol. $I_1 = I_2 = I_3 = I_4$ & $I = I_1 + I_3$

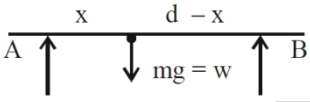
51. A rod of weight w is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at a distance x from A.

(A) the normal reaction at A is $\frac{wx}{d}$ (B) the normal reaction at A is $\frac{w(d-x)}{d}$

(C) the normal reaction at B is $\frac{wx}{d}$ (D) the normal reaction at B is $\frac{w(d-x)}{d}$

Ans. (BC)

Sol.



$N_A d = w(d-x)$

$\therefore N_A = \frac{w(d-x)}{d}$

$N_B = \frac{wx}{d}$

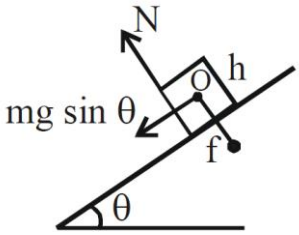
52. A block with a square base measuring a×a and height h, is placed on an inclined plane. The coefficient of friction is μ. The angle of inclination (θ) of the plane is gradually increased. The block will

(A) topple before sliding if $\mu > \frac{a}{h}$ (B) topple before sliding if $\mu < \frac{a}{h}$

(C) slide before toppling if $\mu > \frac{a}{h}$ (D) slide before toppling if $\mu < \frac{a}{h}$

Ans. (AD)

Sol.



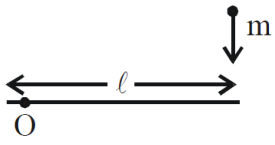
if $f\frac{h}{2} = N\frac{a}{2}$

$\therefore\ \mu Nh = Na\ \therefore\ \mu min = \frac{a}{h}$

53. A particle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground.
(A) Angular momentum of the particle about O is increasing.
(B) Torque of the gravitational force on the particle about O is decreasing.
(C) The moment of inertia of the particle about O is decreasing.
(D) The angular velocity of the particle about O is increasing.

Ans. (ACD)

Sol.



L = mvl
as v↑ $\therefore L\uparrow \Rightarrow \omega\uparrow$
τ same

54. A man spinning in free space changes the shape of his body, eg. by spreading his arms or curling up. By doing this, he can change his
(A) moment of inertia (B) angular momentum
(C) angular velocity (D) rotational kinetic energy

Ans. (ACD)

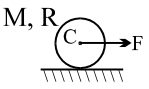
Sol. Angular momentum same

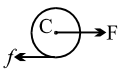
COMPREHENSION BASED QUESTIONS

Paragraph for Question No. 55 to 58

In the following problems, indicate the correct direction of friction force acting on the cylinder, which is pulled on a rough surface by a constant force F.

55. A cylinder of mass M and radius R is pulled horizontally by a force F. The friction force can be given by which of the following diagrams



(A) 

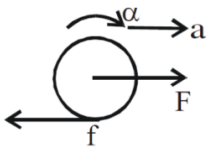
(B)

(C)

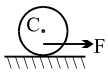
(D) cannot be interpreted

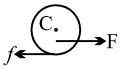
Ans. (A)

Sol.



56. A cylinder is pulled horizontally by a force F acting at a point below the centre of mass of the cylinder, as shown in figure. The friction force can be given by which of the following diagrams



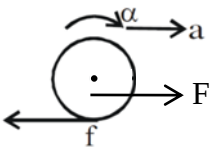
(A) 

(B)

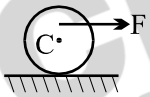
(C)

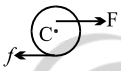
(D) cannot be interpreted

Ans. (A)

Sol. 

57. A cylinder is pulled horizontally by a force F acting at a point above the centre of mass of the cylinder, as shown in figure. The friction force can be given by which of the following diagrams



(A) 

(B)

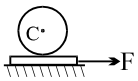
(C)

(D) cannot be interpreted

Ans. (D)

Sol. Friction can act forward or backward according to point of action of force.

58. A cylinder is placed on a rough plank which in turn is placed on a smooth surface. The plank is pulled with a constant force F. The friction force can be given by which of the following diagrams



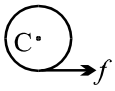
(A)

(B)

(C)

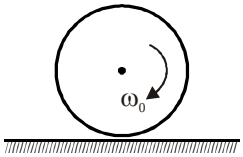
(D) cannot be interpreted

Ans. (B)

Sol. 

Paragraph for Question No. 59 to 61

A solid sphere of mass m and radius R is gently placed on a rough horizontal ground with an angular speed ω_0 and no linear velocity. Coefficient of friction is μ.

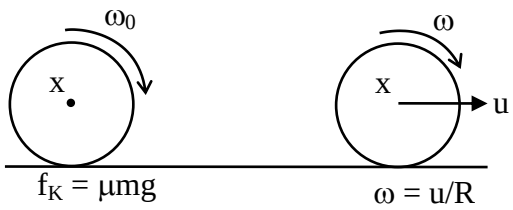


59. Find the time t when the slipping stops.

(A) $\frac{2R\omega_0}{5\mu g}$ (B) $\frac{2R\omega_0}{7\mu g}$ (C) $\frac{2R\omega_0}{3\mu g}$ (D) $\frac{2R\omega_0}{14\mu g}$

Ans. (B)

Sol.



$a = +\mu g$

$v = u + at$

$v = \mu g t$ …(1)

$\tau = I\alpha = \mu mgR$

$\left(\frac{2}{5}MR^2\right)\alpha = \mu mgR$

$\alpha = \frac{5\mu g}{2R}$ (antilock rise)

$\omega = \omega_0 - \frac{5\mu g}{2R}t$ ….(2)

$\omega = \frac{v}{R}$ ….(3)

$\omega_0 R - \frac{5\mu g t}{2} = \mu g t$ (Solving (1), (2) & (3))

$\omega_0 R = \frac{7\mu g t}{2}$

$\boxed{t = \frac{2\omega_0 R}{7\mu g}}$

60. The angular velocity at the end of the slipping is

(A) $\frac{2}{5}\omega_0$ (B) $\frac{2}{7}\omega_0$ (C) $\frac{2}{3}\omega_0$ (D) $\frac{2}{14}\omega_0$

Ans. (B)

Sol. from eq. (2)

$$\omega = \omega_0 - \frac{5\mu g}{2R}\left(\frac{2\omega_0 R}{7\mu g}\right)$$

$$= \omega_0 - \frac{5\omega_0}{7}$$

$$= \frac{2\omega_0}{7}$$

61. The angular momentum of the sphere about the bottommost point at the end of slipping is

(A) $\frac{2}{5}mR^2\omega_0$ (B) $\frac{2}{7}mR^2\omega_0$ (C) $\frac{2}{3}mR^2\omega_0$ (D) None of these

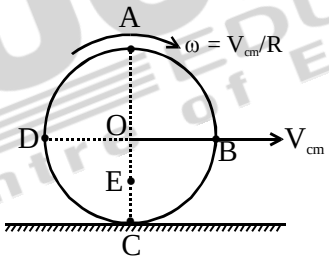
Ans. (A)

Sol. Angular momentum is conserve about the bottommost point.

$$L = \frac{2}{5}MR^2\omega_0$$

MATCH THE COLUMN

62. Consider a body rolling on a horizontal surface as shown in figure (Symbols have their usual meaning)



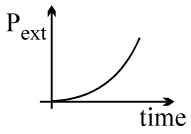
	Column-I		Column-II
(A)	Velocity is zero	(P)	Point A
(B)	Speed in maximum	(Q)	Point B
(C)	$0 <$ speed $< v_{cm}$	(R)	Point C
(D)	$1.3v_{cm} <$ speed $< 2\ v_{cm}$	(S)	Point D
		(T)	Point E

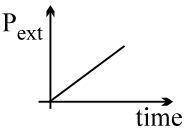
Ans. (A)→ R; (B)→P; (C)→T; (D)→QS;

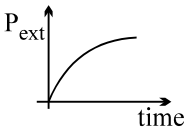
Sol. C is ICOR & speed of point at a distance r from C is $\frac{v_{CM}r}{R}$

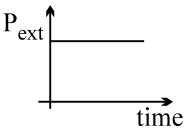
EXERCISE # (O-2)

1. A rod is hinged at its centre and rotated by applying a constant torque starting from rest. The power developed by the external torque as a function of time is :

(A) 

(B) 

(C) 

(D) 

Ans. (B)

Sol. $P = \tau\omega = \tau\frac{\tau}{I}t \qquad \therefore P \propto t$

2. A heavy seesaw (i.e., not massless) is out of balance. A light girl sits on the end that is tilted downward, and a heavy body sits on the other side so that the seesaw now balances. If they both move forward so that they are one-half their original distance from the pivot point (the fulcrum) what will happen to the seesaw ?

(A) The side the body is sitting on will tilt downward

(B) The side the girl is sitting on will once again tilt downward

(C) Nothing ; the seesaw will still be balanced

(D) It is impossible to say without knowing the masses and the distances

Ans. (B)

Sol.

$$m_a g\ell_1 + m_{is}x = m_b g\ell_2 \;\&\; m_a g\frac{\ell_1}{2} + m_{so}x > \frac{m_b g\ell_2}{2}$$

3. A ladder of length L is slipping with its ends against a vertical wall and a horizontal floor. At a certain moment, the speed of the end in contact with the horizontal floor is v and the ladder makes an angle $\alpha = 30°$ with the horizontal. Then the speed of the ladder's center must be

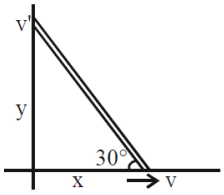
(A) $2v/\sqrt{3}$ (B) v/2 (C) v (D) None

Ans. (C)

Sol. v'cos60 = v cos 30

$\therefore$ v' = $v\sqrt{3}$

COM = $\frac{x}{2}, \frac{v}{2}$



$$\therefore\ v_{cm} = \left(\frac{v}{2}, \frac{v'}{2}\right) = \frac{v}{2}, \frac{\sqrt{3}}{2}$$

$$\therefore\ v_{cm} = \frac{v_2}{2} = v$$

4. In the previous question, if dv/dt = 0, then the angular acceleration of the ladder when $\alpha = 45°$ is

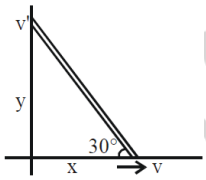
(A) $2v^2/L^2$ (B) $v^2/2L^2$ (C) $\sqrt{2}[v^2/L^2]$ (D) None

Ans. (A)

Sol. $\frac{x}{\cos\theta} = L$

$\Rightarrow v\cos\theta - x\sin\theta\,\omega = 0$

$v\cos\theta = x\sin\theta\,\omega$ $\qquad \therefore\ \omega = \frac{v}{x}\cos\theta$



$$\therefore\ \alpha = -\frac{v}{x}\text{cosec}^2\theta\omega + \cot\theta\left(\frac{xa - v^2}{x^2}\right)$$

5. One ice skater of mass m moves with speed 2v to the right, while another of the same mass m moves with speed v toward the left, as shown in figure I. Their paths are separated by a distance b. At t = 0, when they are both at x = 0, they grasp a pole of length b and negligible mass. For t > 0, consider the system as a rigid body of two masses m separated by distance b, as shown in figure II. Which of the following is the correct formula for the motion after t = 0 of the skater initially at y = b/2?

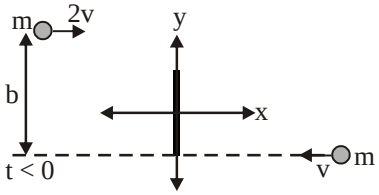


Figure I

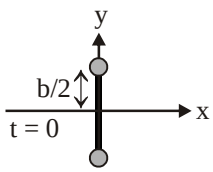


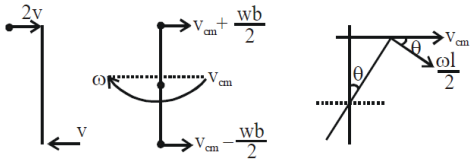
Figure II

(A) x = 2vt, y = b/2

(B) x = vt + 0.5b sin (3vt/b), y = 0.5b cos (3vt /b)
(C) x = 0.5vt + 0.5b sin (3vt/b), y = 0.5b cos (3vt/b)
(D) x = 0.5vt + 0.5b sin (6vt/b), y = 0.5b cos (6vt/b)

Ans. (C)

Sol.

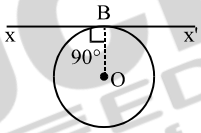


$2mv_{cm} = m2v - mv \qquad \therefore v_{cm} = \frac{v}{2}$

$$2vm\frac{b}{2} + mv\frac{b}{2} = m\left(v_{cm} + \frac{\omega b}{2}\right)\frac{b}{2} - m\left(v_{cm} - \frac{\omega b}{2}\right)$$

$$\therefore\ 3m\frac{vb}{2} = m\omega b\frac{b}{2} \qquad \therefore \omega = \frac{3v}{b}$$

6. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is:



(A) $\rho L^3/8\pi^2$ (B) $\rho L^3/16\pi^2$ (C) $5\rho L^3/16\pi^2$ (D) $3\rho L^3/8\pi^2$

Ans. (D)

Sol. $\frac{3}{2}mR^2 \qquad 2\pi R = L$

$$= \frac{3}{8}\frac{\rho L^3}{\pi 2}$$

7. If a person sitting on a rotating stool with his hands outstretched, suddenly lowers his hands, then his

(A) Kinetic energy will decrease (B) Moment of inertia will decrease
(C) Angular momentum will increase (D) Angular velocity will remain constant

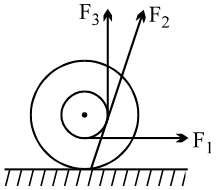
Ans. (B)

Sol. If person lowers his hand, moment of Iertia about the axis of rotation will decrease.
But angular momentum remains constant

$I_1\omega_1 = I_2\omega_2$

$I_2 < I_1 \Rightarrow \omega_2 > \omega_1$

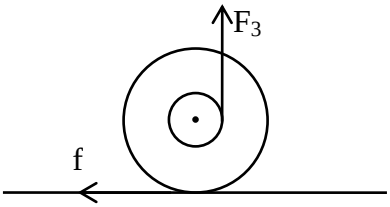
8. A yo-yo is resting on a perfectly rough horizontal table. Forces F_1, F_2 and F_3 are applied separately as shown. The correct statement is



(A) when F_3 is applied the centre of mass will move to the right.
(B) when F_2 is applied the centre of mass will move to the left.
(C) when F_1 is applied the centre of mass will move to the right.
(D) when F_2 is applied the centre of mass will move to the right.

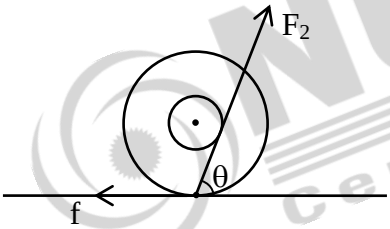
Ans. (C)

Sol. When F_3 is applied



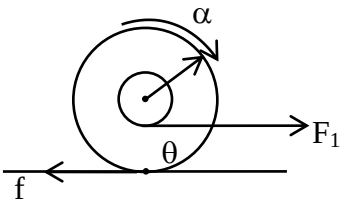
moves towards left

When F_2 is Applied



$f = F\cos\theta$
Body is at rest

When F_1 is applied



Moves right, such that $a_{cm} - R\alpha = 0$

9. A hollow sphere of radius R and mass m is fully filled with non viscous liquid of mass m. It is rolled down a horizontal plane such that its centre of mass moves with a velocity v. If it purely rolls

(A) Kinetic energy of the sphere is $\frac{5}{6}mv^2$

(B) Kinetic energy of the sphere is $\frac{4}{5}mv^2$

(C) Angular momentum of the sphere about a fixed point on ground is $\frac{8}{3}mvR$

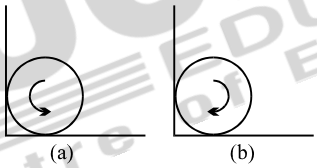
(D) Angular momentum of the sphere about a fixed point on ground is $\frac{14}{5}mvR$

Ans. (C)

Sol. Liquid has translation only.

$$KE = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\frac{2}{3}mR^2\omega^2 + \frac{1}{2}mv^2 = \frac{4}{3}mv^2;\ AM = mvR + mvR + \frac{2}{3}mR^2\omega = \frac{8}{3}mvR$$

10. A sphere is placed rotating with its centre initially at rest in a corner as shown in figure (a) & (b). Coefficient of friction between all surfaces and the sphere is $\frac{1}{3}$. Find the ratio of the frictional force $\frac{f_a}{f_b}$ by ground in situations (a) & (b).



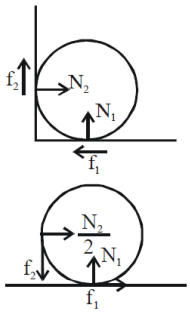
(A) 1 (B) $\frac{9}{10}$ (C) $\frac{10}{9}$ (D) none

Ans. (B)

Sol. $N_2 = f_1 - \mu N_1$

$N_1 + f_2 = mg$

$$\frac{f_1}{\mu} + \mu N_2 = mg = 3f_1 + \frac{1}{3}f_1 = mg$$



$\therefore f_1 = \dfrac{3mg}{10}$

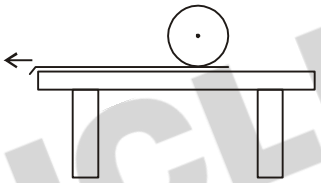
$N_2 = 0$

$\therefore f_2 = 0$

$\therefore f_1 = \mu N_1 = \mu mg = \dfrac{mg}{3}$

$\therefore \dfrac{3}{10 \times \frac{1}{3}} = \dfrac{9}{10}$

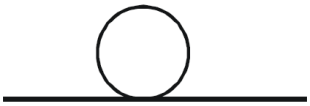
11. A thin table cloth covers a horizontal table and a uniform body of round shape lies on top of it. The table cloth is pulled from under the body, and friction causes the body to slide and rotate. What is the body's final motion on the table? (Assume that the table is so large that the body does not fall off it.)



(A) Body will finally roll towards left
(B) Body will finally roll towards right
(C) Body will finally come to rest
(D) Any of the above is possible depending on shape of body

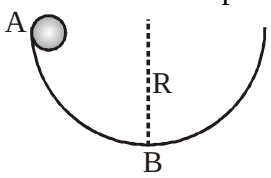
Ans. (C)

Sol.



L always conserved about O. Initial = 0
$\therefore$ final = 0

12. A small sphere A of mass m and radius r rolls without slipping inside a large fixed hemispherical bowl of radius R (>> r) as shown in figure. If the sphere starts from rest at the top point of the hemisphere find the normal force exerted by the small sphere on the hemisphere when it is at the bottom B of the hemisphere.



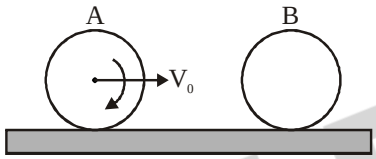
(A) $\frac{10}{7}mg$ (B) $\frac{17}{7}mg$ (C) $\frac{5}{7}mg$ (D) $\frac{7}{5}mg$

Ans. (B)

Sol. $N = mg + \frac{mv^2}{R}$ & $\frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mv^2 = mgR$

$\therefore N = mg + \frac{10}{7}mg = \frac{17}{7}mg$

13. A hollow smooth uniform sphere A of mass 'm' rolls without sliding on a smooth horizontal surface. It collides head on elastically with another stationary smooth solid sphere B of the same mass m and same radius. The ratio of kinetic energy of 'B' to that of 'A' just after the collision is :

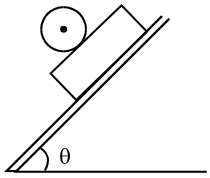


(A) 1 : 1 (B) 2 : 3 (C) 3 : 2 (D) None of these

Ans. (C)

Sol. $\frac{\frac{1}{2}mv^2}{\frac{1}{2}I\omega^2} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}\times\frac{2}{3}m\omega^2} = \frac{3}{2}$

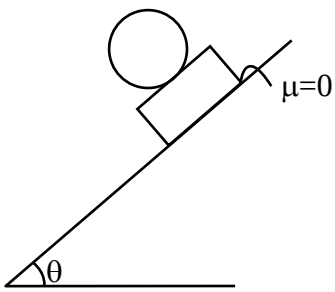
14. A plank of mass M is placed over smooth inclined plane and a sphere is also placed over the plank. Friction is sufficient between sphere and plank. If plank and sphere are released from rest, the frictional force on sphere is:



(A) up the plane (B) down the plane (C) horizontal (D) zero

Ans. (D)

Sol.



Both plank & sphere will move down the inclined with acceleration g sinθ together.

MULTIPLE CORRECT TYPE QUESTIONS

15. A body is in equilibrium under the influence of a number of forces. Each force has a different line of action. The minimum number of forces required is
(A) 2, if their lines of action pass through the centre of mass of the body.
(B) 3, if their lines of action are not parallel.
(C) 3, if their lines of action are parallel.
(D) 4, if their lines of action are parallel and all the forces have the same magnitude.

Ans. (BCD)

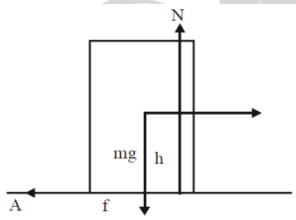
Sol. for equilibrium

$$\Sigma F_{ext} = 0$$
$$\Sigma \tau_{ext} = 0$$

16. A block of mass m moves on a horizontal rough surface with initial velocity v. The height of the centre of mass of the block is h from the surface. Consider a point A on the surface.
(A) angular momentum about A is mvh initially
(B) the velocity of the block decreases at time passes.
(C) torque of the forces acting on block is zero about A
(D) angular momentum is not conserved about A.

Ans. (ABD)

Sol.



AM not conserved $\therefore$ Torque not 0
mg & N form Force couple.

17. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A}\times\vec{L}$ where $\vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about that point. From this it follows that
(A) $d\vec{L}/dt$ is perpendicular to $\vec{L}$ at all instants of time
(B) the components of $\vec{L}$ in the direction of $\vec{A}$ does not change with time
(C) the magnitude of $\vec{L}$ does not change with time
(D) $\vec{L}$ does not change with time

Ans. (ABC)

Sol. $\tau = \vec{A} \times \vec{L} = \frac{d\vec{L}}{dt}$

change in $\vec{L}$ is always perpendicular to $\vec{A}$

$\therefore$ component along $\vec{A}$ does not change.

18. A ring rolls without slipping on the ground. Its centre C moves with a constant speed u. P is any point on the ring. The speed of P with respect to the ground is ν.

(A) $0 \le \nu \le 2u$

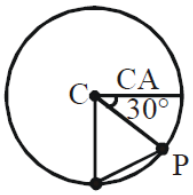
(B) $\nu = u$, if CP is horizontal

(C) $\nu = u$, if CP makes an angle of 30° with the horizontal and P is below the horizontal level of C.

(D) $\nu = \sqrt{2}u$, if CP is horizontal

Ans. (ACD)

Sol.



19. If a cylinder is rolling down the incline with sliding.

(A) after some time it may start pure rolling

(B) after sometime it will start pure rolling

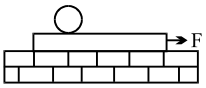
(C) it may be possible that it will never start pure rolling

(D) none of these

Ans. (AC)

Sol. Depends on friction.

20. A plank with a uniform sphere placed on it rests on a smooth horizontal plane. Plank is pulled to right by a constant force F. If sphere does not slip over the plank. Which of the following is correct.



(A) Acceleration of the centre of sphere is less than that of the plank.

(B) Work done by friction acting on the sphere is equal to its total kinetic energy.

(C) Total kinetic energy of the system is equal to work done by the force F

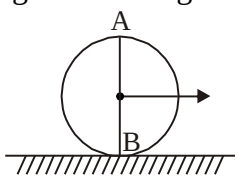
(D) None of the above

Ans. (ABC)

So. work done by friction is 0 on total

$\because$ it does not depend on frame of frame of plank or sphere.

21. A uniform disc of radius R is rolling on a horizontal surface. At a certain instant B is the point of contact on ground and A is at height 2R from ground which doesn't lie on the disc.



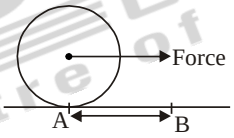
(A) The magnitude of the angular momentum of the disc about B is thrice that about A.
(B) The angular momentum of the disc about A is anticlockwise.
(C) The angular momentum of the disc about B is clockwise
(D) The angular momentum of the disc about A is equal to that about B.

Ans. (ABC)

Sol. about B = $mvr + \frac{mR^2}{2}\omega = \frac{3}{2}mvr$

$A = -mvr + \frac{mR^2}{2}\omega = -\frac{1}{2}mvr$

22. A disc of circumference s is at rest at a point A on a horizontal surface when a constant horizontal force begins to act on its centre. Between A and B there is sufficient friction to prevent slipping, and the surface is smooth to the right of B. AB = s. The disc moves from A to B in time T. To the right of B,



(A) the angular acceleration of the disc will disappear, linear acceleration will remain unchanged
(B) linear acceleration of the disc will increase
(C) the disc will make one rotation in time T/2
(D) the disc will cover a distance greater than s in further time T.

Ans. (BCD)

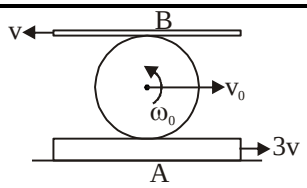
Sol. F – f = ma

From A to B $\quad 2\pi = \frac{1}{2}\alpha T^2 \quad \& \omega = \alpha t$

B to ahead $\quad 2\pi = \omega t \quad \therefore t = \frac{T}{2}$

$\because$ w will be constant

23. The disc of radius r is confined to roll without slipping at A and B. If the plates have the velocities shown, then

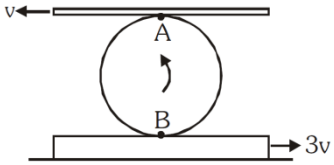


(A) linear velocity $v_0 = v$ (B) angular velocity of disc is

(C) angular velocity of disc is (D) None of these

Ans. (AC)

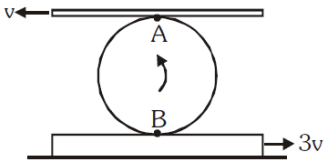
Sol.



$v_A = \omega_0 R - v_0 = v$

$\omega_0 R - v_0 = v$ ....(i)

$v_B = \omega_0 R + v_0 = 3v$ ...(ii)

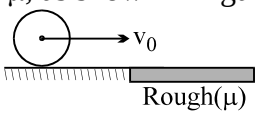


$\omega_0 = \dfrac{2v}{R}$ from (1) $v_0 = v$

COMPREHENSION BASED QUESTIONS

Paragraph for Question No. 24 to 27

A ring of mass M and radius R sliding with a velocity v_0 suddenly enters into rough surface where the coefficient of friction is μ, as shown in figure.



24. Choose the correct statement(s)

(A) As the ring enters on the rough surface, the limiting friction force acts on it

(B) The direction of friction is opposite to the direction of motion

(C) The friction force accelerates the ring in the clockwise sense about its centre of mass

(D) As the ring enters on the rough surface it starts rolling

Ans. (ABC)

Sol. Oral

25. Choose the correct statement(s)

(A) The momentum of the ring is conserved

(B) The angular momentum of the ring is conserved about its centre of mass
(C) The angular momentum of the ring conserved about any point on the horizontal surface
(D) The mechanical energy of the ring is conserved

Ans. (C)

Sol. Oral

26. Choose the correct statement(s)

(A) The ring starts its rolling motion when the centre of mas stationary

(B) The ring starts rolling motion when the point of contact becomes stationary

(C) The time after which the ring starts rolling is $\frac{v_0}{2\mu g}$

(D) The rolling velocity is $\frac{v_0}{2}$

Ans. (BCD)

Sol. $mv_0r = mvr + mr^2\omega = 2mvr$

$\therefore v = \frac{v_0}{2}$

& $\frac{v_0}{2} = v_0 - \mu gt$ $\qquad\qquad$ $\therefore t = \frac{\mu_0}{2\mu g}$

27. Choose the correct alternative(s)

(A) The linear distance moved by the centre of mass before the ring starts rolling is $\frac{3v_0^2}{8\mu g}$

(B) The net work done by friction force is $-\frac{3}{8}mv_0^2$

(C) The loss is kinetic energy of the ring is $\frac{mv_0^2}{4}$

(D) The gain in rotational kinetic energy is $+\frac{mv_0^2}{8}$

Ans. (ACD)

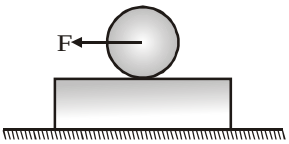
Sol. (A) $V_0t - \frac{1}{2}\mu gt^2 = \frac{v_0^2}{2\mu g} - \frac{1}{2}\mu g\frac{v_0^2}{4\mu g}$

(B) $\Delta KE = \frac{1}{2}mv^2 + \frac{1}{2}mR^2\omega^2 - \frac{1}{2}mV_0^2$

(D) $\frac{1}{2}mR^2\omega^2 = \frac{1}{2}m\frac{v_0^2}{4} = \frac{1}{8}mv_0^2$

Paragraph for Question No. 28 to 29

A disc of mass m and radius R is placed over a plank of same mass m. There is sufficient friction between disc and plank to prevent slipping. A force F is applied at the centre of the disc.

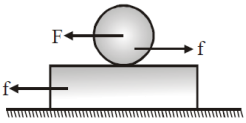


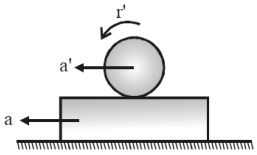
28. Acceleration of the plank is :-

(A) $\frac{F}{2m}$ (B) $\frac{3F}{4m}$ (C) $\frac{F}{4m}$ (D) $\frac{3F}{2m}$

Ans. (C)

Sol.





$F - f = ma'$ $a' - a'R = a$

$fR = \frac{mR^2}{2}\alpha'$

$f = ma$

$\therefore R\alpha' = 2a$ $\therefore a' - 2a = a$ $\therefore a' = 3a$

$\therefore F = m(a' + a) = 4ma \therefore a = \frac{F}{4m}$

29. Force of friction between the disc and the plank is :

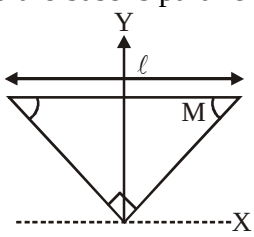
(A) $\frac{F}{2}$ (B) $\frac{F}{4}$ (C) $\frac{F}{3}$ (D) $\frac{2F}{3}$

Ans. (B)

Sol. $f = \frac{F}{4}$

Paragraph for Question No. 30 to 33

The figure shows an isosceles triangular plate of mass M and base L. The angle at the apex is 90°. The apex lies at the origin and the base is parallel to X–axis



30. The moment of inertia of the plate about the z-axis is

(A) $\frac{ML^2}{12}$ (B) $\frac{ML^2}{24}$ (C) $\frac{ML^2}{6}$ (D) None of these

Ans. (C)

31. The moment of inertia of the plate about the x-axis is

(A) $\frac{ML^2}{8}$ (B) $\frac{ML^2}{32}$ (C) $\frac{ML^2}{24}$ (D) $\frac{ML^2}{6}$

Ans. (A)

32. The moment of inertia of the plate about its base parallel to the x-axis is

(A) $\frac{ML^2}{18}$ (B) $\frac{ML^2}{36}$ (C) $\frac{ML^2}{24}$ (D) None of these

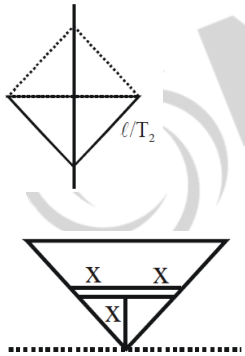
Ans. (C)

33. The moment of inertia of the plate about the y-axis is

(A) $\frac{ML^2}{6}$ (B) $\frac{ML^2}{8}$ (C) $\frac{ML^2}{24}$ (D) None of these

Ans. (C)

Sol. **30-33**



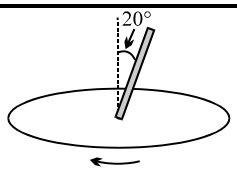
I about y axis = $\frac{m\ell^2}{24}$

$\int_0^{\frac{\ell}{2}} \frac{M}{\frac{\ell^2}{4}} 2x dx$ $x^2 = \frac{m\ell^2}{8}$

$I_z = I_x + I_y = \left(\frac{1}{8} + \frac{1}{24}\right) m\ell^2 = \frac{1}{6} m\ell^2$

Paragraph for Question No. 34 to 35

A uniform rod is fixed to a rotating turntable so that its lower end is on the axis of the turntable and it makes an angle of 20° to the vertical. (The rod is thus rotating with uniform angular velocity about a vertical axis passing through one end.) If the turntable is rotating clockwise as seen from above.



34. What is the direction of the rod's angular momentum vector (calculated about its lower end)?(A) vertically downwards (B) down at 20° to the horizontal
(C) up at 20° to the horizontal (D) vertically upwards

Ans. (B)

Sol. $m\vec{r}_{cm} \times \vec{v}_{cm}$ down at 20°

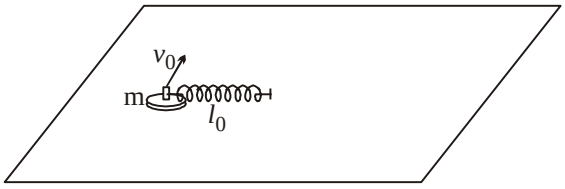
35. Is there a torque acting on it, and if so in what direction?
(A) yes, vertically (B) yes, horizontally
(C) yes at 20° to the horizontal (D) no

Ans. (B)

Sol. Yes of mg & also direction of $\vec{L}$ is changing
$\therefore$ horizontal $\therefore$ both $\vec{r}$ & mg is in vertical plane.

Paragraph for Question No. 36 to 38

A spring having initial unstretched length ℓ_0 is lying on a smooth table. It's one end is fixed and the other one is fastened to a small particle of mass m. The particle is imparted an initial speed v_0 horizontally in a direction perpendicular to the spring.
In the course of the motion in horizontal plane, the maximum elongation of the spring is $\Delta\ell = \ell_0/10$. (Given : m = 2.1 kg, ℓ_0 = 21 cm, v_0 = 1.1 m/s).



36. In the course of motion, which of the quantities relating to spring block system are conserved ?
(A) kinetic energy (B) momentum (C) angular momentum (D) potential energy

Ans. (C)

Sol. oral

37. Which of the following is correct about initial situation and situation at maximum elongation ?
(A) The orientation of spring in both positions should be perpendicular to each other.
(B) The velocity at maximum extension should be zero
(C) The velocity at maximum extension as well as at initial position should be perpendicular to spring.

(D) Acceleration should be zero at the maximum extension as well as at initial position.

Ans. (C)

Sol. At ℓ_{max} , component of velocity along the length of spring is zero.

38. **Student-A :** at maximum extension $mv'(\ell_0 + \Delta\ell) = mv_0\ell_0$.

Student-B : at maximum extension $k\Delta\ell = \frac{mv'^2}{(\ell_0+\Delta\ell)}$ **Student-C :** $\frac{1}{2}mv_0^2 = \frac{1}{2}mv'^2 + \frac{1}{2}k\Delta\ell^2$.

where v' is velocity at instant of maximum extension :

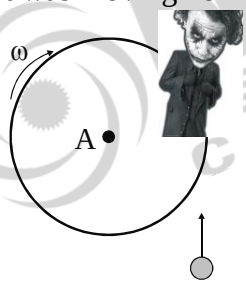
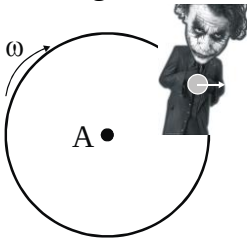
(A) Only Student-A and B are correct (B) Only Student-A and C are correct

(C) Only Student-B and C are correct (D) All are correct

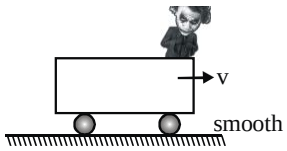
Ans. (B)

Sol. $mv'(\ell_0+\Delta\ell) = mv_0\ell_0$

39. Column-I depicts various situations where some sudden events are taking place. Column-II describes changes in various parameters of systems immediately after the events taking place in column-I.

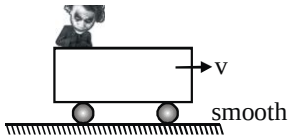
	Column I	**Column II**
(A)	Joker is standing on revolving platform and batman throws the ball and joker catches the ball while it was moving horizontally.  Joker, ball and platform is system.	(P) Linear momentum remains conserved.
(B)	Joker throws the ball horizontally and perpendicular to his motion while standing on the revolving platform.  Joker, ball and platform is system.	(Q) Mechanical energy is conserved
(C)	Joker jumps horizontally towards	(R) Mechanical energy increases.

right from the cart which is moving at speed v on smooth horizontal floor.



Joker and cart is the system

(D) Joker drops himself vertically from the moving cart with no horizontal velocity relative to cart.



Joker and cart is the system

(S) Mechanical energy decreases.

(T) v or ω changes

Ans. (A)→ ST; (B)→R; (C)→PRT; (D)→PQ;

Sol. For (A) : Hinge will apply external impulse hence momentum will not be conserved.

So (P) is not correct.

Mechanical energy reduced hence (S)

ω decreases by conservation of angular momentum hence(T)

For(B) : Hinge will apply external impulse hence momentum will not be conserved.

So (P) is not correct.

Mechanical energy will increase hence(R)

ω does not change by conservation of angular momentum hence (No "T")

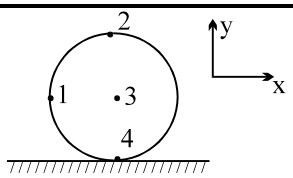
Mechanical energy will increase hence (R)

V will change so (T)

For (D) : There is no external impulse and no change in momentum as velocity of joker w.r.t. car is still same so P Not 'T'

Mechanical energy remains same immediately after joker drops as he has no vertical velocity so (Q)

40. A rigid cylinder is kept on a smooth horizontal surface as shown. If **Column I** indicates velocities of various points (3-centre of cylinder, 2- top point, 4-bottom point, 1- on the level of 3 at the rim) on it shown, choose correct state of motion from **Column II**.



	Column I	Column II
(A)	$\vec{v}_1 = \hat{i} + \hat{j}, \vec{v}_2 = 2\hat{i}$	(P) Pure rotation about centre
(B)	$\vec{v}_1 = \hat{i} + \hat{j}, \vec{v}_3 = -\hat{i}$	(Q) Rolling without slipping to left
(C)	$\vec{v}_2 = \hat{i}, \vec{v}_3 = 0$	(R) Rolling without slipping to right
(D)	$\vec{v}_4 = 0, \vec{v}_1 = -\hat{i} - \hat{j}$	(S) Not possible

Ans. (A)→ R; (B)→S; (C)→P; (D)→Q

Sol. (A) $V_{cm} = l$ & $\omega R = 1$

(B) $v_{rel\ 1/3} \neq 0$ ∴ not a rigid body

(C) P

(D) Q

NUCLEUS EDUCATION Centre of Excellence

EXERCISE # (JM)

1. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected ? **[AIEEE 2004]**

(1) Moment of inertia (2) Angular momentum

(3) Angular velocity (4) Rotational kinetic energy

Ans. (2)

Sol. In free space, neither acceleration due to gravity nor external torque act on the rotating solid sphere.Therefore, taking the same mass of sphere if radius is increased then moment of inertia, rotational kinetic energy and angular velocity will change but according to law of conservation of momentum, angular momentum will not change.

2. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that : **[AIEEE 2004]**

(1) $I_A = I_B$ (2) $I_A > I_B$ (3) $I_A < I_B$ (4) $I_A / I_B = d_A/d_B$

where d_A and d_B are their densities.

Ans. (3)

Sol. Let same mass and same outer radii of solid sphere and hollow sphere are M and R respectively.

The moment the moment of inertia of hollow sphere (spherical shell) B about its diameter

$$I_A = \frac{2}{5}MR^2 \qquad .....(i)$$

Similarly the moment of inertia of hollow sphere (spherical shell) B about its diameter

$$I_B = \frac{2}{5}MR^2 \qquad .....(ii)$$

It is clear from eqs. **(i)** and **(ii)**, $I_A < I_B$

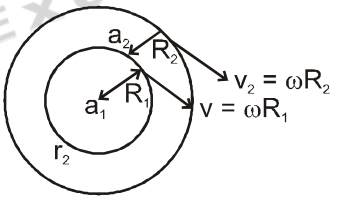
3. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring, $\frac{F_1}{F_2}$ is : **[AIEEE 2005]**

(1) $\frac{R_2}{R_1}$ (2) $\left(\frac{R_1}{R_2}\right)^2$ (3) 1 (4) $\frac{R_1}{R_2}$

Ans. (4)

Sol. $ma_1 = \frac{mv_1^2}{R_1}$ ..... (i)

and $ma_2 = \frac{mv_2^2}{R_2}$ ..... (ii)



and $\frac{F_1}{F_2} = \frac{ma_1}{ma_2}$

$= \frac{R_2}{mR_2^2\omega^2}$ $\therefore \frac{F_1}{F_2} = \frac{R_1}{R_2}$

4. The moment of inertia of uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is : **[AIEEE 2005]**

(1) $\frac{1}{4}Mr^2$ (2) $\frac{2}{5}Mr^2$ (3) Mr^2 (4) $\frac{1}{2}Mr^2$

Ans. (4)

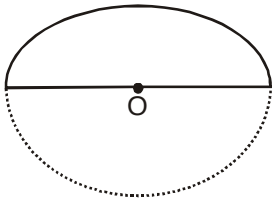
Sol. The mass of complete (circular) disc is

$$M + M = 2M$$

The moment of inertia of disc is

$$I = \frac{2Mr^2}{2}$$

$$= Mr^2$$



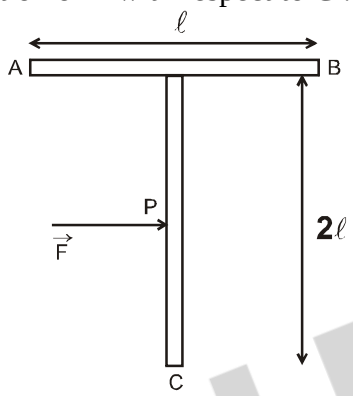
Let the moment of inertia of semicircular disc is I_1.

The disc may be assumed as combination of two semicircular parts.

Thus, $I_1 = I - I_1$

$\therefore \quad I_1 = \frac{I}{2} = \frac{Mr^2}{2}$

5. A 'T' shaped object with dimensions shown in the figure, is lying on a smooth floor. A force '$\vec{F}$' is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C : **[AIEEE 2005]**



(1) $\frac{2}{3}\ell$ (2) $\frac{3}{2}\ell$ (3) $\frac{4}{3}\ell$ (4) ℓ

Ans. (3)

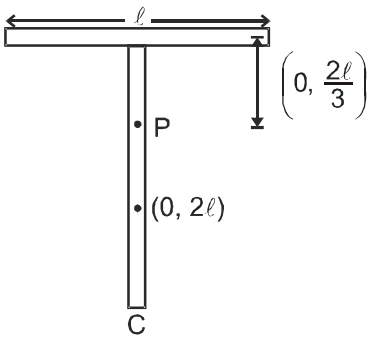
Sol. For pure translatory motion, net torque about centre of mass should be zero. Thus is applied at centre of mass of system.

$$P = \frac{0 \times l + l.2l}{l + 2l} = \frac{2l^2}{3l}$$

$$= \frac{2l}{3}$$

$\therefore \quad PC = \left(l - \frac{2l}{3} + l \right)$

$$= \frac{4l}{3}$$



6. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity ω′ : **[AIEEE-2006]**

(1) $\frac{\omega m}{(m+2M)}$ (2) $\frac{\omega(m+2M)}{m}$ (3) $\frac{\omega(m-2M)}{(m+2M)}$ (4) $\frac{\omega m}{(m+M)}$

Ans. (1)

Sol. $L_i = L_r$

$mR^2\omega = (mR^2 + 2MR^2)\omega'$

$\omega' = \left(\frac{m\omega}{m+2M}\right)$.

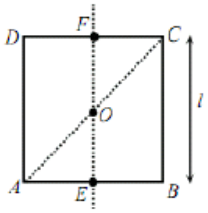
7. Four point masses, each of value m, are placed at the corners of a square ABCD of side ℓ. The moment of inertia about an axis passing through A and parallel to BD is : **[AIEEE-2006]**

(1) $m\ell^2$ (2) $2m\ell^2$ (3) $\sqrt{3}\,m\ell^2$ (4) $3m\ell^2$

Ans. (4)

Sol. $I = 2m\,(l/\sqrt{2})^2 + m(\sqrt{2})^2 = 3ml^2$.

8. For the given uniform square lamina ABCD, whose centre is O, **[AIEEE-2007]**



(1) $\sqrt{2}I_{AC} = I_{EF}$ (2) $I^{AD} = 2I^{EF}$ (3) $I^{AC} = I^{EF}$ (4) $I^{AC} = \sqrt{2}I_{EF}$

Ans. (3)

Sol. $I_{AC} = \frac{1}{2}\left(\frac{Ml^2}{6}\right) = \frac{Ml^2}{12}c$, $I_{EF} = \frac{Ml^2}{12}$, $I_{AC} = I_{EF}$.

9. A round uniform body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ ? with the horizontal. Then its acceleration is : **[AIEEE- 2007]**

(1) $\frac{g\sin\theta}{1+I/MR^2}$ (2) $\frac{g\sin\theta}{1+MR^2/I}$ (3) $\frac{g\sin\theta}{1-I/MR^2}$ (4) $\frac{g\sin\theta}{1-MR^2/I}$

Ans. (1)

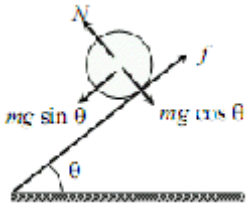
Sol. $mg\sin\theta - f = ma_{CM}$ .....(i)

$f.R = I\alpha$ .....(ii)

$a_{CM} = R\alpha$ .....(iii)

On solving (i),(ii) & (iii)

$a_{CM} = \frac{g\sin\theta}{1+\frac{I}{MR^2}}$.



10. Angular momentum of the particle rotating with a central force is constant due to : **[AIEEE - 2007]**

(1) constant force (2) constant linear momentum

(3) zero torque (4) constant torque

Ans. (3)

Sol. Central force is directed towards a point, therefore torque of the central force is zero.

11. Consider a uniform square plate of side 'a' and mass 'm'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is :

[AIEEE-2008]

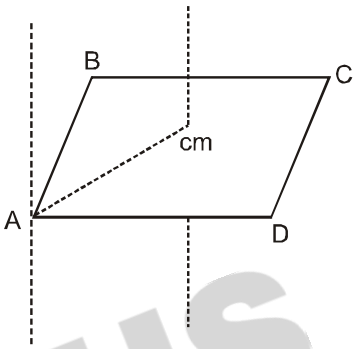
(1) $\frac{1}{12}ma^2$ (2) $\frac{7}{12}ma^2$ (3) $\frac{2}{3}ma^2$ (4) $\frac{5}{6}ma^2$

Ans. (3)

Sol. $I_A = I_{cm} + m\left(\frac{a}{\sqrt{2}}\right)^2$

$= \frac{ma^2}{6} + \frac{ma^2}{2}$

$= \frac{2}{3}ma^2$



12. A thin uniform rod of length ℓ and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of :

[AIEEE-2009]

(1) $\frac{1}{6}\frac{\ell\omega}{g}$ (2) $\frac{1}{2}\frac{\ell^2\omega^2}{g}$ (3) $\frac{1}{6}\frac{\ell^2\omega^2}{g}$ (4) $\frac{1}{3}\frac{\ell^2\omega^2}{g}$

Ans. (3)

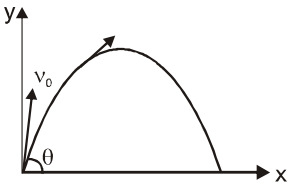
Sol. $\frac{1}{2}I\omega^2 = mgh$

$\frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 = mgh$

$h = \frac{\omega^2 l^2}{6g}$

13. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is

[AIEEE-2010]



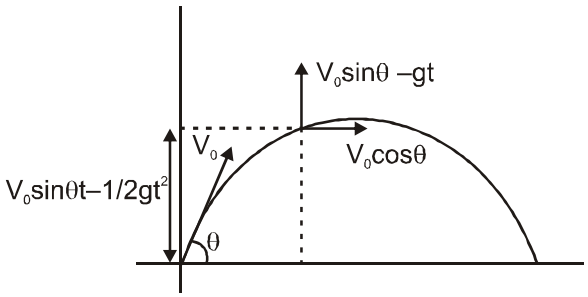
(1) $-mg\, v_0\, t_2 \cos\theta\, \hat{j}$ (2) $mg\, v_0\, t \cos\theta\, \hat{k}$

(3) $-\frac{1}{2}\, mg\, v_0\, t_2 \cos\theta\, \hat{k}$ (4) $\frac{1}{2}mg\, v_0\, t_2 \cos\theta\, \hat{i}$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively.

Ans. (3)

Sol.



angular momentum = $m\left\{(v_0 \sin\theta - gt)(V_0 \cos\theta t) - (V_0 \cos\theta)\left(V_0 \sin\theta t - \frac{1}{2}gt^2\right)\right\}$

$$= -\frac{1}{2} mg\, V_0\, t^2 \cos\theta_0\ \hat{k}$$

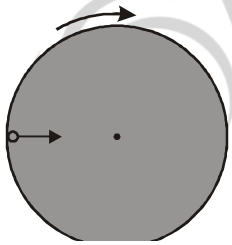
14. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc : **[AIEEE-2011]**

(1) remains unchanged (2) continuously decreases

(3) continuously increases (4) first increases and then decreases

Ans. (4)

Sol.



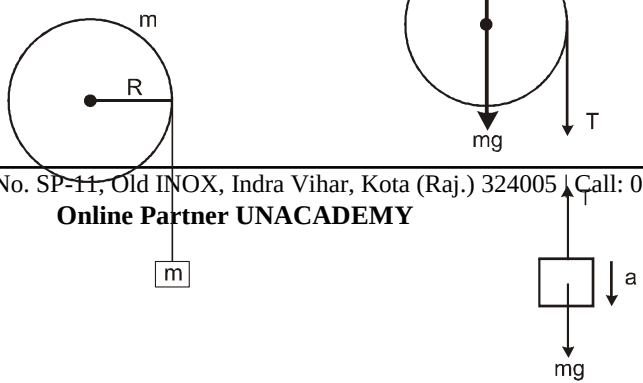
From angular momentum conservation about vertical axis passing through centre. When insect is coming from circumference to center. Moment of inertia first decrease then increase. So angular velocity inecrease then decrease.

15. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is : **[AIEEE - 2011]**

(1) $\frac{3}{2}g$ (2) g (3) $\frac{2}{3}g$ (4) $\frac{g}{3}$

Ans. (3)

Sol. mg – T = ma



$TR = \frac{mR^2\alpha}{2}$

$T = \frac{mR\alpha}{2} = \frac{ma}{2}$

$mg - \frac{ma}{2} = ma$

$\frac{3ma}{2} = mg$

$a = \frac{2g}{3}$ **Ans.**

16. A pulley of radius 2m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion if reversed, is : **[AIEEE - 2011]**

(1) less than 3 (2) more than 3 but less than 6

(3) more than 6 but less than 9 (4) more than 9

Ans. (2)

Sol. To reverse the direction $\int \tau d\theta = 0$ (work done is zero)

$\tau = (20\,t - 5t^2)\,2 = 40t - 10t^2$

$\alpha = \frac{\tau}{I} = \frac{40t - 10t^2}{10} = 4t - t^2$

$\omega = \int_0^t \alpha dt = 2t^2 - \frac{t^3}{3}$

ω is zero at

$2t^2 - \frac{t^3}{3} = 0$

$t^3 = 6t^2$

$t = 6$ sec.

$\theta = \int \omega dt$

$= \int_0^6 (2t^2 - \frac{t^3}{3})dt$

$\left[\frac{2t^3}{3} - \frac{t^4}{12}\right]_0^6 = 216\left[\frac{2}{3} - \frac{1}{2}\right] = 36$ rad.

No of revolution $\frac{36}{2\pi}$ Less than 6

17. A particle of mass 'm' is projected with a velocity υ making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height 'h' is : **[AIEEE 2011]**

(1) zero (2) $\frac{mv^3}{\sqrt{2}g}$ (3) $\frac{\sqrt{3}}{16}\frac{mv^3}{g}$ (4) $\frac{\sqrt{3}}{2}\frac{mv^2}{g}$

Ans. (3)

Sol. $L_0 = Pr \perp$

$L_0 = mv\cos\theta\ H$

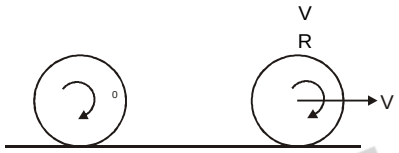
$= mg\frac{\sqrt{3}}{2} \cdot \frac{v^2 \sin^2 30^o}{2g} = \frac{\sqrt{3}mv^3}{16g}$.

18. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip ?
[AIEEE 2013]

(1) $\frac{r\omega_0}{4}$ (2) $\frac{r\omega_0}{3}$ (3) $\frac{r\omega_0}{2}$ (4) $r\omega_0$

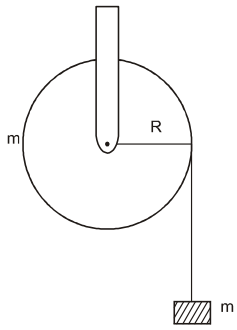
Ans. (3)

Sol.



$mr^2\omega_0 = mvr + mr^2 \times \frac{v_0}{r}$

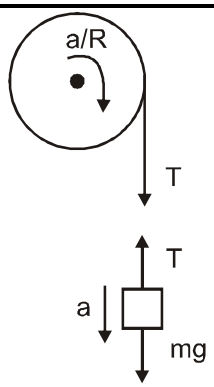
19. A mass 'm' supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release ?
[JEE-Main 2014]



(1) $\frac{2g}{3}$ (2) $\frac{g}{2}$ (3) $\frac{5g}{6}$ (4) g

Ans. (2)

Sol.



$mg - T = ma\frac{a}{R}$ .....(1)

$T.R = mR^2$ .....(2)

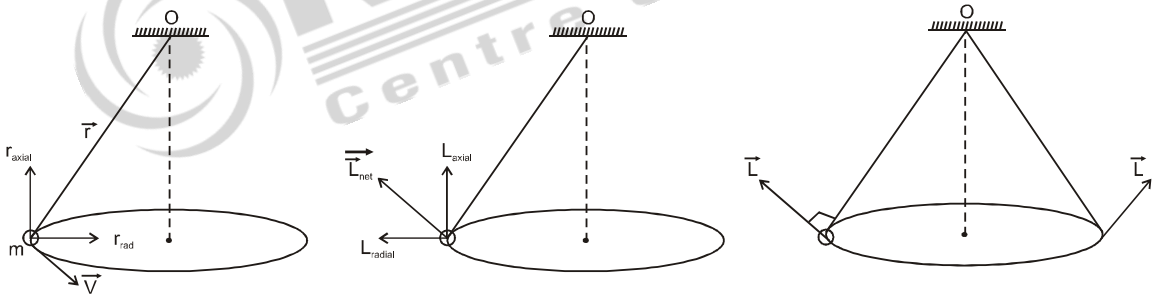
$\frac{g}{2} = a$

20. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with a angular speed ω rad/s about the vertical. About the point of suspension : **[JEE-Main 2014]**

(1) angular momentum is conserved.
(2) angular momentum changes in magnitude but not in direction
(3) angular momentum changes in direction but not in magnitude.
(4) angular momentum changes both in direction and magnitude.

Ans. (3)

Sol.



Angular momentum of the pendulum about the suspension point 'O' is $\vec{L} = m(\vec{r} \times \vec{v})$

Then $\vec{r}$ can be resolved into two components, radial component r_{rad}, and axial component r_{axial}. Due to r_{rad}, L will be axial and due to r_{axial}, L will be radially outwards as shown.

So net angular momentum will be as shown in figure whoose value will be constant ($|L| = mv\ell$). But its direction will change as shown in the figure.

21. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:-

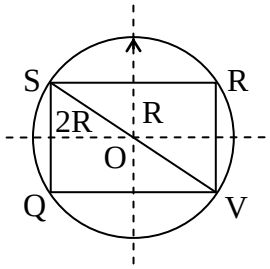
[JEE Mains-2015]

(1) $\frac{4MR^2}{9\sqrt{3\pi}}$ (2) $\frac{4MR^2}{3\sqrt{3\pi}}$ (3) $\frac{MR^2}{32\sqrt{2\pi}}$ (4) $\frac{MR^2}{16\sqrt{2\pi}}$

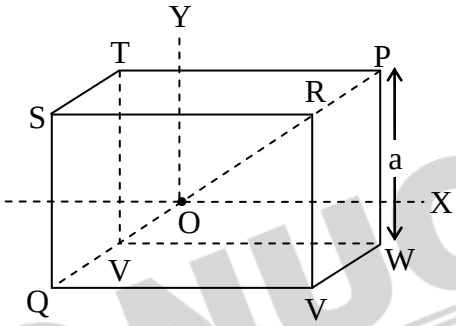
Ans. (1)

Sol. Use geometry of the figure to calculate mass and side length of the cube interms of M and R respectively.

Cross the cross-sectional view of a diametric as shown in the adjacent diagram.



Cross-sectional view of the cube and sphere



Using geometry of the cube

$$PQ = 2R = (\sqrt{3})a \text{ or } a = \frac{2R}{\sqrt{3}}$$

Volume density of the solid sphere

$$\rho = \frac{M}{(4/3)\pi R^3} = \frac{3}{4\pi}\left(\frac{M}{R^3}\right)$$

Mass of cube (m) = $(\rho)\,(a)^3$

$$= \left(\frac{3}{4\pi}\times\frac{M}{R^3}\right)\left[\frac{2R}{\sqrt{3}}\right]^3$$

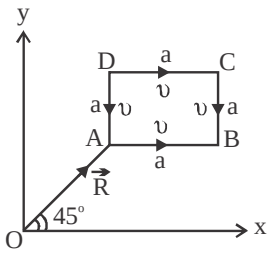
$$= \frac{3M}{4\pi R^3}\times\frac{8R^3}{3\sqrt{3}} = \frac{2M}{\sqrt{3}\pi}$$

Moment of inertia of the cube about given axis is

$$I_Y = \frac{ma^2}{12}(a^2+a^2) = \frac{ma^2}{6}$$

$$I_Y = \frac{ma^2}{6} = \frac{2M}{\sqrt{3}\pi}\times\frac{1}{6}\times\frac{4R^2}{3} = \frac{4MR^2}{9\sqrt{3}\pi}$$

22. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed u in the xy plane as shown in the figure : **[JEE Mains-2016]**



Which of the following statement is false for the angular momentum $\vec{L}$ about the origin ?

(1) $\vec{L}=\frac{m\upsilon}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A

(2) $\vec{L}=-\frac{m\upsilon}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B

(3) $\vec{L}=m\upsilon\left[\frac{R}{\sqrt{2}}-a\right]\hat{k}$ when the particle is moving from C to D

(4) $\vec{L}=m\upsilon\left[\frac{R}{\sqrt{2}}+a\right]\hat{k}$ when the particle is moving from B to C

Ans. (1 or 3)

Sol. From C to D

$$\vec{L}_0=mv\left[\frac{R}{\sqrt{2}}+a\right]\hat{k}$$

from B to C

$$\vec{L}_0=mv\left[\frac{R}{\sqrt{2}}+a\right]\hat{k}$$

from D to A

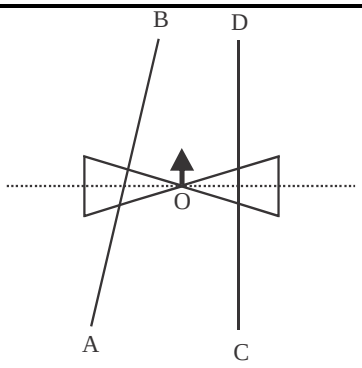
$$\vec{L}_0=\frac{mv}{\sqrt{2}}R(-\hat{k})$$

from A to B

$$\vec{L}_0=\frac{mv}{\sqrt{2}}R(-\hat{k})$$

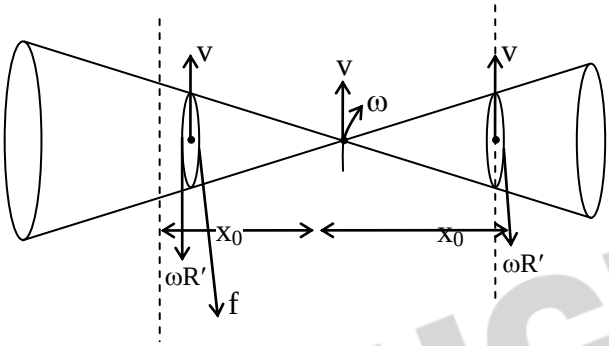
23. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to **[JEE Mains-2016]**

(1) turn left and right alternately.
(2) turen left.
(3) turn right.
(4) go straight.



Ans. (2)

Sol.



At distance x_0 from O $\quad v = \omega R$

distance less than X_0 $\quad v > \omega R$

Initially, there is pure rolling at both the contacts. As the cone moves forward slipping at AB will start in forward direction as radius at left contact decreases.

Thus the cone will start turning towards left. As it moves further slipping at CD will start in backward direction which will also turn the cone towards left.

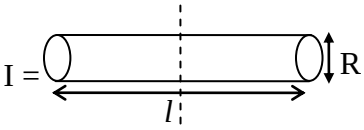
24. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ/R such that the moment of inertia is minimum ?

[JEE Mains-2017]

(1) 1 (2) $\frac{3}{\sqrt{2}}$ (3) $\sqrt{\frac{3}{2}}$ (4) $\frac{\sqrt{3}}{2}$

Ans. (3)

Sol.



$$I = \frac{ML^2}{12} + \frac{MR^2}{4}$$

$$V = \pi R^2 L = \text{const.} \Rightarrow R^2 = \frac{V}{\pi L}$$

$$I = \frac{ML^2}{12} + \frac{MV}{4\pi L}$$

$$I \text{ is minimum} \Rightarrow \frac{dI}{dL} = 0 \Rightarrow L^3 = \frac{3}{2}\frac{V}{\lambda}$$

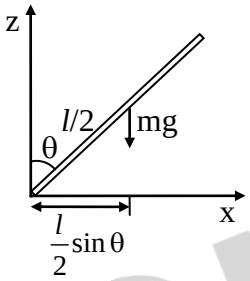
$$\Rightarrow R^2 = \frac{V}{\pi L} = \frac{2\pi L^3}{3(\pi L)} \Rightarrow \frac{R^2}{L^2} = \frac{2}{3}$$

25. A slender uniform rod of mass M and length is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is : **[JEE Mains-2017]**

(1) $\frac{3g}{2\ell}\cos\theta$ (2) $\frac{2g}{3\ell}\cos\theta$ (3) $\frac{3g}{2\ell}\sin\theta$ (4) $\frac{2g}{3\ell}\sin\theta$

Ans. (3)

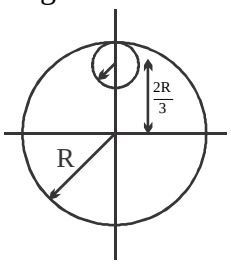
Sol.



$$mg\frac{l}{2}\sin\theta = \frac{ml^2}{3}.\alpha$$

$$\alpha = \frac{3g}{2l}\sin\theta$$

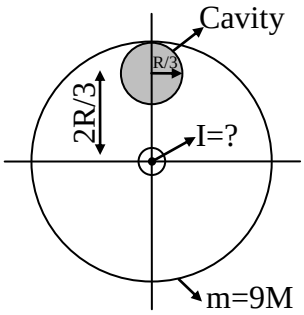
26. From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is : **[JEE Mains-2018]**



(1) $\frac{37}{9}MR^2$ (2) $4\ MR^2$ (3) $\frac{40}{9}MR^2$ (4) $10\ MR^2$

Ans. (2)

Sol.



mass of cavity = M $\left[\begin{array}{l}\pi R^2 \to 9M \\ \pi\dfrac{R^2}{9} \to M\end{array}\right]$

MI of Big Disc = $\dfrac{9MR^2}{2}$

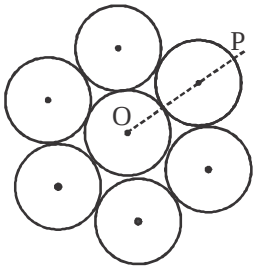
MI of Cavity = $\dfrac{MR^2}{18}$

$\therefore$ MI of final structure = $\dfrac{9MR^2}{2}-\left[\dfrac{MR^2}{18}+\dfrac{4MR^2}{9}\right]$

$= \dfrac{9MR^2}{2}-\left[\dfrac{MR^2+8MR^2}{18}\right]$

$= 4\,MR^2$

27. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is : **[JEE Mains-2018]**



(1) $\dfrac{181}{2}MR^2$ (2) $\dfrac{19}{2}MR^2$ (3) $\dfrac{55}{2}MR^2$ (4) $\dfrac{73}{2}MR^2$

Ans. (1)

Sol. First we calculate M.I about O

M.I of each disc about O

$I \Rightarrow M(2R)^2+\dfrac{1}{2}MR^2$

$\Rightarrow \dfrac{9}{2}MR^2$

So, M.I. of all disc about O

$I_{net} = 6\times\left(\frac{9}{2}MR^2\right)+\frac{1}{2}MR^2$

$\Rightarrow \frac{55MR^2}{2}$

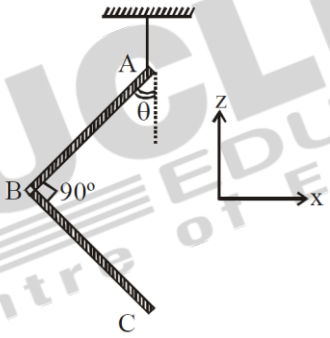
Now, M.I. about point P

$I_p = \frac{55MR^2}{2}+7M(3R)^2$

$\Rightarrow \frac{55MR^2}{2}+63MR^2$

$\Rightarrow \frac{181MR^2}{2}$

28. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then : **[JEE Main-2019]**



(1) $\tan\theta = \frac{2}{\sqrt{3}}$ (2) $\tan\theta = \frac{1}{3}$ (3) $\tan\theta = \frac{1}{2}$ (4) $\tan\theta = \frac{1}{2\sqrt{3}}$

Ans. **(2)**

Sol. Let mass of one rod is m.

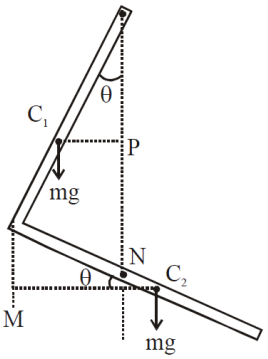
Balancing torque about hinge point.

mg (C_1P) = mg (C_2N)

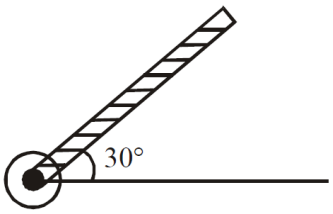
$mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta - L\sin\theta\right)$

$\Rightarrow \frac{3}{2}mgL\sin\theta = \frac{mgL}{2}\cos\theta$

$\Rightarrow \tan\theta = \frac{1}{3}$



29. A rod of length 50cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be ($g = 10ms^{-2}$)

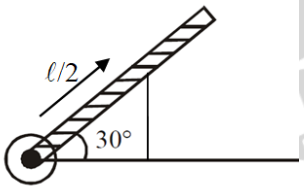


[JEE Main-2019]

(1) $\sqrt{\frac{30}{2}}$ (2) $\frac{\sqrt{20}}{3}$ (3) $\sqrt{30}$ (4) $\frac{\sqrt{30}}{3}$

Ans. **(3)**

Sol. $mgh_{cm} = \frac{1}{2}I\omega^2$



$\Rightarrow mg\frac{\ell}{2}\sin 30° = \frac{1}{2}\frac{m\ell^2}{3}\omega^2$

$\Rightarrow \omega = \sqrt{\frac{3g}{2\ell}} = \sqrt{30}\,rad/s$

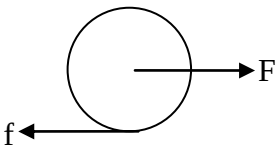
30. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is:

[JEE Main-2019]

(1) $\frac{F}{2MR}$ (2) $\frac{3F}{2MR}$ (3) $\frac{F}{2MR}$ (4) $\frac{2F}{3MR}$

Ans. **(4)**

Sol.



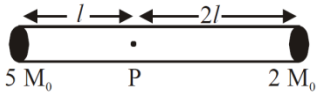
$F - f = ma$

$fR = \frac{MR^2}{2}\alpha$

$\alpha = \frac{a}{R}$ (for Pure Rolling)

On solving, $\alpha = \frac{2F}{3MR}$

31. A rigid massless rod of length 3l has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be :



[JEE Main-2019]

(1) $\frac{7g}{3l}$ (2) $\frac{g}{13l}$ (3) $\frac{g}{3l}$ (4) $\frac{g}{2l}$

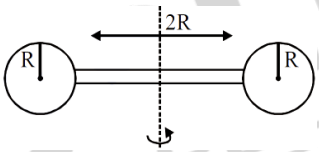
Ans. **(2)**

Sol. $\tau_{net} = 5M_0gl - 2M_0g(2l) = M_0gl$

$I = 2M_0(2l)^2 + 5M_0(l)^2 = 13M_0l^2$

$$\alpha = \frac{\tau_{net}}{I} = \frac{M_0gl}{13M_0l^2} = \frac{g}{13l}$$

32. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is :



[JEE Main-2019]

(1) $\frac{137}{15}MR^2$ (2) $\frac{209}{15}MR^2$ (3) $\frac{17}{15}MR^2$ (4) $\frac{152}{15}MR^2$

Ans. **(1)**

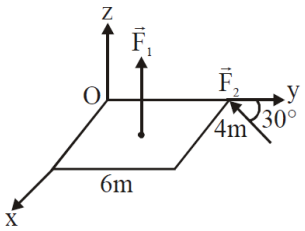
Sol. $$I = 2\left(\frac{2}{5}MR^2 + M(2R)^2\right) + \frac{M(2R)^2}{12}$$

$$I = \frac{44MR^2}{5} + \frac{4MR^2}{12}$$

$$I = \frac{137MR^2}{15}$$

33. A slab is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$ of same magnitude F as shown in the figure. Force $\vec{F}_2$ is in XY-plane while force F_1 acts along z-axis at the point $\left(2\vec{i}+3\vec{j}\right)$. The moment of these forces about point O will be :

[JEE Main-2019]



(1) $\left(3\hat{i}+2\hat{j}-3\hat{k}\right)F$ (2) $\left(3\hat{i}-2\hat{j}-3\hat{k}\right)F$ (3) $\left(3\hat{i}+2\hat{j}+3\hat{k}\right)F$ (4) $\left(3\hat{i}-2\hat{j}+3\hat{k}\right)F$

Ans. **(4)**

Sol. $\vec{I}_{f_1} = \vec{r}_1 \times \vec{f}_1$

$= (2\hat{i}+3\hat{j})\times(F\hat{k})$

$= 3F\hat{i} - 2F\hat{j}$

$\vec{I}_{f_2} = \vec{r}_2 \times \vec{f}_2$

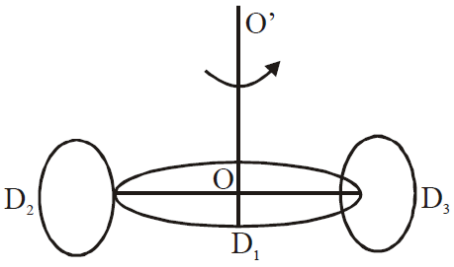
$= 6\hat{j}\times\left(-\frac{F\hat{i}}{2}-\frac{F\sqrt{3}\hat{j}}{2}\right)$

$\vec{I} = \vec{I}_{F_1} + \vec{I}_{F_2}$

$= 3F\hat{i} - 2F\hat{j} + 3F\hat{K}$

$= \left(3\hat{i}-2\hat{j}+3\hat{k}\right)F$

34. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1, as shown in the figure, will be:-



[JEE Main-2019]

(1) MR^2 (2) $\frac{2}{3}MR^2$ (3) $3MR^2$ (4) $\frac{4}{5}MR^2$

Ans. **(3)**

Sol. $I_{net} = \frac{MR^2}{2} + 2\left(\frac{MR^2}{4} + MR^2\right)$

$= 3MR^2$

35. The magnitude of torque on a particle of mass 1kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians) :-

[JEE Main-2019]

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{8}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Ans. **(1)**

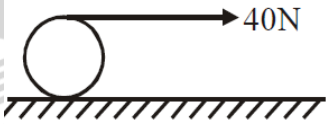
Sol. $\vec{\tau} = \vec{r} \times \vec{F}$

$2.5 = 1 \times 5 \times \sin\theta$

$\boxed{\theta = \pi/6}$

36. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) :-

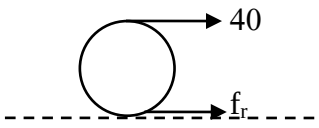
[JEE Main-2019]



(1) 10 rad/s^2 (2) 16 rad/s^2 (3) 20 rad/s^2 (4) 12 rad/s^2

Ans. **(2)**

Sol.



$40 + f = ma$

$40 \times R - f \times R \Rightarrow I \cdot \alpha$

$a = R\alpha$

By solving above 3 equations

$\boxed{\alpha = 16 \text{ rad/sec}^2}$

37. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm), about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is : **[JEE Main-2019]**

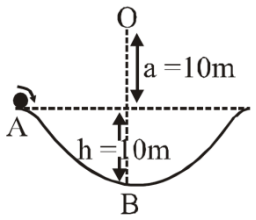
(1) 16 cm (2) 14 cm (3) 12 cm (4) 18 cm

Ans. **(1)**

Sol. $1 = \frac{M(R_1^2 + R_2^2)}{2} = MR^2$

$R = \sqrt{\frac{R_1^2 + R_2^2}{2}} = 16\text{cm}$

38. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be : (Take g = 10 m/s^2) **[JEE Main-2019]**



(1) 6kg-m^2/s (2) 2kg-m^2/s (3) 3kg-m^2/s (4) 8kg-m^2/s

Ans. **(1)**

Sol. $mgh = \Delta K$

$= \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$

$\Rightarrow v_B^2 = v_A^2 + 2gh$

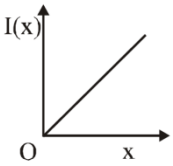
$= 5^2 + 2 \times 10 \times 10 = 225$

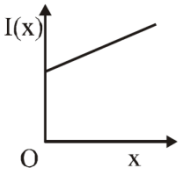
$\Rightarrow v_B = 15$ m/s

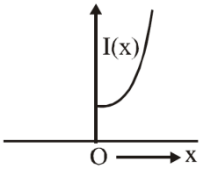
So, $L_0 = mvr_\perp$

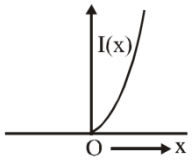
$= \frac{20}{1000} \times 15 \times 20 = 6$ kg-m^2/s

39. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is 'I(x)'. Which one of the graphs represents the variation of I(x) with x correctly? **[JEE Main-2019]**

(1) 

(2) 

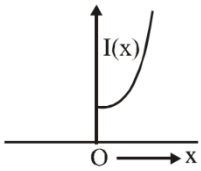
(3) 

(4) 

Ans. **(3)**

Sol. $I(x) = Icm + md^2$

$= \frac{2}{5}mR^2 + m(R+x)^2$



$\therefore$ Parabolic curve and at x = 0, I(x) has finite value.

40. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its center. The moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = a MR^2$. The value of the coefficient a is : **[JEE Main-2019]**

(1) $\frac{3}{2}$ (2) $\frac{8}{5}$ (3) $\frac{1}{2}$ (4) $\frac{3}{5}$

Ans. **(2)**

Sol. $M = \int_0^R \rho_0 r(2\pi r dr) = \frac{\rho_0 \times 2\pi \times R^3}{3}$

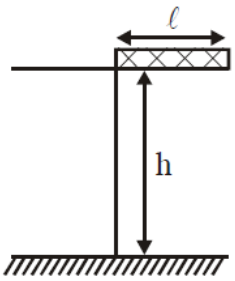
$I_{0\ (\text{MOI about COM})} = \int_0^R \rho_0 r(2\pi r dr) \times r^2 = \frac{\rho_0 \times 2\pi R^5}{5}$

By parallel axis theorem

$I = I_{com} + MR^2 = I_0 + MR_2$

$= \frac{\rho_0 \times 2\pi R^5}{5} + \frac{\rho_0 \times 2\pi R^3}{3} \times R^2 = \rho_0 2\pi R^5 \times \frac{8}{15} = MR^2 \times \frac{8}{5}$

41. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5m. When released, it slips off the table in a very short time $\tau = 0.01$ s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to : **[JEE Main-2019]**



(1) 0.3 (2) 0.5 (3) 0.28 (4) 0.02

Ans. **(2)**

Sol. Angular impulse = change in angular momentum

$\tau\Delta t = \Delta L$

$$mg\frac{l}{2}\times 0.01 = \frac{ml^2}{3}\omega$$

$$\omega = \frac{3g\times 0.01}{2l} = \frac{3\times 10\times 0.01}{2\times 0.3} = \frac{1}{2} = 0.5\ \text{rad/s}$$

Time taken by rod to hit the ground

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 5}{10}} = 1\ \text{sec}$$

In this time angle rotate by rod $\theta = \omega t = 0.5 \times 1 = 0.5$ radian

42. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is: **[JEE Main-2019]**

(1) $\frac{2k}{I}\theta$ (2) $\frac{k}{2I}\theta$ (3) $\frac{k}{4I}\theta$ (4) $\frac{k}{I}\theta$

Ans. **(1)**

Sol.

Kinetic energy KE = $\frac{1}{2}\ I\omega^2 = k\theta^2$

$$\Rightarrow \omega^2 = \frac{2k\theta^2}{1} \Rightarrow \omega = \sqrt{\frac{2k}{1}}\theta \qquad ....(1)$$

Differentiate (1) wt time $\rightarrow$ d

$$\frac{d\omega}{dt} = \alpha = \sqrt{\frac{2k}{1}}\left(\frac{d\theta}{dt}\right)$$

$$\Rightarrow \quad \alpha = \sqrt{\frac{2k}{1}}\cdot\sqrt{\frac{2k}{1}}\ \theta \ \{\text{by (1)}\}$$

$$\Rightarrow \ a = \frac{2k}{1}\theta$$

43. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be : **[JEE Main-2019]**

(1) $\frac{M\,\omega_0}{M+m}$ (2) $\frac{M\,\omega_0}{M+2m}$ (3) $\frac{M\,\omega_0}{M+6m}$ (4) $\frac{M\,\omega_0}{M+3m}$

Ans. **(3)**

Sol. Applying angular momentum conservation, about axis of rotation

$L_i = L_f$

$$\frac{ML^2}{12}\omega_0 = \left(\frac{ML^2}{12} + m\left(\frac{L}{2}\right)^2 \times 2\right)\omega$$

$$\Rightarrow \quad \omega = \frac{M\omega}{M+6m}$$

44. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is : **[JEE Main-2019]**

(1) $\frac{MR^2}{3}$ (2) $\frac{MR^2}{6}$ (3) $\frac{2MR^2}{3}$ (4) $\frac{MR^2}{2}$

Ans. **(3)**

Sol. $dI = dmr^2$

$dI = \sigma \cdot 2\pi\, rdr \cdot r^2$

$dI = 2\pi k\, r^5\, dr$

$$I = \int_O^R dI = \frac{2\pi KR^6}{6}$$

$dm = \sigma \cdot 2\pi rdr = 2\pi kr^3 dr$

$$M = \int dm = \frac{2\pi KR^4}{4}$$

$2\pi KR^4 = 4M$

$$I = \frac{4MR^2}{6} = \frac{2MR^2}{3}$$

45. The time dependence of the position of a particle of mass m = 2 is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time t = 2 is : **[JEE Main-2019]**

(1) $-48\hat{k}$ (2) $48(\hat{i}+\hat{j})$ (3) $-34(\hat{k}-\hat{i})$ (4) $36\hat{k}$

Ans. **(1)**

Sol. $\vec{L} = \vec{r} \times \vec{p}$

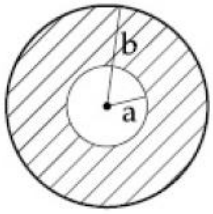
$= m(\vec{r}\times\vec{v})$

$= m(2t\hat{i}-3t^2\hat{j})\times(2\hat{i}-6t\hat{j})$

$= 2(4\hat{i}-12\hat{j})\times(2\hat{i}-12\hat{j})$

$= -48\hat{k}$

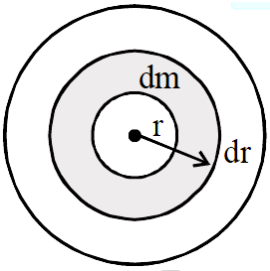
46. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is **[JEE Main-2019]**



(1) $\sqrt{\frac{a^2+b^2+an}{3}}$ (2) $\frac{a+b}{2}$ (3) $\frac{a+b}{3}$ (4) $\sqrt{\frac{a^2+b^2+ab}{2}}$

Ans. **(1)**

Sol.



$dI = (dm)r^2$

$= (\sigma\, dA)r^2$

$= \left(\frac{\sigma_0}{r}2\pi r dr\right)r^2$

$= (\sigma_0 2p)r^2\, dr$

$I = \int dI = \int_a^b \sigma_0 2\pi r^2 dr$

$= \sigma_0 2\pi\left(\frac{b^3-a^3}{3}\right)$

$m = \int dm = \int \sigma dA$

$= \sigma_0 2\pi\int_a^b dr$

$m = \sigma_0\, 2\pi(b-a)$

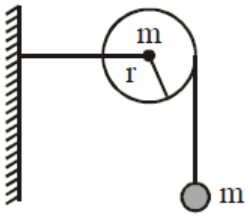
Radius of gyration

$$k = \sqrt{\frac{I}{m}} = \sqrt{\frac{(b^3 - a^3)}{3(b-a)}}$$

$$= \sqrt{\left(\frac{a^2 + b^2 + ab}{3}\right)}$$

47. As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be :

[JEE Main-2020]



(1) $r\sqrt{\frac{3}{2gh}}$ (2) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$ (3) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$ (4) $r\sqrt{\frac{3}{4gh}}$

Ans. (2)

Sol. Apply mechanical energy conservation

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}IW^2$$

$$v = RW$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{MR^2}{2}\frac{V^2}{R^2}$$

$$mgh = \frac{3}{4}MW^2R^2$$

$$w = \sqrt{\frac{4gh}{3R^2}} = \frac{1}{R}\sqrt{\frac{4hg}{3}}$$

48. The radius of gyration of a uniform rod of length *l*, about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it, is : **[JEE Main-2020]**

(1) $\frac{1}{4}l$ (2) $\sqrt{\frac{7}{48}}l$ (3) $\sqrt{\frac{3}{8}}l$ (4) $\frac{3}{8}l$

Ans. **(2)**

Sol. $I = \frac{Ml^2}{12} + M\left(\frac{l}{4}\right)^2$

$$I = \frac{7Ml^2}{98} \quad \ldots(1)$$

$$I = MK^2 \quad \ldots(2)$$

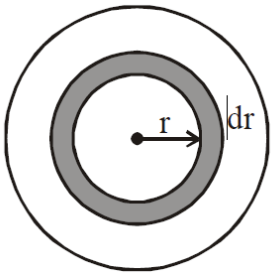
$$MK^2 = \frac{7Ml^2}{98}$$

$$K = \sqrt{\frac{7}{8}}l$$

49. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular t6o the plane and passing through its centre is : **[JEE Main-2020]**

(1) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (2) $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$ (3) $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (4) $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$

Ans. (3) **Sol.**



$dI = dmr^2$

$dI = \sigma 2\pi r\, dr\, r^2$

$dI = 2\pi(A + Br)\, r3\, dr$

$$\int dI = 2p\int_0^a (Ar^3 + Br^4)\, dr$$

$$I = 2\pi a^4\left(\frac{A}{4}+\frac{B9}{5}\right)$$

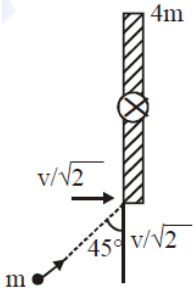
50. Consider a uniform rod of mass M = 4m and length l pivoted about its centre. A mass m moving with velocity v making angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is:

[JEE Main-2020]

(1) $\frac{3}{7\sqrt{2}}\frac{v}{l}$ (2) $\frac{3}{7}\frac{v}{l}$ (3) $\frac{4}{7}\frac{v}{l}$ (4) $\frac{3\sqrt{2}}{7}\frac{v}{l}$

Ans. (4)

Sol.



Let angular velocity of the system after collision is ω.

Applying conservation of angular momentum about the hinge:

$$m\left(\frac{v}{\sqrt{2}}\right)\left(\frac{l}{2}\right)=\left[\frac{(4m)l^2}{12}+\frac{ml^2}{4}\right]\omega$$

On solving

$$\omega=\frac{3\sqrt{2}}{7}\left(\frac{v}{l}\right)$$

51. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is: **[JEE Main-2020]**

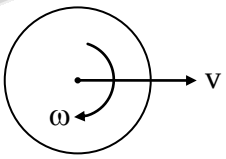
(1) 8.75×10^{-4} J (2) 6.25×10^{-4} J (3) 1.13×10^{-3} J (4) 8.75×10^{-3} J

Ans. **(1)**

Sol. For pure rolling ,

$$v = r\omega$$

$$\text{K.E.} = \frac{1}{2}mr^2+\frac{1}{2}\cdot\frac{2}{5}mR^2\cdot\frac{v^2}{R^2}$$

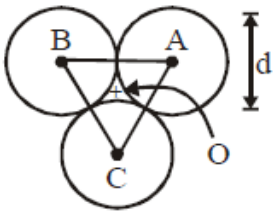


$$\text{K.E.} = \frac{1}{2}m\cdot\frac{7}{5}v^2$$

$$\text{K.E.} = \frac{1}{2}\times\frac{1}{2}\times\frac{7}{5}\times\frac{5}{100}\times\frac{5}{100}$$

$$\text{K.E.} = 8.75 \times 10^{-4} \text{ J}$$

52.



Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio I_0 / I_A of momentum of inertia I_0 of the system about an axis passing the centroid and about centre of any of the spheres I_A and perpendicular to the plane of the triangle is : **[JEE Main-2020]**

(1) $\frac{13}{23}$ (2) $\frac{15}{13}$ (3) $\frac{23}{13}$ (4) $\frac{13}{15}$

Ans. **(1)**

Sol. From parallel axis theorem

$$I_0 = 3\times\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M\left(\frac{d}{\sqrt{3}}\right)^2\right] = \frac{13}{10}Md^2$$

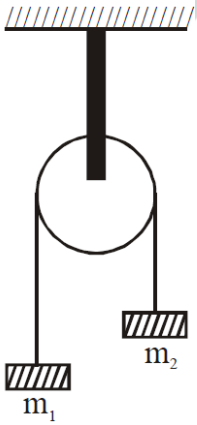
$$I_A = I_0 + 3M\left(\frac{d}{\sqrt{3}}\right)^2$$

$$= \frac{13}{10}Md^2 + Md^2$$

$$= \frac{23}{10}Md^2$$

$$\frac{I_0}{I_A} = \frac{13}{23}$$

53. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig). A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m, descents by a distance h is: **[JEE Main-2020]**

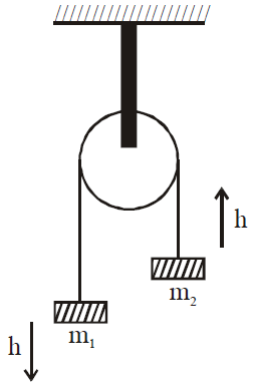


(1) $\left[\frac{2(m_1+m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$ (2) $\left[\frac{m_1+m_2}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}gh$

(3) $\left[\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$ (4) $\left[\frac{(m_1-m_2)}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}gh$

Ans. **(3)**

Sol.



by using work energy theorem

$W_g = \Delta KE$

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)V^2 + \frac{1}{2}I\omega^2$$

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)(\omega R)^2 + \frac{1}{2}I\omega^2$$

$$(m_1 - m_2)gh = \frac{\omega^2}{2}[(m_1 + m_2)R^2 + I]$$

$$\omega = \sqrt{\frac{2(m_1 + m_2)gh}{(m_1 + m_2)R^2 + I}}$$

EXERCISE # (JA)

1. A particle is moving in a horizontal uniform circular motion. The angular momentum of the particle is conserved about the point : **[JEE'(Scr)2003]**

(A) Centre of the circle (B) Outside the circle

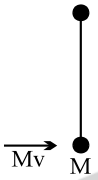
(C) Inside the circle (D) Point on circumference

Ans. (A)

Sol. $\vec{L} = \vec{r} \times \vec{p}$; since $|\vec{p}|$ = constant

$\therefore$ L is constant when $|\vec{r}|$ is constant.

2. Two particles each of mass M are connected by a massless rod of length *l*. The rod is lying on the smooth surface. If one of the particle is given an impulse MV as shown in the figure then angular velocity of the rod would be : **[JEE'(Scr)2003]**



(A) v/*l* (B) 2v/*l* (C) v/2*l* (D) None

Ans. (A)

Sol. Applying conservation of angular momentum,

$$\frac{mvL}{2} = \left(\frac{mL^2}{4} + \frac{mL^2}{4}\right)\omega$$

$$\therefore \quad \omega = \frac{v}{L}$$

3. A child is standing with folded hands at the center of a platform rotating about its central axis. The kinetic energy of the system is K. The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is **[IIT JEE-2004]**

(A) 2K (B) K/2 (C) K/4 (D) 4K

Ans. (B)

Sol. From conservation of angular momentum ($I\omega$ = constant), angular velocity will remain half. As,

$$K = \frac{1}{2} I\omega^2$$

The rotational kinetic energy will become half. Hence, the correct option is (B).

4. A horizontal circular plate is rotating about a vertical axis passing through its centre with an angular velocity ω_0. A man sitting at the centre having two blocks in his hands stretches out his hands so that the moment of inertia of the system doubles. If the kinetic energy of the system is K initially, its final kinetic energy will be **[IIT JEE-2004]**

(A) 2 K (B) K/2 (C) K (D) K/4

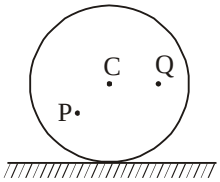
Ans. (B)

Sol. From conservation of angular momentum ($I\omega$ = constant), angular velocity will remain half. As,

$$K = \frac{1}{2} I\omega^2$$

The rotational kinetic energy will become half. Hence, the correct option is (B).

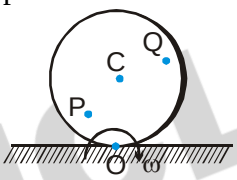
5. A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is **[IIT JEE-2004]**



(A) $v_Q > v_C > v_P$ (B) $v_P > v_C > v_Q$ (C) $v_P = v_C$, $v_Q = v_C/2$ (D) $v_P < v_C > v_Q$

Ans. (A)

Sol. In case of pure rolling bottommost point is the instantaneous centre of zero velocity.



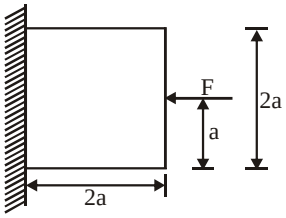
Velocity of any point on the disc, $v = r\omega$, where r is the distance of point from O.

$$r_Q > r_C > r_P$$

$\therefore$ $v_Q > v_C > v_P$

Therefore, the correct option is (A).

6. A block of mass m is held fixed against a wall by a applying a horizontal force F. Which of the following option is incorrect: **[JEE'2005 (Scr)]**

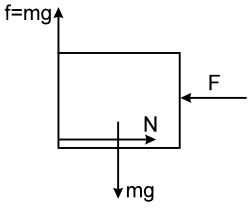


(A) friction force = mg (B) F will not produce torque

(C) normal will not produce torque (D) normal reaction = F

Ans. (C)

Sol.



From equilibrium,

friction = mg N = F

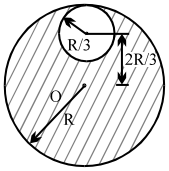
about centre of mass

$\tau = 0 \Rightarrow$ mg a = torque due to normal

$\therefore$ Normal will produce torque

since F passes through centre its torque is zero.

7. A disc has mass 9m. A hole of radius R/3 is cut from it as shown in the figure. The moment of inertia of remaining part about an axis passing through the centre 'O' of the disc and perpendicular to the plane of the disc is: **[JEE'2005 (Scr)]**



(A) $8\ mR^2$ (B) $4\ mR^2$ (C) $\frac{40}{9} mR^2$ (D) $\frac{37}{9} mR^2$

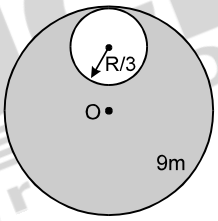
Ans. (B)

Sol. $I_0 = I_1 - I_2$

where I_1 = (M.I. of full disc about O)

I_2 (M.I. of small removed disc about O)

since mass α area



$$\frac{\text{mass of cut disc}}{\text{mass of total}} = \frac{\frac{R^2}{9}}{R^2} = \frac{1}{9}$$

$\therefore$ mass of cut disc = m

$\therefore$ $I_0 = \frac{(9m)R^2}{2} - m\left[\frac{\left(\frac{R}{3}\right)^2}{2} + \left(\frac{2R}{3}\right)^2\right]$ (by theorem of parallel axis.)

$$= \frac{9mR^2}{2} - mR^2\left[\frac{1}{18} + \frac{4}{9}\right] = \frac{9mR^2}{2} - \frac{mR^2}{2} = \frac{8mR^2}{2} = 4mR^2.$$

8. A particle moves in circular path with decreasing speed. Which of the following is correct **[JEE'2005 (Scr)]**

(A) $\vec{L}$ is constant

(B) only direction of $\vec{L}$ is constant

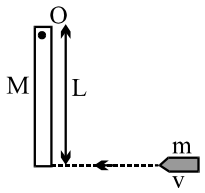
(C) acceleration $\vec{a}$ is towards the centre

(D) it will move in a spiral and finally reach the centre

Ans. (B)

Sol. Only direction of $\vec{L}$ (angular momentum) is constant because the direction of rotation is unchanged.

9. A wooden rod of mass M and length L is hinged by a frictionless nail at O. A bullet of mass m strikes with velocity v and sticks to it. Find angular velocity of the system immediately after the collision about O. **[IIT JEE- 2005]**



Ans. $\omega = \dfrac{3mv}{(3m+M)L}$

Sol. Initial angular momentum about fixed point = mvL

final angular momentum = $I\omega = \left(\dfrac{ML^2}{3} + mL^2\right)\omega$

where I is moment of inertia of the system about the fixed point and ω is angular velocity about the fixed point.

angular momentum before collision = angular momentum after collision

$$mLv = L^2\left(\frac{M}{3}+m\right)\omega \qquad \omega = \frac{mv}{L\left(\frac{M}{3}+m\right)} = \frac{3mv}{(M+3m)\times L}$$

10. A cylinder of mass m and radius R rolls down an inclined plane of inclination θ. Calculate the linear acceleration of the axis of cylinder. **[IIT JEE- 2005]**

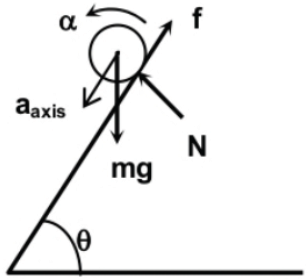
Ans. $a_{axis} = \dfrac{2g\sin\theta}{3}$

Sol. $mg\sin\theta - f = ma_{axis}$ (1)

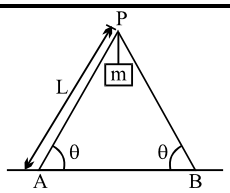
$fR = I_{axis}\,\alpha$ (2)

$a_{axis} = R\alpha$ (3)

$$a_{axis} = \frac{2g\sin\theta}{3}$$



11. Two identical ladders, each of mass M and length L are resting on the rough horizontal surface as shown in the figure. A block of mass m hangs from P. If the system is in equilibrium, find the magnitude and the direction of frictional force at A and B. **[IIT JEE- 2005]**

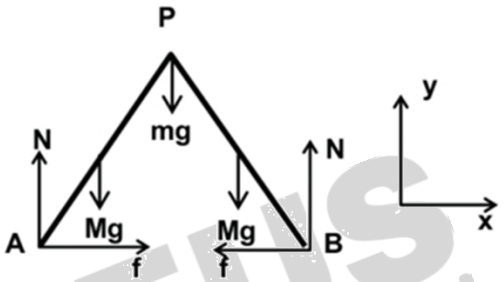


Ans. $f = (M+m)\, g \dfrac{\cot\theta}{2}$

Sol. For equilibrium of whole system,

$\Sigma F_y = 0$

$\Rightarrow \quad N = \left(\dfrac{2M+m}{2}\right) g$



For rotational equilibrium of either ladder

Calculating torque about P

$NL\cos\theta - Mg\dfrac{L}{2}\cos\theta - fL\sin\theta = 0$

$\Rightarrow \quad f = (M+m)\, g \dfrac{\cot\theta}{2}$

12. A solid sphere of mass M, radius R and having moment of inertia about an axis passing through the centre of mass as I, is recast into a disc of thickness t, whose moment of inertia about an axis passing through its edge and perpendicular to its plane remains I. Then, radius of the disc will be **[IIT JEE-2006]**

(A) $2R/\sqrt{15}$ (B) $R\sqrt{2/15}$ (C) $4R/\sqrt{15}$ (D) R/4

Ans. (A)

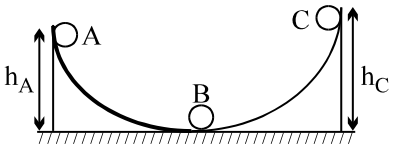
Sol. $2/5\ MR^2 = 1/2\ Mr^2 + Mr^2$

$\therefore \quad 2/5\ MR^2 = 3/2\ Mr^2$

$r^2 = \dfrac{4}{15} R^2$

$r = \dfrac{2R}{\sqrt{15}}$

13. A ball moves over a fixed track as shown in the figure. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C, respectively. Then **[IIT JEE -2006]**



(A) $h_A > h_C$; $K_B > K_C$ (B) $h_A > h_C$; $K_C > K_A$

(C) $h_A = h_C$; $K_B = K_C$ (D) $h_A < h_C$; $K_B > K_C$

Ans. (AB)

Sol. As total mechanical energy at points A,B and C will be constant

$\therefore \quad \varepsilon_A = \varepsilon_B = \varepsilon_C$

$\Rightarrow \quad mgh_A + K_A = K_B = mgh + K_C$

$\therefore \quad K_B > K_A \qquad (mgh_A + K_A = K_B)$

and $\quad K_B > K_C \qquad (mgh_C + K_C = K_B)$

Also $\quad h_A - h_C = \dfrac{K_C - K_A}{mg} \quad$ when $\quad mgh_A + K_A = mgh_C + K_C$

if $\quad h_A > h_C \qquad \Rightarrow \qquad K_C > K_A$

(if LHS is positive then RHS have to be positive)

14. A solid cylinder is rolling down a rough inclined plane of inclination θ. Then **[IIT JEE -2006]**

(A) The friction force is dissipative

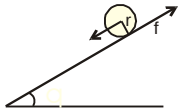
(B) The friction force is necessarily changing

(C) The friction force will aid rotation but hinder translation

(D) The friction force is reduced if θ is reduced

Ans. (CD)

Sol.



necessary torque for rolling $\quad \tau = fr$, (frictional force provides this torque)

as $\quad mg \sin\theta - f = ma$

but $\quad a = r\alpha \qquad \Rightarrow \qquad mg \sin\theta - f = mr\alpha$

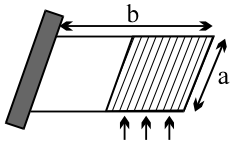
as $\quad \tau = fr = I\alpha \qquad \Rightarrow \qquad \alpha = fr/I$

$\therefore \quad mg \sin\theta - f = mrfr/I = 5f/2 \left(I = \dfrac{2mr^2}{5} \right)$

$\therefore \quad mg \sin\theta = \dfrac{7f}{2}$

thus friction increases the torque in hence the angular velocity and decreases the linear velocity. If θ decreases friction will decrease.

15. There is a rectangular plate of mass M kg of dimensions (a × b). The plate is held in horizontal position by striking n small balls each of mass m per unit area per unit time. These are striking in the shaded half region of the plate. The balls are colliding elastically with velocity v. What is v? It is given n = 100, M = 3 kg, m = 0.01 kg; b = 2 m; a = 1m; g = 10 m/s^2. **[IIT JEE-2006]**



Ans. 10 m/s

Sol. (As collision is elastic)

$\therefore \quad F = \dfrac{dP}{dt} = \dfrac{2mV}{1} = 2mV$

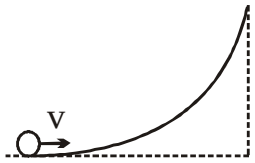
$\therefore$ torque about hinge = $2mV \times \left(\frac{b}{2}+\frac{b}{4}\right) \times 100$

$= 2mV\ \frac{3b}{4} \times 100 = Mg\frac{b}{2}$

$\therefore$ V = 10 m/s

16. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches upto maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is

[IIT JEE -2007]



(A) ring (B) solid sphere (C) hollow sphere (D) disc

Ans. (D)

Sol. From the conservation of energy

loss in KE of body = Gain in potential energy

$$\frac{1}{2}\ mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2 = mg\frac{3}{4}\frac{v^2}{g}$$

on solving

$$I = \frac{mr^2}{2}$$

$\therefore$ The body is a disc

Comprehension Questions for Q. No. 17 to 19

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and 2I respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction. **[IIT-JEE 2007]**

17. The ratio $\frac{x_1}{x_2}$ is

(A) 2 (B) $\frac{1}{2}$ (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

Ans. (C)

Sol. $\frac{1}{2}\times I\times(2\omega)^2 = \frac{1}{2}kx_1^2$

$\frac{1}{2}\times 2I\times\omega^2 = \frac{1}{2}kx_2^2 \qquad \Rightarrow \frac{x_1}{x_2} = \sqrt{2}$

18. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

(A) $\frac{2I\omega}{3t}$ (B) $\frac{9I\omega}{2t}$ (C) $\frac{9I\omega}{4t}$ (D) $\frac{3I\omega}{2t}$

Ans. (A)

Sol. Apply conservation of angular momentum

$$(I \times 2\omega) + (2I \times \omega) = (I + 2I)\,\omega'$$

$$\Rightarrow \quad \omega' = \frac{4\omega}{3}$$

For Disc A

$$\tau\, t = I \times (2\omega - \omega')$$

$$\Rightarrow \quad \tau = \frac{2I\omega}{3t}$$

19. The loss of kinetic energy during the above process is

(A) $\frac{I\omega^2}{2}$ (B) $\frac{I\omega^2}{3}$ (C) $\frac{I\omega^2}{4}$ (D) $\frac{I\omega^2}{6}$

Ans. (B)

Sol. Initial Kinetic Energy $k_1 = \frac{1}{2} \times I \times (2\omega)^2 + \frac{1}{2} \times 2I \times \omega^2$

Final Kinetic Energy $k_2 = \frac{1}{2} \times I \times \omega'^2 + \frac{1}{2} \times 2I\,\omega'^2$

Loss of Kinetic Energy = $k_1 - k_2 = \frac{I\omega^2}{3}$

20. **Statement–I :** Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first. **[IIT JEE-2008]**

and

Statement–2 : By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1

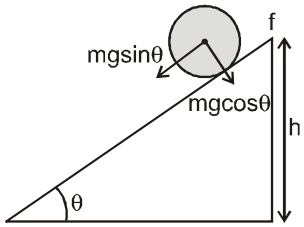
(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Ans. (D)

Sol. The acceleration of centre of mass of either cylinder

$a = \dfrac{g\sin\theta}{1+\dfrac{K^2}{R^2}}$ where K is radius of gyration.



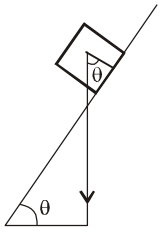
So acceleration of centre of hollow cylinder is less than that of solid cylinder. Hence time taken by hollow cylinder will be more.
So statement-1 is wrong.

21. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then : **[IIT-JEE 2009]**
(A) at θ =30°, the block will start sliding down the plane
(B) the block will remain at rest on the plane up to certain θ and then it will topple
(C) at θ=60°, the block will start sliding down the plane and continue to do so at higher angles
(D) at θ=60°, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

Ans. (B)

Sol. Angle of repose $\theta_0 = \tan^{-1}\mu = \tan^{-1}\sqrt{3} = 60^\circ$

$\tan\theta = \dfrac{5}{15/2} = \dfrac{2}{3}$. $\theta < 45^\circ$.



Block will topple before it starts slide down.

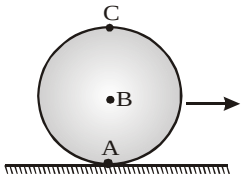
22. If the resultant of the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **[IIT JEE-2009]**
(A) linear momentum of the system does not change in time
(B) kinetic energy of the system does not change in time
(C) angular momentum of the system does not change in time
(D) potential energy of the system does not change in time

Ans. (A)

Sol.: (A) Since there in no resultant external force, linear momentum of the system remains constant.
(B) Kinetic energy of the system may change.

(C) Angular momentum of the system may change as in case of couple, net force is zero but torque is not zero. Hence angular momentum of the system is not constant.
(D) Potential energy may also change.

23. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then **[IIT-JEE 2009]**

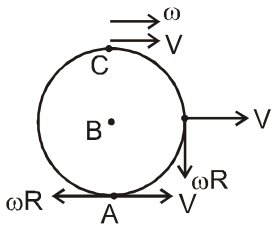


(A) $\vec{v}_C - \vec{v}_A = 2(\vec{v}_B - \vec{v}_C)$ (B) $\vec{v}_C - \vec{v}_B = \vec{v}_B - \vec{v}_A$

(C) $|\vec{v}_C - \vec{v}_A| = 2|\vec{v}_B - \vec{v}_C|$ (D) $|\vec{v}_C - \vec{v}_A| = 4|\vec{v}_B|$

Ans. (BC)

Sol.



$\vec{V}_A = V(\hat{i}) + \omega R(-\hat{i})$; $\vec{V}_B = V\hat{i}$; $\vec{V}_C = V\hat{i} + \omega R\hat{i}$

$\vec{V}_C - \vec{V}_A = 2\omega R\hat{i}$

$2\left[\vec{V}_B - \vec{V}_C\right] = 2[V(\hat{i}) - V(\hat{i}) - \omega R(\hat{i})] = -2\omega R(\hat{i})$

Hence $\vec{V}_C - \vec{V}_A = -2(\vec{V}_B - \vec{V}_C)$

so $|\vec{V}_C - \vec{V}_A| = -2(\vec{V}_B - \vec{V}_C)$

$\vec{V}_C - \vec{V}_B = \omega R(\hat{i})$

$\vec{V}_B - \vec{V}_A = \omega R(\hat{i})$

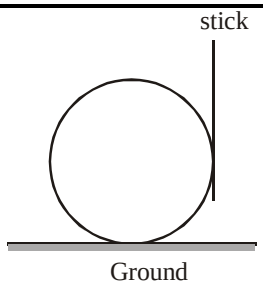
$\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

Hence $\vec{V}_C - \vec{V}_A = 2\omega R(\hat{i})$

$\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$; $4\vec{V}_B = 4V(\hat{i}) = 4\omega R(\hat{i})$

Hence $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B)$

24. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is **[IIT JEE-2011]**



Ans. 4

Sol. II Law$\Rightarrow$ $2 - f = 2\,[0.3]$

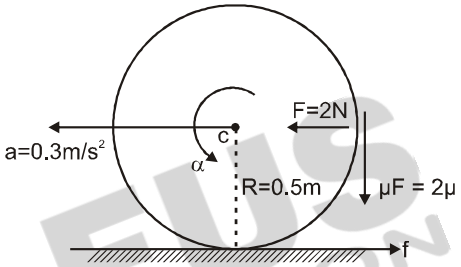
$\Rightarrow$ $f = 2 - 0.6$

$f - 1.4$ Nx ...(i)

$a = R\alpha$

$\Rightarrow$ $0.3 = \alpha\,[0.5]$

$\Rightarrow$ $\alpha = \frac{3}{5}$ rad/s ....(ii)



$\tau_c = I_c\,\alpha$

$\Rightarrow$ $fR - 2\mu R = mR^2\,\alpha$

$f - 2\mu = mR\alpha$

$1.4 - 2\mu = \frac{2}{2}\left(\frac{3}{5}\right)$

$1.4 - 0.6 = 2\mu$

$0.8 = 2\mu$ $\Rightarrow$ $\mu = 0.4 = \frac{P}{10}$ $\therefore$ $P = 4$ **Ans.**

25. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kg-m^2, then N is **[IIT-JEE 2011]**

Ans. 9

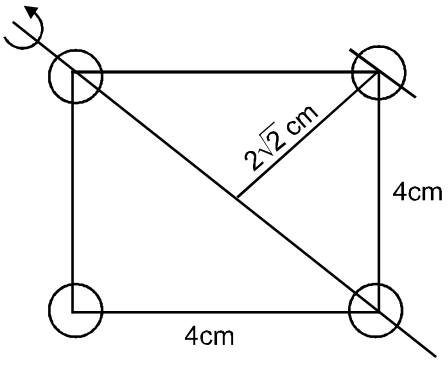
Sol. $I = \left(\frac{2}{5}MR^2\right)2 + \left(\frac{2}{5}MR^2 + Mx^2\right)2$

$= \left(\frac{2}{5}MR^2\right)2 + \left(\frac{2}{5}MR^2\right)2 + (Mx^2)\,2$

$= 4\left(\frac{2}{5}MR^2\right) + 2mx^2$

$= \frac{8}{5}MR^2 + 2mx^2$

$= \left[\frac{8}{5}\times 0.5\times\left(\frac{\sqrt{5}}{2}\right)^2 + 2\times(0.5)\times(4\times 2)\right]10^{-4}$

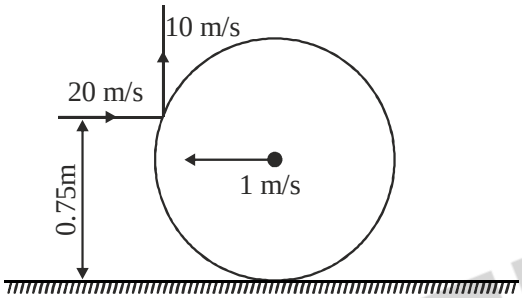


$$= \left[\frac{5}{5}+8\right] \times 10^{-4}$$

$$= 9 \times 10^{-4} = N \times 10^{-4}$$

So, N = 9 **Ans.**

26. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision **[IIT-JEE 2011]**

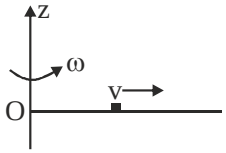


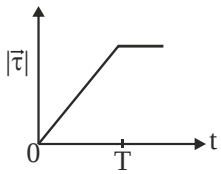
(A) the ring has pure rotation about its stationary CM
(B) the ring comes to a complete stop
(C) friction between the ring and the ground is to the left
(D) there is no friction between the ring and the ground

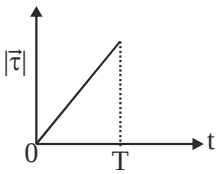
Ans. AC or C

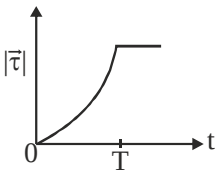
Sol. [friction would be impulsive so we can not apply conservation of momentum]

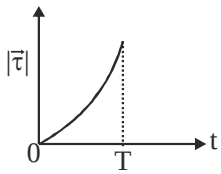
27. A thin uniform rod, pivoted at OP, is rotating in the horizontal plane with constant angular speed ω. as shown in the figure. At time t = 0, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. If reaches the end of the rod at t= T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque (|τ|) on the system about O, as a function of time is best represented by which plot? **[IIT-JEE 2012]**



(A) 

(B) 

(C) 

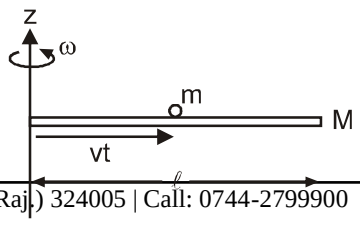
(D) 

Ans. (B)

Sol. $L = [m(vt)^2]\omega$

$L = mv^2\omega t^2$

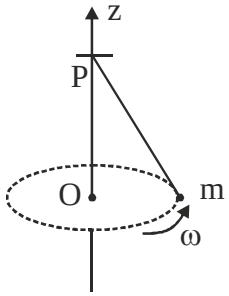
So $\tau = \frac{dL}{dt} = 2mv^2\omega t$



$\tau \propto t$

$\Rightarrow$ straight line passing through (0, 0)

28. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then **[IIT-JEE 2012]**



(A) $\vec{L}_0$ and $\vec{L}_P$ do not vary with time

(B) $\vec{L}_0$ varies with time while $\vec{L}_P$ remains constant

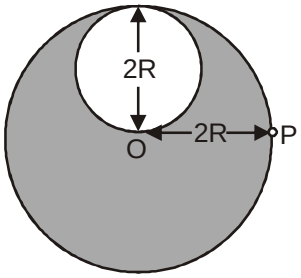
(C) $\vec{L}_0$ remains constant while $\vec{L}_P$ varies with time

(D) $\vec{L}_0$ and $\vec{L}_P$ both vary with time

Ans. (C)

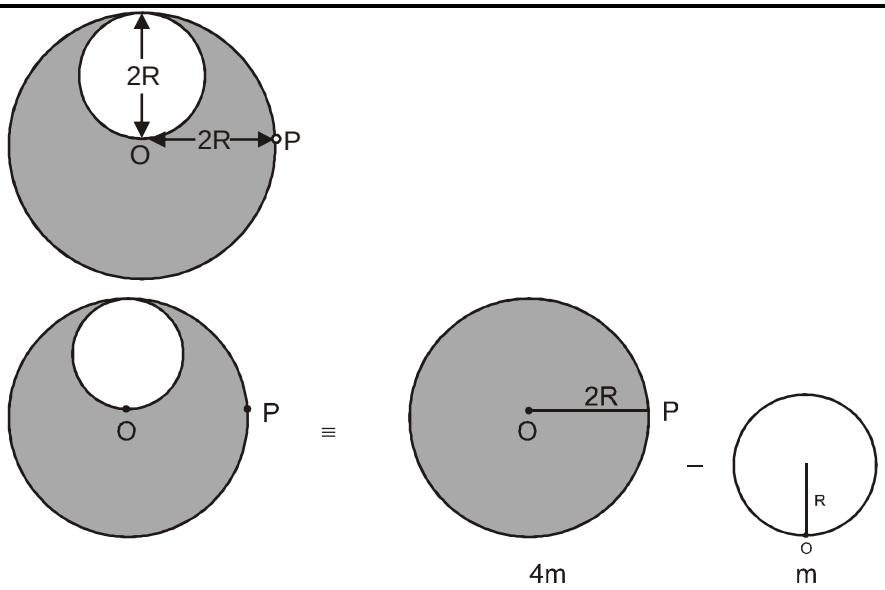
Sol. The net force acting on the mass m is $m\omega^2 r$ acting towards point O and hence net torque about point O is zero but about point P is non-zero. Hence $\vec{L}_0$ remains constant while $\vec{L}_p$ varies with time.

29. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_0 and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_0}$ to the nearest integer is **[IIT-JEE 2012]**



Ans. 3

Sol.



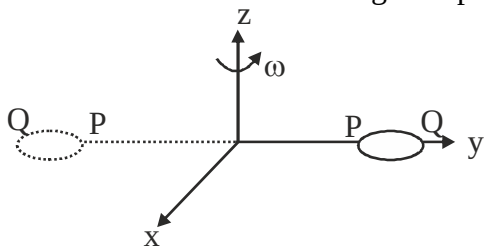
$$I_0 = \frac{(4m)(2R)^2}{2} - \frac{3}{2}mR^2$$

$$= mR^2\left[8 - \frac{3}{2}\right]$$

$$= \frac{13}{2}mR^2$$

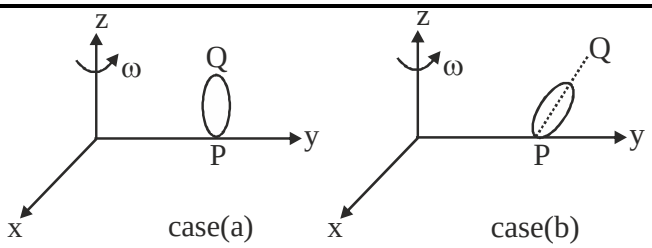
Paragraph for Questions 30 and 31

The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.



Now consider two similar systems as shown in the figure : case (A) the disc with its face vertical and parallel to x-z plane; Case (B) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis.

[IIT-JEE 2012]



30. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

(A) It is $\sqrt{2\omega}$ for both the cases

(B) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b)

(C) It is ω for case (a); and $\sqrt{2\omega}$ for case (b).

(D) It is ω for both the case

Ans. (D)

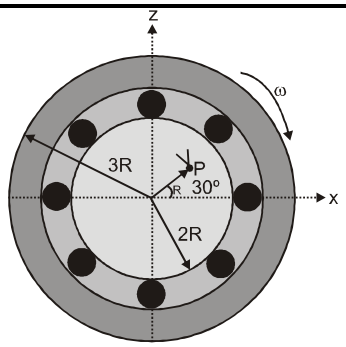
Sol. Angular Velocity of rigid body about any axes which are parallel to each other is same . So angular velocity is ω .
Angular Velocity of rigid body about any axes which are parallel to each other is same . So angular velocity is ω .

31. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?
(A) It is vertical for both the cases (a) and (b).
(B) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case(b).
(C) It is horizontal for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).
(D) It is vertical for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

Ans. (A)

Sol. Since z- coordinate of any particle is not changing with time so axis must be parellel to z axis.

32. The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed ω/2. The ring and disc are separated by frictionless ball bearing. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface



(A) the point O has a linear velocity $3R\omega\,\hat{i}$

(B) The point P has a linear velocity $\frac{11}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$

(C) The point P has a linear velocity $\frac{13}{4}R\omega\hat{i}-\frac{\sqrt{3}}{4}R\omega\hat{k}$

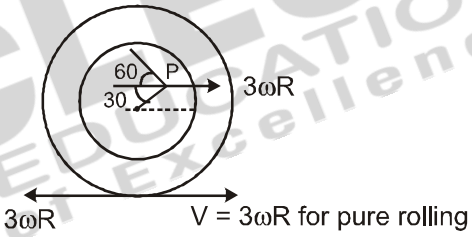
(D) The point P has a linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\hat{k}$

Ans. (AB)

Sol. $V_0 = 3\omega R\,\hat{i}$

$V_P\ (3\omega R - \frac{\omega R}{2}\cos 60^\circ)\ \hat{i}\ +\ \frac{\omega R}{2}\ \sin 60\,\hat{j}$

$=\frac{11\omega R}{4}\hat{i}+\frac{\sqrt{3}\omega R}{4}\hat{i}$



33. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct? **[IIT-JEE 2012]**

(A) Both cylinders P and Q reach the ground at the same time.

(B) Cylinder P has larger acceleration than cylinder Q.

(C) Both cylinders reach the ground with same translational kinetic energy.

(D) Cylinder Q reaches the ground with larger angular speed.

Ans. (D)

Sol. $a = \frac{Mg\sin\theta}{M+\frac{I}{R^2}}$

$a_P = \frac{Mg\sin\theta}{M+\frac{MR^2}{R^2}} \approx \frac{g}{2}$

$a_Q = g\sin\theta$ as $I_Q \sim 0$

$$\therefore \quad \omega_P = \frac{\sqrt{2 \cdot \frac{g}{2} \cdot \ell}}{R}$$

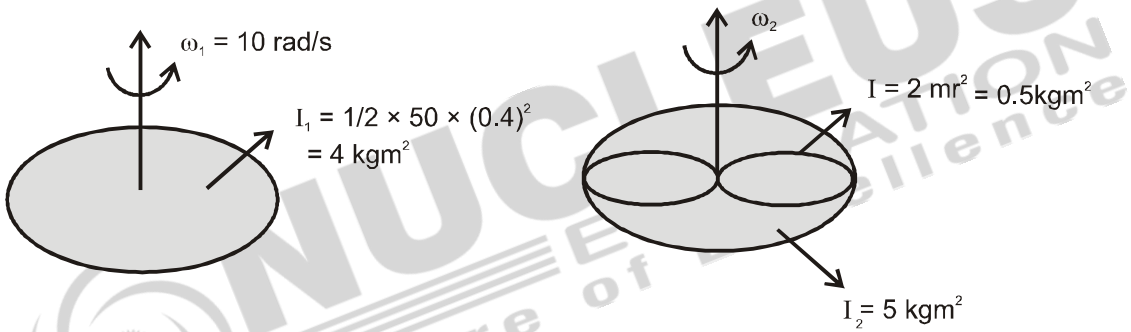
$$\omega_Q = \frac{\sqrt{2.g.\ell}}{R}$$

$$\therefore \quad \omega_Q > \omega_P$$

34. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is **[IIT-JEE 2013]**

Ans. (8)

Sol.

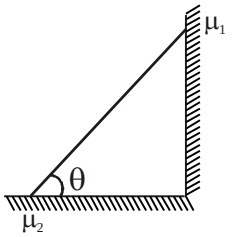


$I_1 \omega_1 = I_2 \omega_2$

$$\omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{4}{5} \times 10 \text{ rad/s} = 8 \text{ rad/s}$$

35. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then

[JEE Advanced-2014]

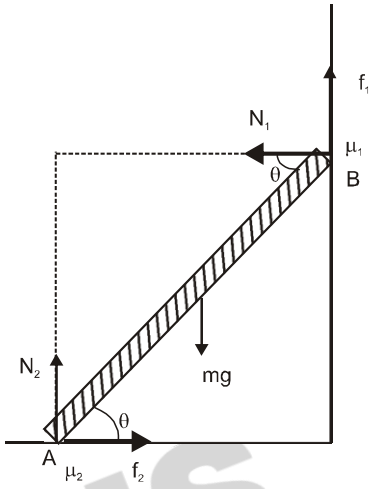


(A) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_2 \tan\theta = \frac{mg}{2}$ (B) $\mu_1 \neq 0$ $\mu_2 = 0$ and $N_1 \tan\theta = \frac{mg}{2}$

(C) $\mu_1 \neq 0$ $\mu_2 \neq 0$ and $N_2 = \dfrac{mg}{1+\mu_1\mu_2}$ (D) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$

Ans. (CD)

Sol. Since rod is about to slip so both friction will be limiting



$f_1 = \mu_1 N_1$

$f_2 = \mu_2 N_2$

In option (A)(D) $\mu_1 = 0$

Net torque about A should be zero

$$mg\cos\theta\ \frac{l}{2} = N_1 \sin\theta\ \ell$$

$$\Rightarrow \quad N_1 = \frac{mg\cot\theta}{2}$$

$$\Rightarrow \quad N_1\tan\theta = \frac{mg}{2}$$

and $N_2 = mg$

(B) $\mu_2 = 0$

There is no force to balance N_1 so rod can not remain in equilibrium

(C) $N_1 = \mu_2 N_2$

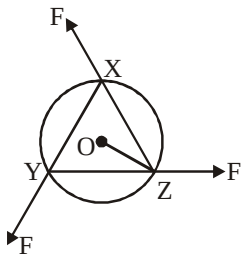
$N_2 + \mu_1 N_1 = mg$

$N_2 + \mu_1 \mu_2 N_2 = mg$

$$N_2 = \frac{mg}{1+\mu_1\mu_2}$$

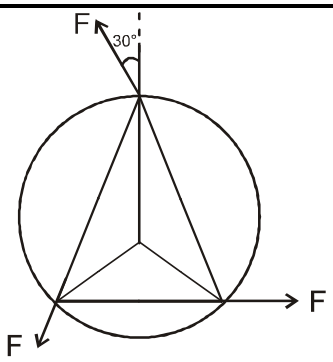
36. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is

[JEE Advanced-2014]



Ans. 2

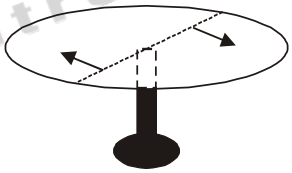
Sol.



$$\omega = \frac{\int \tau dt}{I} = \frac{\int_0^t 3F\sin 30°R\ dt}{I}$$

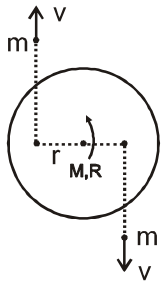
$$= \frac{3.(0.5)\ (0.5)\ (0.5)\ (1)}{\frac{1.5(0.5)^2}{2}} = 2 \text{ rad/s}$$

37. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is **[JEE Advanced-2014]**



Ans. 4

Sol. Applying conservation of angular momentum.



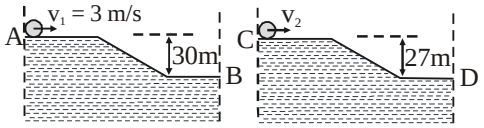
$$2mvr - \frac{MR^2}{2}\omega = 0$$

$$\omega = \frac{4mvr}{MR^2}$$

$$\omega = \frac{(4)(5\times10^{-2})(9)\left(\frac{1}{4}\right)}{45\times10^{-2}\times\frac{1}{4}}$$

ω = 4 rad/s

38. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ m/s, then v_2 in m/s is ($g = 10$ m/s^2) **[JEE Advanced-2015]**



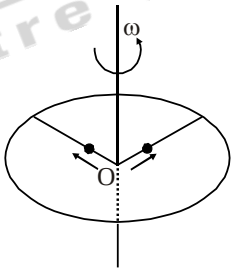
Ans. 7

Sol. Kinetic energy of a pure rolling disc having velocity of centre of mass

$$v = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\frac{v^2}{R^2} = \frac{3}{4}mv^2$$

So, $\frac{3}{4}m(3)^2 + mg(30) = \frac{3}{4}m(v_2)^2 + mg(27) \quad \therefore v_2 = 7$ m/s

39. A ring of mass M and radius R rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant the distance of the other mass from O is **[JEE Advanced-2015]**



(A) $\frac{2}{3}R$ (B) $\frac{1}{3}R$ (C) $\frac{3}{5}R$ (D) $\frac{4}{5}R$

Ans. (C or D or C and D [Proper Answer is D, But C may also be considered true, if someone assumed symmetric friction in both the rods])

Sol. Using conservation of angular momentum

$$mR^2\omega = \left(mR^2 \times \frac{8\omega}{9}\right) + \left(\frac{m}{8} \times \frac{9R^2}{25} \times \frac{8\omega}{9}\right) + \left(\frac{m}{8} \times x^2 \times \frac{8\omega}{9}\right) \Rightarrow x = \frac{4R}{5}$$

40. A uniform wooden stick of mass 1.6 kg and length *l* rests in an inclined manner on a smooth, vertical wall of height h (< *l*) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of

30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio $\frac{h}{l}$ and the frictional force *f* at the bottom of the stick are ($g = 10\ ms^2$) **[JEE Advanced- 2016]**

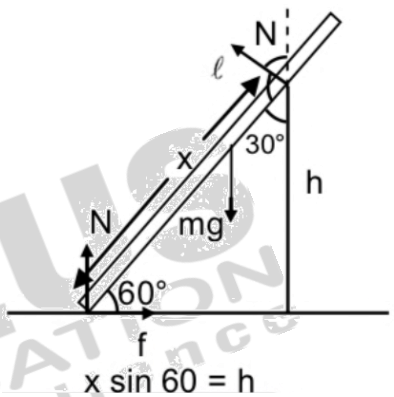
(A) $\frac{h}{l}=\frac{\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}N$ (B) $\frac{h}{l}=\frac{3}{16}, f=\frac{16\sqrt{3}}{3}N$

(C) $\frac{h}{l}=\frac{3\sqrt{3}}{16}, f=\frac{8\sqrt{3}}{3}N$ (D) $\frac{h}{l}=\frac{3\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}N$

Ans. (D)

Sol. On the basis of given data, it is not possible for the rod to stay at rest.

Note : If we take the normal reaction at the wall equal to the normal reaction at the floor, the answer will be D.



balancing torque about lowest point

$$N\frac{h}{\sin 60^\circ}=mg\frac{\ell}{2}\cos 60^\circ \quad \ldots(1)$$

$$N+\frac{N}{2}=mg$$

$$\frac{3}{2}N=mg \Rightarrow N=\frac{2mg}{3}$$

$$\frac{2mg}{3\sqrt{3}}\ 2h=\frac{mg\ell}{4}$$

$$\frac{h}{\ell}=\frac{3\sqrt{3}}{16}$$

$$f = N\sin 60^\circ = \frac{2mg}{3}\frac{\sqrt{3}}{2}=\frac{mg}{\sqrt{3}}=\frac{1.6\times 10}{\sqrt{3}}$$

41. The position vector of a particle of mass m is given by the following equation $\vec{r}(t)=\alpha t^3\hat{i}+\beta t^2\hat{j}$, where $\alpha=\frac{10}{3}\ ms^{-3}$, $\beta = 5\ ms^{-2}$ and m = 0.1 kg . At t = 1 s, which of the following statement(s) is (are) true about the particle? **[JEE Advanced- 2016]**

(A) The velocity $\vec{v}$ is given by $\vec{v}=(10\hat{i}+10\hat{j})\ ms^{-1}$

(B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L}=-\left(\frac{5}{3}\right)\hat{k}$ N m s

(C) The force $\vec{F}$ is given by $\vec{F}=(\hat{i}+2\hat{j})N$

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau}=-\left(\frac{20}{3}\right)\hat{k}$ N m

Ans. (ABD)

Sol. $\vec{v}=\frac{d\vec{r}(t)}{dt}=3\alpha t^2\hat{i}+2\beta t\hat{j},\ \vec{a}=\frac{d\vec{v}}{dt}=6\alpha t\hat{i}+2\beta\hat{j}$

At t = 1 s, $\vec{v}=(10\hat{i}+10\hat{j})\ ms^{-1}$

$\vec{a}=20\hat{i}+10\hat{j}\ ms^{-2}$

$\vec{r}=\frac{10}{3}\hat{i}+5\hat{j}\ m$

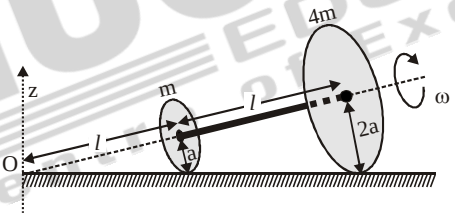
$\vec{L}_0=\vec{r}\times m\vec{v}=\left(-\frac{5}{3}\hat{k}\right)\ N\ ms$

$\vec{F}=m\frac{d\vec{v}}{dt}=\left(2\hat{i}+\hat{j}\right)N$

$\vec{\tau}_0=\vec{r}\times\vec{F}=\vec{r}\times m\vec{a}=\left(-\frac{20}{3}\hat{k}\right)\ N\ m$

42. Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively, are rigidly fixed by a massless, rigid rod of length $\ell=\sqrt{24}\,a$ through their centers. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is(are) true?

[JEE Advanced- 2016]



(A) The center of mass of the assembly rotates about the z-axis with an angular speed of ω/5

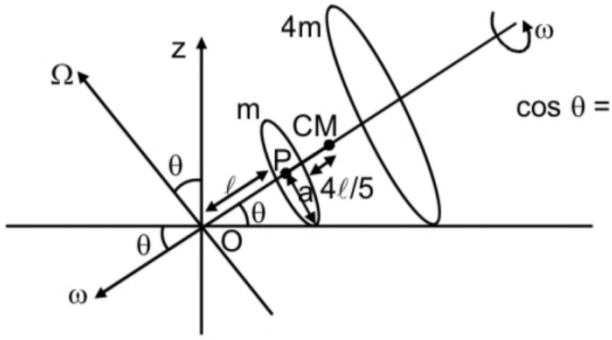
(B) The magnitude of angular momentum of center of mass of the assembly about the point O is 81 $ma^2\omega$

(C) The magnitude of angular momentum of the assembly about its center of mass is 17 $ma^2\omega/2$

(D) The magnitude of the z-component of $\vec{L}$ is 55 $ma^2\omega$

Ans. AC [rotation of axis of rotation is not in IIT JEE syllabus, JEE Advance Syllabus : dynamics of rigid body with fixed axis.]

Sol.



$$\cos\theta = \frac{\ell}{\sqrt{\ell^2 + a^2}} = \frac{\sqrt{24}}{5}$$

Velocity of point P : $a\omega = l\Omega$ then

$\Omega = \dfrac{a\omega}{l}$ = Angular velocity of C.M. w.r.t. point O.

Angular velocity of C.M. w.r.t z axis = $\Omega \cos\theta$

$$\omega_{C.M.-z} = \frac{a\omega}{l}\frac{\sqrt{24}}{5} = \frac{a\omega}{\sqrt{24}a}\frac{\sqrt{24}}{5}$$

$$\omega_{CM-z} = \frac{\omega}{5}$$

$$L_{D-CM} = \frac{ma^2}{2}\omega + \frac{4m(2a)^2}{2}\omega = \frac{17ma^2\omega}{2}$$

$$L_{CM-O} = (5m)\left[\frac{9l}{5}\Omega\right]\frac{9l}{5} = \frac{81ml^2\Omega}{5} = \frac{81ml^2}{5}\times\frac{a\omega}{l}$$

$$L_{CM-O} = \frac{81mla\omega}{5} = \frac{81\sqrt{24}a^2m\omega}{5}$$

$$L_Z = L_{CM-0}\cos\theta - L_{D-CM}\sin\theta$$

$$= \frac{81\sqrt{24}}{5}a^2m\omega\times\frac{\sqrt{24}}{5} - \frac{17ma^2\omega}{2}\times\frac{1}{\sqrt{24}} = \frac{81\times 24ma^2\omega}{25} - \frac{17ma^2\omega}{2\sqrt{24}}$$

Paragraph for Question Nos. 43 to 45

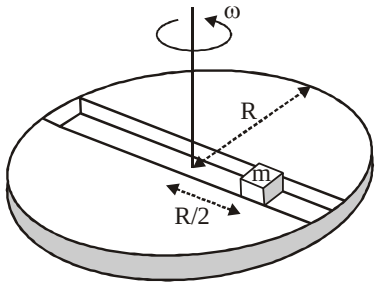
A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{v}_{rot}\times\vec{\omega}) + m(\vec{\omega}\times\vec{r})\times\vec{\omega}$$

where $\vec{v}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a

coordinate system with the origin at the center of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $(\vec{\omega} = \omega\hat{k})$. A small block of mass m is gently placed in the slot at $\vec{r} = \left(\frac{R}{2}\right)\hat{i}$ at t = 0 and is constrained to move only along the slot. **[JEE Advanced- 2016]**



43. The distance r of the block at time t is

(A) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$ (B) $\frac{R}{4}\cos\omega t$ (C) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$ (D) $\frac{R}{2}\cos 2\omega t$

Ans. (A)

Sol. $v\frac{dv}{dr} = \omega^2 r$, where v is the velocity of the block radially outward.

$$\int_0^v v\,dv = \omega^2 \int_{R/2}^r r\,dr$$

$$\Rightarrow v = \omega\sqrt{r^2 - \frac{R^2}{4}}$$

$$\int_{R/2}^r \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \omega\int_0^t dt$$

$$r = \frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$$

44. The net reaction of the disc on the block is

(A) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} + mg\hat{k}$ (B) $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}$

(C) $-m\omega^2 R\cos\omega t\ \hat{j} - mg\hat{k}$ (D) $m\omega^2 R\sin\omega t\ \hat{j} - mg\hat{k}$

Ans. (B)

Sol. $\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{v}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega}$

$$= -m\omega^2 r\hat{i} + 2mv_{rot}\omega\left(-\hat{j}\right) + m\omega^2 r\hat{i}$$

$$= -2mv_{rot}\omega\hat{j}$$

$$v_{rot} = \frac{dr}{dt} = \frac{\omega R}{4}\left(e^{\omega t} - e^{-\omega t}\right)$$

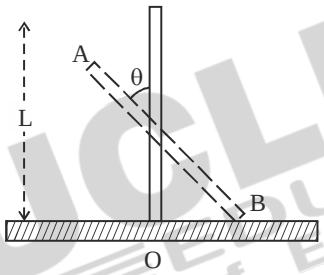
$$\vec{F}_{rot} = -\frac{m\omega^2 R}{2}\left(e^{\omega t} - e^{\omega t}\right)\hat{j}$$

$$\vec{F}_{net} = -\vec{F}_{rot} + mg\hat{k}$$

$$= \frac{m\omega^2 R}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\,\hat{k}$$

45. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct?

[JEE Advanced- 2017]



(A) The midpoint of the bar will fall vertically downward
(B) The trajectory of the point A is a parabola
(C) Instantaneous torque about the point in contact with the floor is proportional to sin θ
(D) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to (1 – cos θ)

Ans. (ACD)

Sol. There is no horizontal force on rod during its motion

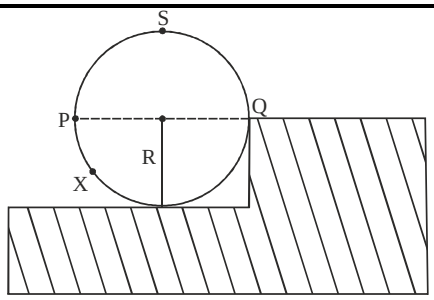
$\Rightarrow$ C.M will fall vertically downwards

Net torque about point B (on the ground) = mgL sin θ

Displacement mid point = $\frac{L}{2} - \frac{L}{2}\cos\theta$

46. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct?

[JEE Advanced- 2017]



(A) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

(B) If the force is applied normal to the circumference at point X then τ is constant

(C) If the force is applied normal to the circumference at point P then τ is zero

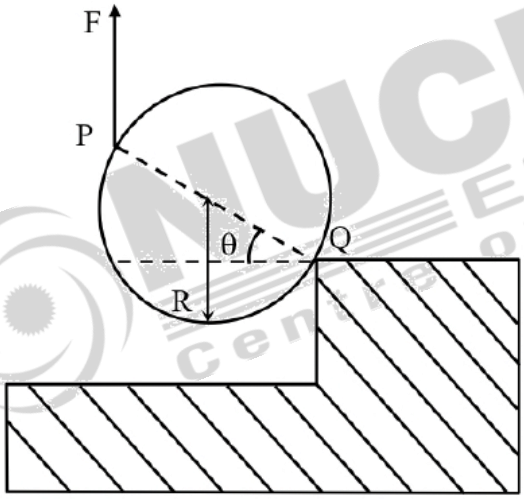
(D) If the force is applied tangentially at point S then $t \neq 0$ but the wheel never climbs the step

Ans. (C) or (C,D)

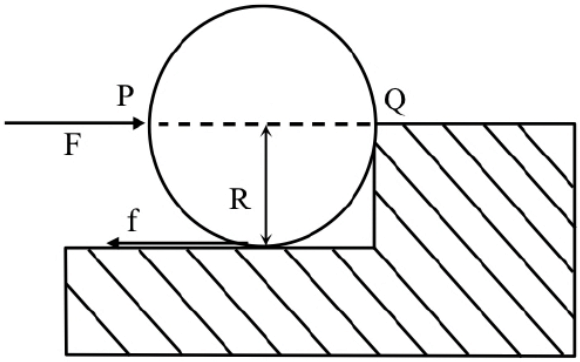
Sol. As the force acts as shown and considering its direction to be constant, then

$\tau_{net} = F(2R \cos\theta) - mgR \cos\theta = R \cos\theta\,(2F - mg)$

Which definitely infers that as θ increases, τ_{net} decreases.



As soon as force is applied as shown, it leads to frictional force between bottom point and horizontal surface, which will produce non–zero torque about point Q.



Paragraph for Q.No. 47 to 48

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (figure-2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

[JEE Advanced- 2017]

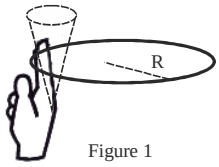


Figure 1

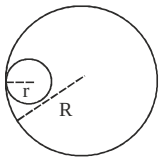


Figure 2

47. The total kinetic energy of the ring is

(A) $M\omega_0^2R^2$ (B) $\frac{1}{2}M\omega_0^2(R-r)^2$ (C) $M\omega_0^2(R-r)^2$ (D) $\frac{3}{2}M\omega_0^2(R-r)^2$

Ans. (Bonus) [rotation of axis of rotation is not in IIT JEE syllabus, JEE Advance Syllabus : dynamics of rigid body with fixed axis.]

48. The minimum value of ω_0 below which the ring will drop down is

(A) $\sqrt{\frac{g}{\mu(R-r)}}$ (B) $\sqrt{\frac{2g}{\mu(R-r)}}$ (C) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (D) $\sqrt{\frac{g}{2\mu(R-r)}}$

Ans. (A)

Sol. $\mu N = mg$ …(i)

& $N = m(R-r)\omega^2$ …(ii)

from (i)

$\mu m(R-r)\omega^2 = mg$

$\omega = \sqrt{\frac{g}{\mu(R-r)}}$

49. The potential energy of a particle of mass *m* at a distance *r* from a fixed point *O* is given by $V(r) = kr^2/2$, where *k* is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius *R* about the point *O*. If *v* is the speed of the particle and *L* is the magnitude of its angular momentum about *O*, which of the following statements is (are) true?

[JEE Advanced- 2018]

(A) $v = \sqrt{\frac{k}{2m}}R$ (B) $v = \sqrt{\frac{k}{m}}R$ (C) $L = \sqrt{mk}R^2$ (D) $L = \sqrt{\frac{mk}{2}}R^2$

Ans. (BC)

Sol. $V = \frac{kr^2}{2} \Rightarrow F = -\frac{dV}{dr} = -kr$

$\therefore \quad \frac{mv^2}{r} = kr \Rightarrow v = \sqrt{\frac{k}{m}}R$

Angular momentum $L = mvr = m\sqrt{\frac{k}{m}}R^2 = \sqrt{mk}R^2$

50. Consider a body of mass 1.0 *kg* at rest at the origin at time $t = 0$. A force $\vec{F} = (\alpha t\,\hat{i} + \beta\,\hat{j})$ is applied on the body, where a =1.0 Ns^{-1} and b = 1.0 N. The torque acting on the body about the origin at time $t = 1.0$ s is $\vec{\tau}$. Which of the following statements is (are) true?

[JEE Advanced- 2018]

(A) $|\vec{\tau}| = \frac{1}{3}\text{N}\,m$

(B) The torque is in the direction of the unit vector + $\hat{k}$

(C) The velocity of the body at $t = 1$ s is $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j})\,ms^{-1}$

(D) The magnitude of displacement of the body at $t = 1$ s is $\frac{1}{6}$ *m.*

Ans. (AC)

Sol. $\vec{a} = t\,\hat{i} + \hat{j}$ m/s^2

$\Rightarrow \vec{v} = \frac{t^2}{2}i + t\,\hat{j}$ m/s

$\Rightarrow \vec{r} = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}$ m

So, $\vec{\tau} = \vec{r} \times \vec{F} = \left(\frac{t^3}{2} - \frac{t^3}{6}\right)\left(-\hat{k}\right) = \frac{t^3}{3}\left(-\hat{k}\right)$ Nm

51. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2 - \sqrt{3}) / \sqrt{10}$ s, then the height of the top of the inclined plane, in *metres*, is _____. Take $g = 10\ ms^{-2}$.

[JEE Advanced- 2018]

Ans. 0.75

Sol. $\frac{h}{\sin\theta} = \frac{1}{2}\frac{g\sin\theta}{1 + \frac{I}{mR^2}}t^2$

$\therefore\ t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1 + \frac{I}{mR^2}\right)}$

$\therefore \quad \frac{2-\sqrt{3}}{\sqrt{10}} = \frac{2}{\sqrt{3}}\sqrt{\frac{2h}{10}}\left(\sqrt{2} - \sqrt{\frac{3}{2}}\right)$

$\therefore$ h = 0.75 m

52. In the List- I below, four different paths of particle are given as functions of times. In these function, α and β are positive constants of appropopriate dimensions and $\alpha \neq \beta$. In each case, the force acting on the particle is either zero or oconservative. In List-II, five physical quantities of the particle are mentioned: $\vec{p}$ is the linear momentum, $\vec{L}$ is the angular momentum about the origin. K is the kinite energy U is the potential energy and E is the total energy. Match each path in List -II with those are conserved for that path. **[JEE Advanced- 2018]**

	List - I		**List - II**
(P)	$\vec{r}(t)=\alpha t\,\hat{i}+\beta t\,\hat{j}$	1.	$\vec{p}$
(Q)	$\vec{r}(t)=\alpha\cos\omega t\,\hat{i}+\beta\sin\omega t\,\hat{j}$	2.	$\vec{L}$
(R)	$\vec{r}(t)=\alpha\left(\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j}\right)$	3.	K
(S)	$\vec{r}(t)=\alpha t\,\hat{i}+\frac{\beta}{2}t^2\,\hat{j}$	4.	U
		5.	E

Codes :

	P	**Q**	**R**	**S**
(A)	1,2,3,4,5	2,5	2,3,4,5	5
(B)	1,2,3,4,5	3,5	2,3,4,5	2 5
(C)	2,3,4	5	1,2,4	2,5
(D)	1,2,3,4,5	2,5	2,3,4,5	2,5

Ans. (A)

Sol. 'P' is straight line with zero acceleration
'Q' is elliptical path.
'R' is circular path
'S' is parabolic path

For 'S' $|v|=\sqrt{\alpha^2+\beta^2t^2}$

$dU = -\vec{F}\cdot d\vec{r}$

$U = -\frac{m\beta^2t^2}{2}c$

Total energy = KE + U = $\frac{1}{2}m\alpha^2$ = constant

53. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical?
[g is the acceleration due to gravity] **[JEE Advanced–2019]**

(1) The angular acceleration of the rod will be $\frac{2g}{L}$

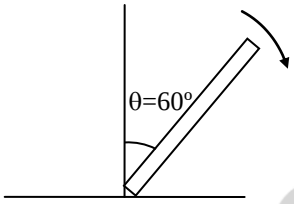
(2) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$

(3) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

(4) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

Ans. **(2,3,4)**

Sol. $mg\frac{l}{2}(1-\cos 60°) = \frac{1}{2}\frac{ml^2}{3}\omega^2$



$\omega = \sqrt{\frac{3g}{2l}}$

$\alpha = \frac{mg\frac{l}{2}\sin 60^o}{\frac{ml^2}{3}} = \frac{3\sqrt{3}g}{4l}$

$a_c = \frac{\omega^2 l}{2} = \frac{3g}{2l}\cdot\frac{l}{2} = \frac{3g}{4}$

$mg-N = m\left(\omega^2\frac{l}{2}\cos 60^o + \alpha\frac{l}{2}\cos 30°\right)$

$mg-N = m\left(\frac{3g}{8}+\frac{9g}{16}\right)$

$N = \frac{mg}{16}$