Physics Grade-12

UNIT - 5

Steady electrical current and circuit property

Prepared by :Addis Ababa Education Bureau

April,2020

UNIT - 5

Steady current and circuit property

Learning Competencies this Unit

By the end of this unit you should be able to:

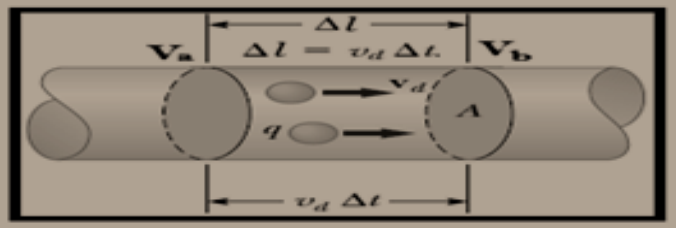
- Define the basic terms resistance, resistivity, conductivity, current density, drift velocity.
- State the Microscopic description of Current and current density.
- Explain how the sources of e.m.f. produce a p.d.
- Express the relationship between e.m.f., terminal p.d. and internal resistance.

- Analyse, in quantitative terms, circuit problems involving potential difference, current and resistance.
- State Kirchhoff's junction rule and loop rule.
- Use Kirchhoff's rules to solve related circuit problems.
- Identify the sign conventions appropriately in applying Kirchhoff's rules.
- Solve problems involving network resistors.
- Describe how a galvanometer can be modified to measure a wide range of currents and potential differences.

- Explain the principle of the Wheatstone bridge and solve problems involving it.
- Solve different problems involving different electrical circuits.

Terms

Suppose current flows through the conductor of length l between the two ends (a) and (b) due to the difference in electric potential (voltage), then there are basic terms in microscopic model of current; and these are



i) *Net charges (ΔQ)*

Suppose (N) number of charges flows through the conductor between a and b, then the net charge is given as $\Delta Q = Nq$

ii) *Charge density (n)*

It is the measure of number of charges (N) per a unit volume (V). $n = \frac{N}{V} \Rightarrow N = nV$ But volume (V) can be given as $V = Al$ then $N = nAl$ therefore the net charge (ΔQ) can be given as $\Delta Q = Nq$

$\Delta Q = (nAl)\,q \qquad \Delta Q = nqAl$

iii) *Drift velocity (V_d)*

it is the measure of average velocity of charges in the conductors for the length l . $V_d = \frac{l}{\Delta t} \Rightarrow$ $l = V_d \Delta t$, since net charge $\Delta Q = nqAl$; $\Delta Q = nqAV_d \Delta t$

iv) *Electric current (I)*

It Is the rate of flow of charges($I = \frac{\Delta Q}{\Delta t}$). i.e from the microscopic view of charges above ($\Delta Q = nqAV_d \Delta t$) the electric current (I) is given as

$I = \frac{\Delta Q}{\Delta t}$ **but** $\Delta Q = nqAV_d \Delta t \qquad I = \frac{nqAV_d \Delta t}{\Delta t} = nqAV_d \Rightarrow \boxed{I = nqAV_d}$

N.B:- The SI unit of electric current is given as $[I] = \frac{[Q]}{[t]} = \frac{coulomb\,(C)}{Second\,(S)} = Ampere\,(A)$. $1A = 1\,\frac{C}{S}$

- ✓ There are two types of current i.e. ***electron current*** (the flow of electrons from negative to positive terminals of a source) and **conventional current** (the flow of positive charges from positive to negative terminals, used before the discovery of electrons.)

V) *Current density (J)*

It is the measure of amount of current (I) to the crossectional area (A).

$J = \frac{I}{A}$ but $I = nqAV_d \qquad J = \frac{nqAV_d}{A} = nqV_d \qquad \boxed{J = nqV_d}$

Example

Find the number of charge carriers per unit volume in a copper wire of cross-sectional area 2 mm^2 carrying a current of 1.5 A if the drift velocity of the charge carriers is 0.00028 m/s. (The elementary charge is 1.6×10^{-19} C.)

Solution

V_d (m/s)	n	A (m^2)	I (a)	q (C)
0.00028	?	2×10^{-6}	1.5	1.6×10^{-19}

$$V_d = \frac{I}{nqA}$$

$$n = \frac{I}{V_d qA}$$

$$= \frac{1.5}{0.00028 \times 1.6 \times 10^{-19} \times 2 \times 10^{-6}}$$

$$= \frac{1.5}{8.96 \times 10^{-26}}$$ (Number of charges per volume)

$n = 1.67 \times 10^{25}$ (The unit is number per cubic meter i.e. $/m^3$)

Ohm's law

It is one of the fundamental laws under electrodynamics which was proposed a Dutch physicist George Simon Ohm. It is stated

"Current density has a direct proportionality with electric field it is produced in a given Ohomic* material."

It is also restated as "The ratio of current density to electric field is always constant for a given materials"

$$J \sim E \Rightarrow J = kE \Rightarrow k = \frac{J}{E}$$



Georg Simon Ohm
German physicist (1789–1854)

Conductivity **is a way of measuring a material's ability to allow flow of current through it**. It is given the symbol σ and its units are ***Siemens per meter (S/m).***
The inverse of conductivity is **resistivity.**
Resistivity is a measure of how much a material resists the flow of an electric current. It is given the symbol *ρ and its units are* ***ohm meters (Ω m).***
A material with a high conductivity will have a low resistivity and a material with a high resistivity will have a low conductivity.
Resistance is a property of a material that controls the amount of current that flows through electrical devices. It is measured in **ohms (Ω).**

where k is a proportionality constant which tells us about the ability of conductors to conduct electric current known as conductivity (σ) , it may also be expressed as resistivity (ρ) *as* the inverse property of conductivity (σ) i.e $\sigma = \frac{1}{\rho}$ thus $K = \sigma = \frac{1}{\rho}$ $\sigma = \frac{J}{E}$ $J = \sigma E$ $\frac{1}{\rho} = \frac{J}{E}$

*Current density (J) is a vector quantity.
*Ohomic materials are materials which obey Ohm's law.

$E = \rho J$

Recall that the potential difference between (a) and (b) of the conductor of length (l) is given as $V_{ab} = V = El \Rightarrow E = \frac{V}{l}$ and $J = \frac{I}{A}$

then $E = \rho J \Rightarrow \frac{V}{l} = \rho \frac{I}{A} \Rightarrow \frac{V}{I} = \frac{\rho l}{A} \Rightarrow$

but $\frac{V}{I}$ is constant known as Resistance (R) thus $\frac{V}{I} = R = \frac{\rho l}{A}$

$R = \frac{\rho l}{A}$ or $R = \frac{V}{I}$

$R = \frac{V}{I}$ Ohm's law

Material	Resistivity[a] (Ω · m)
Silver	1.59×10^{-8}
Copper	1.7×10^{-8}
Gold	2.44×10^{-8}
Aluminum	2.82×10^{-8}
Tungsten	5.6×10^{-8}
Iron	10×10^{-8}
Platinum	11×10^{-8}
Lead	22×10^{-8}
Nichrome[c]	1.50×10^{-6}
Carbon	3.5×10^{-5}
Germanium	0.46
Silicon	640
Glass	10^{10} to 10^{14}

Example

1. Find the approximate current density when an electric field of 12 V/m is applied to a silver conductor. The conductivity of silver is 63.0×10^6 S/m.

Solution:-

$J\,(A/m^2)$	$\sigma\,(S/m)$	$E\,(V)$
?	63.0×10^6	12

$$\text{Use } J = \sigma E$$

$$= 63.0 \times 10^6 \times 12$$

$$J = 7.56 \times 10^8\ A/m^2$$

2. A 1 m length of copper wire of diameter 0.4 mm has a measured resistance of 0.13 Ω. What value does this give for the resistivity of copper?

SOLUTION:-

$$A = \frac{\pi D^2}{4}$$

ρ (Ω m)	R (Ω)	l (m)	A (m^2)
?	0.13	1	$(0.2 \times 10^{-3})^2 \times 3.14$

Use $\rho = \frac{RA}{l}$

$$= \frac{0.13 \times (0.2 \times 10^{-3})^2 \times 3.14}{1}$$

$$\rho = 1.63 \times 10^{-8}\ \Omega\ \text{m}$$

3. Calculate the p.d. across a 40 Ω resistor ? if the current it supplies to the resistor is 0.05 A.

Solution:-

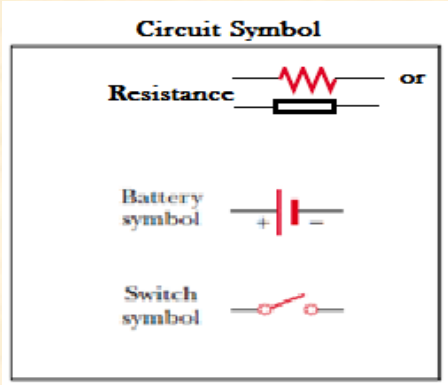
p.d. (V)	I (A)	R (Ω)
?	0.05	40

Use Ohm's law $V = IR$

$= 0.05 \times 40$

$= 2$ V

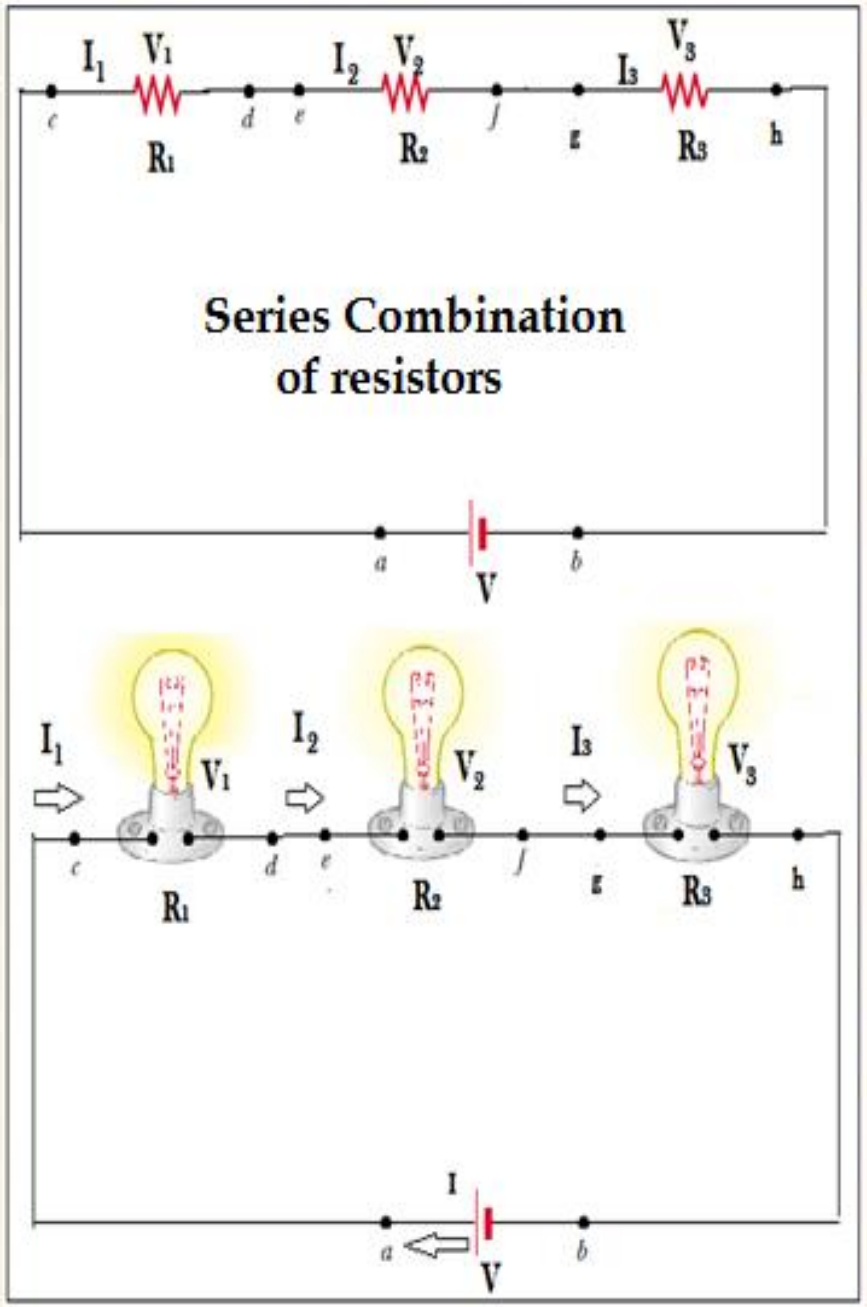
Resistor:- is an electrical component which limits the flow of current. Or it is a device which converts electrical energy to some other forms of energy.

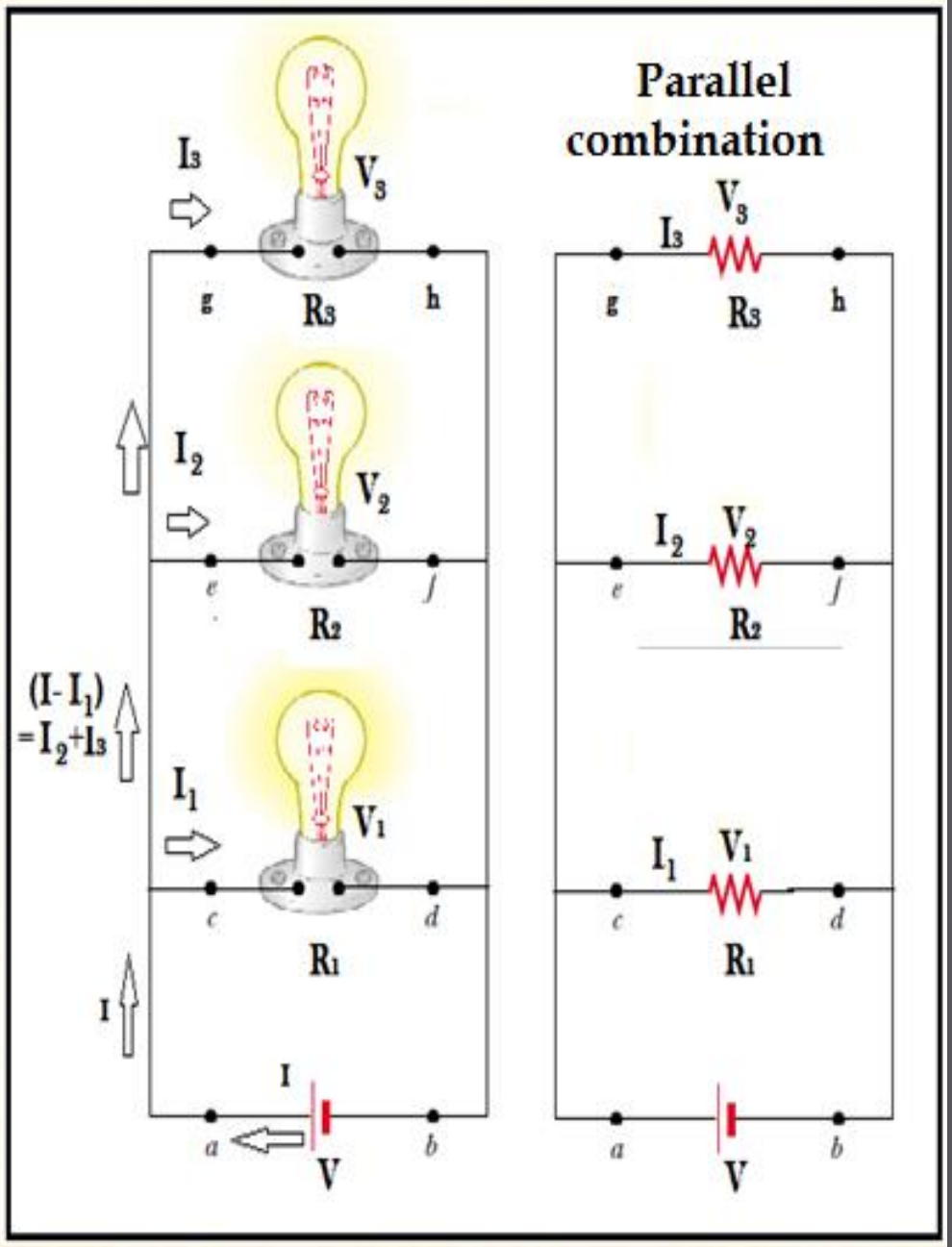


Resistor Networking

There are two combinations of resistors and these are Series and parallel.

Series	parallel
• It is an end to end combination [alternate terminals are connected togeter]	• It is a side by side combination [similar terminals are connected togeter]
• there is only one way of current flow.	• there may be different (alternate) way of current flow.


I1
V1
I2
V2
I3
V3
c
d
e
f
g
h
R1
R2
R3
Series Combination
of resistors
a
b
V
I1
V1
I2
V2
I3
V3
c
d
e
f
g
h
R1
R2
R3
I
a
b
V


Parallel
combination
I3
V3
g
R3
h
I2
V2
e
R2
f
(I- I1)
=I2+I3
I1
V1
c
R1
d
I
I
a
V
b
I3
V3
g
R3
h
I2
V2
e
R2
f
I1
V1
c
R1
d
a
V
b

Let three resistors combine in series as shown below. The left terminal of resistance 1 and the right terminal of resistor 3 are connected to the terminals of a battery. The other three terminals are connected to each other alternately.

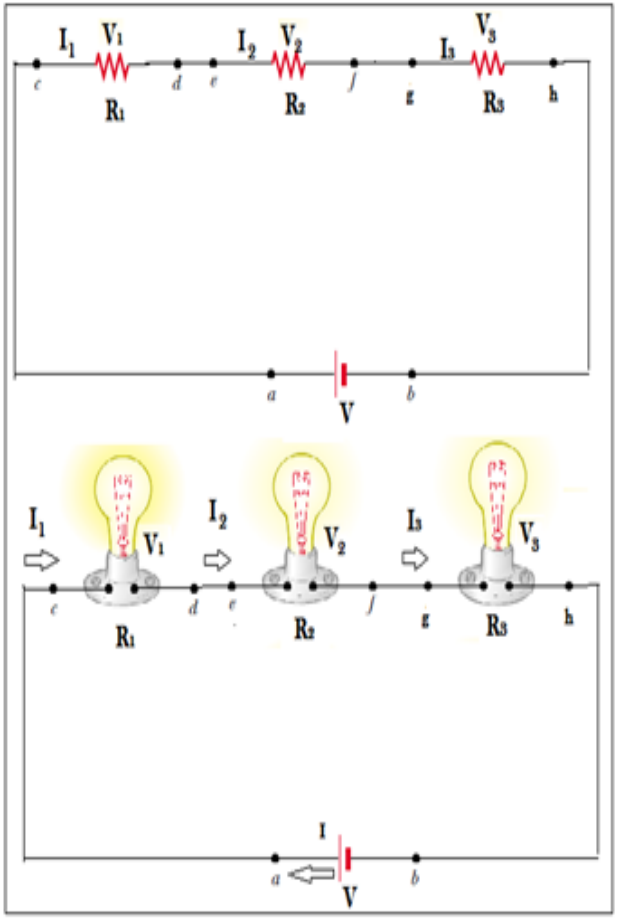
NOTE

- The total amount of electric current (I) is equal in all resistors. That is $I = I_1 = I_2 = I_3$
- The total amount of the Voltage (V) is equals to the linear sum of amount of the voltage dropped across each resistor.
  i.e $V = V_1 + V_2 + V_3$
  To determine the equivalent Resistance (R) use
  $V = V_1 + V_2 + V_3$ Since $R = \frac{V}{I} \Rightarrow V = RI$ and $V_1 = R_1 I_1$, $V_2 = R_2 I_2$, $V_3 = R_3 I_3$

Therefore, $RI = R_1 I_1 + R_2 I_2 + R_3 I_3$ but $I = I_1 = I_2 = I_3$

$$RI = R_1 I + R_2 I + R_3 I = I(R_1 + R_2 + R_3)$$

$$R = R_1 + R_2 + R_3$$



N.B. *The equivalent resistance of a series connection of resistors is the numerical sum of the individual resistances and is always greater than any individual resistance.*

Parallel Combination

Connecting resistors in parallel means connecting similar terminals together (side by side connection).

Let three resistors combine in parallel as shown below. The left terminals of all resistors are connected to the positive terminals of a battery and the right terminals of all resistors connected with the negative terminal of the battery.

NOTE

- The total amount of the current (I) is equal equals to the linear sum of amount of the current passing across each resistors.
  i.e $I = I_1 + I_2 + I_3$
- The total amount of the Voltage (V) is equals to the amount of the voltage dropped across each resistor.
  i.e $V = V_1 = V_2 = V_3$
- To determine the equivalent resistance (R) use
  $I = I_1 + I_2 + I_3$ Since

$$R = \frac{V}{I} \Rightarrow I = \frac{V}{R} \text{ and } I_1 = \frac{V_1}{R_1},\ I_2 = \frac{V_2}{R_2} \text{ and } I_3 = \frac{V_3}{R_3}$$

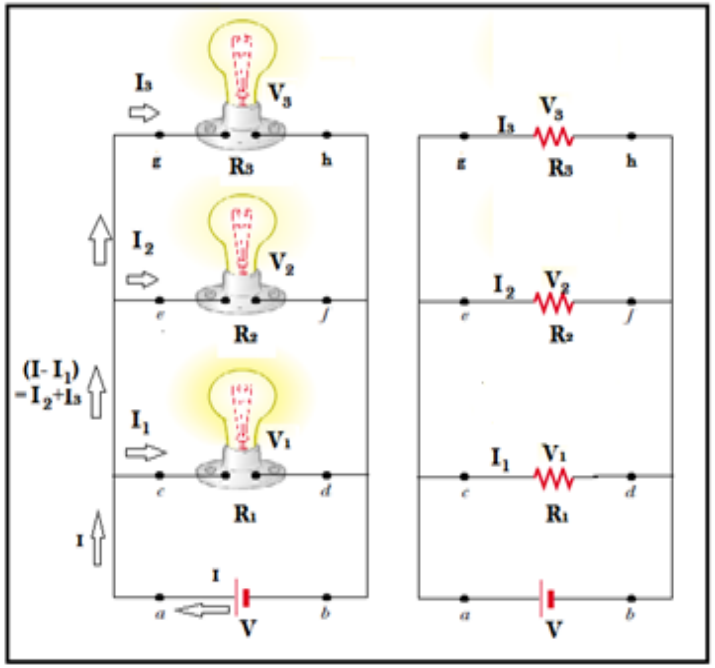
Therefore, but $I = I_1 + I_2 + I_3$

$$I = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \quad \text{but } V_1 = V_2 = V_3 = V$$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \Rightarrow \frac{V}{R} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$



NOTE

For only two resistors R_1 and R_2 connected in parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R = \frac{R_1 R_2}{R_1 + R_2}$$

NOTE: - *The equivalent resistance is always less than the smallest resistance in the group.*

Example

For the circuit shown below,

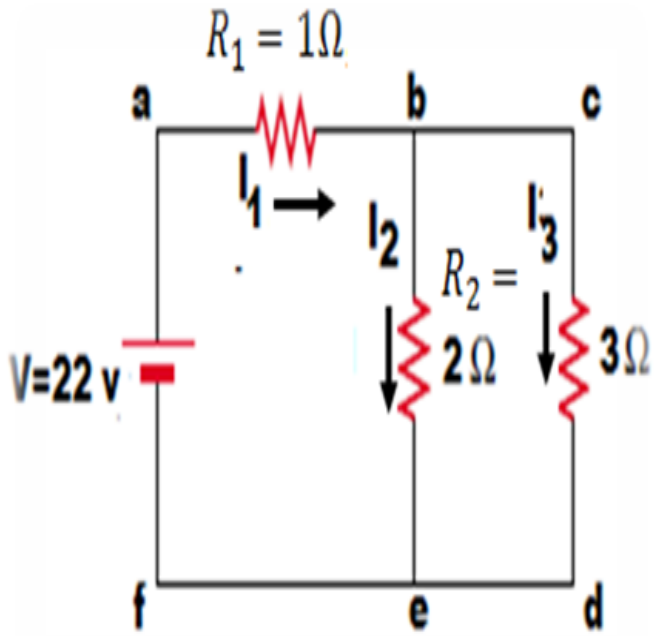
(a) What is the equivalent resistance of the circuit?

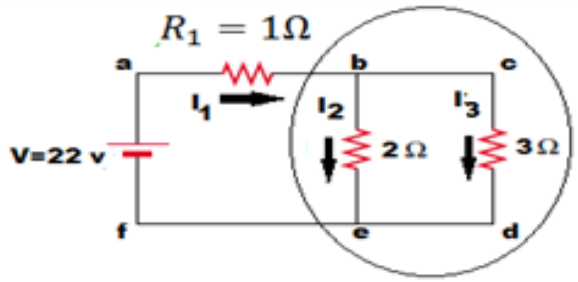
(b) What are the current on each resistors using?

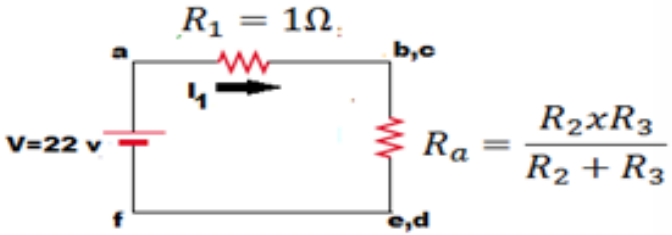
Solution

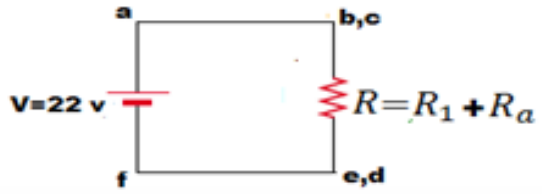
(a) $R_1 = 1\Omega$, $R_2 = 2\Omega$ and $R_3 = 3\Omega$

Let R_a be the parallel combination of R_2 and R_3









$R_a = \frac{R_2 x R_3}{R_2 + R_3}$ $\quad R_a = \frac{2x3}{2+3} = \frac{6}{5}$ $\quad R_a = \frac{6}{5}\Omega$

$R = R_1 + R_a$ $\quad R = 1 + \frac{6}{5} = \frac{11}{5}\Omega$ $\quad R = \frac{11}{5}\Omega$

$R = \frac{V}{I} \Rightarrow I = \frac{V}{R}$ The net current is determined as $I = \frac{22}{\frac{11}{5}}$

$I = 10\ A$, since R_1 *and* R_a are in series the current through each is the same as the total current.

$$I = I_1 = I_a = 10\ A$$

But the voltage across each is given as $V = V_1 + V_a$.

$V_1 = I_1 R_1 = (10)(1) = 10V$

$V_a = I_a R_a = (10)\left(\frac{6}{5}\right) = 12V$

Then $V = V_1 + V_a = 10 + 12 = 22\ V$

Since, R_a is the parallel combination of R_2 and R_3 the voltage on R_2 and R_3 is the same as the voltage dropped on R_a . i.e. $V_a = 12V$

$V_a = V_2 = V_3 = 12V$ *on each resistors.*

Then the current on each resistors (i.e. on R_2 and R_3) can be determined as $I_2 = \frac{V_2}{R_2}$ and $I_3 = \frac{V_3}{R_3}$

$I_2 = \frac{12}{2} = 6V \Rightarrow I_2 = 6V$ and $I_3 = \frac{12}{3} = 4V \Rightarrow I_3 = 4V$ the total current on resistor (R_a) is $I_a = 10\ A$

therefore we can even counter check our work by $I_a = I_2 + I_3 = 6V + 4V = 10V$

Generally the currents on each resistors is given as $\begin{cases} I_1 = 10\ A \\ I_2 = 6V \\ I_3 = 4V \end{cases}$

Electromotive force (e.m.f.) and potential difference (ΔV)

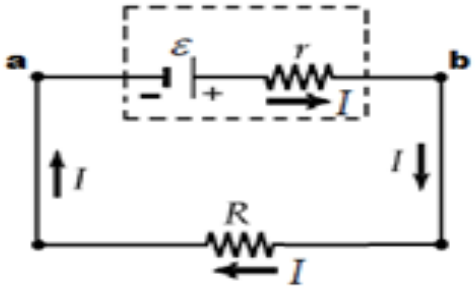
Electromotive force (ε):-is the maximum voltage that a given source can produce between its terminals before the effect of internal resistance (r).

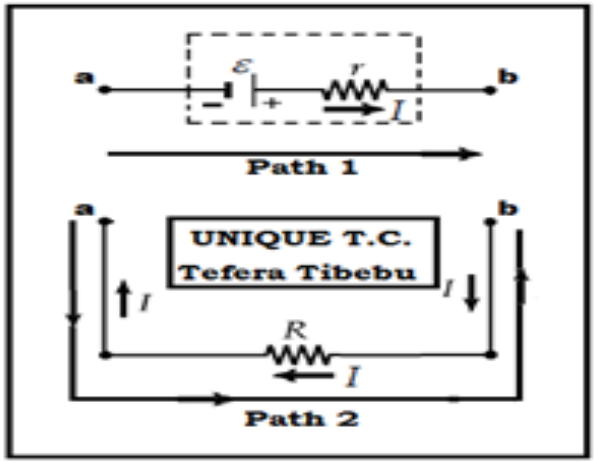
* *(internal resistance /r/ is the resistance of a source produced due to its nature; which is different from load external resistance/R/)*

** *electromotive force is not a force it is a driving energy.*

Potential difference (ΔV):- is the voltage delivered to the external resistance i.e. after the effect of internal resistance (r).

taking a source of voltage connected with load external resistor (R) as shown below; then the potential difference between the two points (a) and (b) can be determined using two paths. i.e.





From path 1 we can write the potential difference between (a) and (b) $\Delta V = V_b - V_a = V_{ba}$ as follows

$V_a + \varepsilon - Ir = V_b \Rightarrow \varepsilon - Ir = V_b - V_a \quad \Delta V = \varepsilon - Ir$

$$\Delta V = \varepsilon - Ir \ldots\ldots eq-1$$

Where $\varepsilon = e.m.f.$, $\Delta V = potential\ difference$, $r = internal\ resistance$; $R = external\ load\ resistor$

From path 2 we can write the potential difference between (a) and (b) $\Delta V = V_b - V_a = V_{ba}$ as follows

$V_a + IR = V_b \Rightarrow IR = V_b - V_a \quad \Delta V = IR$

$$\Delta V = IR \ldots\ldots eq-2$$

Equating the above two equations we can have $IR = \Delta V = \varepsilon - Ir \Rightarrow IR = \varepsilon - Ir \Rightarrow \varepsilon = IR + Ir$

$$\varepsilon = I(R + r)$$

A solar cell generates a potential difference of 0.1 V when a 500Ω resistor is connected across it and a potential difference of 0.15 V when a 1000Ω resistor is substituted. What are the emf and the internal resistance of the solar cell?

A) 0.3 V, 1500Ω B) 0.23V, 1000Ω C) 0.3V, 1000Ω D) 0.35V, 1300Ω

Given: - $\Delta V_1 = 0.1V$ when $R_1 = 500\Omega$ is connected and **required**:- $r = ?$ *and* $\varepsilon = ?$

$\Delta V_2 = 0.15V$ When $R_2 = 1000\Omega$ is connected

Use $\Delta V = IR$ for each of the resistors and re-substitute the current on $\varepsilon = I(R + r)$

$$\Delta V_1 = I_1 R_1 \Rightarrow I_1 = \frac{\Delta V_1}{R_1} \Rightarrow I_1 = \frac{1}{5000} \text{ and } \Delta V_2 = I_2 R_2 \Rightarrow I_2 = \frac{\Delta V_2}{R_2} \Rightarrow I_2 = \frac{3}{20{,}000}$$

$$\varepsilon = I_1(R_1 + r) \ \& \ \varepsilon = I_2(R_2 + r) \Rightarrow r = 1000\Omega \text{ and } \varepsilon = 0.3\,V$$

Answer: - Choice C

Kirchhoff's Rules

It is the procedure of analyzing more complex electrical circuits named after a German physicist R. G. Kirchhoff.
It has two principles called Kirchhoff's rules:

1. ***Junction rule.***

It is also known as Kirchhoff's current law [KCL]
It is stated as
"The algebraic sum of all the currents entering a ***junction*** point is zero"
Junction (j) :- is a point where two or more than two currents meet or split).

$$\sum I_j = 0 \quad \Rightarrow \quad \sum I_{entering} + \sum I_{leaving} = 0$$

Conventionally the current entering to a junction may be considered as negative i.e

$$\sum I_{entering} - \sum I_{leaving} = 0 \quad \Rightarrow \quad \sum I_{entering} = \sum I_{leaving}$$

Thus it may also be restated as

"The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction".

- This rule is a statement of conservation of electric charge i.e. charges neither created nor destroyed rather it transferred from one body to the other.

If we apply this rule to the junction shown below

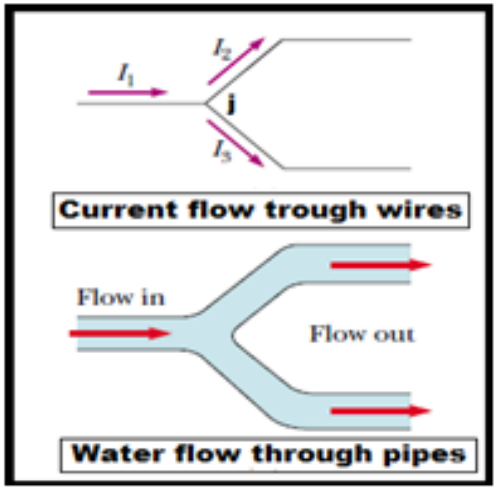
$$\sum I_j = 0 \quad \Rightarrow \quad \sum I_{entering} + \sum I_{leaving} = 0$$

$$\sum I_{entering} = I_1 \quad \text{and} \quad \sum I_{leaving} = I_2 + I_3$$

Therefore $I_1 + I_2 + I_3 = 0 \Rightarrow I_1 - (I_2 + I_3) = 0 \Rightarrow$

$$I_1 = (I_2 + I_3) \qquad \text{KCL (junction rule)}$$





2. ***Loop rule***

It is also known as Kirchhoff's voltage law [KVL]

It is stated as

"The algebraic sum of all the voltage around a ***loop*** is zero".

Loop is a complete path for the flow of current.

- The sum of the potential differences across all elements around any closed circuit loop must be zero:

This rule follows from the law of conservation of energy i.e. "energy is neither created nor destroyed". Let us imagine moving a charge around a closed loop of a circuit.
When the charge returns to the starting point, the charge–circuit system must have the same total energy as it had before the charge was moved. The sum of the increases in energy as the charge passes through some circuit elements must equal the sum of the decreases in energy as it passes through other elements. The potential energy decreases whenever the charge moves through a potential drop (*IR*) across a resistor or whenever it moves in the reverse direction through a source.

$$\sum V = 0 \quad \Rightarrow \quad \sum V_{source} + \sum V_{Resistors} = 0 \Rightarrow \quad \sum V_S + \sum V_R = 0$$

but $V_R = IR$ (are negative since they despite or drop electrical energy)

$$\sum V_S + \sum IR = 0 \quad \sum V_S - \sum IR = 0 \quad \sum V_S = \sum IR$$

$$\sum V_S = \sum IR \qquad \text{[KVL]}$$

Sign Convention

$\sum V_S + \sum IR = 0$	$\sum V_S = \sum IR$
For Current (I) or	**For Current (I) or**
** *For voltage dropped on resistors* ($V_R = IR$)	** *For voltage dropped on resistors* ($V_R = IR$)
➢ When current has the same direction with the chosen loop direction then take it as **Negative**.	➢ When current has the same direction with the chosen loop direction then take it as **Positive**.
➢ When current has the opposite direction with the chosen loop direction then take it as **Positive**.	➢ When current has opposite direction with the chosen loop direction then take it as **Negative**.
For Voltage/potential difference/ (v)	**For Voltage/potential difference/ (v)**
** *For voltage source* (V_S)	** *For voltage source* (V_S)
→ As you traverse around the chosen loop direction, if you pass over from negative (-) to positive (+) terminals of the source take the voltage to be *positive.*	→ As you traverse around the chosen loop direction, if you pass over from negative (-) to positive (+) terminals of the source take the voltage to be *positive.*
→ As you traverse around the chosen loop direction, if you pass over from positive (+) to negative (-) terminals of the source take the voltage to be *Negative*	→ As you traverse around the chosen loop direction, if you pass over from positive (+) to negative (-) terminals of the source take the voltage to be *Negative*

Example

For the circuit solved above try to determine the currents across each resistors ?

Now let us determine those currents using Kirchhoff's rules following all the rules listed above.
First after naming all the resistors and source voltages just as shown in the figure above,

- Identify the number of unknown variables to be solved. i.e. I_1, I_2 *and* I_3 as in this case. Thus to solve for three unknowns we need three equations.
- Identify the number of junctions and apply the rule (KCL). i.e. there are six points (a, b, c, d, e, and f) but not all points are junctions among those there are only two junctions that are point (b) and (e).

Then apply KCL at each junctions.

i) At point b

$$\sum I_b = 0 \quad \Rightarrow \quad \sum I_{entering} + \sum I_{leaving} = 0$$

$$\sum I_{entering} = I_1 \text{ and } \sum I_{leaving} = I_2 + I_3$$

Therefore $I_1 + I_2 + I_3 = 0 \Rightarrow I_1 - (I_2 + I_3) = 0 \Rightarrow$

$$I_1 = I_2 + I_3$$

ii) At point e

$$\sum I_e = 0 \quad \Rightarrow \quad \sum I_{entering} + \sum I_{leaving} = 0$$

$$\sum I_{entering} = I_2 + I_3 \text{ and } \sum I_{leaving} = I_1$$

Therefore $I_2 + I_3 + I_1 = 0 \Rightarrow (I_2 + I_3) - I_1 = 0 \Rightarrow (I_2 + I_3) = I_1$

$$I_2 + I_3 = I_1 \text{ ----- equation 1}$$

But from the two junctions, we have got only one equation. i.e. $I_1 = I_2 + I_3$

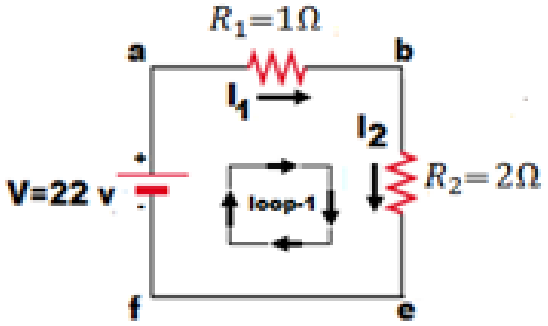
Since we just finish the number of junctions, we cannot get any more equations from junctions. Thus, we need the rest two equations from loop rule.

- Identify the number of loops from the circuit, and give name. in this case we have three loops from the above circuit
  (abefa) as loop-1,
  (bcdeb) as loop-2, and
  (acdfa) as loop-3.
- Choose the direction or traverse to write our loop equations i.e. clockwise or counter-clockwise.
  *** for example take loop-1 as we traverse along (abefa) is clockwise but along (afeba) is anti- clockwise.
- Apply loop rule (KVL) for any of the two loops since we just left only for two equations.
  - Apply KVL for loop-1 let us take clockwise (abefa)

We can use any of the techniques i.e.

$\sum V = 0 \quad \Rightarrow \quad \sum V_S + \sum IR = 0$ or

$\sum V_S = \sum IR$



For $\sum V_S + \sum IR = 0$ just write using addition at the beginning the think about the sign conventions

* $V + I_1R_1 + I_2R_2 = 0$

from the sign convention the source (v) is positive (+) but I_1 and I_2 are negative (-) for the same direction. so the above equation becomes.

$(+)V + (-)I_1R_1 + (-)I_2R_2 = 0 \Rightarrow V - I_1R_1 - I_2R_2 = 0 \quad \Rightarrow \quad V = I_1R_1 + I_2R_2$

OR

$\sum V_S = \sum IR$ just write using addition at the beginning the think about the sign conventions

* $V = I_1R_1 + I_2R_2$

from the sign convention the source (v) is positive (+) but I_1 and I_2 are positive (+) for the same direction. so the above equation becomes.

$(+)V = (+)I_1R_1 + (+)I_2R_2 \Rightarrow V = I_1R_1 + I_2R_2 \quad \Rightarrow \quad V = I_1R_1 + I_2R_2$ the two equations are the same.

Therefore our second equation becomes

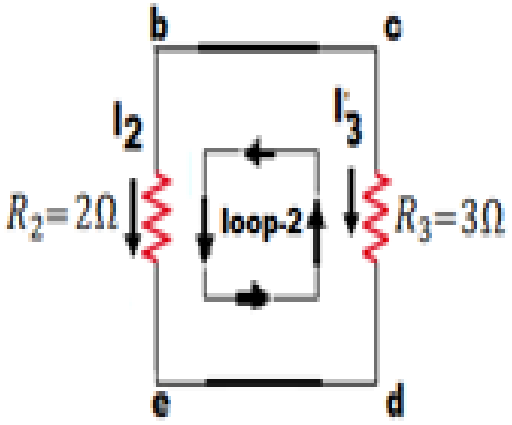
$$V = I_1R_1 + I_2R_2 - - - -equation\ 2$$

- Apply KVL for loop-2 Now let us take Anti- clockwise direction (bedcb)

We may again use any of the techniques i.e.

$\sum V = 0 \Rightarrow \sum V_S + \sum IR = 0$ or

$\sum V_S = \sum IR$



For $V_S + I_2R_2 + I_3R_3 = 0$ just write using addition at the beginning the think about the sign conventions, and note that there are no source around the loop $V_S = 0$

* $I_2R_2 + I_3R_3 = 0$

from the sign convention I_2 is negative(-) since it is the same direction as the loop whereas I_3 is positive (+) since it is opposite direction as the loop. so the above equation becomes.

$(-)I_1R_1 + (+)I_2R_2 = 0 \Rightarrow -I_2R_2 + I_3R_3 = 0 \Rightarrow I_3R_3 = I_2R_2$

OR

$\sum V_S = \sum IR$ just write using addition at the beginning the think about the sign conventions

* $V_S = I_2R_2 + I_3R_3$

note that there are no source around the loop $V_S = 0$

* $0 = I_2R_2 + I_3R_3$

from the sign convention I_2 is positive (+) since it is the same direction as the loop whereas I_3 is negative(-)since it is opposite direction as the loop. so the above equation becomes.

$0 = (+)I_2R_2 + (-)I_3R_3 \Rightarrow \quad I_3R_3 = I_2R_2$

For both cases the equation is identical.

$$I_3R_3 = I_2R_2 \text{ ----- equation 3}$$

- Collect and solve all the three equations simultaneously $\begin{cases} I_2 + I_3 = I_1 \text{ ----- } equation\ 1 \\ V = I_1R_1 + I_2R_2 \text{ --- } equation\ 2 \\ I_3R_3 = I_2R_2 \text{ ----- equation 3} \end{cases}$

$\begin{cases} I_2 + I_3 = I_1 \text{ ---- } equation\ 1 \\ 22 = I_1 + 2I_2 \text{ --- } equation\ 2 \\ 3I_3 = 2I_2 \text{ ----- equation 3} \end{cases}$

Substitute equation (1) on to equation (2)

$22 = (I_2 + I_3) + 2I_2$

$22 = I_3 + 3I_2 \text{ ----} equation\ 4$ but from equations substitute equation (3) on to equation (4) $I_3 = \frac{2}{3}I_2$

$22 = \left(\frac{2}{3}I_2\right) + 3I_2 = \frac{11}{3}I_2 \quad 22 = \frac{11}{3}I_2 \Rightarrow I_2 = 6v\ ,\ I_3 = \frac{2}{3}I_2 \Rightarrow I_3 = \frac{2}{3}(6V) \Rightarrow I_3 = 4V$ **and** from equation (1) $I_2 + I_3 = I_1$

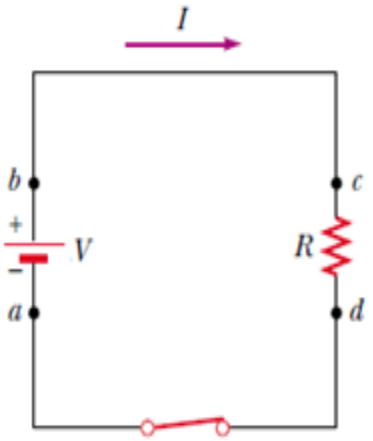
$I_1 = 6V + 4V = 10V \quad I_1 = 10V$

Therefore it is exactly the same as the common series and parallel method solved above.

$\begin{cases} I_1 = 10\,A \\ I_2 = 6V \\ I_3 = 4V \end{cases}$

Electrical energy and power

If a battery is used to establish an electric current in a conductor, there is a continuous transformation of chemical energy in the battery to kinetic energy of the electrons to internal energy in the conductor, resulting in an increase in the temperature of the conductor.



In typical electric circuits, energy is transferred from a source such as a battery, to some device, such as a light bulb or a radio receiver.

Suppose a source of potential difference (V) is connected with a load resistor (R) and current (I) flows across the circuit, as the charges flow across the resistor electrical energy is converted to some kind of energies.

$\Delta U = V\Delta q$ for a steady flow of charges

$dU = Vdq$ then taking the rate of electrical energy we can get $\frac{dU}{dt} = V\frac{dq}{dt}$ but $\frac{dU}{dt} = power\ (P)$ and $\frac{dq}{dt} = electric\ current\ (I)$

$$P = IV$$

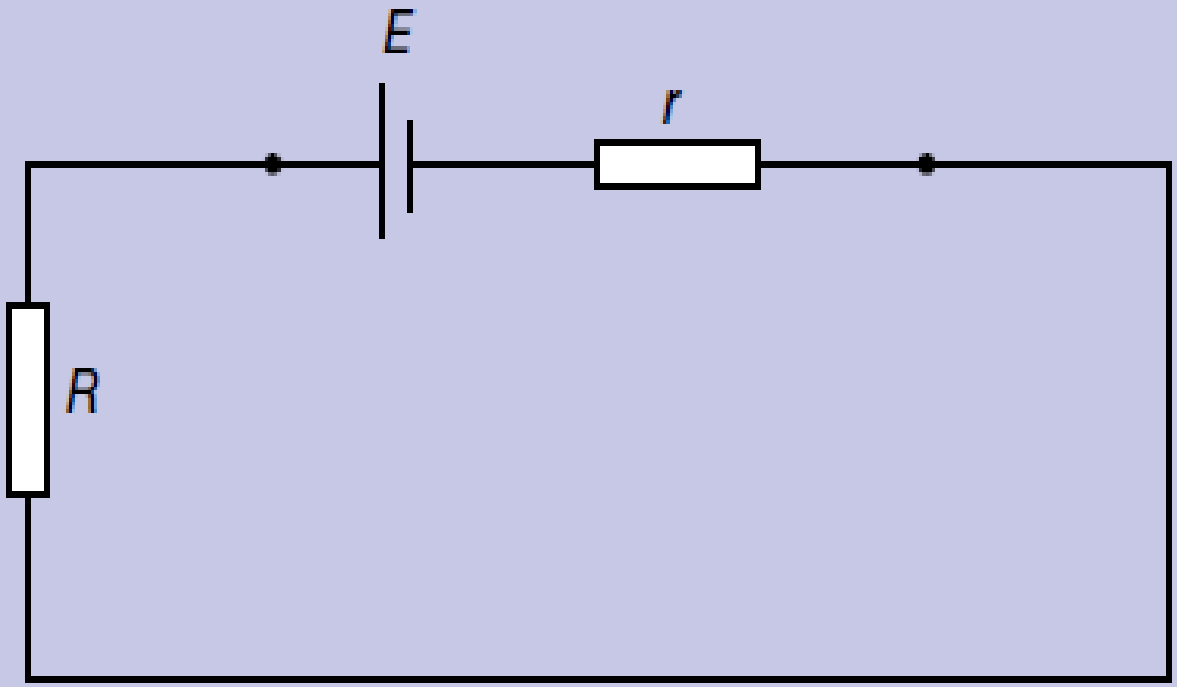
It is also possible to use it interms of resistors as. $P = IV$ but

$$R = \frac{V}{I} \Rightarrow V = IR \quad P = I(IR) = I^2R \quad P = I^2R$$

$$P = I^2R$$

Or since $I = \frac{V}{R} \Rightarrow$ P= IV $P = \left(\frac{V}{R}\right)V = \frac{V^2}{R}$ $P = \frac{V^2}{R}$

$$P = \frac{V^2}{R}$$



The battery has an e.m.f. of 12 V and an internal resistance of 3 Ω. Calculate:

a) the current it supplies to the resistor, *R*, with value 12 Ω

b) the power used in the external resistor

c) the percentage of the total power wasted in the internal resistance.

a)

Total resistance $R + r$ (Ω)	E.m.f. (V)	Current I (A)
15	12	?

Use Ohm's law $V = IR$

In this case

$$\text{Current} = \frac{\text{e.m.f.}}{\text{total resistance}}$$

$$I = \frac{12}{15} = 0.8 \text{ A}$$

b)

Power (W)	Current (A)	R (Ω)
?	0.8	12

Use power $= I^2R$

$= 0.8^2 \times 12$

$= 7.68$ W

c) We need to find the power in internal resistance, then total power, then percentage of total power wasted in internal resistance

For internal resistance:

Power (W)	Current (A)	r (Ω)
?	0.8	3

Use power $= I^2R$

$= 0.8^2 \times 3$

$= 1.92$ W

Total power $= 1.92 + 7.68$ W

$= 9.6$ W

Percentage wasted in internal resistance $= \frac{1.92 \times 100}{9.6} = 20\%$

Measuring Instruments

Current and voltage are measurable quantities.

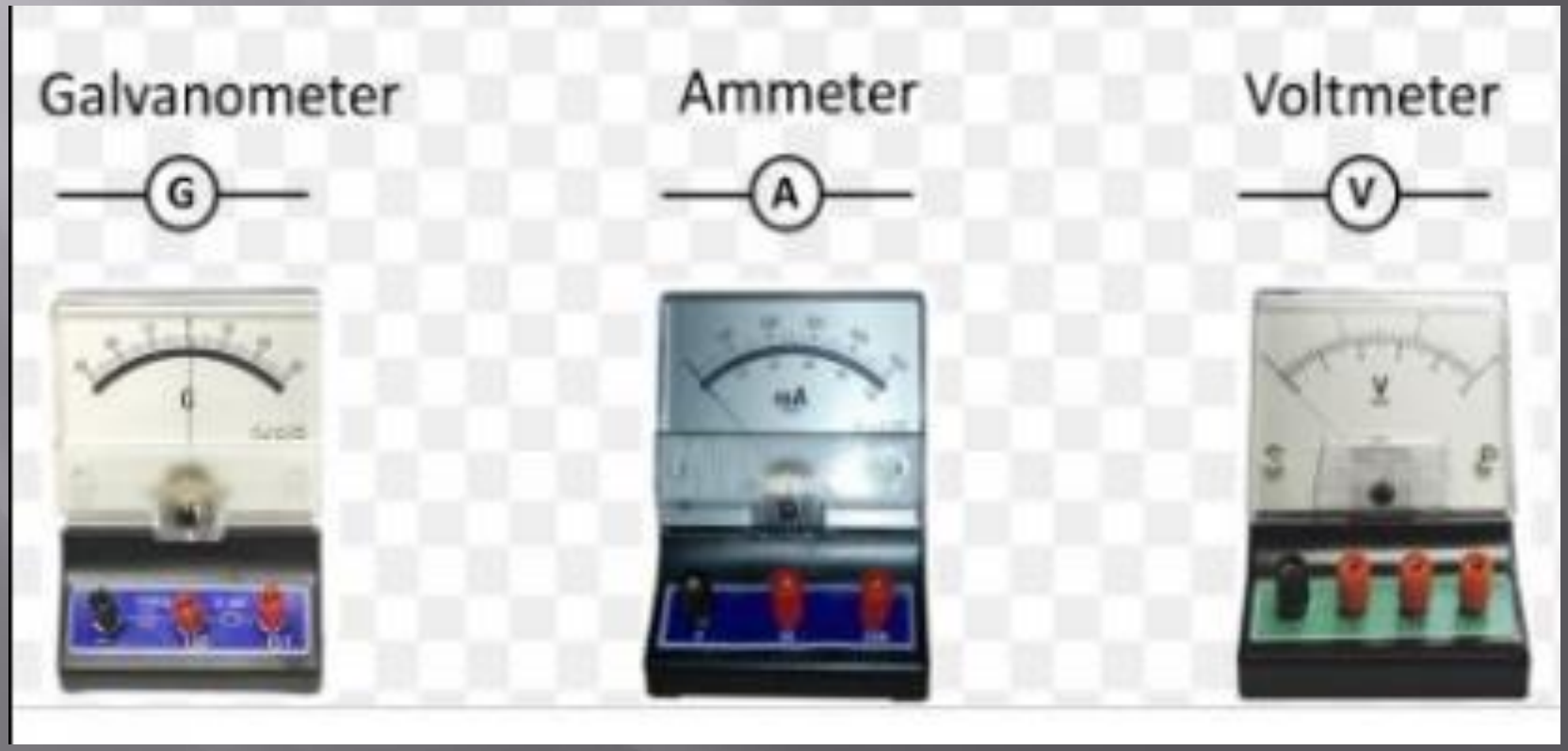
Current is measured by **Ammeter**

Voltage is measured by **Voltmeter**.

*both can be constructed using galvanometer.

Galvanometer

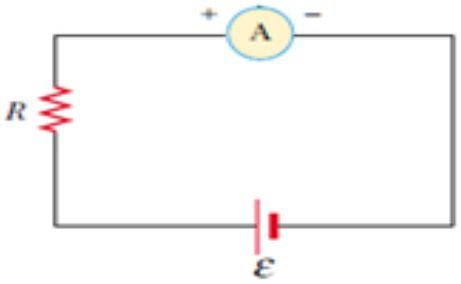
is a very sensitive device used to measure a very small amount of current.



The Ammeter

A device that measures current is called an ammeter. The charges constituting the current to be measured must pass directly through the ammeter, so the ammeter must be connected in **series** with other elements in the circuit.

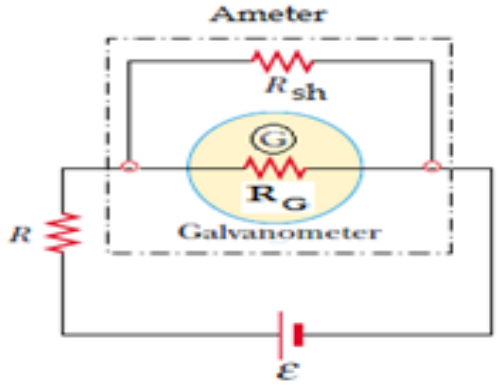
When using an ammeter to measure direct currents, you must connect it so that charges enter the instrument at the positive terminal and exit at the negative terminal.



- ***Ideally, an ammeter should have zero resistance so that the current being measured is not altered.***

In the circuit shown below, this condition requires that the resistance of the ammeter be much less than the load resistor *R*. Because any ammeter always has some internal resistance, the presence of the ammeter in the circuit slightly reduces the current from the value it would have in the meter's absence.

A typical off-the-shelf galvanometer is often not suitable for use as an ammeter, primarily because it has a resistance of about $50 - 60\Omega$. An ammeter resistance this great considerably alters the current in a circuit. You can understand this by considering the following example. The current in a simple series circuit containing a 3-V battery and a 3Ω resistor is 1 A. If you insert a 60Ω galvanometer in this circuit to measure the current (1A), the total resistance becomes 63Ω and the current is reduced to 0.02 A.



A second factor that limits the use of a galvanometer as an ammeter is the fact that a typical galvanometer gives a *full-scale deflection* (maximum reading) for currents on the order of 1 mA or less. Consequently, such a galvanometer cannot be used directly to measure currents greater than this value.

However, it can be converted to a useful ammeter by placing a shunt resistor R_{sh} *in* ***parallel*** *with the galvanometer,* as shown in the fig below. The value of R_{sh} must be much less than the galvanometer resistance so that most of the current to be measured is directed to the shunt resistor.

Since R_G and R_{sh} are connected in parallel potential difference across each remain constant i.e.

$V_G = V_{sh}$ but $V_G = R_G I_G$ and $V_{sh} = R_{sh} I_{sh}$ therefore $R_G I_G = R_{sh} I_{sh} \Rightarrow \frac{R_G I_G}{I_{sh}} = R_{sh}$

$R_{sh} = \frac{R_G I_G}{I_{sh}}$ since $I = I_G + I_{Sh} \Rightarrow I - I_G = I_{Sh}$

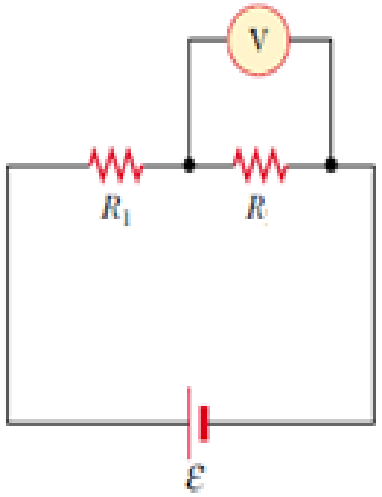
$$R_{sh} = \frac{R_G I_G}{I - I_G}$$

Where R_{sh} is shunt resistance,

I_G is the full scale reading of the galvanometer,

R_G is the safety resistor of the galvanometer $(50 - 60\Omega)$

And I is the maximum reading of the Ammeter needed to be constructed.



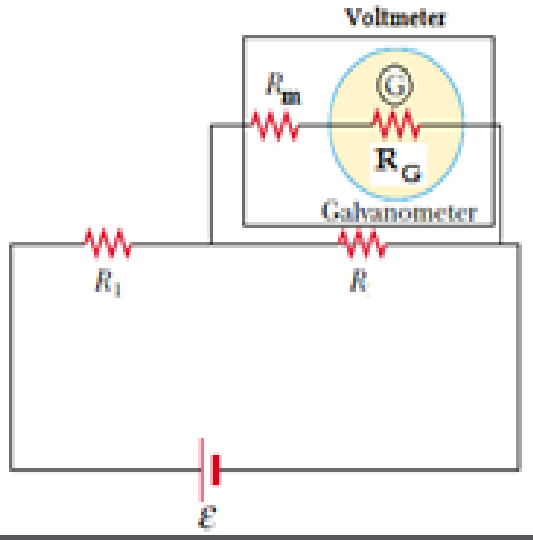
The Voltmeter

A device that measures potential difference is called a voltmeter. The potential difference between any two points in a circuit can be measured by attaching the terminals of the voltmeter between these points without breaking the circuit, as shown in Figure below. The potential difference across resistor R is measured by connecting the voltmeter in parallel with it. Again, it is necessary to observe the polarity of the instrument. The positive terminal of the voltmeter must be connected to the end of the resistor that is at the higher potential, and the negative terminal to the end of the resistor at the lower potential.

- ***An ideal voltmeter has infinite resistance so that no current exists in it.***

In Figure below, this condition requires that the voltmeter have a resistance much greater than R In practice, if this condition is not met, corrections should be made for the known resistance of the voltmeter.

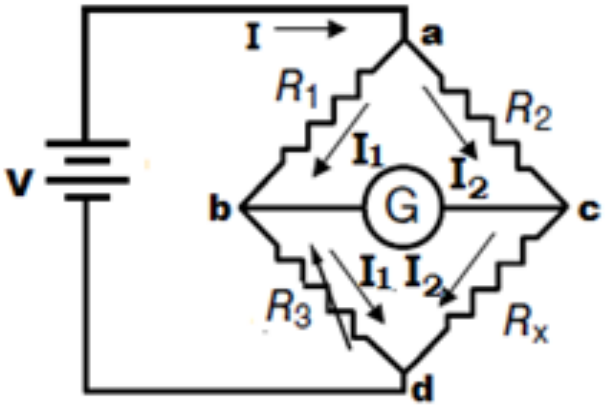
A galvanometer can also be used as a voltmeter by adding an external resistor R_m (multiplier resisters) in **series with it,* as shown in Figure below. In this case, the external resistor (R_m) must have a value much greater than the resistance of the galvanometer to ensure that the galvanometer does not significantly alter the voltage being measured.



The Wheatstone bridge

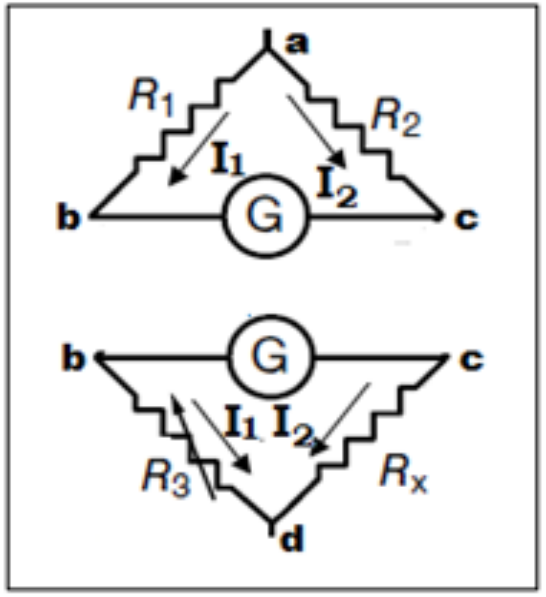
It is the basic electrical circuit used to determine the unknown values of resistors and capacitors. It was invented by Samuel Hunter Christie but it was more familiarized by Sir Charles Wheatstone.

In its classic form, a galvanometer is connected between the output terminals, and is used to monitor the current flowing from one voltage divider to the other. If the two voltage dividers have exactly the same current flow across each portion of circuit; the circuit the bridge is said to be *balanced* and no current flows in either direction through the galvanometer.



If one of the resistors changes even a little bit in value, the bridge will become unbalanced and current will flow through the galvanometer. Thus, the galvanometer becomes a very sensitive indicator of the balance condition.

In its basic application, a D.c. voltage (V) is applied to the Wheatstone bridge, and a galvanometer (G) is used to monitor the balance condition. The values of R_1 and R_2 are precisely known, but do not have to be identical. R_3 is a calibrated variable resistance, the current value of which may be read from a dial or scale. An unknown resistor, R_x, is connected as the fourth side of the circuit, as shown in Figure below, and power is applied. R_2 is adjusted until the galvanometer, G, reads zero current. At this point,



From Δabc writing the loop rule for a complete path

$$-R_1 I_1 + I_2 R_2 + I_G R_G = 0 \quad \text{Since } I_G = 0 \text{ after Balance}$$

$$-R_1 I_1 + I_2 R_2 = 0 \quad R_1 I_1 = I_2 R_2 \Rightarrow \quad \boxed{\frac{I_1}{I_2} = \frac{R_2}{R_1} \ldots\ldots eq\ 1}$$

From Δdbc writing the loop rule for a complete path

$$-R_3 I_1 + I_2 R_x + I_G R_G = 0 \quad \text{Since } I_G = 0 \text{ after Balance}$$

$$-R_3 I_1 + I_2 R_x = 0 \quad R_3 I_1 = I_2 R_x \Rightarrow \quad \boxed{\frac{I_1}{I_2} = \frac{R_x}{R_3} \ldots\ldots eq\ 2}$$

Equating the above two equations we can get the basic equation of Wheatstone bridge.

$$\frac{R_x}{R_3} = \frac{R_2}{R_1} \qquad R_x = \frac{R_2}{R_1}(R_3)$$

$$R_x = \frac{R_2 R_3}{R_1}$$

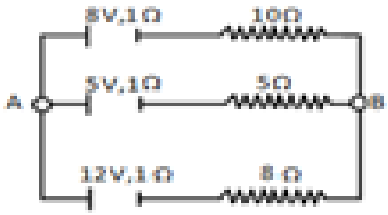
ETHIOPIAN UNIVERSITY ENTERANCE EXAM (EUEE) ON

Kirchhoff's rule and e.m.f.

1. A cell supplies a current of 0.9A through a 2Ω resistor and a current of 0.3A through a 7 Ω resistor. What is the internal resistance of the cell is

   A) 0.5 Ω B) 10 Ω C)1.2 Ω D) 2.0 Ω

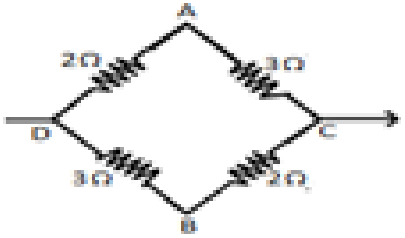
2. Determine the current in the 5 Ω resistor in the figure shown below



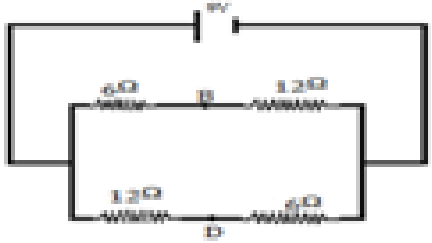
   A) 7.85A
   B) 1.09A
   C) 0.48A
   D) 0.18A

3. There is a current of 2A in the system of conductors shown in the figure below. The potential difference V_A-V_B

   A) -2v
   B) +1V
   C) -1V
   D) +2V



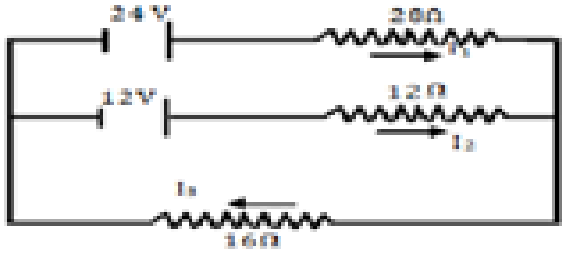
4. In the circuit sown below, what is the potential difference between points B and D?



   A) 1V C) 3V

   B) 2V D) 4V

5. In figure below the current in the 16 Ω resistor is



   a) 0.64 A
   b) 0.15A
   c) 0.49A
   d) 1.28A

Thank you for watching!!!