ELECTRO-SPACE®
ACADEMY

0.1: Physical Constants

Speed of light	c	3×10^8 m/s
Planck constant	h	6.63×10^{-34} J s
	hc	1242 eV-nm
Gravitation constant	G	6.67×10^{-11} m^3 kg^{-1} s^{-2}
Boltzmann constant	k	1.38×10^{-23} J/K
Molar gas constant	R	8.314 J/(mol K)
Avogadro's number	N_{A}	6.023×10^{23} mol^{-1}
Charge of electron	e	1.602×10^{-19} C
Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$ N/A^2
Permitivity of vacuum	ϵ_0	8.85×10^{-12} F/m
Coulomb constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N m^2/C^2
Faraday constant	F	96485 C/mol
Mass of electron	m_e	9.1×10^{-31} kg
Mass of proton	m_p	1.6726×10^{-27} kg
Mass of neutron	m_n	1.6749×10^{-27} kg
Atomic mass unit	u	1.66×10^{-27} kg
Atomic mass unit	u	931.49 MeV/c^2
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/(m^2 K^4)
Rydberg constant	R_∞	1.097×10^7 m^{-1}
Bohr magneton	μ_B	9.27×10^{-24} J/T
Bohr radius	a_0	0.529×10^{-10} m
Standard atmosphere	atm	1.01325×10^5 Pa
Wien displacement constant	b	2.9×10^{-3} m K

1 MECHANICS

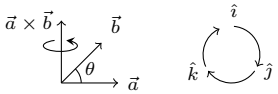
1.1: Vectors

Notation: $\vec{a} = a_x\,\hat{\imath} + a_y\,\hat{\jmath} + a_z\,\hat{k}$

Magnitude: $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Dot product: $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z = ab\cos\theta$

Cross product:



$$\vec{a}\times\vec{b} = (a_y b_z - a_z b_y)\hat{\imath} + (a_z b_x - a_x b_z)\hat{\jmath} + (a_x b_y - a_y b_x)\hat{k}$$

$$|\vec{a}\times\vec{b}| = ab\sin\theta$$

1.2: Kinematics

Average and Instantaneous Vel. and Accel.:

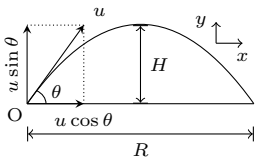
$$\vec{v}_{\mathrm{av}} = \Delta\vec{r}/\Delta t, \qquad \vec{v}_{\mathrm{inst}} = d\vec{r}/dt$$
$$\vec{a}_{\mathrm{av}} = \Delta\vec{v}/\Delta t \qquad \vec{a}_{\mathrm{inst}} = d\vec{v}/dt$$

Motion in a straight line with constant a:

$$v = u + at, \quad s = ut + \tfrac{1}{2}at^2, \quad v^2 - u^2 = 2as$$

Relative Velocity: $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$

Projectile Motion:



$$x = ut\cos\theta, \quad y = ut\sin\theta - \tfrac{1}{2}gt^2$$
$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$
$$T = \frac{2u\sin\theta}{g}, \quad R = \frac{u^2\sin 2\theta}{g}, \quad H = \frac{u^2\sin^2\theta}{2g}$$

1.3: Newton's Laws and Friction

Linear momentum: $\vec{p} = m\vec{v}$

Newton's first law: inertial frame.

Newton's second law: $\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}, \quad \vec{F} = m\vec{a}$

Newton's third law: $\vec{F}_{\mathrm{AB}} = -\vec{F}_{\mathrm{BA}}$

Frictional force: $f_{\mathrm{static,\ max}} = \mu_s N, \quad f_{\mathrm{kinetic}} = \mu_k N$

Banking angle: $\frac{v^2}{rg} = \tan\theta$, $\frac{v^2}{rg} = \frac{\mu+\tan\theta}{1-\mu\tan\theta}$

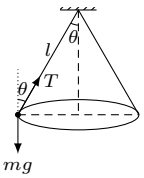
Centripetal force: $F_{\mathrm{c}} = \frac{mv^2}{r}, \quad a_{\mathrm{c}} = \frac{v^2}{r}$

Pseudo force: $\vec{F}_{\mathrm{pseudo}} = -m\vec{a}_0, \quad F_{\mathrm{centrifugal}} = -\frac{mv^2}{r}$

Minimum speed to complete vertical circle:

$$v_{\mathrm{min,\ bottom}} = \sqrt{5gl}, \quad v_{\mathrm{min,\ top}} = \sqrt{gl}$$

Conical pendulum: $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$



1.4: Work, Power and Energy

Work: $W = \vec{F}\cdot\vec{S} = FS\cos\theta, \quad W = \int \vec{F}\cdot\mathrm{d}\vec{S}$

Kinetic energy: $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Potential energy: $F = -\partial U/\partial x$ for conservative forces.

$$U_{\mathrm{gravitational}} = mgh, \quad U_{\mathrm{spring}} = \tfrac{1}{2}kx^2$$

Work done by conservative forces is path independent and depends only on initial and final points: $\oint \vec{F}_{\mathrm{conservative}}\cdot\mathrm{d}\vec{r} = 0$.

Work-energy theorem: $W = \Delta K$

Mechanical energy: $E = U + K$. Conserved if forces are conserved in nature.

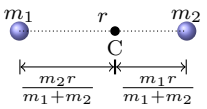
Power $P_{\text{av}} = \frac{\Delta W}{\Delta t}, \quad P_{\text{inst}} = \vec{F} \cdot \vec{v}$

1.5: Centre of Mass and Collision

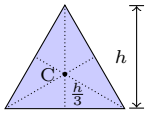
Centre of mass: $x_{\text{cm}} = \frac{\sum x_i m_i}{\sum m_i}, \quad x_{\text{cm}} = \frac{\int x \mathrm{d}m}{\int \mathrm{d}m}$

CM of few useful configurations:

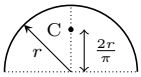
1. m_1, m_2 separated by r:



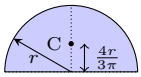
2. Triangle (CM ≡ Centroid) $y_c = \frac{h}{3}$



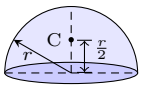
3. Semicircular ring: $y_c = \frac{2r}{\pi}$



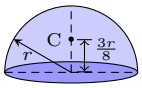
4. Semicircular disc: $y_c = \frac{4r}{3\pi}$



5. Hemispherical shell: $y_c = \frac{r}{2}$



6. Solid Hemisphere: $y_c = \frac{3r}{8}$



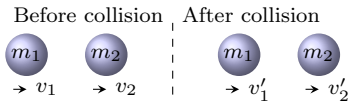
7. Cone: the height of CM from the base is $h/4$ for the solid cone and $h/3$ for the hollow cone.

Motion of the CM: $M = \sum m_i$

$$\vec{v}_{\text{cm}} = \frac{\sum m_i \vec{v}_i}{M}, \quad \vec{p}_{\text{cm}} = M\vec{v}_{\text{cm}}, \quad \vec{a}_{\text{cm}} = \frac{\vec{F}_{\text{ext}}}{M}$$

Impulse: $\vec{J} = \int \vec{F}\,\mathrm{d}t = \Delta\vec{p}$

Collision:



Momentum conservation: $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$
Elastic Collision: $\frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 = \frac{1}{2}m_1{v_1'}^2 + \frac{1}{2}m_2{v_2'}^2$
Coefficient of restitution:

$$e = \frac{-(v_1' - v_2')}{v_1 - v_2} = \begin{cases} 1, & \text{completely elastic} \\ 0, & \text{completely in-elastic} \end{cases}$$

If $v_2 = 0$ and $m_1 \ll m_2$ then $v_1' = -v_1$.
If $v_2 = 0$ and $m_1 \gg m_2$ then $v_2' = 2v_1$.
Elastic collision with $m_1 = m_2$: $v_1' = v_2$ and $v_2' = v_1$.

1.6: Rigid Body Dynamics

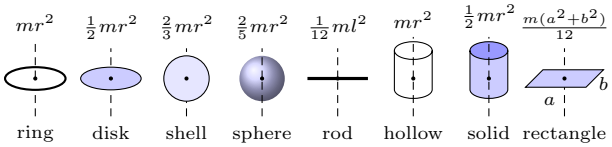
Angular velocity: $\omega_{\text{av}} = \frac{\Delta\theta}{\Delta t}, \quad \omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}, \quad \vec{v} = \vec{\omega} \times \vec{r}$

Angular Accel.: $\alpha_{\text{av}} = \frac{\Delta\omega}{\Delta t}, \quad \alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}, \quad \vec{a} = \vec{\alpha} \times \vec{r}$

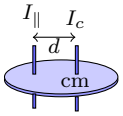
Rotation about an axis with constant α:

$$\omega = \omega_0 + \alpha t, \quad \theta = \omega t + \tfrac{1}{2}\alpha t^2, \quad {\omega}^2 - {\omega_0}^2 = 2\alpha\theta$$

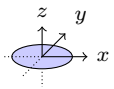
Moment of Inertia: $I = \sum_i m_i {r_i}^2, \quad I = \int r^2 \mathrm{d}m$



Theorem of Parallel Axes: $I_{\parallel} = I_{\text{cm}} + md^2$



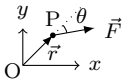
Theorem of Perp. Axes: $I_z = I_x + I_y$



Radius of Gyration: $k = \sqrt{I/m}$

Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}, \quad \vec{L} = I\vec{\omega}$

Torque: $\vec{\tau} = \vec{r} \times \vec{F}, \quad \vec{\tau} = \frac{\mathrm{d}\vec{L}}{\mathrm{d}t}, \quad \tau = I\alpha$



Conservation of $\vec{L}$: $\vec{\tau}_{\text{ext}} = 0 \implies \vec{L} = \text{const.}$

Equilibrium condition: $\sum \vec{F} = \vec{0}, \quad \sum \vec{\tau} = \vec{0}$

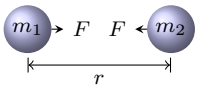
Kinetic Energy: $K_{\text{rot}} = \frac{1}{2}I\omega^2$

Dynamics:

$$\vec{\tau}_{\text{cm}} = I_{\text{cm}}\vec{\alpha}, \quad \vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}, \quad \vec{p}_{\text{cm}} = m\vec{v}_{\text{cm}}$$
$$K = \tfrac{1}{2}m{v_{\text{cm}}}^2 + \tfrac{1}{2}I_{\text{cm}}\omega^2, \quad \vec{L} = I_{\text{cm}}\vec{\omega} + \vec{r}_{\text{cm}} \times m\vec{v}_{\text{cm}}$$

1.7: Gravitation

Gravitational force: $F = G\frac{m_1 m_2}{r^2}$



Potential energy: $U = -\frac{GMm}{r}$

Gravitational acceleration: $g = \frac{GM}{R^2}$

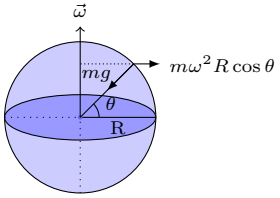
Variation of g with depth: $g_{\text{inside}} \approx g\left(1 - \frac{h}{R}\right)$

Variation of g with height: $g_{\text{outside}} \approx g\left(1 - \frac{2h}{R}\right)$

Effect of non-spherical earth shape on g:
$g_{\text{at pole}} > g_{\text{at equator}}$ $(\because R_{\text{e}} - R_{\text{p}} \approx 21 \text{ km})$

Effect of earth rotation on apparent weight:

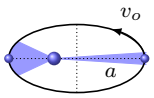
$mg'_\theta = mg - m\omega^2 R\cos^2\theta$



Orbital velocity of satellite: $v_o = \sqrt{\frac{GM}{R}}$

Escape velocity: $v_e = \sqrt{\frac{2GM}{R}}$

Kepler's laws:



First: Elliptical orbit with sun at one of the focus.
Second: Areal velocity is constant. ($\because \mathrm{d}\vec{L}/\mathrm{d}t = 0$).
Third: $T^2 \propto a^3$. In circular orbit $T^2 = \frac{4\pi^2}{GM}a^3$.

1.8: Simple Harmonic Motion

Hooke's law: $F = -kx$ (for small elongation x.)

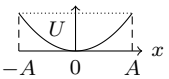
Acceleration: $a = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{k}{m}x = -\omega^2 x$

Time period: $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

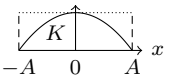
Displacement: $x = A\sin(\omega t + \phi)$

Velocity: $v = A\omega\cos(\omega t + \phi) = \pm\omega\sqrt{A^2 - x^2}$

Potential energy: $U = \frac{1}{2}kx^2$

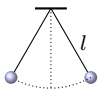


Kinetic energy $K = \frac{1}{2}mv^2$



Total energy: $E = U + K = \frac{1}{2}m\omega^2 A^2$

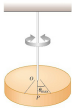
Simple pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$



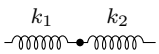
Physical Pendulum: $T = 2\pi\sqrt{\frac{I}{mgl}}$



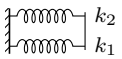
Torsional Pendulum $T = 2\pi\sqrt{\frac{I}{k}}$



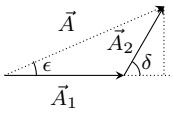
Springs in series: $\frac{1}{k_{\text{eq}}} = \frac{1}{k_1} + \frac{1}{k_2}$



Springs in parallel: $k_{\text{eq}} = k_1 + k_2$



Superposition of two SHM's:



$$x_1 = A_1\sin\omega t, \qquad x_2 = A_2\sin(\omega t + \delta)$$
$$x = x_1 + x_2 = A\sin(\omega t + \epsilon)$$
$$A = \sqrt{{A_1}^2 + {A_2}^2 + 2A_1A_2\cos\delta}$$
$$\tan\epsilon = \frac{A_2\sin\delta}{A_1 + A_2\cos\delta}$$

1.9: Properties of Matter

Modulus of rigidity: $Y = \frac{F/A}{\Delta l/l}$, $B = -V\frac{\Delta P}{\Delta V}$, $\eta = \frac{F}{A\theta}$

Compressibility: $K = \frac{1}{B} = -\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}P}$

Poisson's ratio: $\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\Delta D/D}{\Delta l/l}$

Elastic energy: $U = \frac{1}{2}$ stress $\times$ strain $\times$ volume

Surface tension: $S = F/l$

Surface energy: $U = SA$

Excess pressure in bubble:

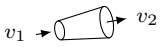
$$\Delta p_{\text{air}} = 2S/R, \quad \Delta p_{\text{soap}} = 4S/R$$

Capillary rise: $h = \frac{2S\cos\theta}{r\rho g}$

Hydrostatic pressure: $p = \rho gh$

Buoyant force: $F_B = \rho Vg =$ Weight of displaced liquid

Equation of continuity: $A_1v_1 = A_2v_2$

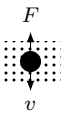


Bernoulli's equation: $p + \frac{1}{2}\rho v^2 + \rho gh =$ constant

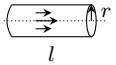
Torricelli's theorem: $v_{\text{efflux}} = \sqrt{2gh}$

Viscous force: $F = -\eta A\frac{\mathrm{d}v}{\mathrm{d}x}$

Stoke's law: $F = 6\pi\eta rv$



Poiseuilli's equation: $\frac{\text{Volume flow}}{\text{time}} = \frac{\pi pr^4}{8\eta l}$



Terminal velocity: $v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$

2 Waves

2.1: Waves Motion

General equation of wave: $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$.

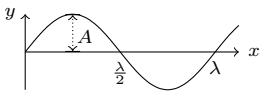
Notation: Amplitude A, Frequency ν, Wavelength λ, Period T, Angular Frequency ω, Wave Number k,

$$T = \frac{1}{\nu} = \frac{2\pi}{\omega}, \quad v = \nu\lambda, \quad k = \frac{2\pi}{\lambda}$$

Progressive wave travelling with speed v:

$$y = f(t - x/v), \rightsquigarrow +x; \quad y = f(t + x/v), \rightsquigarrow -x$$

Progressive sine wave:



$$y = A\sin(kx - \omega t) = A\sin(2\pi\,(x/\lambda - t/T))$$

2.2: Waves on a String

Speed of waves on a string with mass per unit length μ and tension T: $v = \sqrt{T/\mu}$

Transmitted power: $P_{\text{av}} = 2\pi^2\mu v A^2\nu^2$

Interference:

$$y_1 = A_1\sin(kx - \omega t), \quad y_2 = A_2\sin(kx - \omega t + \delta)$$

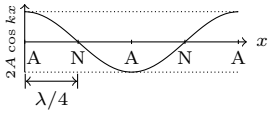
$$\boxed{y = y_1 + y_2 = A\sin(kx - \omega t + \epsilon)}$$

$$A = \sqrt{{A_1}^2 + {A_2}^2 + 2A_1A_2\cos\delta}$$

$$\tan\epsilon = \frac{A_2\sin\delta}{A_1 + A_2\cos\delta}$$

$$\delta = \begin{cases} 2n\pi, & \text{constructive;} \\ (2n+1)\pi, & \text{destructive.} \end{cases}$$

Standing Waves:

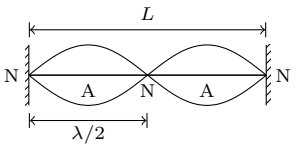


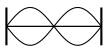
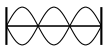
$$y_1 = A_1\sin(kx - \omega t), \quad y_2 = A_2\sin(kx + \omega t)$$

$$\boxed{y = y_1 + y_2 = (2A\cos kx)\sin\omega t}$$

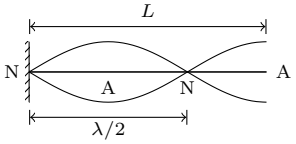
$$x = \begin{cases} \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, & \text{nodes;} \quad n = 0, 1, 2, \ldots \\ n\frac{\lambda}{2}, & \text{antinodes.} \quad n = 0, 1, 2, \ldots \end{cases}$$

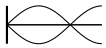
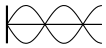
String fixed at both ends:



1. Boundary conditions: $y = 0$ at $x = 0$ and at $x = L$
2. Allowed Freq.: $L = n\frac{\lambda}{2}$, $\nu = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$, $n = 1, 2, 3, \ldots$.
3. Fundamental/1st harmonics: $\nu_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ 
4. 1st overtone/2nd harmonics: $\nu_1 = \frac{2}{2L}\sqrt{\frac{T}{\mu}}$ 
5. 2nd overtone/3rd harmonics: $\nu_2 = \frac{3}{2L}\sqrt{\frac{T}{\mu}}$ 
6. All harmonics are present.

String fixed at one end:



1. Boundary conditions: $y = 0$ at $x = 0$
2. Allowed Freq.: $L = (2n+1)\frac{\lambda}{4}$, $\nu = \frac{2n+1}{4L}\sqrt{\frac{T}{\mu}}$, $n = 0, 1, 2, \ldots$.
3. Fundamental/1st harmonics: $\nu_0 = \frac{1}{4L}\sqrt{\frac{T}{\mu}}$ 
4. 1st overtone/3rd harmonics: $\nu_1 = \frac{3}{4L}\sqrt{\frac{T}{\mu}}$ 
5. 2nd overtone/5th harmonics: $\nu_2 = \frac{5}{4L}\sqrt{\frac{T}{\mu}}$ 
6. Only odd harmonics are present.

Sonometer: $\nu \propto \frac{1}{L}$, $\nu \propto \sqrt{T}$, $\nu \propto \frac{1}{\sqrt{\mu}}$. $\nu = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

2.3: Sound Waves

Displacement wave: $s = s_0\sin\omega(t - x/v)$

Pressure wave: $p = p_0\cos\omega(t - x/v)$, $p_0 = (B\omega/v)s_0$

Speed of sound waves:

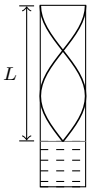
$$v_{\text{liquid}} = \sqrt{\frac{B}{\rho}}, \quad v_{\text{solid}} = \sqrt{\frac{Y}{\rho}}, \quad v_{\text{gas}} = \sqrt{\frac{\gamma P}{\rho}}$$

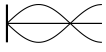
Intensity: $I = \frac{2\pi^2 B}{v}{s_0}^2\nu^2 = \frac{{p_0}^2 v}{2B} = \frac{{p_0}^2}{2\rho v}$

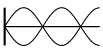
Standing longitudinal waves:

$$p_1 = p_0\sin\omega(t - x/v), \quad p_2 = p_0\sin\omega(t + x/v)$$
$$p = p_1 + p_2 = 2p_0\cos kx\sin\omega t$$

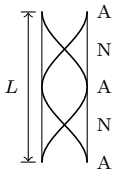
Closed organ pipe:

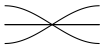
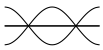
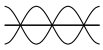


1. Boundary condition: $y = 0$ at $x = 0$
2. Allowed freq.: $L = (2n+1)\frac{\lambda}{4}$, $\nu = (2n+1)\frac{v}{4L}$, $n = 0, 1, 2, \ldots$
3. Fundamental/1st harmonics: $\nu_0 = \frac{v}{4L}$ 
4. 1st overtone/3rd harmonics: $\nu_1 = 3\nu_0 = \frac{3v}{4L}$ 

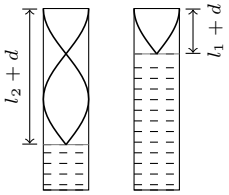
5. 2nd overtone/5th harmonics: $\nu_2 = 5\nu_0 = \frac{5v}{4L}$ 
6. Only odd harmonics are present.

Open organ pipe:



1. Boundary condition: $y = 0$ at $x = 0$
   Allowed freq.: $L = n\frac{\lambda}{2},\ \nu = n\frac{v}{4L},\ n = 1, 2, \ldots$
2. Fundamental/1st harmonics: $\nu_0 = \frac{v}{2L}$ 
3. 1st overtone/2nd harmonics: $\nu_1 = 2\nu_0 = \frac{2v}{2L}$ 
4. 2nd overtone/3rd harmonics: $\nu_2 = 3\nu_0 = \frac{3v}{2L}$ 
5. All harmonics are present.

Resonance column:



$$l_1 + d = \tfrac{\lambda}{2}, \quad l_2 + d = \tfrac{3\lambda}{4}, \quad v = 2(l_2 - l_1)\nu$$

Beats: two waves of almost equal frequencies $\omega_1 \approx \omega_2$

$$p_1 = p_0 \sin\omega_1(t - x/v), \quad p_2 = p_0 \sin\omega_2(t - x/v)$$
$$p = p_1 + p_2 = 2p_0 \cos\Delta\omega(t - x/v) \sin\omega(t - x/v)$$
$$\omega = (\omega_1 + \omega_2)/2, \quad \Delta\omega = \omega_1 - \omega_2 \quad \text{(beats freq.)}$$

Doppler Effect:

$$\nu = \frac{v + u_o}{v - u_s}\nu_0$$

where, v is the speed of sound in the medium, u_0 is the speed of the observer w.r.t. the medium, considered positive when it moves towards the source and negative when it moves away from the source, and u_s is the speed of the source w.r.t. the medium, considered positive when it moves towards the observer and negative when it moves away from the observer.

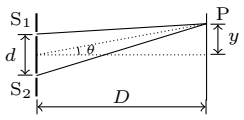
2.4: Light Waves

Plane Wave: $E = E_0 \sin\omega(t - \frac{x}{v}),\ I = I_0$ 

Spherical Wave: $E = \frac{aE_0}{r} \sin\omega(t - \frac{r}{v}),\ I = \frac{I_0}{r^2}$ 

Young's double slit experiment

Path difference: $\Delta x = \frac{dy}{D}$



Phase difference: $\delta = \frac{2\pi}{\lambda}\Delta x$

Interference Conditions: for integer n,

$$\delta = \begin{cases} 2n\pi, & \text{constructive;} \\ (2n+1)\pi, & \text{destructive,} \end{cases}$$

$$\Delta x = \begin{cases} n\lambda, & \text{constructive;} \\ \left(n + \frac{1}{2}\right)\lambda, & \text{destructive} \end{cases}$$

Intensity:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta,$$
$$I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2,\ I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$
$$I_1 = I_2 : I = 4I_0 \cos^2\tfrac{\delta}{2},\ I_{\max} = 4I_0,\ I_{\min} = 0$$

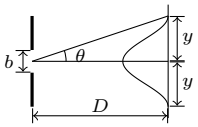
Fringe width: $w = \frac{\lambda D}{d}$

Optical path: $\Delta x' = \mu\Delta x$

Interference of waves transmitted through thin film:

$$\Delta x = 2\mu d = \begin{cases} n\lambda, & \text{constructive;} \\ \left(n + \frac{1}{2}\right)\lambda, & \text{destructive.} \end{cases}$$

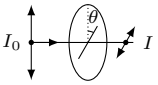
Diffraction from a single slit:



For Minima: $n\lambda = b\sin\theta \approx b(y/D)$

Resolution: $\sin\theta = \frac{1.22\lambda}{b}$

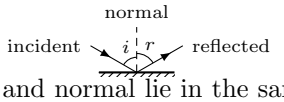
Law of Malus: $I = I_0 \cos^2\theta$



3 Optics

3.1: Reflection of Light

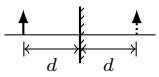
Laws of reflection:



(i)

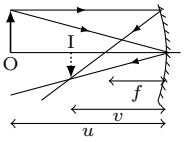
Incident ray, reflected ray, and normal lie in the same plane (ii) $\angle i = \angle r$

Plane mirror:



(i) the image and the object are equidistant from mirror (ii) virtual image of real object

Spherical Mirror:

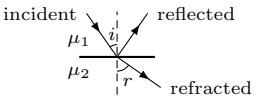


1. Focal length $f = R/2$
2. Mirror equation: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$
3. Magnification: $m = -\frac{v}{u}$

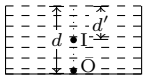
3.2: Refraction of Light

Refractive index: $\mu = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}} = \frac{c}{v}$

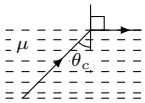
Snell's Law: $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$



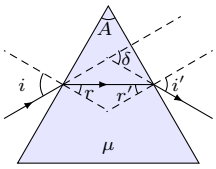
Apparent depth: $\mu = \frac{\text{real depth}}{\text{apparent depth}} = \frac{d}{d'}$



Critical angle: $\theta_c = \sin^{-1}\frac{1}{\mu}$



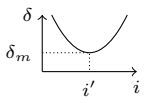
Deviation by a prism:



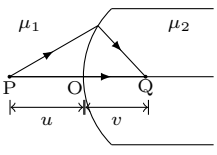
$\delta = i + i' - A$, general result

$\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$, $i = i'$ for minimum deviation

$\delta_m = (\mu - 1)A$, for small A



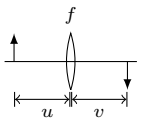
Refraction at spherical surface:



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}, \quad m = \frac{\mu_1 v}{\mu_2 u}$$

Lens maker's formula: $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

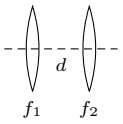
Lens formula: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, $m = \frac{v}{u}$



Power of the lens: $P = \frac{1}{f}$, P in diopter if f in metre.

Two thin lenses separated by distance d:

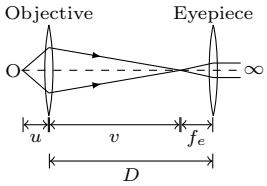
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$



3.3: Optical Instruments

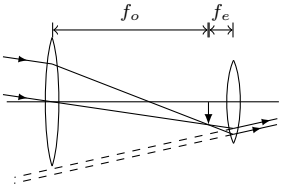
Simple microscope: $m = D/f$ in normal adjustment.

Compound microscope:



1. Magnification in normal adjustment: $m = \frac{v}{u}\frac{D}{f_e}$
2. Resolving power: $R = \frac{1}{\Delta d} = \frac{2\mu\sin\theta}{\lambda}$

Astronomical telescope:



1. In normal adjustment: $m = -\frac{f_o}{f_e}$, $L = f_o + f_e$
2. Resolving power: $R = \frac{1}{\Delta\theta} = \frac{1}{1.22\lambda}$

3.4: Dispersion

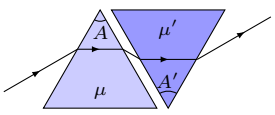
Cauchy's equation: $\mu = \mu_0 + \frac{A}{\lambda^2}$, $A > 0$

Dispersion by prism with small A and i:

1. Mean deviation: $\delta_y = (\mu_y - 1)A$
2. Angular dispersion: $\theta = (\mu_v - \mu_r)A$

Dispersive power: $\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} \approx \frac{\theta}{\delta_y}$ (if A and i small)

Dispersion without deviation:



$(\mu_y - 1)A + (\mu'_y - 1)A' = 0$

Deviation without dispersion:

$(\mu_v - \mu_r)A = (\mu'_v - \mu'_r)A'$

4 Heat and Thermodynamics

4.1: Heat and Temperature

Temp. scales: $F = 32 + \frac{9}{5}C, \quad K = C + 273.16$

Ideal gas equation: $pV = nRT, \quad n$: number of moles

van der Waals equation: $\left(p + \frac{a}{V^2}\right)(V - b) = nRT$

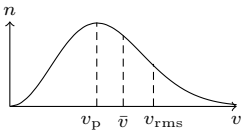
Thermal expansion: $L = L_0(1 + \alpha\Delta T)$,
$A = A_0(1 + \beta\Delta T)$, $V = V_0(1 + \gamma\Delta T)$, $\gamma = 2\beta = 3\alpha$

Thermal stress of a material: $\frac{F}{A} = Y\frac{\Delta l}{l}$

4.2: Kinetic Theory of Gases

General: $M = mN_A$, $k = R/N_A$

Maxwell distribution of speed:



RMS speed: $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$

Average speed: $\bar{v} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}}$

Most probable speed: $v_p = \sqrt{\frac{2kT}{m}}$

Pressure: $p = \frac{1}{3}\rho v_{rms}^2$

Equipartition of energy: $K = \frac{1}{2}kT$ for each degree of freedom. Thus, $K = \frac{f}{2}kT$ for molecule having f degrees of freedoms.

Internal energy of n moles of an ideal gas is $U = \frac{f}{2}nRT$.

4.3: Specific Heat

Specific heat: $s = \frac{Q}{m\Delta T}$

Latent heat: $L = Q/m$

Specific heat at constant volume: $C_v = \left.\frac{\Delta Q}{n\Delta T}\right|_V$

Specific heat at constant pressure: $C_p = \left.\frac{\Delta Q}{n\Delta T}\right|_p$

Relation between C_p and C_v: $C_p - C_v = R$

Ratio of specific heats: $\gamma = C_p/C_v$

Relation between U and C_v: $\Delta U = nC_v\Delta T$

Specific heat of gas mixture:

$$C_v = \frac{n_1C_{v1} + n_2C_{v2}}{n_1 + n_2}, \quad \gamma = \frac{n_1C_{p1} + n_2C_{p2}}{n_1C_{v1} + n_2C_{v2}}$$

Molar internal energy of an ideal gas: $U = \frac{f}{2}RT$, $f = 3$ for monatomic and $f = 5$ for diatomic gas.

4.4: Theromodynamic Processes

First law of thermodynamics: $\Delta Q = \Delta U + \Delta W$

Work done by the gas:

$$\Delta W = p\Delta V, \quad W = \int_{V_1}^{V_2} p\,\mathrm{d}V$$

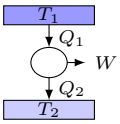
$$W_{\text{isothermal}} = nRT\ln\left(\frac{V_2}{V_1}\right)$$

$$W_{\text{isobaric}} = p(V_2 - V_1)$$

$$W_{\text{adiabatic}} = \frac{p_1V_1 - p_2V_2}{\gamma - 1}$$

$$W_{\text{isochoric}} = 0$$

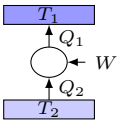
Efficiency of the heat engine:



$$\eta = \frac{\text{work done by the engine}}{\text{heat supplied to it}} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta_{\text{carnot}} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

Coeff. of performance of refrigerator:



$$\text{COP} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

Entropy: $\Delta S = \frac{\Delta Q}{T}$, $S_f - S_i = \int_i^f \frac{\Delta Q}{T}$

$$\text{Const. } T: \Delta S = \frac{Q}{T}, \quad \text{Varying } T: \Delta S = ms\ln\frac{T_f}{T_i}$$

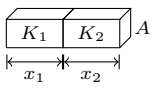
Adiabatic process: $\Delta Q = 0$, $pV^\gamma = \text{constant}$

4.5: Heat Transfer

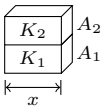
Conduction: $\frac{\Delta Q}{\Delta t} = -KA\frac{\Delta T}{x}$

Thermal resistance: $R = \frac{x}{KA}$

$$R_{\text{series}} = R_1 + R_2 = \frac{1}{A}\left(\frac{x_1}{K_1} + \frac{x_2}{K_2}\right)$$

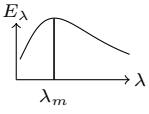


$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{x}(K_1A_1 + K_2A_2)$$



Kirchhoff's Law: $\frac{\text{emissive power}}{\text{absorptive power}} = \frac{E_{\text{body}}}{a_{\text{body}}} = E_{\text{blackbody}}$

Wien's displacement law: $\lambda_m T = b$



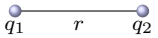
Stefan-Boltzmann law: $\frac{\Delta Q}{\Delta t} = \sigma eAT^4$

Newton's law of cooling: $\frac{\mathrm{d}T}{\mathrm{d}t} = -bA(T - T_0)$

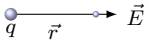
5 Electricity and Magnetism

5.1: Electrostatics

Coulomb's law: $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$



Electric field: $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$

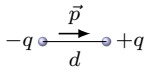


Electrostatic energy: $U = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$

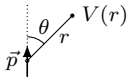
Electrostatic potential: $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

$$\mathrm{d}V = -\vec{E}\cdot\vec{r}, \quad V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E}\cdot\mathrm{d}\vec{r}$$

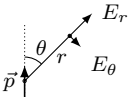
Electric dipole moment: $\vec{p} = q\vec{d}$



Potential of a dipole: $V = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$



Field of a dipole:



$$E_r = \frac{1}{4\pi\epsilon_0}\frac{2p\cos\theta}{r^3}, \quad E_\theta = \frac{1}{4\pi\epsilon_0}\frac{p\sin\theta}{r^3}$$

Torque on a dipole placed in $\vec{E}$: $\vec{\tau} = \vec{p}\times\vec{E}$

Pot. energy of a dipole placed in $\vec{E}$: $U = -\vec{p}\cdot\vec{E}$

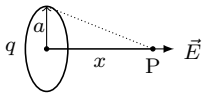
5.2: Gauss's Law and its Applications

Electric flux: $\phi = \oint \vec{E}\cdot\mathrm{d}\vec{S}$

Gauss's law: $\oint \vec{E}\cdot\mathrm{d}\vec{S} = q_{\text{in}}/\epsilon_0$

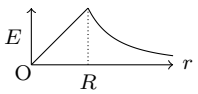
Field of a uniformly charged ring on its axis:

$$E_P = \frac{1}{4\pi\epsilon_0}\frac{qx}{(a^2+x^2)^{3/2}}$$

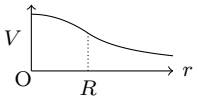


E and V of a uniformly charged sphere:

$$E = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}, & \text{for } r < R \\ \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}, & \text{for } r \geq R \end{cases}$$

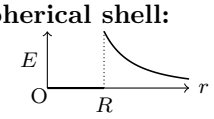


$$V = \begin{cases} \frac{Q}{8\pi\epsilon_0 R}\left(3 - \frac{r^2}{R^2}\right), & \text{for } r < R \\ \frac{1}{4\pi\epsilon_0}\frac{Q}{r}, & \text{for } r \geq R \end{cases}$$

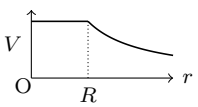


E and V of a uniformly charged spherical shell:

$$E = \begin{cases} 0, & \text{for } r < R \\ \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}, & \text{for } r \geq R \end{cases}$$



$$V = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{Q}{R}, & \text{for } r < R \\ \frac{1}{4\pi\epsilon_0}\frac{Q}{r}, & \text{for } r \geq R \end{cases}$$



Field of a line charge: $E = \frac{\lambda}{2\pi\epsilon_0 r}$

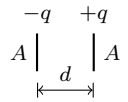
Field of an infinite sheet: $E = \frac{\sigma}{2\epsilon_0}$

Field in the vicinity of conducting surface: $E = \frac{\sigma}{\epsilon_0}$

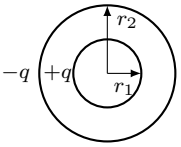
5.3: Capacitors

Capacitance: $C = q/V$

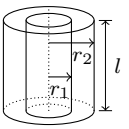
Parallel plate capacitor: $C = \epsilon_0 A/d$



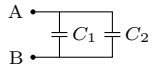
Spherical capacitor: $C = \frac{4\pi\epsilon_0 r_1 r_2}{r_2 - r_1}$



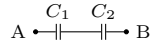
Cylindrical capacitor: $C = \frac{2\pi\epsilon_0 l}{\ln(r_2/r_1)}$



Capacitors in parallel: $C_{\text{eq}} = C_1 + C_2$



Capacitors in series: $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$



Force between plates of a parallel plate capacitor: $F = \frac{Q^2}{2A\epsilon_0}$

Energy stored in capacitor: $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$

Energy density in electric field E: $U/V = \frac{1}{2}\epsilon_0 E^2$

Capacitor with dielectric: $C = \frac{\epsilon_0 KA}{d}$

5.4: Current electricity

Current density: $j = i/A = \sigma E$

Drift speed: $v_d = \frac{1}{2}\frac{eE}{m}\tau = \frac{i}{neA}$

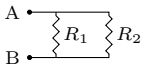
Resistance of a wire: $R = \rho l/A$, where $\rho = 1/\sigma$

Temp. dependence of resistance: $R = R_0(1 + \alpha\Delta T)$

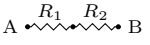
Ohm's law: $V = iR$

Kirchhoff's Laws: *(i) The Junction Law:* The algebraic sum of all the currents directed towards a node is zero i.e., $\Sigma_{\text{node}} I_i = 0$. (ii) *The Loop Law:* The algebraic sum of all the potential differences along a closed loop in a circuit is zero i.e., $\Sigma_{\text{loop}} \Delta V_i = 0$.

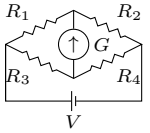
Resistors in parallel: $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$



Resistors in series: $R_{\text{eq}} = R_1 + R_2$



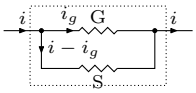
Wheatstone bridge:



Balanced if $R_1/R_2 = R_3/R_4$.

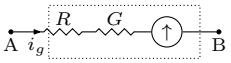
Electric Power: $P = V^2/R = I^2 R = IV$

Galvanometer as an Ammeter:



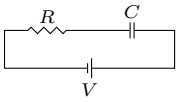
$$i_g G = (i - i_g) S$$

Galvanometer as a Voltmeter:



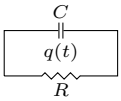
$$V_{\mathrm{AB}} = i_g (R + G)$$

Charging of capacitors:



$$q(t) = CV\left[1 - e^{-\frac{t}{RC}}\right]$$

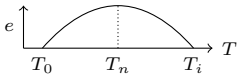
Discharging of capacitors: $q(t) = q_0 e^{-\frac{t}{RC}}$



Time constant in RC circuit: $\tau = RC$

Peltier effect: emf $e = \frac{\Delta H}{\Delta Q} = \frac{\text{Peltier heat}}{\text{charge transferred}}$.

Seeback effect:



1. Thermo-emf: $e = aT + \frac{1}{2}bT^2$
2. Thermoelectric power: $\mathrm{d}e/\mathrm{d}t = a + bT$.
3. Neutral temp.: $T_n = -a/b$.
4. Inversion temp.: $T_i = -2a/b$.

Thomson effect: emf $e = \frac{\Delta H}{\Delta Q} = \frac{\text{Thomson heat}}{\text{charge transferred}} = \sigma \Delta T$.

Faraday's law of electrolysis: The mass deposited is

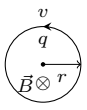
$$m = Zit = \tfrac{1}{F} Eit$$

where i is current, t is time, Z is electrochemical equivalent, E is chemical equivalent, and $F = 96485$ C/g is Faraday constant.

5.5: Magnetism

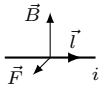
Lorentz force on a moving charge: $\vec{F} = q\vec{v} \times \vec{B} + q\vec{E}$

Charged particle in a uniform magnetic field:



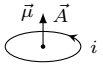
$r = \frac{mv}{qB}$, $T = \frac{2\pi m}{qB}$

Force on a current carrying wire:



$$\vec{F} = i\,\vec{l} \times \vec{B}$$

Magnetic moment of a current loop (dipole):

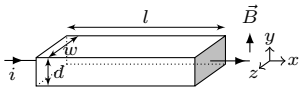


$\vec{\mu} = i\vec{A}$

Torque on a magnetic dipole placed in $\vec{B}$: $\vec{\tau} = \vec{\mu} \times \vec{B}$

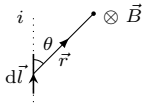
Energy of a magnetic dipole placed in $\vec{B}$:
$U = -\vec{\mu} \cdot \vec{B}$

Hall effect: $V_w = \frac{Bi}{ned}$

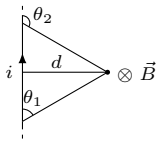


5.6: Magnetic Field due to Current

Biot-Savart law: $\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,\mathrm{d}\vec{l} \times \vec{r}}{r^3}$



Field due to a straight conductor:



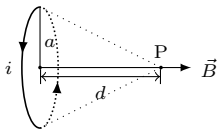
$$B = \frac{\mu_0 i}{4\pi d}(\cos\theta_1 - \cos\theta_2)$$

Field due to an infinite straight wire: $B = \frac{\mu_0 i}{2\pi d}$

Force between parallel wires: $\frac{\mathrm{d}F}{\mathrm{d}l} = \frac{\mu_0 i_1 i_2}{2\pi d}$

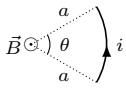


Field on the axis of a ring:



$$B_P = \frac{\mu_0 i a^2}{2(a^2+d^2)^{3/2}}$$

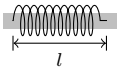
Field at the centre of an arc: $B = \frac{\mu_0 i \theta}{4\pi a}$



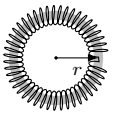
Field at the centre of a ring: $B = \frac{\mu_0 i}{2a}$

Ampere's law: $\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I_{\text{in}}$

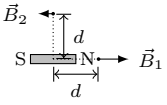
Field inside a solenoid: $B = \mu_0 n i$, $n = \frac{N}{l}$



Field inside a toroid: $B = \frac{\mu_0 N i}{2\pi r}$

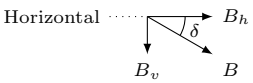


Field of a bar magnet:



$$B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d^3}, \quad B_2 = \frac{\mu_0}{4\pi}\frac{M}{d^3}$$

Angle of dip: $B_h = B\cos\delta$



Tangent galvanometer: $B_h \tan\theta = \frac{\mu_0 n i}{2r}$, $\quad i = K\tan\theta$

Moving coil galvanometer: $niAB = k\theta$, $\quad i = \frac{k}{nAB}\theta$

Time period of magnetometer: $T = 2\pi\sqrt{\frac{I}{MB_h}}$

Permeability: $\vec{B} = \mu\vec{H}$

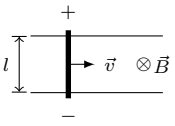
5.7: Electromagnetic Induction

Magnetic flux: $\phi = \oint \vec{B} \cdot \mathrm{d}\vec{S}$

Faraday's law: $e = -\frac{\mathrm{d}\phi}{\mathrm{d}t}$

Lenz's Law: Induced current create a B-field that opposes the change in magnetic flux.

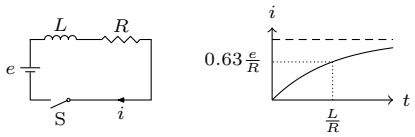
Motional emf: $e = Blv$



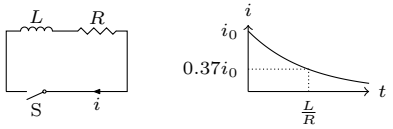
Self inductance: $\phi = Li, \quad e = -L\frac{\mathrm{d}i}{\mathrm{d}t}$

Self inductance of a solenoid: $L = \mu_0 n^2(\pi r^2 l)$

Growth of current in LR circuit: $i = \frac{e}{R}\left[1 - e^{-\frac{t}{L/R}}\right]$



Decay of current in LR circuit: $i = i_0 e^{-\frac{t}{L/R}}$



Time constant of LR circuit: $\tau = L/R$

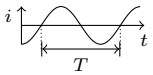
Energy stored in an inductor: $U = \frac{1}{2}Li^2$

Energy density of B field: $u = \frac{U}{V} = \frac{B^2}{2\mu_0}$

Mutual inductance: $\phi = Mi, \quad e = -M\frac{\mathrm{d}i}{\mathrm{d}t}$

EMF induced in a rotating coil: $e = NAB\omega \sin \omega t$

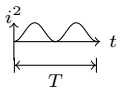
Alternating current:



$i = i_0 \sin(\omega t + \phi), \quad T = 2\pi/\omega$

Average current in AC: $\bar{i} = \frac{1}{T}\int_0^T i \, \mathrm{d}t = 0$

RMS current: $i_{\mathrm{rms}} = \left[\frac{1}{T}\int_0^T i^2 \, \mathrm{d}t\right]^{1/2} = \frac{i_0}{\sqrt{2}}$



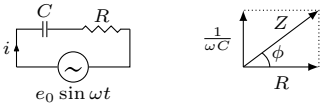
Energy: $E = {i_{\mathrm{rms}}}^2 RT$

Capacitive reactance: $X_c = \frac{1}{\omega C}$

Inductive reactance: $X_L = \omega L$

Imepedance: $Z = e_0/i_0$

RC circuit:



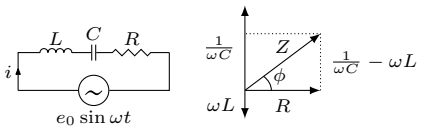
$Z = \sqrt{R^2 + (1/\omega C)^2}, \quad \tan\phi = \frac{1}{\omega CR}$

LR circuit:



$Z = \sqrt{R^2 + \omega^2 L^2}, \quad \tan\phi = \frac{\omega L}{R}$

LCR Circuit:

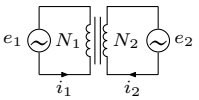


$Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}, \quad \tan\phi = \frac{\frac{1}{\omega C} - \omega L}{R}$

$\nu_{\mathrm{resonance}} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

Power factor: $P = e_{rms} i_{rms} \cos\phi$

Transformer: $\frac{N_1}{N_2} = \frac{e_1}{e_2}, \; e_1 i_1 = e_2 i_2$



Speed of the EM waves in vacuum: $c = 1/\sqrt{\mu_0 \epsilon_0}$

6 Modern Physics

6.1: Photo-electric effect

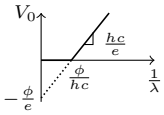
Photon's energy: $E = h\nu = hc/\lambda$

Photon's momentum: $p = h/\lambda = E/c$

Max. KE of ejected photo-electron: $K_{\max} = h\nu - \phi$

Threshold freq. in photo-electric effect: $\nu_0 = \phi/h$

Stopping potential: $V_o = \frac{hc}{e}\left(\frac{1}{\lambda}\right) - \frac{\phi}{e}$



de Broglie wavelength: $\lambda = h/p$

6.2: The Atom

Energy in nth Bohr's orbit:

$$E_n = -\frac{mZ^2e^4}{8\epsilon_0{}^2h^2n^2}, \quad E_n = -\frac{13.6Z^2}{n^2}\ \text{eV}$$

Radius of the nth Bohr's orbit:

$$r_n = \frac{\epsilon_0 h^2 n^2}{\pi m Z e^2}, \quad r_n = \frac{n^2 a_0}{Z}, \quad a_0 = 0.529\ \text{Å}$$

Quantization of the angular momentum: $l = \frac{nh}{2\pi}$

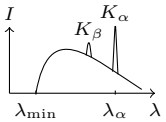
Photon energy in state transition: $E_2 - E_1 = h\nu$



Wavelength of emitted radiation: for a transition from nth to mth state:

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$$

X-ray spectrum: $\lambda_{\min} = \frac{hc}{eV}$



Moseley's law: $\sqrt{\nu} = a(Z - b)$

X-ray diffraction: $2d\sin\theta = n\lambda$

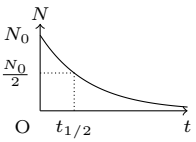
Heisenberg uncertainity principle:
$\Delta p \Delta x \geq h/(2\pi), \qquad \Delta E \Delta t \geq h/(2\pi)$

6.3: The Nucleus

Nuclear radius: $R = R_0 A^{1/3}, \quad R_0 \approx 1.1 \times 10^{-15}$ m

Decay rate: $\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N$

Population at time t: $N = N_0 e^{-\lambda t}$



Half life: $t_{1/2} = 0.693/\lambda$

Average life: $t_{\text{av}} = 1/\lambda$

Population after n half lives: $N = N_0/2^n$.

Mass defect: $\Delta m = [Zm_p + (A - Z)m_n] - M$

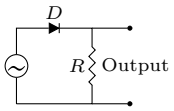
Binding energy: $B = [Zm_p + (A - Z)m_n - M]\,c^2$

Q-value: $Q = U_i - U_f$

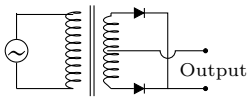
Energy released in nuclear reaction: $\Delta E = \Delta m c^2$
where $\Delta m = m_{\text{reactants}} - m_{\text{products}}$.

6.4: Vacuum tubes and Semiconductors

Half Wave Rectifier:



Full Wave Rectifier:



Triode Valve:

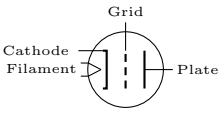


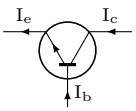
Plate resistance of a triode: $r_p = \left.\frac{\Delta V_p}{\Delta i_p}\right|_{\Delta V_g = 0}$

Transconductance of a triode: $g_m = \left.\frac{\Delta i_p}{\Delta V_g}\right|_{\Delta V_p = 0}$

Amplification by a triode: $\mu = -\left.\frac{\Delta V_p}{\Delta V_g}\right|_{\Delta i_p = 0}$

Relation between r_p, μ, and g_m: $\mu = r_p \times g_m$

Current in a transistor: $I_e = I_b + I_c$



α and β parameters of a transistor: $\alpha = \frac{I_c}{I_e}$, $\beta = \frac{I_c}{I_b}$, $\beta = \frac{\alpha}{1-\alpha}$

Transconductance: $g_m = \frac{\Delta I_c}{\Delta V_{be}}$

Logic Gates:

		AND	OR	NAND	NOR	XOR
A	B	AB	A+B	$\overline{\text{AB}}$	$\overline{\text{A}+\text{B}}$	$\text{A}\bar{\text{B}} + \bar{\text{A}}\text{B}$
0	0	0	0	1	1	0
0	1	0	1	1	0	1
1	0	0	1	1	0	1
1	1	1	1	0	0	0