

Frictional force

Frictional force is a force exerted between two bodies when one body moves or tries to move over another body.

Frictional force is caused by tiny bumps between surfaces. It is always opposite to the motion of a body.

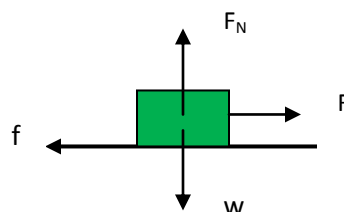
* Frictional force depends up on:

- the normal force(F_N)
- Coefficient of friction (μ) means the roughness of a surface.

$$f = \mu F_N$$

μ - coefficient of friction (unit less)

In horizontal surface, $F_N = mg$



Types of frictional force

- Static friction** is the force of frictional force between two bodies that are in contact and trying to move past each other but not yet moving.

Limiting friction is the maximum value of static friction.

- Kinetic (dynamic) friction** is the force of frictional force between two bodies sliding over each other.

❖ Frictional force does **NOT** depend on the *contact area* of the interacting surfaces.

Examples

- A 12kg block is moving along a leveled surface with a constant speed. The coefficient of friction between the block and the surface is 0.2. What is the force of friction?

$$f = \mu F_N = \mu mg = 0.2 \times 12\text{kg} \times 10\text{m/s}^2 = 24\text{N}$$

- A 30kg wooden block is placed on a horizontal surface. The maximum coefficient of friction between the block and the surface is 0.5. What minimum force should be applied to move the block?

$$f = \mu F_N = (0.5) \times (30\text{kg}) \times (10\text{m/s}^2) = 150\text{N}$$

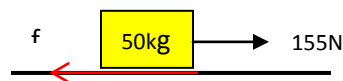
$$F_{\text{applied}} = f_{\text{maximum}} = 150\text{N}$$

- Look at the figure. A block of mass 50kg moves over a leveled surface where the coefficient of friction is 0.25. How much is the acceleration of the block?

$$f = \mu F_N = \mu F_N = \mu mg = (0.25) \times (50\text{kg} \times 10\text{m/s}^2) = 125\text{N}$$

$$F_{\text{net}} = F - f = 155\text{N} - 125\text{N} = 30\text{N}$$

$$a = \frac{F_{\text{net}}}{m} = \frac{30\text{N}}{50\text{kg}} = 0.6 \text{ m/s}^2$$



4. As shown in the figure below, a girl pulls a 10kg concrete using a rope making 37° to the horizontal by applying 40N force on the rope. The coefficient of friction is 0.2.

Calculate the acceleration of the concrete. ($\sin 37^\circ = 0.6$, $\cos 37^\circ = 0.8$)

The x- component of the force accelerates the concrete.

$$F_x = F \cos 37^\circ = 40\text{N} \times 0.8 = 32\text{N} \text{ and } F_y = F \sin 37^\circ = 40\text{N} \times 0.6 = 24\text{N}$$

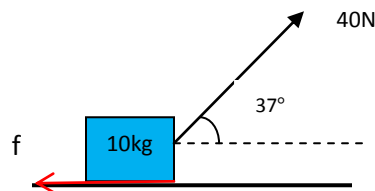
$$F_N + F_y = mg$$

$$F_N = mg - F_y = 100\text{N} - 24\text{N} = 76\text{N}$$

$$f = \mu F_N = (0.2) (76\text{N}) = 8\text{N}$$

$$F_{\text{net}} = F_x - f = 32\text{N} - (0.2 \times 76)\text{N} = 16.8\text{N}$$

$$a = \frac{F_{\text{net}}}{m} = \frac{16.8\text{N}}{10\text{kg}} = 1.68\text{m/s}^2$$



Frictional force on an inclined plane

$$F_N = mg \cos \theta$$

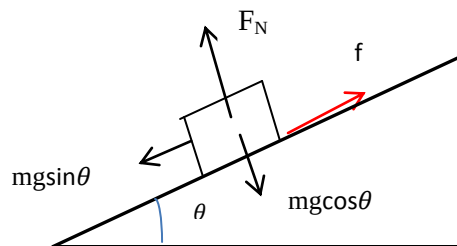
$$f = \mu F_N = \mu mg \cos \theta$$

- i. If the block accelerates downward in the absence of friction, the resultant force acts on the block is given by:

$$F_{\text{net}} = mg \sin \theta - f$$

$$F_{\text{net}} = mg \sin \theta$$

And the acceleration $a = g \sin \theta$



- ii. If there is a force of friction between the block and the inclined plane,

$$F_{\text{net}} = mg \sin \theta - f$$

$$F_{\text{net}} = mg \sin \theta - \mu mg \cos \theta$$

$$a = g (\sin \theta - \mu \cos \theta)$$

Examples

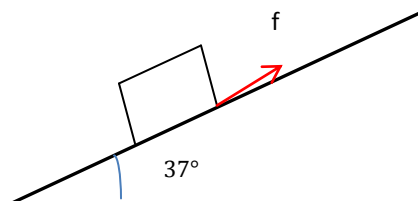
1. Look at the figure. A 20kg block is on an inclined plane at an angle of 37° .

($\sin 37^\circ = 0.6$ and $\cos 37^\circ = 0.8$)

- a. How much is the normal force?

$$F_N = mg \cos \theta = (20\text{kg}) (10\text{m/s}^2) \cos 37^\circ = 200 \times 0.8 = 160\text{N}$$

- b. What is the value of the friction force between the block



and the inclined plane if $\mu = 0.5$?

$$f = \mu F_N = (0.5) \times 160\text{N} = 80\text{N}$$

c. Neglecting friction if the block is sliding down, what will be the acceleration of the block?

$$a = g \sin \theta = 10\text{m/s}^2 \times \sin 37^\circ = 10\text{m/s}^2 \times 0.6 = 6\text{m/s}^2$$

d. If the block is sliding down with friction ($\mu = 0.5$), what will be the acceleration of the block?

$$a = g (\sin \theta - \mu \cos \theta) = 10(0.6 - (0.5) \times (0.8)) = 2\text{m/s}^2$$

Methods of reducing friction

It is impossible to avoid frictional force from a surface but we can reduce it by:

- polishing the surface
- lubrication
- using rolling instead of sliding.

Advantages and disadvantages of friction

Some of the **advantages** are:

- to walk,
- to write,
- a car to move,
- a moving car to stop .

Some of the **disadvantages** are:

- causes clothes and shoes to wear out.
- causes to wastage of energy.
- produce sound in machines.

3. Newton's third law

Newton's third law states that “*To every action there is always an equal but opposite reaction*”.

$$\mathbf{F}_A = -\mathbf{F}_R$$

If a body **A** exerts a force on body **B**, body **B** will also exerts an equal but opposite force on body **A**.

* Action and reaction forces have the following characteristics.

- equal in magnitude
- same type
- act on different bodies
- exert in pair

Application of Newton's third law

Some of the applications are:

- launching a rocket
- firing a bullet
- walking, jumping

- moving the tires of a car.

Conservation of linear momentum

Linear momentum

Linear momentum is the measure of how hard it is to stop a moving object.

Mass and velocity are the factors that affect the momentum.

$$\vec{P} = m\vec{v}$$

Momentum is vector quantity and its SI unit is kg m/s.

Examples

1. What will happen to the momentum of a body if both the mass and the velocity of the body be doubled?
→ *the momentum will increase four times.*

2. What is the momentum of a 150kg car moving at 30m/s due south?

$$\vec{P} = m\vec{v} = 150\text{kg} \times 30\text{m/s} = \mathbf{4,500\text{kgm/s}}, \text{ south.}$$

3. The linear momentum of a body moving at 18m/s is 720kg m/s. What is the mass of the body?

$$m = \frac{\vec{P}}{\vec{v}} = \frac{720\text{kgm/s}}{18\text{m/s}} = \mathbf{40\text{kg}}$$

4. A 60gram object moves with a velocity of 90m/s and a 24gram object moves with 160m/s. Which object has the greater momentum?

$$\vec{P} = m\vec{v} = 0.06\text{kg} \times 90\text{m/s} = \mathbf{5.4\text{kgm/s}} \quad \text{and} \quad \vec{P} = m\vec{v} = 0.024\text{kg} \times 160\text{m/s} = \mathbf{3.84\text{kgm/s}}$$

So, the 60g object has greater momentum.

Law of conservation of linear momentum

The law of conservation of linear momentum states that “*In a closed system the total linear momentum must remain constant*”.

Total momentum before collision = Total momentum after collision

$$\sum P_i = \sum P_f$$

$$m_1v_1 + m_2v_2 = m_1v_1 + m_2v_2$$

Examples

1. A 2kg ball moving at 40m/s collides with a stationary ball of mass 8kg. After the collision if the 2kg ball is moving at 30m/s in the same direction, what will be the velocity of second ball?

$$(2\text{kg} \times 40\text{m/s}) + 0 = (2\text{kg} \times 30\text{m/s}) + (8\text{kg} \times v)$$

$$80\text{kg m/s} = 60\text{kg m/s} + 8v$$

$$v = 2.5\text{m/s}$$

2. 1.2kg ball is moving at 35m/s towards a 5.8kg block at rest. After collision they stick and move together. What is their common velocity?

$$(1.2\text{kg} \times 35\text{m/s}) + 0 = (1.2\text{kg} + 5.8\text{kg}) \times v$$

$$42\text{kg m/s} = 7\text{kg} (v)$$

$$v = 6\text{m/s}$$

3. A car of mass 600kg is moving at a speed of 20m/s. It collides with a stationary car of mass 900kg. If the first car bounces back at 4m/s at what speed does the second car move after the collision?

$$(600\text{kg} \times 20\text{m/s}) + 0 = (600\text{kg} \times (-4\text{m/s})) + (900\text{kg} \times v)$$

$$12,000\text{kg m/s} = -2,400\text{kg m/s} + 900\text{kg} \times v$$

$$14,400\text{kg m/s} = 900\text{kg} \times v$$

$$v = 16\text{m/s}.$$

Explosion

When a gun is fired, an explosion occurs inside the gun and the bullet fires off at a high speed.

The bullet has small mass and high speed, moves to the right.

The gun has large mass and low speed, moves to the left.

The momentum of the bullet = the momentum of the gun

Examples

1. A bullet of mass 5 gram is shot with a velocity of 400m/s from a 5kg gun. What is the recoil velocity of the gun?

$$m_1v_1 + m_2v_2 = m_1v_1 + m_2v_2$$

$$0 + 0 = (0.005\text{kg} \times 400\text{m/s}) + (5\text{kg}) \times v$$

$$v = - 0.4\text{m/s}$$

The negative sign shows that the gun moves backward when the bullet moves forward.

Impulse

Impulse is the change in momentum. It is the product of force and the time for which the force acts.

$$I = \Delta p = p_f - p_i = mv - mu$$

The resultant force acting on an object is directly proportional to the rate of change of linear momentum.

$$F_R = ma = \frac{m(v-u)}{t} = \frac{\Delta p}{t}$$

$$\therefore I = \Delta p = F\Delta t$$

The SI unit of impulse is kg m/s or N s.

✓ The direction of the impulse is in the direction of the resultant force.

Examples

1. A 700g ball strikes a wall with 30m/s and bounces back with the same speed. What is the change in momentum of the ball?

$$\Delta p = m(v - u) = 0.7\text{kg}(30 - (-30)) = 420\text{kg m/s}$$

2. The velocity of a 1,800kg car increases from 4m/s to 16m/s in 8 seconds.

- What is the change in momentum of the car?
- How much is the impulse?
- What is the force impacted to the car?

Collisions

Collisions can be classified into elastic and inelastic.

- a. **Elastic collision** is a collision between two bodies where both linear momentum and kinetic energy are conserved.

$$m_1v_1 + m_2v_2 = m_1v_1 + m_2v_2$$

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

- b. **Inelastic collision** is a collision between two bodies where linear momentum is conserved but kinetic energy is not conserved. This is because some energy is converted into other form of energy during collision.

$$m_1v_1 + m_2v_2 = m_1v_1 + m_2v_2$$

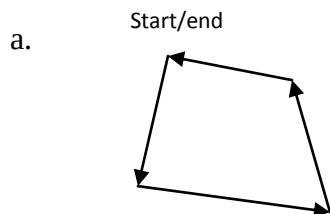
1. A 200gram ball moving at 8m/s collides with a 500gram ball moving at 3m/s in the same direction. After the collision if the 500gram ball is moving at 1m/s, what is the velocity of the second ball?

Is the collision elastic or inelastic?

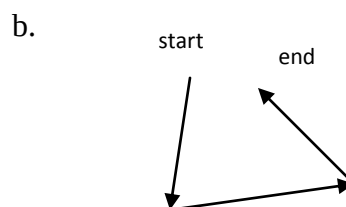
The first condition of equilibrium

- * A body is said to be in the first condition of equilibrium, the vector sum of all forces acting on the body is zero. It means that the body is at rest or moving with constant velocity.

$$\sum F_{\text{net}} = 0 \quad \text{that is } \sum F_x = 0 \text{ and } \sum F_y = 0$$



It is in equilibrium because $F_{\text{net}} = 0$



It is not in equilibrium because $F_{\text{net}} \neq 0$

Exercise

1. Look at the figure below. The mass attached with the two strings is in equilibrium.

Find the tensions in the strings. (T_1 and T_2)

