

Physics

Student Textbook
Grade 11

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Measurement and practical work

Unit 1

Contents

Section	Learning competencies
1.1 Science of measurement (page 2)	• Explain the importance of measurement. • Identify and use appropriate units for data that will be collected. • Describe what is meant by the term significant figures and how it is related to precision. • Identify rules concerning the number of significant figures that a numeral has. • Define the term scientific method. • State the steps of scientific methods. • State the uncertainty in a single measurement of a quantity. • Identify the orders of magnitude that will be appropriate and the uncertainty that may be present in the measurement of data.
1.2 Errors in measurement (page 9)	• Distinguish between random error and systematic error. • Describe sources of errors. • Identify types of errors. • Distinguish between random uncertainties and systematic errors.
1.3 Precision, accuracy and significance (page 12)	• Distinguish between precision and accuracy. • State what is meant by the degree of precision of a measuring instrument. • Use scientific calculators efficiently.
1.4 Report writing (page 17)	• Describe the procedures of report writing. • Use terminology and reporting styles appropriately and successfully to communicate information and understanding. • Present information in tabular, graphical, written and diagrammatic form. • Report concisely on experimental procedures and results.

One of the most important developments in the history of science was the scientific method, the procedure scientists use to acquire knowledge in any field of science.

Science is all about observing and experimenting. We need a framework to add relevance to observations and experiments.

Experiments may vary in size and expense, including using huge particle accelerators, making observations using space telescopes, testing simple circuits, or even just bouncing a ball! In order to be considered scientific, they must follow a key set of principles and be presented in a suitable manner. This section looks at how experiments should be represented and how the data should be analysed before drawing conclusions.



Figure 1.1 *The Hubble space telescope*

1.1 Science of measurement

By the end of this section you should be able to:

- Explain the importance of measurement.
- Identify and use appropriate units for data that will be collected.
- Describe what is meant by the term significant figures and how it is related to precision.
- Identify rules concerning the number of significant figures that a numeral has.
- Define the term scientific method.
- State the steps of scientific methods.
- State the uncertainty in a single measurement of a quantity.
- Identify the orders of magnitude that will be appropriate and the uncertainty that may be present in the measurement of data.

The scientific method

The scientific method is exceptionally important to the process of science. It ensures a rigorous, evidence-based structure where only ideas that have been carefully tested are accepted as scientific theory.

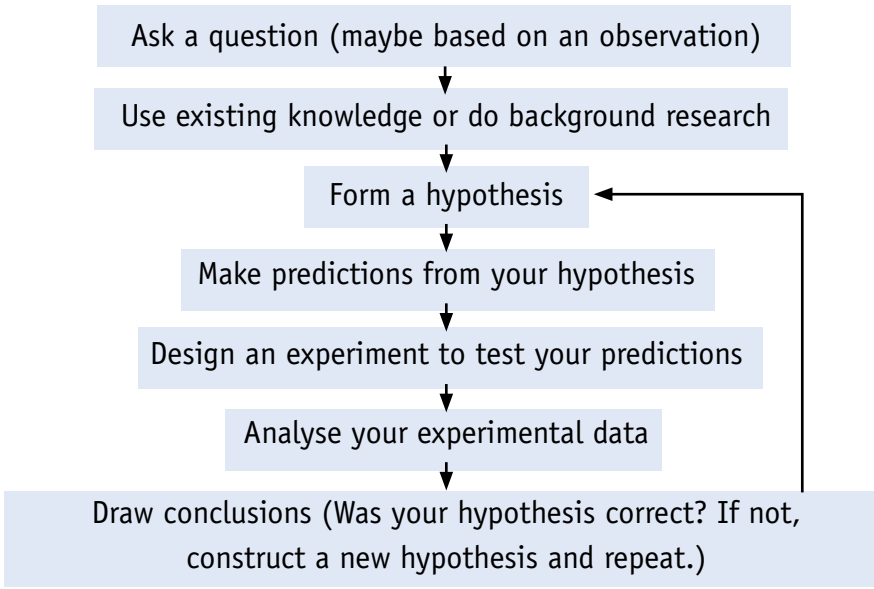


Figure 1.2 *The scientific method*

Your hypothesis will have to be tested by others before it becomes an accepted scientific theory. This process of peer review is very important and prevents scientists making up data.



Figure 1.3 *This scientist is researching ways of making energy-efficient lighting.*

The process of science begins with a question. For example: Why is the sky blue? Why does the Sun shine? Scientists are curious about the world around them; it is this curiosity that is the spark of the process.

When you have a question, scientists may have already looked into it and devised an explanation. So the first step is to complete some preliminary research into the existing theories. These theories may provide answers to your question. It is quite probable most of the questions you encounter in your physics course already have answers. However, there are still some big unanswered questions in physics. They are waiting for someone like you to answer them! Are there any questions that science cannot answer?

Using your existing knowledge or information collected via research, the next step is to form a **hypothesis**.

A hypothesis is just an idea that might provide an answer to your question. A scientific hypothesis is based on scientific knowledge, not just made up!

For example: *Why does the Sun shine?*

You might form two hypotheses:

- Nuclear fusion reactions in the Sun release heat and light.
- There is a large lamp in the centre of the Sun powered by electricity.

The first is clearly a more scientific hypothesis using some thoroughly tested existing ideas. That is not to say you shouldn't be creative in making a hypothesis but you should include some scientific reasoning behind your ideas.

It is important to note your hypothesis might be incorrect, and this is what makes science special. An investigation must be carried out to test the ideas before you either rule out an idea or accept it.

Why does the Sun shine?

Once you have a hypothesis, you should use it to form a series of predictions that can be tested through **experiment**. These may range from 'easy to test' to 'hugely complex' predictions. You then design an experiment to test your predictions.

Discussion activity

Some simple questions lead to massively complex experiments costing billions of dollars. One of these is the ATLAS project in Switzerland. This costs billions of dollars; do you think it is right to spend such a large amount of money on scientific experiments?



Figure 1.4 *Scientists investigating renewable energy*

Activity 1.1: Using the scientific method

Look around your classroom or outside. Make three observations, and using your existing scientific knowledge, form a hypothesis for each of them. Discuss these with a partner.



Figure 1.5 *We all know that the Sun shines. But why?*

KEY WORDS

hypothesis *a proposed explanation for an observation*

experiment *a test under known conditions to investigate the truth of a hypothesis*

KEY WORDS

analyse *examine in detail to discover the meaning of a set of results*

conclusions *the overall result or outcome of an experiment. The hypothesis being tested may be supported by the results or may be proven incorrect*

significant figures *the number of digits used in a measurement, regardless of the location of the decimal point*

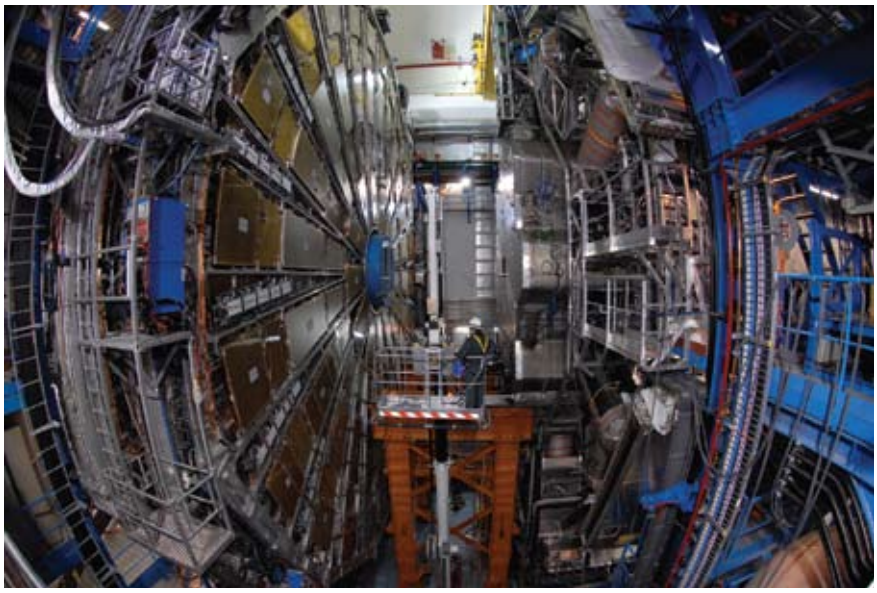


Figure 1.6 *The ATLAS detector in Switzerland*

It is important that your experiment is clearly planned. This will enable others to test your experiment and check your ideas (more detail on this can be found in Section 1.4).

Once you have carefully conducted your experiment, you will need to **analyse** your results and draw **conclusions**. At this stage you need to decide if your results support your prediction. If they do, then perhaps your hypothesis was correct. This will need to be confirmed by several other scientists before it becomes accepted as scientific fact.

If your results do not support your prediction, then perhaps your hypothesis was wrong. There is nothing wrong with that, you just go back and form a different hypothesis. This process continues and it may take years to come up with a correct hypothesis!

Activity 1.2: Choosing the right unit

In a group, discuss what units you would use when measuring each of the following:

- length of a football pitch, width of this book, diameter of a small seed, width of a finger nail.
- area of a page in this book, area of the floor in your classroom, area of a football pitch.
- volume of this book, volume of your classroom, volume of a bottle, volume of a soccer ball.

Making measurements

As part of your experiment you will have to make measurements and collect data. This process is very important and needs to be conducted carefully.

Choosing units

When you are planning your experiment, you need to choose units that are appropriate to the size of the quantity you are measuring. For example, you would measure the length of a finger in centimetres.

Explain why you have chosen the units.

Significant figures

All digits in a number that are not zero are called **significant figures**. For example, the number 523 has three significant figures and the number 0.008 has one significant figure. The zeroes in 0.008 are not significant figures but they are important as they tell you how big or small the number is.

When a number is given in standard form, the number of digits tells you how many significant figures there are. For example, 0.008 in standard form is 8×10^{3} – it's now much more obvious that the number has one significant figure.

For any measurement you take, the number of significant figures (s.f.) must be consistent with the instrument precision. For example, if you are measuring length with a 1 m ruler that has mm on it, then all readings should be expressed to the nearest mm. For example:

0.6 m ✗

0.64 m ✗

0.643 m ✓

If your reading is exactly on an increment this still applies!

0.5 m ✗

0.50 m ✗

0.500 m ✓

You must be consistent with your use of significant figures in your results tables. If your data is to two significant figures, so should be your average. For example:

Reading one: 62 *Reading two: 61* *Average: ??*

The average here should be 62, not 61.5 as this is going from two to three significant figures.

If you then go on to calculate something using your data, you must express your answer to the lowest number of significant figures in your data. For example, if you are calculating average speed you might have the following:

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

Distance travelled = 4.345 m

Time taken = 1.2 s

The distance travelled is to four significant figures but the time taken is only to two. This means your answer should only be to two significant figures:

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$\text{Average speed} = \frac{4.345\ m}{1.2\ s}$$

Average speed = 3.62083333…

Average speed = 3.6 m/s (to 2 significant figures)

Zeroes between the significant figures and the decimal point are important in showing the magnitude of the quantity, but are not significant figures.

***Table 1.1** Prefixes and their symbols*

Prefix	Power of 10	Symbol
giga-	10^{9}	G
mega-	10^{6}	M
kilo-	10^{3}	k
hecto-	10^{2}	h
deca-	10	da
deci-	10^{-1}	d
centi-	10^{-2}	c
milli-	10^{-3}	m
micro-	10^{-6}	μ
nano-	10^{-9}	n

Discussion activity

Is 100 V to one, two or three significant figures? The answer is it could be any of those! If your reading is to a whole number you may need to specify its number of significant figures.

KEY WORDS

uncertainty *the amount of doubt in a measurement*

Activity 1.3: Measuring bounce height of a ball

Collect a ball and ruler. In small groups drop the ball from various heights (around ten different heights) and record how high it bounces. Notice how difficult it is to determine the bounce height to anything less than the nearest cm.

If you were using the same ruler to measure the width of a piece of paper, you might also get 16.0 cm; however, your uncertainty would be much less than 1.0 cm! This is because the piece of paper is not moving, so measuring is easier than with the bouncing ball. Perhaps you would write 16.0 cm ± 0.1 cm.

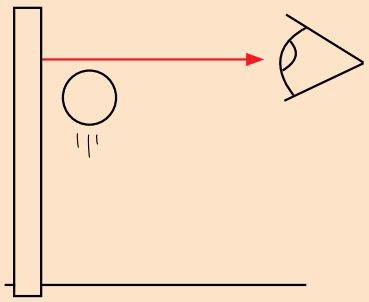


Figure 1.7 *Measuring bounce height of a ball*

Uncertainties

Every measurement you take will have an **uncertainty** associated with it. It does not mean it is wrong, it is just a measure of your confidence in your measurement. If you were measuring the height of a friend, you might write:

1.80 m

Does this mean exactly 1.8 m? Does it mean 1.8000000000000000000000000000 m? When you write 1.80 m you mean your friend's height is between 1.805 m and 1.795 m. You could write this as:

1.80 m ± 0.005 m

The 0.005 m is the uncertainty in your reading. You have measured the height to the nearest 5 mm. You should try to keep this uncertainty as small as possible (more on this in Section 1.3).

The uncertainty in every measurement will be related to the nature of the task and the precision of the instrument you are using.

For example, measuring the height of a ball bouncing is very difficult. You might measure the height using a ruler with mm on it as 16.0 cm, but what is the uncertainty?

16.0 cm ± 0.1 cm → Even though the ruler may measure to mm it is very hard to measure the height to the nearest mm as this is too small.

16.0 cm ± 0.5 cm → This is possible, if you ensure to measure the bounce height carefully; by getting down to eye level and doing a test drop it might be fair to say you can measure the height to the nearest ½ cm.

16.0 cm ± 1.0 cm → This is a realistic uncertainty for this experiment.

16.0 cm ± 2.0 cm → This might be a little unrealistic, but is also acceptable, since determining the bounce height by eye is quite difficult.

16.0 cm ± 5.0 cm → Hopefully you have designed the experiment to enable you to measure to more than the nearest 5 cm!

Which uncertainty you use is a judgement you will have to make depending on the results.

Discussion activity

Can you think of some other examples of measurements you might take using a ruler and what the uncertainty might be in each case?

Percentage uncertainties

You may need to calculate the percentage uncertainty in one of your readings. This is just the uncertainty of the reading expressed as a percentage. If you measured the current through a bulb, you might express your measurement as:

4.32 A ± 0.05 A

So the percentage uncertainty would be:

$\frac{0.05}{4.32} \times 100 = 1.157407\ldots\%$

So you would write 4.32 A ± 1.2%.

As a rule of thumb, percentage uncertainties should be to two significant figures.

You should always aim to keep your percentage error under 10%, although this may not always be possible.

Whenever possible you should measure **multiple values** instead of just one. For example, the time for 20 swings of a pendulum rather than just one, or several thicknesses of card rather than just one. This has the effect of reducing the percentage uncertainty as shown below:

One piece of card	Ten pieces of card
1.03 ± 0.05 mm → 4.9%	10.41 ± 0.05 mm → 0.48%

Notice the uncertainty is still the same but the percentages are very different.

Calculations

You need to determine the uncertainty in a quantity you have calculated. For example, when calculating resistance from values of p.d. (potential difference) and current:

p.d. = 4.32 V ± 1.157407…% *Current = 2.3 A ± 4.534641…%*

$Resistance = \frac{p.d.}{current}$

$Resistance = \frac{4.32\ V}{2.3\ A}$

Resistance = 1.9 Ω (2 s.f., as the value of the current is to 2 s.f.).

To find the percentage uncertainty in the resistance you just add the percentage uncertainties of the p.d. and current. This gives 5.692047…%, and so you would write: 1.9 Ω ± 5.7%. Do not round up until the end!

You could express the resistance (1.9 Ω ± 5.7%) as 1.9 ± 0.1 Ω. This is because 5.7% of 1.9 is 0.108...Ω and also as the resistance reading is to 1/10 of an ohm, you would write 0.1 Ω not 0.11 Ω.

This is same if you are multiplying quantities together. For example, calculating distance travelled using

distance travelled = average speed × time taken.

Activity 1.4: Percentage uncertainty for the bouncing ball

Using the data collected on the bounce height of the ball, calculate the percentage uncertainty in each case. What do you notice about the smaller readings? The larger your reading, the smaller the percentage uncertainty. So small distances have a greater percentage uncertainty than large distances, if measured with the same instrument.

KEY WORDS

multiple values *several readings of the same measurement*

Activity 1.5: Uncertainty in the dimensions of a book

Using a ruler, measure the dimensions of a book in centimetres. Write down the dimensions, the uncertainty for each, and express this as a percentage.

Use your readings to calculate the volume of the book. Calculate the percentage uncertainty in the volume and express this in cm^3.

Average speed: ± *4.1….%* *Time taken:* ± *3.4…%*

Therefore: distance travelled: ± *7.5%*

Be careful if there is a square in the equation. For example, area of a circle = πr^2. If r has a percentage uncertainty of 2.312…%, then the area will have a percentage uncertainty of 4.6% as the equation is effectively: area = $\pi \times r \times r$. So the error in r must be counted twice (2.312…% + 2.312…% = 4.624…% so 4.6% to 2 s.f.).

Summary

In this section you have learnt that:

- The scientific method includes: observing, researching, hypothesising, predicting, experimenting, analysing and concluding.
- Measurements must always be recorded to an appropriate number of significant figures (this depends on the equipment you are using).
- All measurements have an uncertainty associated with them. This is effectively a quantification of the amount of doubt in a measurement.
- To determine the uncertainty of a calculated value, you add the percentage uncertainties of the quantities used to perform the calculation.

Review questions

1. Describe each part of the scientific method. Explain why it is important to follow this structure when conducting a scientific investigation.
2. How many significant figures do the following numbers have:
   a) 258 b) 0.2 c) 12 000 d) 0.084
3. How can you reduce the percentage uncertainty in measurements that you make?
4. Nishan and Melesse have measured the voltage across a resistor to be 5.26 V and the current flowing through it to be 0.41 A. They work out the resistance.

   Nishan says that the resistance is 12.8 Ω. Melesse disagrees and says that the resistance is 13 Ω.
   Who is correct? Explain your answer.
5. A bulb is connected as part of a circuit. The following data is collected:

   Electric current: 3.2A ± 0.1 A
   Potential difference: 12.3 V ± 0.1 V

Use this data and the equation

$$\text{Resistance} = \frac{\text{potential difference}}{\text{electric current}}$$

to determine the resistance. Express the uncertainty in your answer.

1.2 Errors in measurement

By the end of this section you should be able to:

- Distinguish between random error and systematic error.
- Describe sources of errors.
- Identify types of errors.
- Distinguish between random uncertainties and systematic errors.

What are errors?

An experimental error (or just referred to as an error) is not the same as a mistake. An example of a mistake would be to measure the height of a desk when asked to measure the height of a chair. It is just plain wrong!

The measurements you take as part of your investigations will contain experimental errors, but hopefully no mistakes. Errors occur in every scientific investigation; they affect your measurements, making them different from the **accepted value** (sometimes called **true value**) of the item being measured. There are several different types of experimental error.

KEY WORD

accepted/true value *the actual value of the property being measured, made without any experimental errors*

random errors *unpredictable errors that have no pattern or bias and which may be above or below the true value*

Accepted or true value

This is the actual value of the physical property you are measuring. It is the value you would get if it were possible to make the measurement with no experimental errors.

Random error

Random errors are errors with no pattern or bias. They cause measurements to vary in an unpredictable manner. Importantly, they cause your measurements to be sometimes above the accepted value, sometimes below the accepted value.

For example, if you were measuring the acceleration due to gravity, random errors will cause your readings to vary both above and below the accepted value.

Accepted value for acceleration due to gravity = 9.80665 m/s^2 (to 6 s.f.)

Recorded values (m/s^2)						
9.81	9.78	9.65	9.87	9.80	9.86	9.83

Activity 1.6: Testing random error with a ruler

Make yourself a ruler by cutting a strip of card 15.0 cm long. Use a ruler to carefully mark on the centimetre divisions.

Use your ruler to make several different length measurements of items in your classroom. You must resist the temptation to record your readings to the nearest mm. A suitable uncertainty will be to the nearest 0.5 cm.

Repeat the experiment using a real ruler. You will find about half of your readings were too high, the other half too low. This kind of random error happens with all measurements. Even those taken with the real ruler will either be 0.5 mm too high or 0.5 mm too low.

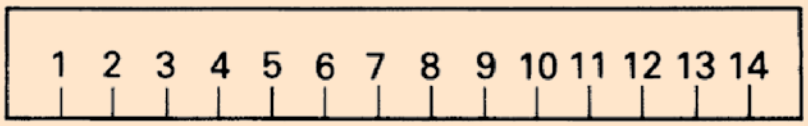


Figure 1.8 *Home-made ruler*

Another example of a random error could be encountered when completing investigations into heat. The surrounding temperature will vary depending on the time of day and general weather conditions. If you are conducting an experiment over a number of days, this will produce random errors in your measurements.

To reduce the effect of random errors, wherever possible you should take several reading and average them. The more repeats you take, the lower the impact of random errors.

Parallax errors with scales

The use of a ruler for length is not without its problems at times. If you wanted to measure the diameter of a table-tennis ball, how might you do it?

When the object and the scale lie at different distances from you, it is essential to view them from directly above if you are to avoid what we call parallax errors (Figure 1.9).

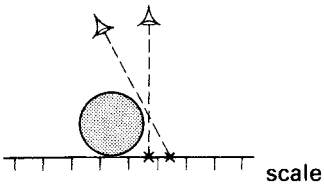


Figure 1.9 *Viewing from directly above avoids a parallax error.*

A clock in a public place has to be read from many different angles. A neat way of avoiding parallax errors in that case is shown in Figure 1.10.

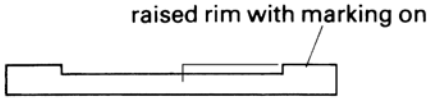


Figure 1.10 *How does this arrangement avoid parallax errors?*

With an instrument designed to be read by a single experimenter, you must take care to position your head correctly. Two ways of achieving the same thing with a current meter are shown in Figure 1.11.

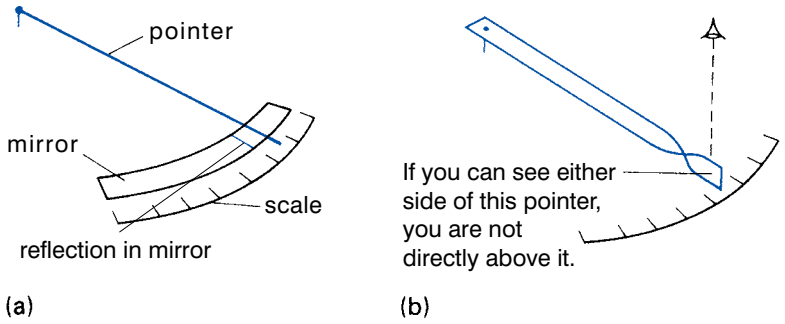


Figure 1.11 *Two ways of preventing parallax errors. (a) You know you are looking straight down on the pointer when it is hiding its own reflection in the mirror. (b) A flat pointer is twisted so it is upright at the tip.*

Returning to the question of the diameter of the table-tennis ball, two rectangular wooden blocks would help (Figure 1.12).

Discussion activity

A little thought is still needed for the best possible result. What if the blocks are not quite parallel? The doubt can be removed by measuring both ends of the gap as shown in the drawing; if the two lengths differ slightly, their average should be taken.

DID YOU KNOW?

Parallax is the name we give to an effect that you are familiar with in everyday life. As you travel along a road, objects in the distance seem to shift position relative to one another. Because of your movement, a distant house may disappear behind a nearer clump of trees, but as you travel further along it comes back into view the other side of them. That is parallax.

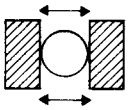


Figure 1.12 *Measuring using wooden blocks*

Figure 1.13 shows some calipers, which can do the same job as the ruler and the two blocks. They may be made of steel, and the part drawn shaded will slide along the main part. It must fit snugly, so that the shaded prong A is always at right angles to the arm B.

The arrow engraved on the sliding part indicates the diameter of the ball, on the millimetre scale. If the ball is removed and the jaws are closed, that arrow should then lie on the zero of the scale.

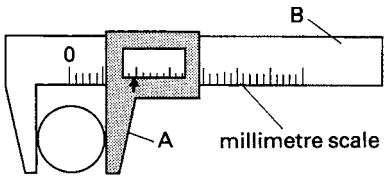


Figure 1.13 *Measuring using the Vernier calipers*

Systematic errors

A **systematic error** is a type of error that shows a bias or a trend. It makes your readings too high every time, or too low every time. Taking repeated readings will not help account for this type of error.

A simple example might be an ammeter that always reads 0.4 A too low. So if your reading was 6.8 A, the true value for the current would be 7.2 A. More complex examples include ignoring the effect of friction in Newton's second law experiments, or not measuring to the centre of mass of a simple pendulum.

The problem with systematic errors is that they can be quite hard to spot! When you have found the source you then either redesign the experiment or account for the error mathematically.

KEY WORD

systematic errors *errors caused by a bias in measurement and which show a bias or trend*

This is quite easy to do. Take for example a voltmeter where each reading is 0.2 V too large. To find the corrected value you need to subtract 0.2 V from each of your readings.

Recorded value (v)	Corrected value (v)
2.8	2.6
6.4	6.2
10.8	10.6
15.4	15.2
20.7	20.5

Zero errors

Zero errors are special examples of a systematic error. They are caused by an instrument giving a non-zero reading for a true zero value. For example, the ammeter mentioned above is a type of zero error. When the current is 0 A it would read –0.4 A.

KEY WORDS

zero errors *errors caused by equipment that has not been correctly zeroed*

Summary

In this section you have learnt that:

- Experimental errors cause readings to be different from their true value.
- Random errors cause readings to be above and below the true value.
- Systematic errors cause a bias in your readings (they are all either too high or too low).
- Parallax errors can cause your readings to be less accurate because of the position of your eye.
- A zero error is a type of systematic error caused by equipment not being zeroed properly.

Review questions

1. Explain the meaning of the term error.
2. Describe different types of errors, give examples, and explain how the effect of these errors might be reduced.

1.3 Precision, accuracy and significance

By the end of this section you should be able to:

- Distinguish between precision and accuracy.
- State what is meant by the degree of precision of a measuring instrument.
- Use scientific calculators efficiently.

What does 'accurate' mean?

Accuracy means how close a reading is to the true value. The more accurate a reading, the closer it is to the true value.

Again using the acceleration due to gravity as an example:

Accepted value for acceleration due to gravity = 9.80665 m/s^2 (to 6 s.f.)

If you took three readings you might get:

9.76 9.87 9.82

The most accurate reading is the third one; it is closest to the true value.

In order to obtain more accurate measurements you must ensure you have minimised random errors, taken into account systematic errors and conducted the experiment as carefully as you can.

KEY WORDS

accuracy *the closeness of a measurement to its true value*

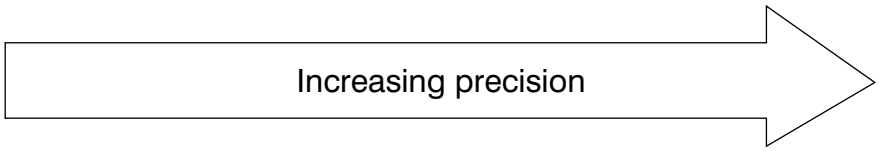
precision *the quality of being exact and the degree to which repeated measurements under the same conditions give the same value*

significance *the number of significant figures used in a reading, which should be appropriate to the precision of the measuring instrument*

Precision and significance

The **precision** of your reading is a measure of the degree of 'exactness' of your value; this is sometimes related to the number of significant figures in the reading. The more precise a reading is, the smaller the uncertainty. A series of precise measurements will have very little variation; they will all be very similar.

For example, dealing with lengths:



1.0 m	1.00 m	1.000 m
± 0.1 m	± 0.01 m	± 0.001 m

The **significance** of your reading is indicated by the number of significant figures you can express your data to. This was discussed in Section 1.1

It is very important not to overstate the significance of your readings. If your ruler measures to mm, then your readings should be to mm; it would not be right to give a length of 1.2756 m.

This is particularly true when calculating values. Take, for example, calculating the resistance of a light bulb:

$$\text{Resistance} = \frac{\text{potential difference}}{\text{electric current}}$$

Potential difference = 10.0 V ± 0.1 V (so a 1.0% uncertainty)

Current = 3.0 A ± 0.1 A (so a 3.3% uncertainty)

This shows a series of imprecise measurements, they are all quite spread out. In addition, the readings are inaccurate as they are not clustered around the true value. This is what you want to try to avoid!

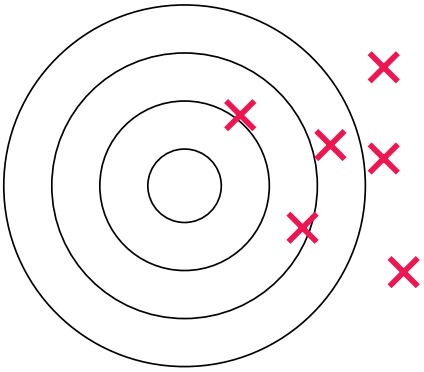


Figure 1.14 *Imprecise and inaccurate*

Put this into your calculator and you would get:

$$\text{Resistance} = \frac{\text{potential difference}}{\text{electric current}}$$

$$\text{Resistance} = \frac{10.0\ \text{V}}{3.0\ \text{A}}$$

Resistance = 3.333333333 Ω

As previously discussed you would answer 3.3 Ω to two significant figures, but why did the calculator give 3.3333333?

The answer is to do with how calculators treat values. When you enter 10.0 V, you mean 10.0 V ± 0.1 V but the calculator takes the value to be exactly 10. That is 10.0000000000000000…. The same sort of thing is true for your current reading.

To express the resistance as 3.333333333 Ω would be wrong. It implies the reading is more precise than it actually is.

Accurate and precise

Accuracy and precision are often confused. A common analogy to help overcome this involves using a target. The centre of the target represents the true value and each shot represents a measurement.

This shows a series of precise measurements; there is very little variation in the readings. However, they are also inaccurate as they are quite far from the true value (centre). A systematic error may give these kinds of results.

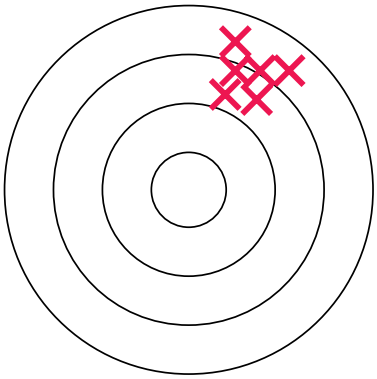


Figure 1.15 *Precise but inaccurate*

Here the measurements are imprecise as there is quite a large spread of readings, but at the same time they are accurate (they are all gathered around the true value). A large random error may cause this.

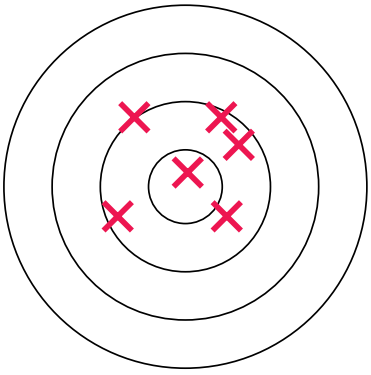


Figure 1.16 *Accurate but imprecise*

This is what we are aiming for! High precision, little spread from readings and all close to the true value.

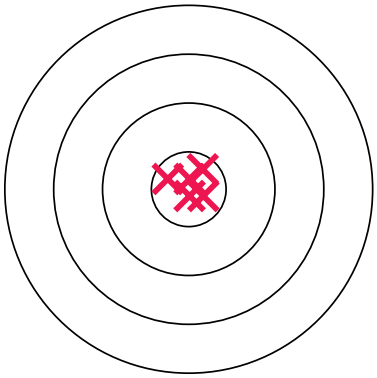


Figure 1.17 *Accurate and precise*

Instrument precision

The precision of an instrument is given by the smallest scale division on the instrument. A normal ruler may have a precision of 1 mm but a screw gauge micrometer has a precision of 0.01 mm.

When you are taking single readings, the precision of the piece of equipment you are using usually determines the uncertainty. For example, if you are using an ammeter with a precision of 0.01 A, then your readings might be:

$$0.32 \pm 0.01 \text{ A or } 2.61 \pm 0.01 \text{ A}$$

An exception to this rule would be if the nature of the task meant that there are other random errors that produce an uncertainty greater than the precision of the equipment.

For example, the bouncing ball experiment described earlier, or measuring the time of a pendulum swinging. A stopwatch may have a precision of ± 0.001 s but your reaction time is much greater. As a result it might be better to express the uncertainty as ± 0.1 s.

However, when you are taking multiple readings, for example recording p.d. with a voltmeter which has a precision of ± 0.01 V, you may obtain the following:

4.32 V 4.36 V 4.27 V

The average would be 4.32 V (to 3 s.f., as the other readings). To express this average as 4.32 ± 0.01 V would not be right as you can tell by looking at your results the variation is more than ± 0.01 V.

In this case you would use half the range as your uncertainty. In the example above, the range from 4.27 V to 4.36 V is 0.09 V, so therefore half this range is 0.05 V. The average reading would be written as 4.32 ± 0.05 V. If you have no variation in your repeats, then you would use the precision of the instrument as the uncertainty.

Solving physics problems

When you are solving physics problems you need to make sure that you use the correct units. For example, one length may be given in feet and another in metres. You need to convert the length in feet to metres using a **conversion factor**. When you do the calculation, check that the units are correct using dimensional analysis.

If there are any intermediate stages in the calculation, keep all of the figures on your calculator screen for later stages in the calculation. You should only round your calculation to the appropriate number of significant figures at the end of the calculation.

Always check your calculations, because it is easy to make mistakes. Check that you are using the conversion factor correctly by making sure that when the units cancel, you are left with the units that you think you should have.

Activity 1.7: Determining instrument precision for different instruments

Look at a range of different pieces of measuring equipment. Determine the instrument precision in each case.

Activity 1.8: Uncertainty in the swing of a pendulum

Using a piece of string and some plasticine, make a simple pendulum. Working in pairs, time how long it takes to complete one swing for various different lengths. Repeat this three times for each length.

Calculate the average time of one swing for each length, and determine the uncertainty in this reading.

Discussion activity

How could you reduce the percentage error in the timing of the pendulum experiment?

KEY WORDS

conversion factor *a numerical factor used to multiply or divide a quantity when converting from one system of units to another*

Remember that when you are talking about an order of magnitude for an answer, this is much less precise than saying that it is approximately equal to something. For example, saying that N is ~ (is about) 10^{25} implies that N is in the range 10^{24} to 10^{26}, but saying that $N \approx 10^{25}$ implies that N is in the range, say 9×10^{24} to 1.1×10^{25}. The latter answer is much more precise.

Worked example 1.1

Berihun walks 3000 feet in 10 minutes. What speed is he walking at? Give your answer in metres per second.

First you need to convert the distance to metres and the time into seconds.

Conversion factor for feet to metres = 1 metre/3.28 feet

Distance = 3000 feet × 1 metre/3.28 feet

= 914.634 14 metres

Remember to keep all the digits from the conversion for the next stage in your calculation.

Time = 10 minutes = 10 × 60 seconds = 600 seconds

Speed = distance ÷ time

= 914.634 14 metres ÷ 600 seconds = 1.524 390 2 m/s

The greatest number of significant figures is three in the conversion factor, so your answer should be given to three significant figures.

Speed = 1.52 m/s

Summary

In this section you have learnt that:

- Accuracy is a measure of how close a measurement is to the true value of the quality being measured.
- Precision is a measure of the degree of 'exactness' of your value.
- A series of precise measurements will have very little variation.
- The significance of a measurement is indicated by the number of significant figures in your value.
- The precision of an instrument is usually given by the smallest scale division on the instrument. More precise instruments have smaller scale divisions.

Review questions

1. Explain the terms accuracy and precision. Describe how they differ using examples of experiments that you might conduct.
2. Research the precision of a range of instruments in your classroom.
3. Dahnay is 167 cm tall. Abeba is 66 inches tall. Who is taller and by how much.

   The conversion factor for inches to centimetres is 2.54 cm/1 inch.
4. a) What difference would it make to the answer in the worked example if you rounded the answer to the intermediate step to 3 significant figures?

   b) What effect do you think rounding the answer to each step might have in a calculation with several intermediate steps?

1.4 Report writing

By the end of this section you should be able to:

- Describe the procedures of report writing.
- Use terminology and reporting styles appropriately and successfully to communicate information and understanding.
- Present information in tabular, graphical, written and diagrammatic form.
- Report concisely on experimental procedures and results.

Presenting information

Science is a collaborative process. As discussed in the first section, all ideas must be independently tested and verified. It is therefore important to ensure that when you write up reports or write up experiments, you do so carefully.

Your results should be recorded in a clear and organised manner. This will usually be in a tabular format. Your tables should include all your raw data, including repeated readings and, where appropriate, columns for processed data (averages, calculations of resistance, etc). It is up to you if you wish to include clearly incorrect readings in your table, or simply repeat the reading.

Column headings must be labelled with a quantity and unit.
You should use the standard convention for this: Quantity (unit).
For example: Time (s) or Mass (kg).

A sample table can be seen below:

Length (m)	**Current (A)**			
	1st Set	2nd Set	3rd Set	Average

KEY WORDS

framework *an outline structure that can be used as the basis for a report*

A framework for writing up reports

There are lots of different ways to write up scientific investigations. Your teacher may have some suggestions.

A sample 10-point **framework** can be seen here. You might not always include every section for every experiment you write up, but instead just focus on three or four of the sections.

1 Title (and date)

2 Aim

- What theory are you going to test? Or what are you going to investigate, and why? Or what are you going to measure?

3 Theory

- Explain the theory behind your experiment, with all the equations set out and explained clearly.
- If you are going to plot a graph to find a quantity, explain how the graph will enable you to do this.
- You should then be able to refer back to/quote from this section in your method and in your analysis of results.

4 Diagram(s) of experimental arrangement(s)

- These should be BIG (don't be afraid to take up a full page), detailed and fully labelled, and showing how the experiment works.

5 Method

- Don't repeat information that is already in the diagram!
- Give a clear, detailed and step-by-step procedure (bullet-point list), including measurements to be taken, any repeats,

AND:

- Accuracy
- How did you ensure the accuracy of your measurements? (Any zero errors, was the experiment horizontal, etc.)
- How did you choose appropriate instruments to give readings to an appropriate precision? Mention the precision and range of key instruments used. For example, 1 m rule with a 1 mm scale, or a 0–10 A ammeter reading to 0.01 A.
- Did you do repeats? Were you looking from the correct angle when making measurements, etc?

6 Results

- Neatly set out ALL data/measurements recorded in a neat table, and averages (if applicable).
- Don't forget headings/explanations of each table, and don't forget the units either.

7 Analysis

- What does your data show?
- Draw large graphs (with suitable scales, so that points take up a least half of the paper) on graph paper, with titles, labelled axes (units), and best-fit (not necessarily straight) smooth lines drawn through the points.
- Describe what your graphs show.
- When you use information from your graph(s) explain what you are using – e.g. gradient, area, etc.

8 Error/uncertainty analysis

- Identify all possible sources of error in your measurements. Distinguish between random and systematic uncertainties.
- Quantify the uncertainty of these (e.g. using ½-range or instrument precision (see Section 1.3). Express the uncertainty as a percentage for important readings.
- Use these to estimate the uncertainty in your final results.

9 Conclusions

- This should refer back to the aim – i.e. can you answer the question implied by the aim?
- If measuring something, quote final value with experimental uncertainty. If there is an accepted value, comment on the difference between your value and the accepted value.
- Does your experimental data fit the theory within experimental uncertainty?

10 Evaluation

- How could you improve the experiment – how could you improve the reliability? Be specific and realistic – just saying 'be more careful' or 'use better equipment' is not enough.

Summary

In this section you have learnt that:

- Your experimental results should be recorded in a clear and organised manner.
- Experimental results are usually recorded in tabular form.
- You can use a 10-point framework to write up scientific investigations.

End of unit questions

1. Explain the importance of the scientific method.
2. Construct a glossary of all the key terms used in this unit.
3. Use the writing frame on pages 19–20 and complete all sections. Carry out a detailed investigation into one of the following:
   a) How the height a ball is dropped from affects the height it bounces up to.
   b) How the length of a piece of wire affects the electric current passing through it.
   c) How the angle of slope affects the time taken for a ball to roll down the slope.
4. For the two activities in question 3 that you did not carry out, identify:
   a) Possible sources of error.
   b) The sizes of the uncertainties in the measurements you would take.
5. Makeda says that you should write down all the numbers on the calculator display when recording the final result of a calculation. Is Makeda correct? Explain your answer.

Vector quantities

Unit 2

Contents

Section	Learning competencies
2.1 Types of vector (page 22)	• Demonstrate an understanding of the difference between scalars and vectors and give common examples. • Explain what a position vector is. • Use vector notation and arrow representation of a vector. • Specify the unit vector in the direction of a given vector.
2.2 Resolution of vectors (page 26)	• Determine the magnitude and direction of the resolution of two or more vectors using Pythagoras's theorem and trigonometry.
2.3 Vector addition and subtraction (page 27)	• Add vectors by graphical representation to determine a resultant. • Add/subtract two or more vectors by the vector addition rule.
2.4 Multiplication of vectors (page 33)	• Use the geometric definition of the scalar product to calculate the scalar product of two given vectors. • Use the scalar product to determine projection of a vector onto another vector. • Test two given vectors for orthogonality. • Use the vector product to test for collinear vectors. • Explain the use of knowledge of vectors in understanding natural phenomena.

You have studies vectors in grade 9. An understanding of vectors is essential for an understanding of physics.

They help physicists and engineers to build amazing structures and to design spacecraft, and they also help you find your way home!

2.1 Types of vector

By the end of this section you should be able to:

- Demonstrate an understanding of the difference between scalars and vectors and give common examples.
- Explain what a position vector is.
- Use vector notation and arrow representation of a vector.
- Specify the unit vector in the direction of a given vector.

Introduction and recap of basic vectors

All physical quantities are either scalar or vector quantities:

- A vector quantity has both **magnitude** (size) and direction.
- A **scalar** quantity has magnitude only.

All vector quantities have a direction associated with them. For example, a displacement of 6 km to the West, or an acceleration of 9.81 m/s^2 down. Scalars are just a magnitude; for example, a mass of 70 kg or an energy of 600 J.

Table 2.1 *Some examples of vector and scalar quantities*

Vector quantities	Scalar quantities
Forces (including weight)	Distance
Displacement	Speed
Velocity	Mass
Acceleration	Energy
Momentum	Temperature

Representing vectors

All vector quantities must include a direction. For example, a displacement of 8 km would not be enough information. We must write 8 km South.

Vectors can be represented by arrows, the magnitude (size) of the vector is shown by the length of the arrow. The direction of the arrow represents the direction of the vector.

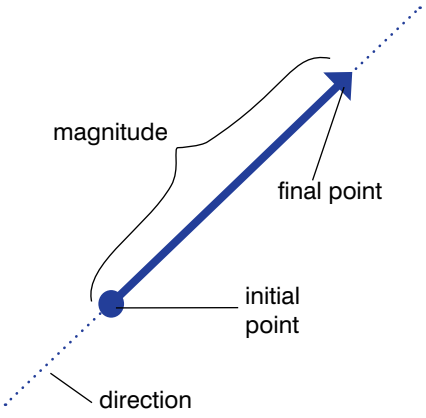


Figure 2.2 *A vector has size (magnitude) and direction.*

Representing vectors as arrows

Vectors are sometimes written in lowercase boldface, for example, **a** or ***a***. If the vector represents a displacement from point A to point B, it can also be denoted as:

$\overrightarrow{AB}$

KEY WORDS

magnitude *the size of a value*

scalar *a quantity specified only by its magnitude*

Discussion activity

Which of the following do you think are scalars and which are vectors? Electric current, moment, time, potential difference, resistance, volume, air resistance and charge.

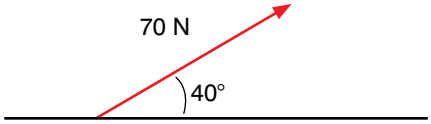


Figure 2.1 *An arrow representing a force of 70 N at about 40° to the horizontal. Is this a vector or a scalar?*

DID YOU KNOW?

In 1881 vectors appeared in a publication called *Vector Analysis* by the American J. W. Gibbs. They have been essential to maths and physics ever since!

Activity 2.1: Drawing vector diagrams

Draw four vector arrows for the following (you will need to use different scales):

- 140 km North
- 2.2 N left
- 9.81 m/s^2 down
- 87 m/s at an angle of 75° to the horizontal.

KEY WORDS

collinear vectors *vectors that are parallel to each other and which act along the same line*

coplanar vectors *vectors that act in the same two-dimensional plane*

position vector *a vector that represents the position of an object in relation to another point*

unit vector *a vector with a length of one unit*

Types of vector

There are several different types of vector to consider. These are outlined below.

Position vector

A **position vector** represents the position of an object in relation to another point.

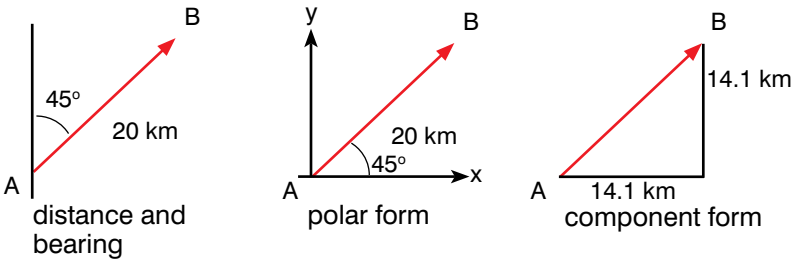


Figure 2.3 *Position vectors*

B is 20 km North East of A. Alternatively this could be written as B is 20 km from A on a bearing of 45°. Remember that bearings are specified as an angle going clockwise from north. The vector can be given in polar form. The angle is given from the positive x-axis, going anticlockwise. The angle can be in degrees or radians. The vector B from A is: B = (20, 45°)

The vector can also be given in component form, where it is given in terms of the components in the x, y and z directions. The vector B is: B = (14.1 km, 14.1 km, 0 km)

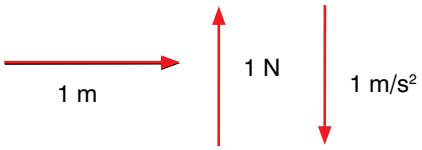


Figure 2.4 *Unit vectors*

Unit vector

A **unit vector** is a vector with a length equal to one unit. For example, Figure 2.4 contains three examples of unit vectors, one each for displacement, force and acceleration.

Unit vectors can also have direction. There are three unit vectors which are used to specify direction, as shown in Figure 2.5:

- unit vector **i** is 1 unit in the x-direction
- unit vector **j** is 1 unit in the y-direction
- unit vector **k** is 1 unit in the z-direction.

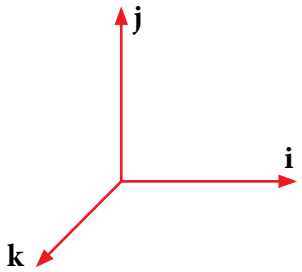


Figure 2.5 *Unit vectors* **i**, **j** *and* **k**

Collinear vector

Collinear vectors are vectors limited to only one dimension. Two vectors are said to be collinear if they are parallel to each other and act along the same line. They can be in the same direction or opposite directions.

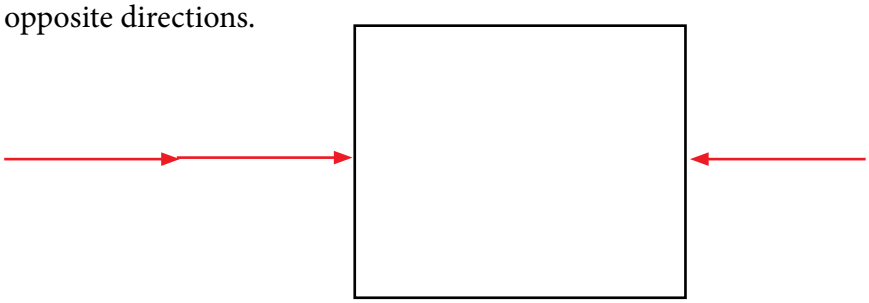


Figure 2.6 *These three vectors are collinear*

Coplanar vector

This refers to vectors in the same two-dimensional plane. This may include vectors at different angles to each other. For example, Figure 2.7 shows two displacement vectors when viewed from above.

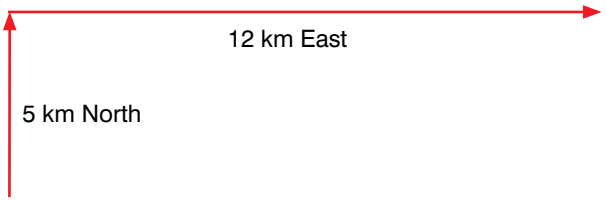


Figure 2.7 *Coplanar displacement vectors*

A more complex example might involve three forces acting on a cube.

A and B are both in the same plane (the *xy* plane) so they might be described as coplanar. C is in a different plane and so is not coplanar.

B and C are in the same plane (the *xz* plane) so they might be described as coplanar. A is in a different plane and so is not coplanar.

A and C are in the same plane (the *yz* plane) so they might be described as coplanar. B is in a different plane and so is not coplanar.

A, B and C cannot be considered to be coplanar with each other as they are in different planes.

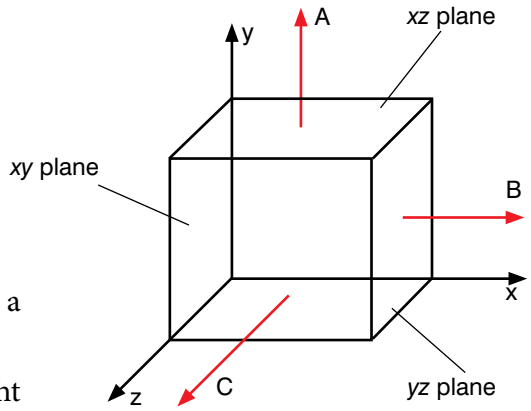


Figure 2.8 *Three vector forces acting on a cube*

Summary

In this section you have learnt that:

- A vector quantity has both magnitude (size) and direction.
- A scalar quantity has magnitude only.
- Vectors are often represented by arrows.
- Different types of vectors include position vectors, unit vectors, collinear vectors (along the same line) and coplanar vectors (in the same two-dimensional plane).

Review questions

1. Define the terms vector and scalar. Give five examples of each.
2. Explain the differences and similarities between position vectors, unit vectors, collinear vectors and coplanar vectors. Give examples for each.

KEY WORDS

component vectors *two or more vectors that, when combined, can be expressed as a single resultant vector*

resolving *splitting one vector into two parts that, when combined, have the same effect as the original vector*

2.2 Resolution of vectors

By the end of this section you should be able to:

- Determine the magnitude and direction of the resolution of two or more vectors using Pythagoras's theorem and trigonometry.

What is resolution?

Resolving means splitting one vector into two **component vectors**. This may be a component in the *x* direction (horizontal) and another in the *y* direction (vertical). The two components have the same effect as the original vector when combined.

An example can be seen in Figure 2.9, the velocity of 25.0 m/s can be resolved into two component vectors that, when combined, have the same effect.

The component vectors can be made to form the sides of a right-angled triangle. They make up the opposite and adjacent sides of the triangle.

As we know the size of the hypotenuse (in this case 25.0 m/s) and the angle (in this case 65°), we can then use trigonometry to find their relative sizes.

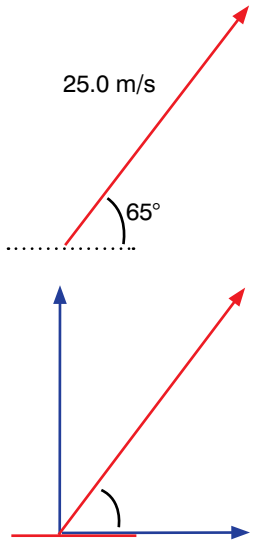


Figure 2.9 *Component vectors of the main vector are shown in blue*

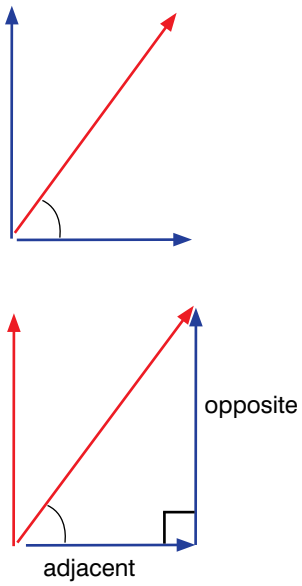


Figure 2.10 *Component vectors as a right-angled triangle*

Using trigonometry to resolve vectors

You will probably remember the following rules from your maths class:

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

In the case of Figure 2.9:

- hypotenuse × sin θ = opposite
- 25.0 m/s × sin 65 = 22.7 m/s in the *y* direction
- hypotenuse × cos θ = adjacent
- 25.0 m/s × cos 65 = 10.6 m/s in the *x* direction

You can check your working by using Pythagoras's theorem to recombine the vectors.

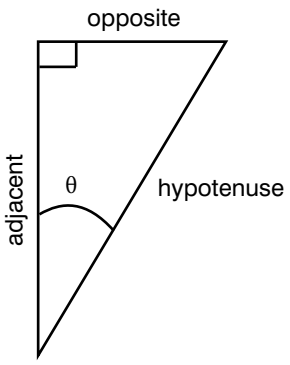


Figure 2.11 *The basic rules of trigonometry*

Pythagoras's theorem

For a right-angled triangle, Pythagoras's theorem states:

$a^2 = b^2 + c^2$

We can use this to work out the magnitude of two coplanar vectors. We can use trigonometry to work out the direction of the two vectors.

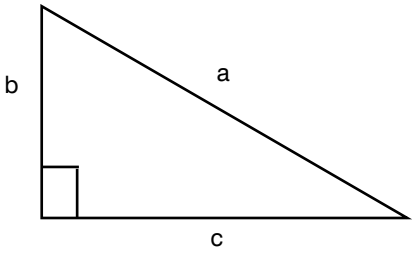


Figure 2.12 *A right-angled triangle demonstrates Pythagoras's theorem*

Worked example 2.1

What is the a) magnitude and b) direction of the two coplanar vectors in Figure 2.13?

a) $D^2 = 3^2 + 4^2 = 9 + 16 = 25$

$D = \sqrt{25} = 5$ m

b) $\tan \theta$ = opposite/adjacent = 4/3 = 1.333 ...

$\theta = \tan^{-1} 1.333 \ldots = 53°$

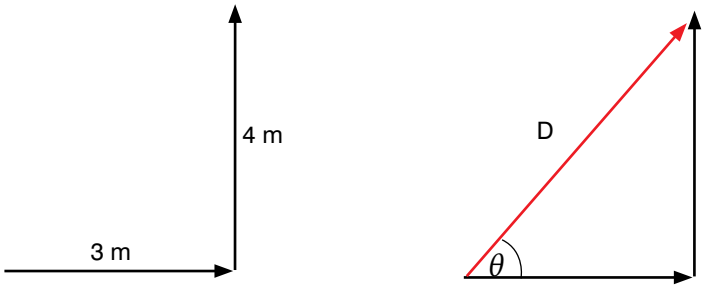


Figure 2.13 *Two perpendicular coplanar vectors form a right-angled triangle.*

Summary

In this section you have learnt that:

- Resolving means splitting a vector into two perpendicular components.
- The components have the same effect as the original vector.
- Trigonometry can be used to determine the magnitude of the components.
- Vectors can be added mathematically using Pythagoras's theorem and trigonometry.

Review questions

1. Explain what it means to resolve a vector.
2. Draw simple vector diagrams and resolve them into two components.
   a) 60 N at an angle of 30° from the horizontal.
   b) 45 m/s at an angle of 80° from the horizontal.
   c) 1900 km at an angle of 40° from the vertical.

DID YOU KNOW?

Vectors are used in computer games to determine the movement of a character. Software will convert the commands from the games controller into a three-dimensional vector to describe how the character should move.

Activity 2.2: Drawing scale diagrams

Draw a scale diagram to find the resultant displacement from the following:

- 12 km at an angle of 0° to the vertical
- 24 km at an angle of 90° to the vertical
- 6 km at an angle of 120° to the vertical
- 30 km at an angle of 210° to the vertical

2.3 Vector addition and subtraction

By the end of this section you should be able to:

- Calculate vectors by graphical and mathematical methods.
- Appreciate the parallelogram rule and the triangle rule.
- Solve more complex examples of vectors.

Adding vectors

It is often necessary to add up vectors to find the resultant vector acting on a body. This may be the resultant velocity of an object, the resultant force acting on an object, or even the resultant displacement after several legs of a journey.

Vector diagrams

The first technique for vector addition involves carefully drawing diagrams. This can only be applied to collinear or coplanar vectors (this is because your diagrams will be two-dimensional only!).

There are three slightly different techniques that could be used.

Scale diagrams

Whenever you are drawing vector diagrams you should draw them to a scale of your own devising. Scale diagrams are very simple:

- Select a scale for your vectors.
- Draw them to scale, one after the other (in any order), lining them up head to tail ensuring the directions are correct.
- The resultant will then be the arrow drawn from the start of the first vector to the tip of the last.

For example:

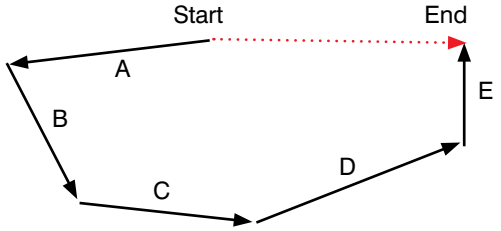


Figure 2.14 *Scale diagram showing a resultant vector (the red arrow) for a series of coplanar vectors*

It does not matter in which order you draw your vectors. Check them for yourself!



Figure 2.15 *Adding three collinear vectors*

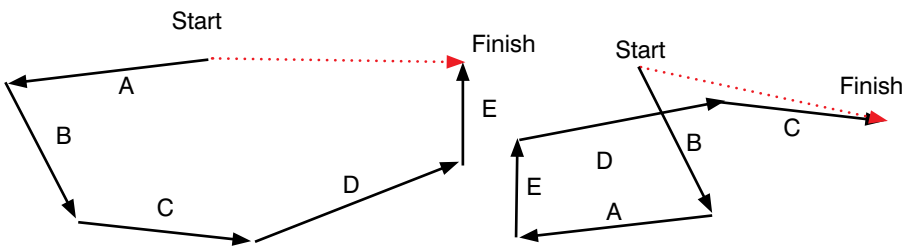


Figure 2.16 *The resultant vector remains the same*

If you end up where you started, then all the vectors cancel out and there is no resultant vector.

The fact that you can add vectors in any order and get the same resultant vector is called the **commutative law**.

Parallelogram rule

If you have two coplanar vectors, you could use the parallelogram rule. This involves drawing the two vectors with the same starting point. The two vectors must be drawn to a scale and are made to be the sides of the parallelogram. The resultant will be the diagonal of the parallelogram.



Figure 2.18 *Two perpendicular coplanar vectors*

If the vectors are perpendicular, the parallelogram will always be a rectangle.

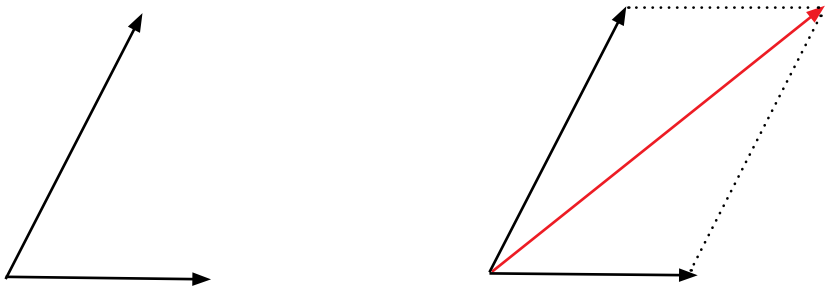


Figure 2.19 *Two non-perpendicular coplanar vectors*

If the vectors are still coplanar but not perpendicular, the parallelogram will not be a rectangle.

Triangle rule

This is a very similar technique, it involves drawing the two coplanar vectors but this time drawing them head to tail. The two vectors must again be drawn to a scale. The resultant will be the missing side from the triangle.

If the vectors are perpendicular, the triangle will be a right-angled triangle.



Figure 2.20 *Two perpendicular coplanar vectors*

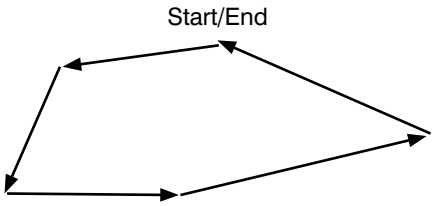


Figure 2.17 *Scale diagram showing no resultant force*

KEY WORDS

commutative law *a process obeys the commutative law when it does not matter which order the quantities are in. For example, the addition of numbers obeys the commutative law.*

Activity 2.3: Finding an unknown force

There are three forces acting on an object, A, B and C. This object is at equilibrium (there is no resultant force acting on it). Draw a scale diagram to find the magnitude and direction of the unknown force.

- Force A, 45 N at an angle of 0° to the horizontal
- Force B, 30 N at an angle of 300° to the vertical
- Force C, unknown

Discussion activity

If the triangle is a right-angled triangle, we could use trigonometry to determine the sides and angles mathematically. What if the triangle is not a right-angled triangle?

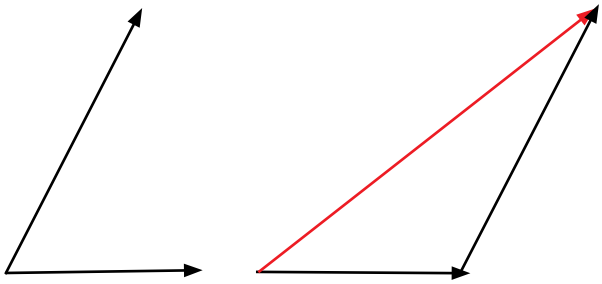


Figure 2.21 *Two non-perpendicular coplanar vectors*

If the vectors are still coplanar but not perpendicular the triangle will not be a right-angled triangle.

Activity 2.4: Determining resultant velocity

Use the parallelogram rule to determine the resultant velocity of the following two velocities:

- Velocity A, 30 m/s at an angle of 45° to the horizontal.
- Velocity B, 40 m/s at an angle of 80° to the horizontal.

Repeat, this time use the triangle rule.

Activity 2.5: Adding vectors

Vector A is 6 m/s along the horizontal and vector B is 9 m/s at 90° to the horizontal.

- Add vector B to vector A.
- Add vector A to vector B.

Does the order you add the vectors make any difference to the resultant?

Adding coplanar vectors mathematically

To add coplanar vectors we use more complex mathematics.

Since two perpendicular coplanar vectors form a right angled-triangle, they can be added using Pythagoras's theorem and trigonometry. Pythagoras's theorem determines the magnitude, and trigonometry can be used to determine the direction.

Component method

We can also express vectors as components in the x, y and z directions.

The resultant vector shown in Figure 2.13 can be expressed in component form as (3, 4), where the first number is the magnitude in the x-direction and the second number is the magnitude in the y-direction. The vector can also be given in the column form $\begin{bmatrix}3\\4\end{bmatrix}$.

We can also add and subtract vectors in this form.

Activity 2.6: Adding forces

You are going to pull on a block of wood with two forces. You will find the resultant of the two forces, and then check your findings by adding the vectors.

- Find a suitable block of wood and three forcemeters (newtonmeters or spring balances). Place the block on a sheet of plain paper.
- Attach two of the forcemeters (A and B) to one end of the block as shown in Figure 2.22. Attach the third (C) to the opposite end.
- One person pulls on each forcemeter. A and B should be at an angle of 90° to each other. C is in the opposite direction. Pull the forcemeters so that their effects balance.
- On the paper, record the magnitudes and directions of the three forces.
- Now add forces A and B together using vector addition.

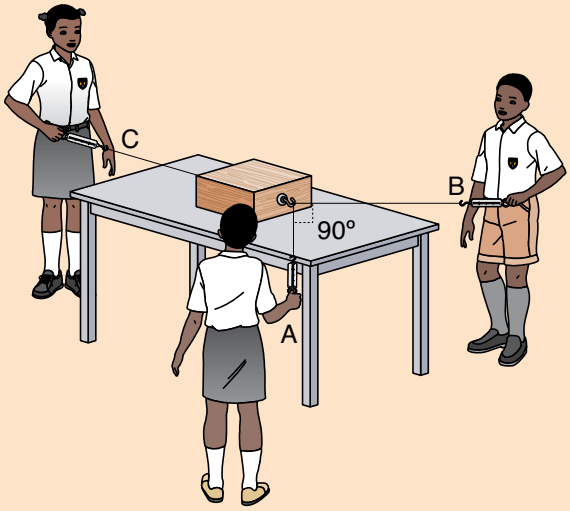


Figure 2.22 *Testing vector addition of forces*

- Because force C balances forces A and B, it must be equal and opposite to the resultant of A and B. Did you find this?
- Repeat the experiment with different forces at a different angle.

Worked example 2.2

Nishan and Melesse are trying to drag a box. Nishan is using a force of (15, 8, 6) N and Melesse is using a force of (12, 10, –6) N.

What is the resultant force on the box?

First draw a diagram to show the forces.

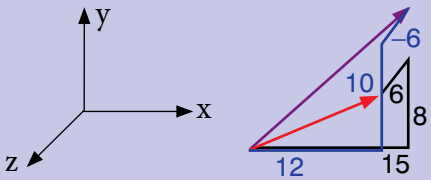


Figure 2.23 *Forces acting on the box*

Add the components of the forces together using:

$F_A + F_B = (A_x + B_x, A_y + B_y, A_x + B_z)$

$F_A = (15, 8, 6)$ N

$F_B = (12, 10, -6)$ N

So $F_A + F_B = (15 + 12, 8 + 10, 6 + - 6)$ N

$= (27, 18, 0)$ N

Worked example 2.3

We can also use the cosine rule and the sine rule to work out the magnitude and direction of the sum of two vectors.

Consider two vectors **a** (5 m along horizontal) and **b** (6 m at an angle of 60° above the horizontal).

First draw a diagram showing the vector addition.

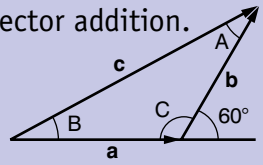


Figure 2.24

We can use the cosine rule to calculate the magnitude of the resultant using

$c = \sqrt{(a^2 + b^2 - 2ab \cos C)}$

where a and b are the magnitudes of the two vectors and c is the magnitudes of the resultant.

We can then use the sine rule to calculate the angle B of the resultant vector c

$\sin B = (b \sin C)/c$

From the diagram, we can see that angle C is 180°– 60° = 120°

Substituting the values in to the equation to find the magnitude of c:

$c = \sqrt{(5^2 + 6^2 - 2 \times 5 \times 6 \cos 120°)}$
$= \sqrt{(25 + 36 - 2 \times 5 \times 6 \times -0.5)}$
$= \sqrt{(25 + 36 + 30)}$
$= \sqrt{91}$
$= 9.54$ m

Substituting into the equation to find the angle

$\sin B = (6 \times \sin 120°)/9.54$
$= 6 \times 0.866/9.54$
$= 0.545$
so $B = \sin^{-1} 0.545 = 33.0°$

So **a** + **b** = 9.54 m at an angle of 33.0° to the horizontal

Summary

In this section you have learnt that:

- Vectors can be added graphically by drawing scale diagrams.
- Vectors can be expressed as components.

Review questions

1. Vector **p** is 6 m in the x-direction. Vector **q** is 10 m in the y-direction.
   a) Use the parallelogram method to work out **p** + **q**.
   b) Use Pythagoras's theorem and trigonometry to work out **p** – **q**.
2. A car travels 3 km due North then 5 km East. Represent these displacements graphically and determine the resultant displacement.
3. Two forces, one of 12 N and another of 24 N, act on a body in such a way that they make an angle of 90° with each other. Find the resultant of the two forces.
4. Two cars A and B are moving along a straight road in the same direction with velocities of 25 km/h and 30 km/h, respectively. Find the velocity of car B relative to car A.
5. Two aircraft P and Q are flying at the same speed, 300 m s^{-1}. The direction along which P is flying is at right angles to the direction along which Q is flying. Find the magnitude of the velocity of the aircraft P relative to aircraft Q.
6. Three vectors are: $\mathbf{a} = \begin{bmatrix}1\\4\end{bmatrix}$, $\mathbf{b} = \begin{bmatrix}-3\\5\end{bmatrix}$, $\mathbf{c} = \begin{bmatrix}2\\-9\end{bmatrix}$
   Work out the following:
   a) **a** + **b** b) **a** + **c** c) **b** – **c** d) **a** – **c** e) **a** + **b** + **c**
   What does the answer to part e) mean?
7. Work out the magnitude and direction of the resultant force in the worked example on page 31.
8. An aeroplane flies (1500, 3000, 200) m to point A and then (2000, –5000, –100) m to point B.
   a) Work out the final displacement of the aeroplane.
   b) Work out the magnitude of the displacement.
9. Add the following pairs of vectors together, using the cosine and sine rules to work out the resultant vector.
   a) **a** is 4 m due west, **b** is 8 m at an angle of 50° above the horizontal
   b) **a** is 6 m due north, **b** is 4 m at an angle of 30° above the horizontal
   c) **a** is 7 m due west, **b** is 5 m at an angle of 65° below the horizontal

2.4 Multiplication of vectors

By the end of this section you should be able to:

- Use the geometric definition of the scalar product to calculate the scalar product of two given vectors.
- Use the scalar product to determine projection of a vector onto another vector.
- Test two given vectors for orthogonality.
- Use the vector product to test for collinear vectors.
- Explain the use of knowledge of vectors in understanding natural phenomena.

Multiplying by a scalar

Vectors can be multiplied by scalars. When you multiply a vector by a scalar, the magnitude of the vector changes, but not its direction.

Figure 2.25 shows the vector **a**, which has a magnitude of 5 at an angle of 53° to the x-direction. It is multiplied by 2, which gives the vector 2**a**. The diagram shows that the magnitude has doubled but its direction is unchanged.

If we break **a** down into its components and express it in column form, it becomes $\begin{bmatrix}3\\4\end{bmatrix}$.

Multiplying it by 2 to give 2**a** gives $\begin{bmatrix}6\\8\end{bmatrix}$. If θ is the direction of the vector relative to the x-direction, we can see that tan θ is the same for both **a** and 2**a**

For **a** tan θ = 4/3

For 2**a** tan θ = 8/6 = 4/3

If you multiply a vector by a negative scalar, the direction of the vector is reversed.

For example, if **a** is multiplied by –2, then –2**a** is [–6, –8]

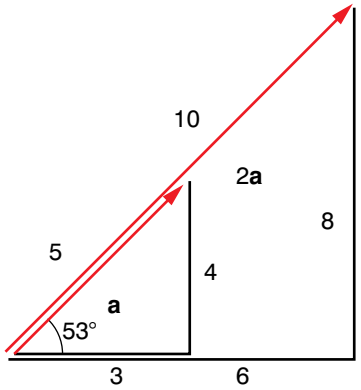


Figure 2.25 *Multiplying a vector by a scalar*

Scalar product

The scalar product of two vectors is when they are multiplied together to give a scalar quantity. The scalar product is also known as the dot product.

The scalar product of two vectors is defined as

$$\mathbf{a} \bullet \mathbf{b} = a_x b_x + a_y b_y$$

where the vectors are given in component form and are $\mathbf{a} = \begin{bmatrix}a_x\\a_y\end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix}b_x\\b_y\end{bmatrix}$. a_x and b_x are the components in the x-direction and a_y and b_y are the components in the y-direction.

Worked example 2.4

a is the vector $\begin{bmatrix}4\\4\end{bmatrix}$ and **b** is the vector $\begin{bmatrix}7\\3\end{bmatrix}$.

Work out the scalar product of **a** and **b**.

$\mathbf{a} \bullet \mathbf{b} = a_x b_x + a_y b_y =$

$(4 \times 7) + (4 \times 3) =$

$28 + 12 = 40$

The scalar product can also be expressed as:

$$\mathbf{a} \bullet \mathbf{b} = |a|\,|b| \cos\theta$$

where $|a|$ and $|b|$ are the magnitudes of the vectors **a** and **b**, respectively, and θ is the angle between the two vectors.

By rearranging this equation, we can calculate the angle between two vectors:

$$\cos\theta = \frac{(\mathbf{a} \bullet \mathbf{b})}{(|a|\,|b|)}$$

Worked example 2.5

What is the angle between the two vectors $\mathbf{a} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$?

Draw a sketch of the vectors, as shown in Figure 2.26.

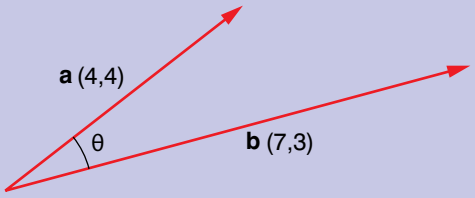


Figure 2.26 *Vectors* **a** *and* **b**

Work out the magnitudes of **a** and **b** using Pythagoras's theorem.

$|a| = \sqrt{(4^2 + 4^2)} = \sqrt{(16 + 16)} = \sqrt{32}$

$|b| = \sqrt{(7^2 + 3^2)} = \sqrt{(49 + 9)} = \sqrt{58}$

We already know from the previous worked example that $a \bullet b = 40$

So $\cos\theta = 40/(\sqrt{32} \times \sqrt{58}) = 40/\sqrt{1856} = 0.93$

$\theta = \cos^{-1} 0.93 = 21.8°$

We can also use the scalar product to work out the scalar projection of one vector onto another vector. If the vector **a** is projected on to vector **b** as shown in Figure 2.27, the scalar projection gives the magnitude of the component of **a** that is in the direction of **b**.

The scalar projection of **a** onto **b** is given by:

$$|a| \cos\theta$$

Worked example 2.6

Two vectors are $\mathbf{a} = \begin{bmatrix}4\\4\end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix}7\\3\end{bmatrix}$. What is the scalar projection of **a** on to **b**?

Sketch the two vectors and the projection of **a** on to **b** as shown in Figure 2.27.

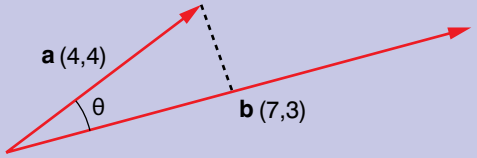


Figure 2.27 *Scalar projection of* **a** *on to* **b**

From worked example 2.5, $|a| = \sqrt{32}$ and $\cos\theta = 0.93$

So the scalar projection of **a** on to **b** is $\sqrt{32} \times 0.93 = 5.25$

KEY WORDS

orthogonal *at right angles. When two vectors are at right angles to each other, they are said to be orthogonal.*

Vector product

The vector product of two vectors is when two vectors are multiplied together to produce another vector. If is given by the formula:

$$\mathbf{a} \times \mathbf{b} = |a|\,|b| \sin\theta\, \hat{n}$$

where $|a|$ and $|b|$ are the magnitudes of **a** and **b**, respectively, θ is the smaller angle between **a** and **b** (θ is between 0° and 180°) and $\hat{n}$ is a unit vector, which is perpendicular to the plane that **a** and **b** are in.

The vector product can also be expressed as:

$$\mathbf{a} \times \mathbf{b} = (a_x b_y - a_y b_x)\,\hat{n}$$

The direction of $\hat{n}$ is given by the right-hand rule, as shown in Figure 2.28.

If the vectors are in three dimensions, the vector product is a bit more complicated:

$$\mathbf{a} \times \mathbf{b} = (a_y b_z - a_z b_y)\mathbf{i} + (a_z b_x - a_x b_z)\mathbf{j} + (a_x b_y - a_y b_x)\mathbf{k}$$

Activity 2.7

Consider the vectors

$\mathbf{a} = \begin{pmatrix}4\\6\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}-9\\6\end{pmatrix}$, $\mathbf{c} = \begin{pmatrix}3\\5\end{pmatrix}$

and $\mathbf{d} = \begin{pmatrix}10\\-6\end{pmatrix}$

What is the angle between **a** and **b**?

What is the angle between **c** and **d**?

Can you see an easy way of checking to see if vectors are **orthogonal**?

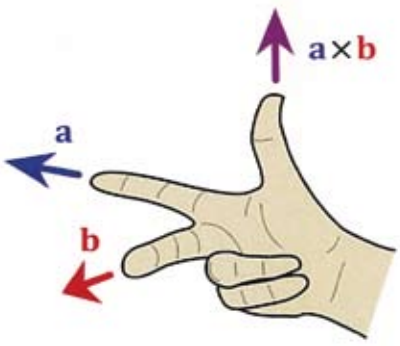


Figure 2.28 *The right-hand rule for finding the direction of the unit vector in the vector product of two vectors*

Activity 2.8

The vectors **g** and **h** are $\begin{bmatrix}7\\0\end{bmatrix}$ cm and $\begin{bmatrix}5\\3\end{bmatrix}$ cm, respectively.

a) Draw the vectors to scale.

b) Find the resultant by drawing a parallelogram.

c) Find the area of the parallelogram.

d) Find the vector product of the two vectors

What do you notice?

Project work

A ladder rests against a wall.

Plan and carry out an investigation into the forces being exerted on the ladder.

What directions are they acting in?

Are they in equilibrium?

Write your results up as a report using the writing frame on pages 19–20.

We can work out the size of the angle by rearranging the equation for the vector product:

$$\sin\theta = \frac{(\mathbf{a} \times \mathbf{b})}{|a|\,|b|\,\hat{n}}$$

But we also know that $\mathbf{a} \times \mathbf{b} = (a_x b_y - a_y b_x)\,\hat{n}$, so

$$\sin\theta = \frac{(a_y b_x - a_y b_x)}{|a|\,|b|}$$

Activity 2.9

1. The vectors **d**, **e** and **f** are $\begin{bmatrix}-4\\1\end{bmatrix}$, $\begin{bmatrix}1\\4\end{bmatrix}$ and $\begin{bmatrix}4\\-1\end{bmatrix}$, respectively.

   Find:

   a) **d** × **e**.

   b) the angle between vectors **d** and **e**.

   c) the area of the parallelogram formed by the resultant of **e** and **f**.

2. When vectors are collinear, they are either in the same direction as each other or in the opposite direction. So, the angle between them will be either 0° or 180°.

   Can you find an easy way to test if vectors are collinear?

Applications of vectors

Vectors have many applications. They are extremely useful in physics and many other areas. Some applications are as follows.

- Analysing forces on a bridge.
- Analysing the motion of an aeroplane.
- Programming motion or the position of an object in a computer game or animation.
- Displaying graphics (in the form of vector graphics) so that the diagram can be resized easily without any loss of quality.
- Modelling and planning the trajectory (path) of a space probe.
- Analysing the motion of planets.
- Analysing magnetic fields.

These are just a few examples – there are many more. We can use vectors whenever there is a variable that has direction as well as magnitude.

Discussion activity

In small groups, discuss other possible applications of vectors.

Report the results of your discussion back to the rest of the class.

Summary

- Multiplying a vector by a scalar changes the magnitude but not the direction of a vector.
- The scalar product of two vectors is $\mathbf{a} \bullet \mathbf{b} = a_x b_x + a_y b_y + a_z b_z = |a|\,|b| \cos\theta$
- The vector product of two vectors is $\mathbf{a} \times \mathbf{b} = |a|\,|b| \sin\theta\, \hat{n} = (a_x b_y - a_y b_x)\, \hat{n}$.

Review questions

1. Vector **d** is 1 N at 50° to the x-direction.

   Vector **e** is 3 N in the x-direction and 2 N in the y-direction.

   Vector **f** is (6, 2).

   Work out the following:

   a) **2d**

   b) 3**e**

   c) –2f

   d) 1/2**d**

   e) –3/4**e**

   f) 0.25**f**

2. Four vectors are $\mathbf{a} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, $\mathbf{c} \begin{bmatrix} -6 \\ 15 \end{bmatrix}$, $\mathbf{d} \begin{bmatrix} 15 \\ -6 \end{bmatrix}$

   a) Work out:

      i) $\mathbf{a} \bullet \mathbf{b}$

      ii) $\mathbf{c} \bullet \mathbf{d}$

      iii) the angle between **c** and **d**

      iv) the projection of **c** on to **a**

   b) Are the vectors **a** and **d** orthogonal?

3. a) Express the unit vectors **i**, **j** and **k** in column form.

   b) Work out $\mathbf{i} \bullet \mathbf{j}$, $\mathbf{j} \bullet \mathbf{k}$ and $\mathbf{i} \bullet \mathbf{k}$. What does this tell you about the unit vectors?

4. Work out $\mathbf{i} \times \mathbf{i}$, $\mathbf{j} \times \mathbf{j}$ and $\mathbf{k} \times \mathbf{k}$. Explain your answers.

End of unit questions

1. Construct a glossary of the key terms in this unit. You could add it to the one you made for Unit 1.
2. What is the scalar product of two vectors?
3. What is the vector product of two vectors?
4. How can you test to see if vectors are:

   a) orthogonal b) collinear?
5. Give some applications of vectors.

Kinematics

Unit 3

Contents

Section	Learning competencies
3.1 Motion in a straight line (page 39)	• Describe motion using vector analysis. • Define the term reference frame. • Explain the difference between average speed (velocities) and instantaneous speed (velocity). • Solve numerical problems involving average velocity and instantaneous velocity. • Define instantaneous acceleration. • Solve problems involving average and instantaneous acceleration. • Solve quantitative and qualitative kinematics problems related to average and instantaneous velocity and acceleration. • Derive equations of motion for uniformly accelerated motion. • Apply equations of uniformly accelerated motion in solving problems. • Draw graphs from the kinematics equations. • Interpret *s*–*t*, *v*–*t* and *a*–*t* graphs. • Solve numerical kinematics problems. • Relate scientific concepts to issues in everyday life. • Explain the science of kinematics underlying familiar facts, observations and related phenomena. • Describe the conditions at which falling bodies attain their terminal velocity.
3.2 Motion in a plane (page 51)	• Analyse and predict, in quantitative terms, and explain the motion of a projectile with respect to the horizontal and vertical components of its motion. • Derive equations related to projectile motion. • Apply equations to solve problems related projectile motion. • Define centripetal force and centripetal acceleration. • Identify that circular motion requires the application of a constant force directed toward the centre of the circle. • Distinguish between uniform and non-uniform circular motion. • Analyse the motion of a satellite. • Identify that satellites are projectiles that orbit around the Earth. • Analyse and predict, in quantitative terms, and explain uniform circular motion in the horizontal and vertical planes with reference to the forces involved. • Describe Newton's law of universal gravitation, apply it quantitatively and use it to explain planetary and satellite motion. • Determine the relative velocities of bodies moving at an angle relative to each other. • Use the relative velocity equation to convert from one measurement to the other in reference frames in relative motion.

You should be familiar with the four equations of motion from your work in Grade 10:

$s = \frac{1}{2}(u + v)t$

$s = ut + \frac{1}{2}at^2$

$v = u + at$

$v^2 = u^2 + 2as$

where s = distance or displacement, v = final speed or velocity, u = initial speed or velocity, a = acceleration and t = time.

In this unit you will be considering motion in more than one direction, using vectors. Understanding how an object moves and being able to predict how it will move is vital to planning things such as the launch of rockets into space.

3.1 Motion in a straight line

By the end of this section you should be able to:

- Describe motion using vector analysis.
- Define the term reference frame.
- Explain the difference between average speed (velocities) and instantaneous speed (velocity).
- Solve numerical problems involving average velocity and instantaneous velocity.
- Define instantaneous acceleration.
- Solve problems involving average and instantaneous acceleration.
- Solve quantitative and qualitative kinematics problems related to average and instantaneous velocity and acceleration.
- Derive equations of motion for uniformly accelerated motion.
- Apply equations of uniformly accelerated motion in solving problems.
- Draw graphs from the kinematics equations.
- Interpret *s*–*t*, *v*–*t* and *a*–*t* graphs.
- Solve numerical kinematics problems.
- Relate scientific concepts to issues in everyday life.
- Explain the science of kinematics underlying familiar facts, observations, and related phenomena.
- Describe the conditions at which falling bodies attain their terminal velocity.

KEY WORDS

frame of reference *a rigid framework or coordinate system that can be used to measure the motion of an object*

Frame of reference

You have already met several **frames of reference** in your studies – for example, coordinate grids in maths. Different people in different parts of Ethiopia speak different languages and have different cultures. They have different frames of reference. For example, people who speak Amharic as their first language have a different frame of reference from people who speak Afan or Orono as their first language.

Discussion activity

Discuss the story in small groups. What implications does it have for your work in physics?

Report the results of your discussion back to the rest of the class.

The seven blind men and the elephant

There is an old Indian story about seven blind men and an elephant. The men had not come across an elephant before. Each man was asked to describe it. The first felt its side and described it as a wall; the second felt its trunk and described it as a snake; the third felt its tusk and described it as a spear; the fourth felt its leg and described it as a tree trunk; the fifth felt its tail and described it as a piece of rope and the sixth felt the effects of its ear and described it as a natural fan. The seventh blind man took the time to investigate the elephant more fully – he felt all of the parts and knew exactly what the elephant was like.

Activity 3.1

Student A and student B stand facing each other about 2 m apart. Student C walks between the two students (Figure 3.1).

- How do students A and B describe the movement of student C?
- How do the rest of the class describe the movement of student C?
- Do they describe it in the same way? If not, why not?
- Which description is most useful?
- How does your frame of reference affect your observations?
- Repeat the activity, but this time with student B walking towards student A, so that student B walks in front of student C.

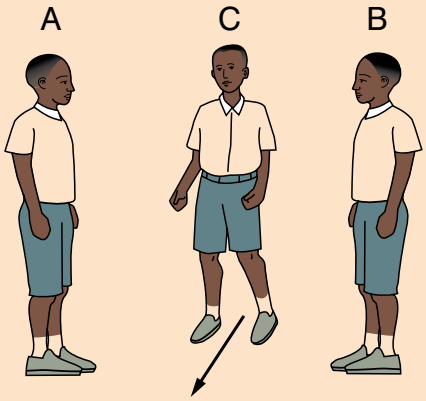


Figure 3.1 *Students observing the motion of a third student*

When you observe something, you use a frame of reference. You should have concluded from Activity 3.1 that you need an agreed frame of reference so that everyone can understand each other's observations. If people use different frames of reference, their observations will not be the same.

Activity 3.2

A body has a displacement of 10 m at an angle of 30° to the horizontal.

- Is this enough information to describe the displacement uniquely?
- What additional information do you need?
- What would be a better way of expressing the displacement?

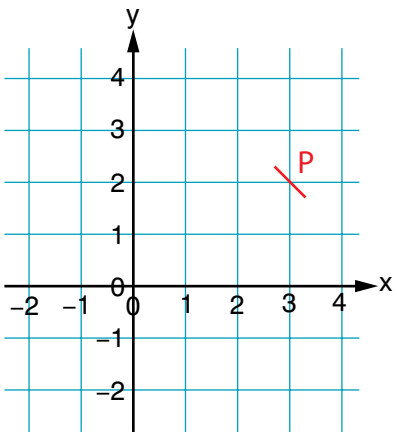


Figure 3.2 *A coordinate grid is a frame of reference. You give the horizontal component of the coordinates of the* point P *before the vertical component.*

You should have discovered from Activity 3.2 that there are four possible positions for the body. When we are giving the displacements and velocities of objects, we need a frame of reference that will describe the vector uniquely. One way of doing this is to give the vector in component form – give the distance in the x-direction and the distance in the y-direction. We use the coordinate grid in all four quadrants.

Average and instantaneous velocity

In everyday speech, velocity is another name for speed, but remember that in physics they are not the same. Speed is a scalar quantity – it has magnitude but no directions, whereas velocity is a vector and has both magnitude *and* direction.

Activity 3.3

One student walks between two points 4 m apart at a constant rate in a straight line. The student should take 3 seconds to cover the distance.

Another student walks between the same two points taking 3 seconds, but in a path that is not a straight line.

Repeat, but this time with both students travelling at constant rates – one travels in a straight line, the other does not.

Observe the motion of the two students. What are the average and instantaneous velocities of the two students? Are they the same?

KEY WORDS

average velocity *difference in displacement between two points divided by the time taken to travel between the two points*

instantaneous velocity *velocity of an object at a point*

Average velocity is the total displacement, or distance travelled in a specified direction, divided by the total time taken to travel the displacement. This is shown in Figure 3.3.

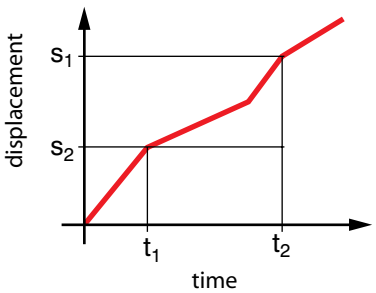


Figure 3.3 *Calculating average velocity*

Expressed mathematically the average velocity is

$$v_{av} = \frac{s_2 - s_1}{t_2 - t_1}$$

As the difference in displacement decreases, the two points get much closer together and the average velocity tends towards the **instantaneous velocity**, which is the velocity at a point. Expressed mathematically, this is

$$v_{inst} = \Delta s / \Delta t \text{ as } \Delta t \rightarrow 0$$

KEY WORDS

average acceleration *change in velocity divided by the time taken for the change to happen*

instantaneous acceleration *acceleration of an object at a point*

Worked example 3.1

A bus travels 60 km due north in 1 hour. It then travels 75 km due east in 2 hours.

What is the average velocity of the bus?

First draw a sketch to show the displacement of the bus (Figure 3.4).

The total displacement of the bus is (75, 60) km.

The total time taken is 1 + 2 = 3 hour.

The average velocity of the bus is (75/3, 60/3) = (25, 20) km/h.

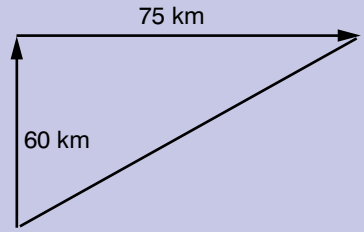


Figure 3.4 *Displacement of bus*

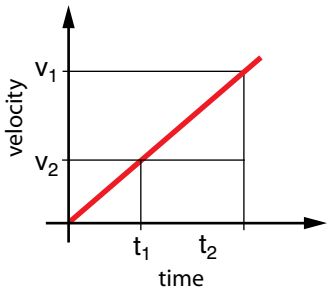


Figure 3.5 *Calculating average acceleration*

Average and instantaneous acceleration

Average acceleration is the change in velocity, divided by the total time taken for the change in velocity. This is shown in Figure 3.5.

Expressed mathematically the average acceleration is:

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1}$$

As the difference in velocity decreases, the two points get much closer together and the average acceleration tends towards the **instantaneous acceleration**, which is the acceleration at a point. Expressed mathematically, this is:

$$a_{inst} = \Delta v/\Delta t \text{ as } \Delta t \rightarrow 0$$

Average and instantaneous acceleration can be quite different. For example, consider the journey of a bus. The initial velocity of the bus is 0 m/s, as it starts off. When it reaches its destination, its final velocity is also 0 m/s. So the average acceleration over the whole journey is 0 m/s^2.

During the course of the journey the instantaneous acceleration at different times will vary a great deal. As the bus pulls away at the start, its instantaneous acceleration will be positive – probably about +1 m/s^2. When the bus slows down, the acceleration will be negative, perhaps about –1 m/s^2. If it has to do an emergency stop, the negative acceleration will be much higher.

As the time interval over which you are measuring the change in velocity gets smaller and smaller, you approach the instantaneous acceleration. In the limit, the instantaneous acceleration is when the time interval is infinitely small.

Activity 3.4

Repeat Activity 3.3, but this time consider the acceleration of the students.

Are they accelerating? Explain why you think they are or are not.

Motion with constant acceleration

Discussion activity

Consider the following velocities and accelerations of a particle. In each case, decide if the particle speeding up or slowing down.

Velocity (m/s)	Acceleration (m/s^2)
+4	+2
+4	−2
−4	+2
−4	−2

What conclusions can you draw about how you can tell if a particle is speeding up or slowing down when you know its velocity and acceleration?

You should have come to the conclusion that if the velocity and acceleration of an object have the same sign, the object is speeding up. If they have different signs, the object is slowing down.

Activity 3.5

Your teacher will set up a ramp. You should record how far a toy car travels down the ramp in equal intervals of time (Figure 3.6).

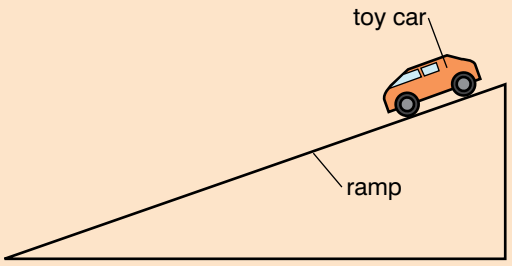


Figure 3.6 *Ramp for Activity 3.5*

- Plot graphs of distance against time and distance against $(time)^2$. What do you notice about the graphs?
- Work out the average velocity and acceleration for different time periods. What do you notice?
- Write a report setting out how you carried out the activity and your results.
- Use the framework given on pages 19–20.

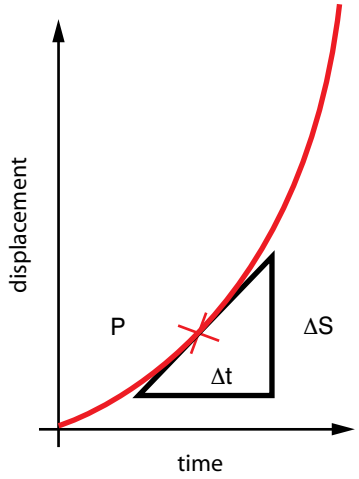


Figure 3.7 *Finding the instantaneous velocity from a distance–time graph*

The graph of displacement against time for a toy car moving down the ramp will have the form shown in Figure 3.7. If we draw a line that is parallel to the curve at a particular point (a tangent to the curve), we can find the instantaneous velocity by finding the gradient of the line.

Worked example 3.2

The driver of a train travelling at 40 m/s applies the brakes as the train enters a station. The train slows down at a rate of 2 m/s. The platform is 400 m long.

Will the train stop in time?

First, extract the information that we know:
initial velocity, $u = 40$ m/s
final velocity, $v = 0$ m/s
$a = -2$ m/s^2
We need to find the stopping distance of the train, s.

We need to select the appropriate equation, which is:
$v^2 = u^2 + 2as$
(Look back at page 39, if you are not sure)
Rearranging the equation to make s the subject:
$2as = v^2 - u^2$
$s = (v^2 - u^2)/2a$

Substituting in the values:
$s = ((0)^2 - (40)^2)/(2 \times -2)$
$s = -1600/-4$
$s = 400$ m

Check that the dimensions are correct:
$s = ((\text{m/s})^2 - (\text{m/s})^2)/(\text{m/s}^2) = \text{m}$
So the dimensions are correct.

The train will stop in time.

KEY WORDS

free body diagram *a simplified diagram of an object showing all the forces acting on it. It can also show the size and direction of the forces.*

Freely falling bodies

In Grade 9 you used a weight falling under gravity to pull a person on a board or a bicycle. The weight was being accelerated by the force due to gravity.

A falling object will show almost constant acceleration on its journey. The acceleration is the acceleration due to gravity, which we call g. Close to the surface of the Earth its value is about 9.8 m/s^2.

The acceleration is 'almost' constant because in real life air resistance will oppose the object's motion and so reduce its acceleration. If the object starts from rest, this air resistance will be zero initially, but as the object speeds up the air resistance on it will increase.

We can also show the forces acting on a body using a **free body diagram**. Figure 3.8 shows free body diagrams for a falling particle at different times in its fall.

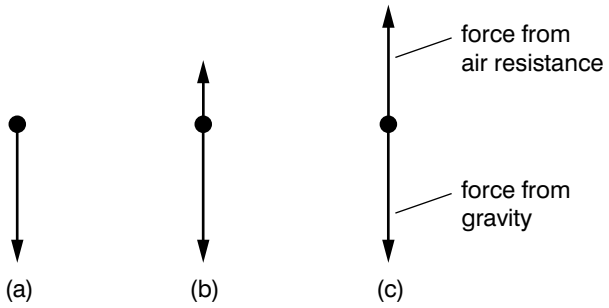


Figure 3.8 *A free body diagram showing the forces on a particle at different times during its fall: (a) at the start of the fall; (b) during the fall; (c) at the end of the fall*

If you are asked to do a calculation on a falling body, you will have to assume that air resistance has a negligible effect on its progress unless you are told otherwise.

Worked example 3.3

A stone is dropped down a well. You hear a splash after 2 s. Work out the depth of the well. What assumptions have you made in working out your answer?

Assume that movement upwards is positive.

In this case, $u = 0$ m/s, $t = 2$ s and $g = -9.8$ m/s^2.

We need to find s. The equation linking u, a, t and s is $s = ut + \frac{1}{2}at^2$

Substituting the values into the equation, we get:
$s = (0 \times 2) + (\frac{1}{2} \times 9.8 \times 2^2) = 0 - 19.6$ m.

So the well is about 20 m deep.

Our assumptions were that air resistance on the stone was negligible and also that the speed of sound is so fast that the time taken to hear the splash at the top of the well can be ignored.

Activity 3.6

Place two marbles or steel balls with different masses on a table. Flip both balls together with the ruler, so that they both go off the edge of the table at the same time.

- What do you notice about when the balls hit the floor?
- How do you explain this?

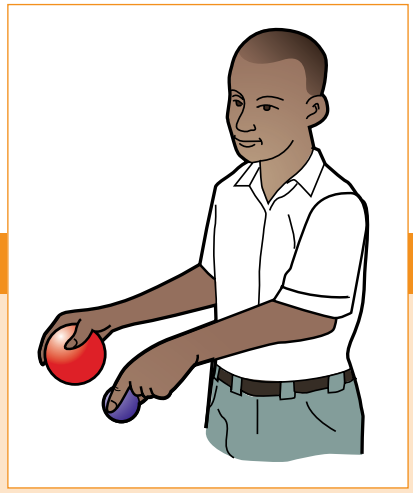


Figure 3.9 *The balls are dropped at the same time. Will they hit the ground at the same time?*

Activity 3.7

Punch two holes near the bottom of a plastic cup on opposite sides. Put the cup in a bowl and fill the cup with water.

- What happens to the water?

Fill the cup with water again, while holding it over the bucket. Drop the cup into the bucket from a height of at least 2 m.

- What happens to the water while the cup is falling?
- Can you explain why this happens?

Galileo's thought experiment

Galileo Galilei (1564–1642) did much of his work as 'thought experiments'. In one of them, he thought about what would happen if he dropped two balls with different masses at the same time from the leaning tower of Pisa. Which one would hit the ground first?

There are three possible outcomes:

- both balls hit the ground at the same time
- the heavier ball falls faster than the lighter one
- the lighter ball falls faster than the heavier one.

He proposed that if the lighter ball covered the distance in a certain time, then if the heavier ball had twice the mass it would cover the distance in half the time.

What happens if we tie the balls together? Suppose that the heavier ball falls faster than the lighter one – it will be slowed down by the lighter ball, which will act a bit like a parachute. But once the balls are tied together, the masses of the two balls are combined and so the two balls should fall faster (think of two balls being tied very closely together).

According to these lines of thinking, when the two balls are tied together they will fall both faster and slower, which is a contradiction! There is only one way to solve this. Both balls fall with the same velocity and will land on the ground at the same time.

Terminal velocity

Figure 3.8 shows that as a particle falls, the force from air resistance increases during its fall. This force increases as the velocity of the particle increases. If the particle is falling for long enough, the force from air resistance will be the same as the force from gravity. As there is no net force on the particle, there is no net acceleration on it and its velocity will not increase any more. This velocity is known as the terminal velocity. You will return to the subject of terminal velocity in Unit 8.

Graphical representation of motion

Displacement–time graphs

Figure 3.10 might represent two people who start on a straight-line race. Every second their distance from the starting point is recorded, and the results plotted on a graph.

One of the runners sets a fast pace so his distance away from the start increases rapidly with time. The other one is much slower; by the time his opponent has reached the end he has covered only half the distance and promptly stops for a rest.

It should be apparent from this example that the steeper the slope, the faster the velocity. A uniform slope means a constant (steady velocity).

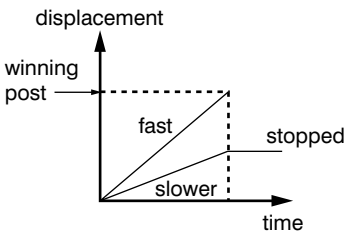


Figure 3.10 *The steeper the slope, the faster the velocity*

Worked example 3.4

What was the velocity of the journey shown in Figure 3.11?

To work out the velocity of the journey shown in Figure 3.11 find the gradient of the graph as follows:

velocity = displacement/time taken = 30 m/5 s = 6 m/s

This is both the average velocity and the instantaneous velocity at every moment in the journey because the velocity was constant.

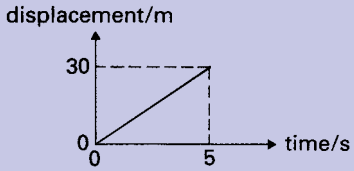


Figure 3.11 *Displacement–time graph for a journey*

Velocity–time graphs

An alternative way of recording a journey is to describe not where you are but what your velocity is at each moment. This will give you a velocity–time graph (Figure 3.12).

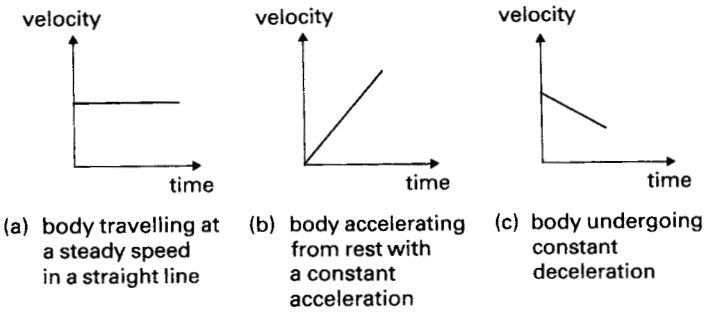


Figure 3.12 *Velocity–time graphs*

The slope of the graph is your acceleration. The steeper the upwards slope, the greater the acceleration.

It is also possible to deduce how far you have travelled by using a velocity–time graph. Figure 3.14 illustrates the straightforward case of something going at a constant velocity of 2 m/s^2 for 10 s. The displacement will of course be 20 m.

Worked example 3.6

What is the distance travelled in the journey shown in Figure 3.14?

The area of the shaded rectangle is its length multiplied by its width. Read the lengths of the sides from the scales of the graph – do not use a ruler! The area is 2 m/s × 10 s, which works out to be 20 metres.

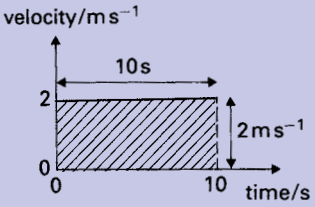


Figure 3.14 *You can use a velocity–time graph to find out how far you have travelled.*

Activity 3.8

Draw displacement–time, velocity–time and acceleration–time graphs to show the motion of each of the following:

a) a bus moving at a constant velocity

b) a car accelerating at a constant rate

c) a car decelerating at a constant rate.

Compare what each graph shows you for the three types of motion.

Worked example 3.5

What is the acceleration shown on the graph in Figure 3.13?

acceleration = change in velocity/time taken = (8 m/s – 2 m/s)/2 s = 3 m/s^2

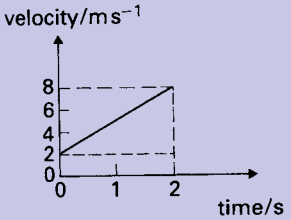


Figure 3.13 *A velocity–time graph*

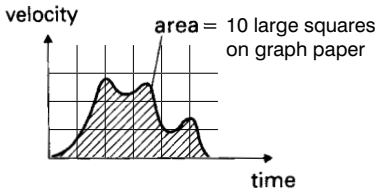


Figure 3.15 *The shaded area represents the displacement*

This is a particular example of a general rule: the area under a velocity–time graph is the displacement.

Look now at Figure 3.15. You can estimate the area under the graph by counting the number of squares on the graph paper that are under the line. Let us suppose you estimate the area under the line to be 10 large squares on the graph paper. To convert this to find the displacement the journey has produced, you must look at the scales used. Suppose the scale on the time axis is 'one unit represents 5 s', while on the vertical axis, one unit represents a velocity of 2 m/s. On this scale, each of the unit squares on the graph paper will have an area of 2 m/s × 5 s = 10 m. Therefore the 10 large squares under the graph mean a displacement of 100 m.

Worked example 3.7

A body starts from rest with a constant acceleration of 3 m/s^2. How far will it have travelled after 4 s?

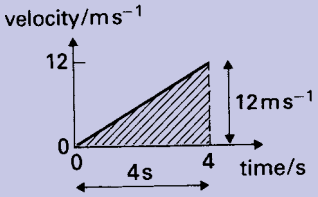


Figure 3.16 *How far has the body travelled after 4 seconds?*

Method 1: sketch the velocity–time graph, but it does not have to be to scale (Figure 3.16).

displacement = area under velocity–time graph=
area of triangle = ½ base × height
= ½ × 4 s × 12 m/s = 24 m

Method 2: reason the problem like this. Over the 4 s, the body speeds up steadily from 0 to 12 m/s. Therefore:

Average velocity over the whole journey = (0 + 12)/2 m/s = 6 m/s

If you travel at an average velocity of 6 m/s for 4 s:

displacement = average velocity × time = 6 m/s × 4 s = 24 m.

Activity 3.9

A ball is thrown vertically upwards with a speed of 20 m/s.

Use the equations of motion to work out its velocity and displacement every 0.5 seconds.

Plot graphs of the following:

a) displacement against time

b) velocity against time

c) the path of the ball.

What similarities and differences are there between your graphs?

Worked example 3.8

A train is at rest at a station. The train then moves away from the station and after 20 seconds, its velocity is 15 m/s and moves at this velocity for 60 seconds. Its velocity increases to 20 m/s over 5 seconds. It moves at a velocity of 20 m/s for 120 seconds and then slows down to come to a stop after a further 30 seconds.

a) Draw a velocity–time graph for the train's journey.

b) During which period was the acceleration of the train the highest?

c) How far did the train train travel during this time?

a) The velocity–time graph is shown in Figure 3.17

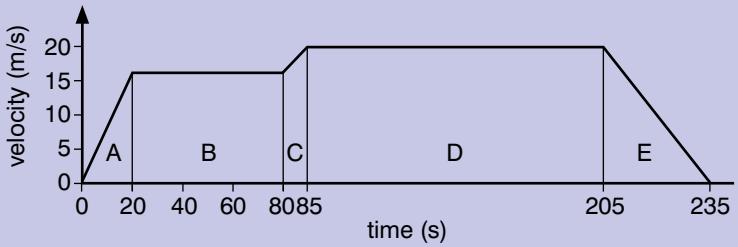


Figure 3.17

b) acceleration = change in velocity ÷ time taken.

The acceleration of the train is the highest where the slope of the graph is the steepest.

In areas B and D, the graph is flat, and there is no increase or decrease in velocity, so the acceleration is zero.

In area A, acceleration = 15/20 = 0.75 m/s^2

In area C, acceleration = 5/5 = 1 m/s^2

In area E, acceleration = –20/30 = –0.67 m/s^2

So the acceleration was highest in area C, from 80 to 85 seconds after the start of the journey.

c) Displacement = area under the graph = area A + area B + area C + area D + area E

= (½ × 20 s × 15 m/s) + (60 s × 15 m/s) + (½ ×5 s × 5 m/s + 5 s × 15 m/s) + (120 s × 20 m/s) + (½ × 30 s × 20 m/s)

= 150 m + 900 m + 12.5 m + 75 m + 2400 m + 300 m

= 3837.5 m

Activity 3.10

Draw some simple graphs of motion to illustrate the following:

- displacement–time
- velocity–time.

Practise acting out the motion represented by your graph until you can do it without any pauses.

How did you know to move in the way that you did when acting out the motion of your graphs?

Summary

In this section you have learnt that:

- A frame of reference is needed when recording measurements.
- When the velocity and acceleration of an object have the same signs, the object is speeding up; when they have opposite signs, the object is slowing down.
- Gravity provides a uniform acceleration.
- Two freely falling objects that are dropped at the same time fall at the same velocity when there is no air resistance.
- The forces on a falling object can be shown on a free body diagram.
- The motion of objects can be represented on displacement–time and velocity–time graphs.
- The displacement of an object is given by the area under a velocity–time graph.

Review questions

1. After 2 hours, the displacement of a car is 150 km north. The initial displacement of the car is 0. What is the average velocity of the car?
2. A car travels 100 km due East in 2 hours. It then travels 50 km South in 1 hour. What is its average velocity?
3. A man runs 300 m West in 60 seconds. He then runs 100 m North-west in 20 seconds.
   What is his average velocity in metres per second?
4. To get to school, a girl walks 1 km North in 15 minutes. She then walks 200 m South-west in 160 seconds.
   What is the girl's average velocity for her walk to school?
5. A body sets off from rest with a constant acceleration of 8.0 m/s^2. What distance will it have covered after 3.0 s?
6. A car travelling at 5.0 m/s starts to speed up. After 3.0 s its velocity has increased to 11 m/s.
   a) What is its acceleration? (Assume it to be uniform.)
   b) What distance does it travel while speeding up?
7. An aeroplane taxis onto the runway going at 10 m/s. If it can accelerate steadily at 3.0 m/s^2 and its take-off speed is 90 m/s, what length of runway will it need?
8. A motorist travelling at 18 m/s approaches traffic lights. When he is 30 m from the stop line, they turn red. It takes 0.7 s before he can react by applying the brakes.



Figure 3.18 *An aeroplane taking off from Addis Ababa airport*

The car slows down at a rate of 4.6 m/s^2. How far from the stop line will he come to rest and on which side of it?

9. A falling stone accelerates at a constant rate of 10 m/s^2. It is dropped from rest down a deep well, and 3 s later a splash is heard as it hits the water below.
   a) How fast will it be moving as it hits the water?
   b) What will be its average speed over the three seconds?
   c) How deep is the well?
   d) What have you assumed about the speed of sound?
10. A ball is thrown at a velocity of 15 m/s vertically upwards.
   a) What height will the ball reach before it starts to fall?
   b) How long will the ball take to reach this maximum height?
11. Draw displacement–time and velocity–time graphs to illustrate the motion of the ball in question 10.
12. Abeba walks to school. She walks 1 km in 15 minutes. She meets her friend Makeda – they talk for 5 minutes and then carry on walking to school. They walk 800 m in 10 minutes.
   a) Draw a displacement–time graph to show Abeba's journey to school.
   b) What was the average velocity of Abeba's journey? Give your answer in m/s
   c) When was Abeba walking the fastest? Explain your answer.
13. Dahnay travels on a bus to school. He gets on the bus, which then accelerates from rest to 18 m/s in 20 seconds. The bus travels at this velocity for 60 seconds. The bus slows down and comes to rest in 15 seconds. It is stationary for 30 seconds while more people get on the bus. The bus then accelerates to a velocity of 20 m/s in 25 seconds. It travels at 20 m/s for 5 minutes and then slows down to come to rest in a further 20 seconds, where Dahnay gets off the bus.
   a) Draw a velocity–time graph to show the journey of the bus.
   b) Between which times is the acceleration of the bus the greatest?
   c) How far does Dahnay travel on the bus?

3.2 Motion in a plane

By the end of this section you should be able to:

- Analyse and predict, in quantitative terms, and explain the motion of a projectile with respect to the horizontal and vertical components of its motion.
- Derive equations related to projectile motion.
- Apply equations to solve problems related projectile motion.

- Define centripetal force and centripetal acceleration.
- Identify that circular motion requires the application of a constant force directed toward the centre of the circle.
- Distinguish between uniform and non-uniform circular motion.
- Analyse the motion of a satellite.
- Identify that satellites are projectiles that orbit around the Earth.
- Analyse and predict, in quantitative terms, and explain uniform circular motion in the horizontal and vertical planes with reference to the forces involved.
- Describe Newton's law of universal gravitation, apply it quantitatively and use it to explain planetary and satellite motion.
- Determine the relative velocities of bodies moving at an angle relative to each other.
- Use the relative velocity equation to convert from one measurement to the other in reference frames in relative motion

KEY WORDS

projectile *an object that is propelled through space by a force. The action of the force ceases after the projectile is launched.*

trajectory *the path a moving object follows through space*

Projectile motion

A **projectile** is an object that has been launched into the air by the action of a force. A ball that is thrown or a football which is kicked into the air are both projectiles. Javelins thrown in athletics and bullets fired from guns are also projectiles. If the force continues, for example, in a rocket, the object is not a projectile. However, if the force stops, the rocket then becomes a projectile.

You came across the motion of projectiles in Grade 10. Activity 3.11 will remind you of what you learned earlier.



Figure 3.19 *This ball becomes a projectile when the force stops acting on it.*

Activity 3.11

Position a ruler at the edge of a table, with one end over the edge. Put one coin on the end of the ruler that is over the edge of the table, as shown in Fiure 3.20. Place the other coin on the table by the ruler. Quickly pivot the ruler about the other end so that the coin on the table is hit by the ruler. This coin will be fired off the table with a horizontal velocity. The other coin should fall off the ruler and drop straight downwards.

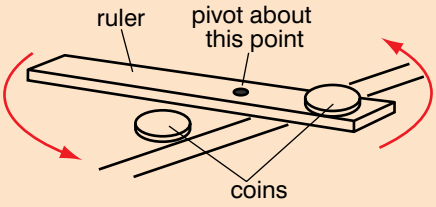


Figure 3.20

When do the coins hit the floor?

Try again with a different height and a different initial horizontal velocity.

Worked example 3.9

A ball is thrown with a velocity of 18 m/s at an angle of 56° above the horizontal.

Neglecting air resistance
a) how high will it rise?
b) how long will the ball spend in the air before it hits the ground?
c) what is the range of the ball?

First we need to resolve the vector into its vertical and horizontal components (Figure 3.21).

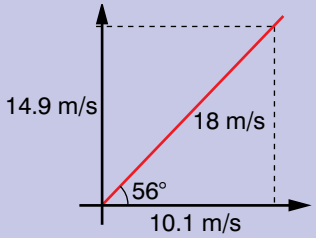


Figure 3.21 *Resolving the horizontal and vertical components of the vector*

vertical velocity, v_v = 18 sin 56° = 14.9 m/s
horizontal velocity, v_h = 18 cos 56° = 10.1 m/s

The **trajectory** of the ball is shown in Figure 3.22.

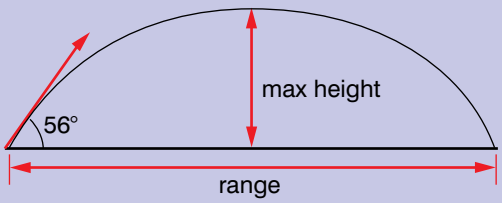


Figure 3.22 *Trajectory of the ball*

a) At the maximum height, the ball has stopped moving upwards and is momentarily at rest.

We only need to consider the vertical component of the motion.

We know the following:
u = 14.9 m/s
v = 0
a = –9.8 m/s^2 (the minus sign is there because the force of gravity is *slowing the ball down* while it is moving upwards, which is the same as decelerating.

We need to find s, so must use $v^2 = u^2 + 2as$.
Substituting in the values
$0^2 = 14.9^2 + (2 \times -9.8 \times s)$
$19.6s = 220.01$
s = 11.3 m

b) To find the time of the flight, we must realise that the upward journey and the downward one are symmetrical – in other words, the time to go up is the same as the time taken to descend. To find the complete time taken, all we do is work out how long it takes to reach the top then double it.

The simplest way will be to use $v = u + at$.
In this case, to find the time at the top:
$0 = 14.9 + (-9.8t)$
$9.8t = 14.9$
t = 1.52 s.

Thus the full time of the flight is 1.52 × 2 = 3.04 s.

c) To find the range of the ball, we need to consider the horizontal component of the motion.

Horizontally, in the absence of air resistance, there is no force to slow the ball down so it keeps travelling along at the same horizontal speed. This will not change until the ball hits something that can supply a force to stop it.

We have found that the ball will hit the ground after 3.04 s.

So the range is 3.04 s × 10.1 m/s = 30.7 m

KEY WORDS

maximum height *the vertical distance to the highest point reached by a projectile and the point at which the projectile is momentarily at rest*

range *the horizontal distance travelled by a projectile*

time of flight *the duration of a projectile's motion from launch to landing*

DID YOU KNOW?

Projectile motion can be used to model the motion of a stone when it is fired from a catapult and the motion of a frog when it jumps to catch a fly.

Activity 3.12

A ball is thrown at a speed of 15 m/s at an angle of 30° to the horizontal.

Resolve the movement of the ball into its horizontal and vertical components.

Use the equations of motion to work out the components of the ball's velocity and displacement every 0.5 seconds.

Plot graphs of the following:

a) displacement against time, for each component

b) velocity against time for each component

c) the path of the ball.

What similarities and differences are there between your graphs?

The two components of velocity are independent of each other – the vertical component changes because of the effects of gravity, but the horizontal component is unaffected.

We can use the equations of motion to predict and describe the motion of the ball in the worked example above in vector form. The velocity of the ball could be given as $(14.9 - 9.8t, 10.1)$ and the displacement as $(14.9t - 4.9t^2, 10.1t)$.

We can also use the equations of motion to derive equations which will give the **maximum height** and **range** of the projectile, and the total **time of flight** of the projectile.

Using the equation $v^2 = u^2 + 2as$, at the maximum height, $v_{vert} = 0$, and $a = -g$.
So, $0 = u_{vert}{}^2 - 2gs$

Rearranging the equation gives:
$s = u_{vert}{}^2/2g$

To derive an equation for the time of the flight of the projectile, use the equation $v = u + at$.
At the maximum height, $v_{vert} = 0$ and $a = -g$.
So $0 = u_{vert} - gt$
and $t = u_{vert}/g$

As the time in the air is twice this, the expression is: $t = 2u_{vert}/g$

The range is given by the time in the air multiplied by the horizontal velocity:
range, $r = 2u_{vert}/g \times u_{hor} = 2u_{vert}u_{hor}/g$

If the initial velocity is given in the form of a velocity at an angle θ above the horizontal, the three equations can be given as:
maximum height, $s = (v \sin \theta)^2/2g$
time in air, $t = 2v \sin \theta/g$
range, $r = 2v^2 \sin \theta \cos \theta/g$

The trajectory shown in Figure 3.22 is symmetrical. The angle with the ground at which the projectile starts its motion is the same as the angle just before the projectile hits the ground, and the magnitude of the vertical component of the initial velocity is the same as the vertical component of the velocity just before the projectile hits the ground.

Activity 3.13

A monkey is hanging from a branch of a tree. A hunter aims a rifle at the monkey and fires.

At the instant the rifle fires, the monkey lets go of the branch and begins to fall.

What happens? Use the equations of motion to find out.

What assumptions do you make?

Uniform circular motion

When an object is moving in a circle, there are two ways we can describe how fast it is moving. We can give its speed as it moves around the circle, or we can say that it is going round the circle at a certain number of revolutions per minute.

KEY WORDS

uniform circular motion
when a body is moving at a constant speed in a circle

Radian measure for angles

For all practical work, we would measure angles in degrees. For theoretical work, such as this topic on circular motion, it is often far more convenient to measure angles in radians. A radian is a larger angle than a degree – it is just over 57°. Where does it come from?

Radians are derived directly from circles. An angle of 1 radian provides a sector of a circle of radius r such that the length around the arc of the circle is also r, as shown in Figure 3.23.

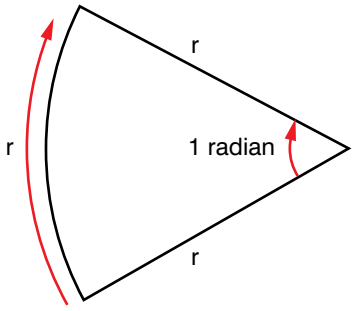


Figure 3.23 *The sector of a circle that is formed by an angle of 1 radian*

Since the circumference of a circle is $2\pi r$, you can fit 2π radians into a complete turn. As a complete turn is also 360°, we can see that:

2π radians = 360°

and 1 radian = $360°/2\pi = 57.3°$

Radian measure provides a quick and easy way to work out distances measure round the rim of a sector (Figure 3.24).

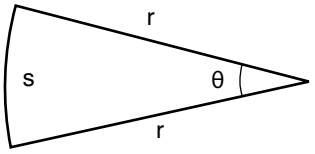


Figure 3.24 *Sector of a circle with angle θ*

When θ is 1 radian, $s = r$. When θ = 2 radians, $s = 2r$. In general, $s = \theta r$.

Angular velocity is given the symbol ω (small omega in the Greek alphabet). In this work we will consider it to be measured not in complete revolutions per minute but in radians per second (rad/s).

There is a connection between the speed v of a body as it goes round a circle of radius r and its angular velocity ω in rad/s.

In time t, the distance travelled round the circle by the body will be vt. The angle traced out will be ωt. We know that $s = \theta r$, so $s = \omega tr$. We also know that $s = vt$, so

$$vt = \omega tr$$

Cancelling the ts gives us:

$$v = \omega r$$

Acceleration is defined as the rate of change of velocity. We can find it by dividing the change in velocity by the time taken. Let us apply that to a body moving in a circle of radius r. Its linear speed (a scalar) is v and its angular velocity is ω.

Figure 3.25 shows the body in two positions separated by a small time t. The angle covered in that time will be ω. The original velocity is v_1, the later velocity is v_2 (same speed, different direction).

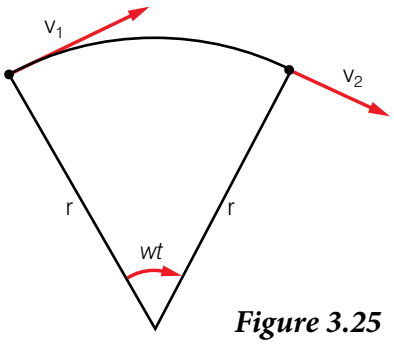


Figure 3.25

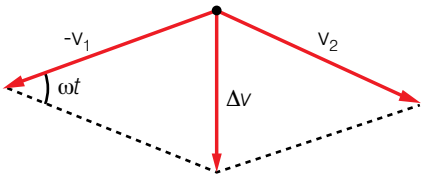


Figure 3.26

By how much has the velocity changed over time t? We need to find Δv, given by $v_2 - v_1$. These are vectors, so we need to consider their directions when working this out. Remember that $-v_1$ has the same magnitude as v_1, but is in the opposite direction. We know how to add and subtract vectors from Unit 2 – we can draw a parallelogram, as shown in Figure 3.26.

The first thing to notice is that the direction of Δv, and therefore of the acceleration, is directly towards the centre of the circle.

What is the magnitude of Δv? This needs a bit more thought. Consider the triangle that makes up the left-hand part of the parallelogram. The body had turned through an angle of ωt, so that is the angle between v_1 and v_2, and is marked on Figure 3.26.

This is a moment when radian measure helps us. We have an angle ωt in radians, and the sector of the circle has a radius v which is the speed. (It is true that Δv is a straight line and not an arc of a circle, but time t is a very short interval in which case the difference vanishes.)

In terms of magnitudes $s = \theta r$ gives us:

$\Delta v = \omega t\, v$

Acceleration is given by $\Delta v/t$, the acceleration a will be:

$a = \Delta v/t = \omega t\, v/t = \omega v$

But $v = \omega r$, so we can express the acceleration as:

$a = \omega^2 r = v^2/r$ towards the centre of the circle.

Worked example 3.10

A boy is riding a bicycle at a velocity of 4 m/s. The bicycle's wheels have a diameter of 0.8 m.

a) What is the velocity of a point on the rim of the wheel? (Ignore the effects of the forward motion of the bicycle)

b) What is the angular velocity of the wheel?

c) What is the acceleration of a point on the rim of the wheel? In what direction is it acting?

a) First draw a diagram to show the wheel.

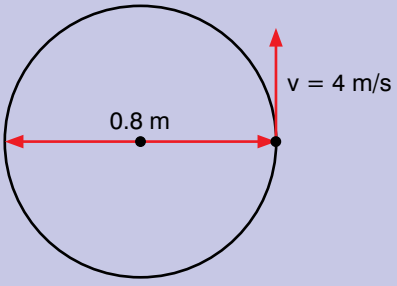


Figure 2.27

The bicycle is travelling at 4 m/s, so in 1 second a point on the rim of the wheel will travel 4 m.

The linear velocity of a point on the rim of the wheel is 4 m/s.

b) We need to use the equation $v = \omega r$

v = 4 m/s and r = 0.8 ÷ 2 = 0.4 m

Substituting the values into the equation:

$\omega = v/r$ = 4 m/s ÷ 0.4 m = 10 rad/s

c) acceleration = $\omega^2 r$

= (10 rad/s)^2 × 0.4 m = 40 m/s^2

The direction is towards the centre of the wheel.

We could also use the equation acceleration = v^2/r

v = 4 m/s, r = 0.4 m

acceleration = (4 m/s)^2 ÷ 0.4 m = 40 m/s^2

Newton's first law of motion tells us that if no resultant force acts on a moving body, the body will just keep moving at the same speed in the same direction.

When an object is moving in a circle, its direction is constantly changing. We have just shown that there is an acceleration towards the centre of the circle. We can also say that there must be a resultant (unbalanced) force on the object which acts inwards towards the centre of the circle.

This is known as a **radial force**.

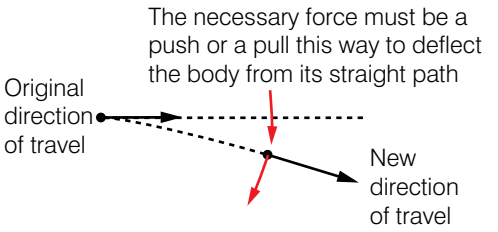


Figure 3.28 *There must be a force that is pulling or pushing the body into a circular path*

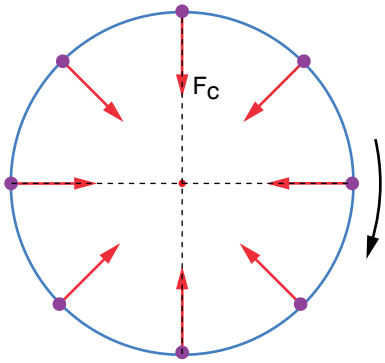


Figure 3.29 *When an object is moving in a circle, there is a radial force towards the centre of the circle that changes direction constantly, while the magnitude of the velocity stays the same.*

When a car is going round a bend (Figure 3.30), that force must be due to friction between the tyres and the ground: if the road is slippery with spilt oil, such a frictional force may not be available, in which case the car will continue at a steady speed in a straight line!

KEY WORDS

radial force *the force acting on a body moving in a circle which is directed towards the centre of the circle.*

tangential acceleration *when a particle is moving in circular motion, the component of a particle's acceleration at a tangent to the circle*

radial acceleration *acceleration towards the centre of the circle when a particle is moving in a circle*

centripetal force *the force acting on a body moving in a circle which is directed towards the centre of the circle*

Worked example 3.11

A bus of mass 2500 kg goes round a corner of radius 50 m at a speed of 5 m/s.

What force is needed for the bus to go round the corner?

Acceleration = v^2/r
v = 5 m/s and r = 50 m
So $a = 5^2/50$
$= 25/50 = 0.5$ m/s^2

Force = mass × acceleration
= 2500 × 0.5
= 1250 N

Motion in a vertical circle

When a body is moving in a horizontal circle, the magnitude of the velocity (or the speed) stays constant. However, when a body moves in a vertical circle, the speed does not stay constant – it decreases as the body moves towards the top of the circle and increases as it moves towards the bottom of the circle.

Figure 3.31 overleaf shows the forces acting on a body moving in a vertical circle. There is **centripetal force** towards the centre of the circle and the force due to gravity, which is always downwards. At the top of the circle, the forces are acting in the same direction, and



Figure 3.30 *There is a force on this racing car that enables it to go round the bend*

at the bottom they are acting in opposite directions. Halfway up the circle, the two forces are acting at right angles (Figure 3.31).

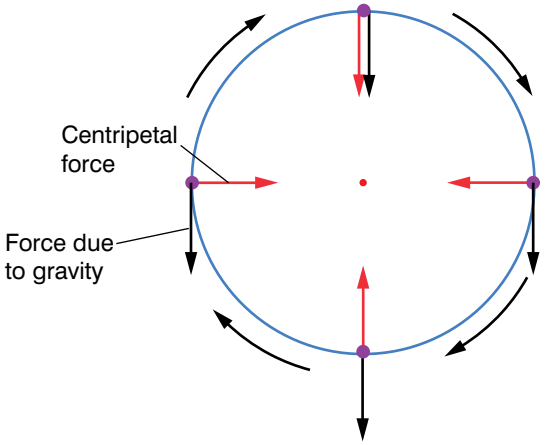


Figure 3.31 *Free body diagram for a body moving in a circle*

A pendulum moves in an arc of a vertical circle. Its speed is not high enough for it to move round a complete circle, so it oscillates about the bottom of the circle. The period (time for one complete swing backwards and forwards) of a pendulum is given by $T = 2\pi\sqrt{(l/g)}$ where l is the length of the pendulum.

This equation is only true for small angles of swing (less than 1 radian).

Activity 3.14

You are going to use a simple pendulum to find a value for g.

- Set up the pendulum as shown in Figure 3.32.

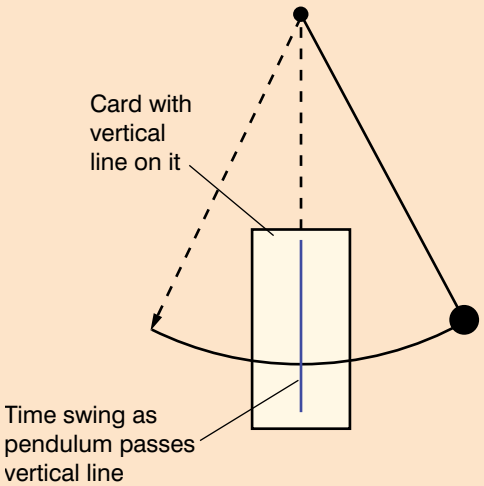


Figure 3.32 *Pendulum*

- Investigate how the period of the pendulum varies for different lengths of the pendulum.
- Plot a graph of T^2 against l.
- Find the gradient of the graph – this will be equal to $4\pi^2/g$.
- From this find a value for g. How does this compare with the actual value of g?
- Write a report describing how you carried out your experiment, including what you did to make sure your test was fair, and details of your experimental results. Use the writing frame on pages 19–20 for the structure of your report.

Worked example 3.12

A girl is swinging a bucket on a piece of rope in a vertical circle with a radius of 1 m. What is the minimum speed needed at the top of the circle so that the bucket stays moving in the circle.

At the top of the circle, the forces acting on the bucket are the force due to gravity and the centripetal force, F.

We also know that force = mass × acceleration
The acceleration of the bucket is v^2/r

Draw a diagram to show the forces.

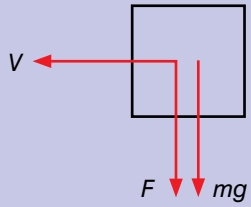


Figure 3.33 *Free-body diagram showing forces acting on bucket at the top of the circle.*

At the top of the circle:
$F + mg = mv^2/r$

At the point that the bucket stops moving in a circle, $F = 0$, and $v_y = 0$ so
$mg = v^2/r$
$v^2 = rg$
$v = \sqrt{(rg)}$

We know that the radius is 1 m and
$g = 9.8$ m/s^2, so
$v = \sqrt{(1 \times 9.8)} = 3.1$ m/s

Worked example 3.13

An object is moving in avertical circle attachd to a piece of rope which is anchored in the centre of the circle. The circle has a radius of 1 m and the object is moving at a steady speed of 5 m/s. The mass of the object is 3 kg. Assume the rope to be massless.

Work out the tension in the rope at the bottom, top and halfway between the bottom and top of the circle.

First draw a free-body diagram to show the forces acting on the object and water.

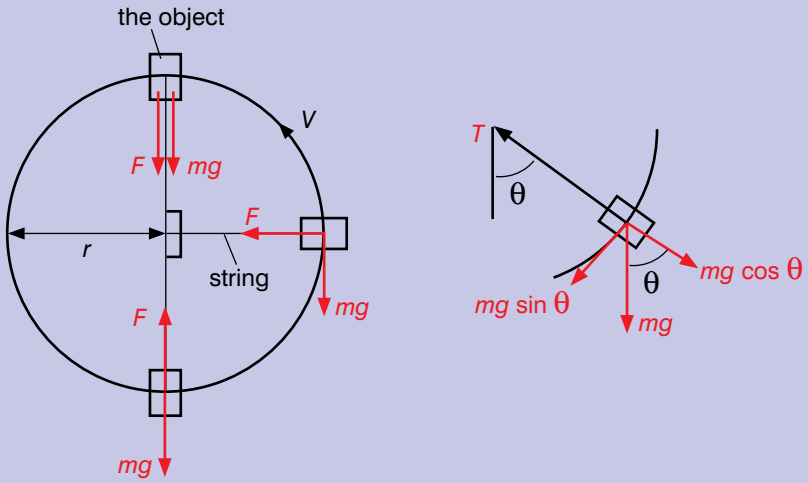


Figure 3.34

The net force to the centre is given by the tension in the rope minus the component of the weight of the object and water that is acting in the same line as the centripetal force:

$F = T - mg \cos \theta$

Now we know that $F = mv^2/r$

So the tension in the rope is given by

$T = mv^2/r + mg \cos \theta$

We know the following values:

$r = 1$ m, $v = 5$ m/s and $m = 3$ kg

At the bottom of the circle, $\theta = 0$ rad, so:

$T = 3 \text{ kg} \times (5 \text{ m/s})^2 \div 1 \text{ m} + 3 \text{ kg} \times 9.8 \text{ m/s}^2 \times \cos 0$

$= 75 + 29.4 = 104.4$ N

At the top of the circle, $\theta = \pi$, so

$T = 3 \text{ kg} \times (5 \text{ m/s})^2 \div 1 \text{ m} + 3 \text{ kg} \times 9.8 \text{ m/s}^2 \times \cos \pi$

$= 75 - 29.4 = 45.6$ N

Halfway between bottom and top, $\theta = \pi/2$, so

$T = 3 \text{ kg} \times (5 \text{ m/s})^2 \div 1 \text{ m} + 3 \text{ kg} \times 9.8 \text{ m/s}^2 \times \cos \pi/2$

$= 75 - 0 = 75$ N

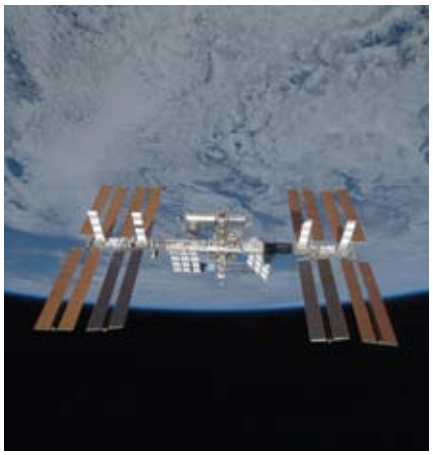


Figure 3.35 *The force of gravity keeps the International Space Station in its orbit of the Earth.*

DID YOU KNOW?

Electrical forces are larger than gravitational forces by a factor of 10^{39} times!

Discussion activity

Satellites are all launched from sites as close to the equator as possible. Why do you think this is?

Motion of a satellite

You should remember from work you did in Grade 10 that when two masses M_1 and M_2 are a distance r apart, there is a gravitational force between them which is given by

$$F = \frac{GM_1M_2}{r^2}$$

where G is the gravitational constant and is equal to 6.67×10^{-11} N m^2 kg^{-2}

This is known as Newton's law of universal gravitation. Gravitation is solely an attractive force and is very small. Gravity is the force that keeps the Earth in its orbit of the Sun, the Moon in its orbit of the Earth and the International Space Station in its orbit of the Earth.

Worked example 3.14

The Hubble Space Telescope is in orbit 559 km above the surface of the Earth.

a) What is its angular velocity?

b) How long does it take to complete an orbit of the earth?

The radius of the Earth is about 6400 km. The mass of the Earth is 5.97×10^{24} kg.

a) The radius of the orbit of the telescope is 559 + 6400 = 6959 km = 6.959×10^3 m

The gravitational force using Newton's law of gravitation is:

$$F = \frac{GM_1M_2}{r^2}$$

As this telescope is in orbit, this is also equal to $F = M_2\omega^2 r$ where M_2 is the mass of the telescope.

So

$$M_2\omega^2 r = \frac{GM_1M_2}{r^2}$$

$$\omega^2 = \frac{GM_1}{r^3}$$

$= (6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2} \times 5.97 \times 10^{24}\ \text{kg})/(6.959 \times 10^3\ \text{km})^3$
$= 1.182 \times 10^{-6}$
$\omega = \sqrt{1.182 \times 10^{-6}} = 1.087 \times 10^{-3}$ rad/s

b) Time taken for one orbit = $2\pi/\omega$
$= 2\pi/1.182 \times 10^{-6}$
= 5780 seconds
= 96.33 minutes

Relative velocity

When the velocity of a certain body is given, the value of this velocity is given as noted by an observer. When an observer who is stationary records some velocity, then the velocity is referred to as **absolute velocity**, or simply velocity. On the other hand, any velocity of a body noted by a non-stationary observer is more correctly termed **relative velocity**; that is, it is the velocity if the body relative to the moving observer. For example, if one bus is overtaking another, a passenger in the first bus sees the overtaking bus as moving with a very small relative velocity. But if the passenger was in a stationary bus, then the overtaking bus would be seen to be moving much more quickly.

If the observer is not stationary, then to find the velocity of body B relative to body A, subtract the velocity of body A from the velocity of body B. If the velocity of A is v_A and that of B is v_B, then the velocity of B with respect to A is:
relative velocity $v_{BA} = v_B - v_A$

Worked example 3.15

Nishan is running eastwards at a speed of 10 km/h and at the same time Melesse is running northwards at a speed of 9 km/h. What is the velocity of Nishan relative to Melesse?

We will use the standard frame of reference – velocities to the East are positive, as are velocities to the North.

In vector form, the velocities are:

$$v_A = \begin{bmatrix} 10 \text{ km/h} \\ 0 \text{ km/h} \end{bmatrix}$$

$$v_B = \begin{bmatrix} 0 \text{ km/h} \\ 9 \text{ km/h} \end{bmatrix}$$

Calculate their relative velocity:

$$\text{Relative velocity } v_{AB} = v_A - v_B = \begin{bmatrix} 10 - 0 \\ 0 - 9 \end{bmatrix} = \begin{bmatrix} 10 \\ -9 \end{bmatrix} \text{ km/h}$$

The magnitude is $\sqrt{(100 + 81)} = 13.45$ km/h
The direction is $\tan^{-1}(-9/10) = -42°$
The direction is –42° to the east.

So Nishan is moving at 13.45 km/h at an angle of –42° to the east, relative to Melesse. Melesse is moving at 13.45 km/h at an angle of 42° to the east, relative to Nishan.

KEY WORDS

absolute velocity *velocity observed from a stationary frame of reference*

relative velocity *the vector difference between the velocities of two objects*

Activity 3.15

Student A and student B walk at normal walking speed across the classroom, at right angles to each other.

- What is the velocity of the two students walking at from the point of view of the rest of the class?
- What is the velocity of student A from the point of view of student B?
- What is the velocity of student B from the point of view of student A?

Summary

In this section you have learnt that:

- To describe motion using vectors.
- To analyse, predict and explain the motion of a projectile.
- To analyse, predict and explain uniform circular motion in the horizontal and vertical planes.
- To describe and apply Newton's law of universal gravitation.
- To use Newton's law of universal gravitation to explain the motion of the planets and satellites.
- That in circular motion there is a constant force towards the centre of the circle.

Review questions

Take g to be 9.8 m/s^2 and ignore air resistance in questions 1–5.

1. An aircraft flying at a steady velocity of 70 m/s eastwards at a height of 800 m drops a package of supplies.
   a) Express the initial velocity of the package as a vector. What assumptions have you made about the frame of reference?
   b) How long will it take for the package to reach the ground?
   c) How fast will it be going as it lands? Express your answer as a vector.
   d) Describe the path of the package as seen by a stationary observer on the ground.
   e) Describe the path of the package as seen by someone in the aeroplane.

2. A stone is thrown upwards with an initial velocity of 25 m/s at an angle of 30° to the ground.
   a) Show that the vertical component of the velocity at the start is 12.5 m/s upwards.
   b) Without doing any calculations, state what the stone's vertical velocity will be when it again reaches ground level.
   c) Using the answer to a), show that the stone will rise to a height of about 8 m.
   d) How long will it take for the stone to reach its maximum height?
   e) How long will it take between when the stone was thrown and when it comes back to the ground?
   f) Explain why the horizontal component of the stone's velocity at the start is 21.7 m/s.
   g) What will the stone's horizontal velocity be just before it lands?
   h) Use the answers to parts e)–g) to work out the stone's horizontal range.

3. Ebo throws a ball into the air. Its velocity at the start is 18 m/s at an angle of 37° to the ground.
   a) Express the initial velocity in component vector form.
   b) Work out the velocity of the ball as it lands. Give your answer in component vector form.
   c) Work out the range of the ball.
   d) What assumptions have been made about the frame of reference?
4. Ebo throws the ball again at an initial velocity of 18 m/s but this time at an angle of 53° to the ground.
   a) Work out the velocity in component vector form.
   b) Why will the ball spend a longer time in the air than it did before.
   c) Calculate the range of the ball.
5. Look at the equations for maximum height, time of flight and range. Check the dimensions of each of these, by putting them into the equations.
6. A mass on the end of a length of rope is being swung in a circle of radius 3.2 m at an angular velocity of 0.71 rad/s. How fast is it moving?
7. A spinning top has a diameter of 10 cm. A point on the outer rim of the top moves through an angle of 8π radians each second.
   a) What is the angular velocity of the point?
   b) What is the distance moved by the point in 5 seconds?
   c) What is the velocity of the point?
   d) What is the acceleration of the point?
8. A car of mass 800 kg goes round a corner of radius 65 m at a speed of 10 m/s.
   a) What size force is needed to achieve this?
   b) Suggest how this force is likely to be obtained.
   c) What force would be needed if the driver approached the bend at twice the speed?
9. A fishing line will break when the tension in it reaches 15 N. A 3.1 m length of it is used to tie a model aeroplane of mass 280 g to a post so it goes round in circles. What is the fastest speed the aeroplane can reach before the line breaks.

   Give your answer both as an angular velocity and in m/s.
10. A centrifuge consists of a container held at a distance of 0.20 m from its axis. When turned on, the centrifuge spins the container and its contents round at 9000 revolutions per minute.
    a) Find its angular velocity.
    b) A small mass of 10 g is placed in the drum. Work out what force will need to be provided for it to go round at that rate in a circle of radius 0.20 m.

c) In which direction must that force act?
d) What supplies that force to the ball?
e) Taking g to be 9.8 N/kg, what size force will the Earth supply to the ball when it is at rest on the floor?

11. A toy car moves around a loop-the-loop track. The loop is 0.5 m high.What is the minimum speed of the car at the top of the loop for it to stay on the track?

12. A boy is swinging a toy on a piece of string in a vertical circle. The toy has a mass of 150 g and the radius of the circle is 0.8 m.

    a) He swings the toy with a linear velocity of 2 m/s. Will the toy move in a circle? Explain your answer.

    b) Another boy swings the toy with a linear velocity of 3.5 m/s.

    Work out the tension in the string at the top of the circle, at the bottom of the circle and halfway between the top and the bottom of the circle.

13. A geostationary satellite for communications seems to be in a fixed spot above the equator because it has the same angular velocity as the earth.
    a) Show that if it goes round once a day, its angular velocity ω is a little over 7×10^{-5} rad/s.
    b) Geostationary satellites are placed in orbits of radius 4.2×10^{4} m. Use this information to deduce g, the acceleration of a freely falling body, at this height.

14. At ground level g is 9.8 m/s^2. Suppose the Earth started to increase its angular velocity. How long would a day be when people on the equator were just 'thrown off'? Why is the expression 'thrown off' a bad one?

15. Two cars A and B are moving along a straight road in the same direction with velocities of 25 km/h and 30 km/h, respectively. Find the velocity of car B relative to car A.

16. Two aircraft P and Q are flying at the same speed, 300 m s^{-1}. The direction along which P is flying is at right angles to the direction along which Q is flying.
    a) Find the magnitude of the velocity of the aircraft P relative to aircraft Q.
    b) Find the direction of the velocity.

17. A river flows at 2 m/s. The velocity of a ferry relative to the shore is 4 m/s at right angles to the current. What is the velocity of the ferry relative to the current?

End of unit questions

1. Construct a glossary of the key terms in this unit. You could add it to the one you made for Units 1 and 2.
2. Describe what happens to a ball when you drop it from a height of 2 metres.
3. Explain the difference between average velocity and instantaneous velocity.
4. Explain, in terms of forces and acceleration, what happens when a body is moving in uniform horizontal circular motion.
5. How do the forces on a body moving in a vertical circle vary?
6. What is the difference between radial and tangential acceleration?
7. What is relative velocity?