

Chapter 2

Kinematics and Dynamics of Particles

Newtonian Dynamics and Planetary Motion (6hrs)

CONTENT

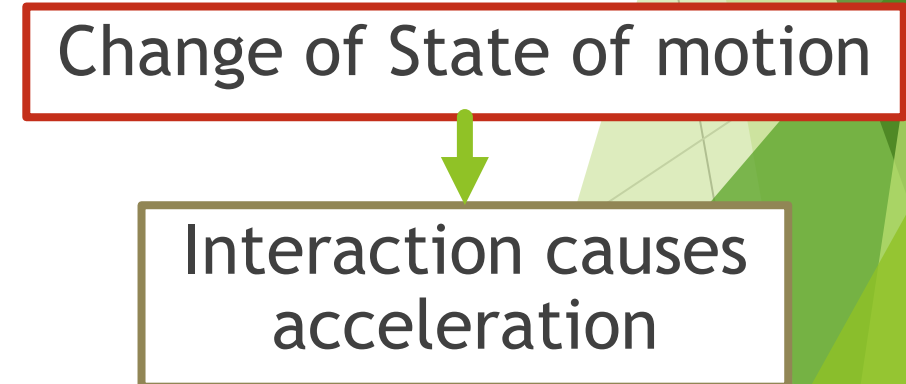
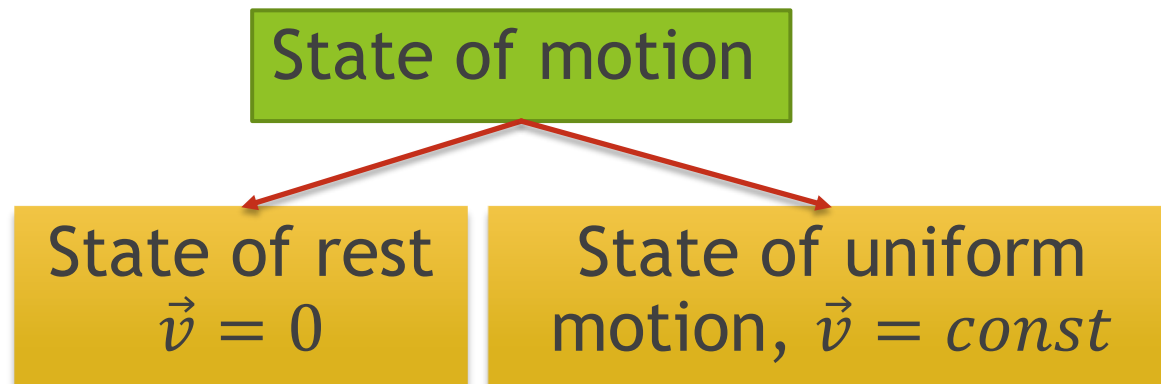
- The Concept of Force as a Measure of Interaction
- Types of forces
- Newton's Laws of Motion and Applications
- Circular Motions
- Newton's Law of Universal Gravitation and Examples
- Kepler's laws, Satellites motion and weightlessness

The Concept of Force as a Measure of Interaction

- ▶ So far we discussed **Kinematics**
 - ▶ the study of the motion of objects
 - ▶ *how far* (**distance** and **displacement**),
 - ▶ *how fast* (**speed** and **velocity**) and
 - ▶ *how fast the motion changes* (**acceleration**)
 - ▶ *a set of kinematic equations* linking these concepts
 - ▶ No concern for the **causes** of motion

The Concept of Force as a Measure of Interaction

- ▶ We now discuss **dynamics**
 - ▶ the study of motion and its **causes**
- ▶ Objects change their **state of motion** because they **interact** with each other



The Concept of Force

- ▶ How do we measure the strength of interactions?
 - ▶ Introduce the **concept of Force**
 - ▶ Do **quantitative** measurement
 - ▶ Observe: the **stronger** the **interaction**, the **larger** the **force**.

See Interacting Masses

Types of Forces

- 4 types of fundamental forces depending on the type of interaction:

Force	Interaction between	Strength	Range
Strong	Nucleons/Quarks	1	10^{-15}
Electromagnetic	Charges	$1/137$	∞
Weak	Quarks/Leptons	10^{-18}	10^{-6}
Gravity	Particles with Mass	6×10^{-39}	∞

Types of Forces

contact & non-contact (action-at-a-distance)

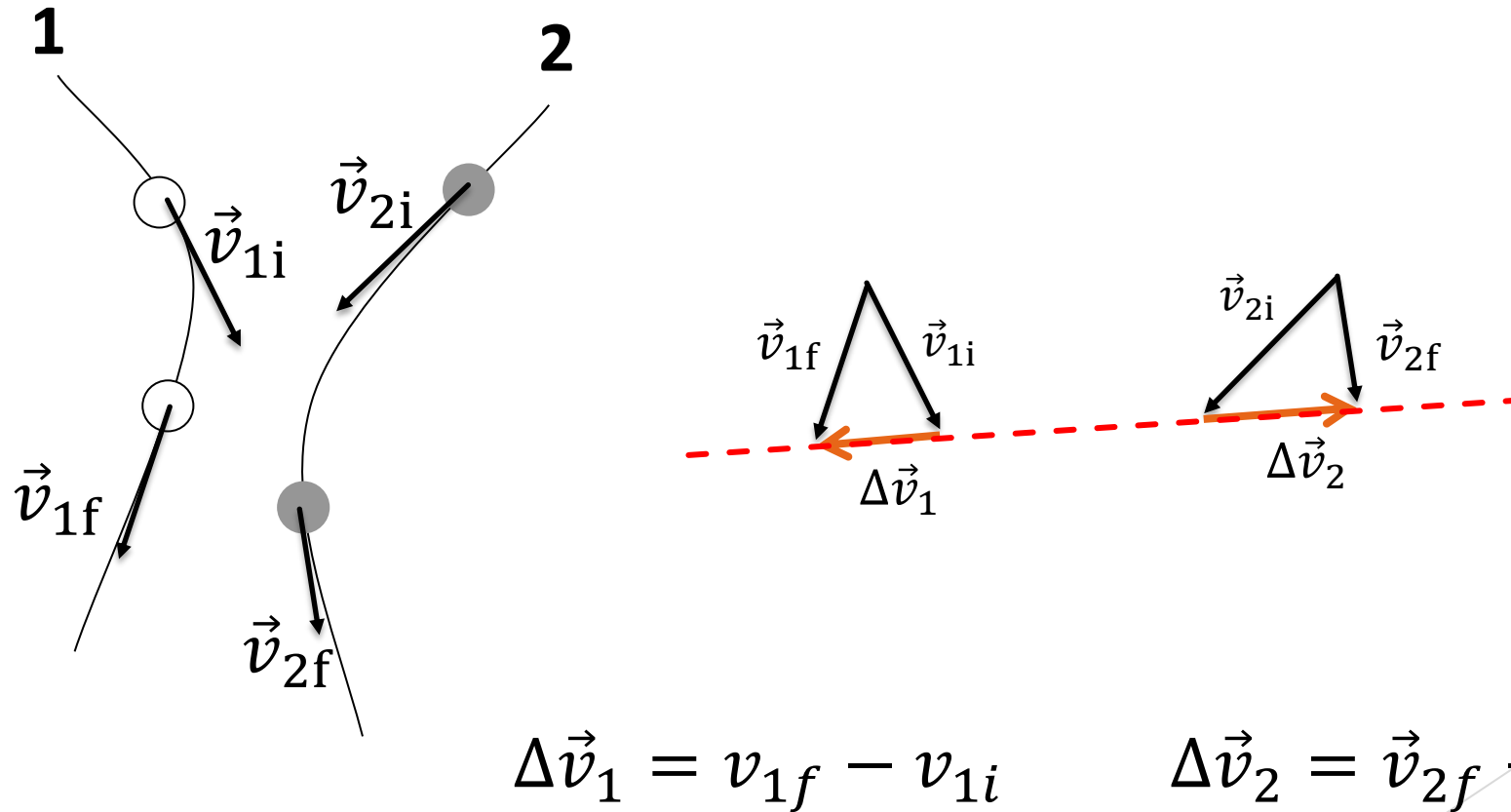
Contact	Non-contact
Frictional forces	Gravitational forces
Tensile forces,	Electric forces
Normal forces	Magnetic forces
Fluid resistance forces	Nuclear forces
Applied forces	

Derivation of Newton's Laws

- ▶ Newtonian mechanics is based on the **particle model**.
- ▶ What is a particle?
 - ▶ A particle is a **mental construct** of a real object that is deprived of its dimensional extension
- ▶ With this in mind
 - ▶ consider two particles engaged in their own **mutual interactions** but isolated otherwise.

Derivation of Newton's Laws

Consider the following experiment.



Experimental Observations

- a. The changes in velocities $\Delta\vec{v}_1$ and $\Delta\vec{v}_2$ are opposite in direction.
- b. The ratio of the magnitudes $|\Delta\vec{v}_1|$ and $|\Delta\vec{v}_2|$ is the same all the time.

So, for a two-particle isolated system:

$$\Delta\vec{v}_1 = -k_{12}\Delta\vec{v}_2$$

Introducing a third particle (denote it by 's')

$$\Delta \vec{v}_1 = -k_{1s} \Delta \vec{v}_s \text{ for particles 1 and s}$$

$$\Delta \vec{v}_2 = -k_{2s} \Delta \vec{v}_s \text{ for particles 2 and s}$$

- Relation between the constants:

$$k_{12} = \frac{k_{2s}}{k_{1s}}$$

- Therefore, for particles 1 and 2:

$$\Delta \vec{v}_1 = -\frac{k_{2s}}{k_{1s}} \Delta \vec{v}_2 \quad \text{or} \quad k_{1s} \Delta \vec{v}_1 = -k_{2s} \Delta \vec{v}_2$$

Defining mass

► But $k_{1s}|\Delta\vec{v}_1| = k_{2s}|\Delta\vec{v}_2| = \text{constant}$

► Or $|\Delta\vec{v}_1| \sim \frac{1}{k_{1s}}$

- A little change in the state of motion (velocity) corresponds to large k , and vice versa.
- That is, a particle with large k resists a change in its state of motion.
- This resistance is called **inertia**.
- The constant k is, therefore, a measure of the inertia of the particle.
- k is given a special name, the **mass** of the particle.

Linear Momentum

- ▶ Using $k = m$: $m_{1s}\Delta\vec{v}_1 = -m_{2s}\Delta\vec{v}_2$
- ▶ Choosing particle 's' as a standard of unit mass:

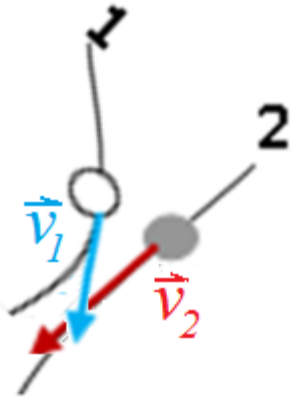
$$m_1\Delta\vec{v}_1 = -m_2\Delta\vec{v}_2$$

- ▶ Or $\Delta(m_1\vec{v}_1) = -\Delta(m_2\vec{v}_2)$

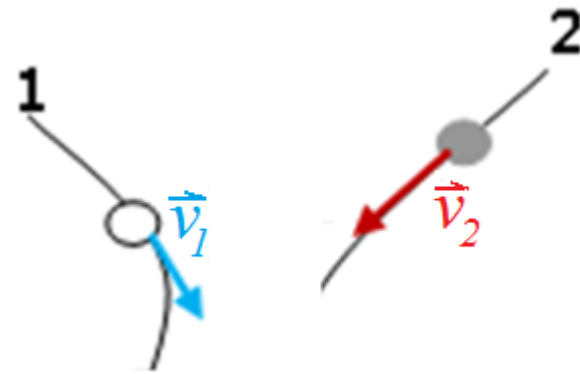
- ▶ Definition: Linear Momentum = $\vec{p} = m\vec{v}$

- ▶ Then: $\Delta\vec{p}_1 = -\Delta\vec{p}_2$ for the same Δt

Conclusions:



- The closer the particles, the stronger the interaction
- $\vec{p}$ changes at a faster rate



- The farther the particles, the weaker the interaction
- $\vec{p}$ changes at a slower rate

Newton's Second Law

- ▶ The rate of change of momentum of either particle is directly related to the magnitude of the interaction between them.
- ▶ The magnitude of the interaction is called Force

$$\vec{F}_1 = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{p}_1}{\Delta t} \quad \text{for particle 1}$$

$$\vec{F}_2 = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{p}_2}{\Delta t} \quad \text{for particle 2}$$

- ▶ Newton's 2nd law

$$\vec{F} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{p}}{\Delta t} \quad \text{where } \vec{F} \text{ is the net force on the particle}$$

Newton's Second Law

► If $m = \text{const}$

$$\vec{F} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{p}}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\Delta(m\vec{v})}{\Delta t} = m \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = m\vec{a}$$

$$\vec{F} = m\vec{a}$$

Newton's First Law

► For a free particle:

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} = 0$$

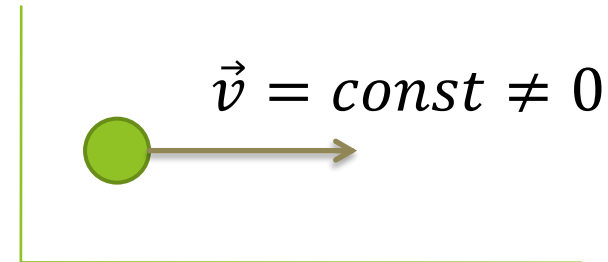
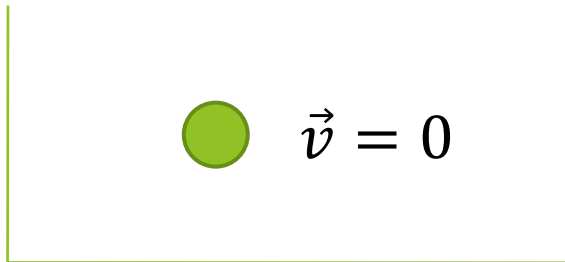
$$\Rightarrow \vec{p} = m\vec{v} = \text{const} \Rightarrow \vec{v} = \text{const}$$

A **free** particle is either at **rest** or moves with **constant** velocity

Newton's First Law (The law of inertia)

Where is Newton's first law true?

- ▶ Newton's First Law holds in an **inertial frame of reference**.
- ▶ No single, absolute inertial reference frame exists



Newton's Third Law

- From the two-particle interaction:

$$\Delta \vec{p}_1 = -\Delta \vec{p}_2$$

- Dividing by Δt and taking the limit as $\Delta t \rightarrow 0$:

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{p}_1}{\Delta t} = - \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{p}_2}{\Delta t}$$

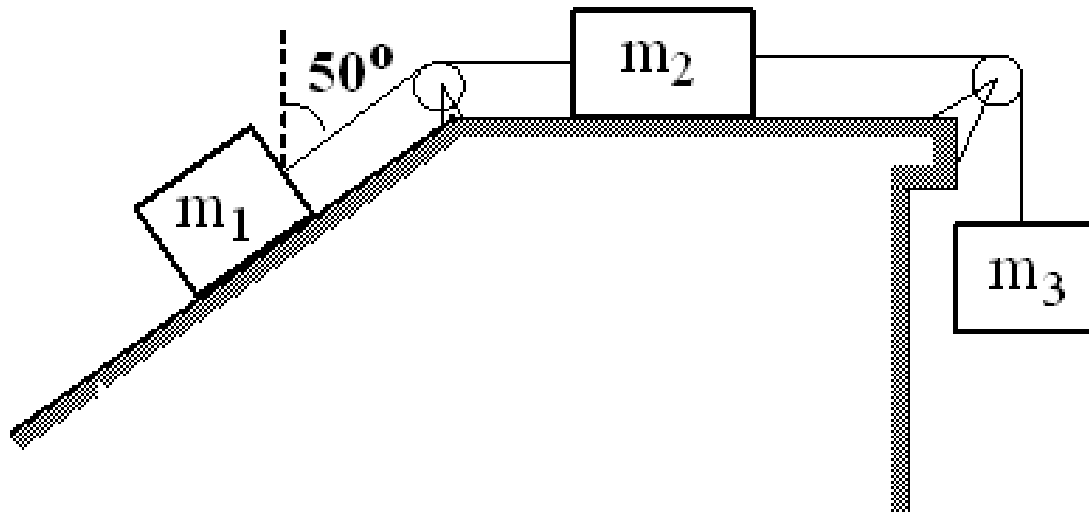
- Using the definition of force:

$$\vec{F}_1 = -\vec{F}_2$$

Applications of Newton's Laws of Motion

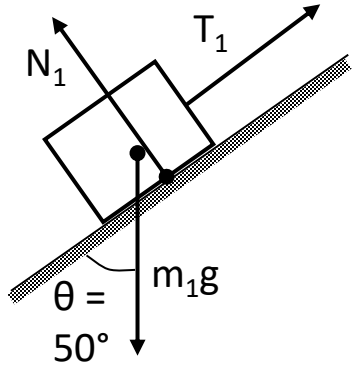
Example 1

Masses $m_1 = 6 \text{ kg}$, $m_2 = 4 \text{ kg}$ and $m_3 = 8 \text{ kg}$ are connected as shown. Assuming frictionless surfaces and weightless pulleys, determine the motion of the system and find the tension in each string.



Solution to example 1

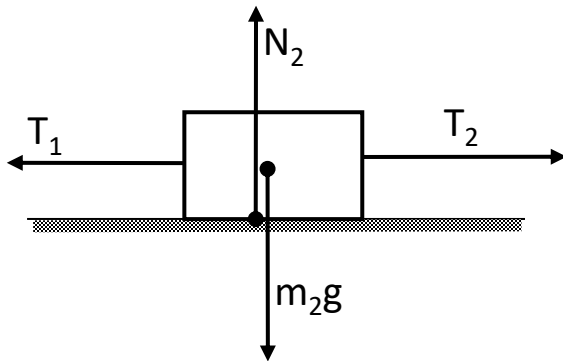
► Apply Newton's 2nd law to m_1



$$N_1 - m_1g \sin \theta = 0 \quad \perp \text{ to the incline}$$

$$m_1g \cos \theta - T_1 = m_1a \quad \parallel \text{ to the incline (1)}$$

• Apply Newton's 2nd law to m_2

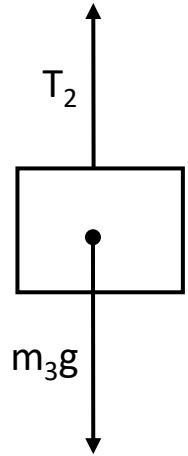


$$N_2 - m_2g = 0 \quad \perp \text{ to the horiz. surface}$$

$$T_1 - T_2 = m_2a \quad \parallel \text{ to the horiz. surface (2)}$$

Solution to example 1

- Apply Newton's 2nd law to m_3



$$T_2 - m_3g = m_3a \quad (3)$$

- Adding equations 1, 2 and 3:

$$m_1g \cos \theta - T_1 = m_1a$$

$$T_1 - T_2 = m_2a$$

$$T_2 - m_3g = m_3a$$

$$m_1g \cos \theta - m_3g = (m_1 + m_2 + m_3)a$$

$$\text{From which } a = \frac{m_1 \cos \theta - m_3}{m_1 + m_2 + m_3} g$$

Solution to example 1

► Inserting numerical values:

$$a = \frac{6 \cos 50^\circ - 8}{6 + 4 + 8} g = -0.23g = -2.26 \text{ m/s}^2$$

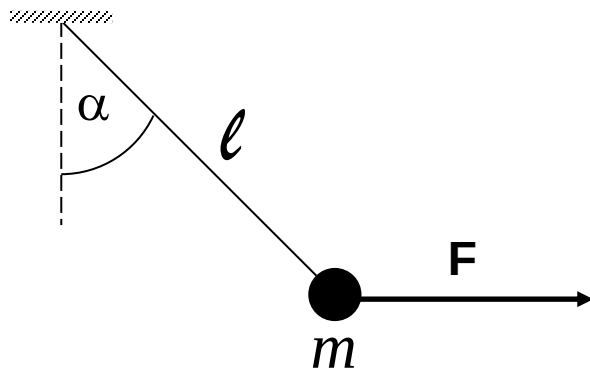
- What is the meaning of the minus sign?
- Use equations (2) and (3) to calculate the tensions

$$T_2 = m_3 (g + a) = m_3 (g - 0.23g) = 0.77m_3g = 60.4 \text{ N}$$

$$T_1 = T_2 + m_2a = 60.4 - 4 \times 2.26 = 51.4 \text{ N}$$

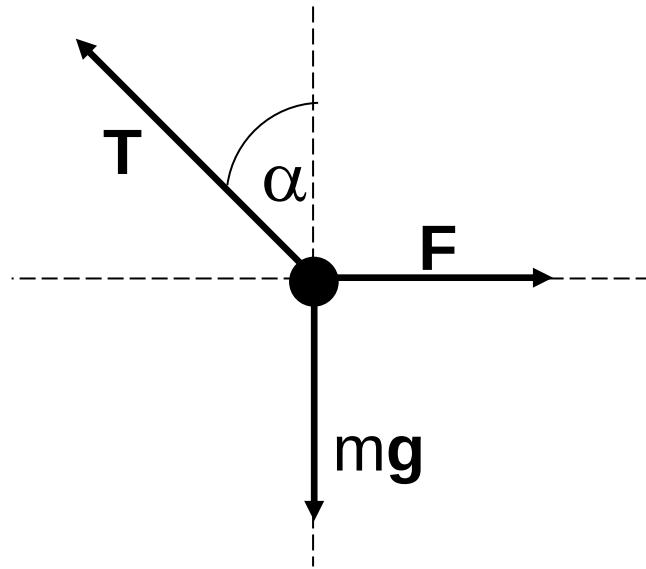
Example 2

A ball with a mass $m = 1.5 \text{ kg}$ is attached to a string anchored in a rigid ceiling as shown. The length of the string is $\ell = 2.2 \text{ m}$. A horizontal force draws the ball aside until the string makes an angle $\alpha = 20^\circ$ with the vertical. In this position the ball is at rest. Determine F . If the force is removed what will be the initial acceleration of the ball?



Solution to example 2

► Forces acting on the mass



• Equations from Newton's 2nd law

$$T \sin \alpha - F = 0$$

horizontal direction

$$T \cos \alpha - mg = 0$$

vertical direction

Solution to example 2

Solving for F : $F = mg \tan \alpha$

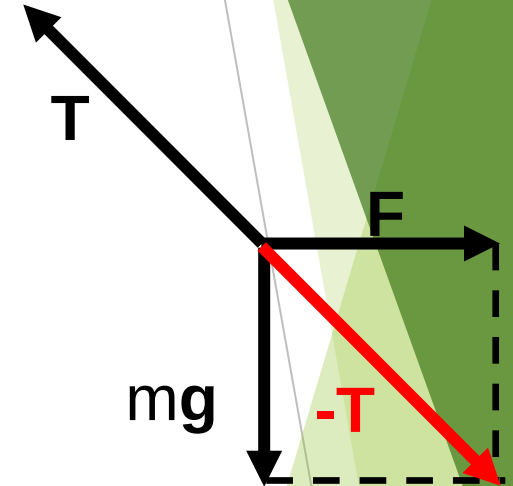
Putting in numerical values:

$$F = 1.5 \times 9.8 \times \tan 20^\circ = 5.4 \text{ N}$$

The value of T just before F is removed:

$$\vec{T} + \vec{F} + m\vec{g} = 0 \quad (\text{equilibrium})$$

$$\Rightarrow T = \sqrt{F^2 + (mg)^2} = \sqrt{5.4^2 + 14.7^2} = 15.6 \text{ N}$$



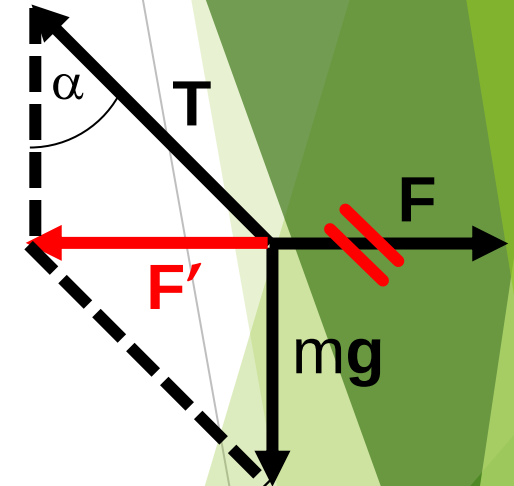
Solution to example 2

The net force just after F is removed:

$$\begin{aligned} F' &= \sqrt{T^2 + (mg)^2 - 2Tmg \cos \alpha} = F \\ &= \sqrt{(15.6)^2 + (14.7)^2 - 2 \times 15.6 \times 14.7 \cos 20^\circ} = 5.3 \text{ N} \end{aligned}$$

Initial acceleration:

$$a = \frac{F'}{m} = \frac{5.3 \text{ N}}{1.5} = 3.6 \text{ m/s}^2$$



Example 3

A rope has a length of 12 m and a mass of 16 kg. The rope hangs from a rigid support. A man whose mass is 80 kg slides down the rope at a constant speed of 0.8 m/s.

- a) What is the tension in the rope at a point 6 m from the top when the man has slid to this point?
- b) What will be the tension if the man suddenly stops at the same point? Assume the stoppage time is 0.1 sec.

Solution to example 3

Part a: Tension at the middle of the rope

$$\begin{aligned} T &= W_{man} + \frac{1}{2} W_{rope} \\ &= \left(m_{man} + \frac{1}{2} m_{rope} \right) g = (80 + 8) \times 9.8 = 862.4 \text{ N} \end{aligned}$$

Part b: Δp of man when he stops suddenly

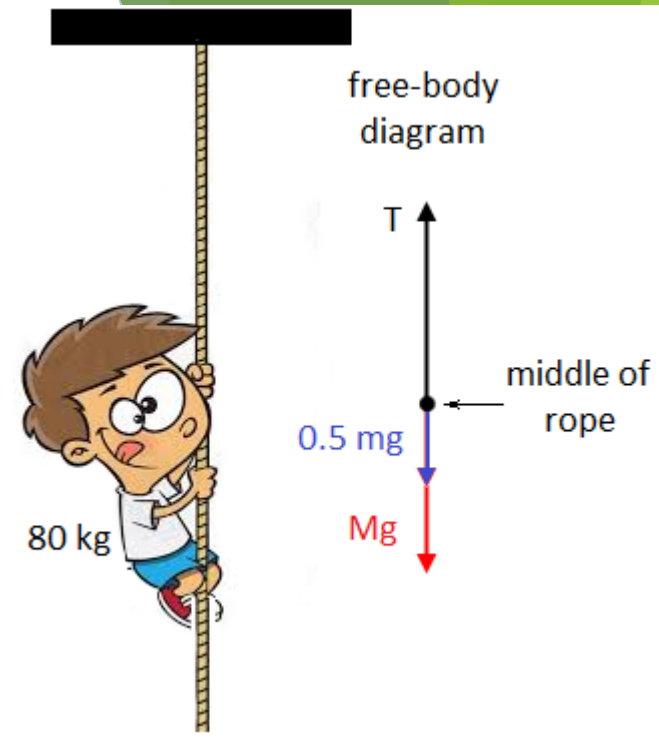
$$\Delta p = m_{man} \Delta v = 80(0 - 0.8) = -64 \text{ kg.m/s}$$

Part b: stopping force

$$F = \frac{\Delta P}{\Delta t} = \frac{64}{0.1} = 640 \text{ N}$$

Part b: Tension in the string:

$$T = F + \frac{1}{2} W_{rope} = 640 \text{ N} + 78.4 = 718.4 \text{ N}$$

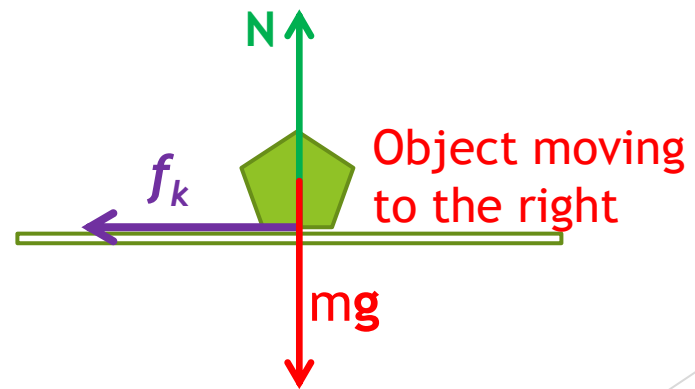
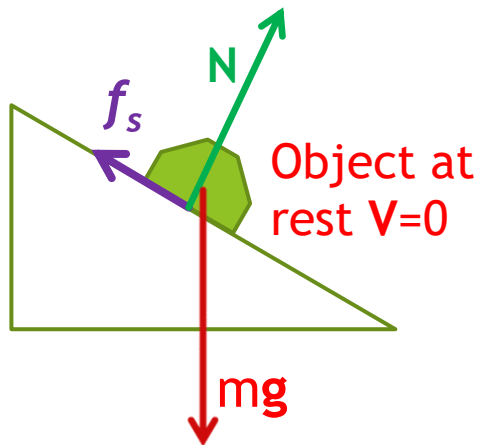


Motion in the Presence of Resistive Forces

► Four types of damping (resistive) forces

1. Dry frictional force

- Acts between two dry surfaces
- Generally, independent of relative speed
- acts opposite to the tendency or direction of motion



Motion in the Presence of Resistive Forces

2. Viscous force

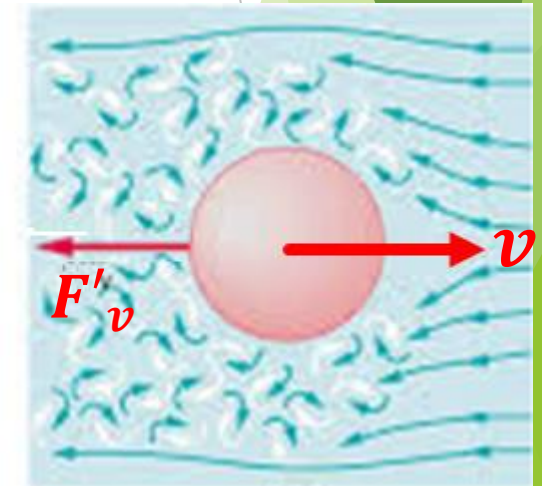
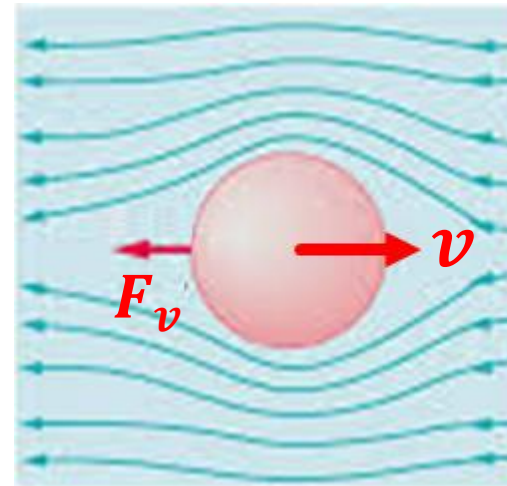
- ▶ Acts on objects moving slowly through a fluid (laminar flow).
- ▶ proportional to the speed of the object
- ▶ Acts opposite to the velocity direction.



Turbulent
(low viscosity)
e.g., water



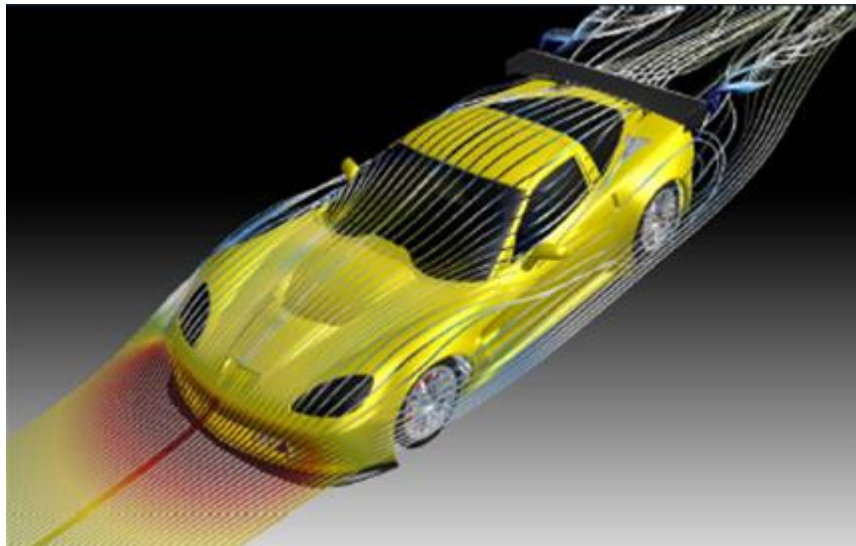
Laminar
(high viscosity)
e.g., oil



Motion in the Presence of Resistive Forces

3. Turbulent force

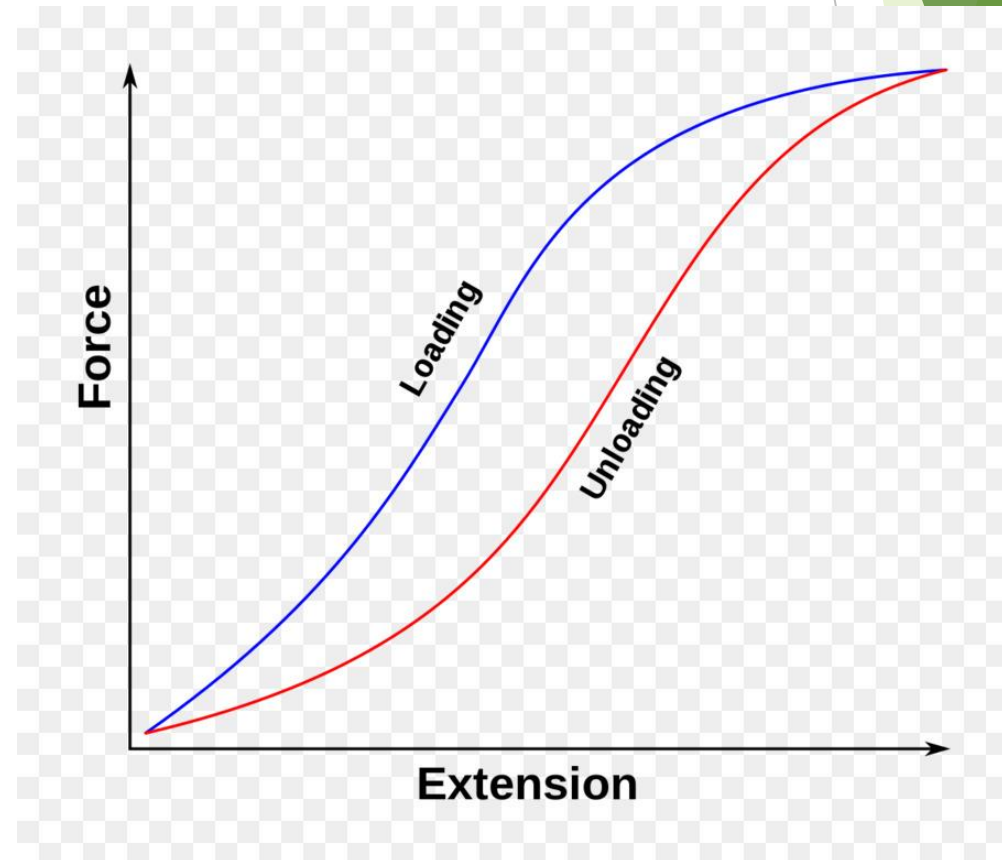
- ▶ Acts on objects moving fast through fluids
- ▶ proportional to the square of the speed
- ▶ Acts opposite to the velocity direction.



Motion in the Presence of Resistive Forces

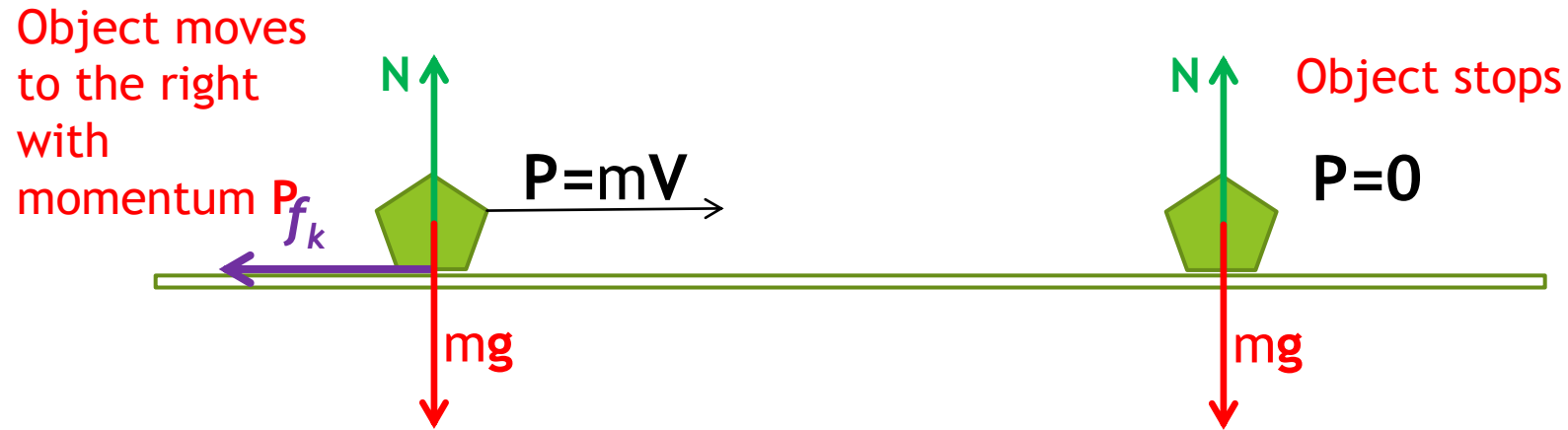
4. Hysteretic force (beyond scope)

- ▶ internal friction or hysteresis loss when a solid is strained.



Dry Friction

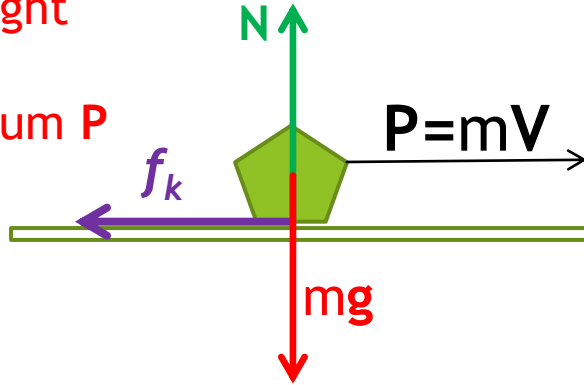
- An object pushed across a level surface comes to rest. Why? Discuss using Newton's laws.



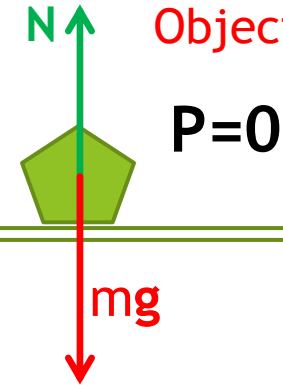
- Momentum changes: $\Delta \vec{p} = \vec{p}_f - \vec{p}_i = 0 - \vec{p} = -\vec{p}$

Dry Friction

Object moves
to the right
with
momentum P



Object stops



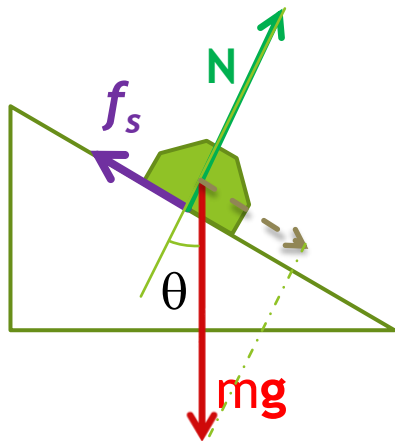
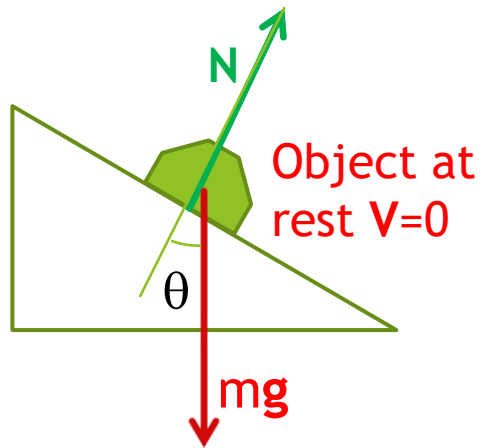
- By the 2nd law, there must be a force acting on the object to the left:

$$\vec{F}_{av} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{t_f - t_i} = \frac{0 - \vec{p}}{t - 0} = -\frac{\vec{p}}{t}$$

$$\vec{F}_{av} = \vec{f}_k$$

Dry Friction

- An object is at rest on an inclined plane. Discuss what holds it in place.



- Down the incline the force is

$$mg \sin \theta$$

- Since the object is at rest, $mg \sin \theta$ must be balanced by an equal but opposite force:

$$|\vec{f}_s| = mg \sin \theta$$

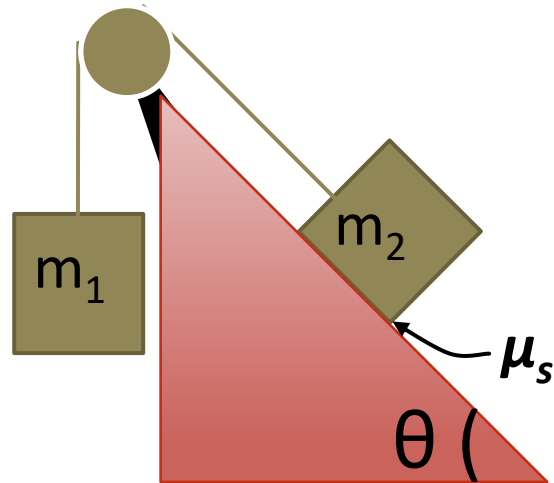
Dry Friction

► Experimental result

$$\begin{array}{ccc} & f = \mu N & \\ \swarrow & & \searrow \\ f_s = \mu_s N & & f_k = \mu_k N \\ & \mu_s > \mu_k & \end{array}$$

Dry Friction Example

- What is the minimum and maximum values of θ for which the system below remains in equilibrium?
- Take $\mu_s = 0.2$, $m_1 = 5$ kg and $m_2 = 10$ kg. Assume weightless and frictionless pulley.



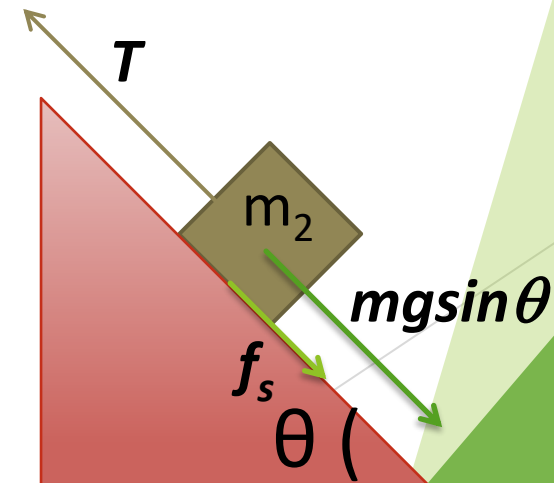
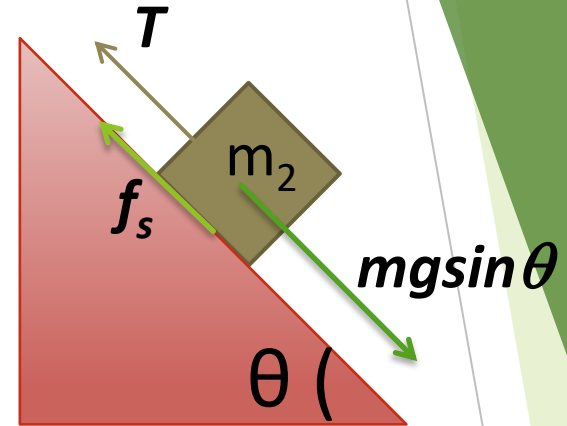
Solution

- If m_2 has a tendency to move down,

$$m_2 g \sin \theta - T - f_s = m_2 a = 0$$

- If m_2 has a tendency to move up,

$$T - m_2 g \sin \theta - f_s = m_2 a = 0$$



Solution

- Moreover, we have,

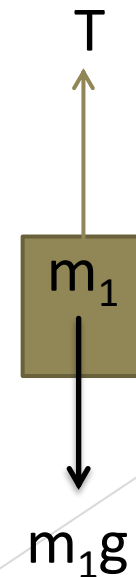
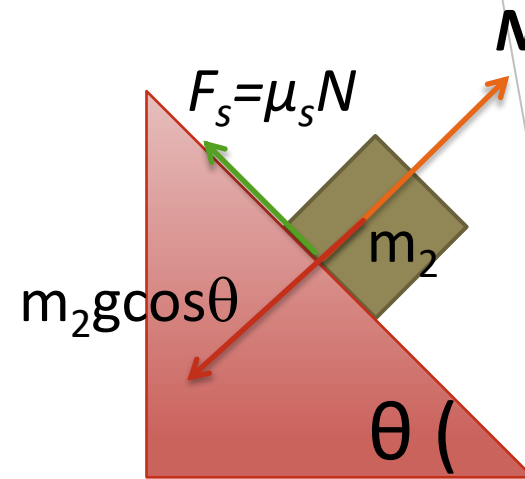
$$T = m_1 g$$

$$f_s = \mu_s N = \mu_s m_2 g \cos \theta$$

- After substitution, we have

$$m_1 = m_2 (\sin \theta \mp \mu_s \cos \theta)$$

- From this equation, the student can determine θ_{min} and θ_{max} .



Circular Motions

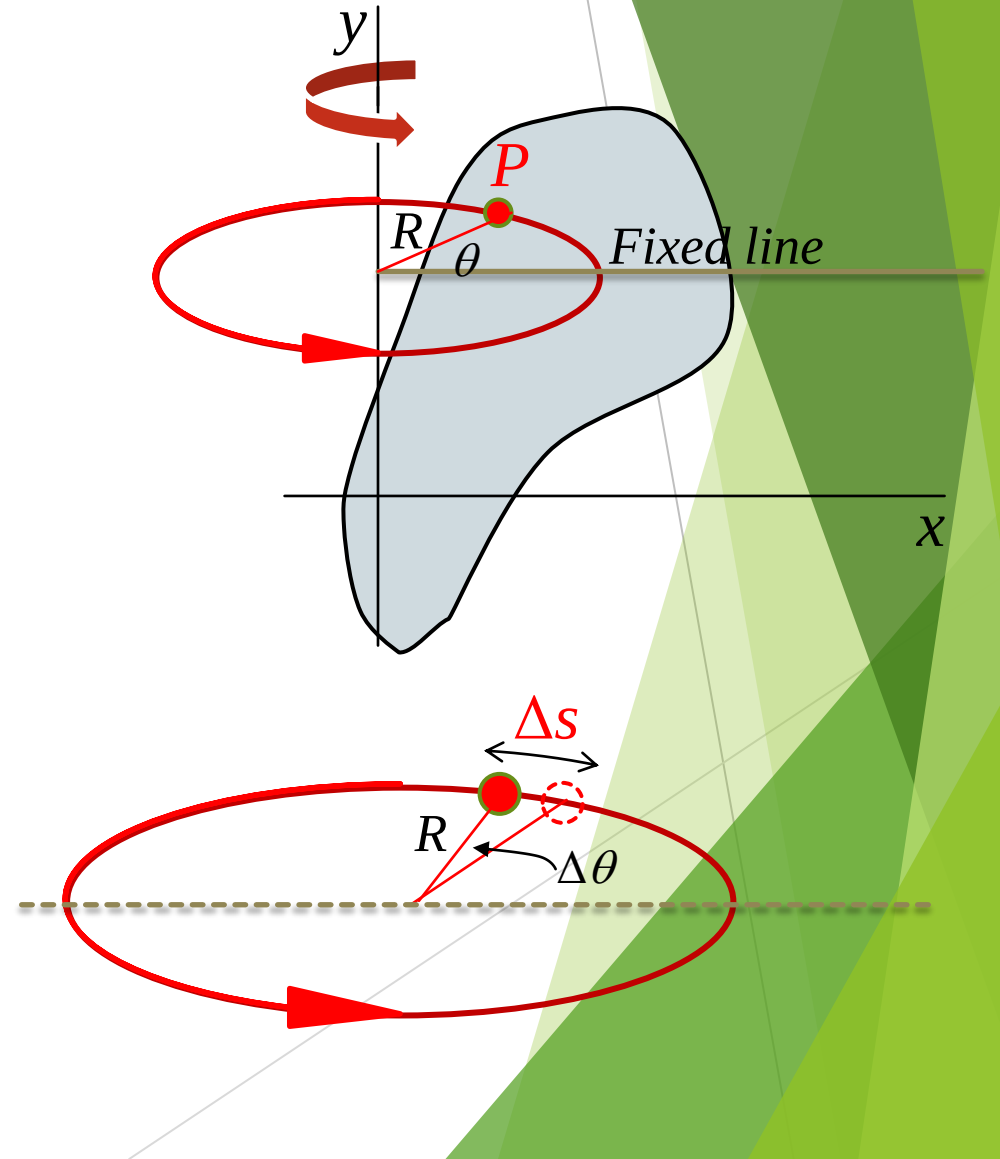
- ▶ Angular displacement $\Delta\theta$

$$\Delta\theta = \frac{\Delta s}{R} \quad (\text{dimensionless})$$

SI auxiliary unit: radian (rad)

- ▶ Finite angles are not vector quantities!
 - ▶ Violate commutative rule of vector addition

$$\theta_x \hat{i} + \theta_y \hat{j} \neq \theta_y \hat{j} + \theta_x \hat{i}$$



Kinematics of Uniform Circular Motion

Uniform circular motion: motion in a circle of constant radius at constant speed

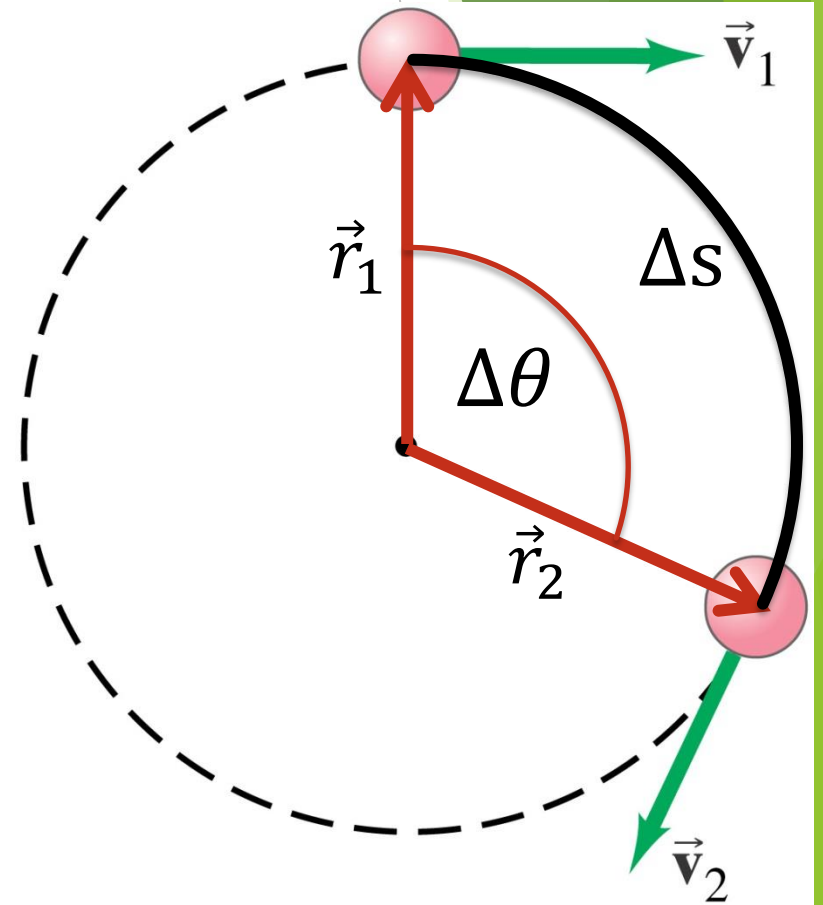
$\vec{r}$ changes but $|\vec{r}| = R = \text{constant}$

$\vec{v}$ changes but $|\vec{v}| = \text{constant}$, $\omega = \text{constant}$

$$\Delta s = R\Delta\theta \quad \frac{\Delta s}{\Delta t} = R \frac{\Delta\theta}{\Delta t} \rightarrow \bar{v} = R\bar{\omega}$$

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = R \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} \rightarrow v = R\omega$$

Instantaneous velocity is always tangent to the circle.



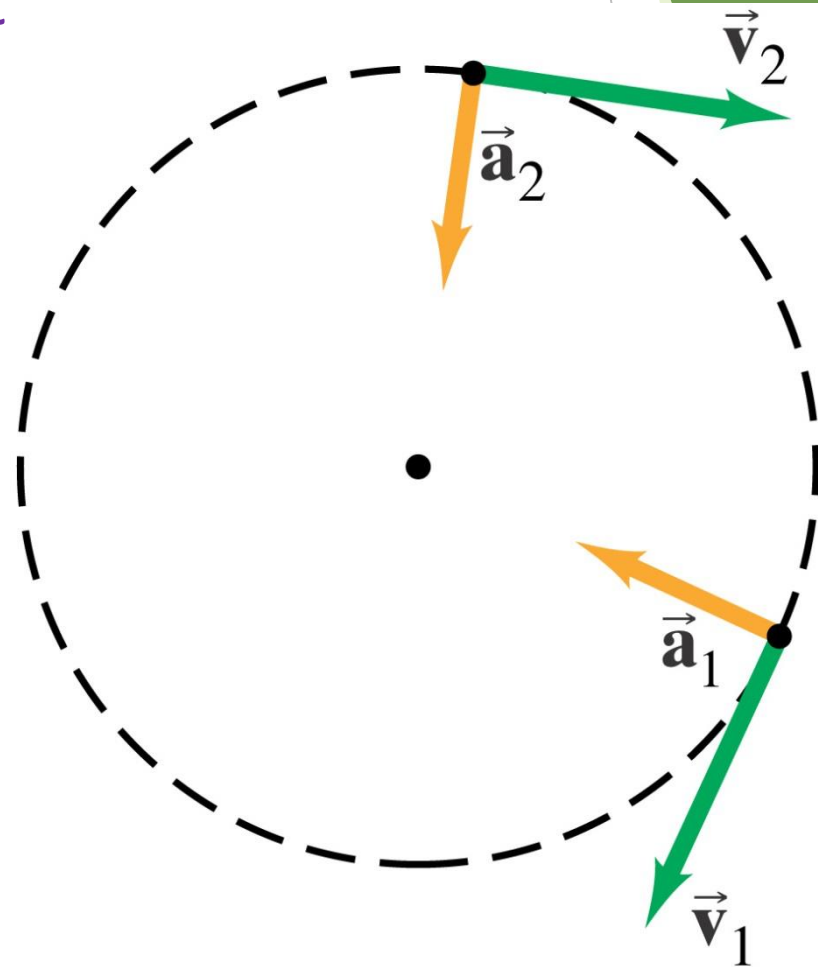
Kinematics of Uniform Circular Motion

In uniform circular motion, $\omega = \text{constant}$
Angular acceleration is zero

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t} = 0$$

But, there is centripetal (radial) acceleration, pointing towards the center of the circle.

See simulation



Kinematics of Uniform Circular Motion: Definitions and Formulae

Frequency (f):

Number of revolutions per second

Unit: per second (s^{-1})

Period (T): $T = 1/f$

Time it takes for one revolution

Unit: second (s)

Angular frequency (ω): $2\pi f = 2\pi/T$

Unit: per second (s^{-1})

Average speed (one cycle)

$$v = \frac{\Delta s}{\Delta t} = \frac{2\pi R}{T}$$

Acceleration

$$a_c = \frac{v^2}{R}$$

Combine:

$$a_c = \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2 R}{T^2}$$

velocity not required

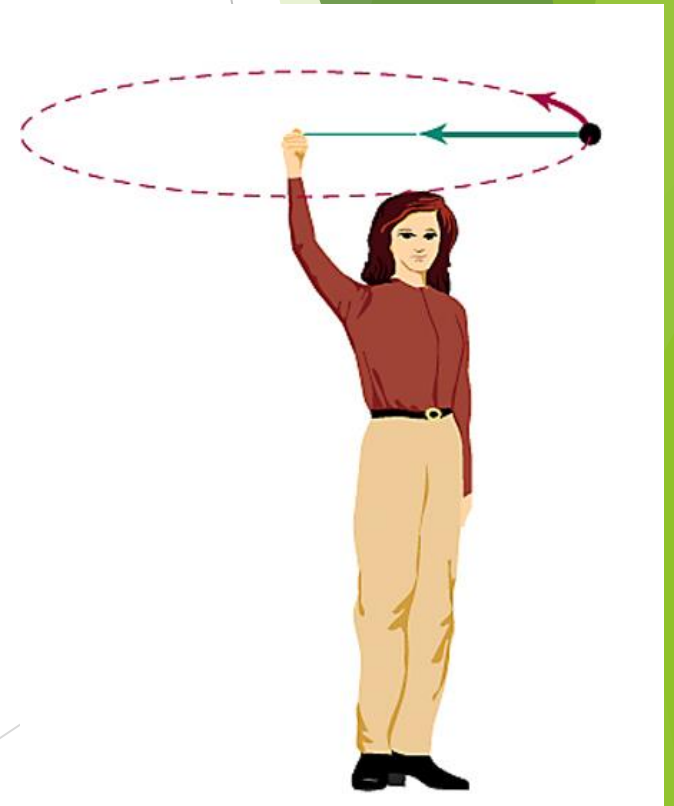
Example: Acceleration of a revolving ball

- ▶ A ball at the end of a string rotates uniformly in a horizontal circle of radius 0.600 m, making 2.00 revolutions in a second. Find
(a) the frequency of rotation,

Solution

$$1 \text{ cycle} = 1 \text{ revolution}$$

$$f = 2 \text{ rev/s} = 2 \text{ cycles/s} = 2 \text{ s}^{-1}$$

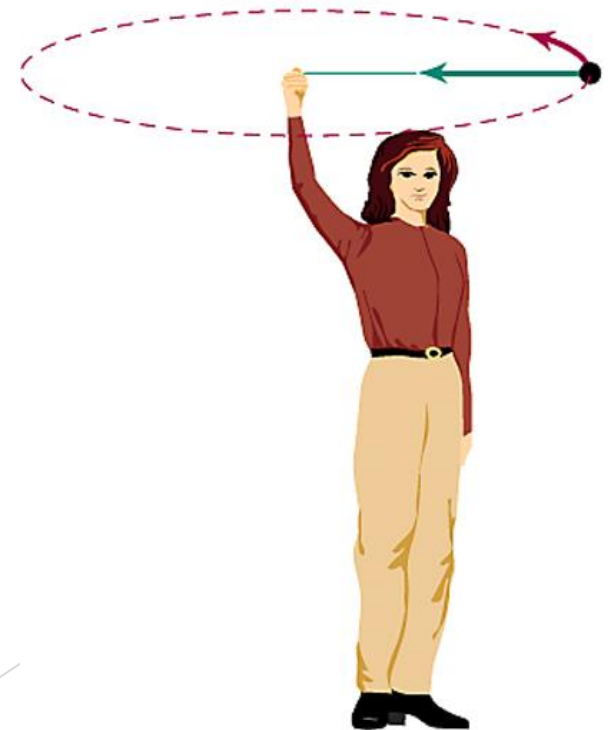


Example: Acceleration of a revolving ball

- ▶ A ball at the end of a string rotates uniformly in a horizontal circle of radius 0.600 m, making 2.00 revolutions in a second. Find
(b) the period of rotation,

Solution

$$T = \frac{1}{f} = \frac{1}{2} = 0.5 \text{ s}$$



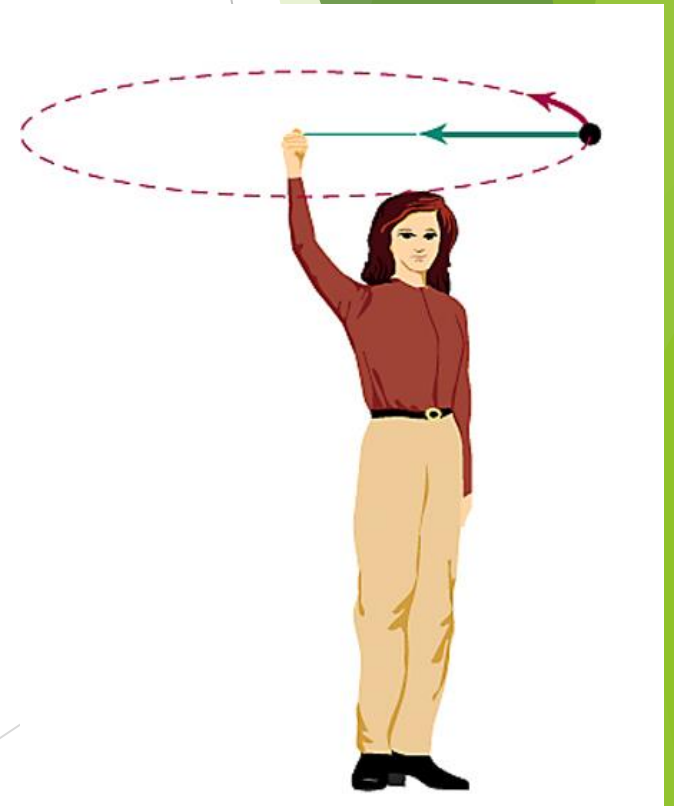
Example: Acceleration of a revolving ball

- ▶ A ball at the end of a string rotates uniformly in a horizontal circle of radius 0.600 m, making 2.00 revolutions in a second. Find
(c) the angular frequency,

Solution

$$\omega = 2\pi f = 2\pi \times 2 \text{ s}^{-1} = 4\pi \text{ s}^{-1}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5 \text{ s}} = 4\pi \text{ s}^{-1}$$

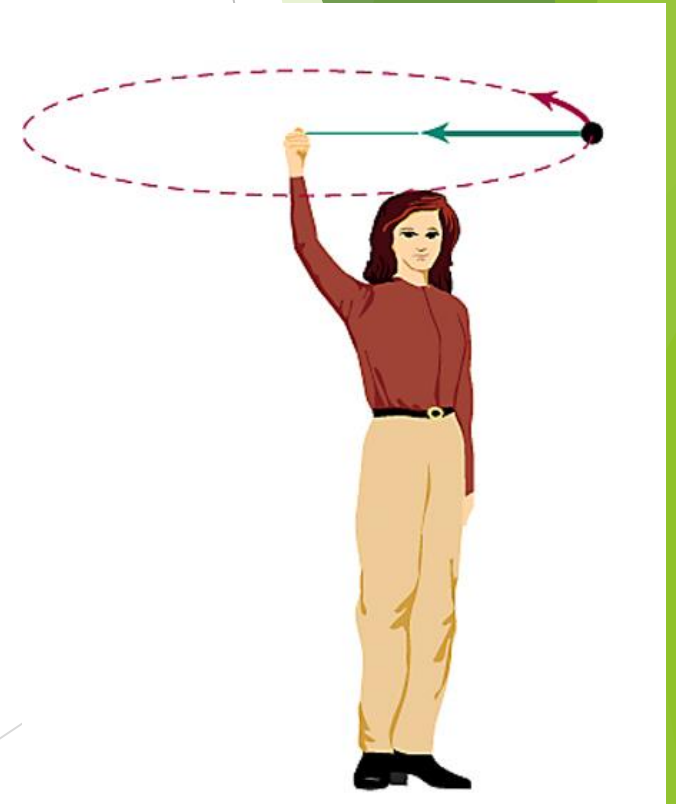


Example: Acceleration of a revolving ball

- ▶ A ball at the end of a string rotates uniformly in a horizontal circle of radius 0.600 m, making 2.00 revolutions in a second. Find
(d) the speed of the ball,

Solution

$$v = \frac{2\pi R}{T} = \frac{2\pi \times 0.600\text{m}}{0.5\text{s}} = 7.54\text{ m/s}$$



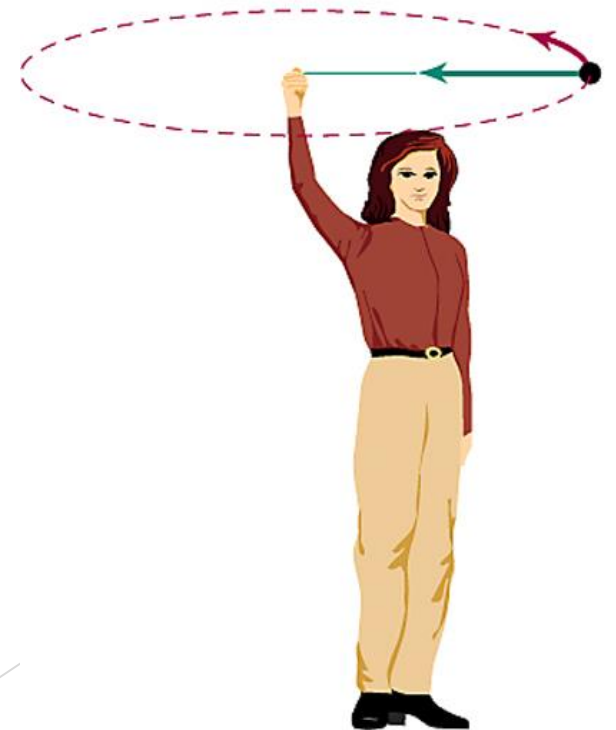
Example: Acceleration of a revolving ball

- ▶ A ball at the end of a string rotates uniformly in a horizontal circle of radius 0.600 m, making 2.00 revolutions in a second. Find

(e) the centripetal acceleration

Solution

$$a_c = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2 \times 0.600 \text{ m}}{0.5^2 \text{ s}^2} = 94.75 \text{ m/s}^2$$



Rotation with constant α

- If $\alpha \neq 0$, there is tangential acceleration

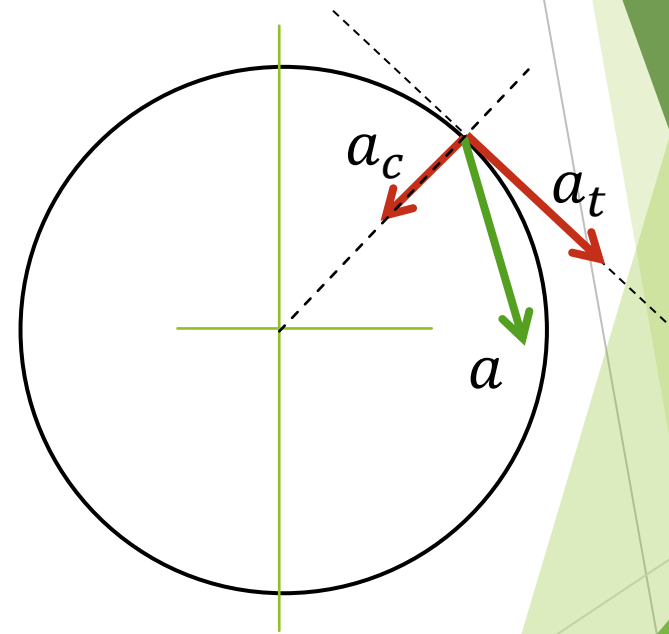
$$a_t = \frac{\Delta v}{\Delta t} = R \frac{\Delta \omega}{\Delta t} = R\alpha$$

- Total acceleration: $a = \sqrt{a_c^2 + a_t^2}$
- Equations of motion:

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$



Example

- ▶ A wheel rotates with a constant angular acceleration of 3.50 rad/s^2 . If the angular velocity of the wheel is 2.00 rad/s at $t = 0$,
 - (a) through what angle does the wheel rotate between $t = 0$ and $t = 2.00 \text{ s}$?
Give your answer in radians and in revolutions.

- ▶ Solution

(a)
$$\Delta\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\Delta\theta = 2.00 \frac{\text{rad}}{\text{s}} \times 2.00 \text{ s} + \frac{1}{2} \times 3.50 \frac{\text{rad}}{\text{s}^2} \times (2.00 \text{ s})^2 = 11.0 \text{ rad}$$

$$\Delta\theta = (11.0 \text{ rad}) \left(\frac{1.00 \text{ rev}}{2\pi \text{ rad}} \right) = 5 \text{ } 1.75 \text{ rev}$$

Example

- ▶ A wheel rotates with a constant angular acceleration of 3.50 rad/s^2 . If the angular velocity of the wheel is 2.00 rad/s at $t = 0$,
(b) What is the angular velocity of the wheel at $t = 2.00 \text{ s}$?

- ▶ Solution

(b) $\omega = \omega_0 + \alpha t$

$$\omega = 2.00 \frac{\text{rad}}{\text{s}} + 3.50 \frac{\text{rad}}{\text{s}^2} \times 2.00 \text{ s} = 9.00 \text{ rad/s}$$

Example

- ▶ A wheel rotates with a constant angular acceleration of 3.50 rad/s^2 . If the angular velocity of the wheel is 2.00 rad/s at $t = 0$,
 - (c) What angular displacement (in revolutions) results while the angular velocity found in part (b) doubles?

- ▶ **Solution**

(c) $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

$$\left(2 \times 9.00 \frac{\text{rad}}{\text{s}}\right)^2 = \left(9.00 \frac{\text{rad}}{\text{s}}\right)^2 + 2 \left(3.50 \frac{\text{rad}}{\text{s}^2}\right) \Delta\theta$$

$$\Delta\theta = 134.7 \text{ rad} \times \frac{1.00 \text{ rev}}{2\pi \text{ rad}} = 25.52 \text{ rev}$$

Relationship between linear and angular quantities

► $v = R\omega$ $a_t = R\alpha$ $a_c = R\omega^2$

- A racetrack is constructed as shown in the figure. In a particular trial run, a driver travels at a **constant** speed of 50 m/s for one complete lap.

1. The ratio of the tangential acceleration at A to that at B is

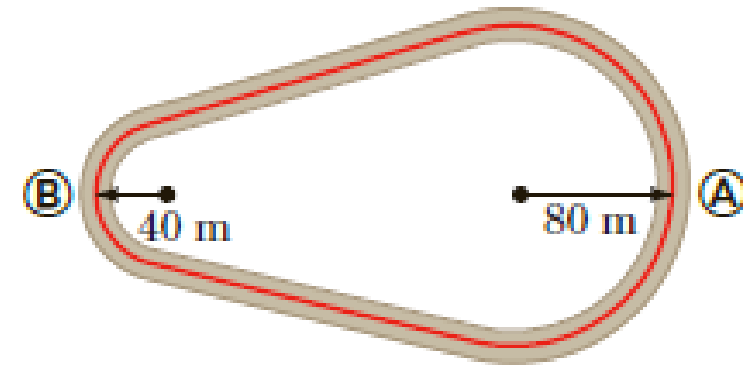
(a) 1/2

(b) 1/4

(c) 2

(d) 4

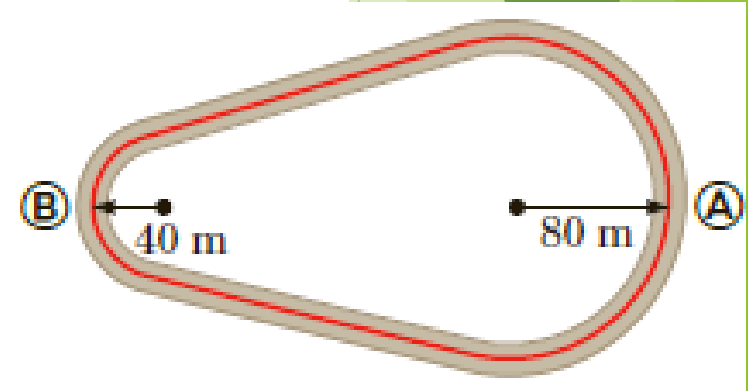
(e) The tangential acceleration is zero at both points



Relationship between linear and angular quantities

► $v = R\omega$ $a_t = R\alpha$ $a_c = R\omega^2$ [$a_c = v^2/R$]

- A racetrack is constructed as shown in the figure. In a particular trial run, a driver travels at a **constant** speed of 50 m/s for one complete lap.
- 2. The ratio of the centripetal acceleration at A to that at B is
- (a) 1/2 ←
 - (b) 1/4
 - (c) 2
 - (d) 4
 - (e) The centripetal acceleration is zero at both points.



Relationship between linear and angular quantities

► $v = R\omega$ $a_t = R\alpha$ $a_c = R\omega^2 = v^2/R$

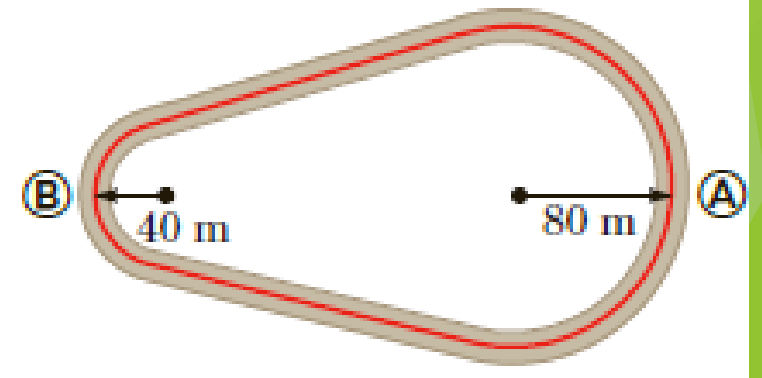
- A racetrack is constructed as shown in the figure. In a particular trial run, a driver travels at a **constant** speed of 50 m/s for one complete lap.

3. The angular speed is greatest at

(a) A

(b) B ←

(c) It is equal at both A and B.



Dynamics of Uniform Circular Motion

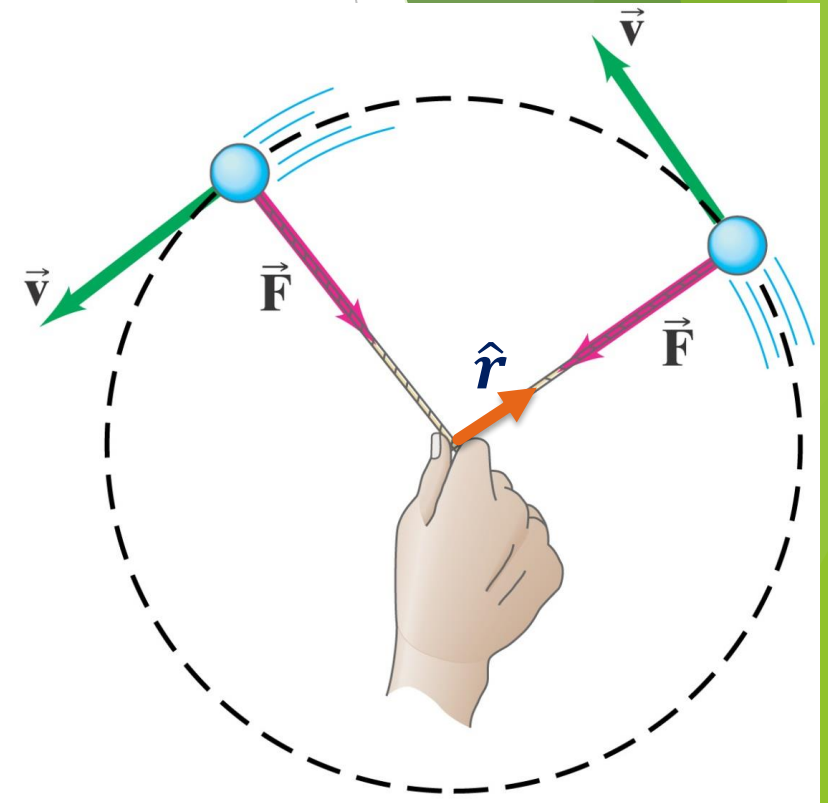
An object in uniform circular motion experiences a centripetal force

The centripetal force is the sum of all forces in the radial direction with a resultant pointing to the center

$$\sum F_r = F_c = ma_c = m \frac{v^2}{R}$$

Direction: radially in is negative ($-\hat{r}$)

$$\vec{F}_c = -m \frac{v^2}{R} \hat{r}$$



Dynamics of Uniform Circular Motion

We can see that the force must be inward by thinking about a ball on a string.

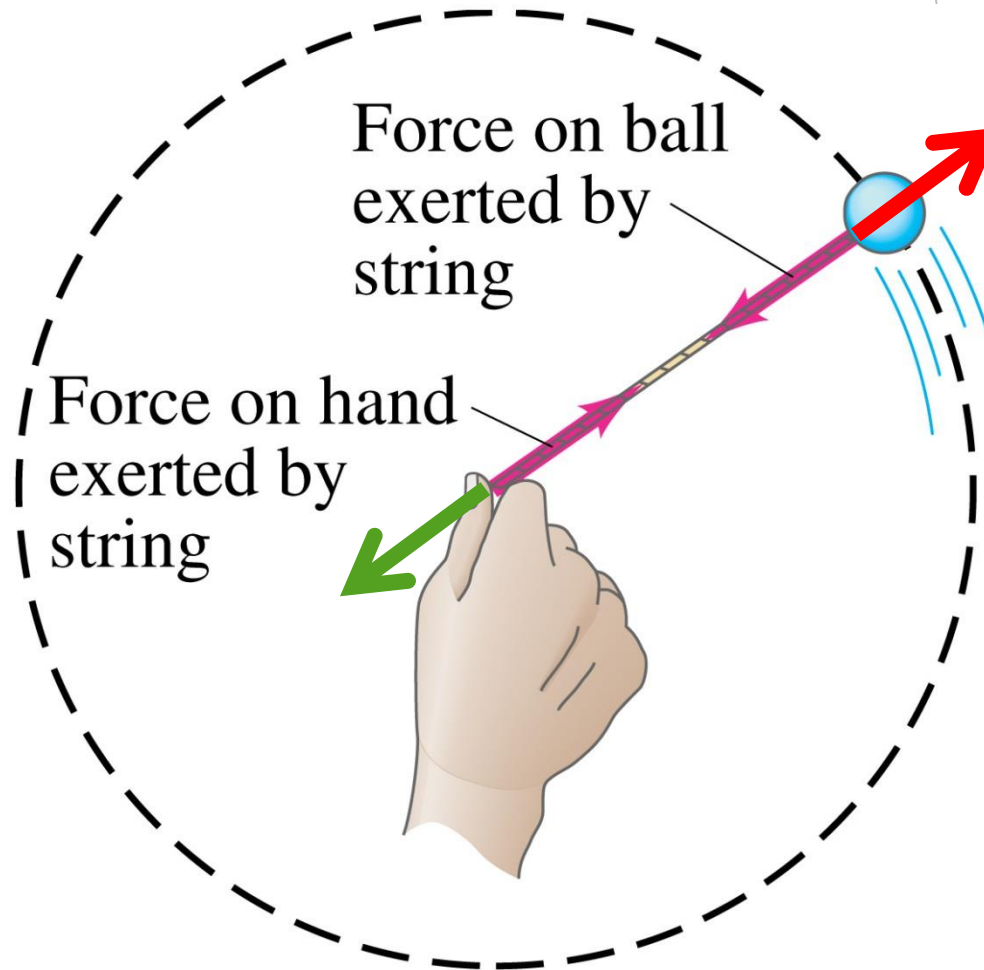
Are these forces
action and
reaction?

YES

X

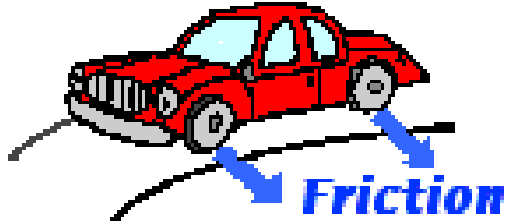
NO

✓

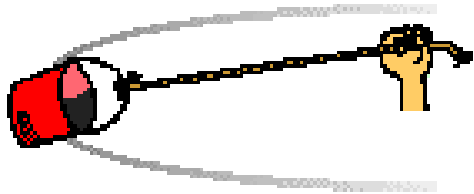


Dynamics of Uniform Circular Motion

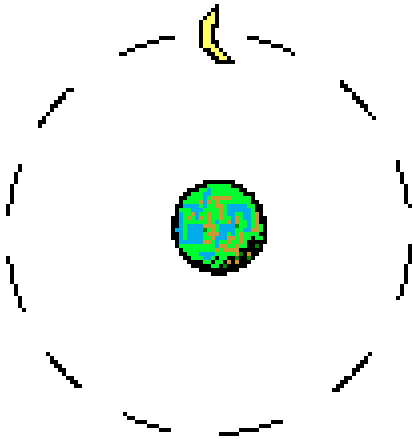
Net forces that can cause centripetal acceleration



Friction of tires on road



Tension of cord

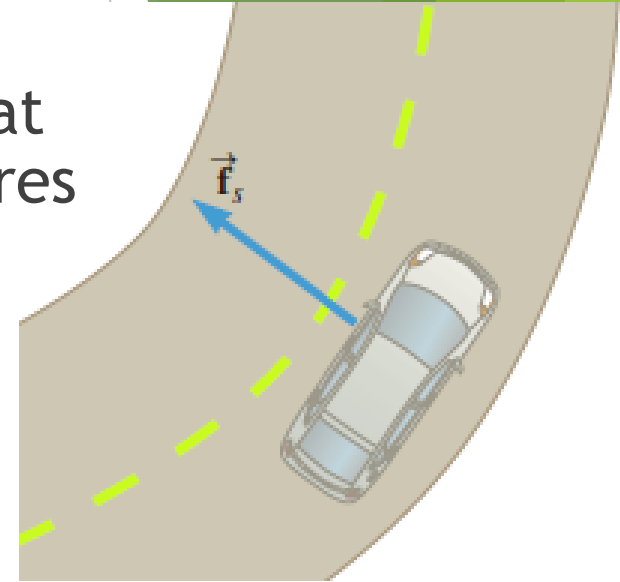


Gravity keeping object in orbit

Source: physicsclassroom.com

Examples

- ▶ A car travels at a constant speed of 13.4 m/s on a level circular turn of radius 50.0 m, as shown in the figure. What **minimum** coefficient of static friction, μ_s , between the tires and roadway will allow the car to make the circular turn without sliding?



- ▶ Solution

Draw a free-body diagram

Radial direction: $F_c = m \frac{v^2}{R}$

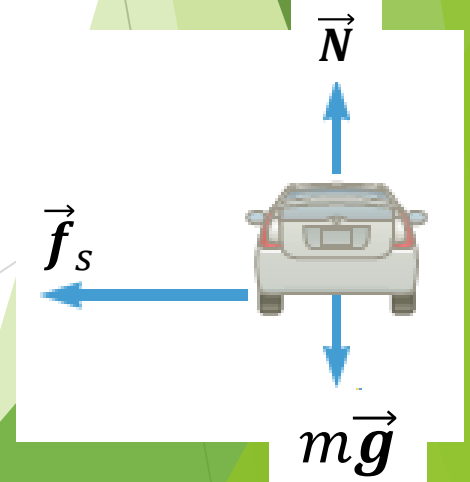
Vertical direction: $N = mg$

Also, $F_c = f_s = \mu_s N$

Combining:

$$\mu_s = \frac{v^2}{gR}$$

$$\mu_s = \frac{13.4^2}{9.8 \times 50.0} = 0.366$$



- In the previous example, at what **maximum** speed can a car negotiate a turn on a wet road with coefficient of static friction 0.230 without sliding out of control? The radius of the turn is 25.0 m.

- Solution

use the relation $\mu_s = \frac{v^2}{gR}$ from previous example.

Solve for v :

$$v = \sqrt{\mu_s g R}$$

Insert numerical values:

$$v = \sqrt{0.230 \times 9.8 \frac{m}{s^2} \times 25m} = \mathbf{7.51 \text{ m/s}}$$

Banked road

- A race car moves along a 31.0° banked curve, with a radius of 316 m. If the car negotiates the curve too slowly, it tends to slip down the incline of the turn, whereas if it's going too fast, it may begin to slide up the incline.
- (a) Find the necessary centripetal acceleration on this banked curve so the car won't tend to slip down or slide up the incline. (Neglect friction.)
- (b) Calculate the speed of the race car.

Solution:

$$\text{Horizontal: } F_c = N \sin \theta$$

$$\text{Vertical: } mg = N \cos \theta$$

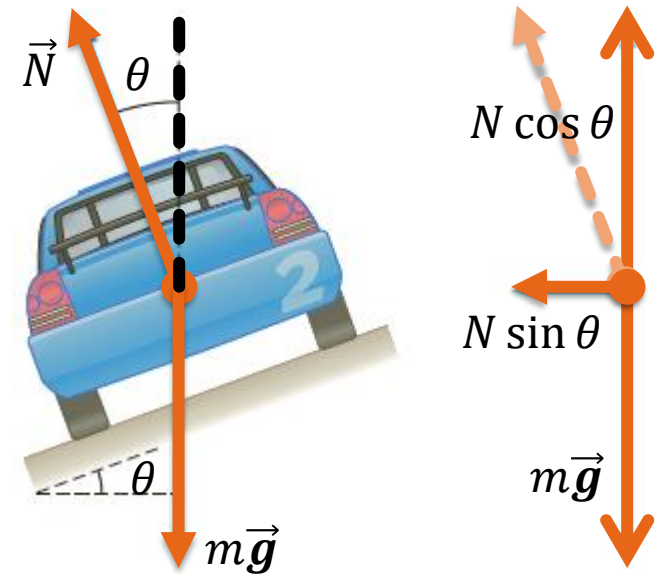
$$\Rightarrow F_c = mg \tan \theta$$

$$a_c = \frac{F_c}{m} = g \tan \theta$$

$$a_c = 9.8 \times \tan 31^\circ = 5.89 \text{ m/s}^2$$

$$a_c = \frac{v^2}{R} \Rightarrow v = \sqrt{R a_c}$$

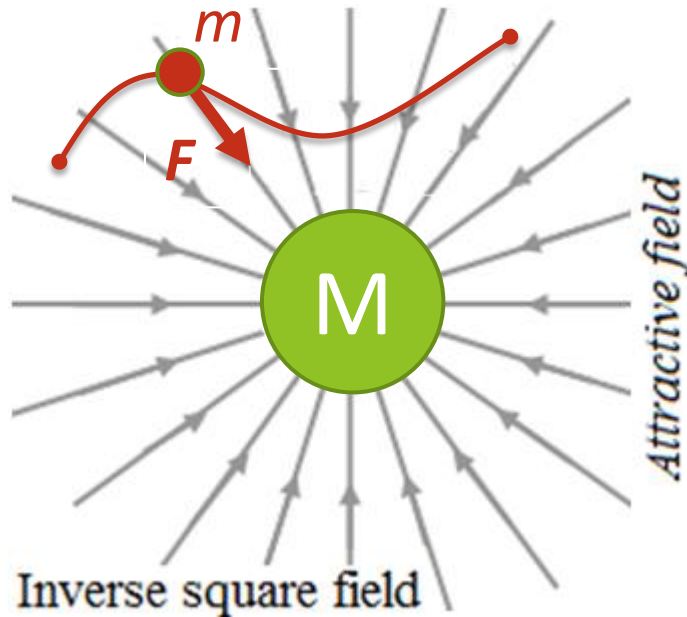
$$v = \sqrt{316 \times 5.89} = 43.1 \text{ m/s}$$



Newton's Law of Universal Gravitation and Examples

WHAT IS GRAVITY?

- ▶ Attractive force between objects with mass
- ▶ The weakest force ever known
- ▶ Obeys an inverse square law; it forms a vector field



- **g is the gravitational field strength, that is, the gravitational force per unit mass**

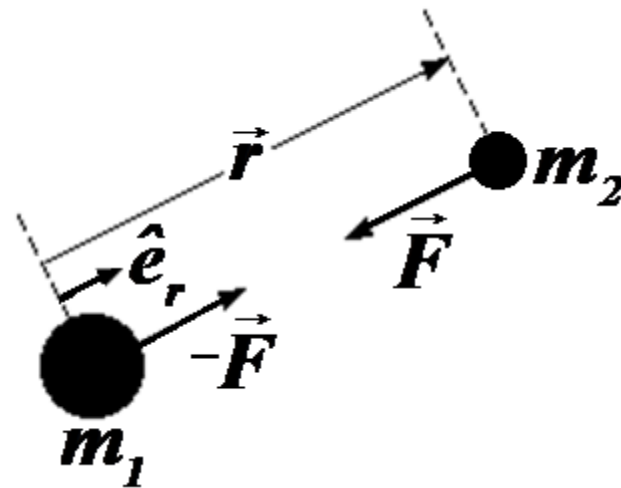
$$\vec{g} = \frac{\vec{F}}{m}$$

NEWTON'S LAW OF UNIVERSAL GRAVITATION

$$\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{e}_r$$

$$G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$

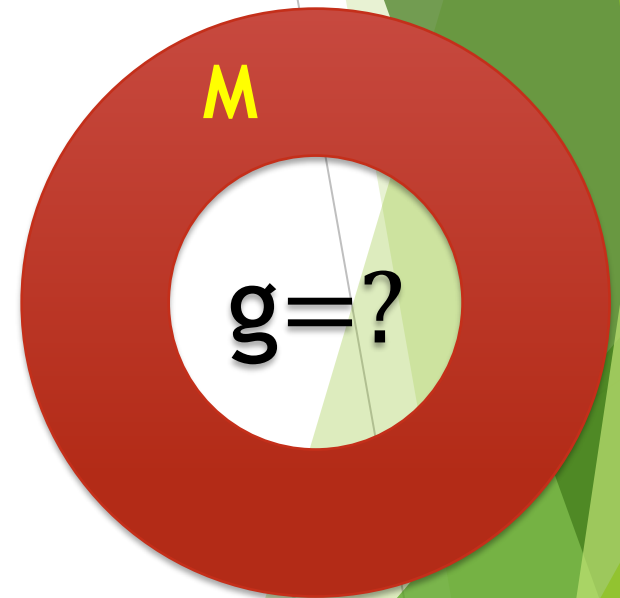
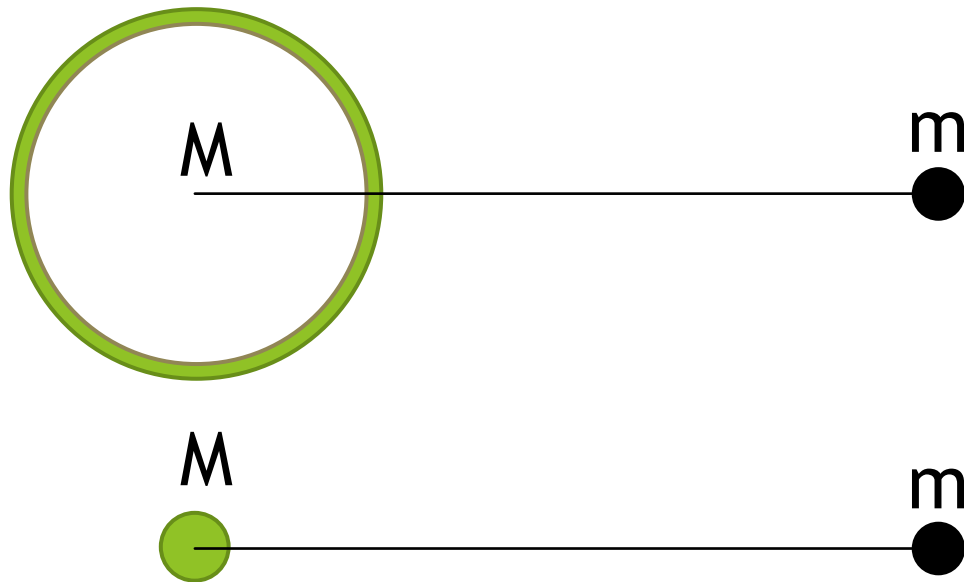
- The gravitational forces between two particles act along the line joining them, and form an action-reaction pair.



- In reality, we deal with not particles but with extended objects.
- To evaluate the gravitational force between extended objects, the *shell theorem* can be used.

SHELL THEOREM (LEFT TO THE STUDENT)

- ▶ A uniform shell of matter attracts an external particle as if all the shell's mass were concentrated at its center



aka Gauss Law: the gravitational force exerted by a uniform sphere on a particle outside the sphere is the same as the force exerted if the entire mass of the sphere were concentrated at its center.

THE VALUE OF g

- ▶ For a particle of mass m on the surface of the earth

$$F = G \frac{mM}{R_e^2} = mg \quad \Rightarrow \quad \frac{F}{m} = g = G \frac{M}{R_e^2}$$

Same for all masses
on or near the
earth's surface.

- ▶ In reality, the situation is more complicated:
 - ▶ The **earth's crust is not uniform**. Precise measurements of the variations of the free-fall acceleration give information about non-uniformities in the density of the earth.
 - ▶ The **earth is not a sphere**. The earth is an ellipsoid. It is flattened on the poles and bulging at the equator (difference in radius is 21 km). The free-fall acceleration is larger at the poles than it is at the equator.
 - ▶ The **earth is rotating**. The centripetal acceleration will change the free-fall acceleration.

Check Yourself

► **DOES THE MOON ATTRACT THE EARTH GRAVITATIONALLY?**

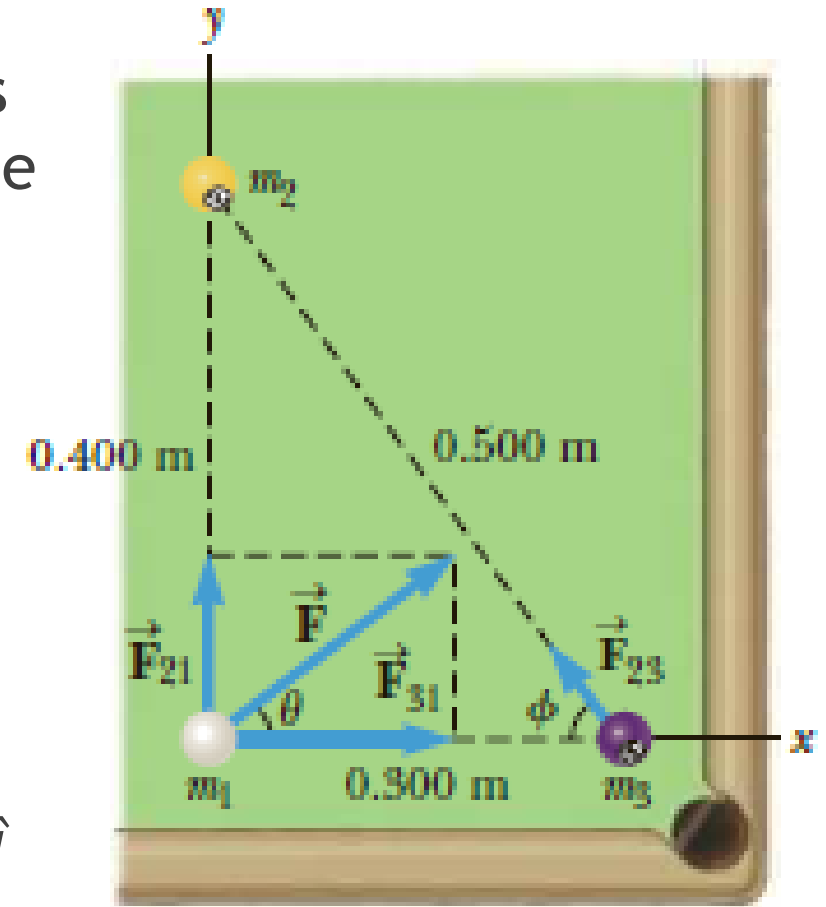
- A. No
- B. Yes, but with a much weaker force than the Earth attracts the Moon.
- C. Yes, with the same force the Earth attracts the Moon.

Example

- ▶ Three 0.300-kg billiard balls are placed on a table as shown. Find the net gravitational force on m_1 due to its interaction with other two balls.

Solution:

- ▶ Gravitational force on m_1
- ▶ $\vec{F}_1 = \vec{F}_{31} + \vec{F}_{21} = Gm_1m_2 \left(\frac{1}{r_{31}^2} \hat{i} + \frac{1}{r_{21}^2} \hat{j} \right)$
- ▶ $\vec{F}_1 = 9G \left(\frac{\hat{i}}{0.300^2} + \frac{\hat{j}}{0.400^2} \right) = 6.67 \times 10^{-9} \hat{i} + 3.75 \times 10^{-9} \hat{j}$



Kepler's laws, Satellites motion and weightlessness

Planetary Motion

▶ Kepler's Laws

▶ THE LAW OF ORBITS

- ▶ *All planets move in elliptical orbits having the sun as one focus.*

▶ THE LAW OF AREAS

- ▶ *A line joining any planet to the sun sweeps out equal areas in equal times.*

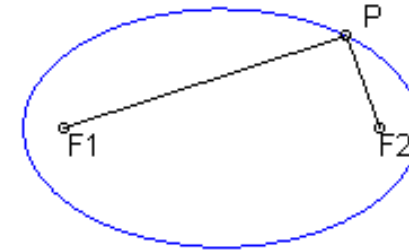
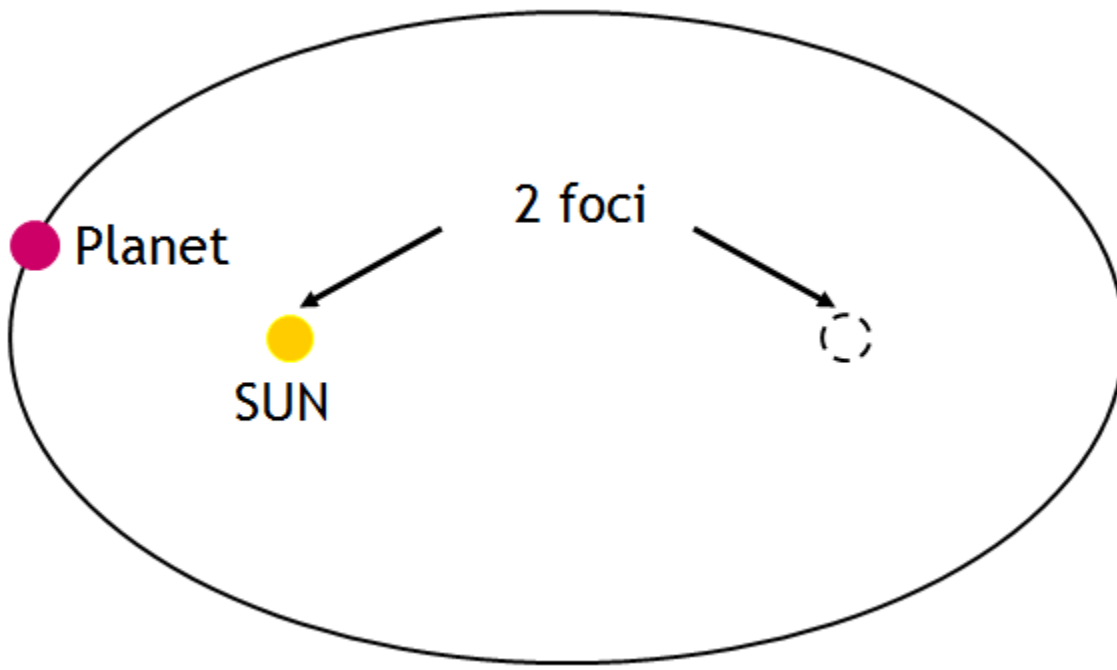
▶ THE LAW OF PERIODS

- ▶ *The square of the period of any planet about the sun is proportional to the cube of its mean distance from the sun.*

THE LAW OF ORBITS

All planets move in elliptical orbits having the sun as one focus.

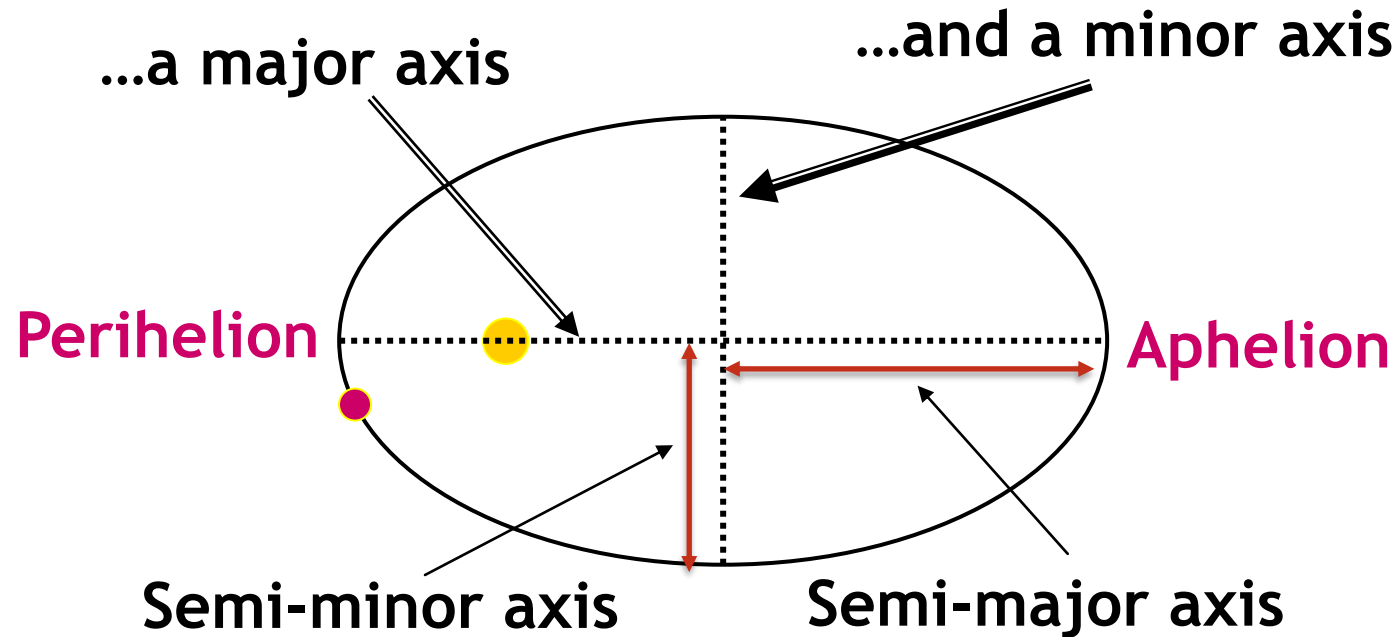
What is an **ellipse**?



An **ellipse** is a geometric shape with **2 foci** instead of 1 central focus, as in a circle. The **sun** is at one **focus** with nothing at the other **focus**.

THE LAW OF ORBITS

An **ellipse** also has...



Perihelion: planet's closest position to the **sun**.

Aphelion: planet's farthest position from the **sun**.

THE LAW OF ORBITS

- ▶ Perihelion distance: $R_p = a(1 - e)$
- Aphelion distance: $R_a = a(1 + e)$
- Equation of ellipse

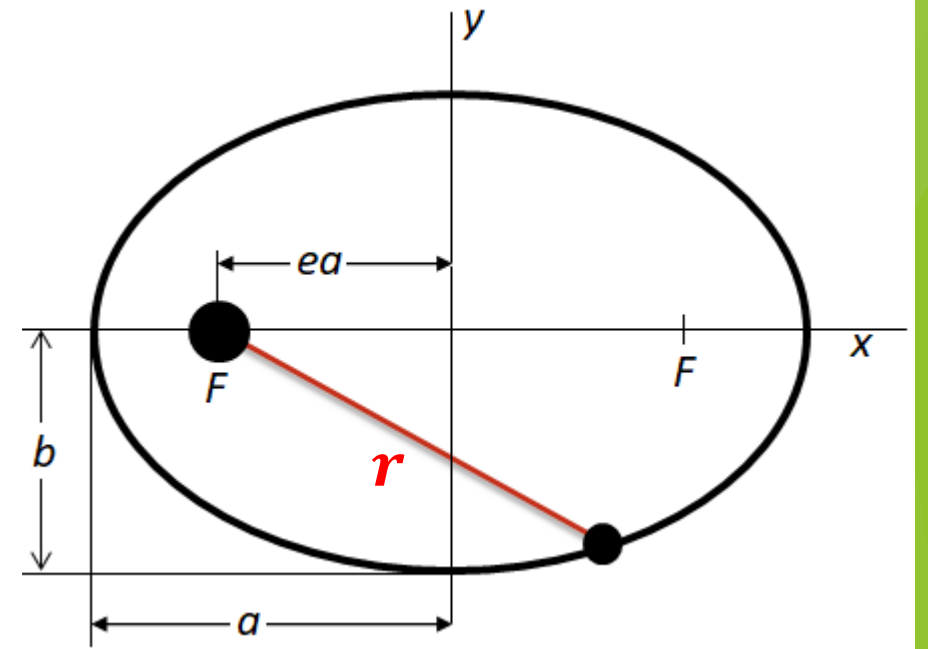
- In cartesian coordinates

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

- In plane-polar coordinates

$$r = \frac{a(1 - e^2)}{1 + e \cos \theta}$$

$$e = \frac{1}{a} \sqrt{a^2 - b^2}$$



The Law of Areas

A line joining any planet to the sun sweeps out equal areas in equal times.

► Swept area $\Delta A = \frac{1}{2} r^2 \Delta \theta$

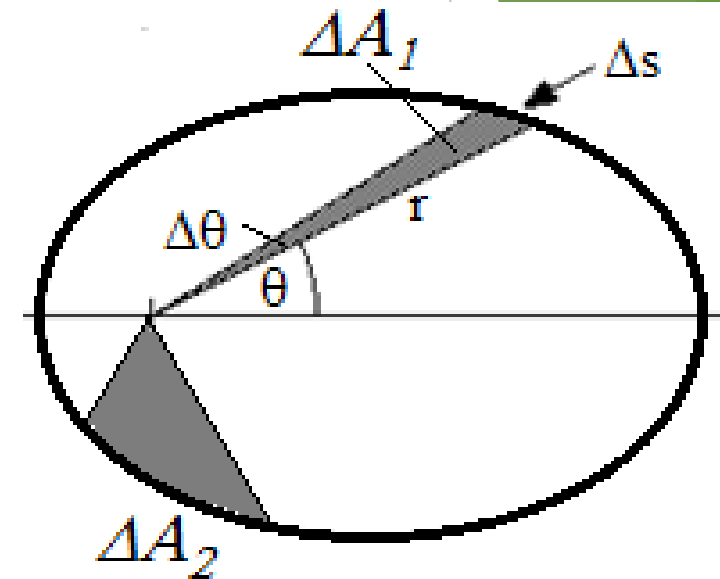
$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \frac{\Delta \theta}{\Delta t} = \frac{1}{2} r^2 \omega$$

- Swept area depends on angular momentum

$$L = m r^2 \omega$$

- Combining

$$\frac{\Delta A}{\Delta t} = \frac{L}{2m}$$

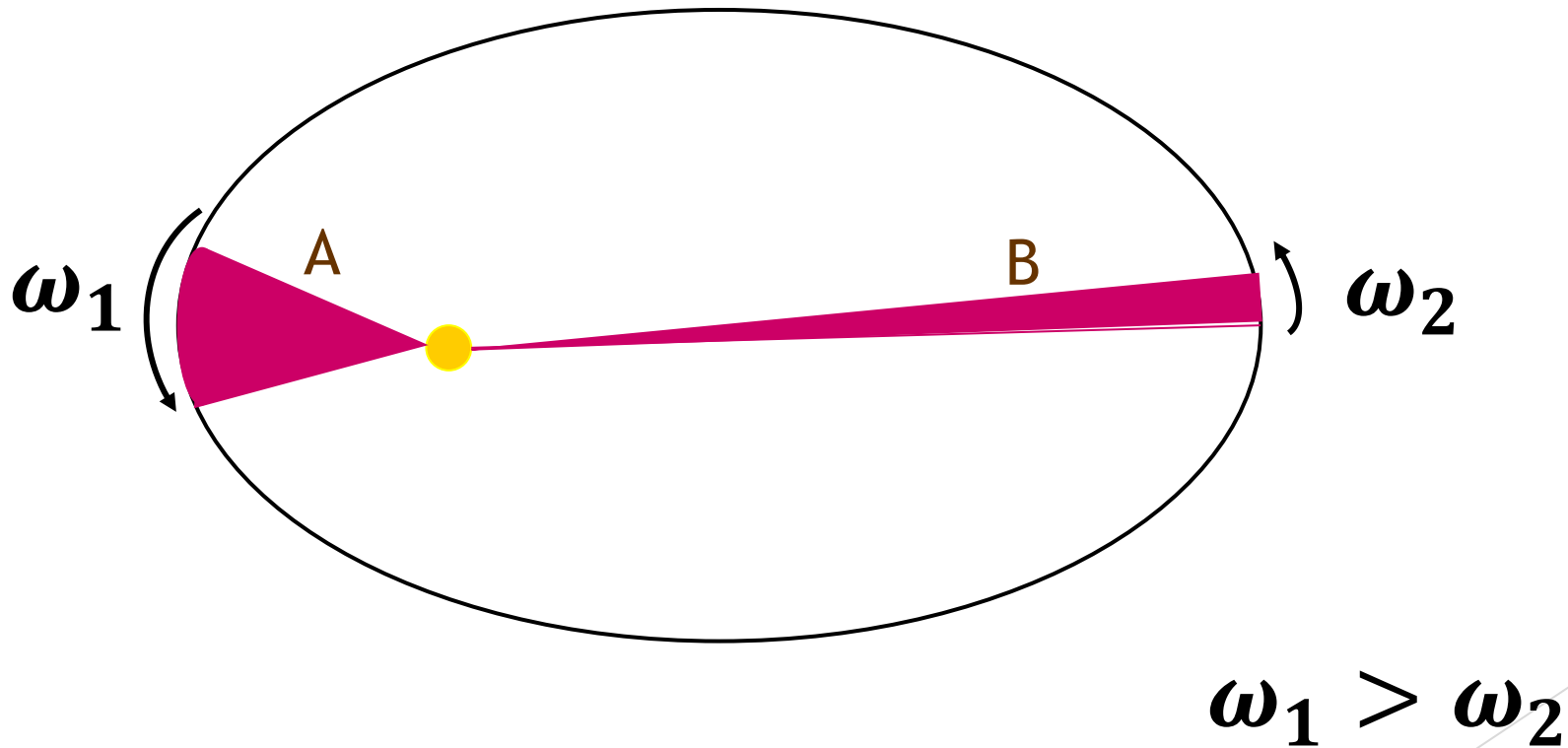


For equal time intervals
 $\Delta A_1 = \Delta A_2$

Since no external torques are acting on the sun-planet system, the angular momentum of the system is constant. Therefore, $\Delta A / \Delta t = \text{const}$

The Law of Areas

The planet changes speed as it orbits around the **sun**: faster when closer to the **sun**, slower when farther from the **sun**...



The Law of Periods

The square of the period of any planet about the sun is proportional to the cube of its mean distance from the sun

- Circular orbit of radius r

$$F_g = F_c$$

$$G \frac{M_S m}{r^2} = m \left(\frac{4\pi^2 r}{T^2} \right)$$

- Solving for T^2

$$T^2 = \left(\frac{4\pi^2}{GM_S} \right) r^3$$

For elliptical orbit of semi-major axis a

$$T^2 = \frac{4\pi^2}{GM_S} a^3$$

The constant depends only on the mass of the sun (M) and the gravitational constant (G)

$$K_S = \frac{4\pi^2}{GM_S} = 2.97 \times 10^{-19} \text{ s}^2/\text{m}^3$$

The Law of Periods

Useful units:

1 *Earth year* = 3.156×10^7 seconds

1 **astronomical unit (AU)** = average distance from the **Earth** to the **sun**
= $1.496 \times 10^{11} \text{ m}$

In these units Kepler's Third Law takes the form:

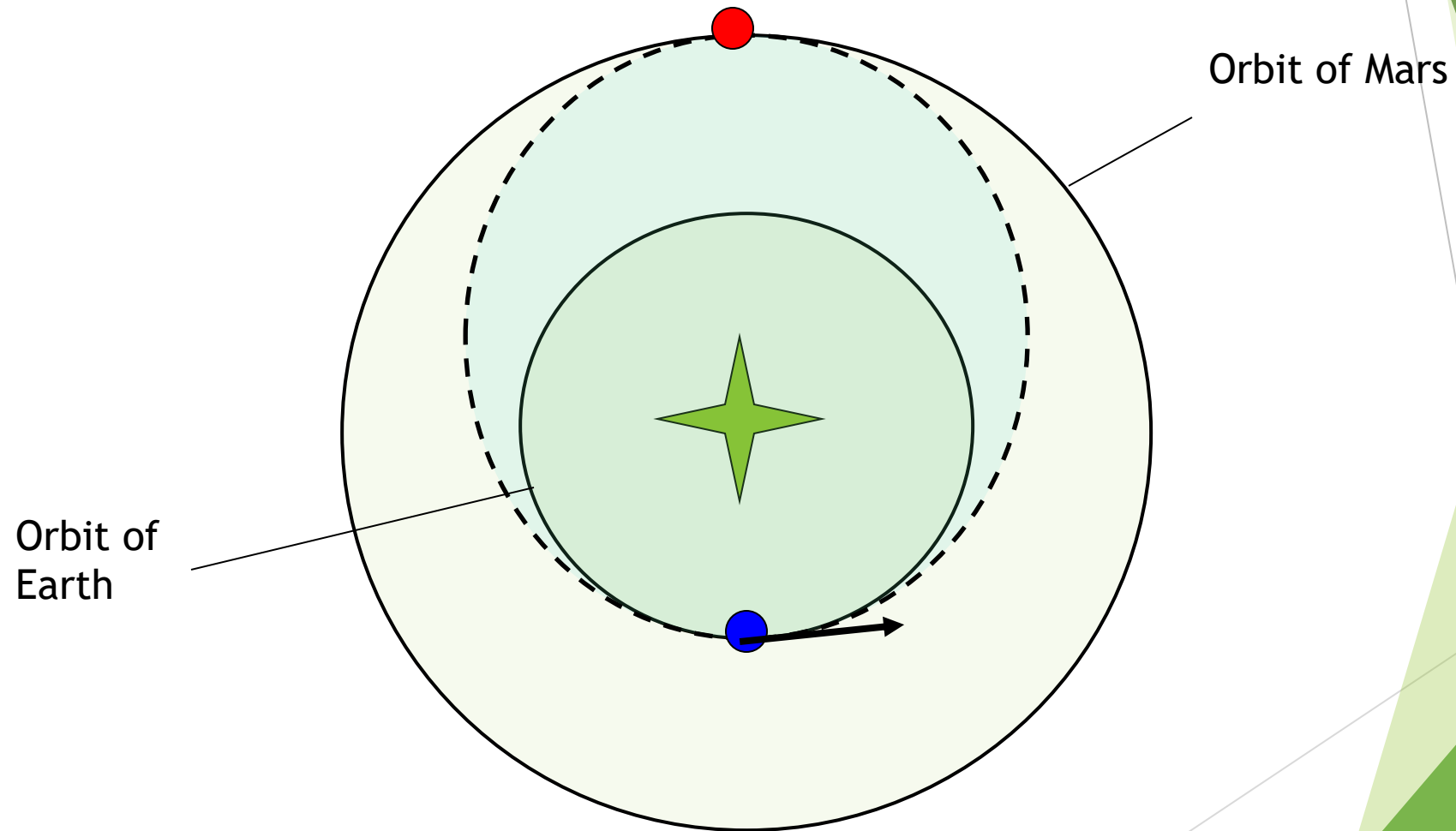
$$T^2 = a^3$$

e.g., light from Alpha Centauri takes 4.3 years

e.g., distance of Alpha Centauri from us is 268,770 AU

Conversion: 1 light year = 63241 AU

How long does it take for a trip to Mars?



Solution

► For spaceship orbit:

Major axis of spaceship = semi-major axis of Earth + semi-major axis of Mars

$$\text{Major-axis} = 1.00 \text{ au} + 1.52 \text{ au} = 2.52 \text{ au}$$

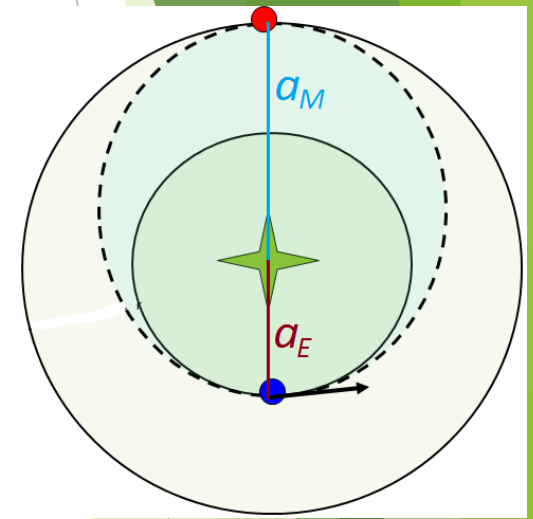
$$\text{sem-major axis} = \frac{2.52}{2} = 1.26 \text{ au}$$

$$T^2 = a^3 = (1.26 \text{ au})^3 = 2.00$$

$$T = 1.41 \text{ years}$$

► The trip takes a time equal to half of the period from Earth to Mars NOT from Mars to Earth.

$$t = \frac{1.41}{2} = 0.707 \text{ years} = 8.5 \text{ months} = 258 \text{ days}$$



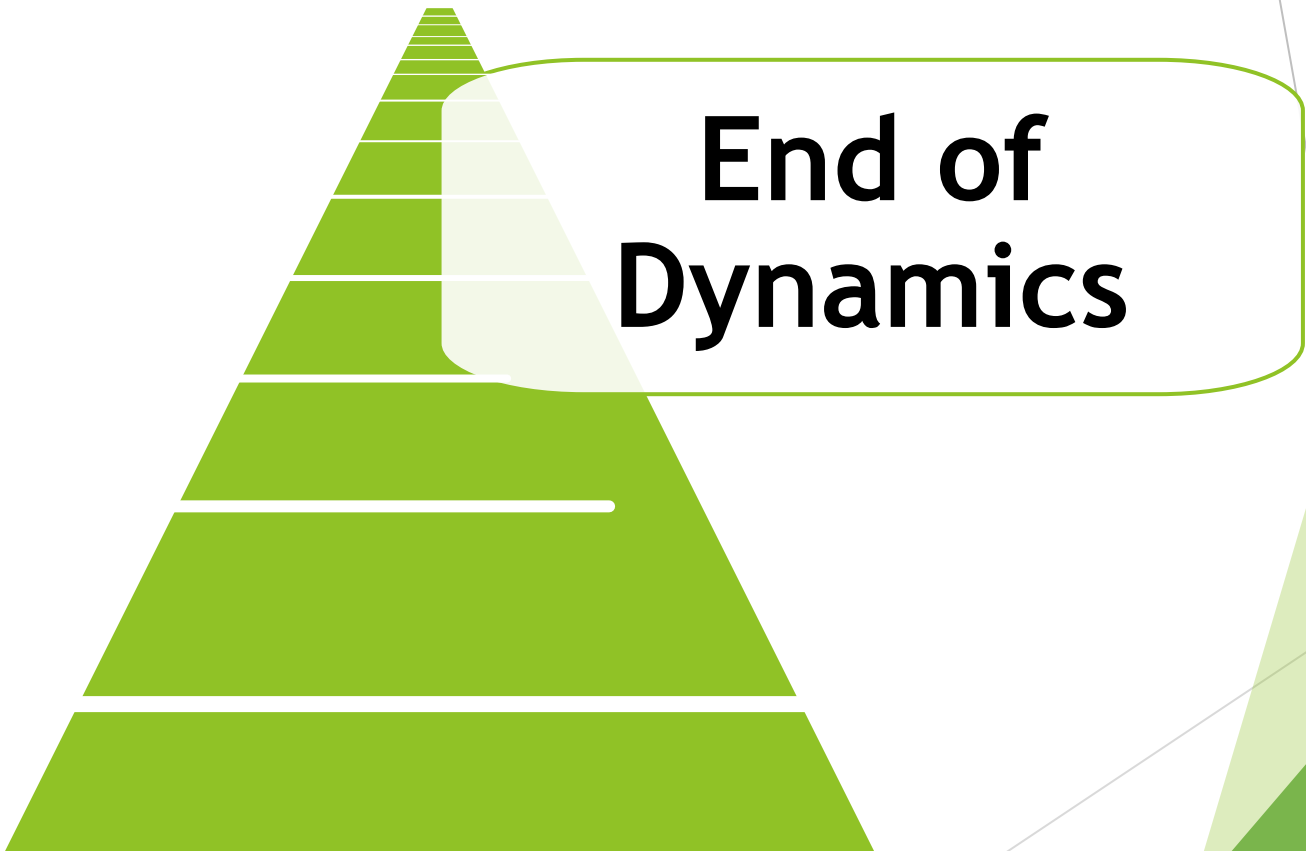
Example

- ▶ Mars rotates on its axis once every 1.02 days.
 - (a) Find the distance of the satellite from the center of Mars if it would remain in one spot over the Martian surface.
 - (b) Find the speed of the satellite.

▶ Solution

$$T^2 = \frac{4\pi^2}{GM_M} a^3$$
$$(1.02 \times 24 \times 3600)^2 = \frac{4\pi^2 a^3}{6.67 \times 10^{-11} \times 6.42 \times 10^{23}}$$

$$a = 2.03 \times 10^7 \text{ m}$$



**End of
Dynamics**