Dear students, during the normal class, we have covered unit 7 up to pure inductive, pure resistive and pure capacitive circuits, and also solved problems. Now, I will give you the solutions of the problems on the remaining portions of unit 7 with their respective solutions. Please, study all other sections by yourself and meanwhile refer solutions for each section.

Resistive AC circuit

For a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor.

Capacitive AC circuit

For a sinusoidally applied voltage, the current always leads the voltage across a capacitor by 90° or Π/2.

Inductive AC circuit

For a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90° (one-quarter cycle in time).

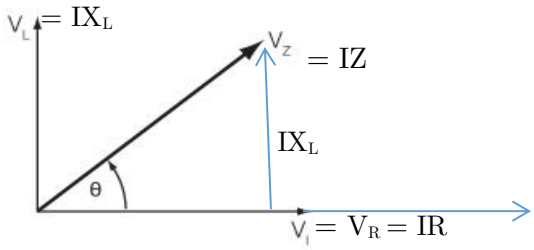
The power factor in an a.c. circuit

power factor = cos θ

where θ is the phase difference between the p.d. and the current.

Series RL AC circuit

The p.d. in an inductive circuit leads the current. The phase d/ce b/n V and I is not Π/2, but an angle θ.



If $\theta = 90^0$, then it is pure inductive

If $\theta = 0^0$, then it is pure resistive

$I_R = I_L$ (In series connection current is constant)

Using Pythagoras's theorem, the total resistance from the resistor and inductor is called Impedance (Z). It is given by: $Z = \sqrt{R^2 + \omega^2 L^2}$

and the phase difference θ is given by: $\theta = \tan^{-1}\frac{\omega L}{R}$

Q. An inductor of 0.8 H is connected in series with a 100 Ω resistor. It is connected to an alternating supply of 12 V at 50 Hz.

a) Calculate the reactance of the inductor.

$X_L = wL = (2\Pi f)\ L = 2\ \Pi\ (50)(0.8)\ \Omega = 80\Pi\ \Omega = 251.3\ \Omega$

b) Calculate the impedance of the circuit.

$Z = \sqrt{R^2 + \omega^2 L^2} = \{100^2 + (251.3)^2\}^{1/2}\ \Omega = 270.5\ \Omega$

c) Work out the current drawn from the supply.

Using ohm's law: V = I Z

$I = V/Z = (12/270.5)\ A = 0.044\ A$

d) i) Find the phase angle between the current and the voltage.

$\theta = \tan^{-1}\frac{\omega L}{R} = \tan^{-1}(270.5/100)$ {since $wL = X_L$}

$\Theta = 69.7^0$

ii) Which is ahead – the current or the voltage?

→ The voltage

e) Find the power factor for this circuit.

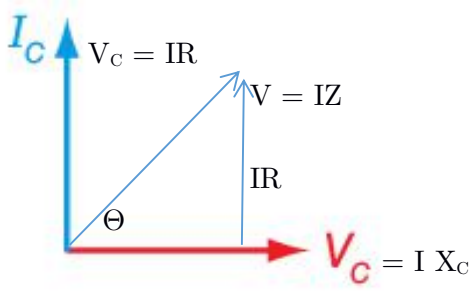
Power factor = cos Θ

Power factor = cos 69.7^0

Power factor = 0.35

Series RC AC circuit

The current in a capacitive circuit leads the voltage. The phase d/ce b/n V and I is not Π/2, but an angle θ.



If θ is 90^0, then it is pure capacitive
If θ is 0^0, then it is pure resistive

The impedance, Z, is given by

$$Z = \sqrt{R^2 + (X_c)^2}$$

and the phase angle θ is given by

$$\theta = \tan^{-1}\frac{X_c}{R}$$

Q. An alternating supply of 12 V and frequency 50 Hz is used in a circuit which has a resistor of 5 Ω and a capacitor of 100 μF in series. Calculate

a) the total impedance of the circuit

$X_C = 1/wC = 1/(2\Pi f)C = 1/\{(100\Pi)(100\mu)\}\ \Omega = 100/\Pi\ \Omega = 31.8\ \Omega$

$$Z = \sqrt{R^2 + (X_c)^2} = \{5^2 + (31.8)^2\}^{1/2}\ \Omega = 32.2\ \Omega$$

b) the current in the circuit

Using ohm's law: V = I Z

$I = V/Z = (12/32.2)\ A = 0.37\ A$

c) the phase angle between the supply and the current

$$\theta = \tan^{-1}\frac{X_c}{R} = \tan^{-1}(31.8/5)$$

$\Theta = 81.1^0$

d) the power factor for the circuit

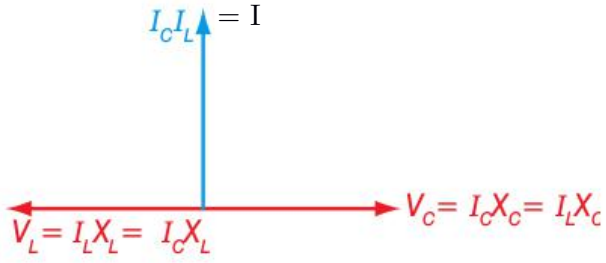
Power factor = cos Θ

Power factor = cos 81.1^0

Power factor = 0.16

Series LC AC circuit

When the power supply is connected, there will be oscillations between the inductor and the capacitor. The current through the capacitor, I_C , is the same as the current through the inductor, I_L .



For pure inductive V_L leads I by 90^0
For pure capacitive circuit I leads V_C by 90^0

Since V_L and V_C are in opposite directions, the impedance of this circuit is given by

$$Z^2 = (X_L - X_C)^2$$

There will therefore by a frequency, f, at which $X_L = X_C$ and the impedance will be zero. This will happen when

$$2\pi fL = \frac{1}{2\pi fC}$$

The value of f is called the resonant frequency and is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Q. Find the resonant frequency for a circuit containing a 30 μH conductor and a 2 pF

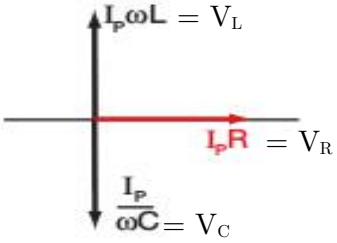
$$f = \frac{1}{2\pi\sqrt{LC}} = 1/(2\Pi)\ 1/(30\ \mu)(2\ p) = (1/12\Pi)\ x\ 10^{15} Hz = 0.0265\ x\ 10^{15} Hz$$

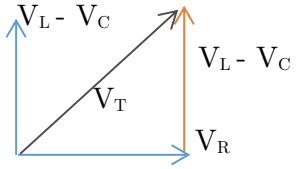
$$f = 2.65\ x\ 10^{13} Hz$$

The RLC Series Circuit

The peak potential differences for a circuit such as the one shown in Fig. below are given by:

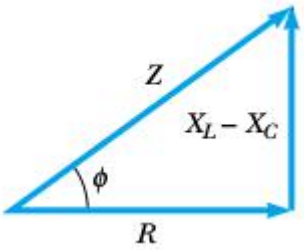
$$V_R = I_{peak} R$$
$$V_L = I_{peak} X_L$$
$$V_C = I_{peak} X_C$$





The sum of the p/c across the circuit, using Pythagoras theorem, may be written as:

$$V_T = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$
$$= \sqrt{(I_{peak}R)^2 + (I_{peak}X_L - I_{peak}X_C)^2}$$
$$= I_{peak}\sqrt{R^2 + (X_L - X_C)^2} = I_{peak} Z$$



Therefore the impedance of the circuit may be written as:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The phase angle between V and I for the circuit is given by:

$$\tan\theta = \frac{X_L - X_C}{R}$$

When $X_L > X_C$ the phase angle is positive so the current lags behind the applied p.d.
When $X_L < X_C$ the phase angle is negative and the current leads the applied p.d.
When $X_L = X_C$ the phase angle is zero and it becomes pure resistive. The resonant frequency is given by:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Q. A series RLC AC circuit has R = 425Ω, L = 1.25 H, C = 3.50 μF, w = 377 1/s , and ΔV max = 150 V.

a) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.

$X_L = wL = (377)(1.25)\ \Omega = 471.25\ \Omega$

$X_C = 1/wC = 1/\{(377)(3.5\mu)\}\ \Omega = 757.86\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \{(425)^2 + (471.25 - 757.86)^2\}^{1/2}\ \Omega = 512.61\ \Omega$

b) Find the maximum current in the circuit.

Using ohm's law: V = I Z

$I_{max} = V_{max}/Z = (150/512.61)\ A = 0.29\ A$

c) Find the phase angle between the current and voltage.

$\tan\theta = \frac{X_L - X_C}{R} = (471.25 - 757.86)/(425) = -1.74$

$\Theta = \tan^{-1}(-1.74) = -60.1^0$

d) Find both the maximum voltage and the instantaneous voltage across each element.

The maximum voltage across each element;

$V_{(max)R} = I_{max}R = (0.29)(425)\ V = 123.25\ V$

$V_{(max)L} = I_{max}X_L = (0.29)(471.25)\ V = 136.66\ V$

$V_{(max)C} = I_{max}X_C = (0.29)(757.86)\ V = 219.78\ V$

The instantaneous voltage across each element;

$V(t) = V_{(max)R}\sin(wt) = 123.25\sin(377t)\ V$

$V(t) = V_{(max)L}\sin(wt) = 136.66\sin(377t)\ V$

$V(t) = V_{(max)C}\sin(wt) = 219.78\sin(377t)\ V$

7.4 Power in a.c. Circuits

Average power in RLC circuit is given by:

$$P_{av} = V_{rms}\, I_{rms} \cos(\theta)$$

where the quantity $\cos(\theta)$ is called the power factor.

Note:-

1) In a purely capacitive circuit, we know that $\theta = 90°$ and $\cos 90° = 0$ so $P_{av} = 0$.

2) In a purely inductive circuit, we know that $\theta = 90°$ and $\cos 90° = 0$ so $P_{av} = 0$.

4) In a purely resistive circuit, we know that $\theta = 0°$ and $\cos 0° = 1$ so $P_{av} = V_{rms}\, I_{rms}$.

Real, apparent and ideal power in an RLC circuit

Real power power transferred when the load is purely resistive.

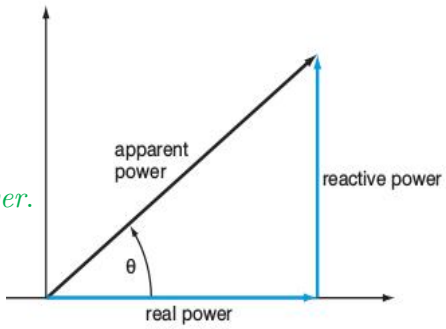
The value of the real power is given by:

real power = current × resistance = supply p.d. × I_{rms}

If the load is purely reactive (a capacitor or an inductor)

then the current and the supply voltage are out of phase by 90°

and there is no net flow of energy to the load. *The power is reactive power.*



Apparent power is the vector sum of real and reactive power,

as shown in Fig. below. We can calculate apparent power using:

$$\textit{apparent power} = (I_{rms})^2\, Z$$

where Z is the impedance in the circuit.

The *ideal power* is where apparent power and true power to be equal, since the difference between real power and apparent power is wasted. In this case, the power factor is 1. This can occur when there is no reactance. To have no reactance X_L and X_C must be equal, so that the reactance = $X_L - X_C = 0$.

Q. A series RLC circuit is as shown in the diagram.

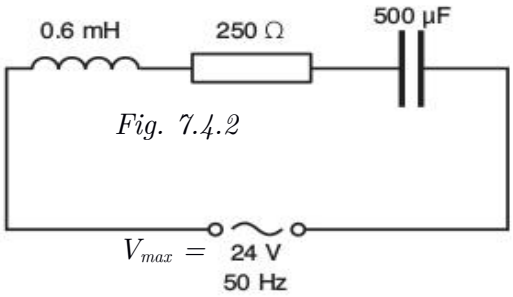


Fig. 7.4.2

Find:

a) the average power for the RLC circuit.

$X_L = wL = (2\Pi fL) = (2\Pi)(50)(0.6m)\ \Omega = 0.06\Pi\ \Omega = 0.19\ \Omega$

$X_C = 1/wC = 1/(2\Pi fC)\ \Omega = 1/\{(2\Pi)(50)(500\mu)\} = 100/5\Pi\ \Omega = 20/\Pi\ \Omega = 6.37\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \{(250)^2 + (0.19 - 6.37)^2\}^{1/2}\ \Omega = 512.61\ \Omega = 250\ \Omega$

$\tan\theta = \frac{X_L - X_C}{R} = (0.19 - 6.37)/(250) = -0.025$

$\Theta = \tan^{-1}(-0.025) = -1.42$

$V_{rms} = V_{max}/\sqrt{2} = 24/\sqrt{2}\ V = 16.97\ V$

$I_{rms} = V_{rms}/Z = 16.97/250\ A = 0.068\ A$

$P_{av} = V_{rms}\, I_{rms} \cos(\theta)$

$P_{av} = (16.97)(0.068)\cos(-1.42)\ W = 1.15\ W$

b) the power factor for the circuit

Power factor = $\cos(\theta) = \cos(-1.42) = 0.99$

c) the apparent power for the circuit

apparent power = $(I_{rms})^2\, Z = (0.068)^2\ (250)\ W = 1.16\ W$

d) the value of C needed to make the power in this circuit ideal.

$X_L = X_C \rightarrow X_L = 1/wC \rightarrow C = 1/wX_L = 1/\{(2\Pi f)(0.19)\}\ F = 1/19\Pi\ F = 0.017\ F$

Additional problems

1. *In a purely inductive circuit, the current lags the voltage by 90^0*

   Ans B. one-fourth of a cycle

2. *A 35μF capacitor is connected to a source of sinusoidal emf with a frequency of 400 Hz and a maximum emf of 20 V. The maximum current is:*

   $X_C = 1/wC = 1/(2\Pi fC) = 1/(800*35\Pi\mu)\ \Omega = 11.37\ \Omega$

   Using ohm's law: $V = I\ Z$

   $I_{max} = V_{max}/Z = (20/11.37)\ A = 1.8\ A$

   Ans. C.

3. *The impedance of an RLC series circuit is definitely increased if R increases*

   $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

   If X_L increases, we don't have any idea on the value of X_C for comparison.

   Ans. D.

4. *An electric motor, under load, has an effective resistance of 30 Ω and an inductive reactance of 40 Ω. When powered by a source with a maximum voltage of 420 V, the maximum current is:*

   RL series circuit

   $Z = \sqrt{R^2 + (X_c)^2} = (30^2 + 40^2)^{1/2}\ \Omega = 50\ \Omega$

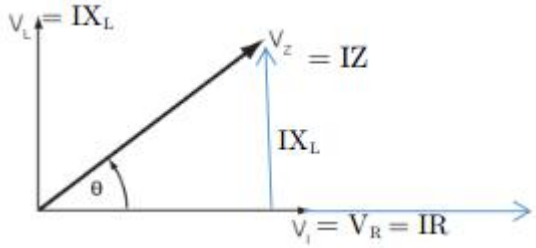
   Using ohm's law: $V = I\ Z$

   $I_{max} = V_{max}/Z = (420/50)\ A = 8.5\ A$

   Ans. B

5. *An RL series circuit is connected to an ac generator with a maximum emf of 20 V. If the maximum potential difference across the resistor is 16 V, then the maximum potential difference across the inductor is:*

   RL series circuit



   $V_T^2 = V_R^2 + V_L^2$

   $(20)^2 = (16)^2 + V_L^2$

   $400 = 256 + V_L^2$

   $V_L^2 = 400 - 256 = 144\ V^2$

   $V_L = 12\ V$

   Ans. C.

6. *An ac generator produces 10 V (rms) at 400 rad/s. It is connected to a series RL circuit (R = 17.3 Ω, L = 0.025 H). The rms current is:*

   RL series circuit

   $X_L = wL = 400*0.025\ \Omega = 10\ \Omega$

   $Z = \sqrt{R^2 + (X_c)^2} = \{17.3^2 + 10^2\}^{1/2}\ \Omega = 19.98\ \Omega$

   $$\theta = \tan^{-1}\frac{\omega L}{R}$$

   $\theta = \tan^{-1}(10/17.3)$

   $\theta = 30^0$

   Using ohm's law: $V = I\ Z$

   $I_{rms} = V_{rms}/Z = (10/19.98)\ A = 0.50\ A$

   Ans. A

7. *A sinusoidal voltage $\Delta v(t) = (40V)\ sin(100t)$ is applied to a series RLC circuit with L = 160 mH, C = 100 μF, and R = 70 Ω.*

   (a) What is the impedance of the circuit? (w = 100 rad/s)

   $X_L = wL = 100*0.16\ \Omega = 16\ \Omega$

   $X_C = 1/wC = 1/\{100*100\mu\}\Omega = 100\ \Omega$

   $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

   $= \{70^2 + (16 - 100)^2\}^{1/2}\ \Omega$

   $= 109.3\ \Omega$

   (b) What is the maximum current?

   Using ohm's law: $V = I\ Z$

   $I_{max} = V_{max}/Z = (40/109.3)\ A = 0.37\ A$

(c) Find the average power delivered to the circuit.

$$\tan\theta = \frac{X_L - X_C}{R} = (16 - 100)/(70)$$

$$= -1.2$$

$$\Theta = \tan^{-1}(-1.2) = -50.2^0$$

$$\cos(\Theta) = \cos(-50.2^0)$$

$$= 0.64$$

$$(V_{rms})(I_{rms}) = (V_{max})(I_{max})/2$$

$$= (40)(0.37)/2$$

$$= 7.4 \text{ W}$$

Thus,

$$P_{av} = (V_{rms}\ I_{rms})\cos(\theta)$$

$$P_{av} = (7.4 \text{ W})(0.64) \text{ W}$$

$$= 4.74 \text{ W}$$

8. A transformer has $N_1 = 350$ turns and $N_2 = 2\,000$ turns. If the input voltage is $\Delta v(t) = (170\ V)\cos wt$, what rms voltage is developed across the secondary coil?

Transformer equation

$$N_1/N_2 = V_1/V_2$$

$$350/2000 = V_{max}1/V_{max}2$$

$$35/200 = 170/V_{max}2$$

$$V_{max}2 = (200)(170)/35$$

$$V_{max}2 = 971.4\ V$$

$$V_{rms}2 = V_{max}2/\sqrt{2}$$

$$V_{rms}2 = (971.4)/\sqrt{2}\ V$$

$$V_{rms}2 = 686.9\ V$$

Dear students, please read all the contents from the handout seriously and do all the questions by referring the solutions. Next time we will discuss and solve problems on unit 8 (Atomic physics).

See you soon !!!! ***Mr. Mesay T.***