

6.1 Rotation about a fixed axis

Purely rotational motion occurs if every particle in the body moves in a circle about a single line. This line is called **the axis of rotation**.

Axis of rotation: the axis about which a body rotates

For instance, the Earth's axis of rotation goes between the geographic north and south poles. One of the term that describe the rotational motion is **angular velocity** ($\omega = \text{omega}$), which is measured in radians per second (rad / sec).

If the flywheel turns through an angle θ in a time t , its angular velocity ω is defined by

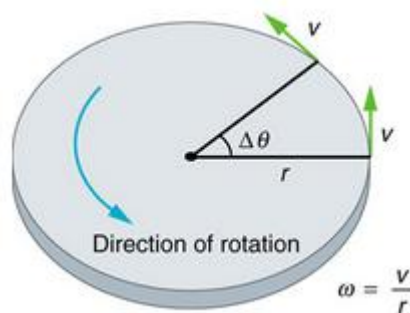
$$\omega = \theta/t$$

Remember that **radians** are dimensionless units.

The linear speed v and angular speed ω of a point on a rotating body are connected by the equation:

$$v = r\omega$$

where r is the distance of the point from the axis of rotation.



Problems

- 1] A car engine rotates at 3600 revolutions per minute. What is its angular velocity in rad/s?
- 2] The linear velocity of a point on a music CD is a constant 1.0 m/s when it is being played. At the start of a CD, this point is 25 mm from the axis of rotation. At the end it is 50 mm from the axis of rotation. What is the angular velocity of the point at the start and the end?
- 3] A car is travelling at 16 m/s. The car wheel has a diameter of 0.8 m. What is the angular velocity of the wheel? Give your answer in rad/s.
- 4] A turbine is rotating at 3000 rev/min.
 - a) What is the angular velocity of the turbine? Give your answer in rad/s.
 - b) A turbine blade is 0.5 m long. Calculate the linear velocity of a point on the end of a turbine blade and halfway down a blade.

6.2 Torque and angular acceleration

Torque: the turning effect of a force round a point; it is also a force

It is the total moment acting on that body about the axis of rotation and is measured by multiplying the force by its perpendicular distance from the axis (see Figure below).

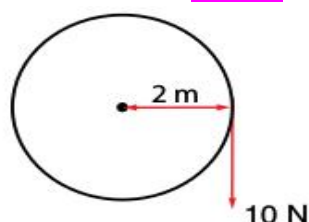
That is,

$$\text{Torque} = \text{Force} \times \text{Perpendicular distance from the axis}$$

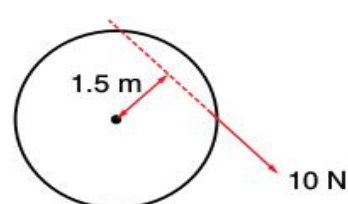
$$\tau = r F$$

The value of a torque depends on the distance from the axis.

The linear equivalent of a **torque** is a **force**.



$$\text{Torque about axis} = 10 \text{ N} \times 2 \text{ m} = 20 \text{ N m}$$



$$\text{Torque about axis} = 10 \text{ N} \times 1.5 \text{ m} = 15 \text{ N m}$$

Since torque, the applied force, the displacement from the axis and the torque are all vectors, we can say that torque is also the vector product of two vectors:

$$\tau = r \times F = |r| |F| \sin \theta \, n$$

where n is the unit vector that is perpendicular to the plane of the two vectors.

In three dimensions, it can be written as:

$$\tau = (r_y F_z - r_z F_y) \hat{i} + (r_z F_x - r_x F_z) \hat{j} + (r_x F_y - r_y F_x) \hat{k}$$

Rotational work done

In linear terms the work done by a force is $F \cdot s$, the force F multiplied by the displacement s it moves in that direction.

The rotational equivalent of linear displacement (s) is angular displacement (θ = the angle through which the torque acts).

The angular equivalent of force is torque, τ .

Thus, substitute τ and θ into the equation for work done ($W = F \cdot s$):

$$W = \tau \cdot \theta$$

Torque has the same units as work done, newton metres (N m), but the units are not called joules, unlike for work done.

Angular acceleration

Consider a flywheel that is rotating with an angular velocity ω . When the rate of rotation of the flywheel begins to increase, it is experiencing an angular acceleration α .

If its angular velocity increases uniformly by $\Delta\omega$ over a time t , this is defined by:

angular acceleration $\alpha = \Delta\omega/t$

The units of angular acceleration are rad/s^2 .

Problems

- 1] A CD in a CD drive in a computer accelerates from rest to 2000 revolutions per minute in 4 seconds. What is its angular acceleration?
- 2] A mechanic has two spanners, one 15 cm long and the second 20 cm long. He applies a force of 30 N with the first spanner, and 20 N with the second. The forces are perpendicular to the spanner.
 - a) With which spanner does he produce the greatest torque?
 - b) How much work does he do with each spanner if he turns a nut one quarter of a turn?
- 3] A force $(2, -5)$ m acts at a displacement of $(3, 1)$ from a point.
What is the torque exerted by the force?
- 4] A hard drive in a computer accelerates from rest to 5400 rev/min in 2 seconds. What is the angular acceleration of the hard drive?
- 6] A tap in a hospital has a rod attached to the top of it which is 0.25 m long. The rod is perpendicular to the axis of rotation of the tap. It takes a force of 25 N quarter of a turn to turn the tap on.
 - a) What is the torque exerted on the tap?
 - b) How much work is done in turning on the tap?
- 7] A circular saw is powered by a motor. When the saw is used to cut wood, the wood exerts a torque of $0.5 \text{ N} \cdot \text{m}$ on the saw blade. If the blade rotates with a constant angular velocity of 20 rad/s the work done on the blade by the motor in 1.0 min is:
- 8] A disk starts from rest and rotates around a fixed axis, subject to a constant net torque. The work done during the first 5 s -----work done by the torque during the second 5 s is
 - A. the same
 - B. twice as much
 - C. half as much
 - D. four times as much
 - E. one-fourth as much

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