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MATHEMATICS(NATURAL.S) 2012/2013

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In all the places you can,
At all the times you can,
To all the people you can,
As long as ever you can."

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ETHIOPIAN UNIVERSITY ENTRANCE EXAMINATION (EUEE)
MATHEMATICS FOR NATURAL SCIENCE YEKATIT 2012/13 MARCH 2020/21

BOOKLET CODE: 48

SUBJECT CODE: 02
TIME ALLOWED: 3:00 HOURS

GENERAL DIRECTIONS

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IN THIS EXAMINATION THERE ARE A TOTAL OF **65 MULTIPLE CHOICE ITEMS**. CAREFULLY SELECT THE BEST ANSWER FROM THE SUGGESTED OPTIONS AND BLACKEN ONLY THE LETTER OF YOUR CHOICE ON THE SEPARATE ANSWER SHEET PROVIDED. FOLLOW THE INSTRUCTIONS ON THE EXAMINATION PAPER AND THE ANSWER SHEET CAREFULLY. USE ONLY **PENCIL** TO MARK YOUR ANSWERS. YOUR ANSWER MARK SHOULD BE **HEAVY** AND **DARK**, COVERING THE ANSWER SPACE COMPLETELY. PLEASE ERASE ALL UNNECESSARY MARKS COMPLETELY INCLUDING ANSWERS YOU HAVE CHANGED FROM YOUR ANSWER SHEET.

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MATHEMATICS FOR NATURAL SCIENCE STREAM

YEKATIT 2012/13 MARCH 2020/21

BOOKLET CODE: 48 **SUBJECT CODE: 02**

Number of Items: 65 **Time Allowed: 3 hours**

1. A Geometric Series with first term a and common ratio r is convergent if

   A. $r \geq 1$ C. $r \leq 1$

   B. $r > 1$ D. $r < 1$

2. Which of the following is an upper bound for the sequence $\left\{\frac{n+1}{n}\right\}_{n=1}^{\infty}$?

   A. 1 C. 2

   B. 0 D. $\frac{4}{3}$

3. The general term of a sequence is $a_n = \left(\frac{1}{2}\right)^{1-n}$. Then which of the following is true about this sequence?

   A. 0 is one of its terms. C. Its terms are odd numbers.

   B. Its terms are non-positive. D. Its terms are natural numbers.

4. The distance between point $(4, -3)$ and the line $l: x + y - 7 = 0$ is _____

   A. $4\sqrt{2}$ C. $\frac{3\sqrt{2}}{2}$

   B. $2\sqrt{2}$ D. $3\sqrt{2}$

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1. Solution

- Geometric series is the sum of an infinite geometrical series of numbers $\{G_n\}_{n=1}^{\infty}$ is denoted by $\sum_{n=1}^{\infty} G_n = G_1 + G_2 + G_3 + \cdots + G_n$
- If the sum of all terms of the sequence is unique or finite number then, the the Geometric series $\sum_{n=1}^{\infty} G_n$ is called Convergent, Otherwise its diverges.
- If $\{G_n\}_{n=1}^{\infty}$ is a geometric series with '$G_1 = a$' and with common ratio 'r', then its n^{th} partial sum is given by

$$S_n = \frac{a(1-r^n)}{1-r}, \quad r \neq 1$$

Now, * If $|r| < 1$ that means $-1 < r < 1$, the geometric series

Converges to: $S = \frac{a}{1-r}$, since $1 - r^n \to 1$ as $n \to \infty$

* If $|r| \geq 1$ Diverges

thus there is no Answer in the given Multiple choice.
since $|r| < 1$ is not equal to $r < 1$

Answer: NO ANSWER

2. Solution

Let $\{a_n\}_{n=1}^{\infty}$ be sequence of number Then:

* A number M is is called an upper bound of $\{a_n\}_{n=1}^{\infty}$ if M is greater than or equal to every number in $\{a_n\}_{n=1}^{\infty}$ i.e. $\forall a_i \in \{a_n\} \Rightarrow M \geq a_i$.

Now, to find the upper bound for the sequence $\left\{\frac{n+1}{n}\right\}_{n=1}^{\infty}$ divide both the numerator and denominator by 'n'

$$\left\{\frac{n+1}{n}\right\}_{n=1}^{\infty} = \left\{\frac{\frac{n}{n}+\frac{1}{n}}{\frac{n}{n}}\right\}_{n=1}^{\infty} = \left\{\frac{1+\frac{1}{n}}{1}\right\}_{n=1}^{\infty} = \left\{1+\frac{1}{n}\right\}_{n=1}^{\infty}$$

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As $n \to \infty$ (very large) $\frac{1}{n} \to 0$

then $\{1 + \frac{1}{n}\} = 1$ as $n \to \infty$ and when $n=1$, $1 + \frac{1}{1} = 2$

thus $1 \leq \left\{\frac{n+1}{n}\right\}_{n=1}^{\infty} \leq 2$

Therefore $[2, \infty)$ the set of all upper bound

ANSWER: C

3. Solution

Given $a_n = \left\{\frac{1}{2}\right\}^{1-n}$

A. a_n is Not one of its terms, because $\left(\frac{1}{2}\right)^{1-n}$ never equal to '0' (zero) for all n.

B. Its terms are positive $\forall n$ (for all n). since $\frac{1}{2}$ is positive.

C. Its terms are odd numbers. FALSE

Its terms are even number for $n>1$ and 1 for $n=1$

D. Its terms are Natural numbers, $\forall n$.

ANSWER: D

4. Given $P(4, -3)$ and line $l: x + y - 7 = 0$

by using distance between point and line formula

$$d = \frac{|1(4) + 1(-3) + (-7)|}{\sqrt{1^2 + 1^2}}$$

$$= \frac{|4 - 3 - 7|}{\sqrt{2}} = \frac{|-6|}{\sqrt{2}} = \frac{6}{\sqrt{2}}$$

$$= \frac{6}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

ANSWER: D

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5. Let $f(x) = \frac{x^2 - x}{2x^2 - x - 1}$, which of the following is **NOT** true about f and its graph?

A. $x = -\frac{1}{2}$ is a vertical asymptote.

B. The graph of f has a hole at $(1, \frac{1}{3})$.

C. f is an even function.

D. $y = \frac{1}{2}$ is a horizontal asymptote.

6. Which of the following statements is **NOT** true?

A. When the variance of a data is close to zero, the data shows less variability.

B. Variance indicates the variability of a set of numerical data items.

C. If two data have equal mean, the data with less variance shows less variability.

D. A large value of variance shows that the data values are closer to the mean.

7. The following is the frequency distribution of a grouped data.

Class Intervals	Frequency (f)
3 – 7	2
8 – 12	2
13 – 17	10
18 – 22	6

What is the mean of the data?

A 15 C 12.5

B 14 D 13

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5. Solution:

$$f(x) = \frac{x^2 - x}{2x^2 - x - 1}, \text{ let } f(x) = \frac{P(x)}{Q(x)} = \frac{x^2 - x}{2x^2 - x - 1}$$

→ the domain of f is all real numbers except $x = -1/2$ and $x = 1$

* $P(-1/2) \neq 0$ and $Q(-1/2) = 0$. Thus, $x = -2$ is a vertical asymptote of $f(x)$
* $P(1) = Q(1) = 0$. Thus, f has hole at $(1, 1/3)$
* Cancelling out the common factor $x-1$, we obtain

$$g(x) = \frac{x}{2x+1}, \text{ and } g(1) = \frac{1}{3}$$

* There the degree of numerator ($n = 2$) and the degree of denominator ($m = 2$). Since $n = m = 2$, the line $y = \frac{1}{2}$ is a horizontal asymptote of the graph of the function.
* $f(x) = \frac{x^2 - x}{2x^2 - x - 1}$ is neither even nor odd, since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

NB: A function f is said to be:
i) Odd if and only if, for all $x \in A$, $f(-x) = -f(x)$
ii) Even if and only if, for all $x \in A$, $f(-x) = f(x)$

ANSWER:

7. Solution

Steps to find the mean from a grouped distribution

From a grouped frequency distribution

1. Find the class mark (mid-point) x_c of each, by

$$\frac{\text{Lower class limit} + \text{upper class limit}}{2}$$

2. Multiply x_c by its corresponding frequency and add
3. Divide the sum obtained in step 2 by the sum of the frequencies.

$$\bar{x} = \frac{f_1 x_{c_1} + \cdots + f_n x_{c_n}}{f_1 + f_2 + f_3 + \cdots + f_n} = \frac{\sum_{i=1}^{n} f_i x_{c_i}}{\sum_{i=1}^{n} f_i}.$$

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Now,

class interval	frequency (f)	class midpoint (x_c)	fx_c
3-7	2	5	10
8-12	2	10	20
13-17	10	15	150
18-22	6	20	120
total	$\Sigma f = 20$		$\Sigma fx_c = 300$

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_{ci}}{\sum_{i=1}^{n} f_i} = \frac{300}{20} = 15$$

ANSWER: A

6. Solution

- When the variance of data is close to zero the data shows less variability.
- Variance indicates the variability of a set of numerical data items
- If two data have equal mean the data with less variance shows less variability.
- A large value of variance shows that the values are far apart from the mean

ANSWER: D

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8. If p: "the rainy season is very good this year" and q: "rivers are rising", then the statement "It is not true that neither the rainy season is very good this year nor rivers are rising" denoted by

A. $(\neg p \Rightarrow \neg q) \vee \neg q$

B. $(p \Rightarrow q) \vee (\neg p \Rightarrow \neg q)$

C. $\neg p \wedge \neg q$

D. $\neg(\neg p \vee \neg q)$

9. $\lim_{x \to \infty} \left(\frac{x-2}{x}\right)^{-2x+1} =$ _____

A. e^{4}

B. e^{-4}

C. e^{2}

D. e^{-2}

10. Given $0 < a_n < \frac{1}{n}$ for each $n = 1, 2, 3, \ldots$. Which of the following is true?

A. $\lim_{n \to \infty} a_n < 0$

B. $\lim_{n \to \infty} a_n = 0$

C. $\lim_{n \to \infty} a_n$ does not exist

D. $\lim_{n \to \infty} a_n > 0$

11. Let $f(x) = x^2 + 2x + 3$ find a number $c \in (1, 3)$ such that $f'(c) = \frac{f(3) - f(1)}{2}$?

A. -6

B. 6

C. -2

D. 2

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8. Solution

Given p: "the rainy season is very good this year"

q: "rivers are rising"

Statement: "It is not true that neither the rainy season is very good this year nor rivers are rising" denoted by

$$\neg(\neg p \vee \neg q)$$

ANSWER: E

9. Solution

$$\lim_{x\to\infty}\left(\frac{x-2}{x}\right)^{-2x+1} = \lim_{x\to\infty}\left(\frac{x-2}{x}\right)^{-2x} \cdot \lim_{x\to\infty}\left(\frac{x-2}{x}\right)^{1}$$

$$= \lim_{x\to\infty}\left[\left(1-\frac{2}{x}\right)^{x}\right]^{-2} \cdot \lim_{x\to\infty}\left(1-\frac{2}{x}\right)$$

$$= \left[\left(e^{-2}\right)^{-2}\right] \cdot 1 = e^{4}$$

ANSWER: A

10. Solution: Given $0 < a_n < \frac{1}{n}$ for each $n = 1, 2, 3 \cdots$

then $\lim_{x\to\infty} a_n = 0$

ANSWER: B

11. Solution: Given $f(x) = x^2 + 2x + 3$

and $f'(c) = \frac{f(3) - f(2)}{2}$, but $f'(x) = 2x + 2$

then $f'(c) = 2c + 2$

and $f(3) = (3)^2 + 2(3) + 3 = 18$ and $f(1) = (1)^2 + 2(1) + 3 = 6$

then $f'(c) = 2c + 2 = \frac{f(3) - f(2)}{2} = \frac{18 - 6}{2}$

$\Rightarrow 2c + 2 = \frac{12}{2} \Rightarrow 2c + 2 = 6$

$\Rightarrow 2c = 6 - 2 = 4$

$\Rightarrow \underline{c = 2}$

ANSWER: B

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12. For what value of c the conclusion of Rolle's theorem is satisfied for the function $f(x) = 2x - x^2 - x^3$ on $[-2, 1]$?

A. $\frac{-1\pm\sqrt{5}}{3}$ C. $\frac{1\pm\sqrt{5}}{3}$

B. $\frac{1\pm\sqrt{7}}{3}$ D. $\frac{-1\pm\sqrt{7}}{3}$

13. Let $z_1 = 2 + i$, $z_2 = 3 + i$ and $z_3 = 1 - i$, then what are the real and imaginary parts of $z_1z_2z_3$ respectively?

A. 6 and 3 C. −1 and 1

B. −6 and −3 D. 10 and 0

14. What is the value of a and b if $\frac{a+2i}{3i} = \frac{b+i}{4}$?

A. $a = 3/4$, $b = -8/3$ C. $a = -3/4$, $b = 8/3$

B. $a = 3/4$, $b = 8/3$ D. $a = -3/4$, $b = -8/3$

15. The table shown below is a simple frequency distribution of data with variable x.

x	1	3	4	5	7
Frequency	2	5	6	5	2

What is the variance of the data?

A. 3 C. 3.2

B. $\sqrt{3}$ D. 2.3

16. The second derivative of the function $f(x) = xe^{-x}$ is _____.

A. $-xe^{-x}$ C. $(x-2)e^{-x}$

B. $(2-x)e^{-x}$ D. $(x+2)^2e^{-x}$

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12. Solution

→ Rolles theorem states:

*Let f be function that satisfies all the following three conditions: i) f is continuous on the closed interval [a,b]

ii) f is differentiable on the open interval (a,b)

iii) f(a) = f(b)

Then there Exist a Number c in (a,b) such that f'(c) = 0

* Now in our case $f(x) = 2x - x^2 - x^3$ is continuous on [-2,1] and differentiable on (-2,1) since f(x) is polynomial fun. Moreover, since f(-2) = f(1) = 0, there Exist c in (-2,1) such that f'(c) = 0

$\therefore f'(x) = 2 - 2x - 3x^2 \Rightarrow f'(c) = 0 \Rightarrow f'(c) = 2 - 2c - 3c^2 = 0$

$\Rightarrow 2 - 2c - 3c^2 = 0$

Solve it by using quadratic formula

$$c = \frac{2 \pm \sqrt{4 + 4(2)(-3)}}{2(-3)} = \frac{2 \pm \sqrt{4 + 24}}{-6} = \frac{2 \pm \sqrt{28}}{-6}$$

$$c = \frac{2 \pm 2\sqrt{7}}{-6} = \frac{-1 \pm \sqrt{7}}{3}$$

$$c_1 = \frac{-1 + \sqrt{7}}{3} \text{ & } c_2 = \frac{-1 - \sqrt{7}}{3}$$

Now $c_1 \in (-2,1)$ but $c_2 \notin (-2,1)$

thus $c = \frac{-1 \pm \sqrt{7}}{3}$

ANSWER: D

13. Solution

$z_1 = 2 + i$, $z_2 = 3 + i$ and $z_3 = 1 - i$

$z_1 z_2 z_3 = ?!$

first let we Multiply z_1 and z_2 then z_3

let $w = z_1 z_2$ and $u = z_1 z_2 z_3 = w z_3$

$w = z_1 z_2 = (2+i)(3+i) = 6 + 2i + 3i - 1 = 5 + 5i$

now $u = z_1 z_2 z_3 = w z_3 = (5+5i)(1-i) = 5 + 5i - 5i + 5$

$= 10 + 0i$

therefore Real Part 10 and imaginary part 0

ANSWER: D

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14. Solution

Given $\frac{a+2i}{3i} = \frac{b+i}{4}$ then $a = ?$ and $b = ?$

thus $\Rightarrow \frac{a+2i}{3i} = \frac{b+i}{4} \Rightarrow 4(a+2i) = 3i(b+i)$

$\Rightarrow 4a + 8i = 3bi - 3$

$\Rightarrow 4a + 8i = -3 + 3bi$

now by using Equality of complex number rules

$4a = -3$ and $8 = 3b$

$a = -3/4$ and $b = 8/3$

ANSWER: C

15. Solution

Given

x	1	3	4	5	7
f	2	5	6	5	2

* first find mean of the data

$$\bar{x} = \frac{1*2 + 3*5 + 4*6 + 5*5 + 7*2}{2+5+6+5+2}$$

$$\bar{x} = \frac{80}{20} = 4$$

then

x	1	3	4	5	7
f	2	5	6	5	2
$x - \bar{x}$	-3	-1	0	1	3
$(x-\bar{x})^2$	9	1	0	1	9
$f(x-\bar{x})^2$	18	5	0	5	18

$\sum f = n = 20$

$\sum f(x-\bar{x}) = 18 + 5 + 0 + 5 + 18$

$= 46$

therefore Variance, $s^2 = \frac{\sum f(x-\bar{x})^2}{\sum f} = \frac{46}{20} = 2.3$

ANSWER: D

16. Solution

Given $f(x) = xe^{-x}$

then $f'(x) = e^{-x} - xe^{-x}$ ← first derivative of $f(x)$

$f''(x) = -e^{-x} - e^{-x} + xe^{-x} = -2e^{-x} + xe^{-x}$

$= e^{-x}(-2+x) = e^{-x}(x-2)$ ← second derivative of $f(x)$

ANSWER: C

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17. If p and q are propositions, then which one of the following pairs of compound propositions are equivalent?

A. $p \Rightarrow \neg q$ and $\neg p \wedge q$

B. $\neg(p \wedge q)$ and $\neg p \vee q$

C. $p \Leftrightarrow q$ and $\neg p \Leftrightarrow \neg q$

D. $\neg p \Rightarrow q$ and $q \Rightarrow \neg p$

18. The volume of a cube is increasing at a rate of $9cm^3/sec$. How fast is the surface area increasing when the length of an edge is 10cm?

A. $3.6cm^2/sec$

B. $36cm^2/sec$

C. $90cm^2/sec$

D. $6cm^2/sec$

19. The product of two positive real numbers is 100, such that the sum of two times the first number and eight times the second number is minimum. Which of the following pairs of numbers are the first and the second numbers respectively?

A. 25 and 4

B. 20 and 5

C. 1 and 99

D. 50 and 2

20. The maximum profit that a company can make if the profit function is given by $P(x) = 36 + 72x - 18x^2$ is

A. 124

B. 31

C. 2232

D. 108

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17. Solution

Let assume p is true(T) and q is false(F)

A. $p \Rightarrow \neg q$ and $\neg p \wedge q$, that is $T \Rightarrow T \equiv T$ and $F \wedge T \equiv F$, Not equivalent.

B. $\neg(p \wedge q)$ and $\neg p \vee q$, that is $\neg(T \wedge F) \equiv T$ and $F \vee F \equiv F$, Not equivalent

C. $p \Leftrightarrow q$ and $\neg p \Leftrightarrow \neg q$, that is $T \Leftrightarrow F \equiv F$ and $F \Leftrightarrow T \equiv F$, Equivalent.

D. $\neg p \Rightarrow q$ and $q \Rightarrow \neg p$, that is $F \Rightarrow F \equiv T$ and $F \Rightarrow F \equiv T$, but not equivalent. incase of p is False and q is False that is $T \Rightarrow F \equiv F$ and $F \Rightarrow T \equiv T$, thus not equivalent.

ANSWER: C

18. Solution.

Volume, $V = x^3$ and $x = 10$

$\Rightarrow \frac{dV}{dt} = 9\ cm^3/sec$, but $\frac{dV}{dt} = 3x^2\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{1}{3x^2} \cdot \frac{dV}{dt}$

thus $\frac{dx}{dt} = \frac{1}{3x^2} \cdot \frac{dV}{dt} = \frac{1}{3*10^2} * 9 = \frac{3}{100}\ cm/sec = 0.03\ cm/sec$

And surface Area, $A = 6x^2$

$\frac{dA}{dt} = 12x\frac{dx}{dt} = 12*10*(0.03)$

$\frac{dA}{dt} = 3.6\ cm^2/sec$

ANSWER: A

19. Solution

let the two positive Numbers are x and y

then • $xy = 100$ --- Eq: (1)

• $s = 2x + 8y$ ← minimum.

• $x \geq 0$ and $y \geq 0$

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then from eqn ①

$y = \frac{100}{x}$ ... eqn ②

Substitute eqn ② into S equation

then $S(x) = 2x + 8\left(\frac{100}{x}\right) = 2x + \frac{800}{x}$ ← to be minimum

$\Rightarrow S'(x) = 2 - \frac{800}{x^2}$

$\Rightarrow S'(x) = 0 \Rightarrow 2 - \frac{800}{x^2} = 0 \Rightarrow 2x^2 = 800$

$\Rightarrow x^2 = 400 \Rightarrow x = \pm\sqrt{400} = \pm 20$

but $x \geq 0$ thus $x = 20$ (Positive)

then $y = \frac{100}{x} = \frac{100}{20} = 5$

∴ the two Numbers are $x = 20$ and $y = 5$ whose $2x + 8y$ is minimum.

ANSWER: B

20. Solution

Given $P(x) = 36 + 72x - 18x^2$

first find the critical points

$P'(x) = 72 - 36x = 0$

$36x = 72$

$x = 2$

then substitute the critical value into the $P(x)$

then $P(2) = 36 + 72(2) - 18(2)^2$

$= 36 + 144 - 72$

$= 108$

ANSWER: D

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21. If $f(x)=\begin{cases} k(2^x), & \text{if } x>2 \\ x^3-x+1, & \text{if } x\le 2 \end{cases}$, then for what value of k is f continuous at $x=2$?

A. $\frac{7}{4}$

B. 4

C. 2

D. $\frac{7}{2}$

22. Which of the following is equal to $\sin^{-1}(\sin\frac{5\pi}{4})$?

A. $-\frac{\pi}{4}$

B. $\frac{5\pi}{4}$

C. $-\frac{\sqrt{2}}{2}$

D. $\frac{\sqrt{2}}{2}$

23. Which one of the following is true about $f(x)=-2+3\sin(\frac{1}{2}x+3)$?

A. The period is 2π

B. The amplitude is 1

C. The range is $[-5, 1]$

D. The phase shift is 6

24. If the 1st and 4th terms of a geometric progression are 3 and 81 respectively, what is the common ratio for this progression?

A. 1/3

B. 9

C. 1/9

D. 3

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21. Solution

$$f(x) = \begin{cases} k(2^x), & \text{if } x > 2 \\ x^3 - x + 1, & \text{if } x \le 2 \end{cases}$$

f is continuous at $x=2$, $k=?$

* Since f is continuous at $x=2$.

$\Rightarrow \lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) \Rightarrow \lim_{x\to 2^+} k(2^x) = \lim_{x\to 2^-} (x^3 - x + 1)$

$\Rightarrow \lim_{x\to 2^+} k(2^x) = \lim_{x\to 2^-} (x^3 - x + 1)$

$\Rightarrow k(2^2) = (2^3 - 2 + 1) \Rightarrow 4k = 7 \Rightarrow k = 7/4$

ANSWER: A

22. Solution

* $\sin^{-1}(\sin 5\pi/4) = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$= -\pi/4$, since $\sin 5\pi/4 = -\frac{\sqrt{2}}{2}$

* TIP * $\sin(\sin^{-1} x) = x$, if $-1 \le x \le 1$

* $\sin^{-1}(\sin x) = x$, if $-\pi/2 \le x \le \pi/2$

ANSWER: A

23. Solution

$f(x) = -2 + 3\sin\left(\frac{1}{2}x + 3\right)$

Compare it with $f(x) = C + B\sin(Ax + b)$

* Period $= \frac{2\pi}{k} = \frac{2\pi}{1/2} = 4\pi$

* Amplitude, $|a| = |3| = 3$

* Range $[c - |a|, c + |a|] = [-2-3, -2+3] = [-5, 1]$

* Phase shift $= \frac{-b}{k} = \frac{-3}{1/2} = -6$

ANSWER: C

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24. Solution

Given $G_1 = 3$ and $G_4 = 81$

You know that $G_n = G_1 r^{n-1}$

thus $G_4 = G_1 r^{4-1} \Rightarrow G_4 = G_1 r^3$

$\Rightarrow 81 = 3 \cdot r^3$

$\Rightarrow r^3 = \frac{81}{3} \Rightarrow r = \sqrt[3]{\frac{81}{3}} = \sqrt[3]{27} = 3$

ANSWER: D

25. Solution

Let $A = \begin{bmatrix} x & 2x \\ -x & x \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

You know that $A \cdot A^{-1} = I$ from properties of Matrix.

then $\begin{bmatrix} x & 2x \\ -x & x \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -x+2x & -2x+2x \\ x+x & 2x+x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x & 0 \\ 2x & 3x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

from Equality of Matrix

$x = 1$, $0 = 0$, $2x = 0$, $3x = 1$

$x = 1$, $0 = 0$, $x = 0$, $x = \frac{1}{3}$

since we have got different value of "x" the question has some Error

ANSWER: NO ~~SOLUTION~~ ANSWER

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25. If the inverse of the matrix $\begin{pmatrix} x & 2x \\ -x & x \end{pmatrix}$ is $\begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix}$, then which one of the following is the value of x?

A. -1
B. 1
C. -1/3
D. 1/3

26. What is the set of values of k so that the system $\begin{aligned} x - y + z &= 3 \\ 2x + y - z &= 4 \\ kx - 2y - z &= 5 \end{aligned}$ has a solution?

A. $\mathbb{R}$
B. $\mathbb{R}\backslash\{1\}$
C. $\mathbb{R}\backslash\{-1\}$
D. $\mathbb{R}\backslash\{-1, 1\}$

27. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$. Then which of the following is equal to 3A + 2B?

A. $\begin{pmatrix} 7 & 4 \\ 7 & 18 \end{pmatrix}$
B. $\begin{pmatrix} 7 & 1 \\ 7 & 7 \end{pmatrix}$
C. $\begin{pmatrix} 3 & 1 \\ 2 & 7 \end{pmatrix}$
D. $\begin{pmatrix} 3 & 4 \\ 2 & 18 \end{pmatrix}$

28. Which of the following is an interval at which the function $f(x) = \frac{1}{x-1} - \sqrt{4 - x^2}$ is differentiable?

A. $(-2, 1) \cup (1, 2)$
B. $(-2, 2)$
C. $(-\infty, -2) \cup (2, \infty)$
D. $[-2, 1) \cup (1, 2]$

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26. $x - y + z = 3$
$2x + y - z = 4$
$kx - 2y - z = 5$ has solution for $k = ?$

To find the value of k let we use Gussion Elimination Method (more hint about Gussion Elimination Method refer Question Number 34 answer).

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 2 & 1 & -1 & 4 \\ k & -2 & -1 & 5 \end{array}\right]$$ ← AUGMENTED Matrix.

Now $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 2 & 1 & -1 & 4 \\ k & -2 & -1 & 5 \end{array}\right] \xrightarrow[R_3 \to R_3 - kR_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 3 & -3 & -2 \\ 0 & k-2 & -(k+1) & 5-3k \end{array}\right]$$

$$\xrightarrow{R_3 \to 3R_3 - (k-2)R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 3 & -3 & -2 \\ 0 & 0 & -9 & 11-7k \end{array}\right]$$

Since, the coefficient of z in the third row is independent of the constant k.
Thus, the system has solution for whatever value of k. i.e $k \in \mathbb{R}$

ANSWER: A

27. Solution

Given $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$

$$3A + 2B = 3\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2\begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 4 & -2 \\ -2 & 6 \end{pmatrix} = \begin{pmatrix} 7 & 4 \\ 7 & 18 \end{pmatrix}$$

ANSWER: A

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28 Solution

Given $f(x) = \frac{1}{(x-1)} - \sqrt{4-x^2}$

to find interval of differentiation simple derivate the function first and find its domain. the domain is the interval of differentiation

thus $f'(x) = \frac{-(x-1)'}{(x-1)^2} - \frac{(4-x^2)'}{2\sqrt{4-x^2}}$

$= \frac{-1}{(x-1)^2} - \frac{(-2x)}{2\sqrt{4-x^2}}$

$= \frac{-1}{(x-1)^2} + \frac{2x}{2\sqrt{4-x^2}}$

$f'(x) = \frac{-1}{(x-1)^2} + \frac{x}{\sqrt{4-x^2}}$

now $f(x)$ is differentiable when $(x-1)^2 \neq 0$ and $4-x^2 > 0$

$(x-1)^2 \neq 0 \Rightarrow x \neq 1$ and

$\Rightarrow 4-x^2 > 0 \Rightarrow -x^2 > -4 \Rightarrow x^2 < 4$

$\Rightarrow x < \pm 2 \Rightarrow -2 < x < 2$

$\Rightarrow -2 < x < 2$

therefore $f(x)$ is differentiable on;

$-2 < x < 2$ and $x \neq 1$

that means $(-2, 1) \cup (1, 2)$

ANSWER: A

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29. The function $f(x) = (x-1)^{\frac{2}{3}}$ is differentiable on

A. $[1, \infty)$

B. $(-\infty, 1]$

C. $(-\infty,-1) \cup (-1, \infty)$

D. $(-\infty, 1) \cup (1, \infty)$

30. What is the slope of the line tangent to the graph of $f(x) = x^2 + \tan x$ at $(\pi, f(\pi))$?

A. 2π

B. 2

C. $2\pi+1$

D. $2\pi-1$

31. If $f(x) = 4 - x^2 \sin x$, then which of the following expressions describes $f'(x)$?

A. $-x(x\cos x + 2\sin x)$

B. $1+\cos x + 2\sin x$

C. $x(x\cos x - 2\sin x)$

D. $1-\cos x - 2\sin x$

32. Consider the sequence whose terms are $a_1 = 6$, and $a_{n+1} = -\frac{1}{3}(a_n)$ for $n \in \mathbb{N}$. Then what is the sum of the first ten terms is

A. $9\left(1-\left(-\frac{1}{3}\right)^{10}\right)$

B. $\frac{9}{2}\left(1-\left(-\frac{1}{3}\right)^{10}\right)$

C. $9\left(1-\left(-\frac{1}{3}\right)^{11}\right)$

D. $\frac{9}{2}\left(1-\left(-\frac{1}{3}\right)^{11}\right)$

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29. Solution

NB: A function f is differentiable on (a,b) if f is differtiable at all Number inside the number inside the interval (a,b) i.e $a < x_0 < b$

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \quad \text{exist}$$

x_0 is likely or possibly is the zeros of f and point of discontinuity

thus, $$\lim_{x \to 1} \frac{(x-1)^{2/3} - (1-1)^{2/3}}{x-1} = \frac{(x-1)^{2/3}}{x-1} = \frac{1}{(x-1)^{1/3}}$$

f is not differentiable at $x=1$ only

therefore f is differentiable on $(-\infty, 1) \cup (1, \infty)$

* OR simply derivate the function and find it's domain.

ANSWER: D

30. Solution

We know that $f'(x_0)$ is the slope of line tangent to the graph of f at $(x_0, f(x_0))$.

thus, $f(x) = x^2 + \tan x$

$f'(x) = 2x + \sec^2 x$

therefore, slope $= f'(\pi) = 2(\pi) + \sec^2(2\pi)$

$= 2\pi + 1$

ANSWER: C

31. Solution:

$f(x) = 4 - x^2 \sin x$

$\Rightarrow f'(x) = 0 - [(x^2)' \sin x + x^2 (\sin x)']$

$= 0 - 2x \sin x - x^2 \cos x$

$= 0 - x(2\sin x + x\cos x)$

$= -x(2\sin x + x\cos x)$

ANSWER: A.

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32. Solution:

$a_1 = 6$, $a_{n+1} = -\frac{1}{3}(a_n)$, for $n \in \mathbb{N}$

then $a_1 = 6$

$a_2 = -\frac{1}{3}(6) = -2$

$a_3 = -\frac{1}{3}(-2) = 2/3$

$a_4 = -\frac{1}{3}(2/3) = -2/9$

$r = \frac{a_4}{a_3} = \frac{a_3}{a_2} = \frac{a_2}{a_1} = -\frac{1}{3}$ thus its geometric sequence

$\Rightarrow S_n = \frac{a_1(1-r^n)}{1-r}$

$\Rightarrow S_{10} = \frac{6\left(1-\left(-\frac{1}{3}\right)^{10}\right)}{1+\frac{1}{3}} = \frac{6\left(1+\left(\frac{1}{3}\right)^{10}\right)}{4/3} = \frac{9}{2}\left[1-\left(-1/3\right)^{10}\right]$

ANSWER: B

33. Solution

* A matrix is in Row reduced Echelon form if and only if,
  1. It is in echelon form
  2. the first non-zero element in each non-zero row is the only non-zero element in its column.

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 4 & -1 & 4 \end{pmatrix} \xrightarrow[R_3 \to R_3 - 2R_2]{R_2 \to 2R_1 - R_2} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & -3 & -2 \end{pmatrix} \xrightarrow{R_1 = R_1 - R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & -3 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & -3 & -2 \end{pmatrix} \xrightarrow{R_3 = R_3 + 3R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & -5 \end{pmatrix} \xrightarrow{R_3 = -\frac{1}{5}R_3} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow[R_2 = R_2 + R_3]{R_1 = R_1 - 2R_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

ANSWER: E

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33. Which one of the following is a row reduced echelon form of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 4 & -1 & 4 \end{pmatrix}$?

A. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

B. $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

D. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

34. For what value of k does the system $\begin{cases} 2x - y + 4k = 6 \\ kx - y - z = 2 \\ x + y + 2z = 8 \end{cases}$ has a unique solution?

A. $k \neq -\frac{1}{2}$

B. $k \neq -\frac{1}{3}, \frac{1}{2}$

C. $k \neq -\frac{1}{6}$

D. $k \neq \frac{1}{2}$

35. Let $A = \begin{pmatrix} 2 & 3 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 5 & -4 \\ 3 & 2 & 6 \end{pmatrix}$. The AB is?

A. $\begin{pmatrix} 8 & 13 & -18 \\ -1 & 8 & -2 \end{pmatrix}$

B. $\begin{pmatrix} 13 & 1 \\ 16 & 8 \\ 10 & -14 \end{pmatrix}$

C. $\begin{pmatrix} 8 & -1 \\ 13 & 8 \\ 18 & -2 \end{pmatrix}$

D. $\begin{pmatrix} 13 & 16 & 10 \\ 1 & 8 & -14 \end{pmatrix}$

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34. Solution

$$\begin{cases} 2x - y + 4z = 6 \\ kx - y - z = 2 \\ x + y + 2z = 8 \end{cases}$$ has unique solution when $k = ?$

Let us solve it using Gussion Elimination method.

The Gussion Elimination Method

Let $AX = B$ be a linear system of equation then

1. Write down the augmented Matrix of the system of linear Equation
2. Find an echelon form of the augmented Matrix using Elementary row operations
3. Write down the system of equations corresponding to the echelon form
4. Use back substitution to arrive at the solution.

Let come back to our question.

$$\left(\begin{array}{ccc|c} 2 & -1 & 4 & 6 \\ k & -1 & -1 & 2 \\ 1 & 1 & 2 & 8 \end{array}\right)$$ ← augmented Matrix.

Swap R_2 with R_3 →

$$\left[\begin{array}{ccc|c} 2 & -1 & 4 & 6 \\ 1 & 1 & 2 & 2 \\ k & -1 & -1 & 8 \end{array}\right]$$

Now find an echelon form.

$$\left[\begin{array}{ccc|c} 2 & -1 & 4 & 6 \\ 1 & 1 & 2 & 2 \\ k & -1 & -1 & 8 \end{array}\right] \xrightarrow[R_3 \to R_3 - kR_2]{R_2 \to R_1 - 2R_1} \left[\begin{array}{ccc|c} 2 & -1 & 4 & 6 \\ 0 & -3 & 0 & -4 \\ 0 & -1-k & -1-2k & 8-2k \end{array}\right] \xrightarrow{R_3 \to 3R_3 + (-1-k)R_2}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 4 & 6 \\ 0 & -3 & 0 & -4 \\ 0 & 0 & -3-6k & 28-2k \end{array}\right]$$

The system has unique solution if the coefficient of z in the third row is different from 0 (zero)

thus, $-3 - 6k \neq 0 \Rightarrow -3 \neq 6k \Rightarrow k \neq -\frac{3}{6} \neq -\frac{1}{2}$

$\Rightarrow k \neq -\frac{1}{2}$

ANSWER: A

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Note After you write the augmented matrix of the system in Echelon form. for Example for 3 by 3 (3x3) matrix

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a_{22} & a_{23} & b_2 \\ 0 & 0 & a_{33} & b_3 \end{array}\right]$$

the system has:-

* Unique solution if and only if the coefficient of z in the third row is dif from zero
  i.e $a_{33} \neq 0$
* infinite solution
  if and only if $a_{33} = 0$ and $b_3 = 0$
  i.e both a_{33} and b_3 are zero
* No solution
  if and only if $a_{33} = 0$ but $b_3 \neq 0$

5. Solution

Given $A = \begin{pmatrix} 2 & 3 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 5 & -4 \\ 3 & 2 & 6 \end{pmatrix}$

then $AB = \begin{pmatrix} 2 & 3 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 2 & 5 & -4 \\ 3 & 2 & 6 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 4+9 & 10+6 & -8+18 \\ 4-3 & 10-2 & -8-6 \end{pmatrix} = \begin{pmatrix} 13 & 16 & 10 \\ 1 & 8 & -14 \end{pmatrix}$$

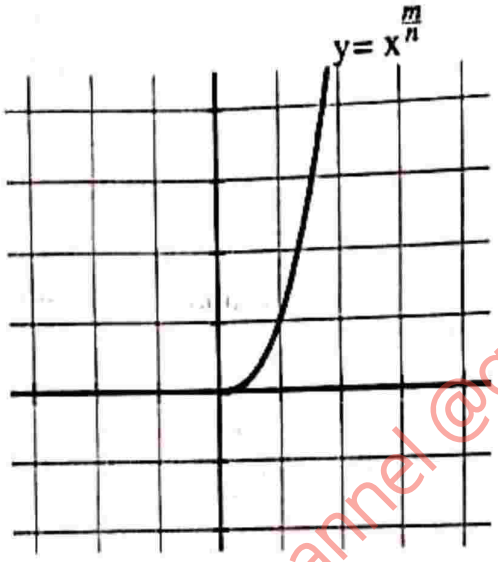
$$\therefore A \cdot B = \begin{pmatrix} 13 & 16 & 10 \\ 1 & 8 & -14 \end{pmatrix}$$

ANSWER: D

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36. The following graph is the graph of the function $y = x^{\frac{m}{n}}$, where m and n are positive integers and $n \neq 0$.



Which of the following is true about m and n?

A. m is odd, n is even and $m > n$.

B. m is even, n is odd and $m < n$.

C. m is odd, n is even and $m < n$.

D. m is even, n is odd and $m > n$.

37. Consider the experiment of rolling a die whose sample space is {1,2,3,4,5,6}. If two dice are rolled simultaneously, what is the probability that a prime number turns up on one of the dice and a composite number on the other?

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{5}{6}$

D. $\frac{1}{6}$

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36. Solution

~~[illegible]~~

M is even, n is odd and $M > n$

ANSWER: D

37. Solution

⊛ Sample space: $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1) \cdots (6,6)\}$

total number of possible outcomes $= n(S) = 36$

⊛ Event: $E = \{(2,3), (2,5), (4,3), (4,5), (6,3), (6,5), (3,2), (5,2), (3,4), (5,4), (3,6), (5,6)\}$

number of Event $= n(E) = 12$

Then $P(E) = \frac{n(E)}{n(S)} = \frac{12}{36} = \frac{1}{3} = 1/3$

ANSWER: A

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38. Consider the following frequency distribution of grouped data.

values	Frequency
95–99	4
90–94	6
85–89	10
80–84	16
75–79	9
70–74	5

Which of the following statements is true about this frequency distribution?

A. The class interval is 4

B 95.5 is the upper limit of the second class.

C. 85.5–90.5 is one of the class boundaries.

D. 82 is the class midpoint of the fourth class.

39. Let $f(x) = \frac{1}{(1-\sqrt{x})^2}$. Then $f'(4)$ is equal

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 1

D. 2

40. Given the function $f(x) = 2x^2 - 3x + 1$, what is the slope of the line passing through the points $(-1, f(-1))$ and $(2, f(2))$?

A. -1

B. $-\frac{1}{3}$

C. $\frac{1}{3}$

D. 1

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38. Solution

* correcting factor is $\frac{95-94}{2} = 0.5$ (uniform for all classes)
* the upper limit of second class is 94
* the midpoint of fourth class is:

$$\frac{80+84}{2} = 82$$

* class interval is $99.5 - 94.5 = 5$

ANSWER: D

39 Solution.

Given $f(x) = \frac{1}{(1-\sqrt{x})^2}$, then $f'(4) = ?$

first $f'(x) = \frac{-[(1-\sqrt{x})^2]'}{(1-\sqrt{x})^4} = \frac{-2(1-\sqrt{x})(1-\sqrt{x})'}{(1-\sqrt{x})^4}$

$= \frac{-2\left(-\frac{1}{2\sqrt{x}}\right)}{(1-\sqrt{x})^3} = \frac{1}{(1-\sqrt{x})^3 \cdot \sqrt{x}}$

then $f'(4) = \frac{1}{(1-\sqrt{4})^3 \cdot \sqrt{4}} = \frac{1}{(1-2)^3 \cdot 2} = \frac{1}{-1 \cdot 2} = -\frac{1}{2}$

$\therefore f'(4) = -\frac{1}{2}$

ANSWER: B

40. Solution

Given $f(x) = 2x^2 - 3x + 1$

then $f(-1) = 2(-1)^2 - 3(-1) + 1 = 6$

$f(2) = 2(2)^2 - 3(2) + 1 = 3$

i.e $(-1, f(-1)) = (-1, 6)$ and $(2, f(2)) = (2, 3)$

then the slope of line passing through the points $(-1, 6)$ & $(2, 3)$

$$m = \frac{3-6}{2-(-1)} = \frac{-3}{3} = -1$$

ANSWER: A

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41. Let $\vec{u}=(2,0)$ and $\vec{v}=(-1,3)$ be two vectors in the plane and $\vec{w}=-3\vec{u}+2\vec{v}$.

Which one of the following is the unit vector in the direction of $\vec{w}$, in terms of the standard unit vectors $\mathbf{i}$ and $\mathbf{j}$?

A. $\frac{4}{5}\mathbf{i}-\frac{3}{5}\mathbf{j}$

B. $-\frac{4}{5}\mathbf{i}+\frac{3}{5}\mathbf{j}$

C. $\frac{3}{5}\mathbf{i}-\frac{4}{5}\mathbf{j}$

D. $-\frac{3}{5}\mathbf{i}+\frac{4}{5}\mathbf{j}$

42. Which of the following is **NOT** true about the scalar product of vectors?

A. The scalar product is commutative.

B. The scalar product of non-zero parallel vectors is zero.

C. The scalar product is distributive over addition of vectors.

D. The scalar product of perpendicular vectors is zero.

43. Which one of the following is **NOT** true about transformation?

A. Rotations move triangles in to congruent triangles.

B. Translations move angles to congruent angles.

C. Rotation is a rigid motion.

D. Reflection is not a rigid motion.

44. If $z=-1+i$ and $w=1-3i$, what is the simplified form of $\frac{5z}{w}+|z|^2\overline{w}$?

A $4i$

B $5i$

C $4+5i$

D $5+4i$

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41. Solution

Given $\vec{u} = (2,0) = 2i + 0j$

$\vec{v} = (-1,3) = -i + 3j$

$\vec{w} = -3\vec{u} + 2\vec{v} = -3(2i+0j) + 2(-i+3j)$

$= -6i + 0j + (-2i + 6j)$

$= -6i - 2i + 6j$

$= -8i + 6j$

$|\vec{w}| = \sqrt{(-8)^2 + (6)^2} = 10$

the unit vector in the direction of $\vec{w}$ is given by

$\hat{u}_{\vec{w}} = \frac{\vec{w}}{|\vec{w}|} = \frac{-8i+6j}{10} = -\frac{4}{5}i + \frac{3}{5}j$

ANSWER: B

42. Solution

Scalar product of vectors has the following properties:

- its commutative ie $A \cdot B = B \cdot A$
- The scalar product of perpendicular vector is zero
- The scalar product of Non-zero parallel vectors is not zero
- The scalar product is distributive over addition of vectors

ANSWER: B

43. Solution

* Rotation is Rigid motion.
* Reflection is Rigid motion
* Translations move angles to congruent angles
* Rotations move triangles into congruent triangles.

ANSWER D

44. SOLUTION Given $z = -1 + i$ and $w = 1 - 3i$ then $\frac{5z}{w} + |z|^2 \bar{w} =$ ——

* $|z| = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$, $\bar{w} = 1 + 3i$

then, $\frac{5z}{w} + |z|^2 \bar{w} = \frac{5(-1+i)}{1-3i} + (\sqrt{2})^2(1+3i) = \frac{-5+5i}{1-3i} + 2(1+3i)$

$\Rightarrow \frac{-5+5i}{1-3i} \cdot \frac{1+3i}{1+3i} + 2 + 6i = \frac{-5+5i-15i-15}{1+9} + 2 + 6i$

$\Rightarrow \frac{-20-10i}{10} + 2 + 6i = -2 - i + 2 + 6i = 5i$

ANSWER: B

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45. A real number a is the limit of a sequence $\{a_n\}$ if

A. a is closer to every term of the sequence.

B. $a \leq |a_n|$ for all n.

C. a is very close to all terms a_n from some arbitrarily big n onwards.

D. $|a_n| \leq a_n \leq |a|$ for all n.

46. Which one of the following is the standard form of the equation of the ellipse with vertices: (3, 1), (3, 9); and minor axis of length 6

A. $\frac{(x-3)^2}{9} + \frac{(y-5)^2}{16} = 1$

B. $\frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} = 1$

C. $\frac{(x-3)^2}{4} + \frac{(y-3)^2}{36} = 1$

D. $\frac{(x-1)^2}{16} + \frac{(y-9)^2}{64} = 1$

47. What is the image of the ellipse $\frac{(x-3)^2}{6} + \frac{(y-2)^2}{4} = 1$ when it is rotated through 90° about the origin?

A. $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{6} = 1$

B. $4(x+3)^2 + 6(y-2)^2 = 24$

C. $\frac{(x+2)^2}{6} + \frac{(y-3)^2}{4} = 1$

D. $6(x+2)^2 + 4(y-3)^2 = 24$

48. Let p and q be propositions with truth value True and False respectively. Then which of the following is correct?

A. $(p \vee \neg q) \Rightarrow$ q is True.

B. $\neg p \Rightarrow q$ is False

C. $p \vee (q \wedge \neg p)$ is True.

D. $(p \Rightarrow q) \wedge$ p is True.

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45. Solution

ANSWER:

46. Solution

Given Vertices: $(3,1)$, $(3,9)$; and Minor axis of length 6

- to find the standard form of the equation of ellipse with the given first identify the major axis parallel to which axis (i.e. x-axis or y-axis).
- So, from the given vertices the first entry (x-value) is the same thus that major axis of the Ellipse is parallel to the y-axis

Now the standard equation of Ellipse with Major-axis parallel to the y-axis is:

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

from the question $b = \frac{6}{2} = 3$, $h = 3$

the vertices is given by $V(h, k-a)$ and $V(h, k+a)$

thus $V(3,1) = V(h, k-a)$ and $V(3,9) = V(h, k+a)$

$3 = h$, $1 = k-a$ and $3 = h$, $9 = k+a$

$$\begin{cases} k-a = 1 \\ +\ k+a = 9 \end{cases}$$ Solve simultaneously

$$2k = 10$$

$$k = 5 \quad \text{and} \quad a = k-1 = 5-1 = 4$$

Now the equation is given by

$$\frac{(x-3)^2}{9} + \frac{(y-5)^2}{16} = 1$$

∴

ANSWER: A

NB You can choice the answer without doing something if you are intelligent by using the length of the Minor axis!!! Think!!!

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47. Solution

To rotate any given Ellipse rotate its centre through given angle and about given point and find its new Equation with the new centre.

Given Ellipse Equation $\frac{(x-3)^2}{6} + \frac{(y-2)^2}{4} = 1$

its centre is $(3,2) = (h,k)$

then $h' = h\cos\theta - k\sin\theta$, $k' = h\sin\theta + k\cos\theta$, $\theta = 90°$

thus $h' = 3\cos 90° - 2\sin 90° = -2$, $k' = 3\sin 90° + 2\cos 90° = 3$

then $R(h', k') \xrightarrow{90°} (h', k') = (-2, 3)$

the Equation of the image of the Ellipse is

$$\frac{(x+2)^2}{6} + \frac{(y-3)^2}{4} = 1$$

ANSWER: C

48. Solution

Given $P \equiv T$

$q \equiv F$

A. $(P \vee \neg q) \Rightarrow q \equiv (T \vee T) \Rightarrow F \equiv T \Rightarrow F \equiv F$

B. $\neg P \Rightarrow q \equiv F \Rightarrow F \equiv T$

C. $P \vee (q \wedge \neg P) \equiv T \vee (F \wedge F) \equiv T \vee F \equiv T$

D. $(P \Rightarrow q) \wedge P \equiv (T \Rightarrow F) \wedge T \equiv F \wedge T \equiv F$

ANSWER: C

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49. Let $f(x) = \sqrt{x+2}$ and $g(x) = x^2 - 1$. What are the domain and range of the composition of f with $g, f \circ g$ respectively?

A. $\mathbb{R}$ and $[1, \infty)$
B. $\mathbb{R}$ and $[0, \infty)$
C. $[0, \infty)$ and $[1, \infty)$
D. $[0, \infty)$ and $[0, \infty)$

50. Given a rational function $f(x) = \frac{2x+4}{x-1}$ which of the following is NOT true about its graph? The graph has;

A. a horizontal asymptote, $y = 2$.
B. a vertical asymptote, $x = 1$.
C. $y - intercept\ (0, -4)$.
D. $x - intercept$ at $(-4,\ 0)$

51. $\lim_{x \to \infty} \left(1 + \frac{5}{x}\right)^x$ is equal to

A. $\frac{1}{e}$
B. e^5
C. $5e$
D. e^{-5}

52. Which of the following is true about limit of a function?

A. Limit of a constant function is always zero.
B. Right hand and left hand limits are the same for every function.
C. Intuitively, the limit of a function f is a y-value L to which $f(x)$ approaches as x approaches to some specified number.
D. Whenever the limit exists for a function it is the same as functional value.

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49. Solution

Given $f(x) = \sqrt{x+2}$ and $g(x) = x^2 - 1$

$f \circ g = f(g(x)) = f(x^2-1) = \sqrt{(x^2-1)+2}$

$f \circ g = f(g(x)) = \sqrt{x^2+1}$

Domain of $f \circ g = \mathbb{R}$ and Range of $f \circ g = [1, \infty)$

ANSWER: A

50. Solution

Given $f(x) = \dfrac{2x+4}{x-1}$

the graph of $f(x)$ has:

- horizontal asymptote, $y = \frac{2}{1} = 2$ since the degree of denominator & numerator is equal.
- Vertical asymptote, $x = 1$, since the denominator is zero at $x = 1$ but the numerator is $\neq 0$ at $x = 1$
- y- intercept $(0, -4)$, by setting $x = 0$ and solving for y
- x- intercept $(-2, 0)$, by setting $y = 0$ and solving for x

ANSWER: D

51. Solution

$$\lim_{x \to \infty} \left(1 + \frac{5}{x}\right)^x = e^5$$

NB: $\lim_{x \to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$, $\lim_{x \to \pm\infty} \left(1 + \frac{n}{x}\right)^x = e^n$, $\lim_{x \to 0} (1 + nx)^{1/x} = e^n$

ANSWER: B

52. Solution:
- Whenever the limit exists for a function it is true
- limit of constant function is constant (may be zero or not zero)
- Right hand and left hand limits are may not be some for every function
- intuitively, the limit of a function f is a y-value L to which $f(x)$ approaches as x approaches to some specified number

ANSWER: C

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53. Which of the following is equal to $\lim_{n\to\infty} \frac{\sin n}{n^2}$?

A. -1
B. 1
C. π
D. 0

54. If the functions f and g are continuous at $x = c$, then which one of the following combinations is continuous at $x = c$?

A. $\sqrt{g}$
B. $\frac{f}{g}$
C. f^{-1}
D. f^2

55. What is the value of $\lim_{x\to 0} \frac{\sin 4x}{\sin 6x}$?

A. $\frac{3}{2}$
B. $\frac{2}{3}$
C. 24
D. 6

56. Which of the following is a zero for the function $f(x) = x^3 - 3x^2 + 3x - 1$?

A. 0
B. 1
C. -2
D. -1

57. If $f(x) = \frac{\ln 2}{x^2}$, then $f'(x)$ is equal to

A. $\frac{1}{2x^2}$
B. $\frac{-2\ln 2}{x^3}$
C. $\frac{-2}{x^4}$
D. $\frac{1+2\ln x}{x^3}$

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53. Which of the following is equal to $\lim_{\Lambda \to \infty} \frac{\sin \Lambda}{\Lambda^2}$?

Solution $\lim_{\Lambda \to \infty} \frac{\sin \Lambda}{\Lambda^2}$

$\Rightarrow \lim_{\Lambda \to \infty} \frac{1}{\Lambda} \cdot \frac{\sin \Lambda}{\Lambda} = \lim_{\Lambda \to \infty} \frac{1}{\Lambda} \cdot \lim_{\Lambda \to \infty} \frac{\sin \Lambda}{\Lambda}$

$\Rightarrow 0 \times 1 = 0$

Solution : D

54. Solution

if the function f and g are continuous at $x = c$, then the following functions are also continuous at $x = c$

(i) $f + g$ (ii) $f - g$ (iii) $f \cdot f = f^2$

ANSWER: D

55. Solution

$$\lim_{x \to 0} \frac{\sin 4x}{\sin 6x} = \lim_{x \to 0} \frac{(\sin 4x)'}{(\sin 6x)'} = \lim_{x \to 0} \frac{4 \cos 4x}{6 \cos 6x} = \frac{4 \cos 0}{6 \cos 0}$$

$$= \frac{4}{6} = 2/3$$

NB: We can use other method to solve the Limit but for now I use L'HOSPITAL'S RULE see 2021 Qubee Exambook Question number 37

ANSWER: B

56. Solution

Given $f(x) = x^3 - 3x^2 + 3x - 1$

$\Rightarrow x^3 - 3x^2 + 3x - 1 = (x-1)(x^2 - 2x + 1) = (x-1)(x-1)(x-1)$

$\Rightarrow f(x) = (x-1)^3 = 0 \Rightarrow x = 1$

NB: ON Exam by substituting the value of x in the function you can get the Answer. this is the simple way.

ANSWER: B

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57. Solution

GIVEN $f(x) = \frac{\ln 2}{x^2}$

$f'(x) = \ln 2 \cdot \left(\frac{1}{x^2}\right)'$ since $\ln 2$ is constant.

$= \ln 2 \left(\frac{-2x}{(x^2)^2}\right) = \ln 2 \cdot \left(\frac{-2x}{x^4}\right) = \ln 2 \cdot \left(\frac{-2}{x^3}\right)$

$f'(x) = \frac{-2}{x^3} \ln 2 = \frac{-2\ln 2}{x^3}$

ANSWER: B.

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58. Let $x^2 + y^2 + kx = 1$ be equation of a circle for some $k \in \Re$. What is the radius of the circle if its center is $(2, 0)$?

A 5 C $\sqrt{2}$

B $\sqrt{5}$ D 2

59. The equation of a parabola is given by $y^2 + 6y + 8y + 25 = 0$. Which one of the following is **NOT** true about this parabola?

A. Its vertex is $(-2, -3)$ C. It opens to the right

B. Its directrix is the y-axis D. Its focus lies at $(-4, -3)$

60. What is the slope of the line tangent to the graph of $f(x) = x^2 + \tan x$ at $(\pi, f(\pi))$?

A. 2π C. $2\pi+1$

B. 2 D. $2\pi-1$

61. Suppose the measurement of height (in meter) of twelve students is:
1.72, 1.65, 1.70, 1.56, 1.72, 1.70, 1.65, 1.70, 1.65, 1.65, 1.70, 1.72
Which one is true about the mode of the data?

A. The data is unimodal with mode 1.65.

B. The data is unimodal with mode 1.70.

C. The data is bimodal with mode 1.65 and 1.70.

D. The data is multimodal.

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58. Solution

Given equation ⇒ circle $x^2+y^2+kx=1$, $k\in\mathbb{R}$

and center $C(2,0)$. $r=?$

* General equation ⇒ circle is given by

$(x-a)^2+(y-b)^2=r^2$ where (a,b) is center.

Now write $x^2+y^2+kx=1$ in General form

$\Rightarrow x^2+y^2+kx=1$

$\Rightarrow x^2+kx+y^2=1$

$\Rightarrow x^2+\frac{k}{2}x+\left(\frac{k}{2}\right)^2+y^2=1+\left(\frac{k}{2}\right)^2$ ← (completing the square method)

$\Rightarrow \left(x+\frac{k}{2}\right)^2+y^2=1+\frac{k^2}{4}$

where $\left(-\frac{k}{2},0\right)$ and $r^2=1+\frac{k^2}{4}$

for the given center $(2,0)=\left(-\frac{k}{2},0\right)$

$a=\frac{-k}{2}$

$k=-4$ --- solve for k

Now $r^2=1+\frac{k^2}{4} \Rightarrow r^2=1+\frac{(-4)^2}{4}=1+4$

$\Rightarrow r^2=5 \Rightarrow \underline{r=\sqrt{5}}$

ANSWER: B.

59 solution

Given equation ⇒ parabola $y^2+6y+8x+25=0$

(the question has may some error but in this case consider the first two terms as a variable of y and the third as the variable of x).

Now let Rewrite it in general form.

$y^2+6y+8x+25=0$

$y^2+6y+25+8x=0$

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$\Rightarrow y^2 + \frac{6}{2}y + \left(\frac{6}{2}\right)^2 = -8x - 25 + \left(\frac{6}{2}\right)^2$

$\Rightarrow y^2 + 3y + 9 = -8x - 25 + 9$

$\Rightarrow (y+3)^2 = -8x + 16$

$\Rightarrow (y+3)^2 = -8(x+2)$

Hence $-4p = -8$

$p = 2$

then :

- Its vertex is $V(h,k) = V(-2,-3)$
- Its directix is $x = h+p = -2+2 = 0$, that means y-axis
- ~~Since~~ Since the sign infront of p is negative, the parabola opens to the left
- Its focus lies $F(h-p, k) = F(-2-2, -3) = F(-4,-3)$

ANSWER: C

60. Solution

You know that $f'(x_0)$ is the slope of line tangent to the graph of f at $(x_0, f(x_0))$

thus, $f(x) = x^2 + \tan x$

$f'(x) = 2x + \sec^2 x$

therefore, slope $= f'(\pi) = 2\pi + \sec^2(2\pi) = 2\pi + 1$

ANSWER: C

61. Solution

first arrange the data in increasing order:

1.56, 1.65, 1.65, 1.65, 1.70, 1.70, 1.70, 1.70, 1.72, 1.72, 1.72

* the data is Bimodal with Mode 1.65 and 1.70

ANSWER: C.

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62. Which of the following is correct about f(x) = cotx?

A. Its range is $(-\infty,1]\cup[1,\infty)$

B. Its domain is $\{x \in \mathbb{R}: x \neq n\pi, n \in \mathbb{Z}\}$

C. Its domain is $\left\{x \in \mathbb{R}: x \neq \frac{n}{2}\pi, n \in \mathbb{Z}\right\}$

D. Its period is 2π

63. Let $f(x) = 3 - 2x$. What is the inverse of f?

A. $f^{-1}(x) = 2x - 3$

B. $f^{-1}(x) = 3 + 2x$

C. $f^{-1}(x) = \frac{3}{2} - \frac{x}{2}$

D. $f^{-1}(x) = \frac{x}{2} - \frac{3}{2}$

64. Let $\{a_n\}$ be a sequence of real numbers. Then which of the following is true about the sequence?

A. M is an upper bound of $\{a_n\}$ if $M \leq a_n$ for all a_n.

B. m a lower bound of $\{a_n\}$ if $a_n \leq m$ for all a_n.

C. The sequence $\{a_n\}$ is bounded if it has both an upper bound and a lower bound.

D. The sequence $\{a_n\}$ is bounded if there exists $k > 0$ such that $k \leq |a_n|$ for all a_n.

65. Which of the following is the lower bound for the sequence $\left\{\frac{n}{n-1}\right\}_{n=2}^{\infty}$?

A. $\frac{9}{5}$

B. 1

C. $\frac{7}{4}$

D. $\frac{5}{3}$

THE END

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62. Solution

Given $f(x) = \cot x$

∴ Domain of $f(x) = \cot x$, $D_f = \{x \in \mathbb{R} : x \neq n\pi, n \in \mathbb{Z}\}$

∴ Range of f: Range $= \mathbb{R}$

∴ Period of f: Period $= \pi$

ANSWER: B

63. $f(x) = 3 - 2x$

* To find the INVERSE FUNCTION of a one-to-one function $f(x)$

STEP 1: Write $y = f(x)$

STEP 2: Solve this equation for x in terms of y (if possible)

STEP 3: To express f^{-1} as a function of x, interchange x and y. The resulting equation is $y = f^{-1}(x)$

Therefore $y = f(x) = 3 - 2x$

$\Rightarrow y = 3 - 2x \Rightarrow \frac{y-3}{-2} = x$ — solve for x

$\Rightarrow \frac{3-y}{2} = x$

$\Rightarrow y = \frac{3-x}{2} = \frac{3}{2} - \frac{x}{2}$ — interchange x and y (step 3)

therefore $\Rightarrow f^{-1}(x) = y = \frac{3}{2} - \frac{x}{2}$

ANSWER: C

64. Solution

Let $\{a_n\}_{n=1}^{\infty}$ be sequence of Numbers Then:

(i) A Number M is an upper bound of $\{a_n\}$ if $M \geq a_n$, for all a_n

(ii) A number m is lower bound of $\{a_n\}$ if $m \leq a_n$ for all a_n

(iii) The sequence $\{a_n\}$ is bounded if it has both an upper bound and a lower bound.

ANSWER: C

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65. Solution

$$\left\{\frac{n}{n-1}\right\}_{n=2}^{\infty} = \{2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \ldots\}$$

$$\therefore \left(\text{i.e } \lim_{n\to\infty} \frac{n}{n-1} = \lim_{n\to\infty} \frac{\frac{n}{n}}{\frac{n-1}{n}} = \lim_{n\to\infty} \frac{1}{1-\frac{1}{n}} = 1\right)$$

Therefore $1 \le \left\{\frac{n}{n-1}\right\}_{n=2}^{\infty} \le 2$.

thus lower bound of the sequence is 1.

ANSWER: B

The End!!

Thank you

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